INTERNATIONALLY DIVERSIFIED PORTFOLIOS:
WELFARE GAINS AND CAPITAL FLOWS

By Herbert G. Grubel*

The models of portfolio balance developed by Markowitz [5] and Tobin [8] explain the real world phenomenon of diversified asset holdings elegantly and properly. The models have been criticized, extended, and empirically tested; by now their basic content has become economic orthodoxy. Strangely, however, the analysis has not yet been applied explicitly to the explanation of long-term asset holdings that include claims denominated in foreign currency.¹

The present paper fills this gap and yields some interesting results. First, the international diversification of portfolios is the source of an entirely new kind of world welfare gains from international economic relations, different from both the traditional "gains from trade" and increased productivity flowing from the migration of the factors of production. This specific theoretical proposition is illustrated with some calculations based on empirical data drawing on ex post realized rates of return from investment in 11 major stock markets of the world.

Second, the theoretical model shows that international capital movements are a function not only of interest rate differentials but also of rates of growth in total asset holdings in two countries. As a result, capital may flow between countries when interest rate differentials are zero or negative and may not flow when a positive interest differential exists. Third, the analysis has some important policy implications in a growing world where monetary and fiscal policies are mixed to achieve internal and external balance.

I. The Static Model

Consider a world consisting of two countries, A and B, each with independent monetary and fiscal authorities and initially economically isolated from each other. Populations, income, and wealth are constant through time. There are only three forms of holding wealth: real assets, money, and bonds. The latter are issued by the government to provide investors with an interest-bearing instrument that allows bridging individuals' periods of net savings and dissavings over their lifetimes. In addition, the quantity of bonds in the market and the interest rate they

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¹ The importance of the real world phenomenon is exemplified by the recent report in [10].
fetch are regulated by the government in such a manner as to maintain full employment. For example, if there is unemployment, the government purchases bonds, paying for them with newly issued money. As a result of the increased money holdings and the lower yield of bonds, real assets are relatively more attractive than money and bonds and individuals try to adjust their portfolio imbalance through the purchase of more real assets, which has the desired upward effect on employment.

Assume that initially domestic portfolio balance exists at interest rates on bonds of $R_A$ and $R_B$, and variances and covariances of returns of $\sigma^2_A$, $\sigma^2_B$, $\sigma_{A,B}$, where the subscripts $A$ and $B$ refer to the two countries and are measured from the point of view of Country $A$. That is $R_B$, $\sigma^2_B$, and $\sigma_{A,B}$ include an adjustment for exchange risk stemming from past variations in some shadow price of foreign exchange. Furthermore, assume for analytical convenience that when economic relations between the two countries are opened up only bonds and consumer goods can be exchanged so that the opening of trade does not affect the return and variance from holding real assets and money. Consequently, attention can be focused on the changes in bond holdings resulting from the opening of trade.

Before trade the expected rate of return $E(R_A)$ and risk $V(R_A)$ on the “average” investor's bond portfolio in Country $A$ and $B$ are:

1. $E(R_A) = R_A$
2. $V(R_A) = \sigma_A^2$
3. $E(R_B) = R_B$
4. $V(R_B) = \sigma_B^2$

After diversification a portfolio containing bonds of both Countries $A$ and $B$ has the following expected rate of return:

$E(R_{A,B}) = P_A R_A + P_B R_B$

Where $P_A$ and $P_B$ are the proportions of bonds of country $A$ and $B$ respectively held in the average portfolio of Country $A$, $P_A$ plus $P_B$ must sum to one and neither may be negative. The variance of the diversified portfolio is

$V(R_{A,B}) = P_A^2 \sigma_A^2 + 2 P_A P_B \sigma_{A,B} + P_B^2 \sigma_B^2$

As the two equations show, investors have the opportunity to choose from a whole range of combinations of expected rates of return and variance by picking the appropriate sizes of $P_A$ and $P_B$. Which specific combinations of risk and return they choose depends on their personal preferences, as has been demonstrated by Markowitz [5] and Tobin [8].

While the exact diversification is not important for the present pur-
poses of analysis, it is useful to demonstrate with the help of a numerical example that diversification results in portfolios superior to one-asset portfolios of either kind of bonds. Assume that \( R_A = R_B = 5 \) per cent. Therefore, before trade, \( E(R_A) = E(R_B) = 5 \). Diversification of the nature \( P_A = P_B = .5 \) yields an expected rate of return:

\[
E(R_{A,B}) = E(R_{B,A}) = 5
\]

Assume that the variances of expected returns on Country A and B's properly adjusted for exchange rate fluctuations are \( \sigma_A^2 = \sigma_B^2 = 10 \), with a correlation between the two rates of return of \( r = .3 \). The variances on undiversified portfolios are \( V(R_A) = V(R_B) = 10 \) but the variance on the portfolio containing both assets is

\[
V(R_{A,B}) = 6.5
\]

Thus, holding both assets does not change the expected rate of return but does reduce the riskiness of the portfolio as compared with the one-asset portfolio. By similar calculations and data it can be shown that the exchange of financial assets can lead to higher expected rates of return with equal risks and other combinations of returns and risks, all of which are superior to those from undiversified portfolios and, therefore, make the holders of wealth better off than they were without the opportunity for international diversification. The same principles apply to the residents of Country A and Country B.

The quantity of foreign bonds demanded by the residents of Country A and Country B after the opening of trade in this model depends on five primary factors. First, the size of total wealth assets held by the public: Since the variables \( P_A \) and \( P_B \) represent proportions, the absolute size of bond holdings is greater the greater the stock to which these proportions are applied.

Second, the size of the interest rate differential: Given the variances and covariance of the two-asset returns for any risk avoider, the trade-off between return and risk is more favorable the greater the foreign interest rate and, therefore, the more of the foreign asset will be held in the portfolio. Third, the size of the risk differential: For a given earnings differential and covariance of returns the foreign asset is more attractive the smaller the risk attached to it, given the riskiness of the domestic asset.

Fourth, the degree of correlation of returns on domestic and foreign assets: As can be seen from equation (6) the variance of a diversified

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2 The discussion of circumstances under which diversification does not take place, i.e., investors are risk lovers, domination of assets, perfect correlation of returns, etc., go beyond the scope of this paper and it is assumed that investors and assets in both countries meet the necessary requirements for diversification to take place.
portfolio is smaller the smaller the correlation of returns. Thus, given the earnings differential and variance of each asset independently, diversification reduces portfolio variance more and, therefore, is more desirable the smaller the covariance. Fifth, the tastes of the public: The combination of risk and return actually chosen from among the combinations made possible by diversification depends on wealth holders' preferences with respect to risk and return and current vs. future consumption.

Given the magnitudes of the five determinants of the demand for foreign bonds, the opening up of economic relations between the two countries is assumed to lead to a mutual exchange of bonds by the private wealth holders. Only if tastes, returns, variances and relative sizes of total wealth holdings are equal are the demands generated by each country equal. In the following analysis the empirically most relevant and theoretically most interesting assumption is made that the potential demand for foreign bonds by the residents of Country A exceeds that by the residents of Country B. The real effects of such a net excess demand are analyzed first, under the assumption of rigidly pegged exchange rates, and second, under the assumption of perfectly flexible exchange rates.

First, at pegged rates Country A's excess demand for bonds tends to depress its exchange rate and official sales of B's currency are required to keep it stable. We assume that the government of A obtains this foreign exchange from the government of B in return for its official IOU's.

When international relations are opened, the private residents of A sell off some of the bonds issued by their own government and acquire those of Country B. Some of A's bonds thus offered are purchased by residents of Country B, but under the present assumption of an excess demand for bonds by Country A, that government must purchase some of its old obligations to maintain aggregate portfolio balance and full employment. At the same time B's government issues a net supply of new bonds to the residents of A.

All of these changes in the balance sheets of governments and private wealth holders are completed a certain time after opening of international relations. The length of the adjustment depends on institutional arrangements in the bond markets and is not important for the present analysis. In the new equilibrium the excess demand for foreign exchange ceases and along with it the need for official intervention.\textsuperscript{3} The excess demand by Country A's residents has caused the government of A to be indebted to the government of B rather than to its own citizens. B's

\textsuperscript{3} The lower the risk or higher the return on bond holdings in both countries, the more likely an increase in the total demand for assets. Under these circumstances, savings will increase, causing a fall in the interest rate and requiring changes in employment policy. We neglect these effects by assuming that they are likely to be small. On the same grounds we disregard balance of payments and income problems arising from net interest payments.
government finds its obligation to A's private citizens matched by claims on A's government.

At no time between the two points of asset equilibrium did the exchange rate move and since full employment in both countries has been maintained there have been no income or price effects on the balance of trade and no real resources transferred between the two countries. The new pattern of asset holdings involves a net transfer of resources only if the interest rate on the official IOU's issued by government A and held by government B is different from that paid on the bonds issued by government B and held by the public in A, assuming equal liquidity and other service yields on each type and assuming equal taxation rates.

Second, under flexible exchange rates the net demand for B's bonds causes a lowering of A's exchange rate, the appearance of a balance of trade surplus for A, which persists until real resources equal in value to A's excess demand for bonds is realized. Then the exchange rate returns to its previous level under the present assumptions of a static world.

Assuming that neither government changes the quantity of its bonds outstanding, the net demand for B's bonds from the residents of A tends to raise the prices and lower the yields on B's bonds, inducing the residents of B to substitute real assets transferred from A for these bonds in their portfolios. There is a tendency for the return on real capital to fall in B and rise in A, reducing what ceteris paribus would have been the net excess demand for bonds in A. However, given the other determinants of this demand, total asset holdings and tastes, there is no necessity for this net asset demand to be moved to zero.

As long as the interest rate paid by Country B on the bonds held by the residents of A is equal to the marginal productivity of the resources transferred to B, the real income in both countries is the same as before the opening of international relations, except for the welfare gains accruing to the wealth holders from the diversification of their portfolios.

The model just presented gives rise to the possibility that real capital flows away from the country with the higher to the one with lower physical productivity of capital. Such an event occurs if the size of total asset portfolios in Country A is greater than that in Country B so that even at the initial interest rate differential in favor of A a net demand for B's bonds is created. Under flexible exchange rates these conditions result in a transfer of real resources to Country B through the process described in the preceding paragraphs.

It is clear that the welfare gains accruing to wealth holders through international diversification of their portfolios are different in nature from those known from the traditional literature in international economics, i.e., the Ricardo-Heckscher-Ohlin gains from trade and the classical gains from factors moving to higher productivity employment.
II. Some Empirical Estimates of Potential Gains From Diversification

In order to demonstrate the range of possible gains to American investors from international diversification of their portfolios, information on rates of return from portfolio-investment in common stock market averages of 11 major countries (see Table 1) was collected, covering the

<table>
<thead>
<tr>
<th>Country</th>
<th>Per cent Value of $100 Standard Correlation</th>
<th>Value of $100 Standard Correlation</th>
<th>Correlation (R) with USA</th>
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<td>5.95 158.82 41.19 0.7025*</td>
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<td>9.59 208.00 65.28 0.24148*</td>
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<td>4.27 139.69 49.60 0.19388*</td>
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<td>8.12 186.74 103.33 0.1465</td>
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<td>5.14 149.35 86.34 0.2107*</td>
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<td>16.54 340.21 92.52 0.1149</td>
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<td>9.44 205.75 34.87 0.0585</td>
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<tr>
<td>South Africa</td>
<td>8.47 191.60 61.92 −0.1620</td>
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</table>

* Statistically significant at the 5 per cent level.

Note: For computational methods see text.


The share price indices for the United Kingdom, West Germany, France, Italy, Belgium, and the Netherlands are from the industrial series of the Allgemeines Stat. Bull., European Economic Communities, various issues. The share price indices for Japan and Australia are industrial series from Internat. Fin. Stat., International Monetary Fund, various issues. The price index for South Africa is a gold mining shares index from the Quart. Bull., South Africa Reserve Bank, various issues.

The industrial dividend yields for the United States are from Moody's Indus. Manual, June 1967. The dividend yield on industrials series for the United Kingdom, West Germany, France, Italy, and the Netherlands are from Allgemeines Stat. Bull., European Economic Communities, various issues; for Belgium, from personal correspondence with the Dredietbank; for Japan, from The Oriental Economist, various issues; for Australia, from personal correspondence with the Reserve Bank of Australia; for Canada, from one published by Moss Lawson and adapted to the Toronto Stock Exchange Industrial Index, from personal correspondence with the Toronto Stock Exchange.

The dividend yield on gold mining shares series for South Africa is from personal correspondence with the South Africa Reserve Bank.

The exchange rates for all countries are taken from Internat. Fin. Stat., International Monetary Fund, various issues.
period from January 1959 to December 1966. For each of these eleven markets the following monthly observations were obtained: Indexes of common share prices \( P \), dividend yields on the shares in the index \( Y \) expressed as per cent per year, and the dollar exchange rate \( X \), defined as the price of one dollar. Subscripts 0 and 1 used below refer to the beginning and end of each monthly investment period; the share price index and exchange rate at the end of the current month is considered to be the price at which the next month’s investment is made.

The monthly rates of return were calculated on the basis of the following considerations. The dollar price of one foreign stock market index unit is \( PE_0 = P_0/X_0 \). The dollar value of the investment at the end of the first month, \( VE_1 \), is equal to the foreign currency value of dividends received \( DP_1 = P_0 Y_0/12 \) plus the foreign currency value of one unit of the index at the end of the month \( P_1 \) converted to dollars at the exchange rate \( X_1 \), i.e.,

\[
VE_1 = \left[ \frac{P_0 Y_0}{12} + P_1 \right]/X_1
\]

The problem then becomes to find the solution value for \( r_1 \) in the equation

\[
VE_1 = PE_0 (1 + r)^{1/12}
\]

which after some manipulation and substitution becomes

\[
r_1 = \left[ \left( \frac{Y_0/12 + \frac{P_1}{P_0}}{X_0/X_1} \right)^{12} - 1.0 \right]
\]

A matrix of correlation among the eleven countries’ monthly returns was computed and the variances and covariances were used in the subsequent calculations. Average rates of return were computed by taking the geometric mean of 95 monthly rates:

\[
R = \left[ \prod_{i=1}^{95} (1 + r_i) \right]^{1/12} - 1.0
\]

This formula, thus, computes the annual rate of return from capital gains due to common stock price and exchange rate changes, under the assumption that dividends are reinvested each month in fractional shares at current prices and that interest is compounded annually. No adjustments were made for withholding taxes on income or transactions.

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4. January 1959 was chosen as a starting point because in December 1958 European currencies became convertible \textit{de jure}. Common stocks rather than bonds were analyzed because of the greater variance around the average returns and across countries found in the former. The theoretical analysis can easily be modified to account for foreign stock purchases.

5. Taking the simple arithmetic mean of the monthly rates vastly overstates the value of the capital gains between the beginning and the end of the period plus the value of the dividends. For a discussion of the biases inherent in the calculation of indices and averages see [1].
costs. It should also be noted that exchange rate variations are assumed to be the only risks attached to foreign investment. Risks on foreign investment stemming from war, confiscation and exchange restrictions could not be quantified and were disregarded. Consequently the variances used in the subsequent calculations understate foreign risk and the estimates of gains from diversification are biased upward.

The empirical calculations are unrealistic in one other important respect. Due to indivisibilities, transactions costs, and limited portfolio sizes, it is virtually impossible for anyone to hold portfolios containing all of the shares making up the indices used in the calculations. Because the portfolio variance decreases with the number of individual stocks held, the underestimate of variance available to investors implicit in the calculation procedure is smaller the more diversified portfolios are in the real world. In general, the bias may not be too large in view of the availability of mutual funds in most of the foreign markets, though more empirical information on the investment patterns, transactions costs, etc., of these funds is needed.

In Table 1, column (1) shows average rates of return calculated in the manner just discussed while column (2) shows the capital value in December of 1966 of $100 invested in January of 1959. Columns (3) and (4) report the standard deviation of monthly returns and the correlation of these fluctuations with those of Moody's industrial average of common stocks. As can be seen, the U. S. yield has been the sixth lowest, but the riskiness of the investment as measured by variance has been the fourth lowest.

Given these historic rates of return and interdependencies of the national stock markets, it is possible to compute rates of return and variances of portfolios which would have accrued to investors who had purchased foreign assets in various combinations. The most interesting of these combinations are those which for any given variance maximize the return. Portfolios which have these characteristics and are attainable with the available set of assets can be found through methods of quadratic programming, for which standard computer algorithms are available.6

Table 2 presents the results of two different calculations for efficient sets of internationally diversified portfolios. Part A is based on rates of return and variances of the eleven industrialized countries mentioned before, while Part B is restricted to the data of the eight countries of the Atlantic Community. The eight portfolios shown for each case are so-called corner portfolios, i.e., those at which further reduction in variance
### Table 2—Efficient Internationally Diversified Portfolios

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*Notes:* For computational method see text.

*Sources:* Same as Table 1.

can be achieved only through the inclusion or omission of additional assets. The rates of returns and standard deviations for the corner portfolios are shown in the last two rows of Parts A and B of Table 2 and are plotted in Figure 1. Other attainable combinations of return and standard deviations can be found by interpolation between corner portfolios, as is done by the lines drawn between the points in Figure 1.

As can be seen, diversification among the assets from the eleven countries in general would have permitted investors to attain higher rates of return or lower variance of their portfolios than they could have by purchasing a portfolio consisting of Moody’s industrial average of common stocks. Which combination of assets given investors would in fact

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7 In the plotting of the data it was more efficient to use standard deviations rather than variances. Throughout this section the two terms are used interchangeably since this leaves substantive conclusions unaffected but facilitates exposition.
have chosen cannot be known since it depends on their individual marginal rate of substitution between risk and return. It can be said unambiguously, however, that if an investor had wanted to maintain the same variability in return found in the New York investment, international diversification would have permitted him to earn 12.6 per cent as against 7.5 per cent, a gain of 68.0 per cent in the annual rate of return. When the opportunities for investment in Japan, South Africa, and Australia are excluded from consideration, the opportunity for gains from diversification are reduced considerably, as can be seen from Part B of Table 2, and the appropriate efficiency frontier in Figure 1. However, the increase in return attainable at the New York variance is from 7.5 per cent to 8.9 per cent, a gain of 18.7 per cent. As can be seen from
Table 2, Part B, Column 3, such a portfolio would consist of the following approximate investments: 26.6 per cent in New York, 63.4 per cent in London and 9.9 per cent in Italy.

Analogous calculations can be carried out to demonstrate the reduction in variance attainable by investing in internationally diversified portfolios with the same expected rate of return as that from investment in New York alone. Such calculations are not shown here; rough estimates can be made by inspection of Figure 1.

In general, the preceding analysis and calculations suggest that recent experience with foreign investment returns would have given rise to substantial gains in welfare to wealth holders. If past experiences are considered to be indicative of future developments, then these data suggest that future international diversification of portfolios is profitable and that more of it will take place.8

III. The Dynamic Model

Some interesting conclusions from the model of internationally diversified portfolios result from the assumption that assets in both countries are growing through time. To simplify the analysis it is assumed that growth occurs in perfect balance, i.e., that income and assets in various forms grow at the same rate \( r_a \) and \( r_b \) for countries A and B respectively, and that exchange rates are pegged rigidly. If \( Q^0_{A,B} \) and \( Q^0_{B,A} \) are the initial stocks of foreign assets held in static equilibrium in Countries A and B respectively, then the gross flows \((\dot{Q}_t)\) at any point in time \( t \) are:

\[
\begin{align*}
\dot{Q}_{A,B}^t &= r_a e^{r_a t} Q_{A,B}^0 \\
\dot{Q}_{B,A}^t &= r_b e^{r_b t} Q_{B,A}^0
\end{align*}
\]

and the net flow from A to B \((\dot{N}_{A,B})\) is

\[
\dot{N}_{A,B}^t = r_a e^{r_a t} Q_{A,B}^0 - r_b e^{r_b t} Q_{B,A}^0
\]

Thus, it can be seen that the net flows of bonds between the two countries is a function of the growth rates and the size of the initial stocks in both countries. It is recalled that the initial stocks are determined primarily by the relative sizes of the two countries' wealth holdings and the existing interest rate differential.

Because of these determinants of bond flows, we have the following

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8 The validity of this statement depends on the interpretation of the results. One could argue that U.S. investors are in equilibrium and that the measures of risk used in the calculations represent an underestimate, which if properly accounted for would show little advantage to be gained from diversification. On the other hand, one could also argue that the calculations show the existence of a disequilibrium, that in fact U.S. investors are in the process of making stock adjustments which are taking time to accomplish. See Part V and footnote 10 for more comments on this possibility.
interesting possibilities. First, gross capital flows can occur between countries even if interest rates differentials are zero at all times. This is true whenever initial stocks of foreign bonds and growth rates are positive. Second, net capital flows into the low interest country (assumed to be Country A) can take place when first, \( r_a > r_b \) and \( Q^0_{A,B} = Q^0_{B,A} \); second, \( Q^0_{A,B} > Q^0_{B,A} \) and \( r_a = r_b \); third, \( r_a > r_b \) and \( Q^0_{A,B} > Q^0_{B,A} \); fourth, \( r_a < r_b \) and \( Q^0_{A,B} > Q^0_{B,A} \). In the last two cases, however, the net flow to Country A occurs only if the growth effect outweighs the stock effect or vice versa.

Under the assumed system of fixed exchange rates there are no equilibrating forces set into motion by net bond flows as long as the government of the country selling the private bonds is willing to accept the other country’s official I.O.U.’s in the manner described in Part I. In the long run, however, these stocks of official I.O.U.’s can become very large and it is doubtful that any governments are willing to accumulate them indefinitely. Pressures for a real transfer of resources will be generated and these will bring into being equilibrating forces.

The nature of these forces can be discerned most readily in the world of perfectly flexible exchange rates, where the net demand for bonds by residents of Country A results in the transfer of real resources to Country B through the generation of a trade surplus for A. This transfer has two effects. First, the rate of real economic growth in B increases while that in A decreases. Second, the marginal productivity of capital falls in B and rises in A. Both the real growth and interest rate effects tend to reduce the gross demand for bonds in A and raise the gross demand for bonds in B. The effects persist until gross bond flows have become equalized. However, such equality does not necessarily occur when the interest rate differential is zero. The differential can be either positive or negative and gross flows can remain equal as long as the products of growth rates times stocks of foreign assets are equal for both countries.

IV. Interest Elasticity of Capital Flows

In this part special attention is given to the role of interest rates in the preceding models, primarily because of some interesting policy conclusions following from the analysis.

In the static model after stock equilibrium has been established bonds cease to flow between the two countries. However, the potential for flows in response to interest rate changes is always present. Thus, if for some domestic policy purpose Country A decides to lower its interest rate, foreigners will decrease their holdings of Country A’s bonds and domestic wealth holders will increase their holdings of foreign bonds. The result is a net demand for bonds by Country A which leads to a transfer of I.O.U.’s to the government of B or to the transfer of real resources
in the manner discussed above. However, it is important to note that in this static model the flow of capital following the interest rate change is a once-and-for-all stock adjustment, which is accomplished within a certain time period, the length of which depends on institutional characteristics of the bond market.

In the dynamic version of the model the change in the interest rate differential calls forth the equivalent of a stock adjustment flow which is superimposed on the flow due to portfolio growth. The duration of this stock-adjustment flow component depends on the institutional characteristics of the bond market, as in the static model. After completion of the stock adjustment flow, the regular transfer of bonds continues to grow at the same rate as before, but the level is different. These points can also be made with the help of the accompanying Figure 2.

![Figure 2](image)

On the horizontal axis we plot time, on the vertical axis the log of gross capital flows from B to A. The line segment \( t_0t_1 \) has a slope \( r_a \), equal to the rate of growth of wealth portfolios in Country A. At period \( t_1 \), Country A lowers its interest rate and the growth rate of foreign bond holdings in A increases, as is shown by the steeper slope of the line segment \( t_1t_2 \). After the completion of the stock-adjustment process, the rate of growth in foreign bond holdings returns to its old level \( r_a \) but the level of bond holdings is raised at any given moment in time by the vertical distance between the solid and broken growth lines as a result of the increased rate differential.

V. Some Implications of the Model

First, the classical theory of factor movements considers rates of return alone as the determinants of international capital flows. In its basic
form, therefore, it cannot explain the real world phenomenon of simultaneous European investments in the United States and U.S. investments in Europe. Direct investment of this nature has been explained as resulting from the cost conditions in oligopolistic industries [2]. The present model provides an additional explanation that is especially applicable to the purchase of foreign bonds and other noncontrol conferring assets.

Second, the present model suggests that the empirical measurement of the interest elasticity of international capital movements can be improved by the inclusion of independent variables representing the growth in total asset portfolios and by studying gross flows of capital from each country. Consider, for example, the case where the interest rate differential is zero, gross flows are positive and large but net flows are zero. In our model it is possible that an increase in the rate of economic growth of one country causes the rise of that country's gross purchases of foreign assets and causes the appearance of a net flow even though the interest rate differential remained at zero.9 A measurement of the interest elasticity of net flows would yield nonsensical results, but the measurement of gross flows and inclusion of total portfolio growth can explain the phenomenon.

Third, the model leads to the hypothesis that the large scale U.S. investments in Europe during the last decade are part of a stock adjustment phenomenon that started when European currencies became convertible de jure in 1958 after having been convertible de facto a few years earlier. If this hypothesis is correct, then there may eventually take place a slowdown in the rate of U.S. capital outflows to Western Europe.10 However, because of the proportionately larger size of U.S. portfolios, normal growth in both continents leads to the expectation of a continued net demand for European assets. If this is so, European governments must either be willing to accept more of the U.S. government's I.O.U.'s or permit a greater trade surplus to occur if the free convertibility of the major Western currencies is to be maintained. Equalization of interest rates will be insufficient to equalize gross flows, because of the different sizes of total asset holdings in Europe and the United States.

Fourth, the portfolio model suggests that a once-and-for-all change in international interest rate differentials leads to only a once-and-for-all stock adjustment, after which gross flows return to their old levels. This implication of the portfolio model leads to an empirically important

9 Harry G. Johnson has suggested a similar dependence of international capital flows on rates of economic growth in his [4].

10 This point has also been made by J. Tobin [9, p. 168].
extension of the arguments over the proper mix of monetary and fiscal policy for the achievement of internal and external balance.

In Mundell's formulation of this argument [6] the domestic interest rate is set at such a level as to attract a quantity of foreign capital sufficient to fill the current account gap in international payments while fiscal policy is set at a level of restrictiveness sufficient to attain domestic full employment. Our model suggests that at the international interest rate differential initially chosen, there will be a stock-adjustment flow of a size that cannot be sustained beyond the attainment of the new stock equilibrium. If the external deficit on current account persists beyond this point of new stock equilibrium, then the interest rate differential has to be raised again to finance the deficit in the next period and so on until it is eliminated by some other policies. If foreign wealth holders run into diminishing returns to international diversification, then the subsequent increments to the interest differential have to be increasingly larger.11

Fifth, the model can be used to explain holding of foreign short-term assets as well as bonds, corporate securities, and direct investment. Continuous and growing international diversification demand for short-term assets has some interesting implications for U.S. balance of payments "deficits" under the liquidity definition. Even if the growing exchange of short-term assets between the United States and the rest of the world is perfectly balanced, the United States would show a continuous and growing balance of payments deficit since the foreign holdings of short-term dollar assets are considered to be a potential claim on U.S. reserves which the balance of payments statistics are designed to reflect. Yet, the model presented suggests that these potential liabilities are counterbalanced by U.S. holdings of foreign short-term assets and that the foreign asset demand is normal and permanent because of the continued welfare gains from holding internationally diversified portfolios. The model thus strengthens the arguments made against the use of the liquidity concept and in favor of the official-reserve-transactions concept.12

REFERENCES


11 I have proved the existence of such diminishing returns to diversification in the traditional quadratic utility function [3, p. 20].
12 These issues have been analyzed in [7].


