THE DYNAMICS OF PRECIOUS METAL MARKETS VAR: A GARCH-TYPE APPROACH

by

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Abstract

The data analysis of the metal markets has recently attracted a lot of attention, mainly because the prices of precious metal are relatively more volatile than its historical trend. A robust estimate of extreme loss is vital, especially for mining companies to mitigate risk and uncertainty in metal price fluctuations. This paper examines the Value-at-Risk and statistical properties in daily price return of precious metals, which include gold, silver, platinum, and palladium, from January 3, 2008 to November 27, 2018. The conditional variance is modeled by different univariate GARCH-type models (GARCH and EGARCH). The estimated model suggests that the two models both worked effectively with the metal price returns and volatility clustering in those metal returns are very clear.

In the second part, backtesting approach is applied to evaluate the effectiveness of the models. In comparison of VaRs for the four precious metals return, gold has the highest and most steady VaR, then is platinum and silver, while palladium has the lowest and most volatile VaR. The backtesting result confirms that our approach is an adequate method in improving risk management assessments.
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1. Introduction

Precious metal markets have been highly volatile in recent years not only due to supply and demand issues, but also due to many other factors such as extreme weather conditions, new financial innovations, and international inflation. In this study, we replicate Z. Zhang & H-K Zhang’s study (2016) on the metal commodity markets. In their study, they examined the VaR and statistical properties in daily price return of precious metals, which include gold, silver, platinum and palladium from January 11, 2000 to September 9, 2016. We generally confirm the original results using the time period used in the original study. However, we expected that using more recent data, especially data from the financial crisis, would change many of those results as GARCH generally does not perform particularly well during extreme events. We find that when including data from Jan. 3, 2008 to Oct. 26, 2018, the GARCH model only performs well on 95% confidence interval, while performs bad on 99% and 99.5% confidence interval. EGARCH model, similar to the previous study, performs well on the data sets. We offer a number of tests of the models, and show that their performance holds part of the conclusions of the previous study.

The quantification of the potential size of losses and assessing risk levels for precious metals and portfolios including them is fundamental in designing prudent risk management and risk management strategies. Value-at-Risk (VaR for short) models is an important instrument within the financial markets which estimate the maximum
expected loss of a portfolio can generate over a certain holding period. Regulators also accept VaR as a basis for companies to set their capital requirements for market risk exposure. GARCH-type models are a common approach to model VaR and estimate volatility and correlations. Yet the standard GARCH model is unable to model asymmetries of the volatility, which means in a standard GARCH model, bad news has the same influence on the volatility as good news. To deal with this problem, there are extension models in GARCH family such as a threshold GARCH (TGARCH) or exponential GARCH (EGARCH). These two models have taken leverage effect into consideration.

In this paper, similar to the previous study, we examine the volatility behavior of four precious metal: gold, silver, platinum and palladium. We contained two models, GARCH(1,1) and EGARCH(1,1), of GARCH family to calculate VaR at different level of confidence interval and estimate 1-day-ahead VaR for both GARCH-type models, and then use violation ratio to examine and compare the accuracy of fitting of the two models. In the previous study, they contained AR(1)-GARCH model and EGARCH model to test the data sets.

This paper is organized as follow. After this introduction, Section 2 provides a literature review. Section 3 introduces the data exploration and statistical analysis. Section 4 presents the methodology implemented in this study. Section 5 provides the result of 1-day-ahead VaR estimation and violation ratio for GARCH-type models. Section 6 is
our conclusion.

2. Literature Review

To offer a comparative view, we summarize the key findings of major studies in the related literature in Table 1, which demonstrates that GARCH and GARCH related models are widely used in the literature to analyze volatility performance and VaR in precious metal markets.
<table>
<thead>
<tr>
<th>Studies</th>
<th>Purposes</th>
<th>Data</th>
<th>Methodology</th>
<th>Main Findings</th>
</tr>
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<tbody>
<tr>
<td>Hammoudeh and Yuan (2008)</td>
<td>This study uses three “two factors” volatility models of the GARCH family to examine the volatility behavior of three strategic commodities</td>
<td>Daily time series for the closing future prices of oil, gold, silver and copper, and for the US three-month Treasury bill rates from January 2, 1990 to May 1, 2006</td>
<td>GARCH, CGARCH, EGARCH</td>
<td>Risk hedging in the gold and silver markets is more pressing than in the copper market.</td>
</tr>
<tr>
<td>Cheng and Hung (2010)</td>
<td>This paper utilizes the most flexible skewed generalized t (SGT) distribution for describing petroleum and metal volatilities that are characterized by leptokurtosis and skewness in order to provide better approximations of the reality.</td>
<td>West Texas Intermediate (WTI) crude oil, gasoline, heating oil, gold, silver, and copper for the period January 2002 to March 2009</td>
<td>GARCH-SGT, GARCH-GED</td>
<td>The SGT distribution appears to be the most appropriate choice since it enables risk managers to fulfill their purpose of minimizing MRA regulatory capital requirements</td>
</tr>
<tr>
<td>Hammoudeh et al. (2011)</td>
<td>This paper uses VaR to analyze the market downside risk associated with investments in four precious metals, oil and the S&amp;P 500 index, and three diversified portfolios.</td>
<td>Daily returns based on closing spot prices for four precious metals: gold, silver, platinum, and palladium from January 4, 1995 to November 12, 2009.</td>
<td>RiskMetrics, asymmetric GARCH type models</td>
<td>The RiskMetrics model is the best performer under the Basel rules in terms of both the number of days in the red zone and the average capital requirements</td>
</tr>
<tr>
<td>Huang et al. (2015)</td>
<td>This paper use generalized Pareto distribution (GPD) to model extreme returns in the gold market</td>
<td>Monthly gold prices from January 1969 to October 2012.</td>
<td>Generalized Pareto distribution (GPD model)</td>
<td>GPD was found to be an appropriate model to describe the conditional excess distributions of a heteroscedastic gold log return series and provides adequate estimations for VaR and ES.</td>
</tr>
<tr>
<td>He et al. (2016)</td>
<td>This paper proposed a new Bivariate EMD copula-based approach to analyze and model the multiscale dependence structure in the precious metal markets</td>
<td>Gold, Platinum, and Palladium closing price from 4 January 1993 to 4 April 2015</td>
<td>Copula GARCH, Bivariate Empirical Mode Decomposition (BEMD) model</td>
<td>There exists multiscale dependence structure, corresponding to different DGPs, in the precious metal markets. The proposed model can be used to identify the significant interdependent relationship among precious metal markets in the multiscale domain</td>
</tr>
</tbody>
</table>
3. Data Exploration and Statistical Analysis

In this study, we tend to estimate risk measures for precious metal market. For this aim, we consider daily closing spot prices of four precious metal: gold, silver, platinum and palladium, same as the previous study did. For the selected series, the data covers from January in 2008 to October in 2018, which is totaling more than 2500 observations. In our opinion, we think the high volatility in precious metal market after 2008 will be typical for the future market, so we removed data before 2008 and extended it to 2018. We collect daily spot price of all four kinds of precious metal from Bloomberg. All the four-precious metal price is based on U.S. dollars. The continuously compounded daily returns are computed as follows:

\[ r_t = 100 \ln \left( \frac{p_t}{p_{t-1}} \right) \]

In this formula, \( r_t \) and \( p_t \) are the return in percentage and the precious metal daily spot price on day \( t \) respectively.

We used the price and return of gold and silver to represent metal market historical tendency since gold and silver are not only a financial indicator that can have impact on other precious metal commodities, but also widely used as a financial instrument for inclusion in portfolios. Fig. 1 provides the time series plots of gold and silver daily spot prices and their log-returns.
Figure 1 indicate that volatility clustering is manifestly apparent for precious metal returns revealing the presence of heteroscedasticity. The number of isolated peaks in both log-return figure is larger than what would be expected from Gaussian series. The statistical results of the four kinds of precious metal returns are shown in Table 2.

As can be seen in Table 2, the mean of all data sets is extremely close to zero, while the
standard deviation is also at a low level. Among the four-precious metal, silver has the highest standard deviation, while gold has the lowest. In the previous study, palladium has the highest standard deviation while gold has the lowest, indicating that silver has became more volatile during recent years. Comparing to the standard normal distribution with skewness 0 together with kurtosis 3, it leads to a conclusion that each data set has a leptokurtic distribution with fat tail. Meanwhile, the result of Jarque-Bera (J-B for short) test supports that we can surely reject the null hypothesis of Gaussian distribution for all returns. According to Augmented Dickey-Fuller (ADF for short) test, the result undoubtedly rejects the hypothesis of unit root for the time series studied. So, we can conclude that precious metal price sample returns all have short memory.

Table 2. Statistical analysis

<table>
<thead>
<tr>
<th></th>
<th>Mean (%)</th>
<th>Standard Deviation</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>J-B test</th>
<th>ADF test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>0.0019</td>
<td>0.0042</td>
<td>2.0381</td>
<td>-3.8546</td>
<td>-0.6039</td>
<td>8.8557</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Silver</td>
<td>-0.0032</td>
<td>0.0076</td>
<td>2.7484</td>
<td>-6.0146</td>
<td>-0.9297</td>
<td>9.6886</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Platinum</td>
<td>-0.0144</td>
<td>0.0047</td>
<td>1.7815</td>
<td>-2.7228</td>
<td>-0.2254</td>
<td>5.1795</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Palladium</td>
<td>0.0138</td>
<td>0.0069</td>
<td>3.7298</td>
<td>-3.5061</td>
<td>-0.2639</td>
<td>5.3379</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes: J-B test results in 1 means reject the null hypothesis that the sample data have the skewness and kurtosis matching a normal distribution. ADF test results in 1 means reject the null hypothesis that a unit root is present in a time series sample.

In conclusion, the statistical analysis for precious metal price return data sets reveals that these precious metal returns are stationary, non-normally distributed, and all have
short memory. This conclusion is same as the previous study.

4. Testing for data features

4.1 Test for Stationary

Similar to the previous research, in order to build an effective model, stationary test is needed on the series to make sure the underlying assumption that all the series must be stationary hold. Only when series is stationary, i.e. has statistical properties that do not change with time, models can be adopted to process those series.

In this paper, we adopted a simple test based on the null hypothesis that the data in vectors x and y comes from independent random samples from normal distributions with equal means and equal but unknown variances. We divided each time series data into equally two vectors x and y. Then we adopt the test on the two vectors to see if test results will reject the null hypothesis.

<table>
<thead>
<tr>
<th></th>
<th>Gold</th>
<th>Silver</th>
<th>Platinum</th>
<th>Palladium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test result</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*result = 0, fail to reject null hypothesis, stationary

The results for the four series are all 0, indicating that we cannot reject the null hypothesis, and all the four data sets are stationary. And this conclusion is consistent with the previous research.
### 4.2 Test for Serial Correlation

In order to perform a detailed modeling on the log returns, we need to perform several tests on the return and variance characteristics of the data sets. In the previous research, AR(1) model is adopted to filter out the autocorrelations of considered metal log-returns. And AR(1) is singled out according to the censored orders of autocorrelation and partial autocorrelation functions graphs through numerous trails. In this paper, we tried to conduct the same analysis and trying to figure out if serial correlation still holds. The first test we performed was about whether the return datasets still exists serial correlation. We adopted the autocorrelation function and partial autocorrelation function in MATLAB.
From the graph, it is evident that there is little influence of past return on today’s return. Thus, we can reach the conclusion that no serial correlation exists, and that the use of AR(1) model in the previous research would be not appropriate anymore.

4.3 Test for Heteroscedasticity

After testing on the serial correlation, we conducted two tests on the heteroscedasticity. The time-varying volatility would interfere the effectiveness of the forecasting process and influence the quality of the data. The previous research has shown that the all the
metal returns showed significant conditional variance feature.

The first test is the autocorrelation function on the variance of the metal prices returns. Like the autocorrelation and partial autocorrelation function on the returns data, this test showing the influence of past volatility (i.e. variance) on today’s volatility.

![Graphs of Conditional Variance for Gold, Silver, Platinum, and Palladium](image)

**Figure 3. Conditional variance for gold, silver, platinum and palladium**

From the graph, we can see that today’s return for all the metal price returns data would be influenced by the previous returns, meaning that conditional variance does exist. This conclusion is consistent with the previous research. Moreover, for Gold and
Platinum, the last day’s variance has the strongest influence, while for Palladium and Silver, other recent variance also has some influence on today’s volatility.

The second test is plotting the return against time to show whether the volatility changes with time. According to the previous research, they found out that return for Palladium has the highest standard deviation, while return for gold has the lowest during the period 2000–2016. Compared with the newest data that we adopt in this paper, we found out that the return volatility for Silver became the highest one, indicating the Silver market in recent years are more volatile than before. And the return for Gold still has the lowest volatility, which means that the market volatility for Gold remained relatively stable and unchanged during the years.
On the other hand, as the previous research point out, the return against time graph for each metal showed strong mean-reverting trend, fluctuating around zero. They also pointed out that the return against time figure indicates heteroscedasticity and volatility clustering behavior. These conclusions still hold with the newest data we adopted based on the following graph.

4.4 Test for Distribution

Due to the fat tail of the metal price returns, it is generally harder for normal distribution to capture the extreme conditions in the metal future market. Thus, we conducted test to modify if the student t distribution would be a better fit for the metal price returns.
According to the qqplot function in MATLAB, we monitored the fitness of the four-data series with 2 different distributions - t distribution and normal distribution. The test result is that t-distribution have much stronger ability to capture the fat tail of the metal price returns. More specifically, the t-distribution can capture most of the extreme increases in metal prices and a considerable amount of extreme decrease in the metal prices. As for the normal distribution, it only captures some of the increases and decreases when the metal market has huge fluctuation. Thus overall, we decided to adopt the t-distribution. Based on different method, the previous research adopted a complicated EVT distribution to capture all extreme conditions. However, because t-
distribution has proven to be able to capture most of the extreme conditions between 2010 ~ 2018. Thus, we adopted t-distribution for this paper.

In conclusion, based on the tests we performed above, the metal price returns are all stationary, meaning that models can be directly used. Then according to the autocorrelation function and partial autocorrelation function, the returns between 2010 and 2018 have no serial correlation, thus ARMA models are not needed anymore. Finally, according to the autocorrelation function on the return variance, the previous variance would have impact on today’s variance, thus heteroscedasticity exists, and GARCH-type models are still needed. The difference between our conclusion and previous research are 1. The AR(1) Model is proven to be not necessary anymore; 2. We adopted t-distribution instead of the EVT distribution in the previous research; 3. The Silver return turned out to be more volatile than palladium and to be the most volatile one in recent years.
5. Model Estimation

To implement an estimation procedure for these measures we must choose a particular model for the dynamics of the conditional volatility.

According to the previous research, the AR(1)-GARCH model was adopted because they found out that historical returns of the metal historical prices are not independent, and variance was not constant either. However, after the 2010, based on the previous test we performed in part2, we found out that the newest returns for the four metals are independent, which means that AR(1) model is no longer needed. And the time-vary volatility still holds.

Thus, in this paper, we use the parsimonious but effective GARCH (1,1) process for the volatility. Moreover, they also conducted EGARCH model to test whether leverage effect exist in the metal returns. Our paper also introduced EGARCH model to forecast VaR by capturing some volatility stylized facts such as asymmetry and leverage effect in the metal price return innovations to see whether EGARCH model still provide good VaR’s computations.

5.1 Defining Value-at-Risk

Exposure to risk can be defined as the worst expected loss over a great horizon within a given confidence level, which VaR is this quantity. The VaR at a given confidence
level Indicates the amount that might be lost in a portfolio of assets over a specified
time period with a specified small failure probability $\alpha$. In this paper, we still adopt time
period as one day.

Similar to the previous research, we suppose that a random variable $X$ characterizes the
distribution of daily returns in some risky financial asset, the left-tail $\alpha$-quantile of the
portfolio is then defined to be the VaR $\alpha$ such that

$$\Pr(X \leq \text{VaR}) = \alpha$$

The VaR is the smallest value for $X$ such that the probability of a loss over a day is no
more than $\alpha$. Although the parameter $\alpha$ is arbitrarily chosen, analysis in this study does
not refer to the process of choosing the parameter which is considered to be $\alpha \in \{0.005, 0.01, 0.05\}$. In the estimation, for each day we estimated 1000 possible returns, so the
VaR for that day would be absolute value the 5th smallest return, 10th smallest return
and 50th smallest return respectively. And this methodology is consistent with the
previous research.

5.2 Estimating $\sigma_{t+1}$ using GARCH-type Model

In 1986, Bollerslev developed the generalized ARCH, or GARCH, to capture the time-
 vary volatility, which relies on modeling the conditional variance as a linear function
of the squared past innovations. By using the log-returns $(x_1, x_2, x_3 \ldots x_{t-1})$ as the input,
the conditional variance of the standard GARCH (1,1) is defined as:

$$\sigma_t^2 = c + \eta * \epsilon_{t-1}^2 + \beta * \sigma_{t-1}^2$$
Where the $c > 0$, $\eta > 0$, $\beta > 0$, $\epsilon_{t-1} = x_{t-1} - \mu_{t-1}$. $\mu$ is the average return during the observing period. The volatility today would be a combination of the mean-adjusted return and variance

However, due to the drawback of standard GARCH that it fails to consider the leverage effect in the volatility of metal price returns. In the GARCH model, the underlying assumption is the volatility are symmetric to the change in return. However, in real world, the increase and decrease in return may bring different volatility changes (asymmetric impacts).

In order to verify whether leverage effect exist in the metal return, the EGARCH model is also included.

The E-GARCH(1,1) is defined as:

$$\ln \sigma_t^2 = c + \eta \left| \frac{\epsilon_{t-1}}{\sigma_{t-1}^2} \right| + \beta \frac{\epsilon_{t-1}}{\sigma_{t-1}^2} + \gamma \ln \sigma_{t-1}^2$$

where $\epsilon_{t-1} = x_{t-1} - \mu_{t-1}$ and $\eta$ depicts the leverage effect.

- The positive return and negative return with same absolute amount of change will have different impact on the volatility prediction
- If $\eta$ is positive and $\beta$ is negative, meaning that negative change in return would bring higher impact on the next day’s volatility
- In contrast to the GARCH model, no restrictions need to be imposed on the
model parameters since the logarithmic transformation ensures that the forecasts of the variance are non-negative.

In conclusion, the previous research adopted AR(1)-GARCH(1,1) and AR(1)-EGARCH(1,1). In our research, we found out that serial correlation do not exist anymore, thus GARCH(1,1) and EGARCH(1,1) are still adopted.

5.3 Estimating Result and Discussion

By observing the autocorrelations in Section 2, we found the heteroscedasticity and volatility clustering behavior in the considered precious metal returns. The four metal returns have significant volatility clustering, so a GARCH-type model needs to be adopted. Because of the fat tail of the return, we chose the t-distribution instead of the normal distribution to better fit the data. GARCH(1,1), and EGARCH(1,1) models with student t distributions are developed so as to further investigate the leverage effect of the precious metal returns.
As can be seen from Table 3, we performed GARCH(1,1) and EGARCH(1,1) to each of the metal returns, and we also recorded the AIC value, DW-test value and adjusted R-Square to compare the fitness of those models.

For the GARCH model estimation result:

$$\sigma_t^2 = c + \eta * \varepsilon_{t-1}^2 + \beta * \sigma_{t-1}^2$$

The previous research indicates that the all the parameters are significant and parameter $\beta$ all exceed 0.86, indicating the strong volatility clustering. And we found out that all
the parameters are still significant while parameter $\beta$ are all greater than 0.93, indicating the volatility clustering in those metal returns is even clearer.

As for the EGARCH model estimation result:

$$
\ln \sigma^2_t = c + \eta \frac{\varepsilon_{t-1}}{\sqrt{\sigma^2_{t-1}}} + \beta \frac{\varepsilon_{t-1}}{\sqrt{\sigma^2_{t-1}}} + \gamma \ln \sigma^2_{t-1}
$$

The precious research got four conclusions, (1) leverage effects coefficient $\gamma$ are all positive and significant at any significant level; (2) the asymmetric volatility behavior is the most significant in palladium while the least significant in gold; (3) the coefficient estimators $\gamma$ in the EGARCH(1,1) conditional variance model are all greater than 0.95, which indicates that over 95% of current variance shock can still be seen in the following period.

As for the three conclusions, according to the parameter estimation result from the conditional variance EGARCH(1,1) equation, we found that all the coefficient $\eta$ are all positive and significant at any significant level.

1. Unlike the previous research that Palladium has the most leverage effect. We found out that the leverage effect coefficient $\eta$ and $\beta$ for Gold and Silver are bigger than the other two metals, indicating that Gold and Silver may suffer more from bad news and benefit less from good news than the other two metals.

2. Similar to the previous research, estimators $\gamma$ is all greater than 0.98, showing that the volatility clustering in those metal returns are still clear. This result is consistent
with the previous research on this topic.

DW-test results are all close to 2, indicating that there is little serial correlation exists in the metal returns.

Overall, based on the minimum AIC value, the GARCH(1,1) and EGARCH(1,1) model both have a relatively small AIC value, indicating that the fitness of the model is quite good. And the value of the AIC for EGARCH model is the smallest, which is consistent with the conclusion of the previous research. Moreover, we also conducted the DW-test. Based on its assumption that the closer the DW-test statistic to 2, the less serial correlation exist, we can also reach the conclusion that those metal price returns have no serial correlation. Similar to the previous research, the volatility clustering in those metal returns is clear, and the decay of the volatility shock is quite slow. Leverage effect does have an impact on the metal price returns.

6. VaR Estimations and Backtesting

According to Basel Committee on Banking Supervision, a financial institution has freedom to use their own model to compute Value-at-Risk (VaR). In this section, we estimate the 1-day-ahead VaRs via the GARCH(1,1) model and E-GARCH(1,1) model and implement backtesting to measure accuracy for each of the two approaches by using violation ratio. As mentioned above in Section 5, we compute VaR by using:
\[ \Pr (X \leq \text{VaR}) = \alpha \]

To define and record the violations of VaR, we use:

\[ I_t(\alpha) = \begin{cases} 1 & \text{if } X < \text{VaR} \\ 0 & \text{else} \end{cases} \]

The recorded violations and 0.5% quantile VaRs using our E-GARCH approach are showed in Fig. 6. Figure for violations and VaRs under 0.01% or 0.005% quantile and using GARCH approach are available upon request.
6.1 One-Day-Ahead VaR Estimations

For the next step, we use the negative standardized residuals to estimate VaR for the four data sets. From Table 4 we can see that at a quantile level of 99.5%, the estimated VaR from our GARCH(1,1) approach is 0.0091 for losses, which means we are 99.5% confidence that the expected market value of gold would not lose more than 0.91% for the worst-case scenario within one-day duration. The reason we choose the GARCH(1,1) model and EGARCH(1,1) model to VaR is that EGARCH model does not have restrictions on nonnegativity constraints as linear GARCH model has. Therefore, we identify EGARCH model as the most proper conditional variance model for the four precious metal returns, and we want to compare its VaR estimations with GARCH model. According to the estimation result for 1-day-ahead VaR. As shown in Table 4, we note that EGARCH model produced lower VaR forecasts than the GARCH model at any quantile levels for any metal price return series, which is same to the previous study.
Table 5. 1-day-ahead VaR estimations

<table>
<thead>
<tr>
<th>Return</th>
<th>Gold</th>
<th>Silver</th>
<th>Platinum</th>
<th>Palladium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimates for 1-day ahead VaRs from the GARCH model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VaR_{T+1} 0.005</td>
<td>0.0091</td>
<td>0.0160</td>
<td>0.0156</td>
<td>0.0185</td>
</tr>
<tr>
<td>VaR_{T+1} 0.01</td>
<td>0.0073</td>
<td>0.0132</td>
<td>0.0113</td>
<td>0.0201</td>
</tr>
<tr>
<td>VaR_{T+1} 0.05</td>
<td>0.0043</td>
<td>0.0072</td>
<td>0.0072</td>
<td>0.0105</td>
</tr>
<tr>
<td>Estimates for 1-day ahead VaRs from the E-GARCH model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VaR_{T+1} 0.005</td>
<td>0.0072</td>
<td>0.0141</td>
<td>0.0119</td>
<td>0.0163</td>
</tr>
<tr>
<td>VaR_{T+1} 0.01</td>
<td>0.0062</td>
<td>0.0104</td>
<td>0.0096</td>
<td>0.0156</td>
</tr>
<tr>
<td>VaR_{T+1} 0.05</td>
<td>0.0041</td>
<td>0.0059</td>
<td>0.0059</td>
<td>0.0104</td>
</tr>
</tbody>
</table>

Then we use a moving window to estimate the 1-day-ahead 5% quantile VaRs using our EGARCH approach to investigate further about the dynamics of VaR for the precious metal return series as shown in Figure 7.
In comparison of VaRs for the four precious metals return, gold has the highest and most steady VaR, then is platinum and silver, while palladium has the lowest and most volatile VaR. It indicates that gold is the safest valuable asset for investment, while palladium is most volatile since it is relatively rare comparing to other three precious
There are also other factors that contribute to the downtrend of VaR. For instance, from 2012 to 2014, there is a long-term bull run in the U.S. stock market, which encouraged investors to use their money in stock investing and lead to the sustained low-level precious metal price.

### 6.2 Results of Violation Ratio

In the previous study, they used likelihood ratio test to do backtesting. Different from the previous study, we used violation ratio to test the accuracy for fitting of GARCH model together with EGARCH model. If the violation ratio is between 0.8 and 1.2, it will be defined as close to 1 which means the model fits the data set at a good level. Otherwise, the violation ratio will be defined as significantly different from 1 which means the model fits the data series at a poor level.

<table>
<thead>
<tr>
<th>Return</th>
<th>Gold</th>
<th>Silver</th>
<th>Platinum</th>
<th>Palladium</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Violation ratio result from GARCH model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VR(_{p=0.005})</td>
<td>0.5944</td>
<td>0.5944</td>
<td>0.4458</td>
<td>0.5458</td>
</tr>
<tr>
<td>VR(_{p=0.01})</td>
<td>1.1174</td>
<td>1.3072</td>
<td>1.4006</td>
<td>1.3401</td>
</tr>
<tr>
<td>VR(_{p=0.05})</td>
<td>1.0401</td>
<td>1.0698</td>
<td>1.0253</td>
<td>1.2184</td>
</tr>
<tr>
<td><strong>Violation ratio result from E-GARCH model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VR(_{p=0.005})</td>
<td>1.3373</td>
<td>1.4821</td>
<td>1.1887</td>
<td>1.4859</td>
</tr>
<tr>
<td>VR(_{p=0.01})</td>
<td>1.0401</td>
<td>1.3373</td>
<td>1.0401</td>
<td>1.0401</td>
</tr>
<tr>
<td>VR(_{p=0.05})</td>
<td>1.0847</td>
<td>1.1738</td>
<td>1.0996</td>
<td>1.1558</td>
</tr>
</tbody>
</table>
Table 5 provides the backtesting results of violation ratio, where the level of confidence interval ranging among 0.5%, 1% and 5%. EGARCH model performs well at 1% and 5% confidence interval, yet the performance for 0.5% confidence interval is poor. GARCH model performs well only at 5% confidence interval. In the previous research, GARCH and EGARCH model do very well in predicting critical loss for precious metal markets. Our result is partly changed in comparison with the previous study.

7. Conclusion

In this paper, we introduce an extension of the original study by Zhang Z. and Zhang H-K. (2016) by including data from the financial crisis to see if this would change many of those results as GARCH generally does not perform particularly well during extreme events.

As the volatility in the metal market increases, it's extremely important to implement an effective risk management system against market risk. In this context, VaR has become the most popular tool to measure risk for institutions and regulators and how to correctly and effectively estimate VaR has become increasingly important. In addition, leverage effect has been proved to be an important influence factor of future prices. In this paper we introduce GARCH and EGARCH model to capture the volatility clustering of metal price returns and conducted back testing to exam the
effectiveness of the two models. Our findings reveal that the GARCH and EGARCH models all worked effectively with the metal price returns and volatility clustering in those metal returns are still clear. After we conduct the estimation for 1-day-ahead VaR, results at any quantile levels for any metal price return series of GARCH are higher than that of EGARCH, which indicate that EGARCH performs better than GARCH. It reveals that taking leverage effect into consideration is more realistic and comprehensive than using GARCH to VaR model. According to the backtesting result, violation ratio for GARCH only performs well at 5% quantile which proves our assumption that GARCH model is inadequate during extreme events. For EGARCH model, at 5% and 1% quantile the model performs good, while at 0.5% quantile the accuracy of fitting has a serious deterioration. We have not yet found out the reason for this question. A detailed analysis of this question is left for future research.
References


Appendix

![T Distribution Fitness, Gold](image1)

![Normal Distribution Fitness, Gold](image2)

![T Distribution Fitness, Silver](image3)

![Normal Distribution Fitness, Silver](image4)

![T Distribution Fitness, Platinum](image5)

![Normal Distribution Fitness, Platinum](image6)

![T Distribution Fitness, Palladium](image7)

![Normal Distribution Fitness, Palladium](image8)
### GOLD_RTN

**Dependent Variable:** GOLD_RTN  
**Method:** ML ARCH - Normal distribution (BFGS / Marquardt steps)  
**Date:** 11/28/18  **Time:** 19:28  
**Sample:** 1/03/2008 10/28/2018  
**Included observations:** 2842  
**Convergence achieved after 25 iterations**  
**Coefficient covariance computed using outer product of gradients**  
**Presample variance: backcast (parameter = 0.7)**  
\[ GARCH = C(1) + C(2)*RESID(-1)^2 + C(3)*GARCH(-1) \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance Equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1.73E-07</td>
<td>3.31E-08</td>
<td>5.217308</td>
<td>0.0000</td>
</tr>
<tr>
<td>RESID(-1)^2</td>
<td>0.037650</td>
<td>0.002193</td>
<td>17.16828</td>
<td>0.0000</td>
</tr>
<tr>
<td>GARCH(-1)</td>
<td>0.952451</td>
<td>0.003014</td>
<td>316.0301</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**R-squared:** -0.000020  **Mean dependent var:** 1.90E-05  
**Adjusted R-squared:** 0.000410  **S.D. dependent var:** 0.004242  
**S.E. of regression:** 0.004241  **Akaike info criterion:** -8.211978  
**Sum squared resid:** 0.041788  **Schwarz criterion:** -8.204551  
**Log likelihood:** 9541.212  **Hannan-Q Quinn criter.:** -8.209722  
**Durbin-Watson stat:** 2.025303

### SILVER_RTN

**Dependent Variable:** SILVER_RTN  
**Method:** ML ARCH - Normal distribution (BFGS / Marquardt steps)  
**Date:** 11/28/18  **Time:** 19:30  
**Sample:** 1/03/2008 10/28/2018  
**Included observations:** 2842  
**Convergence achieved after 29 iterations**  
**Coefficient covariance computed using outer product of gradients**  
**Presample variance: backcast (parameter = 0.7)**  
\[ GARCH = C(1) + C(2)*RESID(-1)^2 + C(3)*GARCH(-1) \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance Equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>4.03E-07</td>
<td>7.54E-08</td>
<td>5.343490</td>
<td>0.0000</td>
</tr>
<tr>
<td>RESID(-1)^2</td>
<td>0.038630</td>
<td>0.002218</td>
<td>17.41679</td>
<td>0.0000</td>
</tr>
<tr>
<td>GARCH(-1)</td>
<td>0.954051</td>
<td>0.002934</td>
<td>325.1244</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**R-squared:** -0.000018  **Mean dependent var:** -3.17E-05  
**Adjusted R-squared:** 0.000409  **S.D. dependent var:** 0.007576  
**S.E. of regression:** 0.007578  **Akaike info criterion:** -7.101887  
**Sum squared resid:** 0.134656  **Schwarz criterion:** -7.094521  
**Log likelihood:** 8333.513  **Hannan-Q Quinn criter.:** -7.099204  
**Durbin-Watson stat:** 1.970823

### PLATINUM_RTN

**Dependent Variable:** PLATINUM_RTN  
**Method:** ML ARCH - Normal distribution (BFGS / Marquardt steps)  
**Date:** 11/28/18  **Time:** 19:31  
**Sample:** 1/03/2008 10/28/2018  
**Included observations:** 2842  
**Convergence achieved after 36 iterations**  
**Coefficient covariance computed using outer product of gradients**  
**Presample variance: backcast (parameter = 0.7)**  
\[ GARCH = C(1) + C(2)*RESID(-1)^2 + C(3)*GARCH(-1) \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance Equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>2.30E-07</td>
<td>5.42E-08</td>
<td>4.232050</td>
<td>0.0000</td>
</tr>
<tr>
<td>RESID(-1)^2</td>
<td>0.025929</td>
<td>0.003565</td>
<td>8.394128</td>
<td>0.0000</td>
</tr>
<tr>
<td>GARCH(-1)</td>
<td>0.960143</td>
<td>0.004805</td>
<td>198.9017</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**R-squared:** -0.000995  **Mean dependent var:** -0.000144  
**Adjusted R-squared:** -0.000508  **S.D. dependent var:** 0.004708  
**S.E. of regression:** 0.004709  **Akaike info criterion:** -7.929182  
**Sum squared resid:** 0.052022  **Schwarz criterion:** -7.921816  
**Log likelihood:** 9303.931  **Hannan-Q Quinn criter.:** -7.926499  
**Durbin-Watson stat:** 1.952190

### PALLADIUM_RTN

**Dependent Variable:** PALLADIUM_RTN  
**Method:** ML ARCH - Normal distribution (BFGS / Marquardt steps)  
**Date:** 11/28/18  **Time:** 19:31  
**Sample:** 1/03/2008 10/28/2018  
**Included observations:** 2842  
**Convergence achieved after 29 iterations**  
**Coefficient covariance computed using outer product of gradients**  
**Presample variance: backcast (parameter = 0.7)**  
\[ GARCH = C(1) + C(2)*RESID(-1)^2 + C(3)*GARCH(-1) \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance Equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>5.68E-07</td>
<td>1.23E-07</td>
<td>4.637682</td>
<td>0.0000</td>
</tr>
<tr>
<td>RESID(-1)^2</td>
<td>0.040761</td>
<td>0.005573</td>
<td>8.750113</td>
<td>0.0000</td>
</tr>
<tr>
<td>GARCH(-1)</td>
<td>0.939608</td>
<td>0.006720</td>
<td>139.8709</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**R-squared:** -0.000401  **Mean dependent var:** 0.000138  
**Adjusted R-squared:** 0.000026  **S.D. dependent var:** 0.008878  
**S.E. of regression:** 0.008877  **Akaike info criterion:** -7.222509  
**Sum squared resid:** 0.110963  **Schwarz criterion:** -7.215143  
**Log likelihood:** 8475.003  **Hannan-Q Quinn criter.:** -7.219826  
**Durbin-Watson stat:** 1.957429
**Dependent Variable: GOLD_RTN**
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)  
Date: 11/28/18 Time: 19:32  
Sample: 1/03/2008 10/28/2018  
Included observations: 2842  
Convergence achieved after 54 iterations  
Coefficient of variance computed using outer product of gradients  
Presample variance: backcast (parameter = 0.7)  
\[
\text{LOG(GARCH)} = C(1) + C(2)\times \text{ABS(RESID(-1)@SQRT(GARCH(-1)))} + C(3) \times \text{RESID(-1)@SQRT(GARCH(-1))} + C(4) \times \text{LOG(GARCH(-1))}
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>-0.280585</td>
<td>0.033943</td>
<td>-8.676827</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(2)</td>
<td>0.111517</td>
<td>0.006582</td>
<td>18.94367</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(3)</td>
<td>-0.012872</td>
<td>0.004987</td>
<td>-2.592992</td>
<td>0.0096</td>
</tr>
<tr>
<td>C(4)</td>
<td>0.983832</td>
<td>0.002904</td>
<td>338.7434</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared: -0.000000  
Adjusted R-squared: 0.000000  
S.E. of regression: 0.004227  
Sum squared resid: 0.049895  
Log likelihood: 964.5787  
Hannan-Quinn crit: -8.218190  
Durbin-Watson stat: 2.025974

**Dependent Variable: SILVER_RTN**
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)  
Date: 11/28/18 Time: 19:33  
Sample: 1/03/2008 10/28/2018  
Included observations: 2842  
Convergence achieved after 50 iterations  
Coefficient of variance computed using outer product of gradients  
Presample variance: backcast (parameter = 0.7)  
\[
\text{LOG(GARCH)} = C(1) + C(2)\times \text{ABS(RESID(-1)@SQRT(GARCH(-1)))} + C(3) \times \text{RESID(-1)@SQRT(GARCH(-1))} + C(4) \times \text{LOG(GARCH(-1))}
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>-0.215845</td>
<td>0.035571</td>
<td>-6.440468</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(2)</td>
<td>0.118866</td>
<td>0.008208</td>
<td>14.48223</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(3)</td>
<td>-0.005109</td>
<td>0.005172</td>
<td>-0.987689</td>
<td>0.3333</td>
</tr>
<tr>
<td>C(4)</td>
<td>0.986872</td>
<td>0.002256</td>
<td>437.4890</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared: -0.000018  
Adjusted R-squared: 0.000049  
S.E. of regression: 0.007576  
Sum squared resid: 0.123456  
Log likelihood: 831.351  
Hannan-Quinn crit: -7.050614  
Durbin-Watson stat: 1.070823

**Dependent Variable: PLATINUM_RTN**
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)  
Date: 11/28/18 Time: 19:34  
Sample: 1/03/2008 10/28/2018  
Included observations: 2842  
Convergence achieved after 67 iterations  
Coefficient of variance computed using outer product of gradients  
Presample variance: backcast (parameter = 0.7)  
\[
\text{LOG(GARCH)} = C(1) + C(2)\times \text{ABS(RESID(-1)@SQRT(GARCH(-1)))} + C(3) \times \text{RESID(-1)@SQRT(GARCH(-1))} + C(4) \times \text{LOG(GARCH(-1))}
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>-0.130877</td>
<td>0.028577</td>
<td>-4.579809</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(2)</td>
<td>0.069023</td>
<td>0.007868</td>
<td>7.629011</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(3)</td>
<td>-0.032591</td>
<td>0.004388</td>
<td>-7.427688</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(4)</td>
<td>0.992084</td>
<td>0.002455</td>
<td>404.1598</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared: -0.000093  
Adjusted R-squared: 0.000050  
S.E. of regression: 0.004709  
Sum squared resid: 0.052022  
Log likelihood: 930.3404  
Hannan-Quinn crit: -7.926861  
Durbin-Watson stat: 1.952190

**Dependent Variable: PALLADIUM_RTN**
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)  
Date: 11/28/18 Time: 19:34  
Sample: 1/03/2008 10/28/2018  
Included observations: 2842  
Convergence achieved after 46 iterations  
Coefficient of variance computed using outer product of gradients  
Presample variance: backcast (parameter = 0.7)  
\[
\text{LOG(GARCH)} = C(1) + C(2)\times \text{ABS(RESID(-1)@SQRT(GARCH(-1)))} + C(3) \times \text{RESID(-1)@SQRT(GARCH(-1))} + C(4) \times \text{LOG(GARCH(-1))}
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>-0.233780</td>
<td>0.035272</td>
<td>-6.827966</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(2)</td>
<td>0.097942</td>
<td>0.018055</td>
<td>5.263898</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(3)</td>
<td>-0.035959</td>
<td>0.006229</td>
<td>-5.773258</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(4)</td>
<td>0.983956</td>
<td>0.002960</td>
<td>332.4076</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared: -0.000401  
Adjusted R-squared: 0.000026  
S.E. of regression: 0.008877  
Sum squared resid: 0.110963  
Log likelihood: 845.5358  
Hannan-Quinn crit: -7.220806  
Durbin-Watson stat: 1.957429