TESTING OUT-OF-SAMPLE PERFORMANCE OF CORRELATION MODELS FOR PORTFOLIO CHOICE AND VALUE-AT-RISK

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Approval

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Degree: Master of Science in Finance

Title of Project: Testing Out-of-Sample Performance of Correlation Models for Portfolio Choice and Value-at-Risk

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Senior Supervisor
Professor of Finance

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Second Reader
Professor of Finance

Date Approved: ____________________________
Abstract

Our research strive to determine the relative out-of-sample performance of constant and dynamic correlation models in the context of portfolio choice and value-at-risk (VaR). We specify and estimate the dynamic conditional correlation (DCC) model proposed by Engle (2002) and the constant conditional correlation model proposed by Bollerslev (1990) and benchmark their performance against unconditional estimates. We use two data sets of daily returns comprised of three broad North American market indices and backtest the models over a period of more than 10 years which has not been done in the previous reviewed literature. Consistent with previous studies, we find DCC outperforms CCC and unconditional estimates, especially in times of changing volatility and correlation. VaR fails to adequately capture market risk during the 2008 financial crisis and the distribution assumption of innovations is more important than the choice of correlation model.

Keywords: Multivariate GARCH; Dynamic Conditional Correlation; Value at Risk; Asset Allocation; Portfolio Choice; Mean Variance Optimization; Applied Econometrics; Econometrics of Financial Markets.
Acknowledgements

We would like to sincerely thank Dr. Peter Klein, Dr. Andrey Pavlov and Dr. Jorge Lopez for their thesis project supervision, guidance and flexibility. We also wish to extend our gratitude to the faculty of the Beedie School of Business at Simon Fraser University. And we express our utmost gratitude to our families and friends for their unconditional support.

Thank you.
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# Glossary

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<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH</td>
<td>Auto-regressive conditional heteroskedasticity</td>
</tr>
<tr>
<td>ARMA</td>
<td>Auto-regressive moving average</td>
</tr>
<tr>
<td>CCC</td>
<td>Constant conditional correlation</td>
</tr>
<tr>
<td>DCC</td>
<td>Dynamic conditional correlation</td>
</tr>
<tr>
<td>DEX</td>
<td>DEX Canadian Universe Bond Index represents the fixed income market in Canada</td>
</tr>
<tr>
<td>FR</td>
<td>Fail Rate</td>
</tr>
<tr>
<td>GARCH</td>
<td>Generalized auto-regressive conditional heteroskedasticity</td>
</tr>
<tr>
<td>LBQ</td>
<td>Ljung-Box test for serial correlation</td>
</tr>
<tr>
<td>MVO</td>
<td>Mean-variance optimization</td>
</tr>
<tr>
<td>P&amp;L</td>
<td>Profit and Loss</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>The S&amp;P 500 Index consists of 500 large-cap constituents traded on stock exchanges in the U.S. Market</td>
</tr>
<tr>
<td>TSX</td>
<td>The S&amp;P/TSX Composite Index consists of constituents traded on the Toronto Stock Exchange</td>
</tr>
<tr>
<td>VaR</td>
<td>Value-at-risk</td>
</tr>
</tbody>
</table>
1 Introduction

Today’s financial markets are highly volatile and dynamic and the quantification of risk is crucial to the competitiveness and survival of financial institutions. The calculation of a portfolio’s standard deviation of returns requires an accurate measure of the covariance matrix. Empirical studies have failed to confirm that correlations are constant and stable over time. An accurate forecast of the covariance matrix between assets is required as an input to risk models such as value-at-risk and to the asset allocation – portfolio choice problem. Previous studies have focused on univariate volatilities and multivariate GARCH models such as Bollerslev’s Constant Conditional Correlation (CCC) model (1990) and Engle’s Dynamic Conditional Correlation (DCC) model (2002).

1.1 Previous Studies and our Innovations

1.1.1 Value-at-Risk

Market risk management and VaR in particular is a very important topic in finance, especially since the financial crisis of 2008. In addition to regulatory attention from the Bank of International Settlement’s Basel Accords, which mandated the use of VaR by banks since 1996, there is a significant body of academic research on the topic. For an interesting comparison of commercial banks’ internal VaR models against simple volatility modelling of portfolio profit and loss (P&L), Berkowitz and O’Brien (2002) use a data set obtained from the Federal Reserve to evaluate the performance of six commercial banks’ own VaR methodologies. They find that a reduced form univariate GARCH model applied to historical portfolio P&L actually achieves a similar fail rate with more independent exceptions than the banks’ sophisticated models. This study is an early example of the difficulty in evaluating out-of-sample market risk using VaR, even with sophisticated modelling techniques.

Lee, Chiou and Lin (2006) apply the DCC model to forecast out-of-sample VaR using a dataset comprised of seven equity indices from G7 countries and compare its efficacy using both Gaussian and student-t innovations to univariate methods and find the DCC model with student-t innovations performs best. Rombouts and Verbeek (2009) compare different multivariate GARCH models including DCC and DVEC and various distributional assumptions and find the
distributional assumption is very important determinant of the efficacy of the VaR measure. They find a semi-parametric distribution obtained using a kernel density technique outperforms student-t and Gaussian however; their evaluation method is completely in-sample which has no basis in practical risk management. Pesaran and Pesaran (2010) evaluate out-of-sample VaR performance during the 2008 financial crisis and is the first prominent paper to do so. Although their research finds the DCC model with student-t innovations is the best statistical fit to the data set, all VaR forecasts perform relatively poorly in estimating risk leading up to the financial crisis. Their research includes a large data set with 17 assets spanning a long period as they use weekly data. Their research and literature review confirms financial time series exhibit excess kurtosis and time varying correlations that increase during times of financial stress. Santos, Nogales and Ruiz (2013), who assess out-of-sample VaR performance using three portfolios of US equities, implement a backtest partition to study the impact of the 2008 financial crisis on their results. They also find VaR models perform relatively poorly in forecasting market risk during the crisis however, dynamic correlation models do capture risk more effectively than univariate methods.

Our key innovations to the existing literature include a unique data set, a rigorous backtest and a comparison of the skewed student-t to the standard student-t and Gaussian innovations of residuals. A portfolio of Canadian equities and fixed income, represented by the S&P/TSX index and DEX Universe Bond index, respectively, has not been studied in the reviewed literature. In terms of the VaR backtest, most studies examine a period of one to four years, with the maximum testing window being 1,000 data points. Our backtest covers 2,979 realized returns so should be able to better assess the performance of the various modelling techniques. Furthermore, we are able to partition the backtest into five equal sub-periods to assess the independence of exceptions through time. Finally, we also implement Hansen’s skewed student-t distribution and compare its ability to forecast out-of-sample VaR. Our dataset and those of previous studies show that many financial time series display significant negative skewness that has a large impact on VaR. Although the skewed student-t distribution has been used in the literature to analyse VaR, given the large dataset and rigorous backtest, this innovation will add clarity to the effect of distributional assumption on forecasting out-of-sample VaR.

1.1.2 Portfolio Choice

The simplest and most utilized approach to portfolio choice is the mean-variance analysis by Markowitz (1959) where optimal allocation is derived by solving a constrained maximization
problem. Finding the optimal allocation involves accurately forecasting the return and risk of a portfolio. Risk is represented by the variance of the portfolio, which makes the forecasted covariance matrix a critical step in the asset allocation process. Recent studies emphasize empirical evidence in favour of time varying variances and time varying correlations being higher during turmoil than in normal market conditions (Clare et al. 1998, Longing and Solnik, 2001), motivating the use of correlation models, especially during times of crisis.

The multivariate GARCH literature is problematic due to the high number of parameters in the models (the BEKK model of Engle and Kroner 1995 and Kroner and Ng 1998). In response, Bollerslev (1990) suggested the Constant Conditional Correlation model where correlations of the standardized residuals are kept constant. Recently, Engle (2002) proposed a new class of model. The DCC model preserves the ease of estimation, but allows correlation dynamics to change over time, following a GARCH-like process.

Past empirical studies of asset allocation involving correlation models include Otranto (2010), who utilizes correlation models in a study of portfolio choice towards the Italian stock market index. Their findings are however, insufficient to conclude the outperformance of any one model due to the limited test range of the study (2000-2003). Wu (2012) employ copula theory to allow for skewness and asymmetry in stock and bond returns and compare the performance of the CCC, DCC and GJR-GARCH models. Although they find that the GJR-GARCH model with student-t copula has the largest explanatory power of volatility in addition to it yielding larger economic gains than other portfolio choice strategies, their out of sample period is limited to only 4 years. This is again insufficient evidence for the outperformance of the correlation models.

The previous work of Billio et al. (2006) has also inadequately shown the superiority of their proposed correlation model, the Flexible Dynamic Conditional Correlation compared to the DCC and CCC models. Although they have a sample size of 12 years of daily observations (roughly 3000 price points), they only report their backtest results of variance for the last two months of the sample, weights for the final estimation and returns over the last year of the sample. This reporting inconsistency makes it unclear that their proposed Flexible DCC model produced superior risk adjusted returns.

Our key innovation is to be consistent in testing the Unconditional, CCC and DCC models. We will test the out-of-sample performance over the entire sample (less the initial estimation window) and report annualized standard deviations, returns and weights for each of the three models.
1.2 Objective and Outline

Our goal is to determine the relative out-of-sample performance of constant and dynamic correlation models in the context of Value-at-Risk and portfolio choice. Section 2 provides an overview of Bollerslev’s CCC model and Engle’s DCC model. Section 3 describes the two data sets subject to analysis. We then present empirical analyses of the three models across both data sets in the context of value-at-risk in Section 4 and portfolio choice in Section 5. Finally, Section 6 concludes.
2 Correlation Models

In this section, we introduce the statistical techniques we will be implementing in our subsequent analysis. We use continuously compounded daily returns, as defined below, in all subsequent analysis:

\[ r_t = \ln(P_t) - \ln(P_{t-1}) \]  \hspace{1cm} (1)

In order for portfolio choice and VaR models to yield effective results, the inputted series must be weakly stationary. As such, we test the log return vectors for serial correlation using the Ljung-Box Q-test (LBQ test) and filter the return series through an ARMA model as follows:

\[ r_t = \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{j=1}^{q} \theta_j a_{t-j} + a_t \]  \hspace{1cm} (2)

A well-known property of daily financial time series is that they display non-constant variance. Therefore we will test for heteroskedasticity using the LBQ test on the residual series and subsequently fit a GARCH (1,1) model as follows:

\[ \sigma_t^2 = \beta_0 + \beta_1 a_{t-1}^2 + \gamma \sigma_{t-1}^2 \]  \hspace{1cm} (3)

Once the GARCH model is estimated and fitted, we will obtain the standardized residuals as follows:

\[ \epsilon_t = a_t / \sigma_t \]  \hspace{1cm} (3a)

The univariate GARCH model, described above, is then used in the estimate of Engle’s (2002) Dynamic Conditional Correlation model. For our analysis we will use a DCC (1,1) model to obtain the conditional covariance matrix:

\[ H_t = D_t R_t D_t \]  \hspace{1cm} (4)

The DCC model is a reparameterization of the covariance matrix into \( R_t \), a conditional correlation matrix and \( D_t \), a diagonal matrix of conditional standard deviations, defined below:
\[
D_t = \begin{pmatrix}
\sqrt{h_{11}} & 0 & \cdots & 0 \\
0 & \sqrt{h_{22}} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sqrt{h_{nn}}
\end{pmatrix}
\text{ where } h_{it} = \sigma_i^2 \text{ of } i^{th} \text{ series} \tag{4a}
\]

\[
R_t = \begin{pmatrix}
1 & q_{12} & \cdots & q_{1n} \\
q_{21} & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
q_{n1} & q_{n2} & \cdots & 1
\end{pmatrix}
\tag{4b}
\]

And the conditional correlation matrix is further reparameterized as follows:

\[
R_t = Q_t^{-1} Q_t \tag{5}
\]

\[
Q_t^{-1} = \begin{pmatrix}
\frac{1}{\sqrt{q_{11}}} & 0 & \cdots & 0 \\
0 & \frac{1}{\sqrt{q_{22}}} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{1}{\sqrt{q_{nn}}}
\end{pmatrix}
\tag{5a}
\]

\[
Q_t = \begin{pmatrix}
q_{11} & q_{12} & \cdots & q_{1n} \\
q_{21} & q_{22} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
q_{n1} & q_{n2} & \cdots & q_{nn}
\end{pmatrix}
\tag{5b}
\]

The dynamics of the \( D_t \) matrix is given by equation (3) while the dynamics of \( Q_t \) is as follows:

\[
Q_t = (1 - \alpha - \beta)\overline{Q} + \alpha \varepsilon_{t-1}\varepsilon_{t-1}' + \beta Q_{t-1} \text{ where } \overline{Q} = Cov(\varepsilon_t, \varepsilon_t') \tag{6}
\]

Once the DCC (1,1) model is estimated and fitted, we will then use the model to estimate the 1 step ahead forecast of the conditional covariance matrix, \( H_{t+1} \). Peters (2004) shows the one-step ahead forecast of correlation is defined as follows:

\[
R_{t+1} = (1 - \alpha - \beta)\overline{R} + (\alpha + \beta)R_t \text{ where } \overline{R} \text{ is the unconditional correlation matrix of the standard residuals} \tag{7}
\]

Then, \( D_{t+1} \) is forecast from equation (3). We will also use Bollerslev’s (1990) Constant Conditional Correlation model which is identical to the DCC model except \( R_t \) from equation (4)
is simply the unconditional estimate of the correlation matrix and therefore constant through time.

We will estimate and fit a CCC (1,1) model which will be used to forecast the one step ahead covariance matrix $H_{t+1}$ as per equation (4) substituting $\tilde{R}$ for $R_{t+1}$. 
3 Data Sets

For our analysis we will use two data sets. The primary data set, called data set 1, is daily price data on two broad Canadian market indices from Nov 23, 2000 to Oct 3, 2014 and contains 3,483 return observations. The S&P/TSX Index (TSX) represents returns on equities and the DEX Universe Canadian Bond Index (DEX) represents returns on fixed income securities. For analysis of Portfolio Choice, we will use a secondary data set, called data set 2, comprised of daily price observations for the TSX and S&P 500 Index (S&P500) in Canadian Dollars from Nov 20, 1994 to Nov 20, 2014 and contains 4,932 return observations. The TSX and S&P500 represent Canadian and US equities, respectively. Log returns, as per equation (1), will be used in all subsequent analysis.

3.1 Summary Statistics

Basic summary statistics of the daily log returns of both data sets, displayed in Table 3.1, show some well-known features of financial time series. Of particular significance for VaR is the presence of excess kurtosis and significant negative skewness.

<table>
<thead>
<tr>
<th>Table 3.1 Descriptive statistics of daily log returns.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data set 1 - Nov 2000 to Oct 2014</strong></td>
</tr>
<tr>
<td>DEX</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Std Dev</td>
</tr>
<tr>
<td>Skewness</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
</tr>
<tr>
<td>Min</td>
</tr>
<tr>
<td>Max</td>
</tr>
<tr>
<td><strong>Data set 2 - Nov 1994 to Nov 2014</strong></td>
</tr>
<tr>
<td>TSX</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Std Dev</td>
</tr>
<tr>
<td>Skewness</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
</tr>
<tr>
<td>Min</td>
</tr>
<tr>
<td>Max</td>
</tr>
</tbody>
</table>
The unconditional estimate of the correlation coefficient over the entire sample is -0.199 and 0.571, respectively for data set 1 and data set 2, however this varies through time using a rolling window unconditional estimate, which motivates the need for a dynamic correlation model in forecasting out of sample correlation.

### 3.2 Results of Correlation Models

We estimated and fitted the correlation models as per equations (1) to (7) and the results of the CCC (1,1) and DCC (1,1) model are displayed in Table 3.2:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>DEX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>β₀</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>β₁</td>
<td>0.025</td>
<td>0.110</td>
</tr>
<tr>
<td>γ</td>
<td>0.973</td>
<td>0.890</td>
</tr>
<tr>
<td>R</td>
<td>-0.199</td>
<td>0.571</td>
</tr>
<tr>
<td>α</td>
<td>0.021</td>
<td>0.044</td>
</tr>
<tr>
<td>β</td>
<td>0.970</td>
<td>0.932</td>
</tr>
<tr>
<td>LL Ratio</td>
<td>1.0013</td>
<td>1.0029</td>
</tr>
<tr>
<td>TSX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>β₀</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>β₁</td>
<td>0.069</td>
<td>0.068</td>
</tr>
<tr>
<td>γ</td>
<td>0.923</td>
<td>0.924</td>
</tr>
<tr>
<td>R</td>
<td>0.000</td>
<td>0.571</td>
</tr>
<tr>
<td>α</td>
<td>0.014</td>
<td>0.012</td>
</tr>
<tr>
<td>β</td>
<td>65.461</td>
<td>66.971</td>
</tr>
<tr>
<td>LL Ratio</td>
<td>1.0029</td>
<td>1.0029</td>
</tr>
</tbody>
</table>

The table includes parameter estimates for DEX and TSX for two different data sets, with columns for Coeff., Value, Std Error, t-stat, and p-val. The last row indicates the LL ratio between DCC and CCC models.
The estimate of $\gamma$ for all time series is very statistically significant, especially for the equities series, indicating strong persistence of the conditional variance through time. Despite the strong evidence of time-varying correlation, the estimate of the $\alpha$ parameter from the DCC(1,1) for data set 1 is not statistically significant. This indicates that the previous day’s covariance of standardized residuals, $\hat{\varepsilon}_{t-1}\hat{\varepsilon}_{t-1}$, does not significantly impact the next day’s conditional correlation. This is also evidenced by the log likelihood ratio between the DCC(1,1) and CCC(1,1) model, with the DCC providing a better fit by a factor of $1.0013$. For data set 2, the $\alpha$ parameter is statistically significant and this is also shown by a larger log likelihood ratio of $1.0029$.

Summary statistics of the standardized residual series from the DCC(1,1) model for both data sets are also displayed in Table 3.3:

Table 3.3  Descriptive Statistics of Standardized Residuals.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DEX</td>
<td>TSX</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.001</td>
<td>0.012</td>
</tr>
<tr>
<td>Std Dev</td>
<td>1.002</td>
<td>1.001</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.437</td>
<td>-0.509</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>2.693</td>
<td>1.086</td>
</tr>
<tr>
<td>Min</td>
<td>-7.733</td>
<td>-5.937</td>
</tr>
<tr>
<td>Max</td>
<td>3.994</td>
<td>3.364</td>
</tr>
</tbody>
</table>

Although the GARCH(1,1) model fits the data well and removes any significant heteroskedasticity, the standardized residuals still display significant excess kurtosis and significant negative skewness in the case of data set 1. This will have implications for forecasting the VaR, described later.


4 Value at Risk

Value-at-risk, or VaR, is an important measure of market risk of a portfolio and is commonly used at banks and other financial institutions to measure the chance of large losses to the portfolio. VaR attempts to determine the amount of loss over a specified time horizon, given a level of confidence, which the portfolio could sustain. Given a vector of portfolio weights, denoted \( w \), VaR is defined as follows:

\[ P(r^\top w < VaR_{1-a}) = a \]  

(8)

4.1 Structure of Estimation and Testing

For the VaR section, we will use data set 1: TSX and DEX from Nov 23, 2000 to Oct 3, 2014. Given the objective of our research is to test out-of-sample performance, we will define an estimation window in order to fit the econometric models described above to forecast the one-step-ahead covariance matrix. Our estimation window will be 504 days, a proxy for 2 calendar years. The one-step-ahead (one day) forecasts of correlation and variance will be used in the calculation of one day ahead VaR. This VaR will be compared against the realized return on that day. The process will be repeated across all 2979 estimation windows which is the total return observations less the estimation window.

VaR will be calculated using different portfolio weights, confidence levels, underlying distribution assumptions of the simulated standardized residuals, and correlation models. A detailed breakdown:

- Portfolio weights: \( w \) = (0.25,0.75), (0.5,0.5), and (0.75,0.25)

- Confidence levels: \( \alpha \) = 0.01, 0.025, 0.05

- Distributions: normal, student-t, skewed student-t (Hansen, 1994)

- Correlation models: DCC(1,1), CCC(1,1), unconditional, historical

The VaR computed across all 108 methods described above and all 2,979 estimation windows will be compared to the actual portfolio return on that day. The total number of exceptions, when the realized portfolio return is less than the forecasted VaR, will be counted and
divided by the number of days in the sample to determine a failure rate (FR). This FR will be analysed using the Kupiec (1995) test statistic to decide whether a certain VaR calculation technique over a given period should be rejected or accepted at the 5% significance level.

The entire data set will be partitioned into 5 equal segments to analyse the independence in the rate of exceptions. Based on the GARCH parameters in Table 3.2, it is clear the return data displays volatility clustering. As such, it is likely extreme losses will be followed by more extreme losses. By implementing this partition, we are able to identify clustering, or serial correlation, of exceptions by comparing the FR across partitions. The partition interval dates are as follows and contain 595 return observations each:

- Nov 28, 2002
- Apr 8, 2005
- Aug 20, 2007
- Jan 4, 2010
- May 16, 2012
- Sep 30, 2014

4.2 Results of Backtest

A summary table of the backtest results is given in Table 4.1.

<table>
<thead>
<tr>
<th>Confidence Level</th>
<th>Normal Distribution</th>
<th>Student's t Distribution</th>
<th>Skewed Student's t Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>w_1</td>
<td>w_2</td>
<td>w_3</td>
</tr>
<tr>
<td>95%</td>
<td>FR p-val</td>
<td>FR p-val</td>
<td>FR p-val</td>
</tr>
<tr>
<td>H</td>
<td>5.3% 0.21</td>
<td>5.1% 0.35</td>
<td>4.8% 0.33</td>
</tr>
<tr>
<td>U</td>
<td>5.7% 0.03</td>
<td>5.6% 0.06</td>
<td>5.0% 0.48</td>
</tr>
<tr>
<td>C</td>
<td>6.7% 0.00</td>
<td>6.8% 0.00</td>
<td>5.8% 0.02</td>
</tr>
<tr>
<td>D</td>
<td>6.6% 0.00</td>
<td>6.6% 0.00</td>
<td>5.4% 0.17</td>
</tr>
<tr>
<td>97.5%</td>
<td>FR p-val</td>
<td>FR p-val</td>
<td>FR p-val</td>
</tr>
<tr>
<td>H</td>
<td>3.0% 0.03</td>
<td>2.8% 0.17</td>
<td>2.5% 0.46</td>
</tr>
<tr>
<td>U</td>
<td>4.0% 0.00</td>
<td>3.9% 0.00</td>
<td>3.0% 0.03</td>
</tr>
<tr>
<td>C</td>
<td>4.4% 0.00</td>
<td>4.1% 0.00</td>
<td>3.5% 0.00</td>
</tr>
<tr>
<td>D</td>
<td>4.4% 0.00</td>
<td>4.1% 0.00</td>
<td>3.3% 0.00</td>
</tr>
<tr>
<td>99%</td>
<td>FR p-val</td>
<td>FR p-val</td>
<td>FR p-val</td>
</tr>
<tr>
<td>H</td>
<td>1.4% 0.02</td>
<td>1.5% 0.00</td>
<td>1.2% 0.15</td>
</tr>
<tr>
<td>U</td>
<td>2.5% 0.00</td>
<td>2.5% 0.00</td>
<td>1.8% 0.00</td>
</tr>
<tr>
<td>C</td>
<td>2.3% 0.00</td>
<td>2.3% 0.00</td>
<td>1.7% 0.00</td>
</tr>
<tr>
<td>D</td>
<td>2.3% 0.00</td>
<td>2.3% 0.00</td>
<td>1.7% 0.00</td>
</tr>
</tbody>
</table>

The VaR Results - Full Sample (DEX/TSX). FR is fail rate. w_1, w_2, w_3 are portfolios of DEX/TSX with weights (0.25,0.75), (0.5,0.5) and (0.75,0.25), respectively. H, U, C, and D correspond to historical, unconditional, CCC, and DCC, respectively. The p-values are obtained from the Kupiec (1995) test and instances where the model fails at the 5% significance level are highlighted in grey.
Probably the most obvious result is that out of sample VaR, as calculated in this research over the whole sample, is inadequate at capturing market risk. Of the 108 combinations of methods, 73 of them failed the Kupiec test at the 5% significance level. Specifically, out of sample VaR significantly understates the amount of market risk inherent in a portfolio based on these results. Every single one of the 73 failures was due to an understatement of the VaR.

Another striking result, when analysing the full sample, is that the simplest method, historical, outperforms all other models including the more sophisticated DCC and CCC methods. In particular, the historical method only fails in 33% of instances compared to 67% for unconditional, 89% for CCC, and 81% for DCC. Consistent with the previous literature, a distribution that more closely matches the properties of the underlying random variables performs much better regardless of the particular correlation model. Across the 27 combinations of methods for each distribution assumption (ignoring the historical method which makes no assumption regarding the underlying distribution), the Hansen’s (1994) skewed student-t density performs much better than the others with a rejection rate of 59% compared with 89% each for normal and student-t.

However, when examining the partitioned results, one finds the full sample results are actually quite misleading with respect to the relative performance of the correlation models. The introduction of partitions to the backtest shows there is strong evidence of serial correlation among the exceptions for the historical and unconditional method. Results for two particular subsets: between Nov 28, 2002 and Apr 8, 2005 (subset 1) and between Aug 20, 2007 and Jan 4, 2010 (subset 2) are good examples of the shortcomings of the historical and unconditional methods. These results are displayed in Table 4.2 and Table 4.3.

### Table 4.2 VaR Results Between Nov 2002 and Apr 2005 – same formatting as previous results tables.
An unconditional method is slow to adjust to the escalating volatility in the portfolio leading up to the actual returns being plotted against the backtest results. Exceptions to VaR are common, particularly during the financial crisis. Santos, Nogales and Ruiz (2013) observed that poor VaR results in their study were primarily driven by the high rate of exceptions of VaR. The Kupiec test, which is used to evaluate the performance of VaR, found that unconditional methods fail in 100% of instances compared with 79% for CCC and 85% for DCC. As before, the CCC and DCC method allow the VaR forecast to adjust more quickly to the lower volatility conditions. As a result, the historical volatility present in the estimation window is causing a persistent underestimation of VaR (in absolute terms). By comparison, the CCC and DCC method yield a VaR that is much quicker to adjust to the lower volatility conditions. As a result, the historical underestimation of VaR is much quicker to adjust to the lower volatility conditions. As a result, the historical overestimation of the VaR (in absolute terms). By comparison, the CCC and DCC method yield a VaR that is much quicker to adjust to the lower volatility conditions. As a result, the historical method fails in 56% of instances compared with 33% for unconditional, 26% for CCC and 22% for DCC.

Conversely, subset 2 is a period of very high volatility due to the 2008 financial crisis, which was preceded by a period of relatively low volatility. The low volatility in the estimation window results in a persistent underestimation of VaR (in absolute terms) using the historical and unconditional methods. As before, the CCC and DCC method allow the VaR forecast to adjust much quicker to the changing dynamics in the equities market. The historical and unconditional method are rejected 100% of the time by the Kupiec test compared with 79% for CCC and 85% for DCC. It should be noted that the out of sample performance of VaR during the financial crisis was abysmal, with a rejection rate of 91%. This result is consistent with the findings of Pesaran and Pesaran (2010) who find out-of-sample performance of VaR, even with dynamic correlation models, is very poor during this crisis. Santos, Nogales and Ruiz (2013) also partition their backtest. They split the evaluation window into two subsets: one leading up to the 2008 crisis and another during the crisis. They find that the poor results of VaR in their study is primarily driven by the high rate of exceptions of VaR during the 2008 financial crisis.

A visual representation of the serial correlation of exceptions is shown in Figure 4.1. The actual returns are plotted against the forecasted VaR by model type. It can be seen that the unconditional method is slow to adjust to the escalating volatility in the portfolio leading up to the
crisis and slow to adapt to the diffusing volatility afterwards. This leads to a large number of exceptions in the first part of the sample and a very small number afterwards.

Figure 4.1  *VaR99 of equal weight DEX/TSX portfolio between 2008 – 2010.*

Analysing the results across all partitions, it becomes clear that the CCC and DCC model outperform the historical method. The historical method is rejected in 62% of instances while the unconditional and CCC method cannot pass the Kupiec test 50% of the time. The DCC model performs incrementally better with a rejection rate of 47%.

What is most convincing is the effect of the distribution assumption on the results of the backtest. Independent of the correlation model and ignoring the historical method which has no distributional assumption, the normal distribution has a rejection rate of 59% compared with 53% for student-t and 37% for the skewed student-t distribution. When considering which VaR methodology to implement to measure the market risk of a portfolio, the effect of the distribution has a much more profound effect on the results than the particular correlation model. Rombouts
and Verbeek (2009) compare the normal distribution, student-t, and a semi-parametric distribution derived using the kernel density method. They find that the normal distribution performs poorly, especially at a higher confidence such as 97.5% or 99%. While the student-t distribution offers an improvement, it is their semi-parametric distribution which performs unequivocally better across all other permutations of VaR parameters. Lee, Chiou and Lin (2006) also use the normal distribution and student-t distribution in their VaR backtest. They find that the student-t distribution performs better than normal.

As a closing remark related to the implementation of the VaR methodology used in this research, one must consider computation expense against the efficacy of the method. Although DCC did offer an incremental benefit compared with the other procedures, it is much more time consuming to implement, especially in the non-trivial case with more than 2 asset series.
5 Asset Allocation - Portfolio Choice

To investigate the performance of the Unconditional, CCC and DCC models, a simulation was conducted in the context of Markowitz mean-variance optimization and the minimum variance portfolio problem:

$$\min_{w_t} w_t^\prime H_t w_t$$

where \( \ell \) is a Nx1 vector of ones

subject to \( w_t^\prime \ell = 1 \)

(9)

The data sets used were chosen to view the regimes of outperformance or underperformance, if any, of any of the three models over an extended period, 12 years for the first data set and 18 years for the second. A comprehensive summary of return and risks are shown over the whole backtest period.

The out-of-sample backtest was constructed by the following procedure: First, the estimation of the previous two years of data was used to capture recent market events as well as have a stable enough data (504 data points) for fitting the parameters of the CCC and DCC models. A one-step ahead forecast of covariance and variance was generated for the estimation window via the models and was fed into a mean-variance optimizer to obtain the global minimum variance portfolio. The asset weights found were then used along with the historical returns to obtain the backtest returns for the simulated portfolios for the following week (5 days). This process was repeated every 5 days until the end of the data set.

The backtest portfolio was chosen assuming no risk free asset, no short selling (i.e. \( w_t \geq 0 \)), no transactions costs and no restrictions on the magnitude of weight change at each of the weekly rebalance points.

5.1 Data Set 1: TSX and DEX (November, 2000 – October, 2014)

5.1.1 The Data Set

The first data set is the TSX index and the DEX Canadian Bond Universe index from November 23, 2000 until October 3, 2014. This was the maximum range for the iShares proxy for
the DEX Universe and was chosen to mimic the asset allocation problem of a balanced Canadian investor who has to make a choice between stocks and bonds.

Table 5.1 shows the summary statistics for the first data set. Over the entire sample, the DEX had a higher annualized return with significantly lower risk. This resulted in a return to risk ratio of nearly 4x that of the TSX.

Figure 5.1 shows the cumulative log returns for the first data set. It is clear that the DEX shows a consistent up-trend while the TSX is more volatile. Our data samples the end of the technology crisis for the TSX, greater returns by the TSX from November 2002 to the peak of the credit crisis in mid 2008 and also from the start of 2009 to 2011 where equities bounced back to pre-crisis levels before the Eurozone debt crisis. The clear periods of return outperformance for each asset allow the optimizer to minimize risk by loading into each asset.

<table>
<thead>
<tr>
<th>Table 5.1</th>
<th>Annualized summary statistics for data set 1, TSX and DEX over the full sample (Nov 2000 – Oct 2014). The DEX stochastically dominates the TSX with a higher return and lower risk.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TSX</td>
</tr>
<tr>
<td>Return</td>
<td>3.72%</td>
</tr>
<tr>
<td>Risk</td>
<td>17.9%</td>
</tr>
<tr>
<td>Return/Risk</td>
<td>0.208</td>
</tr>
</tbody>
</table>
Figure 5.1  Data set 1: Cumulative log returns for the TSX and the DEX. The DEX is stable, while the TSX is more volatile.

5.1.2 Conditional Volatilities and Correlation

Figure 5.2 shows the log returns, conditional volatilities and correlation for the TSX and the DEX. It is observed that the TSX is much more volatile 1% daily volatility, corresponding to a 16% annual volatility. This is compared to the DEX which showed a 0.3% daily volatility corresponding to a 5% annual volatility. A large difference between the two data sets are during the financial crisis of 2008-2009 where the TSX volatility jumps to 4% daily (65% annual) while the DEX stays suppressed at 0.4% (6.4% annual).

The correlations are obtained from the DCC model and can be classified into regimes of volatile correlation for the periods of the start of the sample until November 2005, in the buildup to the credit crisis in 2008 and again from 2010 to the end of the sample in 2014. Stable
correlation periods are observed from mid 2005 to mid 2007 as well from the start of 2009 to October 2010. The full sample correlation was -0.2 indicating the typical diversification effect of the two asset classes.

Figure 5.2  Data set 1: TSX and DEX log returns, conditional volatilities and correlation. The volatility of the TSX is greater than that of the DEX. Correlation decreases in times of turmoil.

5.1.3 Backtest Returns

Figure 5.3 shows the backtest returns and return spreads for data set 1. We observe a very similar return profile from each of the three models. It can be noted that the backtest portfolio followed a very steady uptrend and a much lower drop in the credit crisis compared to the TSX.

To compare the correlation models together against the unconditional model, the volatilities and correlation in Figure 5.2 will be referenced to help explain the features of the backtest return spreads. Taking a look at the return spreads, we see that there are regimes where
the two correlation models outperform the Unconditional model. This included the start of the sample until the end of 2004 where the DEX was (relatively high volatility) and correlations where volatile. The outperformance was moderate at 2%, after which the models underperform the Unconditional model from 2005 until October 2008. This corresponded to a period of stable asset volatilities and correlations. When volatilities and correlations spike in October, 2008 and when correlation decreased greatly in the period leading up to the Eurozone credit crisis, the correlation models massively outperform. This outperformance does not last, however, and quickly deteriorates as correlations stabilize in the 2008 case, and from 2011 until mid 2013 for the Eurozone debt crisis case. The CCC model outperformed in 2014.

For the TSX and the DEX, the correlation models generate higher returns in periods of high volatility and high and changing correlation. The unconditional method outperformed in periods of stable asset volatilities and correlations.
Data set 1: Backtest returns and return spreads for the Unconditional, CCC and DCC models. The correlation models outperform in periods of changing correlation and high and changing volatilities.

5.1.4 Risk

Figure 5.4 breaks down the backtest returns, risk and return to risk ratios by year for each of the three models. Positive returns are observed for most years along with a downward trend in portfolio risk for the periods of 2002 to 2007 with a spike in 2008, then a decline again from 2008 to 2014. We see that from 2002 to 2003, with a relatively higher DEX volatility, the correlation models’ risk is slightly higher than that of the unconditional model. Correlation becomes more volatile in 2005, 2007 and 2008 corresponding to periods where the correlation models’ risk is significantly lower than that of the unconditional model. This however, also contributed to lower returns. This is not the case in 2008 where the correlation models posted both higher returns and lower risk than the unconditional model. The correlation models again post a lower risk in 2010
leading up to the Eurozone crisis, then the performance and risk stay relatively constant as correlation and volatilities stabilized.

Figure 5.4  Data set 1: Backtest returns, risk and return to risk ratio for the Unconditional, CCC and DCC models. The correlation models outperform on a risk-adjusted return basis in 2003-2005, 2008 and 2012-2014.

5.1.5 Weights

Figure 5.5 shows the weights of the TSX in each of the backtest portfolios and confirms the heavy weights in the DEX.

The average weight in the TSX of 15.6% for U, 17.2% for CCC and 17% for the DCC. We observe an initial volatile period from 2002 to 2004.

Table 5.2 shows a summary of the weight statistics. The higher volatility of weights is demonstrated by the higher standard deviation of weights for the correlation models of 10% compared to 8% for the Unconditional model. The volatility of the backtest portfolio weights of the correlation models observed in Figure 5.5 is demonstrated by the average absolute change in
weight from week to week being 2.3% and 2.4% for the correlation models and nearly zero for the Unconditional model. A lower volatility of weights is observed for the period of 2005 – 2009 for all three models. This is due to the DEX’s more dominant outperformance of both risk and return throughout the period leading to a high weighting in the backtest portfolio. It can be noted that the three models were never fully loaded 100% into either asset. This signifies that the mean-variance optimizer found that the two assets together always resulted in a lower portfolio standard deviation due to the diversification effect caused by the low (-0.2) correlation between the TSX and the DEX.

Figure 5.5  Data set 1: Weight in TSX found by mean-variance optimization for the three models. The weights are smooth for unconditional and more volatile due to rebalancing for the correlation models.
Table 5.2  Data set 1: Weights of backtest portfolio summary statistics. The correlation models rebalanced portfolio weight changes on average by 2.3% and 2.4% for the CCC and DCC models.

<table>
<thead>
<tr>
<th></th>
<th>U</th>
<th>CCC</th>
<th>DCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>St. Dev.</td>
<td>8.0%</td>
<td>10.1%</td>
<td>10.0%</td>
</tr>
<tr>
<td>Av(Abs(ΔWeight))</td>
<td>0.2%</td>
<td>2.3%</td>
<td>2.4%</td>
</tr>
<tr>
<td>Full Load 1 Asset</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

5.1.6 Summary of Data set 1 MVO Backtest

The DEX is clearly the outperformer in this data set and as a result, all three models found a high weight in the DEX over the backtest period. The correlation models greatly outperformed for the period 2003-2004, through the start credit crisis 2008 and in late 2010 showing spikes of excess returns of 2% for short bursts over the unconditional model. This corresponded to environments of high volatility in the DEX, TSX (compared to “normal market conditions”) and periods of changing correlation (late 2003, 2008, late 2010). One exception was in 2013 where correlation spikes from -0.4 to 0 where the CCC model maintains its performance relative to the unconditional model, however, the DCC model underperformed.

Over the full sample, the CCC and DCC models outperform the unconditional model with slightly higher returns, lower annualized portfolio standard deviations and higher return/risk ratios.

Table 5.3  Data set 1: Annualized return, risk and return to risk ratio for the three models over the full backtest. The correlation models outperform the unconditional model on a risk-adjusted return basis.

<table>
<thead>
<tr>
<th></th>
<th>U</th>
<th>CCC</th>
<th>DCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>4.20%</td>
<td>4.26%</td>
<td>4.23%</td>
</tr>
<tr>
<td>Risk</td>
<td>4.10%</td>
<td>3.94%</td>
<td>3.95%</td>
</tr>
<tr>
<td>Return / Risk</td>
<td>1.024</td>
<td>1.081</td>
<td>1.071</td>
</tr>
</tbody>
</table>
5.2 Data Set 2: TSX and S&P500 (November 1994 – November 2014)

5.2.1 Data Set

The second data set under investigation included 20 years of returns for the TSX and S&P500 from November 21, 1994 to November 20, 2014. The TSX and SP&P500 were chosen as broad market indicators of the Canadian and US markets. This scenario represents the problem where an investor has to allocate equity between the two markets.

Table 5.4 shows the summary statistics for the second data set. The two assets of second data set are now more closely matched in both return, risk and return to risk ratios compared to the first sample. The 20 year data set now better represents the long term return and risk expectations as the data encapsulates stock market returns over multiple business cycles.

Figure 5.6 shows the data set throughout significant market events including the lead up to the technology bubble from 1994-2000, the bubble burst, then lead up to the financial crisis from 2000-2008. Then the bull market from 2009-2014 including the Eurozone debt crisis downturn. It is clear that there are clear sub-periods of outperformance for the two assets. The S&P500 outperforms from 1994-2000 followed by the TSX from 2002-2008, period leading up to the financial crisis. Lastly the S&P500 outperforms from 2008-2014 as the US market recovers from the financial crisis having suffered from a greater bubble burst. The two assets allow the examination of the backtest return models under two more volatile and higher correlated equity assets.
Table 5.4  Annualized summary statistics for data set 2, TSX and S&P500 over the full sample (Nov 1994 – Nov 2014). The two equity assets are comparable in risk, return and risk-adjusted return.

<table>
<thead>
<tr>
<th></th>
<th>TSX</th>
<th>S&amp;P500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>9.39%</td>
<td>9.01%</td>
</tr>
<tr>
<td>Risk</td>
<td>18.3%</td>
<td>19.3%</td>
</tr>
<tr>
<td>Return/Risk</td>
<td>0.512</td>
<td>0.466</td>
</tr>
</tbody>
</table>

Figure 5.6  Data set 2: Cumulative log return for the TSX and S&P500. There are periods where each asset outperforms the other.

5.2.2  Conditional Volatilities and Correlation

Figure 5.7 shows the log returns, conditional volatilities and correlation for the TSX and the S&P500. We observe that the TSX is more volatile in 1998, and from 1999-2000 in the dot-com bubble. The volatility of the both equity assets jumps in November 2008 due to the collapse

The correlation plot is obtained from the DCC model and shows stable, but high correlation between the two indices from 1996-2000, 2004-2006 and from 2011 to current 2014 levels. Volatile correlations are observed in 2001, Tech bubble and aftermath, as well as spikes in early 2007 and in 2008 for the financial crisis. The full sample correlation was 0.6 indicating the co-movement between the two markets.

Figure 5.7  Data set 2: TSX and S&P500 log returns, conditional volatilities and correlation. The volatility of the TSX is greater than that of the S&P500 leading up to the dot-com crisis. Correlation increases in times of turmoil.
5.2.3 Backtest Returns

Figure 5.8 shows the backtest returns and return spreads for data set 2. A very similar return profile is observed for the backtest returns from each of the three models. The return spreads show that the correlation models gain excess return over the unconditional model over certain periods and to help explain the source of the return spreads, we turn to the volatility and correlation shown Figure 5.7.

The first period of outperformance of the correlation models relative to the unconditional model occurs in 1998 where the volatility of both the TSX and the S&P500 increases sharply. The sustained outperformance continues until the start of 2000 where the TSX is volatile, S&P is less volatile and the correlation between the two is high, but relatively stable. The outperformance then jumps back up to 5% in late 2000 due to increases in volatility of both assets and later by a large downward spike in correlation in 2001. From 2000 to 2007 the performance of the CCC then stays relatively stable, and that of the DCC declines steadily, while they both show both show a spike of outperformance in 2004 caused by a sharp downward (and back up) move in correlation. Both correlation models significantly outperform the unconditional model in 2008 by 12% and 10% for the CCC and DCC models respectively. This is also the period where volatilities are the largest for both assets, and correlations shoot up from 0.6 to 0.8. The outperformance is stable from 2009 until the Eurozone crisis in 2011 where outperformance shoots up again. From 2012 until 2014, volatilities and correlations stabilize and the correlation model’s outperformance steadily decline to finish with values of 5% and 4% for the CCC and DCC model respectively.

The correlation models’ outperformance was observed immediately after sharp market declines, such as in 1998, in 2000 and in 2008 indicating the models’ faster adaptation to changing market volatilities.
5.2.4 Risk

Figure 5.9 breaks down the backtest returns, risk and return to risk ratios by year for each of the three models. The risk of the backtest returns is now examined to complement the return analysis in the previous section. The correlation models’ periods of outperformance in 1998, 2003, 2008 and 2012 are periods also correspond to periods of lower risk compared with those of the unconditional model. For these periods, the correlation models completely outperformed the unconditional model. However, during the periods of 2000, 2001, and 2010, the correlation models demonstrated higher risk and lower returns.


5.2.5 Weights

Figure 5.10 shows the weights of the TSX in each of the backtest portfolios. Clearly, the portfolio weights are more volatile than those of the first data set as there is more switching between the TSX and the S&P500. This can be attributed back to the two assets being very comparable in risk and return, with no clear outperformance of either asset over the other over the whole backtest period.

The greater risk-adjusted performance of the correlation models can be explained by the quicker switching ability of the models’ compared to that of the Unconditional. This involved the capability of a quicker and more accurate risk forecast resulting in faster rebalancing between the assets. The spikes in the volatility and correlation of the assets resulted in quick changes in the weights of the backtest portfolios corresponding to outperformance for the periods of 1998-2000 and 2008-2010.

Table 5.5 shows a summary of the weight statistics. The higher volatility of weights is demonstrated by the higher standard deviation of weights of 26.6% for the CCC model and 27.4% for the DCC model compared to 19% for the Unconditional model. These are significantly larger than those found for the TSX and DEX data set backtest. The mean average absolute change in weight from week to week was 11.4% and 11.7% for the CCC and DCC models.
respectively compared to again nearly zero for the Unconditional model. This is caused by the two equity assets being acting as substitutes to each other with similar risk and return characteristics and a high correlation with one another. We also observe that all three models fully load onto 1 asset a small percentage of the time.

*Figure 5.10* Data set 2: Weight in TSX found by mean-variance optimization for the three models. The weights of the correlation models are highly volatile.
Table 5.5  Data set 2: Weights of backtest portfolio summary statistics. The correlation models rebalanced portfolio weight changes on average by 11.4% and 11.7% for the CCC and DCC models.

<table>
<thead>
<tr>
<th></th>
<th>U</th>
<th>CCC</th>
<th>DCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>St. Dev.</td>
<td>19.1%</td>
<td>26.6%</td>
<td>27.4%</td>
</tr>
<tr>
<td>Av(Abs(ΔWeight))</td>
<td>0.7%</td>
<td>11.4%</td>
<td>11.7%</td>
</tr>
<tr>
<td>Full Load 1 Asset</td>
<td>6.4%</td>
<td>6.9%</td>
<td>9.0%</td>
</tr>
</tbody>
</table>

5.2.6  Summary of Data set 2 MVO Backtest

Table 5.6 shows the backtest results over the full sample. The CCC and DCC models outperform the unconditional model with slightly higher returns, lower annualized portfolio standard deviations and higher return/risk ratios.

Table 5.6  Data set 2: Annualized return, risk and return to risk ratio for the three models over the full backtest. The correlation models outperform the unconditional model on a risk-adjusted return basis.

<table>
<thead>
<tr>
<th></th>
<th>U</th>
<th>CCC</th>
<th>DCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>4.61%</td>
<td>4.75%</td>
<td>4.71%</td>
</tr>
<tr>
<td>Risk</td>
<td>15.29%</td>
<td>14.93%</td>
<td>14.91%</td>
</tr>
<tr>
<td>Return / Risk</td>
<td>0.301</td>
<td>0.318</td>
<td>0.316</td>
</tr>
</tbody>
</table>

While backtest data set 1 showed that high volatility and changing correlation environments drove the outperformance of the correlation models, the backtest on data set 2 revealed a further step in the cause-reaction chain. To elaborate: the correlation models outperformed for the periods of 1998-2000 and from 2008-2010 on a risk-adjusted basis due to quick and large weight changes in the backtest portfolios as the models rapidly adapted to increased and changing levels of volatilities and correlation.

The outperformance of the correlation models is consistent with the previous studies of Wu (2010), Otranto (2009) and Billio (2006), however the results obtained demonstrate the outperformance over a longer time period and with both risk and returns considered throughout the whole backtest period.
6 Conclusion

We use two data sets of daily returns of broad North American stock and bond indices to assess out-of-sample performance of dynamic and constant correlation models in the context of portfolio choice and value-at-risk (VaR). Specifically, we describe and estimate the dynamic conditional correlation (DCC) model of Engle (2002) and the constant conditional correlation (CCC) model of Bollerslev (1990) to forecast the covariance matrix on a rolling estimation window basis. This one-step-ahead covariance matrix is used as an input into mean-variance optimization (MVO) for portfolio choice or a parameter to produce a simulated distribution to calculate VaR.

Our portfolio choice backtest results on two data sets, the TSX/DEX and the TSX/S&P500, showed that the CCC and DCC models generally produced superior results compared to the unconditional estimate of the covariance matrix. We found the correlation models’ outperformed on a risk-adjusted returns basis due to their agile adaption to periods of increased and changing market volatility and correlation.

For VaR, we use the Kupiec (1995) test to determine if the number of exceptions over a given period are acceptable and either accept or reject the model. Using this methodology, we find that DCC has an incrementally better failure rate compared with CCC and unconditional methods, however the distributional assumption of the residual innovations is a much more significant factor when judged by the number of rejections.

Market risk management has assumed a higher profile in the financial industry especially since the global financial crisis. VaR continues to be used as a tool for measuring the market risk of a portfolio and has also been mandated through regulation. There is a large and growing literature analyzing the efficacy of this tool in forecasting the future distribution of portfolio returns. From incorporating dynamic volatility through univariate GARCH and dynamic correlation through the DCC model, the techniques used to forecast covariance have grown in sophistication to attempt to match the statistical properties of financial time series. However, as can be seen from this paper and many others it followed, VaR models are not very reliable in forecasting future market risk especially in times of crisis.
Reference List


