Why do most countries set high tax rates on capital?

Nicolas Marceau\textsuperscript{a,d}, Steeve Mongrain\textsuperscript{b,d,*}, John D. Wilson\textsuperscript{c}

\textsuperscript{a} Université du Québec à Montréal, Canada
\textsuperscript{b} Simon Fraser University, Canada
\textsuperscript{c} CRIME, Canada

ABSTRACT

We consider tax competition in a world with tax bases exhibiting different degrees of mobility, modeled as mobile and immobile capital. An agreement among countries not to give preferential treatment to mobile capital results in an equilibrium where mobile capital is nevertheless taxed relatively lightly. In particular, one or two of the smallest countries, measured by their stocks of immobile capital, choose relatively low tax rates, thereby attracting mobile capital away from the other countries, which are then left to set revenue-maximizing taxes on their immobile capital. This conclusion holds regardless of whether countries choose their tax policies sequentially or simultaneously. In contrast, unrestricted competition for mobile capital results in the preferential treatment of mobile capital by all countries, without cross-country differences in the taxation of mobile capital. Nevertheless, our main result is that the non-preferential regime generates larger expected global tax revenue, despite the sizable revenue loss from the emergence of low-tax countries.

By extending the analysis to include cross-country differences in productivities, we are able to resurrect a case for preferential regimes, but only if the productivity differences are sufficiently large.

1. Introduction

A theme running through the tax competition literature is that jurisdictions face incentives to compete for mobile capital by reducing their tax rates. As a result, tax competition leads to inefficiently low tax rates and public good provision when governments are welfarist, but may constrain the excessive size of governments that act as Leviathans.\textsuperscript{1} One might question the tax-reducing effects of tax competition when examining the effective average capital tax rates in the European Union for the year 1991, which we report in Table 1.\textsuperscript{2} Indeed, note that most of the countries are distributed around an average of 32% – with some variance, possibly explained by differences in preferences for publicly provided goods – while on the other hand, a considerably lower tax rate of only 11% is in effect in Ireland.\textsuperscript{3} An interpretation of such facts is that Ireland was undercutting the other countries, while the rest seemed to act as if it was business as usual. Intuitively, if a country has a comparative advantage at lowering its tax rate to attract mobile capital, it will specialize in this activity. However, the rest of the countries will not attempt to attract mobile capital, and will instead focus on their immobile base to finance their expenditures.

The current paper develops a model of this asymmetric policy response to capital mobility. Countries are assumed to differ in their supplies of an immobile tax base. Taxing this base affects its size, but not the country in which it is located. An interpretation is that some firms have already sunk investments in their host countries, limiting their abilities to relocate in another country, but these firms are able to adjust their investment levels within their host countries. All countries have an opportunity to attract mobile capital by reducing their tax rates. This presence of both mobile and immobile capital in the model is consistent with evidence that while capital is becoming increasingly mobile, a large portion of capital is still subject to limited mobility, as discussed in Gordon and Bovenberg (1996).

We first analyze a non-preferential regime, where each country taxes its mobile and immobile capital at the same rate. Recent policy initiatives have made this case increasingly relevant. In particular, the OECD has become interested in what it calls “harmful tax practices”. In OECD (1998), two sorts of country behavior are viewed as harmful: (a) to impose no or very low taxes on some bases; and (b) to have some preferential features in the tax system that allow part of a given base to escape taxation. For the second sort of behavior, the preferential tax regimes often consist of the foreign-owned portion of a tax base being taxed at a lower rate than the domestic-owned portion, a behavior that is also labeled “discrimination”.

\textsuperscript{1} Corresponding author. Simon Fraser University, Canada.
E-mail addresses: marceau.nicolas@uqam.ca (N. Marceau), mongrain@sfu.ca (S. Mongrain), wilsonjd@msu.edu (J.D. Wilson).

\textsuperscript{2} See Wilson (1999) for a review of the tax competition literature, and Wilson (2005a) for a recent analysis of tax competition with self-interested government officials.

\textsuperscript{3} Effective average capital tax rates measure total taxes paid as a fraction of the relevant tax base. The effective average capital tax rates reported in Table 1 include corporate tax collection as well as personal taxes on capital income. See Sørensen (2000) or Hauffer (2001) for a discussion of these numbers.

\textsuperscript{3} Similar patterns can be observed in the 1981 tax rates, except for the fact that low tax rates.
Some countries – e.g. Canada and the US – have signed mutually advantageous tax treaties, which would be jeopardized if one or the other actor were to start discriminating. And the prohibition of the asymmetric treatment of foreign and domestic firms has been included in treaties in the EU and the OECD. Both the OECD and the EU are active in trying to reduce the extent of discrimination among their members.4

Within our framework, we show that only the two smallest countries compete for mobile capital by reducing their tax rates (assuming there are at least three countries); the rest are content to maximize the revenue obtained from the immobile base.5 This result is proven for a simultaneous-move Nash game, but we similarly find that only the smallest country competes and obtains mobile capital in the sequential-move game. These results have both positive and normative implications, which we next describe.

One positive implication is that it is the relatively small countries that significantly lower their tax rates in an effort to attract capital. Empirical studies of tax havens like the one by Dharmapala and Hines (2006) confirm this prediction. In their comprehensive study, they confirm the fact that most tax havens tend to be small. They also find that tax havens tend to be governed, but for simplicity we abstract from such issues in our paper.

Another positive implication of our results is that tax rates should be considerably higher in most other countries. Table 1 confirms this finding. Chen, Mintz and Tarasov (2007) also comment on the fact that most of the countries with high GDP set high corporate income taxes, often exceeding thirty percent. Our results also help explain the evidence presented by Hines (2005) that corporate tax collections did not decline as a percentage of GDP between 1982 and 1999, despite increasing capital mobility.6 He attributes this finding to a switch in tax burdens from mobile capital to immobile capital, along with an extension of domestic tax bases. But our study suggests that most countries will choose not to compete for mobile capital, in which case increasing mobility need not alter their behavior. Moreover, the finding that most countries set the same tax rates is consistent with the lack of correlation between country size and tax rates. Hines (2005) observes that this correlation had largely disappeared by 1999.

On the normative side, our study addresses the debate about the relative merits of non-preferential and preferential tax regimes. When countries are allowed to individually levy different tax rates on immobile and mobile capital, tax havens do not emerge in our model, but tax competition for mobile capital intensifies, causing a large revenue loss in the form of lower tax rates on mobile capital.7 We conclude that tax competition in the non-preferential regime leads to a lower aggregate loss in tax revenue, compared with tax competition in the preferential regime. The superiority of the non-preferential regime in terms of tax revenue does not require assumptions about whether countries choose their tax rates simultaneously or sequentially. But in all cases, it is only the smallest country that benefits from an expected increase in revenue under the non-preferential regime. The other countries are equally well-off under both regimes. Thus, the non-preferential regime is better from the viewpoint of total revenue, but only because it enables the smallest country to benefit from acting as a tax haven. Thus, fears that the lack of regulation of preferential tax initiatives will cause widespread harm are perhaps unfounded. Moreover, to the extent that this regulation leads to the creation of tax havens, it conflicts with another OECD initiative to limit tax havens.5

Our results also shed light on the theoretical literature on preferential versus non-preferential regimes. This literature gives conflicting results. Janeba and Peters (1999) show that the elimination of preferential regimes leads to higher total levels of tax revenue. On the other hand, Keen (2001) reaches the opposite conclusion. They both analyze simultaneous-move Nash games in tax rates, but their models contain important differences. Janeba and Peters consider two countries that differ in their supplies of an immobile “domestic tax base,” whereas a second base is infinitely elastic with respect to differences in tax rates; it locates in the lowest-tax country. In contrast, Keen assumes that both countries are completely identical and have access to two tax bases that are partially mobile to different degrees. Wilson (2005b) observes, however, that if one of the tax bases in Keen’s paper were made infinitely elastic, as in Janeba and Peters, then a symmetric equilibrium in pure strategies would not exist under the non-preferential regime. By analyzing a mixed-strategy equilibrium, the current paper is able to pinpoint the absence of size differences across jurisdictions, along with restrictions on the elasticity of the tax base, as the driving forces behind Keen’s results.8 But for the non-preferential case, we also find that

### Table 1

<table>
<thead>
<tr>
<th>Effective average tax rate on capital (%)</th>
<th>Productivity of capital (as % of that in US)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>34.5</td>
</tr>
<tr>
<td>Belgium</td>
<td>39.5</td>
</tr>
<tr>
<td>Denmark</td>
<td>48.8</td>
</tr>
<tr>
<td>Finland</td>
<td>35.2</td>
</tr>
<tr>
<td>France</td>
<td>28.4</td>
</tr>
<tr>
<td>West Germany</td>
<td>31.0</td>
</tr>
<tr>
<td>Ireland</td>
<td>11.4</td>
</tr>
<tr>
<td>Italy</td>
<td>25.3</td>
</tr>
<tr>
<td>Netherlands</td>
<td>29.7</td>
</tr>
<tr>
<td>Spain</td>
<td>13.9</td>
</tr>
<tr>
<td>Sweden</td>
<td>47.4</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>66.5</td>
</tr>
<tr>
<td>EU average</td>
<td>33.1</td>
</tr>
<tr>
<td>United States</td>
<td>40.9</td>
</tr>
</tbody>
</table>

Notes: For the effective average tax rate on capital, “EU average” is that of all countries of the EU. For the productivity of capital, “EU average” is that of the countries listed in this table.

Sources: Effective average tax rate on capital: Sørensen (2000, Table 2) or Haufner (2001, Table 4.1). Productivity of capital: Troller (1993, Table 1).

---

4 On this, see OECD (1998).

5 Note that in the asymmetric tax competition literature, small countries have more incentive to lower their tax rate, while larger countries set higher tax rates. The intuition for such a result is that smaller countries face a more elastic tax base so they set lower tax rates. See Bucovetsky (1991), Kanbur and Keen (1993), and Wilson (1991).

6 Note however that for the US, corporate tax collections seem to have declined rapidly between 1960 and 1982. On this, see Auerbach (2005).

7 The term “tax haven” is being applied here to countries that offer low tax rates on real capital investments, rather than countries that facilitate income-shifting for the purpose of reducing taxable income in high-tax countries, independently of the location of physical investments. Slemrod and Wilson (forthcoming) analyze tax competition in the latter setting and conclude that tax havens worsen the tax competition problem, resulting in lower levels of welfare.

8 In 2000 the OECD published a list of 35 countries called “non-cooperating tax havens,” giving them a year to enact fundamental reform of their tax systems and broaden the exchange of information with tax authorities or face economic sanctions. Most of those 35 countries agreed to comply, but a few resisted until recently. However, in April 2009, the countries of the G20 met and addressed explicitly the “problem” of tax havens in the final communiqué of the summit. A few days later, all of the blacklisted tax havens had committed to comply to the OECD’s Memorandum of Understanding agreeing to transparency and exchange of information.

9 Janeba and Smart (2003) generalize both the Janeba–Peters and Keen results to more general settings. But they also must restrict the relevant elasticities to ensure the existence of equilibria. Wilson (2005b) considers only symmetric equilibria for identical countries, whereas the focus of the current paper is on the effects of lower aggregate losses when countries are equally well-off under both regimes. The superiority of the non-preferential regime, the superiority of the non-preferential regime in terms of tax revenue does not require assumptions about whether countries choose their tax rates simultaneously or sequentially. But in all cases, it is only the smallest country that benefits from an expected increase in revenue under the non-preferential regime. The other countries are equally well-off under both regimes. Thus, the non-preferential regime is better from the viewpoint of total revenue, but only because it enables the smallest country to benefit from acting as a tax haven. Thus, fears that the lack of regulation of preferential tax initiatives will cause widespread harm are perhaps unfounded. Moreover, to the extent that this regulation leads to the creation of tax havens, it conflicts with another OECD initiative to limit tax havens.5

10 For the non-preferential case, we also find that
expected tax revenue is lower for the smallest country under the simultaneous-move game than under the sequential-move game, while no country obtains higher expected revenue under the sequential-move game. As a result, we should not be surprised if we find that mixed strategies are not played in practice.

The results reported above, along with previous literature, focus on how total tax revenue compares between the different tax regimes. But we later extend the analysis to also address the question of the revenue-raising capabilities of non-preferential regimes. Table 1 also reports some data on the productivity of capital. Our explanation could account for the case of Ireland — low productivity and low tax rates, and could also help explain the low tax rate chosen by Spain in 1981. Our explanation is also supported by the fact that as Spain’s productivity rose in the eighties, so did its tax rate.10 Note that overall, the correlation between the 1981 tax rates and the 1983 capital productivity is strongly positive at 0.848.

The plan of this paper is as follows. In the next section, we first describe the basic model, with countries of different sizes and no productivity differences. For the case where all countries choose their tax rates simultaneously, we show in Section 3 that there exists a unique simultaneous-move Nash equilibrium in which the smallest two countries play mixed strategies, with their tax rates undercutting the tax rates chosen by the remaining countries. Thus, these two countries obtain all of the mobile capital in equilibrium. We also analyze the case where all countries are identical, finding that there also exist equilibria where more than two countries play mixed strategies and obtain the mobile capital. In Section 4, we introduce a sequential-move game in which each country chooses its tax rate in a specific, randomly determined, order. As noted above, this game produces only a single tax haven and leads to higher global tax revenue than the simultaneous-move game. Section 5 contains the comparison between preferential and non-preferential regimes, along with extending the analysis to the case where countries possess different productivities. Section 6 concludes.

2. The basic model

We consider a variation of the model introduced by Janeba and Peters (1999), but important differences will emerge. Imagine an economy with $J \geq 2$ countries or regions indexed by $j, j = 1, \ldots, J$. In contrast to Janeba and Peters (1999), as well as to the bulk of the existing related literature like Andersson and Konrad (2001), Wilson (2005b), or Konrad (2007), our general framework allows for the possibility of more than two countries competing for mobile capital. Also, we will derive the relation between tax payments and tax rates for a firm located in a particular country, whereas Janeba and Peters (1999) assume a tax function with particular properties. Our approach allows us to investigate what happens when these properties are not satisfied.

In each country or region, a representative citizen owns a constant-return-to-scale technology, $F(K) = yK$, with $y > 0$, which transforms capital into output. Capital owners can be local or mobile.

In particular, the world is populated by $M$ mobile capital owners, who can freely invest in any of the $J$ countries, whereas in country $j$, there are $N_j$ local capital owners, who invest only in $j$. As discussed in the introduction, the capital owned by local owners may be viewed as prohibitively costly to relocate to another country, perhaps because of previous sunk investments in the host country, whereas mobile capital is freely mobile. With these interpretations, it is reasonable to treat mobile and immobile capital as perfect substitutes.

Capital bears a per unit tax of $t_j$ in country $j$. Given the constant-return-to-scale technology in each country, the net return on capital in country $j$ is then simply $(y - t_j)$. The owners of local capital can adjust to taxation by increasing or decreasing the size of their capital investment, denoted $I_j$. The owners of mobile capital can also adjust the size of their investment, but they also choose to locate this investment in a country that offers them the highest net return. Any such country has the lowest tax rate, i.e. country $g$ if $\min(t_j, 1) - 1 = \epsilon_g$. For now, we assume that if $S$ countries have chosen the same lowest tax rate, then all mobile capital owners invest in a country belonging to this set with probability $1/S$. Thus, all of mobile capital always ends up in a single country (i.e. capital investment is bang-bang), and the other countries obtain nothing.12

The general timing of events in this world is as follows. First, countries choose their tax rates $t_j, j = 1, \ldots, J$. Note that the tax rate in a given country applies to the two types of capital; discriminating is simply assumed to be impossible. Second, the owners of mobile capital select the country in which they will invest. Third and finally, owners of local and mobile capital choose the size of their investment. We will in turn consider the case where the countries play simultaneously, and that in which they play sequentially, so the first stage will later be decomposed into $J$ sub-stages.

Owners of capital located in $i$ (be it local or mobile) adjust the size of their investment to maximize their net consumption, which is simply the total return on their investment minus the cost of investment, given by $c(I_j)$, with $c’ > 0$ and $c” > 0$. Thus, for capital owners in $j$, the optimal size of their investment is:

$$I_j = \arg \max l_j = \max I_j (\gamma - t_j) - c(I_j).$$

Of course, given that the owners of mobile capital have decided to invest in $j$, the problem faced by the owners of local capital and those of mobile capital is identical. The first-order condition characterizing the investment decision $l_j(I_j) = (\gamma - t_j) - c’(I_j) = 0$. Using it, we easily find that $l’(I_j) = -1/c’(I_j) < 0$.

Given our focus on the revenue effects of tax competition, it is natural to assume that tax revenue plays a prominent role in government objectives. For simplicity, we assume that governments maximize revenue, but we later argue that our analysis holds more generally.13 Let $m_j$ be an indicator function which takes a value of 1 if all mobile capital owners invest in country $j$, and a value of 0 if they have opted for any other country. Thus, tax revenue for country $j$ is $t_j[N_j + M_m(I_j)l(I_j)]$. We denote by $W(t, m)$ the tax revenue in country $j$ when it has chosen tax rate $t$ and when the indicator variable takes a value of $m$. This tax

10 Data on total factor productivity for Spain in the eighties can be found in Aiyar and Dalggaard (2001).

11 For heuristic reasons, we introduce a different breaking rule when the game is sequential.

12 Our results would obtain even if the assumption that all investments are bunched were relaxed. In our framework, we simply assume that the marginal product of capital is constant in a given country. Departing from the standard assumption of a declining marginal product of capital is frequent in the literature and simplifies our analysis. For papers which investigate the case in which capital tends to agglomerate because of an increasing marginal product, see Baldwin and Krugman (2004), Roadway, Cuff and Marceau (2004), or Kind, Knavik and Schjelderup (2000).

13 Note that Janeba and Peters (1999), a paper which is close to ours, consider a world in which governments maximize tax revenues. Edwards and Keen (1996), Kanbur and Keen (1993), Keen (2001), or Wilson (2005b) make this same assumption.
Revenue function is equivalent to the primitive tax revenue function defined in Janeba and Peters (1999). Thus, we have:

\[ W_i(t, 1) = t[N_j + M]l(t), \]
\[ W_i(t, 0) = tN_jl(t). \]

For future use, denote by \( \hat{t} \) the tax rate that maximizes \( W_i(t, 0) \) and \( W_i(t, 1) \).\(^{14}\) Also note that \( W_i(t, 0) = W_i(t, 1) = 0 \) at both \( t = 0 \) and some \( t > \hat{t} \), where \( \hat{t} \) is the tax rate inducing zero investment \((I = 0)\) by capital owners.\(^{15}\) Finally, we define \( t_j < \hat{t} \) as the tax rate solving \( W_i(t_j, 1) = W_i(\hat{t}, 0) \), i.e. \( t_j(l_j)(M + N_j) = \hat{t}(\hat{t})N_j \). This implies that each country has a specific \( t_j \), that \( t_j = 0 \) when \( N_j = 0 \), and that \( t_j \) increases when \( N_j \) increases. The payoff functions of a given country are represented in Fig. 1.

Irrespective of where mobile capital ends up locating, global tax revenue, summed across all countries, is maximized when tax rates are set at \( t_j = \hat{t} \) \( \forall j \), \( j = 1, \ldots, J \). Thus, because all locations are equivalent, the sole inefficiency that can arise, and that on which we first want to focus, is the under-taxation of capital. In Section 5, we allow the productivity of investment to vary between countries, thereby making some locations better than others, and introducing the possibility of an inefficient location.

### 3. Equilibrium of the simultaneous-move game

Consider first two special cases that are interesting and useful to understand.

In the first special case, there are no mobile capital owners, so \( M = 0 \). In this case, because there is no mobile factor over which the countries can fight, global tax revenue is maximized. In particular, governments set \( t_j = \hat{t} \) \( \forall j \), \( j = 1, \ldots, J \), and each country obtains revenue \( W_i(\hat{t}, 0) \), \( j = 1, \ldots, J \).

As a second special case, suppose there is some mobile capital but no local capital, so \( N_j = 0 \), \( j = 1, \ldots, J \). In such a case, competition will drive tax rates to zero, and no revenue will be generated in equilibrium. This equilibrium is obviously the worse possible outcome in this world.

Note that the allocation in these two special cases does not depend on the timing of the game or on the relative size of the countries. Obviously, in the absence of mobile capital owners, the timing is irrelevant since the decisions made by the countries are essentially independent. As for the case where there is only mobile capital, the equilibrium features zero revenue regardless of the timing.

The general case we now want to consider is one in which there are \( J \) countries differing in their number of local capital owners. Without loss of generality, suppose that \( N_1 \geq N_2 \geq \ldots \geq N_J \). Three preliminary results turn out to be useful. The proofs of all lemmas and propositions are in the Appendix.

**Lemma 1.** Country \( i \) never chooses a strategy \( t_i > \hat{t} \).

Note that \( \hat{t} \) is independent of the relative size of \( N_i \) and \( M \), so that the same upper bound on strategies applies to the countries whether they are identical or different. **Lemma 1** simply states that it does not pay to play a tax rate above \( \hat{t} \), because a lower tax rate can increase tax revenue and the likelihood of attracting mobile capital.

**Lemma 2.** Country \( i \) never chooses a strategy \( t_i < t_j \).

For a given tax rate \( t_j \in [\hat{t}, \hat{t}] \), a country is better off when all mobile capital is invested in it. If the tax rate is lower than \( \hat{t} \), the country prefers to drop off the race and at least get \( W_i(\hat{t}, 0) \). Recall that each country has a specific \( \hat{t} \) and from **Lemma 1** and **Lemma 2**, we now know that the relevant strategy space for country \( i \) is the subset of the real line \([\hat{t}, \hat{t}]\).

**Lemma 3.** The game has no pure strategy equilibrium.

To understand why there is no pure strategy equilibrium, consider an example in which there are only two countries, 1 and 2, with \( N_1 > N_2 \), implying that \( t_1 > t_2 \). From this last inequality, it is clear that country 2 can always undercut country 1. Yet, it is impossible to find a pair of tax rates \((t_1, t_2)\) which would constitute an equilibrium. For any \( t_1 \in [t_2, \hat{t}] \), country 2’s best response is to set \( t_2 \) to just undercut \( t_1 \) (to attract mobile capital). However, given such a \( t_2 \), country 1’s best response is to under cut country 2. For \( t_1 = \hat{t} \), country 2’s best response is again to set \( t_2 \) arbitrarily close to \( t_1 \) (to attract mobile capital). However, given such a \( t_2 \), country 1’s best response is to play \( \hat{t} \). Finally, for \( t_1 = \hat{t} \), country 2’s best response is to set \( t_2 \) arbitrarily close to \( \hat{t} \). However, given such a \( t_2 = \hat{t} \), country 1’s best response is to undercut country 2. Thus, such a game has no pure strategy equilibrium. The argument just developed can be extended to a game with \( J \) countries. In contrast, Janeba and Peters (1999) are able to solve for a pure strategy equilibrium by assuming that their tax revenue functions are such that \( t_j \) is larger than \( \hat{t} \). In our context, this would require sufficient homogeneity in the investment function across countries. Instead of making this assumption, however, we will face the challenge of investigating more complex equilibria.

We are now in a position to characterize the equilibrium of the game. Note that the framework developed in the current paper bears important similarities with that of an all-pay auction, i.e. an auction in which the highest bidder obtains the object for sale and, more importantly, in which all bidders pay their bid to the auctioneer. As it turns out, our results below have the flavor of those found in Baye et al. (1996), who characterize equilibrium in an all-pay auction.\(^{16}\) We here present the case in which \( N_1 > N_2 > \ldots > N_J \) because the general case with \( N_1 \geq N_2 \geq \ldots \geq N_J \) is heavy in terms of notation. However, we present the case in which \( N_1 = N_2 = \ldots = N_J \) in **Proposition 2**.

**Proposition 1.** In a world with \( J \) countries differing in their number of local capital owners (say \( N_1 > N_2 > \ldots > N_J \)), the game has a unique mixed strategy Nash equilibrium in which the equilibrium strategies are as follows.

\(^{14}\) Note that given the present formulation of the model, \( \hat{t} \) maximizes both \( W_i(t, 0) \) and \( W_i(t, 1) \).

\(^{15}\) The exact value of \( \hat{t} \) depends on the \( c(t) \) function. For example, if \( c(t) = t^2 \), then \( \hat{t} = \gamma \).

\(^{16}\) Also note that the equilibrium of our game is reminiscent of the equilibrium obtained in duopoly pricing games with capacity constraints, e.g. Levitan and Shubik (1972) and Kreps and Scheinkman (1983), Varian (1980) characterizes a similar equilibrium in his work on Bertrand price competition when some of the firms’ customers are captive. Dasgupta and Maskin (1986) examine the existence of equilibrium in general discontinuous economic games. They find the conditions under which the equilibrium involves a mixed strategy similar to those obtained here.
Countries \(j = 1, \ldots, J - 2\): play \(\hat{t}\) with probability \(q_1 = 1\).

Country \(J - 1\): with positive probability \(q_{J - 1} \in (0, 1)\), plays \(\hat{t}\); with positive probability \((1 - q_{J - 1})\), plays the interval \([\hat{t}_{J - 1}, \hat{t}]\) with continuous probability distribution \(H_{J - 1}(t)\), with:

\[
q_{J - 1} = 1 - \left[\frac{W'(\hat{t}, 1) - W'(\hat{t}_{J - 1}, 1)}{W'(\hat{t}, 0) - W'(\hat{t}, 1)}\right]
\]

\(H_{J - 1}(t) = \left[\frac{W'(\hat{t}, 1) - W'(\hat{t}_{J - 1}, 1)}{W'(\hat{t}, 0) - W'(\hat{t}, 1)}\right] \frac{W'(\hat{t}, 0) - W'(\hat{t}, 1)}{W'(\hat{t}, 1) - W'(\hat{t}_{J - 1}, 1)}\]

Country \(J\): plays the interval \([\hat{t}_{J - 1}, \hat{t}]\) with continuous probability distribution \(H_J(t)\), with:

\(H_J(t) = \frac{W'(\hat{t}, 1) - W'(\hat{t}_{J - 1}, 1)}{W'(\hat{t}, 0) - W'(\hat{t}, 1)}\)

To understand Proposition 1, first note that because \(N_1 > N_2 > \ldots > N_{J - 1} > N_J\), we have \(0 < \hat{t}_1 < \hat{t}_{J - 1} < \ldots < \hat{t}_2 < \hat{t}_1 < \hat{t}\). The ranking of the \(\hat{t}_s\) reflects the capacity of each country to undercut its opponents. This ranking has a straightforward implication: smaller countries can undercut larger countries. Indeed, the equilibrium described in Proposition 1 is one in which all countries but the two smallest ones \((J - 1)\) put themselves out of the race to attract mobile capital by taxing at rate \(\hat{t}\) with probability one. Country \(J - 1\) puts some mass \(< 1\) on \(\hat{t}\), but it also randomizes over the interval \([\hat{t}_{J - 1}, \hat{t}]\). Finally, country \(J\) randomizes on \([\hat{t}_{J - 1}, \hat{t}]\), and it never plays \(\hat{t}\). It follows from these strategies that mobile capital necessarily locates in country \(J - 1\) or \(J\) (the only countries really participating in the tax competition), and that mobile capital is never taxed at the revenue-maximizing tax rate \(\hat{t}\). Global revenue falls short of its maximum level because mobile capital precisely locates in the countries taxing capital at rates below \(\hat{t}\). Of course, there is a revenue loss also because immobile capital is taxed at a rates below \(\hat{t}\) in \(J\) (for sure) and \(J - 1\) (with probability \(1 - q_{J - 1}\)). In equilibrium, the expected payoff of all countries except \(J\) is equal to what they obtain when unable to attract mobile capital and taxing immobile capital at \(\hat{t}\), i.e. the expected payoff for \(j = 1, \ldots, J - 1\) is \(W'(\hat{t}, 0) = W'(\hat{t}, 0)\). The sole country that does better is country \(J\), the smallest one. It obtains and expected payoff of \(W'(\hat{t}_{J - 1}, 1) = W'(\hat{t}, 1)\), because \(\hat{t}_{J - 1} > \hat{t}\).

As for the uniqueness of the equilibrium, the intuition revolves around the fact that if another equilibrium was to exist, it would require the active participation of more than two countries in the race to attract mobile capital. But, as we show in the proof, if there are more than two active countries, then it is not possible to construct a mixed strategy equilibrium. Indeed, in such a case, it is impossible to ensure that all active participants are indifferent between the pure strategies they play with strictly positive probability. Note that equivalent uniqueness results are obtained in Hillman and Riley (1989) and Baye et al. (1996).

It is useful at this point to introduce a measure of the revenue loss from tax competition. There are of course several ways in which this could be done. We use what we think is a simple and natural measure, the \textit{ex ante} expected foregone tax revenue as a proportion of maximum tax revenue, and we denote it by \(\Phi\). Maximum revenue is obtained when all countries tax all capital at rate \(\hat{t}\). Thus, maximum revenue is \(M \bar{h}(\hat{t}) + \sum_{j \neq \hat{t}} W'(\hat{t}, 0)\). Further, we know from our characterization of the equilibrium that all countries obtain, in expected terms, \(W'(\hat{t}, 0)\), except for country \(J\), which obtains \(W'(\hat{t}_{J - 1}, 1)\). It follows that our measure \(\Phi\) is given by:

\[
\Phi = \frac{W'(\hat{t}, 1) - W'(\hat{t}_{J - 1}, 1)}{M \bar{h}(\hat{t}) + \sum_{j \neq \hat{t}} W'(\hat{t}, 0)}.
\]

It should be clear that although only two countries are effectively competing for mobile capital, all mobile capital is taxed at rates below the revenue-maximizing level, so our measure of expected revenue loss, \(\Phi\), grows larger when \(M\) increases relative to the \(N_J\). Note also that if the size of the economy was doubled (e.g. \(M\) and all the \(N_J\)s are doubled), then the equilibrium tax rates would not change. But the absolute value of foregone expected tax revenues would double, leaving \(\Phi\) unchanged. In next section, we will compare the expected revenue loss associated with tax competition under sequential play with that under simultaneous play.

The special case in which the \(J\) countries are identical yields some interesting insights.

**Proposition 2.** If the \(J\) countries are identical \((N_j = N, \forall j)\), the game has a large number of mixed strategy Nash equilibria. For any \(Q\) where \(0 < Q < J - 2\), there exists an equilibrium with \(Q\) countries playing \(t\) with probability \(1\), and \(J - Q\) countries playing \(t \in [\hat{t}, \bar{t}]\) according to the continuous cumulative function \(H(t)\) and density function \(h(t) = H'(t)\) on \([\hat{t}, \bar{t}]\). For \(t \in [\hat{t}, \bar{t}]\), the mixed strategy \(H(t)\) is given by:

\[
H(t) = 1 - \left[\frac{W(\hat{t}, 0) - W(t, 0)}{W(\hat{t}, 0) - W(\bar{t}, 0)}\right]^{1/(J - Q - 1)}.
\]

In equilibrium, the expected payoff of all countries is \(W(\hat{t}, 0)\).

The following points are worth mentioning. First, if there are more than two countries, then a positive number of them can be playing the revenue-maximizing tax rate, \(\hat{t}\), with probability one. Second, if there are only two countries, then both will play a lower tax rate with a probability approaching one (none will put mass on \(\hat{t}\)). Third, the equilibria are all equivalent in terms of expected revenue. Indeed, our measure of \textit{ex ante} expected revenue loss, \(\Phi\), in the particular context of Proposition 2, yields:

\[
\Phi = \frac{W(\hat{t}, 1) - W(\hat{t}, 0)}{M \bar{h}(\hat{t}) + J W(\hat{t}, 0)}.
\]

All equilibria entail the same \(\Phi\), as all countries obtain the same expected payoff \(W(\hat{t}, 0)\). Note that since, in the context of Proposition 1, country \(J\) does better than \(W(\hat{t}, 0)\), it follows that introducing some heterogeneity in the \(N_J's\) reduces expected revenue losses, as measured by \(\Phi\).

4. Equilibrium of the sequential-move game

We now examine the case in which countries play sequentially in the first stage of the overall game. Let \(\mathcal{J}\) be the set of countries, containing \(J\) countries, each indexed by \(j\), as was the case above. Without loss of generality, suppose that \(N_1 \geq N_2 \geq \ldots \geq N_{J - 1} \geq N_J\). From our discussion above, it must then be that \(\hat{t}_1 \geq \hat{t}_2 \geq \ldots \geq \hat{t}_{J - 1} \geq \hat{t}\). We assume that countries play sequentially, one after the other, but in an order that is independent of a country index \(j\). It is possible to envision that before the countries play, nature chooses with probability \(1/|\mathcal{J}|\) an order of play among the \(\mathcal{J}\) possible orders of play.

Before going further, it is useful to re-formulate our tie breaking rule for the case in which \(S\) countries have chosen the same lowest tax rate. Our assumption is that in such a case, all mobile capital \(M\) locates in the country with the largest index \(j\). For example, if countries 2, 3, and 7 have set the lowest tax rate, then \(M\) locates in country 7. Such an assumption reflects the fact that because \(\hat{t}_1\) is lower (not larger) for a higher index \(j\) (because it has a smaller \(N_j\)), the country with the highest index is that which could ultimately undercut every other countries.

In this sequential game, Lemmas 1 and 2 still hold, so for each country, equilibrium strategies must belong to the real line \([\hat{t}_j, \bar{t}]\). Let \(a_j = 1, \ldots, J - 1\), be an indicator function which takes a value of 1 if

\[\text{This is because the } \hat{t}_s \text{ do not change when } M \text{ and all the } N_j \text{s are doubled. It follows that the equilibrium remains the same.}\]
country \( j \) chooses its tax rate after country \( J \), and a value of 0 if it chooses it before. We denote by \( A \subset \mathcal{J} \) the set of countries who choose their tax rate after \( J \): \( A = \{ j \in \mathcal{J} | a_j = 1 \} \). The following can be obtained.

**Proposition 3.** If \( t_{\gamma} = \min \{ t_j, j \in \mathcal{J} \} \), then the subgame perfect equilibrium of the tax competition game is a strategy profile \( (t_{\gamma}, ..., t_{\gamma}) \) in which all countries play the revenue-maximizing tax rate \( (t_{\gamma} = t, \forall j \neq J) \), except for country \( J \), which plays \( t_J = \min \{ t_k | k \in A \} \), unless \( \langle A = \emptyset \rangle \). If country \( J \) plays last (\( A = \emptyset \)), then \( t_J = t \).

Thus, in all equilibria, mobile capital locates in country \( J \), the one which can undercut every other country. The presence of smallest country \( J \) disciplines all the larger countries, making it useless for them to enter into active tax competition and inducing them to maximize the revenue yield of the immobile base. But the tax rates the smallest country must play to attract mobile capital depend on the order of moves. The worst case scenario occurs when country \( J \) plays last (\( A = \emptyset \)). In this case, of course, \( t_J = t_{\gamma} \), and so \( t_J \) may be significantly smaller than \( t \). The revenue loss stemming from the under-taxation of capital may therefore be quite large. On the other hand, the best-case scenario occurs when country \( J \) plays last (\( A = \emptyset \)). In such a case, \( t_J = t \), so global tax revenue is maximized.

Clearly, the nature of the revenue loss in the sequential game is the same as that in the simultaneous game. Our results can therefore be viewed as being robust to changes in the timing of the game. However, there is only one country taxing capital below \( t \) in the sequential game, and two in the simultaneous game. Also note that in the sequential game, the equilibrium outcome is uncertain ex ante because of the uncertainty regarding the order of play, not because the countries play mixed strategies.

It turns out that calculating the appropriate measure of expected revenue loss in the sequential game in the most general case of \( J \) countries is fairly involved. However, we know that in this sequential game, all countries obtain \( W^s(t, 0) \) for all order of moves, except for \( J \) which, in the worst case scenario, when country \( J \) plays last after country \( J \), obtains a payoff of \( W^s(t_J - 1, 1) \) and does better for any other scenario in which country \( J \) plays before country \( J \). Using our loss measure, \( \Phi \), we can immediately recognize two points: (a) the level of loss in the worst case scenario of the sequential game is equal to the expected loss in the overall simultaneous game; and (b) this level of loss is less than the expected loss in the simultaneous game for any other scenario of the sequential game. Since the worst case scenario occurs with a probability less than one in the sequential game, it follows that there is less revenue loss on average in the sequential game than in the simultaneous game, a result that is intuitive. Note however that the equilibrium outcome of the simultaneous game could entail higher tax rates for some countries and, therefore, larger payoffs.

### 5.2. Preferential versus non-preferential regimes

We now turn to a comparison of preferential regimes – i.e. regimes in which competing countries can set different tax rates on bases of differing mobility – with non-preferential regimes – i.e. regimes in which tax rates are constrained to be the same on all bases. The main advantage of a preferential regime resides in the fact that governments can avoid losing tax revenue on their mobile tax bases by setting an appropriately high tax rate on them, while competing more aggressively on the more mobile ones. On the other hand, a non-preferential regime has the advantage of reducing competition on the mobile tax bases by tying them to the more immobile ones. In other words, a non-preferential regime makes it more costly for governments to lower their tax rates and so reduces harmful tax competition. Depending on the environment, one or the other regime may be desirable. Janeba and Peters (1999), in an environment entailing one perfectly mobile base and one perfectly immobile base, show that a non-preferential regime dominates a preferential regime. On the other hand, Keen (2001) obtains the opposite result when two bases are at least partially mobile. Wilson (2005) generalizes and attempts to reconcile these results within a unified framework.

It turns out that the framework developed in this paper can be used to contribute to this literature. We focus on the simple case in which all countries are equally productive, but will also discuss the impact of adding some heterogeneity toward the end. Since the equilibrium properties depend on whether countries set their tax rates simultaneously or sequentially, we have to study each case in turn.

In the case of a simultaneous game, the equilibrium tax rates for the non-preferential regime are the outcome of a mixed strategy Nash equilibrium in which countries choose their tax rates in the manner stated in **Proposition 1**. In equilibrium, the expected tax revenue of each country is given by \( W^s(t, 0) \) (the same amount they would have received in the non-preferential regime if they had not set different tax rates).

18 Note however that if a country’s productivity is too low (i.e. \( \gamma_j < \max \{ \gamma_i - t_i \} \)), then this country will simply be unable to attract mobile capital at a positive tax rate. Such a possibility is reminiscent of the analysis of Cai and Treisman (2005), in which countries of too low productivity are simply unable to compete for mobile capital.
obtain by maximizing tax revenue from their immobile base only) except for smallest country $J$ which does better on average, obtaining expected tax revenue $W(t_{j-1}) > W(t_{j})$. In the case of a preferential regime, the characterization of the equilibrium is a lot simpler. Since tax rates on mobile and immobile capital are disconnected, our framework can be viewed as a simple first-price auction. Thus, each country sets the revenue-maximizing tax rate $t$ on its immobile base, and competition drives the tax rate on the mobile base to zero. Tax revenue in that case is given by $W(t_{0})$ for all countries. It follows that in the case of a simultaneous game, a non-preferential regime dominates on average a preferential one, since at least one country (country $J$) earns higher ex ante expected tax revenue. Note that such a comparison is based on the ex ante difference in expected tax revenue. Of course, using ex post tax revenue may not yield the same results. For example, under the non-preferential regime, the two smallest (and active) countries are both potentially picking tax rates below $\hat{t}$. The one which ex post picks the lowest tax rate attracts the mobile capital and earns larger tax revenue that it would earn in the preferential regime. But the other country, that which fails to attract mobile capital, earns less tax revenue that it would earn under the preferential regime. Thus, ex post in a non-preferential regime, the winner's gain may or may not compensate for the loser's loss, despite the fact that from an ex ante perspective, the expected gain is clearly larger than the expected loss.

For the case of a sequential game, Proposition 3 establishes that in a non-preferential regime, all countries obtain $W(t_{0})$ except for country $J$ which obtains at least $W(t_{j-1}) > W(t_{j})$ (with a strict inequality if $N_{j} < N_{j-1}$) and even better in a potentially large number of order of moves. The analysis of the preferential regime in the sequential game is identical to that in the simultaneous case. All countries set a tax rate $t$ on immobile capital, but for the mobile base, intense competition implies that the unique equilibrium is where all countries set their tax rate at zero. The expected payoff for all countries is therefore $W(t_{0})$. Thus, because at least one country (country $J$) does better in the non-preferential regime, we again conclude that a non-preferential regime dominates a preferential one.

The presence of heterogeneity in productivity gives rise to new arguments in favour of preferential regimes. For a preferential regime with equally productive countries, tax rates on mobile capital are driven down to zero, and so the location decision of mobile capital becomes purely random. With productivity differences, the most productive country has an advantage, and is not constrained to offer a zero tax rate in order to attract mobile capital. Assuming that country $J$ is the most productive, and that country $J-1$ is the second most productive, we can define $\hat{t} = \gamma_{J-1} - \gamma_{J-2}$ as the highest possible tax rate country $J$ can set and still attract mobile capital, even if country $J-1$ has a zero tax rate. In any simultaneous or sequential equilibrium, country $J$ would pick tax rate $\hat{t}$, and mobile capital would locate in the most productive country. Turning to a non-preferential regime, recall from section 5.1 that mobile capital may locate inefficiently. It follows that non-preferential regimes are better at reducing the under-taxation of capital, but that preferential regimes are better at eliminating the inefficiency associated with the wrong location of mobile capital.

Without differences in productivity, our analysis confirms the main result from Janeba and Peters (1999) that a non-preferential regime generates more tax revenue. However, with productivity differences, such non-preferential regimes can lead to inefficiencies in the capital allocation across countries.

6. Conclusion

The current analysis could be extended in a few directions. First, we could assume that governments care about new investment not only because of the resulting rise in tax revenue, but also because of various external benefits such as employment gains in desirable occupations. Keen (2001) discusses such an extension in his analysis of preferential and non-preferential regimes, showing that his analysis can be generalized to encompass these additional benefits. Similarly, we may amend the objective function to read $W(t_{1}) = (t + b)N_{J} + M(t)$, where $b$ represents the external benefit per unit of investment. This extension reduces the tax rate that is optimal for a country in the absence of mobile capital: countries no longer wish to maximize revenue, because the investment loss resulting from a marginal increase in the tax rate now not only lowers the tax base, but also reduces the external benefits associated with investment. But the previous analysis goes through with $\hat{t}$ now redefined in this manner. In particular, countries other than the two smallest decide not to compete for capital and instead set their tax rates equal to this $\hat{t}$ (Proposition 1). The other results are similarly extended.

Alternatively, the objective function may be specified as a weighted sum of tax revenue and the producers’ surplus received by the suppliers of capital to a country. This extension recognizes that higher tax rates harm capital owners by reducing their income from capital. Once again, the analysis goes through with $\hat{t}$ reduced below its revenue-maximizing level to reflect this harm. Presumably, the weight given to producers’ surplus would reflect the political influence of capital owners, perhaps through lobbying activities. A more complex extension would be to allow the benefits of additional capital to differ between mobile and immobile capital. This asymmetry complicates the calculation of mixed strategies and is therefore left to future research.

Two other extensions appear to us as likely to generate interesting results. The first one would be to introduce labor and political economy considerations in the analysis. Suppose that workers in each country benefit from the presence of productive capital because of the associated larger output and wages, but also because capital is taxed to finance the provision of a public good. Then, if capital is highly mobile and unevenly owned by the workers of various countries, the choice of tax rates on capital in a given country will be driven by strategic international considerations, as in the current paper, but also by the distribution of capital ownership within the country.

A second extension of the current analysis would be to use the framework of Section 4 as the within-period game of a multi-period dynamic game. To simplify, assume that both investors and governments are myopic. Also assume that the countries have the same productivity, but that they differ in terms of their number of local capital owners. Further, suppose that $M_{t}$ new mobile investors are born each period and that the location decision they make at that time is irreversible — in effect, mobile investors locate and transform themselves into local capital investors. Hence, suppose that at time $t$, the countries have local capital $(N_{1}, \ldots, N_{t-1}, N_{t})$. Then, from our previous analysis, and whatever the order of moves within period $t$, if country $i$ is that with the smallest amount of local capital investors, capital investors $M_{t}$ then end up locating in country $i$ at time $t$. Assuming investors are infinitely-lived, it follows that at time $t+1$, the countries will have local capital investors $(N_{t+1} = N_{t}, \ldots, N_{t-2} = N_{t}, \ldots, N_{0} = N_{0})$. Of course, it will again be the country with the smallest number of local capital investors that will attract mobile capital investors $M_{t+1}$. If this process continues, the smaller countries will become larger — while the

---

19 Recall that in the case of a non-preferential regime, in which tax rates are tied, our framework can be interpreted as an all-pay auction.
20 Wilson (1996) and Wildasin and Wilson (1996) construct dynamic models in which factors are freely mobile when they decide on their initial location, but then become partially immobile once they move there. In the first paper, the factor is the capital owned by infinitely-lived firms, whereas an overlapping-generations model with mobile labor is considered in the second paper. Both papers focus on only preferential regimes, whereas our ongoing research involves a dynamic analysis of non-preferential regimes.
large ones will stagnate – and all countries will evolve to be approximately of the same size.\textsuperscript{21} Thus, the environment considered in this paper can generate convergence in the amount of capital located in all countries. However, such a convergence does not seem to be happening in the real world. We speculate that if investors and/or governments were forward-looking – instead of being myopic – then convergence would not necessarily obtain. These extensions of the current paper will be examined in future work.

Acknowledgements

We have benefited from the comments of two referees and from our discussions with Nicolas Bocard, Claude Fluet, Richard Harris, Gordon Myers, Frédéric Rychen, Nicolas Sahuguet, and Michael Smart. We also thank seminar participants at HEC-Montréal, Queen’s University, Simon Fraser University, Université de Cergy-Pontoise, Université de Lille 3, University of Oregon, Canadian Public Economics Group, International Institute of Public Finance, Journées du CIRPÉE, and Les Journées Louis-André Gérard-Varet for their comments. Financial support from FQRSC, RIIM, and SSHRCC is gratefully acknowledged.

Appendix A. Proofs

A.1. Proof of Lemma 1

For any \( t^*_i > \hat{t} \), there exists a \( t^*_i < \hat{t} \) such that \( W^i(t^*_i, m) = W^i(t^*_i, m) \), for \( m \in [0, 1] \). Of course, since \( t^*_i \), the country is more likely to attract the mobile capital, it will always prefer to play \( t^*_i \). QED.

A.2. Proof of Lemma 2

If a country plays \( t_i < \hat{t}_i \) and all the mobile capital locates on its territory, it will get a payoff which is less than what it gets when it taxes at rate \( \hat{t} \) and no mobile capital locates on its territory: \( W^i(t_i, 1) < W^i(\hat{t}_i, 0) \) for \( t_i < \hat{t}_i \). QED.

A.3. Proof of Lemma 3

We already assessed that for any \( N_i \geq N_j \), it must be that \( \hat{t}_i \geq \hat{t}_j \). As Lemma 1 and Lemma 2 apply for any \( N_i \geq N_j \), it follows that the strategies of the countries must belong to the following intervals: \( t_i \in [\hat{t}_i, \hat{t}_i] \) and \( t_j \in [\hat{t}_j, \hat{t}_j] \).

We first show that there is no symmetric (\( t = t_j \)) pure strategy Nash equilibrium and then show that there is no asymmetric (\( t \neq t_j \)) pure strategy Nash equilibrium.

(i) There is no symmetric (\( t = t_j \)) pure strategy Nash equilibrium.

Since \( \hat{t}_j \leq \hat{t}_i < \hat{t} \), a symmetric equilibrium is a pair (\( t, t \)) such that \( t \in [\hat{t}_i, \hat{t}_i] \). Consider such a strategy profile (\( t, t \)). If \( t > \hat{t}_j \), then the payoff of country \( i \) is \( W^i(t, 1) + \frac{t}{2} W(0, 0) \) and that of \( j \) is \( W^j(t, 1) + \frac{t}{2} W(0, 0) \). Clearly, this cannot be an equilibrium as any country, say \( i \), has an incentive to deviate to \( t' = \hat{t}_j \) causing all the capital to locate in \( i \), and ensuring itself a payoff \( W^i = W^i(t - \epsilon, 1) > W^i \).

If \( t = \hat{t}_j \), then the payoff of country \( i \) is \( W^i(\hat{t}_j, 1) + \frac{1}{2} W^i(\hat{t}_j, 0) \) and that of \( j \) is \( W^j(\hat{t}_j, 1) + \frac{1}{2} W^j(\hat{t}_j, 0) \). Clearly, this cannot be an equilibrium as \( i \) has an incentive to deviate to \( t' = \hat{t}_j \) ensuring itself a payoff \( W^i = W^i(\hat{t}_j, 0) > W^i \).

(ii) There is no asymmetric (\( t \neq t_j \)) pure strategy Nash equilibrium.

Without loss of generality, take the case of \( N_1 \geq N_2 \) so that \( \hat{t}_2 \leq \hat{t}_1 < \hat{t} \).

Consider a strategy profile \((t_1, t_2)\) with \( \hat{t}_2 \leq t_1 \leq \hat{t}_2 \leq \hat{t} \). Given those strategies, \( W^1 = W^1(t_1, 1) \) and \( W^2 = W^2(t_2, 0) \). Then, 2 has an incentive to deviate to \( t_2' = t_1 - \epsilon \) to obtain \( W^2 = W^2(t_2 - \epsilon, 1) > W^2 \).

Consider a strategy profile \((t_1, t_2)\) with \( \hat{t}_2 \leq t_2 \leq \hat{t}_1 \leq t_1 < \hat{t}_2 \). Given those strategies, \( W^1 = W^1(t_1, 0) \) and \( W^2 = W^2(t_2, 1) \). Then, 1 has an incentive to deviate to \( t_1 = t_2 - \epsilon \) to obtain \( W^1 = W^1(t_2 - \epsilon, 1) > W^1 \).

Consider a strategy profile \((t_1, t_2)\) with \( \hat{t}_2 \leq t_2 \leq \hat{t}_1 < \hat{t} \) and \( t_1 < \hat{t}_2 \). Given those strategies, \( W^1 = W^1(t_1, 0) \) and 1 has an incentive to deviate to \( t_1 = \hat{t}_1 \) to obtain \( W^1 = W^1(\hat{t}_1, 0) > W^1 \).

Consider a strategy profile \((t_1, t_2)\) with \( \hat{t}_2 \leq t_2 \leq \hat{t}_1 < \hat{t}_1 \leq \hat{t}_2 \). Given those strategies, \( W^2 = W^2(t_2, 1) \) and 2 has an incentive to deviate to \( t_2 = \hat{t}_2 - \epsilon \) to obtain \( W^2 = W^2(\hat{t}_2 - \epsilon, 1) > W^2 \) for \( \epsilon \) small.

The generalization to the case of \( J \) countries with \( N_1 \geq N_2 \geq \ldots \geq N_J \) is tedious but straightforward. This completes the proof. QED.

A.4. Proof of Proposition 1

Recall that because \( N_1 > N_2 > \ldots > N_{J-1} > N_J \), we have \( 0 < \hat{t}_J < \hat{t}_J - 1 < \ldots < t_2 < \hat{t}_1 < \hat{t} \). Also recall that the equilibrium strategies are:

\( \diamond \) Country \( J - 1 \): With positive probability \( q_{J-1} \in [0, 1] \), plays \( t \); with positive probability \( 1 - q_{J-1} \), plays the interval \([\hat{t}_{J-1}, \hat{t}]\) with continuous probability distribution \( H_{J-1}(t) \), with:

\[ q_{J-1} = 1 - \frac{W^J(\hat{t}_J) - W^J(\hat{t}_{J-1})}{W^J(\hat{t}_J) - W^J(\hat{t})} \]

\[ H_{J-1}(t) = \frac{[W^J(\hat{t}_J) - W^J(\hat{t}_{J-1})]W^J(\hat{t}_J) - W^J(\hat{t})}{[W^J(\hat{t}_J) - W^J(\hat{t})][W^J(\hat{t}_J) - W^J(\hat{t}_{J-1})]} \]

\( \diamond \) Country \( J \): Plays the interval \([\hat{t}_{J-1}, \hat{t}]\) with continuous probability distribution \( H_J(t) \), with:

\[ H_J(t) = \frac{W^{J-1}(\hat{t}) - W^{J-1}(\hat{t}_J)}{W^{J-1}(\hat{t}) - W^{J-1}(\hat{t}_{J-1})} \]

We first show that these strategies are actually equilibrium strategies, and we then show that the equilibrium is unique.

The above strategies are such that all countries except the two smallest ones (\( J - 1 \) and \( J \)) put themselves out of the race to attract mobile capital by taxing at rate \( \hat{t} \) with probability one. Mobile capital locates in country \( J - 1 \) or \( J \).

In equilibrium, the expected payoff of all countries (except \( J \)) is equal to what they obtain when unable to attract mobile capital and taxing immobile capital at the revenue-maximizing tax rate, \( \hat{t} \), i.e. the expected payoff for \( J = 1, \ldots, J - 1 \) is \( W^J(\hat{t}_J) = W^J(\hat{t}) \). The sole country which does better is country \( J \), the smallest one. It obtains an expected payoff of \( W^J(\hat{t}_{J-1}) > W^J(\hat{t}_J) = W^J(\hat{t}) \).

The proof that these strategies constitute an equilibrium is simply that given the other countries’ strategy, country \( J \) has no desire to deviate.

To determine \( q_{J-1}, H_{J-1}(t) \), and \( H_J(t) \), the procedure is as follows.

(A) Consider first the payoffs for country \( J \) for some of its pure strategies, given the strategy of country \( J - 1 \). Note that since the other countries always play \( \hat{t} \), they have no impact on the payoff of country \( J \).

\textsuperscript{21} The difference between the size of the largest country and that of the smallest of course depends on the size of the elements of the sequence \( \{M_i, M_{i+1}, \ldots\} \).
A.1 When country $J$ plays $\hat{t}_J - 1$, it obtains $W^J(\hat{t}_J - 1, 1)$:

$$q_J - 1 W^J(\hat{t}_J - 1, 1) + (1 - q_J - 1) \left[ H_{J - 1}(\hat{t}_J - 1) W^J(\hat{t}_J - 1, 0) \right] + (1 - H_{J - 1}(\hat{t}_J - 1)) W^J(\hat{t}_J - 1, 1) = W^J(\hat{t}_J - 1, 1).$$

A.2 For any $t \in [\hat{t}_J - 1, \hat{t}_J]$, country $J$ obtains:

$$q_J - 1 W^J(t, 1) + (1 - q_J - 1) \left[ H_{J - 1}(t) W^J(t, 0) + (1 - H_{J - 1}(t)) W^J(t, 1) \right].$$

Setting this last expression equal to $W^J(\hat{t}_J - 1, 1)$, to ensure that all pure strategies yield the same payoff, we can solve for $H_{J - 1}(t)$:

$$H_{J - 1}(t) = \frac{W^J(t, 1) - W^J(\hat{t}_J - 1, 1)}{(1 - q_J - 1) (W^J(t, 1) - W^J(t, 0))}.$$

It is easily checked that $H_{J - 1}(\hat{t}_J - 1) = 0$. Using the fact that $\lim_{t \to H_{J - 1}}(t) = 1$, we can solve for $q_{J - 1}$ and obtain:

$$q_{J - 1} = \frac{[W^J(\hat{t}_J - 1, 1) - W^J(\hat{t}_J - 1, 1)]}{W^J(\hat{t}_J - 1, 1) - W^J(\hat{t}_J - 1, 0)}.$$

Substituting this value of $q_{J - 1}$ in $H_{J - 1}(t)$ above, we get the following:

$$H_{J - 1}(t) = \frac{[W^J(t, 1) - W^J(\hat{t}_J - 1, 1)] [W^J(\hat{t}_J - 1, 1) - W^J(\hat{t}_J - 1, 0)]}{W^J(t, 1) - W^J(\hat{t}_J - 1, 1) [W^J(\hat{t}_J - 1, 1) - W^J(\hat{t}_J - 1, 0)]}.$$

And it is easily checked that $H_{J - 1}(\hat{t}_J - 1) = 0$ and $\lim_{t \to H_{J - 1}}(t) = 1$.

(B) Consider now the payoffs for country $J - 1$ for any of its pure strategies, given the strategy of country $J$ and that of the other countries.

For any $t \in [\hat{t}_J - 1, \hat{t}_J]$, country $J - 1$ obtains:

$$H_J(t) W^{J - 1}(t, 0) + (1 - H_J(t)) W^{J - 1}(t, 1).$$

In equilibrium, this last expression must equal $W^{J - 1}(t, 0)$, and we can solve for $H_J(t)$:

$$H_J(t) = \frac{W^{J - 1}(t, 1) - W^{J - 1}(t, 0)}{W^{J - 1}(t, 1) - W^{J - 1}(t, 0)}.$$

Note that given $H_J(t)$, country $J - 1$ is indifferent between all its pure strategies (it always obtains $W^{J - 1}(t, 0)$). In particular, country $J - 1$ obtains the same expected payoff for any value of $q_{J - 1}$. Country $J - 1$ is therefore indifferent between putting and not putting some mass on $t$. Since country $J - 1$ does not put mass $q_{J - 1}$ on $\hat{t}$, then country $J$ gets $W^J(\hat{t}_J - 1, 1) > W^J(\hat{t}_J, 1)$.

We now show that the equilibrium is unique. Obviously, no other equilibrium exists in which only two countries (other than $t$ and $J - 1$) are playing tax rates below $\hat{t}$ with positive probability. We now show that no equilibrium exists in which more than two countries play strategy $t$ with probability $q_{J - 1}$. We first consider the case in which the three smallest countries are active, and then extend our argument to the case in which the $Q (2 < Q \leq J)$ smallest countries are active.

The case with three active countries: Imagine that in addition to countries $J$ and $J - 1$, country $J - 2$ also plays tax rates below $\hat{t}$ with positive probability. By playing $t$ with probability one, any country $J$ can always secure a payoff $W^J(\hat{t}_J, 0)$. If country $J - 2$ is willing to play a tax rate below $\hat{t}$ with positive probability, then it must be that the associated expected payoff is at least $W^{J - 2}(\hat{t}_J, 0)$. If country $J$ or $J - 1$ are playing some tax rates less than $\hat{t}_J - 2$ in equilibrium, then, with positive probability, country $J - 2$ does not attract mobile capital when playing $\hat{t}_J - 2$, and obtains an expected payoff which is less than $W^{J - 2}(\hat{t}_J - 2, 1) = W^{J - 2}(\hat{t}_J, 0)$. Country $J - 2$ is therefore better off playing $\hat{t}_J$ with probability one. Thus, the only configurations that remain possible are those in which all three countries play tax rates in $[\hat{t}_J - 2, \hat{t}_J]$. Then, countries $J$ and $J - 1$ earn an expected payoff larger than $W^J(\hat{t}_J, 0)$ when playing $\hat{t}_J - 2$. Now let $q_J = J = J - J - J$. This is the probability with which country $J$ plays $\hat{t}_J$. Clearly, it must be that $q_{J - 1} = q_{J - 0} = 0$ since otherwise, this two countries would earn a larger payoff when playing $\hat{t}_J - 2$ than when playing $\hat{t}_J$. Indeed, when playing $\hat{t}_J$, there would be a chance of not attracting mobile capital because other countries have $q_J > 0$. Thus, because in a mixed strategy equilibrium, all pure strategies played with positive probability must earn the same payoff, it must be that $q_{J - 1} = q_{J - 0} = 0$.

A.5. Proof of Proposition 2

We present the proof for the case of two identical countries. The case of $J - 2$ countries is a straightforward extension.

If the countries have the same number of local capital owners ($N = N_J$), Proposition 2 states that the game has a symmetric mixed strategy Nash equilibrium in which the two countries play $t \in [\hat{t}, \hat{t}]$.
according to the continuous cumulative function $H(t)$ and density function $h(t) = H'(t)$ on $[\tilde{t}, \hat{t}]$. For $t \in [\tilde{t}, \hat{t}]$, the mixed strategy $H(t)$ is given by:

$$H(t) = \frac{W(t, 1) - W(t, 0)}{W(\hat{t}, 1) - W(\hat{t}, 0)}.$$ 

In equilibrium, the expected payoff of the two countries is $W(\hat{t}, 0)$.

We show that when $j$ plays the mixed strategy $H(t)$, $i$ has no incentive to deviate from $H(t)$.

Suppose $j$ plays the mixed strategy $H(t)$. Then, if $i$ plays $t'$, $m_i = 0$ with probability $H(t')$ and $m_i = 1$ with probability $1 - H(t')$.

Before solving for the mixed strategy equilibrium, first note that there are no point masses in equilibrium when there are only two identical countries. The intuition is simple: if the level of tax $t'$ was played with positive probability, there would be a tie at $t'$ with positive probability. Imagine then that country $j$ decides to play $t' - \epsilon$ (instead of $t'$) with the same probability. The cost of such a deviation would be of the order of $\epsilon$, but if the two countries were to tie, then country $j$ would gain a fixed positive amount. The formal proof of this is as follows. Imagine that country $i$ plays $t'$ with positive probability $\omega$, and country $j$ deviates from $t'$ to $t' - \epsilon$ with the same positive probability. The payoff for country $j$ will change by a factor of:

$$\left(1 - \frac{\epsilon}{\omega} H(t' - \epsilon) - \frac{\epsilon}{\omega} H(t') \right).$$

The first terms in curly brackets represent the difference between losing with a tax level $t' - \epsilon$, and losing with a tax level $t'$. As for the second terms in curly brackets, they represent the difference between winning with a tax level $t' - \epsilon$, and winning with a tax level $t'$. It is easy to see that the sum of those terms goes to zero when $\epsilon$ goes to zero. Now, the last terms in curly brackets represent the difference between winning alone with $t' - \epsilon$, and sharing the win with $t'$. Since the sum of these terms is strictly positive when $\epsilon$ goes to zero, it pays to deviate to $t' - \epsilon$ when there is a probability mass at $t'$. This implies that $H(t)$ cannot have a probability mass. And because the cumulative function is continuous, cases in which the countries play $t_i = t_j$ (a tie) occur with probability 0.

We now solve for $H(t)$ knowing that it must be continuous on $[\tilde{t}, \hat{t}]$. Thus, given $j$ plays $H(t)$, when $i$ plays the mixed strategy $H(t)$, its expected payoff is:

$$\int_{\tilde{t}}^{\hat{t}} (H(z)W(z, 0) + (1 - H(z))W(z, 1))dz.$$ 

For $(H(t), H(t))$ to be a mixed strategy Nash equilibrium, it has to be that all pure strategies played with positive probability yield the same payoff. We construct the equilibrium so that the expected payoff of the two countries is $W(\hat{t}, 0)$. Thus, it has to be that:

$$H(t)W(t, 0) + (1 - H(t))W(t, 1) = W(\hat{t}, 0).$$

It follows that for $t \in [\tilde{t}, \hat{t}]$, $H(t)$ is given by:

$$H(t) = \frac{W(t, 1) - W(\hat{t}, 0)}{W(\hat{t}, 1) - W(\hat{t}, 0)}.$$ 

When $j$ plays the mixed strategy $H(t)$, $i$ has no incentive to deviate from $H(t)$ because:

- Changing the probability of playing any $t \in [\tilde{t}, \hat{t}]$ would not affect its payoff as all pure strategies are equivalent by construction.
- Playing $t \in [0, \tilde{t}]$ or $t \in [\hat{t}, \infty]$ with positive probability would decrease $i$'s expected payoff as these strategies are all dominated (Lemma 1 and Lemma 2).

This completes the proof for the case of two identical countries. It is easily shown that for the case of $J > 2$ countries, either all countries play a modified $H(t)$ given by

$$H(t) = 1 - \left(\frac{W(t, 0) - W(\hat{t}, 0)}{W(\hat{t}, 1) - W(\hat{t}, 0)}\right)^{1/(J - 1)}.$$ 

or some of them ($Q \leq J - 2$) put a unit mass on $t$. QED.

A6. Proof of Proposition 3

(A) We first study the case of two countries, 1 and 2, with $N_1 \geq N_2$. We start by examining the case in which country 2 plays first. In that case, the game has a pure perfect Nash equilibrium in which country 2 sets $t_2 = t_1$ and country 1 sets $t_1 = \hat{t}$. In equilibrium, mobile capital locates in 2 and the payoff of country 1 is $W(\hat{t}, 0)$ while that of country 2 is $W(\hat{t}, 1) > W(\hat{t}, 0)$.

To see that this must be true, note that because country 2 has a lower $N_2$, it has a lower $t_1 < t_2$. Consequently, country 2 can always and does undercut country 1 by setting $t_2 = t_1$ (recall our breaking rule). Country 1 then chooses the best tax rate available given it is unable to compete, i.e. the tax rate it chooses when isolated: $\hat{t}$.

Consider now the case in which country 1 plays first. In that case, the game has a pure perfect Nash equilibrium in which both countries play $\tilde{t}$. In equilibrium, mobile capital locates in 2 and the payoff of countries 1 is $W(\tilde{t}, 0)$ while that of country 2 is $W(\tilde{t}, 1)$). To see that this must be true, recall that country 1 can always be undercut by country 2. Country 1 thus sets $t_1 = \tilde{t}$ and country 2, benefiting from the breaking rule, plays $t_2 = \tilde{t}$.

This completes part (A) of the proof.

(B) The generalization of (A) to the case of $J$ countries with $N_1 \geq N_2 \geq \cdots \geq N_J$ is straightforward. QED.

References


