Deterrence in Rank-Order Tournaments

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In a tournament competitors may cheat to gain an advantage. This paper considers the problem of deterrence and finds that tournaments reflect special circumstances that are not present in a traditional model of law enforcement. The traditional model considers sanctions and monitoring as the instruments of deterrence. In a tournament the prize structure plays a critical role in determining both the costs and benefits to cheating. We consider ways in which the prize structure can be manipulated in order to reduce monitoring costs.

"We didn’t get beat, we got out-milligrammed. And when you found out what they were taking, you started taking them." – Tom House (former MLB pitcher) in USA Today

1. INTRODUCTION

The idea that deterrence is a primary function of punishment dates back to the 18th century (Beccaria, 1767; Bentham, 1789). The idea that the economic theory of behavior can be used to analyze the deterrent effect of laws and punishments dates back to the mid-20th century. Formal models developed by economists such as Becker (1968), Ehrlich (1972), and Polinsky and Shavell (2000) consider two instruments for deterrence: a monitoring technology that generates a probability of detection, and a sanction. Individuals are assumed to make decisions about criminal activities by comparing expected benefits to expected costs. These costs and benefits often depend on other aspects of the economy such as the distribution of wealth (Ehrlich, 1973), levels of education (Ehrlich, 1975) and the presence of increasing returns to crime. This paper demonstrates that the prize structure of tournaments has important implications for deterrence.

Tournaments are a commonly used mechanism for the allocation of resources. Examples include promotion tournaments, sporting events, patent races, elections, and the classroom. Rank-order tournaments have many
desirable features, particularly in the work place (see, for example, Bognanno, 2001; Lazear and Rosen, 1981; Prendergast, 1999; Choi and Gulati, 2004). In a tournament, however, competitors may have incentive to engage in activities subverting the goals of the tournament organizer to gain an advantage. In promotion tournaments, workers may falsify their books through “creative accounting” (see Prendergast, 1999; Jacob and Levitt, 2003); athletes may take steroids; politicians running for office may “stuff the ballot box”; and students may bring “cheat sheets” to exams or commit plagiarism. Recently, scandals in tournament environments have made headlines, from steroid use in baseball, international cycling and the US Olympic track team to U.S. Rep. Tom DeLay’s indictment for conspiracy to violate election laws in 2002.

Within the deterrence literature it has been noted that if sanctions are costless and monitoring is not, then optimal deterrence entails setting the sanction arbitrarily high and the probability of detection arbitrarily low. However, if there is a maximum penalty that can be imposed, the second best alternative requires trading off the costs of monitoring against the costs of suboptimal deterrence. In a tournament limited liability adds an additional dimension. When tournament organizers can only strip cheaters of their prize (as is often the case), the expected costs and benefits to cheating depend on other individuals’ decisions to cheat.

The full effect of this interaction depends in part on whether an individual finishing second receives the first place prize after the winner is found to have cheated. We call this re-awarding. If prizes are not re-awarded, then cheating behaviors are strategic complements, which may lead to the co-existence of cheating and honest equilibria. If prizes are re-awarded, then agents’ decisions to cheat may become strategic substitutes. In fact, many tournaments do re-award prizes, one example being the Olympics. While re-awarding prizes may appear to be costly, it can be optimal because it reduces enforcement costs. Monitoring winners and losers differently and the awarding of prizes for runners-up also have such benefits.

Early in the tournament literature, it was noted that competitors have incentive to sabotage others. However, the approach was to consider incentives for sabotage as a function of the tournament design without considering monitoring and punishment (see, for example, Lazear, 1989; Konrad, 2000, 2005; Chen, 2003; Epstein and Hefeker, 2003). Later papers considered enforcement in their analysis (see Berentsen, 2002; Haugen, 2004; Kräkel, 2006; Gilpatric, 2009; Gilpatric and Stowe, 2007), but not as a main focus. This paper presents an in-depth analysis of the effect of the prize structure on the costs of deterrence.
2. THE MODEL

We consider the following model of a rank-order tournament. Two risk neutral contestants, 1 and 2, compete for a first place prize, $A$. We allow for a second place prize, $B$, that may be greater than zero. The probability that contestant $i$ wins the tournament is partially determined by the effort exerted by each of the competitors, $e_1$ and $e_2$, respectively. In addition, there exists another activity, $\theta$, that can increase a player’s chance of winning. Activity $\theta$ is for some reason undesirable and so shall be referred to as “cheating.” This activity is assumed to be a binary decision so that a competitor either cheats or does not, there is no question as to how much to cheat.

The probability that 1 wins the tournament is given by

$$P(e_1, e_2, \theta_{1}, \theta_{2})$$

where $\theta_i$ is an indicator function that takes the value 1 when $i$ cheats and 0 when he/she does not. Thus $P : \mathbb{R}^2 \times \{0, 1\}^2 \to [0, 1]$. We assume that the tournament is symmetric so that $P(e, e'; \theta, \theta') = 1 - P(e', e; \theta', 0)$ for all $e, e', \theta, \theta'$, although the relaxation of this assumption is discussed in Section 5. Let $P'_1(\cdot) > 0$ and $P'_2(\cdot) < 0$ denote the marginal effects of the efforts of 1 and 2, respectively. Cheating increases the probability of winning for the cheater, so that $P(e_1, e_2; 1, \theta_2) > P(e_1, e_2; 0, \theta_2)$ and $P(e_1, e_2; \theta_1, 0) > P(e_1, e_2; \theta_1, 1)$. Finally, it is assumed that effort comes at a cost of $c(e)$ where $c' > 0$ and $c'' \geq 0$, and that cheating is costless (aside from any potential penalty).

This paper focuses on the costs associated with the deterrence of cheating and is agnostic with regards to the determination of the activities that are considered cheating and to their social costs. It is assumed that the participants can be monitored to detect any cheating. Denote by $\pi$ the probability that an agent who cheats is caught. We assume limited liability on behalf of the contestants so that the maximum penalty is the removal of the prize. If the winner is found to have cheated, the other contestant may be awarded the first place prize instead (provided that person was not found to have cheated). We call this “re-awarding.” We consider randomization over whether such re-awarding takes place. This allows us to consider the effects of re-awarding in a more general framework. Let $\lambda$ denote the probability that the loser of the tournament is awarded the first place prize when the winner is caught cheating.

The timing is as follows. First, the prize structure, the probability of detection, and the probability of re-awarding are announced. Players decide their effort and whether to cheat simultaneously. A winner is determined,

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prizes awarded and the organizer audits the players for cheating. If cheaters are detected, then the penalty is imposed. In the analysis that follows, we focus on the minimal levels of monitoring required to induce an unique equilibrium in which neither contestant cheats, which we call an honest equilibrium. This stems in part from the desire to remain agnostic about the social costs of cheating as well as issues of equilibrium selection.

2.1. Honest Equilibrium
We begin by considering the conditions for an honest equilibrium. To begin with, it must be the case that each contestant is choosing a level of effort that is a best response to the other’s choice, given that neither are cheating. In other words, effort choices must solve

\[
\begin{align*}
\max_{e_1} & \quad B + P(e_1, e_2, 0, 0)(A - B) - c(e_1) \\
\max_{e_2} & \quad A - P(e_1, e_2, 0, 0)(A - B) - c(e_2)
\end{align*}
\]

The corresponding first order conditions are

\[
\begin{align*}
P_1(e_1, e_2, 0, 0)(A - B) - c'(e_1) &= 0 \\
-P_2(e_1, e_2, 0, 0)(A - B) - c'(e_2) &= 0
\end{align*}
\]

In a symmetric equilibrium, each player exerts effort \(e^H\) which is characterized by the equation

\[
P_1(e^H, e^H, 0, 0)(A - B) = c'(e^H).
\]

In an honest equilibrium, each contestant has a probability of winning equal to \(\frac{1}{2}\) and an expected payoff of \(EU^H = \frac{Ae^H}{2} - c(e^H)\).

In order for this to be an equilibrium, it must be that neither contestant prefers instead to cheat. Since the game is symmetric, we focus on player 1 in the following analysis, but the same reasoning can be extended to the other contestant. The payoff associated with deviating from this strategy is given by

\[
\max_{e} (1 - \pi)[B + P(e_{1}, e^H, 1, 0)(A - B)] - c(e_{1}).
\]

Denote by \(\hat{e}\) the level of effort that solves this maximization problem and let \(EU\) be the expected utility from deviating from an honest equilibrium. We
therefore have that participant 1 prefers not to cheat if and only if
\[ (1 - \pi)[B + P(\hat{e},e^u,1,0)(A - B)] - c(\hat{e}) \leq \frac{A + B}{2} - c(e^u). \]  
(2.1)

Note that when \( \pi = 0 \), then \( \hat{E}U > EU^H \). Similarly, when \( \pi = 1 \) we have that \( \hat{E}U < EU^H \). Further, \( EU^H \) is independent of \( \pi \) while \( \hat{E}U \) is strictly decreasing in \( \pi \). Denote the minimal level of monitoring required to induce such an equilibrium by \( \pi^H \). It is worth mentioning that \( \pi^H \) is independent of \( \hat{\lambda} \). This is because re-awarding only has an effect on an individual’s decision when the other contestant cheats. Since \( \pi^H \) concerns an individual’s incentives to cheat given that the other is not, the issue of re-awarding has no bearing. Rearranging equation 2.1 and letting it hold with equality gives the following characterization of \( \pi^H \)
\[ \pi^H \frac{A + B}{2} = (1 - \pi^H)\gamma(A - B) + c(e^u) - c(\hat{e}), \]  
(2.2)

where \( \gamma = P(\hat{e},e^u,1,0) - \frac{1}{2} \) is the change in probability of winning the tournament associated with cheating (when the other contestant is not). Note that it is possible for \( \gamma < 0 \). This occurs when an agent reduces his effort when cheating by more than enough to offset the increased chance of winning arising from cheating. In such cases, contestants would cheat not to have a better shot at winning, but to save on effort costs.

2.2. CHEATING EQUILIBRIUM

We now consider an equilibrium in which both contestants cheat. In such an equilibrium, each player’s effort must be a best response to the others given that they are both cheating. Note that limited liability means that if a contestant is caught, her payoff is zero.

\[
\max_{e_i} (1 - \pi)(B + [\lambda \pi + (1 - \lambda \pi)P(e_i,e^u,1,1)](A - B)) - c(e_i)
\]

\[
\max_{e_i} (1 - \pi)(B + [\lambda \pi + (1 - \lambda \pi)(1 - P(e_i,e^u,1,1))](A - B)) - c(e_i)
\]

The first order conditions are
\[
(1 - \pi)(1 - \lambda \pi)P_e(e_i,e^u,1,1)(A - B) - c'(e_i) = 0
\]
\[
- (1 - \pi)(1 - \lambda \pi)P_e(e_i,e^u,1,1)(A - B) - c'(e_i) = 0
\]

In a symmetric equilibrium, denote the optimal effort by \( e^C \), which is
characterized by
\[(1 - \pi)\pi \lambda - p_1(e^c, e^c, 1, 1)(A - B) = c'(e^c).\]
and the equilibrium payoff is
\[EU^c = (1 - \pi)\left(1 + \lambda \pi \right)A + (1 - \lambda \pi)B - c(e^c).\]

There exists an equilibrium in which both competitors cheat as long as neither has incentive to deviate and not cheat. The payoff associated with this type of deviation is given by
\[\max B + [\lambda \pi + (1 - \lambda \pi)P(e^c, e^c, 0, 1)](A - B) - c(e^c).\]

Denote by \(\tilde{e}\) the level of effort that solves this maximization problem and let \(EU\) be the associated payoff. Note that player 1 prefers to deviate\(^9\) if and only if \(EU \geq EU^c\). Recall that \(EU^c\) is a function of the probability of being detected, \(\pi\). Let \(\pi^e\) be the smallest \(\pi\) such that the player prefers not to cheat (meaning that \(EU = EU^c\)). Rearranging the expressions for these two terms yields
\[\pi^e \left(\frac{A + B}{2}\right) = (A - B)\delta + c(\tilde{e}) - c(e^c),\] (2.3)

where \(\delta = \frac{1}{2} - P(\tilde{e}, e^c, 0, 1)\) is the reduction in the probability of winning associated with being honest when the other contestant is cheating. As before, it is possible for \(\delta < 0\). This occurs when a contestant increases her effort enough when deviating to overcome the disadvantage caused by the other’s cheating. Note that \(e^c\), \(\tilde{e}\) and \(\pi^c\) are all functions of \(\lambda\). The effect of \(\lambda\) on \(\pi^c\) is of particular interest and will be examined in section 2.4.

2.3. MULTIPLE EQUILIBRIA
Whenever \(\pi^H < \pi^c\), there exists a range of monitoring, \(\pi \in [\pi^H, \pi^c]\), such that honest and cheating equilibria co-exist. Before considering the existence of multiple equilibria formally, it should be noted that there are two potential types of benefits one derives from cheating. The first is the increase in the probability of winning directly due to cheating. For comparable levels of effort, cheating gives an edge. However, cheaters also reduce their effort, and so it is possible for the probability of coming in first to fall once adjusting for the change in effort.\(^10\) Whenever the probability of winning the tournament increases with cheating (after allowing for effort to adjust), a particular form
of externality is present. When one contestant cheats, it reduces the expected prize for the other, and by doing so reduces the expected penalty associated with cheating for this second contestant. Thus, the benefit of cheating is not independent of the decision of the other player. Thus cheating decisions are strategic complements, and multiple equilibria exist for some levels of monitoring.

Let us first consider the case where prizes are not re-awarded, i.e. $\lambda = 0$. There exist multiple equilibria, if at $\pi^H$, $EU^C > EU$. Combining equations 2.2 and 2.3, this occurs when

$$[\delta - (1 - \pi^H)\gamma](A - B) > c(e^H) - c(\hat{e}) + c(e^C) - c(\hat{e}).$$  \hspace{1cm} (2.4)

**Proposition 1 (Existence of Multiple Equilibria):** When $\lambda = 0$, there exist functions $P(e_1, e_2, \theta_1, \theta_2)$ and $c(e)$ such that multiple equilibria exist for some values of $\pi$.

The demonstration of examples of functions that lead to the existence of multiple equilibria is sufficient to prove this proposition. Two common forms of tournaments are the Lazear-Rosen and Tullock tournaments.

**Example 1 (Lazear-Rosen):** Let $P(e_1, e_2, \theta_1, \theta_2) = a(e_1 - e_2 + \beta(\theta_1 - \theta_2))$, where $a$ is a positive constant.\(^{11}\)

In this case, it is easily verifiable that $e^H = \hat{e}$ and $e^C = \hat{e}$. Thus the right hand side to equation 2.4 is zero. It is also easily shown that $\delta = \gamma > 0$ and so the left hand side is positive, generating multiple equilibria. Note that in this case, the marginal return to effort only depends on whether the contestant is cheating or not. When a player cheats, there is a chance that she will be caught, meaning that her effort has been wasted. Lazear-Rosen tournaments with curvature in the success function also generate multiple equilibria as long as the marginal return to effort does not decrease too much when an individual cheats. $\square$

**Example 2 (Tullock):** Let $P(e_1, e_2, \theta_1, \theta_2) = \frac{(1+\beta\theta_1)e_1}{(1+\beta\theta_1)e_1 + (1+\beta\theta_2)e_2}$ and $c(e) = e$.

While closed form solutions exist for effort levels (and therefore expected utilities), the expression for $\pi^C$ is rather unwieldy. So consider the case where $A = 2, B = 1$ and $= \beta 3$. In this case, $\pi^H = 0.17$ and $\pi^C = 0.19$.\(^{19}\)

Whenever there exist multiple equilibria, the minimum level of monitoring required to ensure that neither agent cheats is $\pi^C > \pi^H$. If monitoring was to be anywhere between those two cutoff points, contestants might still coordinate on the equilibrium in which both cheat. If multiple equilibria are not present, choosing $\pi = \pi^H$ is sufficient to prevent cheating. Note that
conditional on attaining an honest equilibrium, re-awarding comes at no cost. When neither contestant cheats, the value of the prizes handed out is \( A + B \), independent of \( \lambda \). It is worth mentioning at this point that this result can be generalized to a continuous cheating decision. While the issue of multiple equilibria may or may not be present when the decision to cheat is a continuous one, an increase in cheating by one of the contestants would still have the effect of reducing the expected penalty for the other. Thus cheating decisions would still be strategic complements, and a greater degree of monitoring would still be required.

2.4. THE EFFECTS OF RE-AWARDING

When prizes are re-awarded, there exists an additional means for a competitor to win the first place prize. Now, an agent can win the first place prize (assuming they are not found to have cheated) if the other contestant is caught cheating. As noted above, re-awarding has no effect on an individual’s incentive when the other contestant is not cheating, so \( \pi'' \) is independent of \( \lambda \). Let us then consider the effect of cheating on \( \pi^c \). From above, if a tournament organizer wishes to ensure an honest equilibrium, then she must monitor with \( \pi^c \) if multiple equilibria are present.

First, note that re-awarding only affects a contestant when they do not win the tournament. Thus, if players have a higher probability of winning the tournament in a cheating equilibrium than when they deviate, after accounting for adjustments in effort (i.e. \( P(\bar{e}, e^c, 0, 1) < \frac{1}{2} \)), then re-awarding has a greater impact on \( \bar{E}U \) than on \( EU^c \). Re-awarding also reduces the return to effort, since a player does not have to win the tournament in order to get the first place prize. Of particular interest to competitors is that their opponent decreases their effort (\( \partial e^C / \partial \lambda < 0 \)). Since the outcome of a tournament matters to a contestant only when neither is caught, this also tends to have a greater effect on \( \bar{E}U \) than \( EU^c \). The exact effect of \( \lambda \) on \( \pi^c \) can be found by differentiating \( EU^c - \bar{E}U \) with respect to \( \lambda \).

**Proposition 2:** The necessary and sufficient condition for \( \frac{\partial e^C}{\partial \lambda} < 0 \) is

\[
(A - B) \left\{ \pi \left[ -\frac{\pi}{2} - \frac{1}{2} + P(\bar{e}, e^c, 0, 1) \right] - \frac{\bar{e}e^c}{\partial \lambda} (1 - \lambda \pi) \left[ P_2(\bar{e}, e^c, 0, 1) - (1 - \pi)P_2(e^c, e^c, 1, 1) \right] \right\} < 0.
\]

A sufficient condition is that cheaters have a greater probability of winning the tournament \( (P(\bar{e}, e^c, 0, 1) < \frac{1}{2}) \) and that the marginal return to 2’s effort is independent of 1’s
behavior \( P_2(\hat{e}, e^C, 0, 1) = P_2(e^C, e^C, 1, 1) \).

Note that the latter part of this sufficient condition is restrictive. While it describes the example of the linear Lazear-Rosen tournament above, it does not apply to the Tullock example. However, it can be seen that, in both cases, re-awarding reduces the amount of monitoring required.

**Example 1 (Lazear-Rosen):** In the linear example of a Lazear-Rosen tournament above, it is optimal to re-award prizes with probability one.

Since \( P_2(\hat{e}, e^C, 0, 1) = P_2(e^C, e^C, 1, 1) = P_2 \) in this example, we have that \( \frac{\partial P_2}{\partial C} = [A - B] \left( \pi \left[ -\frac{\delta}{2} - \delta \right] - \frac{\pi}{\alpha} (1 - \lambda \pi) P_2 \right) < 0 \), where \( \delta > 0 \) is defined as above.

**Example 2 (Tullock):** In the Tullock example above, re-awarding decreases the amount of monitoring required.

Using the values above (\( A = 2, B = 1 \) and \( \beta = 3 \)), when prizes are re-awarded with probability one, \( \pi^C = 0.15 \). It is worth noting that, in both examples, re-awarding actually eliminates the possibility of multiple equilibria, so that the contestants need only be monitored with \( \pi^H \).

The above result establishes a reason for prizes to be re-awarded. The next section considers the effect that the prizes themselves have on the amount of monitoring required.

### 3. WINNER TAKE ALL, ENTRY FEES AND OTHER FEATURES

Lazear and Rosen (1981) and Tullock (1980) both pointed out that effort is a function of the difference between the first-place and second-place prizes. In particular, it has been noted that winner-take-all formats induce maximal effort at the lowest cost. This stems from the first order conditions for effort, which depend on \( A - B \). Consequently, there is no value to offering generous second prizes (or third and so on) in such a simple environment. However, in reality we often observe contests which do not adopt the winner-take-all approach; the Olympics with gold, silver and bronze medals is an obvious example. Moldovanu and Sela (2001), Singh and Wittman (2000), and Szymanski and Valletti (2005) rationalized the existence of more than one prize by the incentives that the additional prizes create when participants are heterogeneous in their probability of winning a tournament. For example, in an 8-person race, if one contestant is heavily favored to win, the others will have more incentive to exert effort if there is a second-place prize. It should be noted that none of the above papers suggest that if there are \( N \)
contestants, then there should be $N$ prizes. In this paper, there are two symmetric risk neutral contestants, yet there still exists a rationale for a second-place prize.

Consider an environment in which the equilibrium is unique (perhaps due to re-awarding). Choosing a probability of detection equal to $\pi^H$ is thus sufficient to prevent cheating. Without limited liability, the incentive to cheat would simply be an increasing function of $A - B$, for the same reasons that effort increases in the difference between prizes. Tournaments would be winner take all, and maximal fines or punishments would be imposed on cheaters and contestants would be monitored as little as possible, as in the standard Becker (1968) and Polinsky and Shavell (2000) result. With limited liability however, the expected punishment is the expected prize. Since effort depends on $A - B$, if both $A$ and $B$ were to increase, but the difference between them was unchanged, there would be no effect on effort. However, the expected prize, and therefore the expected penalty, would increase. This would enable the tournament organizer to decrease monitoring and still obtain the non-cheating equilibrium, as demonstrated in the following Result.

**Proposition 3:** Increasing $A$ and $B$ such that $A - B$ remains constant leads to the same effort provided by both agents (in the equilibrium with no cheating), but reduces $\pi^H$.

Increasing both prizes is equivalent to increasing the sanction associated with cheating. There is in effect something that can be taken away when bad behavior is detected. Thus if monitoring is particularly costly, the organizer may find it cheaper to offer larger prizes than to increase monitoring in order to prevent contestant cheating. The same results can be derived for $\pi^C$.

If tournaments can charge entry fees, however, then it may actually end up to be costless to reduce monitoring in this fashion. An entry fee would not influence effort choices, nor would it affect the incentives to cheat, all else equal. Such a fee, however, could be used to fund increases in both $A$ and $B$. That is, the entry fee acts as a bond that contestants post before the tournament and do not get back if they are caught cheating. If the difference between the two prizes were held the same, then efforts would be the same, each agent’s expected payoff (net of the entry fee) would be the same, but the organizer could spend less on monitoring costs.\(^{12}\)

### 4. Differential Monitoring

The analysis thus far has assumed that competitors must be monitored with the same probability, $\pi$. This need not be the case. For example, when prizes
are not re-awarded, one could ensure a unique, honest equilibrium by setting
the level of monitoring for competitor 1 at \( \pi^H \) and for competitor 2 at \( \pi^C \). If
this were done, 2 would not be willing to cheat no matter whether 1 were
cheating or not. Conversely, 1 would be willing to cheat if 2 were cheating,
but not if 2 is not cheating. Since 2 definitely will not cheat, 1 will also not
cheat. In this manner, the organizer could achieve the same equilibrium with
less total monitoring.\(^{13}\)

A more common form of differential monitoring is to check for cheating
after the outcome of the tournament has been realized and to monitor the
winner to a greater degree than the loser. For example, urine tests are
mandatory for Olympic medal winners but occur only with some probability
for other athletes.\(^ {14} \) By doing so, the monitoring exploits the fact that when
multiple equilibria are present, deterring one contestant from cheating helps
deter the other. However, this is not the only benefit of differential
monitoring. Since cheating increases one’s chances of winning, additional
monitoring of the winner has a greater impact on one’s decision to cheat. As
a result, the total amount of monitoring required can be reduced by
monitoring the winner more than the loser. This is true even when multiple
equilibria are not present. It should be noted that the second effect is similar
to the rank-based punishment strategy proposed by Berentsen (2002).

Let \( \pi^W \) and \( \pi^L \) denote the monitoring of the winner and loser of the
tournament, respectively. As in the last section, we only consider the case in
which \( \pi^H \) is sufficient to deter cheating so that if \( \pi^W = \pi^L = \pi^H \), nobody
cheats. As before, when neither contestant cheats, their expected payoffs are
independent of any monitoring. The payoff when a contestant deviates,
however, is

\[
(1 - \pi^H)B + P(\hat{e}, e^H, 1, 0) \left[(1 - \pi^W)A - (1 - \pi^H)B\right] - c(\hat{e})
\]

where \( \hat{e} \) is the optimal effort when deviating from an honest equilibrium. The
minimal amount of monitoring required to deter cheating is thus a variation
of equation 2.2:

\[
\frac{A + B}{2} - c(e^w) = (1 - \pi^H)B + P(\hat{e}, e^H, 1, 0) \left[(1 - \pi^W)A - (1 - \pi^H)B\right] - c(\hat{e}).
\]

Using the Implicit Function and Envelope Theorems, we find that

\[
\frac{\partial \pi^H}{\partial \pi^L} = -\frac{B}{A} \cdot \frac{1 - P(\hat{e}, e^H, 1, 0)}{P(\hat{e}, e^H, 1, 0)} < 0.
\]
When $\frac{\partial \pi^W}{\partial \pi^L} > -1$, it is possible to increase the monitoring of the winner by less than the decrease in the monitoring of the loser and still maintain an equilibrium with no cheating.

**Proposition 4**: Suppose total detection costs are a function of the sum of the probabilities with which each contestant is caught ($C(\pi^W + \pi^L)$). Then total detection costs can be reduced by monitoring the winner more than the loser if and only if $[1 - P(\hat{e}, e^H, 1, 0)]B < P(\hat{e}, e^H, 1, 0)A$. A sufficient condition is $P(\hat{e}, e^H, 1, 0) > \frac{1}{2}$, or that cheaters are more likely to win.

Thus there exists an efficiency explanation for the prevalence of differential monitoring. There may, of course, be other reasons. If cheating helps contestants win (net of the reduction in effort), then winners would also be more likely to have cheated. Also, if it is for some reason important that the winner be perceived to have not cheated, then differential monitoring would help foster such perceptions. Note that this may be particularly important for sports.

5. CONCLUSION

This paper examines different issues that arise when deterring tournament participants from undertaking activities (which we refer to as “cheating”) that increase their chances of winning, but that are undesirable. We consider a standard detection and punishment scheme, but with a specific form of limited liability that seems appropriate for most tournaments - contestants can only have their prizes confiscated. In this case, the prize structure becomes an important feature for deterrence. A main result centers on the potential for multiple equilibria. We demonstrate that the tournament environment creates an externality between competitors in their cheating decisions, and so for a given enforcement effort, no cheating or both agents cheating may both be equilibria. We focus on the minimum level of enforcement that would deter cheating with probability one, and then look at ways the prizes can be structured in order to reduce the monitoring intensity and still achieve the same effort from the competitors.

An important feature of this paper is that it focuses on the equilibrium in which no cheating takes place. In reality, it might be desirable to tradeoff costs arising from cheating with prize and monitoring costs. To be properly able to analyze such a problem, one would need a formal objective function for a tournament organizer, something this paper did not undertake in order to focus on the positive aspects of enforcement. Such a question would
certainly be worthy of exploration. What would be the objective function of a tournament organizer be in this context?

In other models, the organizer is assumed to care about output which is a function of effort and a random component. If this is the sole objective, cheating would be undesirable if it displaces effort, or in the case of sabotage, destroys the productive efforts of others. Re-awarding, offering second-place prizes and differential monitoring all reduce efforts, but save on monitoring costs. However, is the displacement of effort the only cost to cheating? Steroid use increases the speed of sprinters, the number of home runs hit or the top weight lifted, but can also have many negative effects. It can jeopardize contestants’ future health, render comparisons of performance impossible (such as the comparison of Barry Bonds and Babe Ruth), or diminish interest in the competition. For example, a July 8, 2002 USA Today poll showed that 86% of baseball fans claim that compulsory testing for steroids would renew their interest in baseball. How these different costs enter the tournament organizer’s objective function would dictate the optimal levels of enforcement, effort, and cheating. We hope that this paper will stimulate research interest on these types of questions.

Finally, there exist some policy implications for the courts. We have thus far considered that the prizes, $A$ and $B$, are determined by the tournament organizer. In many tournaments, however, the rewards to winning extend beyond any prize won directly. For example, in sports the tournament organizer may award medals or cash to the winners, but the benefits may extend to endorsement opportunities, increased future salary and prestige. While it is possible for the tournament organizer to re-award any medals or monetary prizes, it may not be possible to re-award these other benefits. That is, even if the tournament organizer strips a cheater of the first place prize and re-awards it to the runner-up, it may be the case that endorsement possibilities and prestige are diminished when a player wins via default, or simply because cheating is detected at a time when those outside benefits and opportunities are no longer available. Thus the tournament organizer may not be able to re-award the full benefits of winning, thereby reducing the deterrent effect of re-awarding. If contestants who were awarded a prize via re-awarding could receive the full benefits by suing cheaters, then the full deterrent effect would be restored. Thus there are implications of this analysis for the law of torts as well.

On February 15, 2008 Willie Gary, a former football player for the St. Louis Rams who was on the Super Bowl XXXVI losing team, filed a class action suit against the winning team, the New England Patriots. The case was
built on the suspicion that the New England Patriots gained an unfair advantage by spying on the Rams’ pre-game practice. However, the National Football League was not able to find any evidence of wrong-doing\textsuperscript{15} in its own investigation, so on March 13, 2008 the case was dismissed.\textsuperscript{16} Of interest to the analysis here, however is paragraph 52 of the complaint, filed in the Eastern District of Louisiana:\textsuperscript{17}

Each member of the St. Louis Rams 2002 Super Bowl Roster and staff should be compensated for the monetary damages they suffer by losing out on the receipt of (a) the winning bonus (For Super Bowl XXXVI, the winning bonus was $58,000 vs. $33,000 for the loser. This is a $25,000 difference per player.) (b) a Super Bowl winner ring (a ring from Super Bowl XXXVI is on eBay for $125,000), endorsements, and other consequential economic losses.

In other words, the suit was asking for damages equal to the benefits that could not be re-awarded. It is also interesting to note that it seems that a necessary condition for such a suit to proceed is for the defendant(s) to have been stripped of the prize by the tournament organizer. Since the NFL could not find sufficient evidence that the Patriots had cheated, it was deemed that there was insufficient evidence for the lawsuit. This is in stark contrast to a civil suit arising from criminal actions, where a civil trial may be successful even if the evidence was insufficient to garner a criminal conviction.

It is worth considering what would have been the optimal outcome of such a court case, had there been enough evidence to demonstrate that the Patriots had cheated. Surely, the NFL could have re-awarded the Super Bowl to the Rams; it could also have imposed fines on the Patriots and paid the $25,000 difference in bonuses to each of the Rams’ players. The NFL could even have distributed new rings. However, the NFL has no control on the value of those rings, nor does it have control over the endorsement opportunities players get from being part of a Super Bowl winning team. If the value of the rings as well as the endorsement opportunities are lessened because of the cheating, then it would appear that there is a role for civil suits. Given that we have shown that re-awarding is a useful tool in the deterrence of cheating, allowing contestants to sue cheaters would help ensure that there are more appropriate incentives within tournaments, particularly in cases where the tournament organizer does not fully control the benefits to winning.
Appendix

Proof to Proposition 2: Using the Implicit Function Theorem, we have that

\[
\frac{\partial \pi^c}{\partial \lambda} = \frac{\frac{\partial EUC}{\partial \lambda} - \frac{\partial EU}{\partial \lambda}}{\frac{\partial EUC}{\partial \lambda} - \frac{\partial EU}{\partial \lambda}}
\]

Since the denominator is negative, the sign of \( \frac{\partial \pi^c}{\partial \lambda} \) is the same as the sign of the numerator. Thus \( \frac{\partial \pi^c}{\partial \lambda} < 0 \) if and only if

\[
\frac{\partial EUC}{\partial \lambda} - \frac{\partial EU}{\partial \lambda} = (A - B) \left\{ \pi \left[ -\frac{\pi}{2} - \frac{1}{2} + P(\tilde{e}, e^c, 0, 1) \right] - \frac{\partial e^c}{\partial \lambda} (1 - \lambda \pi) [P_2(\tilde{e}, e^c, 0, 1) - (1 - \pi) P_2(e^c, e^c, 1, 1)] \right\} < 0.
\]

Note that if cheaters have a greater probability of winning the tournament even after accounting for changes in effort, \( P(\tilde{e}, e^c, 0, 1) < \frac{1}{2} \), then the first term in the curly brackets is negative. If the marginal return to effort is independent of the other's behavior \( P_2(\tilde{e}, e^c, 0, 1) = P_2(e^c, e^c, 1, 1) \), then the second term (after the minus sign) is positive and the overall effect must be negative. \( \square \)

Proof to Proposition 3: The fact that effort remains the same in the non-cheating equilibrium follows directly from the first order conditions. To examine the effect on \( \pi^H \), we note that if both \( A \) and \( B \) increase by the same amount, the net effect is simply \( \frac{\partial \pi^H}{\partial A} + \frac{\partial \pi^H}{\partial B} \).

\[
\frac{\partial \pi^H}{\partial A} + \frac{\partial \pi^H}{\partial B} = \left[ \frac{\frac{1}{2} - (1 - \pi^H) P(1, 0) - \frac{\partial e^c}{\partial A} [c'(e^H) + (1 - \pi^H) P_3(1, 0) [A - B]]}{B + P(1, 0) (A - B)} \right]
- \left[ \frac{- (1 - \pi^H) P(1, 0) + \frac{\partial e^c}{\partial B} [c'(e^H) - (1 - \pi^H) P_3(1, 0) [A - B]]}{B + P(1, 0) [A - B]} \right]
\]

From the first order conditions for effort choice, it can easily be shown that \( \frac{\partial \pi^H}{\partial A} = -\frac{\partial \pi^H}{\partial B} \). This implies that

\[
\frac{\partial \pi^H}{\partial A} + \frac{\partial \pi^H}{\partial B} = \frac{-1 + 2 \frac{\partial e^c}{\partial A} (1 - \pi^H) P_3(1, 0) (A - B)}{B + P(1, 0) (A - B)} < 0. \quad \square
\]
Endnotes

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1. If there exist increasing returns, then the payoff to crime depends on the choices of others, and multiple equilibria can be present. Increasing returns can be attributed to congestion effects in deterrence as in Sah (1991), or due to social norms and stigma associated with crime as described by Rasmusen (1996), or due to coordination issues surrounding occupational choice as in Burdett et al. (2003) and Murphy et al. (1993).


3. If criminals are risk neutral, it is the expected penalty that determines the level of deterrence. Note that it may be optimal to have the expected penalty be finite, for reasons such as marginal deterrence (see Friedman & Sjostrom, 1993; Mookherjee & Png, 1994; Stigler, 1970).

4. In a rather interesting example, Beckie Scott, a Canadian cross-country skiier, initially won the bronze medal at the 2002 Winter Olympics. After one of the Russian women ahead of her tested positive for a banned substance, she was upgraded to silver. When the other Russian woman tested positive as well, she (after a long legal battle) was awarded the gold, marking the first time an athlete received the bronze, silver and gold in the same event. This case is of interest, because the Olympic Committee debated for a long time whether it was worthwhile to issue another gold medal. This paper suggests that it was.

5. The question as to why some activities are considered undesirable is an interesting one that is left to further research.

6. If cheaters are subject to an additional penalty that is independent of rank in the contest then all of our results still hold, just at lower levels of monitoring.

7. Since \( P(\cdot) \) is assumed to be symmetric, we have that \( P_1(e^{H}, e^{H}, 0, 0) = -P_2(e^{H}, e^{H}, 0, 0) \), and so the two first order conditions are identical.

8. One should note that when one contestant deviates, the effort level provided by the other contestant remains unchanged. This is because we assume that both effort and cheating decisions are simultaneous. This need not be the case; it is possible to imagine an environment in which a player’s cheating decision is observable to the other contestant before the choice of effort. This alternative environment yields qualitatively similar results.

9. We assume that, when indifferent between cheating and not, the player does not cheat.

10. While it may seem odd to cheat if it reduces the probability of winning, it may be worthwhile because of the reduction in effort costs.
11. Strictly speaking, since probabilities cannot be negative or greater than 1, this function should be defined to take on the value of 0 when $e_1 - e_2 + \beta(\theta_1 - \theta_2) < 0$ and 1 when $\alpha(e_1 - e_2 + \beta(\theta_1 - \theta_2)) > 1.$

12. We thank an anonymous referee for pointing this out.

13. We thank Peter Norman for this insight.

14. Note that this does not mean that medal winners are caught with probability one.

15. However, the 2007 Patriots were punished for a similar incident.


17. See Eastern District of Louisiana Complaint Class Action Jury Demand #08-1002, §52.

References


and J. Stowe. 2007. “Cheating in Asymmetric Tournaments: Closing the Gap or Maintaining an Advantage,” manuscript.