A Conversation–Gesture Approach to Recognising Mathematical Understanding in Group Problem Solving
(Teaching from the Sidelines)
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Abstract

Teaching from the sidelines is the practice of allowing students to work in small group settings while actively, but not intrusively, monitoring their progress. This requires that the teacher is looking for and listening to actions that unfold through group talk. By viewing understanding as ‘an ability to go on conversationally’ this research puts forward a method by which teaching from the sidelines can be a productive activity. The idea of knowledge as a basis for action and understanding as a dynamic activity that develops from this knowledge builds on the work of Wittgenstein, Sfard, and Davis. Techniques of conversation and gesture analysis are employed to examine examples of classroom talk to support and develop these ideas. In being more aware of the conversational organization of group talk, teachers can be more attuned to how mathematical understanding is developing. Variations in student gesturing are examined to illustrate how students who are more confident in their knowledge generally use larger gestures, particularly when presenting their ideas. The research also finds that groups of students tend to mimic (echo) each other’s gestures and posture when they are working together to develop a shared understanding.

Keywords: Understanding; Conversation; Gesture; Mimicry, Group work; Prosody
Dedication

To my wife, Karen, for putting up with me over time it has taken to do this.
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Introduction

In a sense, then, mathematical knowledge is like the subject matter of a conversation. It exists only in conversing, and its nature, its structure, and its results can never be anticipated, let alone fixed. (Davis, 1996, p. 23)

In this introductory chapter, I lay out some broad ideas which helped lead me to start this direction of research. My intent is to give a basic account of why I feel an investigation of classroom conversation can be important for teachers. I return to many of these ideas in later chapters in order to unpack them further.

A feature of the reform-based classroom since the early 1990s has been a shift in practice away from procedural understanding and towards conceptual understanding. There has been a push for a deeper understanding of mathematical ideas (e.g. Hiebert & Carpenter, 1992), and a central focus of this has been promoting talk\(^1\) in the classroom (e.g. Williams & Baxter, 1996). Mathematical talk which involves students’ explanation of and the defence of ideas is seen as a hallmark of effective teaching (e.g. Brophy, 1999; Sfard, Forman, & Kieran, 2001), along with observing and listening carefully to students. Recognising what effective talk sounds like, and how it can be used to enhance desirable outcomes, is more problematic. Research into the forms of talk that are common to the classroom suggest that it proceeds differently from everyday conversation and differs further according to the philosophy of the teacher. Cazden (2001), for example, discusses how talk differs between the traditional and non-traditional classrooms, while Skidmore (2003) points to differences within those categories. In addition, there are

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\(^1\) In many research papers the term ‘discourse’ is used when referring to classroom talk. I avoid the use of this practice here, so as to not confuse the term with its more specific use in Discourse Analysis, unless I am quoting directly or making reference to a term outside of this study. By ‘talk’ I mean mathematical talk rather than social chat.
many forms of communication in the classroom, some of which can occur simultaneously. Within a whole-class discussion, there may be several smaller exchanges taking place, while many students may not be engaged at all.

In this research, I want to pay more attention to the informal conversations held by students as they work together. By ‘informal’ I refer to conversations held between students without a teacher being immediately present. My intent is to examine the way the students interact in conversation, and how these conversations are organised, in order to develop a sense of how ways of knowing are shared and how a mutual understanding is developed.

Hiebert & Carpenter (1992) note that “One of the most widely accepted ideas within the mathematics community is that students should understand mathematics” (p. 65). Despite this ‘accepted idea’, my sense from the literature is that interpreting the word ‘understanding’ is far from easy - and relating knowledge and understanding adds another layer of complexity. At a basic level, my interest in starting this research was an attempt to develop a better sense of what it means to understand something, and how that understanding is expressed in a student’s mathematical endeavours.

I was initially intrigued by the work of diSessa (2001) and the use of the ‘Forces Concept Inventory’ (FCI) (Hestenes, Wells, & Swackhamer, 1992) in physics teaching. Through the use of carefully designed questions, the FCI seemed to give insight into the way students thought about forces in a way that standard testing did not. While assessment is not my goal in this research, this initially sparked an interest in the idea of a less formal way to examine a student’s mathematical understanding. Familiar theories of understanding (e.g. Skemp, 1987; Hiebert & Carpenter, 1992; Pirie & Kieren, 1992; Sierpinska, 1994) seem to agree that understanding is an aspect of the individual’s mind, and this was something I began to wonder about. From my experience of working with students I have often found, when talking with them, that they know much more than they
use when facing a problem in mathematics or physics. If understanding were structured as the theories suggested, why was that structure not more readily available to the students? This prompted me to consider the difference between knowledge and understanding. The theories of understanding only seem to go so far in this regard. A further confusion comes from the often interchangeable use of the terms ‘knowledge’ and ‘understanding’. This was something I knew I would need to examine more closely, and I do so in subsequent chapters. ‘Knowing’ the mathematics did not always seem to correspond to the student’s ability to start or to be able to ‘go on’ with a problem. This notion of being able to ‘go on’ developed further as I became familiar with the works of Ludwig Wittgenstein (1967) along with his less-structured views concerning understanding. As I pursued these ideas, I became more focused on the actions of the students; not just their spoken words, but also the organization of their speech and the physical nature of how they were speaking and being involved – or not – in the activity. Bruner (1990), expressing concern about how the cognitive revolution was treating the mind as an information processor rather than as a creator of meanings, described an act of meaning as being somewhere between action and explanation. I began to wonder if a closer analysis of students’ conversations while doing mathematics, particularly in the organisation of these conversations, could give more insight into what was expected from a student in ‘understanding’ something.

There has been a growing awareness that social interaction is a principal way cultures are transmitted or changed, and that mutual understanding and acceptable actions are shaped by communication and conversation. Reform-based mathematics teaching further encourages the development of collective working, questioning, and eliciting student responses. Carpenter et al. (2003), for example, suggest that students cannot learn mathematics with understanding without engaging in talk. Through such talk, students are believed to become more engaged in the thinking that leads to a solution.

In using the terms ‘problem’ and ‘task’ I am guided by Christainsen and Walther (1986): “In view of the educational needs, it seems to be more fruitful to conceive of the field of tasks as a spectrum extending between two poles: tasks, for which a complete procedure leading to the solution is known (often called ‘exercises’); and tasks (with Aporie) for which such a procedure is unknown (often called ‘problems’)” (p. 272).
rather than fixating on the solution itself (Yackel & Cobb, 1996). A number of research studies focus on supporting this process by illustrating how a teacher can develop better questioning techniques to expose students’ thinking (e.g. Herbel-Eisenmann & Breyfogle, 2005). Cazden (2001), for example, has noted that “spoken language is the medium by which much teaching takes place and in which students demonstrate to their teachers much of what they have learned” (p. 432). Such large-scale ‘classroom talk’ has been frequently examined as a means to view learning in the classroom (e.g. Sinclair & Coulthard, 1975). As a result, there is now a large body of research suggesting how important it is that students participate in classroom talk.

It may seem, therefore, that it is less a question of whether to incorporate talk into the classroom, and more a question of how it is best to be done. Sfard (1998), however, does take the time to step back and ask why we believe in learning mathematics through conversation (in the sense of an interactive oral exchange). Sfard frames cognitivist, interactionist, and neo-pragmatist arguments as theoretical frameworks for using classroom talk, before posing to a panel the question “Do you believe that mathematics can and should be learned through conversation?” I shall return to examine these frameworks at a later point, but for now let me share Sfard’s conclusions that the teacher plays a key role in the success of the classroom talk model. She notes:

*There are many ways to turn classroom discussion or group work into a great supplier of learning opportunities; there are even more ways to turn them into a waste of time, or worse than that – into a barrier to learning.*

(Sfard, 1998, p. 50)

If this is the case then it is important that a teacher develops a sense of when classroom group talk is productive and when it is not. Further, in a review of over 100 research articles on such classroom talk, Walshaw and Anthony (2008) conclude that, “teachers who have the intention of developing student understanding will not necessarily produce the desired effect” (p. 539). They further offer that, “effective teachers plan their classroom discussions with many factors in mind, including the individual student’s knowledge and experiences” (p. 539). This again seems to point to the need to have a better sense about what classroom talk is telling us. In contrast, if the goal of the teacher is to give the students space to think and generate their own solutions, the teacher needs
to be away from the focus of the group. The teacher needs to be on the sidelines observing, in the manner of a coach, rather than 'in the activity,' participating – except when there is a need to do so. There is a need for guidelines a teacher can use in order to fulfill the role of actively making sure that the group work is not a waste of time (or worse) and yet not stifle the students’ talk.

If our classroom talk is to be productive, we should take time to examine what it is we are looking for from this talk. Is there something that can give pointers to whether or not a student is using talk in a productive manner? Along the way I became interested in the work of Harvey Sacks in developing Conversation Analysis. Briefly, Sacks was able to determine the seriousness of a caller to a suicide hotline from the way he or she engaged in a conversation. I began to wonder if one pointer to mathematical understanding is the student’s ability to engage in a mathematical ‘conversation’. One feature of Conversation Analysis is to gather data with an open mind, to transcribe the data, and then look for patterns that emerge. With this in mind, I recorded some student talk in the classroom as a trial. While these did not go well in terms of being able to distinguish the students’ utterances from the background noise, I was struck by the gesturing the students employed. As a consequence, I felt it necessary to broaden my outlook to include an analysis of the gestural component of the students’ talk. Conversation Analysis and gesture analysis seemed to share a common goal in revealing something about the thinking of the speaker, and to perhaps help interpret the actions of the speaker (although I later realised that ‘revealing’ and ‘interpreting’ are troublesome notions in themselves). I began to feel that this combination of conversation and gesture may provide what Susan Gerofsky (In private talk, 2013) referred to as ‘embodied evidence of learning’. While these are topics I will return to later in greater detail, I began to recognise that I had opened something of a Pandora’s Box when delving into this area. In particular, my journey would force me to a re-evaluation of my philosophy and beliefs about learning mathematics. What results in this document reads back as a linear progression of ideas, but the reality was far from this. My intention, however, is to try to give some idea of the progression by presenting both my pilot study and my final research as a means to indicate how my research questions evolved. A significant change in thinking, however, came from the realisation that there are no
definitive answers to the questions I pose; at best I hope to have shed more light on the possibilities and to offer a different lens through which to look.

I begin by briefly outlining some features which shape the nature of this research. These are features to which I shall return in later chapters, but are necessary to be able to introduce the pilot study which helped to develop my research questions. I first outline how I view the difference between a conversation and a discussion.

**Conversations and Discussions**

In this section, I will outline an important difference between a conversation and a discussion which I will use to guide the rest of my research.

Sfard (1998) completes her panel conclusion (outlined above) by noting that “perhaps all one can say right now is that for a conversation to be productive, it has to have the characteristics of a true dialogue” (p. 51). She cites the work of Gadamer (1975) for whom a true dialogue is:

> a process of two people understanding each other. Thus it is characteristic of every true conversation that each opens himself to the other person, truly accepts his point of view as worthy of consideration and gets inside the other to such extent that he understands not a particular individual, but what he says. (p. 347)

With Gadamer’s directive in mind, I refer to the distinction Davis (1996) makes between a conversation and a discussion. I will elaborate on this in a later section, but it is important to point out at this stage that, “in the conversation, all the participants are oriented towards deepening their understanding of the issue at hand” (p. 27) while the goal for a discussion is “more towards the articulation of pre-formulated ideas” (p. 27). Davis further notes that a conversation is a fluid thing which meanders towards a non-specific destination, but one that will be commonly known. In a discussion, the participants have a goal to retain their views; they are not open to change. Two points of interest arise here from Davis’s writing: firstly there seems an implication that the interlocutors in the conversation need to be willing to engage in the process; and secondly, in order to be able to engage in the type of conversation both Davis and Sfard hope to encourage, a
student must be able to interpret an interlocutor’s utterances in a meaningful way. One issue Davis points out, however, is that “we can never be aware that a conversation is taking place, (only) that one has taken place” (p. 28, original emphasis). This issue was something I questioned; if teachers could be more aware of how group conversations were organised, then I felt that they might be alerted to when a conversation is taking place. Moreover, it might be possible to recognize when and why the conversational aspect of a group’s talk was breaking down. My thinking was that, in any exchange between students (or between students and their teacher), the organization of the conversation, as examined through Conversation Analysis\(^3\), may prove to be insightful. In the same manner, aspects more related to a discussion – as defined above – may also serve to inform the alert teacher and be used to further guide instruction, either by immediate intervention or in future planning. I recognise, of course, that any findings in this research come from examining video recordings after the event, so in that respect Davis’s point is well made; my goal was to find if there are clear indicators to which, once alerted to, a teacher can, or perhaps does, attend to in real time.

It is important to point out that this is a finer-grained view of the conversation than may often be used. Mason (Sfard, Nesher, Streefland, Cobb, & Mason, 1998), for example, suggests that “conversation is sometimes taken to cover any verbal interchange” (p. 48). This general view can also include the ‘historical detritus of previous conversations’ which shape, as Mason takes from Maturana (1988), the way of being in the world. In contrast, Mason refers to the discussion as a more focused conversation while at the same time noting that it requires genuine student involvement and contains a “great deal of unfocused or off-task interaction” (p. 48). My interpretation of Mason’s view, and by extension the more generally accepted view he refers to, is that he is less concerned with such a distinction and more concerned with fine-tuning the notion of a discussion to the

\(^3\) I leave a more detailed examination of Conversation Analysis to a later section. Briefly though, “CA as practiced by Sacks, Schegloff, Jefferson, Pomerantz and others, is a rigorously empirical approach which avoids premature theory construction. The methods are essentially inductive; a search is made for recurring patterns across many records of naturally occurring conversations” (Levinson, 1983, pp. 286-287) When referring to this method I capitalize the word ‘Conversation’.
point where it consists of a ‘conjecturing atmosphere’. In this atmosphere everything that is said is uttered with the intention of making thoughts external, examining them critically, and modifying them as a result of other people’s comments. While deferring a more detailed analysis of the conversation to a later section, my interpretation of the process of Conversation Analysis (Sacks, 1995) leads me to suggest that it can be applied at the level where the ‘conversation’ is a match to Mason’s ‘conjecturing atmosphere’ and, at the same time, adheres to the distinction between conversation and discussion suggested by Davis.

Before going further down this line, however, I wish to develop the idea of using conversation to help position the student within the learning process. To do this I examine the idea of how a teacher purportedly views a student’s ‘knowledge’ and ‘mathematical understanding’, and consider why this is important. To this point, I note that engaging students in talk is by no means a new form of assessment as historically oral exams were used to assess understanding and, for example, a defence of a thesis is still an oral undertaking. Similarly, applicants for some universities – such as Oxford and Cambridge – still have to participate in an interview before being offered a place. Clearly, there is something extra that can be found in such a process that is seen to have value. Part of this must be the ability of the interviewer to manage the flow of the talk, so in this regard a natural conversation may not take place. However, there may be conversational elements that arise which allow the interviewer(s) and the interviewee to come to what they see as mutual understandings about some topic. My intent is not, however, to develop an assessment method, but to help provide the classroom teacher with more tools to help position the student in the learning process.

In earlier research (Wells, 2009), I was particularly interested in the way students initiate their thinking when problem solving. The entry phase of a problem is one of the four stages identified in most problem-solving heuristics (e.g. Pólya, 1945; Mason et al., 1982; Schoenfeld, 1985; Perkins, 2000) and as such is an important place to look for initial evidence of the development of a conversation.
The Initial Phase

I began by being interested in the initial phase of a student’s learning at the start of a unit or topic. In this phase, it has been suggested that the student may hold many so-called ‘misconceptions’ which can be strongly held and hard to change. Such misconceptions have been thought to play an important role in the development of students’ conceptual ‘understanding’ (Smith, diSessa, & Roschelle, 1993), and identifying these is frequently seen to be an important starting point for educators in determining a course of instruction. Building on current understanding, and making connections to existing knowledge, is considered an important aspect of a constructivist theory of knowledge which has underpinned many of the curriculum changes of the reform movement around the world (e.g. Olssen, 1996). Stevenson (1998), for example, referred to constructivism as “one of the dominant paradigms in mathematics education” (p. 93). Issues with a constructivist approach exist, however, as I shall outline later, but with its pervasiveness in mind, I was initially drawn by the pedagogical question of ‘how can a teacher determine where a student’s level of mathematical understanding is before introducing new material?’ As my research developed, however, I came to see that this question was not so much the wrong question, as an invalid one. However, one purpose of this introductory chapter is to lead to that conclusion and so I shall return to it in due course.

Diagnostic tools are available that purport to allow the teacher to gain some insight into students’ knowledge, but these are often in the form of multiple-choice or short-answer questions, which may not be sufficient to give a clear picture of the student’s conceptual mathematical understanding. Indeed, even the notion of a concept as a static object appeared problematic. What seemed necessary was a shift in thinking about the nature of knowledge and understanding as ideas in themselves. In order to support the ideas taken from researchers such as Sacks and Sfard, as mentioned above, I saw the need to review my ideas around learning. A starting point for such a change came through reading Rorty (1979), who viewed conversation as the ultimate context within which knowledge is to be understood. I interpreted this to mean that understanding is
developed in the conversation, rather than being indicated by the conversation. Further to this idea, in Social Constructivism, conversation is thought to be central to learning (Ernest, 1998). In a classroom where group work and collaborative problem-solving is a focus, conversation⁴ between students is considered a key part of the process. Again, rather than looking at conversation as a way to measure understanding as a state that had been achieved, the implication seemed to be that conversation was a means to understanding. I wondered if paying attention to these conversations might be a productive way to gain fresh insight into a student’s thinking.

In knowing how to look at these ‘conversations’ in the broader sense of classroom talk, Levinson (1983) describes two main approaches as *Discourse Analysis* and *Conversation Analysis*. Discourse Analysis was developed in the 1970s as a focus within Conversation Analysis and “identified some of the distinctive formal characteristics and social purposes of talk in schools” (Williams & Baxter, p. 361). Discourse analysis (DA) extends techniques used in linguistics to beyond the unit of the sentence. For Levinson (1983), DA has “a tendency to take one (or a few) texts and to attempt to give an analysis in depth of all the interesting features of this limited domain” (Levinson, 1983, p. 286). Pimm (1987) introduced discourse analysis to mathematics education research and since then there has been growing interest in mathematics classroom discourse. I will return to discuss this in more detail in Chapter 4, but here I will briefly compare this approach to Conversation Analysis.

Conversation Analysis (CA) has a focus on examining the organising structure of conversation over a large number of recordings in a particular area. A pattern of involvement is suggested in CA which seemed connected to my interest in how students participated in a conversation. In other words, I became interested in seeing if the way

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⁴ Ernest (adopting Grice) describes a spoken conversation as a “sequence of linguistic utterances in a common language made by a number of speakers who take it in turns to speak and who respond with further relevant contributions” (Ernest, 1998, p. 163). While this could also describe a discussion, he later cites Bakhtin (1986) in adding that “Truth is not found inside the head of an individual person, it is born between people collectively searching for truth” (Ernest, 1998, p. 164). With this addition I believe Ernest is using conversation in the same sense as me, and so I retain his reference to the word ‘conversation’.
classroom talk developed in group work had a recognisable pattern. Barwell (2003) adopts a somewhat similar approach in using CA as a framework to look for patterns of attention in students’ talk.

In thinking in this way I was also intrigued by the additional, non-verbal, aspects of conversation such as the use of prosody and body language. It seemed that incorporating the research on this, and particularly that of gesture, could be important. Indeed, McNeil and Duncan (2000) refer to speech and gesture jointly as an enhanced ‘window’ into thinking’. Although McNeil (2005) later moved away from this acquisitionist view of what thinking is, I began to see the possibility of using these two ways to study students’ mathematical understanding.

**Gesture**

While Social Constructivism represented a shift towards recognizing that mathematics may be socially constructed as well as a mental construct, theorists of Embodied Cognition go further and consider that the construction is rooted in and shaped by the body (e.g. Barsalou, 1999; Garbarini & Adenzato, 2004). Many theories reject the mind–body duality of Descartes and instead seek an integration of the sensory and intellectual functions. Abstract conceptual knowledge is considered as built through bodily experience via concrete objects or virtual objects that mirror reality. Bodily experiences ground the abstractions that are the basis of mathematical thought (Lakoff & Núñez, 2000). Doing and communicating mathematics involves a range of actions involving the body, from “committing inscriptions to paper or whiteboard, to speaking, listening, gesturing and gazing” (Edwards, Ferrara, & More-Russo, 2014, p. 1). Although there are variations within the field of study, all versions share the goal of explaining how the mind and body mutually interact with, and influence, each other and the world (Cowart, 2005). The shift towards an embodied view of human experience lends more weight to the consideration of bodily actions such as gesturing and posturing and their role in education (e.g. Goldin-Meadow, 2003). Although our thoughts are private, we all share the same basic cognitive and biological capabilities. As a result, our gesturing may well have a common basis to which we can attach meaning. By attending to a student’s body language, and to gestures in particular, further insights may be possible with regard to a student’s understanding of a mathematics topic.
I will return to the topic of gesture in more detail at a later point as this is a topic that must be unpacked further, particularly in regard to cultural differences. The culture of the classroom, the social environment, and the nationality of the student are all factors which need to be addressed. There is evidence, however, that facial gestures may be universal (Matsumoto & Hwang, 2011) and, given our common physiology, may be biologically innate abilities. If so, spontaneous gesturing may be revealing across all cultures.

**Conversation–Gesture Analysis - Organisation of the dissertation**

My intent, then, was shaping itself up to be an examination of a student's developing understanding of a mathematical topic through a combination of conversation and gesture analysis, which I shall refer to as CGA. I felt CGA might offer possibilities to broaden the platform from which we can gain valuable insight into this area. To make better sense of this idea, I decided that a pilot project would be necessary and this turned out to be important, not only in realising what I needed to be able to do technically to record students in action, but also in helping to shape my ideas about knowledge and understanding. For this reason, I feel the pilot project is important to include, and it is the topic of the first chapter of this dissertation.

The question of ascertaining knowledge and understanding, however, is deep and requires further unpacking. Before I could go forward and investigate the conversational organization of classroom talk in further detail, I realised a need to establish how I interpret the terms I would be working with. The way in which terms such as knowledge, understanding, discourse, etc. are used is often not made clear. Barwell (2013), for example, points out that it is “reasonable to examine what a researcher might understand by ‘knowledge’” (p. 597), while recognising, in the case of research on teacher knowledge, that this has not always been specified. My intent in Chapter 2 is to clarify how I have come to interpret the term ‘knowledge’ in terms of what is developed through the actions and interactions of individuals. In order to reach this viewpoint I examine in more detail some theories of knowledge which have informed me along the way. While knowledge itself is not the focus of study in this research, I feel it is an important discussion to have before going into to a deeper examination of understanding. In this sense, I view clarification of the term knowledge as foundational before looking at understanding.
In the same vein, it seems reasonable to examine how a researcher interprets the term ‘understanding’ and this is the topic of Chapter 3. A key point, for my own development, has been a shift from thinking of knowledge and understanding as processes contained within the head, to seeing them as dynamic processes developed within classroom talk. In Chapter 3, I examine in some detail what I think of as difficulties common to many theories that describe understanding. In short, I see many theories on understanding to be challenged in trying to describe invisible structures within the mind. Such theories have a tendency to get more and more complicated by adding new structures to account for further research findings. I will argue, based on the work of Wittgenstein (1967), that understanding should be seen as developing in the space of classroom talk, and recognised as the ability to ‘go on’ conversationally.

In Chapter 4, I build on these definitions of knowledge and understanding and tie them into the existing literature in order to develop my theoretical framework. For this framework, I tie the work from Chapter 2 and 3 into the work of Sfard (2008) and Davis (1996), among others. The intent in Chapter 4 is to connect my definition of understanding into the ideas around conversation to provide some manner in which such understanding can be recognised and facilitated in the classroom. If understanding is seen as an ‘ability to go on conversationally’, and a connection between mathematical progress and conversation can be established, then there are good grounds for suggesting that mathematical understanding is being developed within the space of classroom talk.

Chapter 5 outlines the method used in this research in order to capture and analyse the classroom talk. This chapter also describes the markup language used in transcribing the recordings in greater detail. Doing transcriptions is seen as important in CA in that it forces you to listen very carefully to the talk. Repeated watching, listening and adding to the transcription enable the researcher to see things that were not immediately obvious (ten Have, 2007). This is particularly true in this research where noticing of gesture and prosody were also seen as important. Chapter 9 illustrates how careful noticing brings out features that were not immediately obvious, in gesture and posture echoing, but might well have important subconscious ramifications.
Chapters 6 through 8 present the main findings of this research, with a focus on the conversational organisation of the talk. In particular, the manner in which progress in a problem is seen to correlate with the conversational aspects of the talk is highlighted. The apparent three-part structure of the group work is also demonstrated using examples taken from the recordings.

Chapter 10 summarises the findings of the research and ties these findings back to the research questions formulated in Chapter 1.

**Summary so far**

Teaching from the sidelines is about making space for students to develop their understanding through classroom talk while at the same time ensuring that this space is not counter-productive. It is about the teacher watching and listening before directly interacting with the students. My intent was to examine examples of both productive and counter-productive talk and determine if a practical set of guidelines could be formulated to help the classroom teacher foster a successful learning environment. My intention was to look for patterns related to how the conversational and gesture space (including posture and body language) was organised and used within group talk. If these patterns exist, they may provide a sense of students' developing mathematical understanding. Recognising how mutual understanding is developed when students talk, both to establish a common ground to build on and to work together mathematically, may alert the classroom teacher (and interlocutors) to the progress of this understanding. Paying attention to these patterns (consciously or subconsciously) may help interlocutors adapt to each other's thinking within the group talk. Being more aware of these patterns might help to know, for both teacher and student, when to intervene and when to stand back in order to allow understanding to develop further.
Chapter 1

Examining Classroom Interactions: An Exploratory Study

“The mind has been pushed out of the head and into social settings.”
(Davis, 1996, pxxii)

In order to give a better sense of the interactions which occur between students when working together in solving a math problem, I felt it was necessary to undertake some exploratory studies. I share these studies here, and what that led to them, because they represent an important development in my own thinking in this area. They situate the background reading from which the research grew, and serve to shape the questions that led to the research that followed. While these studies could be omitted without affecting the final results, I feel their inclusion plays an important role in documenting the journey that is the research. My intent for these studies was to take an open-minded approach to the data with a view to using the results as a guide in further studies. To say that I had no pre-conceived notions going into this research would be an exaggeration, but I also felt it important to adopt an approach which allowed the data to lead me in my findings.

The approach that initially guided these investigations stemmed from my reading and interpretation of Davis (1996), in which he emphasises the importance of listening to the students talk. Davis espouses an enactivist approach to research and I begin by outlining how this informed my early research. Later, in Chapter 4, I outline my theoretical framework in more detail and use that as a more developed way of grounding my full research. The work of Davis, however, was instrumental in guiding the process and I therefore outline it here. I support and develop this section with other readings which were helpful at a later stage but which I feel are more appropriate to include at this point.
The Middle Way

Davis’s framework is Enactivism – a manner of thinking he describes as seeking out the middle way. This favours analogy over logic, embraces complexity, and values language for its capacity to transform reality rather than capture it. The intent is to embrace the past, but to temper it with the new ideas. Davis wants to shift from the purely ocular to include the audible and promote the idea of listening. Sound is seen as lying between what is known and what is not known; listening is described as offering a more generous and compassionate way of ‘seeing’ things along with a wariness of certainty (p. xxvi). Davis adopts the term *bricolage* (from Levi-Straus), referring to a tinkering or delving into possibilities, and seeks for the ‘good enough’ rather than ‘optimal’ teaching method. He avoids saying what math teaching is, but nonetheless seeks to locate it in the realm of personal and collective action. It was this notion of collective action and an open-minded approach to the process that initially caught my attention.

Davis rejects the modernist notion of self as an autonomous entity isolated from others, independent, static, and able to maintain integrity through diverse experience. Drawing on the work of Heidegger, who cautioned against separating what we know from who we are, Davis promotes investigating the conditions that make certain understandings possible – not just what we think but also how we come to think that way and how this affects the way we act now. Understanding is neither objectively fixed nor subjectively constituted – “it is negotiated with others through communicative interaction” (p. 23). Davis quotes Weinsheimer in adding “there is no fixed truth – truth keeps happening” (p.19). I see this being interpreted in the sense that understanding is a product of the conversation rather than being brought to the conversation. During this process, knowledge and subjective truth are also being developed out of the elements of previous knowledge that each participant brings to the conversation. This view of learning and knowledge is, to paraphrase Coles (2013), “intimately linked to action, intention, and expectation – a viewpoint central to the ‘enactive’ approach to cognition” (p. 36). In an enactive approach (Maturana & Varela, 1987), an action is seen to be effective if it allows one to continue in a certain way. As Coles notes:

Any interaction between teacher and student or between student and student must, in the process, alter the structure of both. Enactivism is a
profoundly social theory, we are quite literally changed through interaction with others, or more precisely, we change ourselves through interaction with others who likewise change themselves. (p. 45)

The Hermeneutic Conversation

Davis outlines what he refers to as a “hermeneutic conversation” (p. 26); taking an interpretive approach which recognises the pivotal role of language in understanding, along with the interplay of part and whole in the process of understanding. Important to this process is recognition, drawn from Gadamer, that there is a bi-directional relationship between the research question and the researcher – the way we inquire shapes what we find. The sense here is that a too-rigid approach to a methodology can lead to a particular result which is narrow and reflective of the researcher’s biases, instead of offering a situation where possibilities can occur. There is not only recognition of a theoretical framework pinned on previous research, but also an interrogation of why this is used. There is a need to constantly re-evaluate one’s position in the research. Questions are not so much fixed as they are ideas to be explored, and taken-for-granted modes of speaking and acting are open to question. A key part of this exploration is the conversation, which holds the potential for new (shared) understanding. In a conversation, participants are open to the views of the group and shape the direction of talking. But the conversation is seen as more than just the intertwining of separate voices, and Davis draws on ideas of Merleau-Ponty, Gadamer, as well as Maturana and Varela to develop a sense of participants ‘being joined’ in a conversation with the topic. Illusions of subjectivity are set aside, allowing a collective consciousness to emerge. Davis emphasises that “It is our capacity to listen — that is, our ability to attend to and to interpret what is said — that makes conversation possible. The conversation, enabled by our capacities to listen, is a meeting of (embodied) minds” (p. 42).

Listening

An important aspect of my emerging methodology was in a sense needing to listen carefully for any conversations that emerge as students work on an assigned problem. As Davis points out, the modernist notion of information flow is false; rather than promoting conversation we often simply want people to think the way we think. By
implication then, the teacher needs, at times, to step back and allow the conversation to
develop and be a listener. Davis points out that you can see listening in a conversation
through body interactions: leaning in, reaching out – closeness and lowered inhibitions.
The listener must be active and participatory; questioning and challenging, exhibiting
emotion – connected to each other in what Davis calls a “joining of minds” (p. 38). Davis
thinks of listening as an embodied action and a way of participating rather than a
conscious action. Listening is a ‘reaching activity’ involving our whole body: not only arm
gestures, but by extension posture and mood. If these are indeed features of
conversations involving developing understanding, might they be clearly observable
features?

Davis notes “the plethora of noise picked up in a recording we don’t hear
normally” (p. 45), and this was certainly the case in my initial attempts to capture
students’ talk in the natural setting of the classroom. Talk that could be easily
eavesdropped in situ became impossible to decipher on a recording. Being cut-off bodily
from the real life situation left me unable to pick-out the individual as I could in the
classroom. Davis notes that the notion of ‘voice’ begins to break down as a single
concept; rather, it is a chorus of voices. This shapes what we listen for – what we expect
or anticipate. There is a sense that our perception of an event is determined by how we
listen, how we choose to block out that which we deem unimportant.

Davis outlines three types of listening, to which he refers throughout his book. In
summary, these are:

1. **Evaluative:** contributions are assessed as right or wrong. There is no listening to
   learning, but listening for something in particular. Any response that does not match the
   expected one is brushed off.

2. **Interpretive:** The teacher is encouraged to get at what students are thinking and
   so be willing to revise expectations. Assessment changes to be more subjective rather
   than merely what has been learned. The teacher is aware that reaching out rather than
   just taking in is involved, and that internal biases are projecting onto one’s own
   understandings. In this case the teacher is listening to the students but they still do not
change the lesson plan. The mathematics is still about constructing associations between signifiers but the teacher does not specify every step.

3. **Hermeneutic:** This is more negotiatory, engaging, and messy. It is the “participation in the unfolding of possibilities through collective action” (p. 54). There is an absence of a clearly structured format and a set of prescribed learning outcomes. In this case, class members are jointly exploring mathematical issues. The mathematics is located in the activity and the interaction is dynamic. There is a focus on the possibility of the mathematical (and hermeneutic) interpretation, not on particular concepts. The belief that teaching is a matter of causing learners to acquire, master, or construct particular understandings following a rigid plan is abandoned by the teacher.

It would seem that Davis has the intent of viewing this as a guide to a philosophy of teaching in a more general sense that I am interpreting it for this study. My goal here is not to suggest a teaching method but rather to use these ideas as a way to be open to the possibilities which arise in a particular teaching situation. As Mason (2008) notes, awareness becomes “the base for action” (p. 62). I wanted to be open-minded in interpreting the output from the group situation. Rather than go in with fixed ideas and looking for specific outcomes from the problems posed, I wanted to be open to any sense of a developing understanding emerging through the conversation. I was also aware that in doing so I was interacting with the data in a specific way and so wanted to be cautious about how I interpreted the results. I was aware that I was going into this research with a theoretical framework in mind, that which I examine in detail in Chapter 4, but I wanted to use a pilot study to allow my specific research questions to emerge, and to be constantly open to changing my focus as suggested by the data. I also wanted my approach to be flexible, as Coles (2013) notes, to allow for multiple perspectives and the use of “different frameworks or filters for interacting with data and revising data” (p. 46). In addition, as Coles also notes “I do not assume that I am gaining access to anything stable or fixed” (p. 53). These viewpoints, although later articulated, seem to emphasize the manner of approach I was looking for in investigating the developing nature of understanding within the conversation. An interesting observation by Coles was that there was a sameness in the process of how talk unfolds in the classroom, year after year. Although each year the specific patterns of language would be different, there was “some stability in those patterns across years” (p. 56). Coles refers to this as ‘equifinality’, and draws from this
the idea that, at the end of a particular piece of research the final piece of data should exhibit the most stable (meta) patterns. This data informs future data collection and in this way the analysis of patterns from my initial study informed my analysis for the main study. As Coles writes, in answer to Plato's dilemma of learning:

We learn about the things we do not even know exist by staying alert to the detail of what we see, by sharing the detail of common experiences with others and by deliberately not seeking generalities beyond those that support effective action. (p. 58)

With the above ideas in mind, in the following section I outline my pilot study and the research questions that initially emerged from this process.

A Pilot Study

My intention with this study was to examine the organization of the talk as much as what was being said. As such, I wanted to incorporate the tools of Conversation Analysis along with gesture analysis, but to be open to other aspects as they arose too. I will examine these toolkits in more detail in the later chapter, but it helped to analyse an example at this early stage. Some references are in need of further explaining, and will be expanded more in subsequent chapters, but this section has a more exploratory nature.

My initial research question can be expressed as: “what can an examination of students’ conversation tell us about their understanding of a concept”.

Methodology

The subject of research in this study was a grade 5 class during their regular mathematics lesson. I began by observing the first lesson as a whole and by attempting to record groups working within the confines of the classroom. My focus was initially on my own observations of the students from a perspective other than that of a classroom teacher. I took on a role as a pure observer in that I resisted the temptation to interact with any of the groups I was recording (but I also realise that even my presence in the
classroom automatically turns me into a participant). I also tried to focus on the interactions between the members of the groups by constantly asking myself ‘what is it they are physically doing right now and how is that impacting the other members of the group and the progress of the problem’. I took field notes on what I saw and then later compared these notes to what I saw in the recordings from the camera I had set up in a corner of the room. I hoped to be able to ‘fill in’ the picture by being able to listen more carefully to the group talk and watch the students’ physical interactions more carefully.

There were initially two things that were guiding this approach; I wanted to know if there was anything I could see that stood out and would be worthy of further study, and I wanted to get a sense if my method of recording the students would be effective.

To the latter question, I was surprised at how challenging it was to collect a workable recording of the class in what seemed a typical setting wherein the students were split into small groups around joined tables with the teacher moving between the groups. In situ, the class seemed to be operating within perfectly acceptable noise levels for the task. It was easy for me to tune into individual group talk without being close enough to be a distraction (at least as I perceived it). I could watch students in different groups and focus in on individuals without a problem. The recoding proved to be very different. It was impossible to isolate individual voices from the general din of the room while background noises appeared surprisingly loud and distracting. Even though I often stood near the camera and had no trouble hearing students, it was clear that a better method would need to be employed if recording sound was to be effective. In addition, while a broad sense of students’ gesturing and other body movements could be seen from a few students, too many students had their actions obscured. As a result it was impossible to view interactions in a meaningful way.

In reviewing the methodology and speaking with the classroom teacher, Avio, he suggested that I record students in isolated groups. It was a normal practice for him to disperse students to various quieter areas outside of the classroom when working in groups in any subject area. While we discussed advantages and disadvantages in doing this, most particularly a concern that the ‘eye of the teacher’ was not on the groups, it seemed to be a workable solution to test. The students were familiar with the arrangement so it fitted with my original aim of being a ‘normal’ task for them. My question now was whether I could record all of the activity of the group in a clear and
workable way. Because I wanted my presence to affect the group as little as possible I felt that this arrangement would necessitate that I could not be actively taking field notes at the same time for the group I was recording. I decided to leave the camera to be my eyes and ears with the belief that in a classroom setting I could perform the same actions unobtrusively. Consequently, my recordings needed to be of high quality and to allow me to see all of the group members clearly. With this in mind I set up two cameras in a slightly elevated position to look down on the group.

On viewing the recordings from this session I was happy with the visual effect; all students were clearly visible throughout the session (which lasted approximately 15 minutes) on at least one of the two cameras. However, needing to view separate recordings to capture the entire student talk seemed to take away from the connection between interlocutors. In addition, the audio quality was still not good enough to make an accurate transcription. Too many important words were lost, and there was still too much background noise. I therefore tried a third approach for my next recording. I asked the students to sit in a semi-circle rather than around a table so that I could use a single camera to capture all of the students’ expressions. I then placed a second camera directly in the space in front of the group but positioned to record sound only. I used an identical camera instead of a separate audio recorder in order to make an easier link between the video and sound because it meant that I could use the same software for both. I had initially resisted the idea of placing a microphone in so prominent a position as I did not want it to be a distraction, but it was clear that there had to be some compromise. My aim was to also monitor the effect these cameras had on the group and determine if it was affecting the way in which the students were working.

This third arrangement proved to be satisfactory in allowing a good recording of the students' actions and expressions as well as providing a clear audio. I recorded six groups using this arrangement (three groups at a time over two sessions) and felt confident that it was a workable arrangement technically. While there was some initial interaction with and comments about the cameras, this seemed temporary and the recording devices seemed to be quickly ignored. It cannot be said that the students were unaware of being recorded at this stage, but I felt this may also have helped keep them on-task, in knowing that they were being recorded. At the same time I took field notes on
unrecorded groups with a view to comparing the two sets of groups, recorded and unrecorded, to see if there was any obvious difference.

In the next section I present some of my observations regarding the class. These findings are qualitative observations summarised from my field notes taken in situ and also from a general observation of the recordings. Again, my intent was to look for any patterns or things that stood out which I could expand on in a further study. I follow this with a more formal analysis of a transcript made. The purpose of the transcript was to examine if useful data could be extracted by tying together the students' talk with any gestures made. While this seems a detailed analysis, I was primarily looking for broad strokes at this stage. What stood out from the background that seemed to have significance and could be studied further?

**Initial Findings**

I began by making a first pass through each video recording without stopping, noting down things which immediately stood out. I noted the gender split in case there proved to be any indication that this was affecting the group-talk. Similarly, I made a note of the location when the students were outside of the classroom in case there was evidence of distractions. These were my only prior choices as I wanted anything else to be 'noticed', i.e. to stand out for me. This coarse-grained analysis was similar to the process of observation in the classroom where there is no chance of pausing or rewinding or reflecting on one moment before the next one occurs. I did not use the separate audio recording to start with so as to make these two processes, video recording and in-class listening, as close as possible. Table 1.1 gives an example of these notes.
As can be seen, only a brief note can be made at this level of recording, but by comparing these notes I noticed some common themes:

- The camera did not make an obvious difference. If a student became distracted the camera became an object of interest but in no cases did it seem that it was the object of distraction. In Table 2.1 the boy speaks to the camera after the group decide they want to write. Earlier, a nearby locker took his focus. The whole group became distracted by a brief reference to their school bus.
• There was no obvious difference between the mannerisms of the students in the whole class situation compared to the isolated groups. Students became distracted by other items/activities/topics at some point in all cases.
• Exchanges were brief and often of an incomplete nature, with interruptions.
• Students did not always seem to be listening to each other, but looked down at their papers.
• Students often seemed to adopt a similar posture.
• Students’ gestures were more apparent when they were explaining something.
• Most gestures seemed to point to the paper being held, often tracing out something using a pen. Occasionally the pen would be held to one side and the hand used to gesture a shape.
• Fuller explanations seemed to leave the page and take place in the air. Students made more eye-contact in this situation.
• A lot of time was spent in interpreting the meaning of the question.
• All of these early groups observed seemed to participate in polite behaviour with each other.
• ‘Progress’ towards a solution to the problem varied widely. Some groups seemed unable to establish a starting point. Several students seemed to waste time at the start in ‘organising’ their materials, sharpening pencils, or writing their name. I had a sense that these items were being used as a way to delay thinking about the problem.
• Students who were standing appeared to be gesturing more freely than those who were seated. I wondered if there was a more restricted sense of body space when students were seated on chairs or the floor. I also noticed that the students tended to stand around in small groups to chat before the lesson started. In this talk, students also tended to use their whole body to gesture or enact a scene. In particular, their arms gestured over a much larger space.

Following these initial observations I then re-looked at the recordings and used the pause control to allow more detailed notes to be taken along with snapshots of gestures and posture. I repeated the process with the sound muted to avoid distraction. Table 1.2 gives an example of this process, which was done using the same clip as used in Table 1.1. Only the opening section of the recording is presented here, chosen to give a flavour of the process. As this is a time consuming process my hope was to find trends within the procedure in order to narrow down what I was looking for. One such trend was the very definite opening procedure that became apparent. The students were typically asked to discuss the given problem without writing anything down. The classroom teacher then asked students to discuss the problem for 5 minutes, but was flexible in this depending on how the class seemed to be responding. A closer analysis of this opening
phase seemed to offer an opportunity to examine how students began a problem as a clearly defined stage. This also presented the idea of examining the problem solving session for other apparent stages, suggesting an approach of examining introduction-development-conclusion.

<table>
<thead>
<tr>
<th>Time</th>
<th>Action</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0:00</td>
<td>Two girls and 1 boy are in a group working on a measurement problem. They have been told to discuss the problem without writing anything down or doing any calculations. They are in the hall next to a row of lockers and seated on chairs.</td>
<td></td>
</tr>
<tr>
<td>0:02</td>
<td>The two girls, Susan and Jenny, write their names on their individual sheets. They use a spare chair as a table. The boy, Matthew, uses a window sill as a flat surface. There is soft talking between the girls.</td>
<td></td>
</tr>
<tr>
<td>0:12</td>
<td>Jenny exclaims and the two girls sit back in unison. Matthew turns to face the girls. They are smiling.</td>
<td></td>
</tr>
<tr>
<td>0:21</td>
<td>Matthew asks “about how long is this line, tell me”. Susan pokes him in the arm to get his attention.</td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>Event</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>-------</td>
<td></td>
</tr>
<tr>
<td>0:23</td>
<td>They focus on reading the question. Matthew hums and then exclaims that he is “done”.</td>
<td></td>
</tr>
<tr>
<td>0:44</td>
<td>Matthew turns to fiddle with a lock. There is a brief exchange of comments about the locker.</td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>Susan and Jenny restore Matthew’s attention to the problem.</td>
<td></td>
</tr>
<tr>
<td>1:03</td>
<td>Jenny initiates the discussion about the problem, starting with “So...what do you think...”</td>
<td></td>
</tr>
<tr>
<td>1:08</td>
<td>Jenny uses a hand gesture on the page as she talks. Matthew and Susan look at the gesture.</td>
<td></td>
</tr>
</tbody>
</table>
The ‘pinching’ gesture is repeated. She is telling the others that “an inch is like two times a centimeter, so then…

“..where the two is.. that would be approximately

… one”. Jenny’s hand now gestures in the air.

Her hand moves back to the page. She is scaling the shape with her hands.

Matthew points to his own paper and asks “How long is this?”

Jenny moves her gestures space over to Matthew's page in order to answer his question. She uses the same pinching gestures again. Susan leans in so that all three students share close space.
<table>
<thead>
<tr>
<th>Time</th>
<th>Image</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:39</td>
<td></td>
<td>They lean back and Susan points to her own paper&lt;br&gt;“No see this is one inch..”&lt;br&gt;Jenny shakes her head emphatically while saying “No, no., no…”</td>
</tr>
<tr>
<td>1:44</td>
<td></td>
<td>“Yes it is Jenny.” Susan is firm in her response.</td>
</tr>
<tr>
<td>1:45</td>
<td></td>
<td>“Oh wait, yeah it is..” Jenny gestures to her page again quietly before turning her head to one side as if re-considering and then acknowledging Susan’s answer as correct</td>
</tr>
<tr>
<td>1:48</td>
<td></td>
<td>Susan starts to explain her thinking and in doing so her gesture space enlarges.&lt;br&gt;Matthew adopts similar posture to Jenny in holding the sheet high and sitting back with legs crossed at the knee.</td>
</tr>
<tr>
<td>1:59</td>
<td></td>
<td>Matthew and Jenny make similar pinching gestures on their page as they follow Susan’s explanation.</td>
</tr>
<tr>
<td>Time</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>2:04</td>
<td>It is now Jenny's turn to correct Susan. She is finishing her statement of “No, you see..” and leans forward to gesture to the page again. Her page now occupies the centre of the group.</td>
<td></td>
</tr>
<tr>
<td>2:05</td>
<td>Matthew agrees with Jenny and Susan acknowledges the correction. Matthew then points out something new.</td>
<td></td>
</tr>
<tr>
<td>2:10</td>
<td>Jenny's worksheet has become the common working space as Matthew points to a way forward. “Ohhhh” says Jenny as she realises what he is saying.</td>
<td></td>
</tr>
<tr>
<td>2:15</td>
<td>Susan acknowledges and adds to the conversation. She makes a sweeping gesture across her sheet as she does so.</td>
<td></td>
</tr>
<tr>
<td>2:33</td>
<td>They continue a lively conversation about the measurements, with overlapping and interruptions in speech</td>
<td></td>
</tr>
</tbody>
</table>
“Jenny, can I show you one thing..”

“that’s one inch, two inch, three inch …” She stabs at her paper gently as she speaks.

The two girls debate a point, alternately gesturing both in the air and at Susan’s paper. Matthew becomes frustrated with the talk, eventually sighing and saying “Oh my God”.

Eventually Jenny concedes the point to Susan with a long drawn out “ohhhhhh”.

Matthew makes a face-slapping gesture, but he is smiling. He starts to re-explain, but Jenny cuts him off saying “I understand”. She repeats the statement more softly and then turns towards the teacher asking “can we write?”

The students worked together to successfully solve the problem assigned.

**Table 1-2 Gesturing example #1**

Having established a broad sense of how the students were gesturing and changing posture during the talk, I next set about producing a transcript which could tie the gestures and talk together. In order to do this it was necessary to make multiple passes of the recordings. Initially, I used the video recording, but when it was hard to determine the spoken word I used the higher quality of the audio recording, taking care to match the timings of the video/audio tracks. This was not easy – as well as being time consuming – and I realised I needed a better tool than the default video player on my computer. I therefore invested in video analysis software that allowed the recording to be slowed down and used foot-pedal controlled forward/backward movement of the recordings. This
helped the process and I detail the use of this program further in Chapter 5. In the transcript I wanted to record as many nuances of what was being said as possible, and for this reason I used the conversation markup combined with gesture markup. While the markups used can make reading of a transcript challenging for the casual reader, with practice it gave me the ability to read a transcript at a later time and capture the intonation while gaining a sense of the emotions in the passage. I believe this was important in helping to interpret what was going on in the exchange. I present a transcript of a different session below, to give an example of a different stage of talk compared to the previous session.

**Transcript Example**

These excerpts are taken from talk between two students (referred to as D and M). The students have been doing some examples on estimation; the intent of the teacher was to have them estimate the result of division using rounding. They worked on the problems individually to begin with and were then paired off to share their work and explore different strategies. A camera was set up at a distance to record the images and a sound recorder was placed on the table. These devices were set recording and the students were then left to work without interruption or the presence of an adult during this section of the lesson. The classroom teacher brought the groups together to discuss their results after this recording. The students were initially paired off by the teacher, as was his normal practice, using different pairings or small groups in each interaction.

I begin with a transcript of the first section, which is not marked-up in order to make it easier to gain a sense of what was said:

D: I didn’t really get it.

M: Oh, well, I will explain what I did, so I, it says estimated so I went down to thirty, divided by 6..then I got 4 remainder 1, so I just I just like rounded to the nearest umm ten. Same with this one, then I got 5 remainder 8

D: Umm, I didn’t really understand it..’cuz I just came back..

M: Inaudible (could be “what are your numbers”)
D: Umm.. like I have different, different… and what I tried to do, was I tried to do umm like umm .. you know it’s 8 divide by fi-sixty-five, I said that I could do 2 divided by 65 but do that 4 times, and then just add up all of..

M: You can’t do 2 divided by 65, right? It’s 65 divided by 2.

D: 65 divide by 2.. sorry, and, and then umm and then I could have done like umm and then I could have like umm added all the answers, so….

M: Yeah, that could work. It might have been.. umm., oh yeah never mind. I was saying you could have done it by 4 and then by 4, but it’s easier if you do groups of 2

D: Yeah

M: Yeah

D: Good, cuz you want it to be as easy as possible, but I didn’t I didn’t get it but can I try the rounding like you did?

M: Yeah.

D: Umm..

M: I did just went.. you should probably go to like 60.. go like 60..

D: Wait! Since 65 is in the middle do you do it to 60 or umm 70?

M: you could do it actually, but if you did 60 it would be a lot easier

D: Yeah, it’s just this a little confusing.

On a first pass of the recording I was looking for examples of any evident shift between a discussion, in which students were speaking but not being open to the views of others, and a conversation, in which there was an apparent exchange of views and openness to change. I also wanted to see if/how the students were able to build, or ‘go on’, with the conversation. On a second pass I examined the gestures the students used to accompany their utterances. A third pass was then done to look for any other features worthy of note. My intention was to be as open as possible to notice anything that stood
out in the talk. I do not intend this to be an exhaustive analysis at this point but to serve as an illustration of the process which led to my research questions.

The utterances are displayed in a table format. The first column gives a reference number; the second column is a time marker; the third column displays the utterance using conversation markup, along with gesture markings and a comment on any gesture; the final column gives an image or additional data to illustrate the gesture or specific body language.

<table>
<thead>
<tr>
<th>A brief guide to markup symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>0</code> brackets a quietly spoken utterance.</td>
</tr>
<tr>
<td><code>:</code> indicates a stretching of the sound. Repeated use indicates longer stretching</td>
</tr>
<tr>
<td><code>?</code> indicates a rise in pitch</td>
</tr>
<tr>
<td><code>(..)</code> indicates a short pause</td>
</tr>
<tr>
<td><code>↑</code> represents an increase in volume</td>
</tr>
<tr>
<td><code>&lt;words enclosed as such are spoken more slowly&gt;</code></td>
</tr>
</tbody>
</table>

*Table 1-3 Markup Symbols*
Table 1-4 D & M Transcript 1

1. D is reserved in admitting her lack of understanding. This might indicate a lack of confidence on her part, or recognition that M is regarded as being more capable. She uses ‘really’ and lays emphasis on the word ‘get’ as if suggesting she did not get it fully. The word is stretched and she matches the word with a head gesture while looking directly at her partner. Her lower body position and direct look at her partner is suggestive of a desire for help. This starting approach sets the tone for what is to follow. In other cases a student might flatly state they did not know and close themselves to help. Alternatively, a student might pretend to know, a state which might prevent them from getting help. The opening exchange is something I will examine more closely later.

Table 1-5 D & M Transcript 2

2. M’s tone is one of surprise but her body language and gesture suggests D’s concern is not a problem. There is a sense that she is open to D’s concerns. This may be
important in establishing the following working relationship in which D feels comfortable in expressing her concerns. This response to an opening is another point of interest.

M then proceeds to describe what she did, rather than explain it as she suggested. She is looking down at what she did and this suggests that she is trying to make sense of her work to put it into words as she talks.

There is a hesitation in the word ‘one’ when she states that 30/6 is 4 r1. She is probably aware of the incongruity in the division but wants to make the point about rounding the initial value to the nearest 10. She does not recognize any issue about having a remainder in the estimated value, suggesting that she may be associating estimation with doing division rather than as a means to give an approximate answer as a guide.

<table>
<thead>
<tr>
<th>Time</th>
<th>Transcription</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>D: umm (..) I didn’t really understand it ‘cuz I just came back. [~~~~~~~~~~~~~~/*****/-.-.-] ((she was resting her chin on her hand and removes it, rolls her shoulders at looks M in the eyes on the stroke))</td>
</tr>
<tr>
<td>4</td>
<td>M: Inaudible (could be ‘what are your numbers’)</td>
</tr>
<tr>
<td>5</td>
<td>D: umm.. like (2.1) I have different (1) different (2.3) and what I tried to do? was I tried to do umm like (1) umm (2.4) ((She picks up her paper and is looking down at it)) you know it’s (.) eight divide by fifty-five (.) [~~~~~~~~~~~/*****/hold] ((She moves her hand to point at her paper)) I said that I could do two divided by sixtyfive but do that four times (.) and then just add up [all of]</td>
</tr>
<tr>
<td>6</td>
<td>M: [you] can’t do two divided by sixtyfive right It’s</td>
</tr>
<tr>
<td>Time</td>
<td>Transcription</td>
</tr>
<tr>
<td>------</td>
<td>---------------</td>
</tr>
</tbody>
</table>
| 7:08 | D: (0.9) sixtyfive divide by two (.) sorry (1) and then umm  

~~/~ hold ~~~/~ /----~

((moves her right hand to her ear and raises a flat left hand; her eyes widen))

(1.5) and then I could have d.one like umm (3) and then I could  

~~~ hold  

have done like umm added all the an::sw.ers so….  

/~***//----~

((raises her left hand to her ear and holds it there then rolls her hand out on the stoke )) |
| 8:28 | M: yeah (..) <that could work> (1.5) It might have bee::::n  

~~~~~~~ ~***//----~  

((Lifts her left hand which holds the pen))

umm (..)< oh yeah never mind>  

~~~~~~~ ~***//----~  

((waves both hands from the wrist and shakes her head))

I was s::aying you could have been quicker if you went four (1) sixty-five divided by four but it’s easier if you do in two ( ) |
| 9:41 | D: y::eah (1.5) |
3. D’s response marks her own claim to a lack of understanding (It would be interesting to know how she perceives this term). This could still be a face-saving\(^5\) process at this point, especially as she gives a reason for this stated lack of understanding.

4. D’s response does not match M’s comments. The conversation falters and needs repair, which M has initiated, although not audibly; it seems she is ready to work with D.

5. D is searching for a way to explain her thinking. She is hesitant and both are focusing on D’s paper. M does not interrupt and shows a willingness to listen by moving in closer to D to share the same working space. The point D seems to be making is that she believes division by 8 is the same as dividing by 2 four times.

6. M recognizes a mistake in what D is saying and initiates a repair. Her body language remains inclusive and non-threatening. She does not press the mathematics of what D is saying though, suggesting that she does not notice the error in D’s thinking.

7. D responds to the repair and continues her line of thinking. There is hesitation in her speech and her gestures indicate uncertainty with spread hands. Her thinking indicates that her idea of estimation is also confused. She is trying to find an easy way to do division.

8. M’s response has the characteristic of an adjacency pairing but does not address D’s actual thinking. M seems uncertain of what D was trying to do but again, rather than press for explanation, she replies with her own related thought, seeing that division by 4 would be quicker. This may indicate she has a sense of what D was trying, and the

---

\(^5\) The idea of ‘face’, ‘face-saving’, and repair will be discussed further as part of ‘politeness’ phenomena (Brown & Levinson, 1978).
pauses in her speech indicate she is trying to think it through. She stretches out the word ‘been’ as an indicator to D that she is thinking and this whole process takes about 7 seconds before she lets it drop. Her conclusion is to agree that division by 2 is the easiest way to divide, abandoning her line of thought without resolution. Her final comment is suggestive of a face-saving shield; she has not dismissed the idea but cannot ‘go on’ with it any further. This is a point of interest connected to the ideas discussed earlier. Here we see students who seem to have partial understanding about the notion of repeated division by two but who are unable to develop the idea further. This may be an indicator where, referring to the ideas of Vygotsky, the students are in a zone of proximal development. Intervention by a more expert guide seems to be called for to help clarify this idea, which is subsequently dropped. For a teacher, being able to identify such points of intervention is an important aspect of a reform based classroom and has the potential to maximise learning using this model.

| 11  | 1:44 | D: good (.) cuz you want it to be as easy as possible (.)but I didn’t (.) I didn’t get it= [~~~ /*** hold... ] ((sits up more, raises her head, looks upward)) but can I try the rounding like you did? [~~~~~/***/---.-------------------] ((moves her left arm to touch M then raises it and points her finger up before moving her hand down and rotating it back to her paper)) |
| 12  | 1:50 | M: yea?h |
| 13  | 1:51 | D: umm (.) (Writing on her paper) |
M: I did just went.. <you should pro>bably go to like sixty .. go like [sixty]

D: [.hhh] W?ait (..)but (..) [~~~/***hold/ ***hold] ((raises and flattens her hand then lowers it halfway and curls it towards herself))

Table 1-7 D & M Transcript 11-15

11. D responds to M’s turn in line 8 before initiating a new thread in the discussion. She makes it clear that she still did not get why the division idea she tried did not give a correct answer but is prepared to dismiss this and work with the way M had originally suggested. Her touch and gesture are to maintain M as part of a conversation.

12. M’s response is spoken in a friendly way by a raise in tone. It invites D to continue.

13. D starts to write. She is using the filler ‘umm’ to indicate she is still part of the discussion.

14. M has noticed D is not rounding 65 to 60 and interrupts to point this out. She offers the face-saving ‘you should probably’. Here we see an example of a true conversation as there is clear adjacency pairing directly related to each other, rather than disconnected turn-taking. We also see the first occasion of overlap when M repeats 60, an indication that they may be thinking along the same lines.

15. D responds to M’s turn and we see her pressing for an explanation. In addition, her body language and gestures become more direct as she indicates a pause to reflect.

My sense is that this is an important junction in the discussion, and that this is indicated by a shift to a conversational mode as described earlier. These conversation points are a key, it seemed, to a student demonstrating understanding, or showing a readiness to
learn about the topic in hand. This led to my interest to study examples of where a conversation is initiated, or breaks down, within students' talk. In D's case above, the conversational moment is also a point where her gesturing and body language is strong. This is another connection I was prompted to examine in order to see if any clear conclusions could be drawn.

<table>
<thead>
<tr>
<th>Time</th>
<th>Transcript</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>(D cont.)</td>
</tr>
<tr>
<td></td>
<td>since sixty-five =</td>
</tr>
<tr>
<td></td>
<td>[~~ ~~~ /***/-.-.-.-.]-</td>
</tr>
<tr>
<td></td>
<td>((spreads her fingers and returns))</td>
</tr>
<tr>
<td>17</td>
<td>(cont.)</td>
</tr>
<tr>
<td></td>
<td>is in the middle do you do it to sixty? y =</td>
</tr>
<tr>
<td></td>
<td>[~~~ ~~ /***/-.-]- [~ ~ /**hold]</td>
</tr>
<tr>
<td></td>
<td>((chops down)) ((moves her left hand out and chops again))</td>
</tr>
<tr>
<td>18</td>
<td>(cont.)</td>
</tr>
<tr>
<td></td>
<td>or seventy (1.1)</td>
</tr>
<tr>
<td></td>
<td>[~~ hold /***/-.-.-.-.-.-.-.-.-]</td>
</tr>
<tr>
<td></td>
<td>((moves her right hand out and rolls both hands in an open way, she turns her head to look questioningly at M))</td>
</tr>
<tr>
<td>19</td>
<td>2:04</td>
</tr>
<tr>
<td></td>
<td>M: you could do it actually but if you did sixty it would be a lot easier</td>
</tr>
</tbody>
</table>
D’s gesture now matches her speech as she indicates 5. Gestures which do not match speech have been identified as an indicator of readiness to learn (Goldin-Meadow, 2003).

D uses a chopping gesture to indicate the middle between 60 and 70. She then positions her left hand to indicate the bottom end of the possible range. There is very positive correlation between her gestures and her words at this point. I take this to be an indication that she is clear in her thinking.

D then moves her right hand to indicate 70. Her spread hands reflect the possibilities for the rounding.

M continues the conversation with her turn, replying directly to what D said. She offers a face saving comment in agreeing that 65 could be rounded either way, before pointing out 60 would make an easier division.

D is still confused as to why 60 would be easier. M did not explain her reasoning.

Questions Arising from the Pilot

Taking the opening stage of transcripts and looking at the way students begin their talk has shown some interesting data. By taking a number of these openings and comparing and contrasting where they usually led could help identify useful markers that point to how the talk will proceed.

Some interesting questions which arose from this study were:
a) Does the way a student initiates the group talk tell us all we need to know about how it will continue?

b) Are there consistent markers, such as actions, gestures, phrases which can tell us a lot about how the student will proceed in the talk?

c) In terms of determining the students’ mathematical understanding or ‘ability to continue’, is it sufficient to pose a topic and observe how the students initially try to addresses it? In other words, is it even necessary to have the students’ complete problems?

d) Does the beginning of the student-talk tell us more about the student than it does about their understanding of a concept?

In the middle section of the talk it was observed that students can ‘draw into a conversation’ as particular points coincide with their knowledge.

a) What does this tell us about the students’ understanding? Can they ‘go on’ from that point and develop their understanding further, or is the moment fleeting?

b) Are these points indicated by a change in the way the student gestures or their body language? Can we observe this from a distance?

How the talk terminates was another area of interest.

a) Are the students able to reach a common, or shared, point of understanding?

b) Does the talk end with a conversational exchange? If so, what does this tell us?

c) What body language do the students demonstrate? Can we tell if it relates to their understanding?

At the beginning of the year I made no attempt to find out which students had been considered successful in their mathematics class the previous year. I did not look at any previous report cards or ask the classroom teacher to give an indication which students he felt were more capable. Despite this, I found myself to be drawing conclusions in this regard even after observing only one or two sessions. I was intrigued as to what signs had led me into drawing these conclusions within minutes of students starting their work. Part of my plan going forward was to try to address this question, by observing students
‘from the sideline’, from a non-interactive position (as far as possible) that any classroom teacher could adopt.

Research Sub-questions

I stated at the outset of this chapter that my initial research question was: “what can an examination of students’ conversation (presupposing there is one) tell us about their level of understanding of a concept”. Using the prompts from the previous section I felt able to break this down further into some sub-questions as follows:

1. How can the way a student opens the group talk inform us about the way the student initiates his/her thinking? Is there a logical approach to this process?

2. How can the ‘conversational aspect’ of group talk (or lack thereof) inform us about a student’s developing understanding?

3. How can the nature of a student’s gesturing/body language help inform us about a students’ developing understanding?

Research Question

Ultimately, these questions lead to an over-arching question: are there key moments in the talk, i.e. from the organisation of that talk, which we can observe, that inform us about the development of a student’s understanding?

As mentioned earlier, in order to investigate these questions further I felt I needed to establish clearly what I recognised as knowledge and understanding. In the next two chapters I will investigate this further, building on the ideas formed from this pilot study and relating this to the various theories most commonly used. As mentioned earlier in the overview, knowledge itself is not the focus of study in this research. However, I feel it is an important discussion to have before going into to a deeper examination of understanding. My aim is to clarify the way I view the term knowledge as a foundation to ‘go on’ from and examine understanding. It is also important to point out that the views expressed in chapters 2 and 3 co-emerged with the research. My initial thoughts on
knowledge and understanding changed as I studied the way the students grappled with the tasks they were assigned. While I started off with the view that knowledge was something *possessed* by the students, and that students could ‘go on’ if they *had* understanding, I eventually came to see knowledge as being the way a student uses experience to deal with information, and understanding as formed in the action of going on. Understanding is a verb. Necessarily, this shift in my own thinking does not come across in the linear nature of writing of such a dissertation. Without grappling with ideas around knowledge, a sense of what it means to understand would not have emerged in the same way.
Chapter 2

Theorising Knowledge

Socrates: Try to bring the many sorts of knowledge under one definition.

Theaetetus: I can assure you Socrates, that I have tried very often; but I can neither persuade myself that I have a satisfactory answer to give, nor hear of anyone else who answers as you would have him; and I cannot shake off a feeling of anxiety. (Plato, Theaetetus, p. 141)

In a sense, then, mathematical knowledge is like the subject matter of a conversation. It exists only in conversing, and its nature, its structure, and its results can never be anticipated, let alone fixed. (Davis, 1996, p. 23)

A central aspect of the reform movement has been a shift towards student-centred learning and away from teacher-centred learning. A main focus is on the students, working collaboratively, as the basis of knowledge production. The teacher is positioned more as a facilitator than a transmitter of knowledge. The model is suggestive of students arriving at mathematical knowledge under the guidance of their teacher in an atmosphere of talk wherein students challenge each other’s thinking in a non-threatening way. Such a learning theory points towards a goal to be obtained rather than standing as pedagogical theory, but also opens the question of what a teacher expects to see in terms of knowledge production. My intent in this chapter is to lay down a basis for what I see as knowledge and understanding. I recognise that this is no easy undertaking and is, in reality, an exercise in clarifying these issues for myself before I can go further. In truth, what you read is the result of a shaping of these ideas as this research progressed rather than a chronological development or an early statement to be readdressed in a later chapter. If I am to explore the nature of students’ understanding, then it is evident that I must have some sense of what that means, at least to me. Part of the problem in using
terms such as knowledge and understanding lies in the common interchangeability of their use. Notwithstanding Wittgenstein’s contention that the meaning of a word lies in its use, any deeper analysis using these terms requires a finer distinction.

A dictionary definition of a term can be restrictive in many cases, but I offer it here to illustrate how the words used in relation to this topic are commonly interchanged. The Oxford dictionary (Oxford Dictionaries, 2014), for example, makes the following definition of knowledge: facts, information, and skills acquired through experience or education; the theoretical or practical understanding of a subject (noun). Understanding is defined as: the ability to understand something; comprehension; or as a synonym: the power of abstract thought; intellect (noun). To understand is to perceive the intended meaning of (verb), or to perceive the significance, explanation, or cause of.

My interpretation of these terms is a dynamic one but I will begin by making the following statement before attempting its justification:

Knowledge is the on-going interpretation of the accumulation of experiences and resources that represent one’s life at any given time. It is a source for actions. How one goes on from this platform, to build on this knowledge, represents understanding.

These clarifications are important in following the way I interpret the results of this research. Clearly, it is important to support such claims and in order to do so I shall be drawing on a variety of sources, particularly the writings of Wittgenstein, Davis, and Sfard. However, a good place to begin any attempt to define knowledge would seem to be the work of Plato, if only to establish the difficulty in trying to do so.

The Theaetetus

It is not my intention to offer an in-depth analysis of Plato’s dialogue, this has been done extensively elsewhere, such as in Jowett’s introduction to his translation (Plato, 2013) or by Chappell (2013). Instead I want to discuss what I draw from my own reading of the work and how that relates to the way I am thinking about knowledge.
Can I say I have knowledge about Theaetetus? Certainly I have information about it, which I have gathered from numerous sources. I have read the dialogue, but not in the original Greek so I am restrained by translations to English. I am further restrained by a time gap of almost 2,500 years and the cultural gap that accompanies this. I am restricted in my interpretation of the text by my limited connections to the complete body of Plato's work, a lifetime's study in itself. I am constrained by the influences of other commentaries at the same time as they add to my thinking. I am aware of Unitarian (in keeping with the theory of forms) and Revisionist (a movement away from forms) interpretations of the work (Chappell, 2013) enough to know that there is no single interpretation of the intent of the dialogue. Is it enough that I am able to put this information together in a coherent way or is there a higher demand for any knowledge claim? At what point do I have enough information in order to make the claim that I am knowledgeable about the dialogue?

In order to develop my knowledge I began by reading through the dialogue and then re-reading it until I felt I could form an opinion about what it was saying. This opinion was not based on this dialogue alone, though, as I had previously read other works of Plato and was aware of his ideas regarding Forms. As such, my opinion is further shaped by these previous experiences of Plato’s work as well as other people’s opinions on his work. I followed this by reading Jowett’s introduction to his translation of Theaetetus; confirming some thoughts and further shaping or relinquishing others. Jowett helps to situate the work in history and I recognised the names and writings he addresses; I believed I could follow his argument. Is this enough to claim knowledge? Reading other commentaries on the dialogue gave further insights to its meaning, but was I gathering information or developing knowledge? Must I be able to interpret the information I gather for it to represent knowledge? Furthermore, whose knowledge is it that I can then claim to have?

Socrates’ initial prompting “Herein lies the difficulty which I can never solve to my satisfaction--What is knowledge? “ (Plato, 2013, p. 134) is suggestive that the dialogue is aporetic, setting out to show that there is no clear answer to the question. The dialogue seems designed to say what knowledge is not rather than what it is. What is assumed, however, is that knowledge exists or otherwise it would not require defining. Socrates asks to bring the “many sorts of knowledge under one definition” (p. 141), but the inability
to do so may reflect the fluid nature of knowledge itself. Some knowledge comes via rational thought, some via experience; some is absolute while some is relative; some knowledge is said to be discovered and some lost (as though it could be an object to be held). That I can write this paragraph and it does not appear as nonsense is because the term ‘knowledge’ is often used as if its meaning was well known. This seems to be the root of Socrates’ unease; there is a familiarity with the word that belies its meaning.

Socrates raises the question of how/if knowledge can be demonstrated and this is an important point with regards to a school setting where assessment is common. In this regard an important part of the dialogue comes at the start. Through the character of Theaetetus the idea of learned knowledge, such as what is taught as geometry, is offered but dismissed by Socrates as being too specific. Socrates rejects the use of examples to define knowledge. It seems Socrates wants to get at the core idea of what knowledge is. The details, information, and skills a student demonstrates are seen as the results of knowledge rather than being the “nature of knowledge in the abstract” (p. 136). This would suggest that when a student is able to repeat our teaching, even mimicking a skill, that this is not enough. The student may have knowledge behind their demonstration of their use of mathematics, but this is not the same as saying that their examples demonstrate or constitute knowledge. In the same way, my writing about the Theaetetus is not enough, according to Socrates, to demonstrate that I have knowledge about the dialogue.

But do I want to accept this dismissal of demonstration by example? Is it not sufficient that I can write about the Theaetetus to demonstrate I have knowledge of it? Intuitively, as a teacher, I can recognise times when I want more; that I sense a distinction between a lesser-valued ‘procedural’ and more highly rated ‘conceptual’ knowledge’. The root of this may be my own experiences of being able to perform an action while being aware of not really knowing what was behind it. I have a sense that there is a difference between a ‘deep seated’ and a ‘shallow’ knowledge but I am unable to draw a clear line between the two. In this way, I can associate with Socrates’ unease about a definition of knowledge based on examples. I feel when I observe a student that I want to see more evidence than the recalling of learning before I can say the student has knowledge of mathematics. So, if there is more, what else does Socrates seem to want?
In turn, Socrates dismisses ‘sensible perception’ and ‘true opinion’ as definitions; the former for being ‘outward’ and experiential, and the latter for being insufficient. Theaetetus’s final prompted offering is that knowledge should be “true opinion combined with definition or rational explanation” (p. 263). Socrates concedes that this is “probably correct” but is still not satisfied. He uses a name as illustration, concerned that a user of the name could be aware of the name and the syllables used in pronouncing it yet still are unaware of the letters that make it up. Socrates seems unsatisfied with anything less than a reductionist approach. This may be fine for a philosophical debate (and for a Unitarian to be leading back to Forms) but seems too onerous in practice. There must be a point where we, as teachers, are content to admit that a student has knowledge about a subject, and I would argue, in agreement with the ideas of Davis (1996), that it is based on the actions the student is able to demonstrate in using that knowledge. Depth of knowledge may be conceded in the classroom without operationalising this idea of depth, even though grading rubrics often make attempts to do so.

While it may be possible to state that a particular proposition counts as knowledge if it is true (let us say for a particular mathematical theorem based on accepted axioms), if it can be justified, and if we believe it, the same cannot be said of an individual’s claim to knowledge. In a school setting ‘truth’ is to all intents determined by the intended curriculum, often in the form of ‘Prescribed Learning Outcomes (PLOs)’, but is subject to the interpretation of the teacher and other external influences. It does not matter to the classroom if it is absolute or is corrigible. The performance of students is assessed according to their ability to demonstrate, explain, or otherwise perform some action on what they have learned. I would argue that their ability to perform such actions is what is considered to be their knowledge. The connections we build between bits of information allows for the recognition of similar points, of patterns. Such connections provide a base for action. The actions may show us to be knowledgeable but they do not constitute knowledge itself. Knowledge is not a thing but more a state of affairs. In addition, the dynamic nature of this potential for action means that a person’s knowledge may change from one moment to the next. The knowledge Socrates refers to seems much more static in nature; something acquired and stripped down to the finest detail. For a working definition of knowledge in a school setting, a potential for action seems more appropriate.
Moving on from Plato’s description of knowledge I will turn my attention to theories of knowledge related to a school setting. My purpose now, is to build on the ideas expressed above in order to establish a clearer idea of what mathematical knowledge is, and so to be aware of what it is I am looking for in the classroom. In order to examine these ideas further I will first examine some of the ideas pertaining to knowledge in the constructivist model. While constructivism emerged in the 1980s and 1990s as interest in behaviourism and information processing perspectives declined, (e.g. Mayer, 1996) the ideas of constructivism have a historical basis which can be traced through the history of philosophy (von Glasersfeld, 1990). Of the variants of constructivism, the most prominent are Radical Constructivism and Social Constructivism. I will consider how knowledge is viewed in Radical Constructivism first, before moving on to how ideas about knowledge generation are adapted to Social Constructivism, and then beyond.

**Radical Constructivist Views on Knowledge**

*Mathematical knowledge cannot be reduced to a stock of retrievable “facts” but concerns the ability to compute new results.*

*(von Glasersfeld, 1987, p. 10)*

Constructivism rejects the idea that children are empty vessels to be filled with ‘knowledge’. It is rooted in the cognitive school of psychology that considers what is going on inside the brain. The cognitive approach considers that there is an internal schema and structures which can be retrieved heuristically, an important point of departure from early theories (e.g. Watson, 1913) which had considered this area unknowable. Von Glasersfeld (1990) describes Radical Constructivism as a “theory of knowing rather than a theory of knowledge” (p. 19). He notes that one of the central points of the theory is that ‘truth’ can never be claimed for knowledge produced by human reason. Our ideas and knowledge are derived from experiences, including thinking, acting, and sensing. The constructivist view is that “knowledge is the conceptual means to make sense of experience, rather than a representation of something that is supposed to lie beyond it” (p. 27). We can compare this to the earlier, rationalist, view of knowledge as “being achieved when the inner states of the individual represented matching states of the external world – when mind served as the mirror of
nature” (Steffe & Kieren, 1994, p. 712). The constructivist adaptational sense of knowledge is related back to Piaget and incorporates a concept of viability; an individual’s knowledge must fit with experience. An experiencer-independent world is not denied and this constrains any actions and shapes our knowledge. Von Glasersfeld notes, however, that the viability of an action does not mean it reveals ‘truth’- only that the ‘real world’ leaves us room to act in a certain way. In mathematics education this means that “one is studying the construction of mathematical reality by individuals within the space of their experience” (Steffe & Kieren, 1994, p. 721). It is the student’s way of making sense that determines their knowledge.

In this model, knowledge comes from the assimilation of regularities and disregarding certain differences. A cognizing organism is able to isolate situations in which a certain action leads to a desirable result. If an unexpected result is achieved, ‘perturbation’ is produced which then requires a modification of knowledge or a change in the action. In this way, what von Glasersfeld implies as non-trivial knowledge is not passively received but built up. Trivial knowledge is considered that which focuses on mathematical performance alone, such as may be obtained by rote learning.

**The flow of experience**

A key point in this theory for me is that existing knowledge orders the flow of experience. Mathematical knowledge is operative rather than just facts; it is knowledge of what to do in order to produce an answer (von Glasersfeld, 1984). A student tries to interpret new information in terms of their existing knowledge and learning is seen as the construction of meanings by the learner in order to build knowledge (Duffy & Jonassen, 1991; Cunningham, 1991). If there is too large of a gulf between existing knowledge and newly presented experiences, then the student may be unable to assimilate the experience and this can give rise to problems. The student is said to need a knowledge base to draw upon in order to make connections to the intent of the lesson. Von Glasersfeld writes that two things are required by the teacher in order to help guide the student in the conceptual organisation of experience: “an adequate idea of where the student is at, and an adequate idea of the destination” (von Glasersfeld, 1984, p. 15). It was, originally, the first of these two points that drew me to this area of research. There seems, however, some contradiction in constructivist writings when there is talk of knowledge as an action
but also in having a measurable knowledge base. A student can only be ‘at’ somewhere if knowledge is being accumulated. In this regard it does not seem like an action to be observed, something that is likely to be changing due to the circumstances of the problem the student is being presented with, and their own interaction with the problem on a particular day.

**Criticisms of Radical Constructivism**

Critics of Radical Constructivism (e.g. Olssen, 1996; Meyer, 2008; Kirschner, Sweller, & Clark, 2006) point to problems in completely discarding a correspondence view of knowledge to reality, or relying entirely on minimal guidance and classroom experience. If acknowledgement of a ‘real world’ can only be accepted ‘on faith’ (Olssen, 1996, p. 287), then one student’s construction of knowledge should be as acceptable as any others’, even in contrast to the curriculum taught. While von Glasersfeld (1993) maintains that a teacher can orient learners in a general direction to prevent them from constructing unsuitable realities, Olssen questions the criteria on which this is performed, calling it “suspiciously functionalist” (p. 288). When critics such as Olssen (1996) and Meyer (2008) focus their concerns on the issue of truth, they open up the question of what is meant by ‘truth’, an issue which philosophy has tried and failed to resolve since the time of Plato.

**Misconceptions and examining learning**

Of more concern to this research is how a constructivist view of knowledge affects the way we try to examine a student’s learning. Smith et al. (1993) write of the misconceptions that can arise as students move from ‘novice’ to expert understanding (in this case, I think, using ‘understanding’ and ‘knowledge’ interchangeably). They suggest that concepts are not simply replaced by new ones as a teacher corrects misunderstandings, but may coexist within the students’ schema. Tall and Vinner (1981) similarly point out, when discussing complex numbers, that students can regard the same concept as separate entities depending on the circumstances. Cognitive conflict only occurs when the student tries to “evolve multiple images of the same concept simultaneously” (p. 154). This perturbation in the framework of understanding a student has constructed is the basis of learning described by von Glasersfeld in his exposition of Radical Constructivism. In resolving the conflict the student has to look for a viable
solution which fits the new information into the concepts already developed. This adaptation model of learning has roots in Piaget’s developmental theory but, as Smith et al. point out, it is in conflict with the notion that corrected information replaces misconceptions. Similarly, a teacher may hold onto both instructional models at the same time without internal conflict in much the same way the authors suggest students’ learning develops. While a teacher might accept that knowledge is constructed gradually from where the student is ‘at’, most also feel some exasperation that a student has not corrected a misconception after being told about it. For me, this raises questions about the nature of mental structures supposedly being developed.

It is not, perhaps, a surprising idea that misconceptions and new ideas can exist side-by-side. Examining the history of science and mathematics shows a common theme; an accepted concept can be very difficult to shake off and new data is often interpreted through the eyes of the old model; the earth-centric model of the universe is a well-used example of this. Even as scientists and mathematicians have come to grips with the paradigm shift of the earth orbiting the sun, remnants of the old model can still be seen in statements such as ‘sunrise’. If we accept a constructivist model of learning, then it is legitimate to ask how new understanding can be built from concepts which are erroneous. It would seem that this only occurs if the misconceptions are misinterpretations or extrapolations of concepts which are foundationally correct. If a concept is foundationally false then the student is faced with the difficult action of removing and rebuilding it. As Smith et al. (1993) point out, this is not easy and the misconceptions held from such ideas still resurface. From a pedagogical stance it is important to realize that students will not, and should not, replace their old knowledge with what we simply tell them. Smith et al. consider how misconceptions arise from a novice level of understanding as concepts are stretched beyond applicability. To do this the student must actively engage in the process for which, Smith et al. suggest, confrontation is inappropriate. Rather, they promote “active classroom discourses in which students defend their current understanding” (p. 126) as a means to this goal. It is the incorporation of this discourse (talk) which I explore next with a view to moving ideas about knowledge into the realm of classroom talk.
Social Constructivist Views on Knowledge

Mathematics is cultural knowledge, like the rest of human knowledge. It transcends any particular individual, but not all of humankind, like art, music, literature, religion, philosophy and science. (Ernest, 2010 par. 6)

A common criticism of radical constructivism is its focus on the individual as a learner (e.g. O’Loughlin, 1992). Knowledge, in the form of new meanings rather than actions, is constructed by the individual while social interaction acts more as a stimulus. In ideas of social constructivism, learning is the result of active participation in a ‘community’ where new meanings are co-constructed by the learner as part of a community, and knowledge is the result of consensus (Gruender, 1996; Kovalainen & Kumpulainen, 2007), again making it a thing to be measured. Olssen (1996), for example, refers to Heidegger who, in criticising Kant’s individual agency, pointed out that much knowledge comes by accepting the beliefs and experience of others. To the radical constructivist a textbook is not seen to contain knowledge - von Glasersfeld sees them as scaffolds for readers to build interpretations (von Glasersfeld, 1993) - but I would suggest that sources such as textbooks are often accepted, without question, as a source of knowledge instead of information by a student. A significant role of the teacher is to temper this view and to create a social environment to help students build their interpretations through talk; to be able to perform actions on the information and make it knowledge.

Interaction through language

A central figure in the development of social nature of learning is that of Vygotsky. Vygotsky stressed the importance of social interaction and believed that the learner constructed knowledge through these interactions. In comparison to Piaget, who grounded developmental learning in the individual, Vygotsky recognised the importance of social forces (Vygotsky, 1978). Vygotsky also places an emphasis on the role of the skilful guide to provide a dialogue with the student. In addition, and relevant to this thesis, Vygotsky places more emphasis on the role of language than does Piaget. Besides being the main means by which the teacher transmits information to students, Vygotsky recognized that language becomes a principle tool of intellectual adaptation (Vygotsky, 1934/1986). The learner builds his/her knowledge by interaction with others through language. Vygotsky’s theory suggests learning takes place via person-to-person
interactions, with students learning from each other as well as teachers. Individual learning then occurs through an internalization process that leads to deeper knowledge (Fogarty, 1999), which I would interpret as a greater potential for action.

Much has been written about Vygotsky’s Zone of Proximal Development (ZPD), and it is not my intention to elaborate greatly on the idea. However, I will point to Vygotsky’s definition of the ZPD, which is given as:

\[\text{The distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers. (Vygotsky, 1978, p. 86)}\]

The question arises as to how the teacher is properly able to gauge the ‘distance’ Vygotsky refers to in this definition. If the teacher is to usefully ‘scaffold’ (Wood, Bruner, & Ross, 1976) a learning situation to help a student then a better sense of what constitutes the ZPD for a particular student is essential in this model. Given that language is considered a key aspect of Vygotsky’s theory then it seems reasonable to suggest that it should be through a student’s language that a teacher can acquire a better sense of where a student is ‘at’ in their development.

Paul Ernest

Social constructivists such as Ernest (1998), mentioned in the introduction (see footnote #4), develop the premise that mathematics is “inescapably conversational” (p. 169). This view suggests that conversation is not just a tool for outcomes; rather, language shapes and constrains our experiences as much (or more so) than it reflects them. In diagraming a model of the social construction of mathematical knowledge, Ernest places conversation at the centre of a cyclic process through which collective mathematical knowledge and personal knowledge of mathematics recreate each other; I see this as giving knowledge an active role. Leaning on the work of Wittgenstein, Ernest stresses that personal learning of mathematics is acquired through socially situated conversations. Teachers structure conversations in the classroom based on their own understanding but, as Ernest points out, “sustained two-way participation in such conversations is necessary to generate […] mathematical knowledge and competencies,
and not some partial or distorted version" (p. 221). If conversations form what we perceive to generate a student's mathematical knowledge, then paying attention to the way a student participates in conversations may help inform the teacher about the nature of students' understanding. I would draw attention here to the use of the words sustained and two-way in making the connection back to Davis's (1996) definition of conversation as discussed in earlier. However, the same concerns must be raised with the conversational approach as with any example of student writing, namely a teacher's ability to identify and interpret the conversation, and the students' expectations of their role in classroom talk. Interpreting the nature and role of conversation in the classroom was another starting-off point which led to my interest in this area of research.

**Situated Learning and the shift to participation**

A further development in this vein is the notion of 'situated learning' (e.g. Lave & Wenger, 1991). Situated Learning, however, differs radically from Social Constructivism in its focus on participation instead of mental constructions of schema and representations. Situated Learning examines the context of learning; in particular, the social settings through which the learning occurs. Much of what we learn in school is tightly connected with the environment of that setting and establishing particular types of environment must, then, shape the way the student learns. If a reform-based model of teaching is followed then modifications to the way a teacher assesses a student's knowledge should include consideration of classroom talk. A point to consider, however, is that some students do not find the school setting their primary way of learning, utilizing tutorial or 'learning centres' which may promote a different model of learning. As a result, certain aspects of an institutional setting may not be reflected in the way a student’s knowledge in mathematics is shaped. Views about expressing thoughts, and the implications of wrong answers, can also be strongly culturally shaped. An implication of this can be that certain students are reluctant to engage in a classroom talk. While there may be more tendencies to engage in student-student conversation, it will be important to keep in mind throughout this study that a willingness to engage in conversation may override an ability to engage. One aim of this research is to reduce the impact of the teacher on a conversation by moving him/her away from any direct observation of the students. In doing so the students may be more likely to engage in a conversation and/or say things which they might not if they felt the teacher’s immediate presence.
So far, I have attempted to illustrate a shift from knowledge as being an accurate representation of the world, to something being individually constructed and then being seen as a product of a social environment. The move away from a mental structure to seeing knowledge through participation is taken further by recognising that our whole body is shaping this participation. The whole way students participate in a conversation must then be taken into consideration when we think about their knowledge. Such a view leads to an approach through experience and embodied cognition.

**Embodied Cognition Views about Knowledge**

*Mathematics is a product of the neural capacities of our brains, the nature of our bodies, our evolution, our environment, and our long social and cultural history.* (Lakoff & Núñez, 2000, p. 9)

*According to this view, cognition is not only the basis of action but also a result from it, and 'intelligence is internalized action'.* (Nooteboom, 2006, p. 1)

Theorists of Embodied Cognition consider that mathematics is rooted in and shaped by our bodies (e.g. Barsalou, 1999; Garbarini & Adenzato, 2004). Abstract conceptual knowledge is considered built through bodily experience via concrete objects or virtual objects. The brain is no longer the single store of knowledge but is part of a larger system shaping our actions. Bodily experiences ground the abstractions that are the basis of mathematical thought (Lakoff & Nunez, 2000). Doing and communicating mathematics involves a range of interactions involving the body, from ‘committing inscriptions to paper or whiteboard, to speaking, listening, gesturing and gazing’ (Edwards, Ferrara, & More-Russo, 2011). In a Representational–Computational model of cognition, knowledge is something stored - structured and static symbolic mental representations of the world outside of the brain. The social constructionist model views knowledge as being constructed rather than given, but this model has been criticised for a slide into relativism. By basing cognitive action on bodily actions developed in a shared environment, similar perspectives can be seen to be generated between interlocutors.

Although there are variations within the field of study, all versions of embodied cognition share the goal of ‘developing cognitive explanations that capture the manner in which
mind, body, and world mutually interact and influence one another’ (Cowart, 2005). A result of this can be that embodied cognition “entails a continuum rather than a Cartesian duality between rational evaluation, feelings, and underlying psychological processes in the body” (Nootenboom, 2006, p. 10). Knowledge can then be seen to also include such things as touch, as in the metaphor ‘getting a feel for something’. Importantly missing from constructivist views on knowledge is the impact of emotion, while an embodied view allows meaning to be given to another metaphor, that of emotion clouding judgement.

**Incorporating gesture**

The shift towards an embodied view of human experience also leads to the consideration of bodily actions such as gesturing. If knowledge is indeed embodied, then it would seem reasonable that this would be exposed by subconscious gesturing. Of course, not all gesturing is subconscious, and a distinction needs to be made between gesturing which occurs without deliberate intention, and those gestures which are deliberately made. Deliberate gestures are referred to as emblems (McNeil, 1992), and are generally a cultural product. As such, emblems may be thought of as an extension of language rather than being complementary to it. Emblems are of less interest in considering how gesturing can give information about the way a person is thinking and are as such not a focus of this particular research.

Ideas of embodied cognition reveal a problem that typical written tests are disembodied and as such may not be a sufficient way to gauge a student’s knowledge about mathematics. A typical formal test requires the student to leave a record of what is supposedly ‘within’, to be then interpreted within the examiner’s framework of what the question was intended to expose. But if, as Davis (1996) notes, “our mathematical knowledge [ ..] is neither ‘out there’ nor ‘in here’, but exists and coexists in our acting” (p. 188), then the very act of writing the test is changing the student’s knowledge about a topic. The quotation from Davis at the start of this chapter serves to highlight the fluid nature of this knowledge. If we are to develop a better measure of a student’s mathematical knowledge then it would seem that we need to look at opportunities where a student can demonstrate knowledge in a more embodied and interactive way. By looking for such demonstrations in conversation and body language, we give more opportunity to do so.
Summary so far

In this chapter, I have attempted to provide a basis for thinking of meaning and knowledge as being interpreted through the actions and interactions of individuals in conversation. Knowledge is the on-going interpretation of the accumulation of experiences and resources that represent one’s life at any given time. It is a source for actions. Using this idea as a basis, I return to a deeper look at understanding in the next chapter, as how one goes on from this platform, to build on this knowledge. I will give more support for the idea of thinking of understanding as the ability to ‘go on conversationally’, a theme developed from the writings of Wittgenstein (1967), and to compare some common theories of understanding with a view to being clear on why I see the need to view understanding in this way. From this starting point, my initial goal in this research was to develop a better sense of what students’ participation or lack of participation in the conversational process can tell us about their developing understanding of a topic.
Chapter 3

Theorising Understanding

In this chapter, my intent is to develop further the idea, drawn and developed from Wittgenstein (1967), that a student's understanding is manifested in the ability to ‘go on’ in a given situation. I return to discuss Wittgenstein further at the end of the chapter but I begin with an anecdote before examining other theories of understanding in detail.

Introduction: To Understand

On attending a presentation by Dominik Modlinski, an artist who grew up in the People’s Republic of Poland, I was struck by his reply to a question posed by a member of the audience. The artist was describing his upbringing, during which his artistic talent had been recognised at an early age, and his being sent to an art school rather than a regular school; the question “what would you do if you couldn’t have painted?” was asked. The artist replied without pausing to consider or look at the questioner, stating “I don’t understand the question.” The audience member, perhaps in recognition of the artist’s native tongue, repeated the question by asking again “If you hadn’t been identified at an early age, and trained as an artist, what do you think you might have done?” This time the artist turned to look at the questioner and a slight smile crept across his face. He now replied more deliberately, “I don’t understand the question.”

In the momentary pause before a ripple of light laughter spread across the room, I felt I understood what the artist was saying, that he was unable to envisage a life in which he did not paint; he had no knowledge to base this on. On reflection, I realised that I could not truly understand his viewpoint but rather had inferred an understanding of what he was implying; a different shade of the same colour. This understanding did not seem to occur as an orderly sequence of words, but rather as an atemporal ‘awareness’,

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something that seemed to occur at once, perhaps aided by the body language of the artist and the context of his talk. From the reaction of the audience, I further felt there existed a shared understanding amongst us. My interpretation of the artist’s inability to understand the question – although he clearly knew the intent and meaning of the question, was that he was unable to ‘go on’ in his thinking from the starting point offered. He was unable to realistically envisage any other life, especially a life without painting. Each of us who shared this understanding, I suggest, were able to ‘go on’ in our thinking in order to extrapolate a shade of meaning from the artist’s understated words. Each person’s own experiences would undoubtedly affect how far they were able to go on and the richness of the imagery they would associate with this path. Although there may have been a shared understanding in general, it is unlikely that each person’s understanding was identical. What struck me was that this ‘understanding’ came via a conversation, consisting of both language and gesture and, brief as it was, between the artist and the members of the audience. I realised that a short exchange of related utterances and gestures had the potential to suggest a lot about the understanding of the interlocutors.

The ephemeral nature of this understanding struck me in relation to how a simple stroke of a brush can often say more than a detailed picture as we use our own knowledge to go on from the offered starting point. Further, as I remember this event later in time, my understanding of it may have changed. As I revisit the memory, I add to it as I think about it more; I have ‘gone on’.

**The problem of ‘understanding’**

In mathematics education we are often concerned with students’ ability to understand a topic, such as multiplication, but understanding is an activity that is problematic to operationalize, and can depend on the philosophy of the observer as much as the student to be assessed. The behaviorist, for example, might look for changes in the way the student performs multiplication problems; i.e. getting more questions correct and of an increasing level of difficulty. The cognitivist might study the method used by the student in attaining the answer to the problem and look for evidence of mental structures which support the process. The constructivist might look for evidence that the student can develop their own methods to deal with multiplication, and be able to problem solve...
in unique ways. According to each, what counts as understanding will have quite a different look.

Is ‘understanding’ a requirement in doing mathematics? If the focus of mathematics is the correct answer then understanding may simply be seen as achieving that end. Notable mathematicians such as Joseph Fourier and Werner Heisenberg have suggested as such. It was Heisenberg’s development of quantum mechanics on a mathematical basis without an understanding of what that mathematics was based upon that so concerned Einstein and Schrödinger in the early 20th century (Goldman, 2006). Can it be said that Fourier and Heisenberg’s ability to develop mathematics was sufficient to indicate that they understood it? Clearly they had an ability to use the methods of mathematics in an outstanding way, to be able to see a bigger picture of how these methods tie together. They could take the basic tools at their disposal and go on to produce something unique, even if they had to separate their mathematics from any philosophy as to its meaning outside of mathematics. In this regard, it could be suggested that they understood their mathematics.

**Multiple views and a need for clarification**

In the writing of this chapter, trying to define understanding proved to be – and perhaps still is - a challenging activity. Tall (1978) noted that “there may be as many views as what constitutes understanding as there are mathematics educators” (p. 50), and this, for me, is in itself a concern. It is not obvious how understanding can be inferred from the actions of a student, as even that student’s ability to obtain a solution to a problem may not demonstrate understanding. While the term ‘understanding’ litters the educational landscape there is no consensus on its meaning. A reading of any paper on the subject can offer a dizzying array of prefixes felt necessary to help clarify understanding. A non-exhaustive list often contains some or all of the following: basic understanding; deeper understanding; heighten understanding; show understanding; growth of understanding; development of understanding; proper understanding; collective understanding; good understanding; with a measure of understanding; instrumental understanding; relational understanding; their own understanding; emphatic understanding; acts of understanding and so on. What this suggests is that understanding comes in many guises. At the risk of adding to this confusion, my intention in this chapter is to outline key theories in the field,
while leading towards why I see understanding, from the ideas of Wittgenstein, best
described under the umbrella of the ability to ‘go on’. Having set out this viewpoint,
chapter four will show how this approach can be adapted to classroom situations by
examining the work of Vygostsky; Fleener, Carter and Reeder; Davis; and Sfard.

Background

Henri Poincaré (1908) may have been amongst the first mathematicians to broach the
idea of understanding when, lecturing at the Psychological Society in Paris, he asked
“How does it happen that there are people who do not understand mathematics?” (p. 85).
To Poincaré, understanding was more than the ability to reason or follow the ‘rules of the
game’, it was the ability to put that reason (in the form of syllogisms) in a certain order.
Perhaps more importantly, however, it was to “have a feeling, an intuition, so as to
perceive at a glance the whole” (p. 87). In this, Poincaré seems to be suggesting that
understanding is a synthesis; of being able to grasp a larger idea than individual
eamples can show. If so, this ‘intuition’ does not seem to be a mental process at all as
much as a feeling of being able to move on from the individual elements of the proof (in
Poincaré’s case) in order to put together and develop a sense of what those elements
imply. If understanding is indeed a synthesis, then the question might be less of how to
assess the results of understanding and more of asking ‘what conditions should teachers
put in place for it to occur?’ and ‘how do we recognise such understanding?’

The dynamic viewpoint

Poincaré’s view has a suggestion that understanding is an ‘all or nothing’ situation that
the big picture is either seen or it is not. I think that this need not be the case, however,
as there is still room to have a partial understanding based on the conditions in place at
that time. Rich problems may challenge a student’s understanding by changing the
conditions under which their previous understanding occurred. If understanding is seen
as synthetic then it can expand to encompass these new conditions. In this way
understanding is not static, but rather can be seen as being dynamic. Understanding in
such cases may also be very fleeting, a connection of thoughts not retained. It is often
apparent that a student has understood something in class one day, but is unable to
understanding it the next day. I know that I experience that myself. Understanding seems
to be framed in the conversation – internally or externally – of the moment. What, then, are the conditions necessary to allow for such understanding to occur, and what must be done if this understanding is to be repeated on multiple occasions? I suspect that if understanding was something constructed in the ‘mind’ then it would not be so tenuous. This is something I develop in outlining the development of ideas around the topic of understanding.

Bruner (1969) has suggested that a theory of understanding lies between a theory of instruction and a theory of learning. I take this to mean that while a theory of learning is descriptive and a theory of instruction is prescriptive, instruction should be congruent with the learning theory so as to provide for understanding to occur. In other words, the teaching situation should have understanding as a goal in mind and should therefore know what understanding means. Most attempts to develop such a theory have seemingly taken a more analytical approach in trying to break what it means to understand into more discrete parts. In doing so, I believe that there is a danger of not recognising that its sum may be greater than the parts. For example, a student’s ability to select an appropriate process has been suggested to be a reflection of how understanding is represented internally. However, that there is such a process is by no means clear. Even a student’s performance on an achievement test may not necessarily reflect their understanding (Kulm, 1990). The danger of defining understanding operationally may be to restrict it to a too narrow level of performance, while at the same time, given its complexity, any model of understanding is bound to be inadequate (Pirie, 1988). In turning to the literature, theories of understanding seem plausible as means of describing desired outcomes, but do not seem to capture the dynamic of the process. The aspect of understanding suggested by Poincaré of an overarching synthesis and a ‘human approach’ does not seem to be accounted for.

Before offering a critique of common approaches to understanding, they need to be examined in more detail. In the next section, I look at some of the most commonly cited theories of understanding. I do not presume to claim that these theories are ‘right’ or ‘wrong’, but rather see them as not addressing the dynamic nature of mathematical understanding as developing in the classroom.
Theories of Understanding

John Holt: How Children Fail

Holt (1964) gave what he called a partial list of features he considered demonstrated understanding of something. These included the ability to:

1) state it in my own words; 2) give examples of it; 3) recognise it in various guises and circumstances; 4) see connections between it and other facts or ideas; 5) make use of it in various ways; 6) foresee some of its consequences; 7) state its opposite or converse. (Holt, 1964, pp. 136-137)

This is an impressive list of abilities; it would seem that if a student could perform these actions then most teachers would consider that the student understood the topic. However, Holt’s is more a checklist of actions, a skillset, rather than an indicator of what it means to understand. For example, I have known many students who could apply much of this skillset to functions and yet still claim not to understand what a function really was other than as an equation to solve. Oftentimes it seems that there is a shift in the focus of from the understanding of student to the expectations of the teacher, as in the question posed by Godino (1996): “What object must the student assign to the term ‘function’ so that the teacher may say that he/she understands the object function?” (p. 2). This leads to questions such as ‘what does it mean to understand an object?’ or ‘does the teacher even know what to look for in this regard?’ I question understanding as being in the domain of the educator, wherein it becomes a result to look for, rather than the learner, wherein it is an experience rather than a process. The teacher would then need to look for signs of this experience rather than results of a process. Holt viewed understanding as a continuum with the achievement of each item in the list above as a step along the way. In short though, Holt’s list is a set of behaviours rather than an indicator of understanding.

In considering the question of what makes for a theory of understanding, the research of Sierpinska was helpful. Sierpinska (1994) identified four types of theory for understanding: those which involved a ‘dialectic game between two ways of grasping the object of understanding’; those which envisaged a hierarchy of levels; those involving mental or cognitive models/structures; and those which took a historico–empirical
approach, as she did (p. 119). I propose to examine these types in more detail in the following section, and to pick out some common features which tie these theories to a similar line of difficulty. In doing so, I hope to illustrate why I feel that a different approach is necessary. In the first example, that of Skemp’s theory, I suggest there are inherent difficulties in categorising a dynamic process into distinct types.

**Skemp’s Theory of Understanding (1976)**

An example of Sierpinska’s ‘dialectic’ approach is the work of Skemp (1976), in which the dialectic of instrument and thematization is apparent, and in which a concept can be seen as a tool in solving problems or as an object of study. Sierpinska also identified Sfard’s (1991) operational v. structural understanding to be in this category. At this point it is worth looking at the ideas of Skemp a little more closely.

Richard Skemp’s work has been considered influential in that he was among the first in mathematical education to make a clear distinction between knowledge and understanding (Meel, 2003). Skemp defined and described two types of understanding in his article Relational and Instrumental Understanding. The terms themselves were credited to Steg Mellin-Olsen and Skemp described them as referring to “knowing what to do and why” (relational) and “rules without reason” (instrumental) (p. 152). Instrumental understanding is applicable when there is a specific activity that is required to be done and how this activity connects to other activities is not a concern. Relational understanding is needed when you want to explore ideas further. Skemp noted that, while in many areas (he gave the example of finding one’s way around a new city) these forms of understanding act as different ways of knowing, this would not be the case for mathematics; he suggested there was a “strong case for regarding them as different kinds of mathematics” (Skemp, 1976, p. 25). Skemp revised his definition, building on suggestions by Byers and Herscovics (1977), and included new categories of formal and, later still, symbolic understanding. (A process Tall warned as “start(ing) us on the long slippery slope of introducing more and more refined categories which may not have universal acceptance or universal interpretation” (Tall, 1978, p. 2)) The new definitions, each divided into reflective (looking back and rationalising knowledge) and intuitive (looking beyond what has been understood so far) categories, read as:
Instrumental: the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works.

Relational: the ability to deduce specific rules or procedures from more general mathematical relationships.

Formal: Originally what was to become ‘symbolic’ then later changed to be knowledge of mathematical forms, such as proofs etc.

Symbolic: the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning. (Skemp, 1987, p. 166)

I note here the shift from knowing to ‘abilities’ and the sense that knowledge is being seen in terms of ability to do something. He also writes that “to understand something means to assimilate it into an appropriate schema” (p. 29). This implies that understanding is stored in the mind, becoming an internal process. In this context, a schema is considered to be a group of connected concepts which relate to an object. These schemas can be called up to deal with objects of previous experience, or combined to help solve new problems. Skemp stresses that a schema can be incorrect and need revising (or reconstructing) when new experiences come along, a process that can be challenging. In addition, the schema can become ‘an obstacle’ (p. 27) rather than an aid, especially in situations where the existing schema works for a narrow domain, such as the notion that ‘multiplication’ increases value.

Skemp makes it clear in his reworked scheme that ‘instrumental’ is considered as understanding, something he was initially opposed to. The element that makes it such is the ability to know when and how to apply the learned rules and examples when faced with new situations. If an instrumental ability is to be considered as understanding then it is a fragile process because it lacks the adaptability to integrate new problems, instead taking them on as an increased memory load.

Taking aside the formal and symbolic categories of understanding, the polarisation of mathematical understanding into two basic types, as in Skemp’s instrumental and
relational, has been reworked on many occasions. Haapasalo & Kadijevich (2000) provide the following list illustrating this separation:

- knowing that - knowing how (Skemp, 1979)
- facts/propositional vs. skills/procedural knowledge (Papert, 1980)
- declarative vs. procedural knowledge; (Hofstadter, 1980)
- words specifying concept - mental images/processes (Tall & Vinner, 1981)
- hierarchies of cognitive units - condition-action rules (Anderson, 1983)
- understanding - algorithmic performance (Nesher, 1986)
- conceptual competence - procedural competence (Gelman & Meck, 1986)
- relational representations - condition-action rules (Byrnes & Wasik, 1991)
- rich vs. poor in relationships/algorithms (Hiebert & Wearne, 1986)
- theological vs. schematic knowledge (VanLehn, 1986)
- deductive vs. empirical knowledge (Schoenfeld, 1986)
- meaningful vs. mechanical knowledge (Baroody & Ginsburg, 1986)
- logical/relational vs. instrumental understanding (Skemp, 1987)
- definitions/connections - rules/connotations (Tessmer et al., 1990)
- conceptual vs. practical knowledge (savoir-faire (Piaget; s. Ivic, 1991)
- manifest (structural) vs. instrumental content (Vygotsky; s. Ivic, 1991)
- structural vs. operational thinking (Sfard, 1994)
- connections between conceptions - computational skills (Shimizu, 1996).
- proceptual vs. procedural thinking (Gray & Tall, 1993; 2001)

Observing this list, the use of the terms ‘knowledge’, ‘process’, ‘skills’, ‘competence’, and ‘rules’ stand out. There is the sense that knowledge is the possession of an idea and understanding is a measure of how well these ideas connect. In each case, there is a sense that such an understanding, which is a dynamic and flexible, is more desirable than one in which the student understands how to follow rules. There is a danger, it seems, of confusing procedural knowledge with instrumental understanding; for understanding how and when to follow rules, or not to, does seem to require
understanding. A difficulty with such suggested divisions, as in the above list, is in giving a sense that a student either has one form of these understandings or the other, that there is no interrelationship between the two; that understanding ‘develops’ from instrumental (or the equivalent) to relational. This is not helped when the suggested forms are combined with a ‘versus’ linking them. More appropriately, according to these theories, understanding should be viewed as on a continuous scale, with ‘instrument’ at one end and ‘relational’ at the other.

The variety of attempts to redefine these terms is also suggestive of a sense of unease that they fulfil their intended purpose. Each researcher brings to the table a different interpretation of the ‘process’ based upon their ‘understanding of understanding’. If ‘understanding’ is a process to be viewed, then it would seem that the definition of the terms of study must first be agreed upon. At issue, as Godino (1996) writes, is that “The problem of understanding is, consequently, closely linked to how the nature of mathematical knowledge is conceived” (p. 2). An example of this is the following theory, developed by Pirie and Kieren, which I feel fits into the type Sierpinska (1994) identified as consisting of a hierarchy of levels, but not in a rigid sense such as exemplified by the Van Hiele (1958) model for geometry. Pirie and Kieren’s theory is an attempt to respond to Pirie’s concern that “a single category does not well describe (understanding), nor do such categories capture understanding as a process rather than as a single acquisition. What is needed is an inclusive way of viewing the whole process of gaining understanding”. (Pirie, 1988, p. 7)

Pirie and Kieren’s Theory of Understanding (1989)

Pirie (1988) expressed concerns over theories of understanding such as developed by Skemp. Her concern focussed around a student who demonstrated an understanding of the division of a fraction one day, using pie diagrams, but who was unable to repeat this on a second occasion, even though she could perform division using the standard algorithm. Pirie felt that using categories such as Skemp’s to label a student’s understanding was restrictive and offered that context played a large part in the interpretation of understanding. Pirie’s concern led to a recursive theory of mathematical understanding, developed by Pirie and Kieren (1989). The idea that understanding grows
monotonically was rejected in favour of a model, as suggested by Schoenfeld (1989), wherein understanding can be unstable and retrogressive.

The theory is constructivist in nature, using as a basis Von Glasersfeld’s (1987) interpretation of understanding as a continuing problem-solving process of consistently organizing one’s mathematical structures. Pirie and Kieren couple this with the ideas of Maturana and Varela (1987), to view the knower of mathematics as “self-referencing and self-maintaining in a particular niche of behavioural possibilities” (p. 7). The theory is described as having transcendent recursion because each level of knowing transcends the previous level in sophistication but is similar in structure and contains the levels below. In this way the student has recourse to previous ways of knowing to help them deal with issues of understanding at the current level. Pirie and Kieren (1989) note that “Any previous level is dependent on the forms and processes within and further, is constrained by those without” (p. 8).

Dynamic and recursive

Pirie and Kieren developed a mapping which represents a chart of student’s changes in understanding over time. The levels of understanding wrap round each other, as a series of growing circles, and require access to all previous levels. A key point I see here is that this is suggested to be a dynamic and never-ending process. Pirie and Kieren note that their model is not meant to replace the then contemporary views of understanding, but to provide insight into how understanding grows. Understanding is not seen as a state to be achieved but as a “dynamic and continuously unfolding phenomenon” (Martin, Towers, & Pirie, 2006, p. 151).

A recursive model of learning is appealing in concept as we often need to go back over something in order to go forward. Trying to make sense of a problem, based upon what we already know, perhaps even needing to re-clarify terms or actions within the problem, also relate to problem solving heuristics, such as the work of Pólya (1945), that are commonly suggested for use in classrooms. (Interestingly, Pólya himself identified four levels of knowledge associated with mathematical rules: mechanical, inductive, rational, and intuitive (Pólya, 1962)). The question arises, however, as to where the levels of the model come from and how are they constructed, or why there should not be more. There is a certain sense of imposition about dividing understanding into fixed levels in a
transcendent form, which all students are then thought to naturally construct by themselves. It is not clear how the validity of such a model could be tested, or how it could be usefully used in a classroom. Despite Pirie and Kieren’s addition to the level model there remains the concern that such models are more convenient ways to externally describe the learning process than models of how understanding occurs. Such models also fail to allow for alternate forms of understanding. Finally, but importantly, it does not seem to account for Poincaré’s ‘awareness’ of the problem as a whole. Although the model is dynamic, it still tries to categorise understanding into types without a solid reason for doing so. Difficulties in working with this model to map the growth of understanding in individuals led to further modifications and the development of improvisational models of understanding and the idea that “the growth of mathematical understanding can be observed to emerge at the collective level” (Martin, Towers, & Pirie, 2006, p. 180). This is an area I shall return to in a later chapter.

The third type of theory described by Sierpinska (1994) was built on cognitive models or structures. An example of this type of theory will be described next.

**Hiebert and Carpenter’s Theory of Understanding (1992)**

The basis of this theory also rests on the notion that there exist networks of internal representations within the brain. The degree of understanding is measured by the number and strength of these representations. Knowledge is assumed to be internally structured and that there exists a relationship between these internal structures and external representations. This is an important point to my research because the internal structures are to be inferred from the external representations, based upon how the student chooses to solve a problem in a particular context. In addition, these external representations feed back to further develop the internal structures and create networks between the structures. Understanding is said to grow when existing networks adjoin, which can occur when a student tries to make sense of a new idea. The structures may initially adjoin incorrectly, creating a redundant branch to the network. If the student becomes aware of this, then a reorganization process occurs so that the connections tie together more effectively.
My criticism of this theory is that it seems to describe the storage of information rather than address knowledge or understanding. A student may even have a good knowledge of something without understanding it, as has been mentioned above. Perhaps in response, Carpenter and Lehrer addressed this further in Mathematics Classrooms that Promote Understanding (1999) by writing that “Understanding is not an all-or-nothing phenomena […] but emerges or is developed” (p. 20). The authors characterised understanding in terms of a “mental activity that contributes to the development of understanding rather than as a stable attribute of an individual’s knowledge” (p. 20). This somewhat circular definition views understanding as a way of structuring knowledge, referring to the notion of “acquiring knowledge with understanding” (p. 19). I note here the difference from the ideas discussed above where understanding was a result of structured knowledge. As with Hiebert and Carpenter, (1992) the aim is to create rich integrated mental structures but the notion is extended so that the students are now constructing ‘meaning’. This is done by relating to processes already understood. Carpenter and Lehrer (1999) write “it is not sufficient to think of the development of understanding as the appending of new concepts and processes to existing knowledge… the structuring of knowledge is one of the features that makes learning generative.” (p. 21)

**Introducing Communication**

In such a view it seems, then, that instead of being a result of knowledge, understanding should come with knowledge in order to help the knowledge bond to the internal structures: “Understanding involves the construction of knowledge by individuals through their own actions” (p. 23). Onus is placed on students to adapt what they hear from teachers to their own ends. The authors acknowledge that this is tied to classroom practices in which communication and negotiation of meanings are important factors. This approach argues against the idea of acquiring mastery of a skill first – learning without understanding – and claims that “the rote application of skills often interferes with students’ subsequent attempts to develop understanding” (p. 31). This does not seem to really make clear what understanding is, however, and seems to miss an important aspect whereby students do not ‘get it’ when first presented with a topic but can develop understanding when the topic is subordinated as part of something else (Hewitt, 1996). There is a sense that students need to have what they are expected to achieve. While
this model has features that lead to a more general interpretation of understanding, and is open to an incorporation of a discursive approach, its structure remains vague and hard to justify.

**Some common elements emerging**

Before moving on, there are some common elements to the above theories worth highlighting.

Firstly, each theory places understanding as a process in the individual's mind that can be described and which contains elements that are interconnected. In Skemp's theory, concepts are common properties extracted from experiences and constitute the mental representations. These concepts are then connected, in the form of relations and transformations, to develop schema. It is the regularities in the schema that allow adaptation to new situations. In the Pirie-Kieren theory, the mental representations are images. Images are internalised when the 'image-having level' is reached in the form of general abstractions. Operations are then performed on these abstractions in order to generalize them. Connections within a concept are formed as commonalities are experienced from mental and physical action. An additional feature in the Pirie-Kieren model is the connection between levels as well as concepts. In the Hiebert-Carpenter theory, the whole model is built upon representations and their interconnections within a network.

A second related theme is a need for a basis of understanding upon which to build. In Skemp's model, there must be existing schema. In the Pirie-Kieren theory, the basis is referred to as 'primitive knowing', while Hiebert and Carpenter require an existing system of mental representations and networks. Without an existing base of knowledge, there is an underlying assumption that further development will not occur.

A third common link is the assumption that growth of the mental process occurs by disruption of the existing state by experiencing some kind of problem. Skemp notes that a problem may show the schema to be inadequate. This will result in the schema needing to be reconstructed before the new situation can be understood. The existence of a problem also shapes the progress of understanding in the Pirie-Kieren theory. In this case, the student needs to 'fold-back' to an inner level from where a solution to the
problem can be started by generating new information and ways of operating. In the Hiebert and Carpenter theory, the internal networks grow until a problem is faced. The problem may then force a reconfiguration of the network, or even a complete collapse and rebuilding of the network. This collapse may affect surrounding networks. When the network is eventually rebuilt it “reorganizations (to) yield more richly connected, cohesive networks” (p. 69).

By these models, then, it seems understanding is seen as a process the student has within themselves in the form of interconnected knowledge that grows by being disturbed from its current state. This seems like a very mechanical process, lacking the emotion that often shapes understanding. It could be argued that Watson, the computer that beat all humans at in the game show Jeopardy, could pass a test based upon these distinctions for understanding, yet it is doubtful that Watson had any understanding of what was going on. It is also not clear how the student reconstructs their network of representations or why, having established such a network, this network should not remain in place on a more permanent basis. In some circumstances a student can demonstrate understanding on one day but be unable to demonstrate that understanding the next day.

The issue I am pointing to is that the theories seem incomplete; what is needed is a theory which captures the act of understanding itself and the emotions that go with that understanding, as well as reflecting the broader picture of what it means to understand. A step in this direction, I believe, can be found in the work of Sierpinska (1994), building on the ideas of Bachelard (1938) and Ajdukiewicz (1974). Sierpinska’s work represents a movement towards my own view that understanding is a transient process.

**Sierpinska’s Theory of Understanding**

**Acts of understanding**

Anna Sierpinska opens her book, *Understanding in Mathematics* (1994) with a question that immediately echoed Poincaré’s, discussed earlier. “Why”, she asks, “in spite of all my efforts of good explanation they do not understand and make all these nonsensical errors?” (p. xi). Her introduction and opening chapter outlines her own search for ‘understanding’ and her eventual connection to the work of Ajdukiewicz in Pragmatic
Logic (1974) and the connection between understanding and meaning. Sierpinska’s thoughts, during her search, echoed many of my own, especially in her thoughts about what understanding is not. It is not, for example, just the overcoming of obstacles as suggested by French didactical engineering. What she settles on as core is the notion of an ‘act of understanding’. An act of understanding is “not defined in terms of its impact on cognition; it is not, a priori, judged as valuable or worthless” (p. xiv). The process of understanding is seen as lattices of acts of understanding linked by various reasonings. Good understanding is achieved when the process contains a certain number of especially significant acts, including overcoming obstacles.

Understanding is again described as a process, and Sierpinska’s theory also relates to those mentioned above in that she requires a basis of understanding. At the same time, she sees understanding as developing through, in this case, obstacles to learning. More importantly, though, I feel that Sierpinska has taken a wider view of understanding and has given consideration to such ideas as Gestalt theory – as an “equilibrium in the field of consciousness aimed at the state of mind in understanding” (p. xii) – in developing her ideas. Although linking understanding to such a state of mind, Sierpinska does not always pursue this idea, or incorporate it into her final theory in a tangible way. For example, she notes that things can be understood in different ways and uses the image of a thunderstorm to illustrate her point (p. 5). She refers to the laws of physics and the different states of the atmosphere, but does not incorporate the visceral sense of being in a thunderstorm and how that affects our understanding of it. Mathematics is a more cerebral activity than standing in a storm, but as Sierpinska herself notes, “it is easy to imagine the detrimental effect on education of an attitude that reduces all understanding to knowledge” (p. 23). Sierpinska seems to flit around the human aspect of understanding by further describing understanding as a “grasp of meaning (sense)” (p. 23) and referring to the work of Danto (1969) in noting that understanding does not have to be based on knowledge – we can understand an incorrect sentence without having knowledge that it is incorrect, provided it has sense. Sense, in this case, I would infer as having meaning based on our own experience and ability to follow the arguments of the speaker. In this regard, she seems to be referring to ‘sense’ as action based and knowledge as information. The understanding seems be held in the conversation (and, I would argue, ‘going on’ in the conversation) rather than in the contents of the words.
**Going on**

Sierpinska refers to the work of Greeno (1991) in noting that, while it is common to think of concepts as the objects of understanding, understanding should also encompass actions on notations, procedures, equivalences, relations, and questions (pp. 1-2). I will interpret Sierpinska’s words further by re-wording this section in my own words to read:

I understand a pattern when I can go on to construct a model or generalization.

I understand a concept when I can go on to relate it to other concepts.

I understand a concept when I can go on to relate it to something else in a different situation.

Expressed in this way, I see a connection between some of Sierpinska’s ideas and those of Ludwig Wittgenstein. Sierpinska references Wittgenstein (1958) (to whom I shall return later in discussing the idea of understanding in relation to a mental process) in discussing the notion of ‘seeing-as’, as distinguished from ‘seeing’. Here, Wittgenstein is discussing the optical illusion of a diagram that can be seen as a rabbit or a duck, before relating this to how we might see mathematical shapes in different ways at different times. Wittgenstein also points out that “If you see a leaf in a particular way you use it in such-and-such a way or according to such-and-such rules” (PI §74). In particular, Sierpinska relates this to Poincaré’s response to the question of what understanding is: “they look at the figures, they imagine they have understood when they have only seen” (Poincaré, 1952, p. 119). The ability to be able to see the mathematics ‘as’ something other than just the shapes, or equations, on the page is an important skill, I would suggest. I might say that we are able to ‘go on’ from our initial view of the shape to interpret in a quite different way.

What, then, is an ‘act of understanding’? Sierpinska suggests that it is when the individual is able to mentally relate the object of understanding to another object. She adds that “without a feeling that there is something to understand, it is difficult to talk about any act of understanding to have occurred at all” (p. 32). By mathematical objects, Sierpinska refers to Popper and defines them as creations “brought into existence by our definitions”, something she earlier suggests can be simplifications of real objects, but also quite abstract ideas such as ‘whiteness’ (p. 31). In this passage, Sierpinska seems
to be struggling with the contradiction that objects are often constructed in acts of understanding, while also existing as objects to be understood. This is of interest as it is also tied into Wittgenstein’s notions of the sense of a word being determined by its usage in a language game, a point to which I shall return later.

Sierpinska also notes that “successful communication between two parties is very much like mutual understanding between them” (p. 9), but adds there is also an expectation that some action will be undertaken accordingly by the communicating parties. This stands out, for me, as one of the key principles of ‘having a conversation’, that there is an expectation of ‘going on’ between the interlocutors. Sierpinska later adds, somewhat in contradiction) that social interactions and communications are only steps in a personal process of understanding, and notes that “all (philosophers) agree that understanding is a mental experience: understanding is always in the head” (p. 23). I note, although it may not be her intention, that Sierpinska uses the term ‘experience’ rather than ‘process’, which is a very different idea and one which can be tied into the work of Sfard (2008) in considering thinking as an internal conversation, something I will address in a subsequent chapter.

**Classroom talk**

In contrast to the theories of understanding discussed earlier, Sierpinska also attempts to draw in other aspects into the understanding process besides mental activities. She notes that, for understanding to occur, “some necessary conditions seem quite obvious” (p. 62). In this she is referring to the need for attention and intention. She adds that most students tend to be very passive in their understanding process. The need for the student to be focused on what they are doing was emphasised early on by Locke (1690) and the role of the classroom atmosphere was an integral part of the ‘théorie de situations’ (Brousseau, 1986), where understanding was considered to depend on the didactical contract. Sierpinska again refers to the work of Wittgenstein in pointing out, citing Lambert (1988), that there is no one language in use in the classroom; there exists a mixture of meaning, symbols and notations etc. that may not be compatible. Body language and gestures are also means of communication in play, as is the way teachers capture attention by methods such as highlighting specific points (Pimm, 1992). Sierpinska also examines the role of ‘natural conversation’ in the classroom and why it
can fail in students' discovery. (In this regard she is using ‘conversation’ in the broader sense of classroom talk.) She notes that, often, there is no real communication of ideas or learning and that in many cases the teacher is only looking for the students to use ‘words’ in the right way. This is cited as an example of the ‘funnel pattern’ documented by Bauersfeld (1983) and of leading students to participate in the “meaningless rituals of classroom interaction” (p. 68). I would also relate this to the careful examination of classroom talk in terms of a conversation as a *meaningful exchange of thoughts*. In this way such concerns as raised by Sierpinska may be mitigated if the teacher is vigilant in the process. This is an area where a consideration of Wittgenstein’s ‘language games’ may be of help, and something I shall return to later.

**Community**

What Sierpinska seems to be doing is relating understanding not just to the individual but also to the community and culture in which the individual is placed. Sierpinska uses the work of Piaget and Garcia (1962) in terms of taking a historical view of the development of understanding, but her work also leans heavily on the work of Vygotsky in recognising the cultural and social aspects of learning. In particular, she leans towards Vygotsky’s ideas around the development of concepts. She is moving beyond the idea that understanding is the sole property of the individual mind, in effect broadening its context. I feel she builds her theory on a broader and more flexible base than other theories, but also feel there is still too rigid an approach to what she is suggesting, limited by her retention of understanding as a mental process. By retaining the seat of understanding as being contained within the mind it remains a mysterious process, only to be inferred and based upon our own attempts to describe it. A more flexible approach is contained in the work of Wittgenstein by his adoption of the notion that understanding is not a mental process. In the following section, I shall draw on the ideas of Wittgenstein to show that understanding may usefully be thought of as ‘an ability to go on’. As alluded to earlier, this notion can also be drawn from theories which place understanding as a mental process, but does not require that the brain contains the type of structuring suggested by theories such as those of Hiebert and Carpenter (1992) or Pirie-Kieren (1989).
Wittgenstein

I have made several references above to the work of Ludwig Wittgenstein, one of the most influential 20th-century philosophers, and his ideas about understanding as being the ability to ‘go on’. In this section, I expand on this, situating the idea in Wittgenstein’s own writing and also pointing out concerns with his views. These concerns will ultimately lead back to the work of Sfard and Davis in the next chapter and a more fully developed notion of understanding as the ‘ability to go on conversationally’. I begin by giving a brief background to the work of Wittgenstein before examining his views on understanding.

Wittgenstein, an Austrian national, had a chequered life-story which is of interest in itself. Born in a large and wealthy family (he later was to give away his inheritance to his sister) with three brothers who committed suicide, Wittgenstein worked at Cambridge University with Bertrand Russell on the foundations of mathematics and mathematical logic. In the middle of this he left to fight in the trenches in World War I on the Austrian side, winning medals for bravery. The result of these activities was his book *Tractatus Logico-Philosophicus*, which he regarded as having solved the non-problems of philosophy. The *Tractatus* was highly regarded by the logical positivists, who felt there was no such thing as philosophical knowledge; knowledge was only found through science. Wittgenstein, however, did not share their views and was initially taken by Russell’s logical atomism, the idea that a sentence can be broken down to reveal its logical complexions. Philosophical problems were said to arise from misunderstandings of the logic of language.

Following the publication of *Tractatus* Wittgenstein dropped out of Cambridge and returned to Austria, where he took a position teaching in an elementary school, (Richter, 2004). He returned to Cambridge in 1929 after realising there was more work to do, revising his ideas and leading towards his final work, *Philosophical Investigations*, which was published posthumously. Interestingly, from the point of view of this research, Wittgenstein was reported to have a forceful nature, using characteristic gestures when he spoke; students of Wittgenstein at Cambridge could apparently be recognised by their echoing of these gestures (Edmonds & Eidinow, 2001).
Wittgenstein tended to write in aphoristic ways, leaving much of his meaning open to interpretation. Critics of his work often point to its intent to pull down the edifice of philosophy without putting anything tangible back in place. The abrupt nature of his writing may be a reflection of his character but may also be a way to avoid developing anything that might be interpreted as a structure or process. This aspect of his writing leaves it open to be interpreted and adopted by ideologies he may himself have disagreed with, as was the case with the logical positivists in relation to his earlier work.

It is worth noting that Wittgenstein’s approach to dismantling problems by looking at the language through which they are framed is similar to the approach of Sfard, who establishes what she refers to as ‘quandaries’ before showing how they can be resolved by looking at them in a different way. I shall discuss this further in the next chapter when I tie into Sfard’s work in more detail.

In revising his own work, Wittgenstein’s later writings are characterised by a shift, in my opinion, to a much more readable style, while still leaving much to interpretation. *Philosophical Investigations* was less technical than the *Tractatus* but was itself put together from notes and fragments of writing after his death. His Blue Book and Brown Book, also reflections of his later thinking, were put together from his lectures. The result is that a general philosophy of Wittgenstein cannot be adopted as a general guide because he seemingly worked hard not to have one. His writings read more like a collection of notes than a single structured thesis. While many of Wittgenstein’s ideas are challenged, these challenges often relate to his claims to have solved the problems of philosophy by removing them, in a sense leaving a void to be an answer. While this approach is appealing it also leaves doubt that there ought to be more to it than that.

With particular regard to this research Wittgenstein takes the view that issues with ‘knowledge’ and ‘understanding’ arise when you consider them to refer to ‘things’ and then search for the structure of that thing. I feel his views on knowledge and understanding still have value, and I explore these further in the next section.

**Wittgenstein’s views on understanding**

Rorty (1979) describes Wittgenstein, along with Heidegger and Dewey, as one of the three most influential philosophers of the twentieth century. Rorty’s choice for these three particular philosophers would seem to be his agreement with them that “the notion of
knowledge as accurate representation, made possible by special mental processes, intelligible through a general theory of representation, needs to be abandoned" (p. 6). My intention in summarizing the work of Wittgenstein, in particular, is to use this to illustrate why theories of understanding that similarly rely on mental structures need to be reconsidered, and to discuss an alternative approach to examining understanding. My interpretation of Wittgenstein's work comes from reading his two published works, Tractatus Logico-Philosophicus (1922) and Philosophical Investigations (1967), along with an examination of several commentaries on his work (Goldfarb, 1992; Stern, 1995; Ernest, 1998; Putnam & Conant, 1997; Robinson, 2004; Patterson, 2006). I begin by briefly outlining my understanding of his general philosophy, as I see it, before focusing on that aspect of his writing that pertains to this research.

Wittgenstein's work in philosophy is aimed at exposing errors in thinking caused by our use of language. His two books are interesting in that they represent a complete reversal in the way he approached language. In Tractatus his work was based on a picture theory of meaning (T 2.12), language forming pictures that mirror reality (T 5.511). Wittgenstein's approach was to examine the basic structure of language on the basis that this structure enables us to see the structure of reality (T 4.121). Words had meaning only in the context of the sentence, and only logical statements were sayable. The issue with such a 'picture theory' is that we can examine the knowledge of another person only through their introspective reports. The individual's knowledge is held to be private and separate, to be examined only by that individual. Such a way of thinking led Descartes to his philosophy and the problematic mind–body split; by building outwards from the idea of self-thought as the only reliable grounding, Descartes helped formulate the idea of a disembodied mind which operated separately from its environment. This has given rise to a view which is neglectful of the sense that we are living, embodied beings for whom separation from our environment is impossible. Individuals in cultural isolation can suffer from mental disorders such as schizophrenia (e.g. Faris, 1934) and we necessarily need to be connected to our environment and other people in some way.

Wittgenstein wrote that “The aim of philosophy is to shew (sic) the fly the way out of the bottle” (PI §309). I interpret the ‘bottle’ in this case as a representation of what we refer to as the ‘human mind’, while the ‘fly’ can represent the notion that concepts such as knowledge and understanding are trapped within the ‘mind’. In PI, Wittgenstein's
emphasis became a tool, or ‘use’ metaphor. Instead of saying that the structure of reality determines our language, Wittgenstein switched to believing that the structure of language determines what we think of as reality. With the shift to the ‘use’ metaphor, Wittgenstein moved to the point where he believed that you couldn’t have a conception of the world without language. The meaning of a term is now the sum total of its possible uses and the way to see ‘understanding’ is to examine how such words are used. While the picture only creates an image of one thing for a word, the tool has many uses. The word ‘handle’, for example can be used in many different ways, but it is the way it is used in the context of the person using it that determines its meaning, not a formal definition. To Wittgenstein, in PI, language became infinitely extendable; there was no ‘essence’ that marked a word, just how it was used. A similar idea can be seen in the story of the drunken sailors, cited in Vygotsky’s Thought and Language, from the story in The Diary of a Writer by Dostoevsky (p. 241). In this story a single expletive, repeated between six men, had a separate yet clear meaning to the members in the group, depending on the way it was expressed.

**Removing the mental structures**

Wittgenstein believed it was a great mistake to look for a foundational meaning for a word, going against the traditional idea, stemming from the writings of Plato (as I referred to in Chapter 2) and the notion of Forms, that words get their meaning from ideas in the mind through “non-sensible contact with the ideal and eternal objects of the transcendent world” (Ryle, 1984a, par. 11). The focus of Wittgenstein’s approach was an examination of terms used in philosophy, such as ‘understanding’ in which philosophers had been searching for an ‘essence’, or underlying meaning within the mind. Wittgenstein’s purpose in this was to show that it is a mistake to think of understanding to be a mental process, a structure in the mind which determines what understanding is. Understanding must be interpreted from the way it manifests itself in use, such as in making connections. Wittgenstein considers “a main source of our failure to understand is that we do not command a clear view of the use of our words” (PI §122 original emphasis). However, Wittgenstein does not seem to deny that there is any mental process,

*What we deny is that the picture of the inner process gives us the correct idea of the use of the words ‘to remember’. We say that this picture with its*
ramifications stands in the way of our seeing the use of the word as it is. (Pl §305).

From this I infer that Wittgenstein’s issue is with the attempt to give this process a structure that can be described. The structure ends up becoming the meaning rather than the use of the word, and no structure can fully account for the myriad ways in which the term can be used. In the context of the theories of understanding described earlier, I described how the original theory of Skemp (1987) grew from two types of understanding to four, and how these then grew into a list of ways to describing understanding. The multiple layers of the Pirie-Kieren theory illustrates the point further as the complexity of the model grows in order to account for the many ways ‘understanding’ is interpreted. In addition, if a metaphor is used to describe a state of understanding then this can lead to a need for a state of understanding to produce the metaphor, obscuring the issue (Goldfarb, 1992).

Wittgenstein’s non-denial of some process, but not a structured process, leaves open the possibility of some neural state. Rorty (1979) described how such a process might work when discussing the fictional population called ‘Antipodeans’, for whom the notion of an ‘idea’, or ‘perception’, or ‘mental representation’ was unknown.

When people were given visual illusions they said ‘How odd! It makes neuronic bundle G-14 quiver, but when I look at it from the side I can see that it's not a rectangle at all’. A child moving towards a hot stove is met with the cry ‘He'll stimulate his C-fibres’ (p. 71).

Rorty illustrates how visiting philosophers debate over whether the Antipodeans experience pain, and how they can test for the concept of pain. The responses are the same in every way except that the Antipodeans do not have linguistic mental representations of the sensation, just direct actions. Rorty is making the point that Wittgenstein’s intent was to illustrate how language merely casts ‘linguistic garb’ over our knowledge of what pain is. Our words then render ourselves “forever sceptical about whether the same incommunicable quality is being named when our friends use the same word” (p. 110).
**Language Games**

Wittgenstein recognised that there are many situations where talk is used and, rather than ask for the meaning of a word, the question should be as to how a word is used in the talk. Wittgenstein wants us to get rid of any transcendental foundation for a word and referred to particular discourses as ‘language games’ (PI §7). The notion of a ‘game’ is used to indicate that the players are involved in an activity rather than dealing with something going on inside their heads (PI §23). The language game is a set of social practices which cannot be confined to the individual mind as it is the use of a word in the particular game context, which requires interlocutors, that determines its meaning. This leads to one of the more controversial points of Wittgenstein’s proposals, the idea that you cannot have a private language. Wittgenstein’s reasoning here is that, if you could, you could make up anything as true or false since no one would be there to correct you. Wittgenstein uses the example of a room full of people, who do not know each other and come from no known culture, each of whom claims to have a ‘beetle’ in a box. It would be impossible to know what that person meant by the term ‘beetle’ in such a case since name assigning requires conforming to a socially constructed rule. You cannot make a convention singularly, otherwise how would you know you violated the rule? As a consequence none of the individuals could know what is in their box. From this I infer that Wittgenstein is challenging the notion that there is a private domain to which only the individual has access and claims to be true. Kripke, in an extension to this idea suggested that there could be no such thing as a solitary mathematician; mathematics has to be a communal activity (Kripke, 1982). In relation to the point of my research, this would imply that talk is a necessary part of the process of doing mathematics, and that understanding is a consequence of this talk. What words mean depends on the social aspect; all words are social rather than personal. Our use of language, our understanding, depends on a background of common agreement in what is true and false – not on agreement in opinions, but in ‘form of life’ (§241).

**Action**

Importantly, Wittgenstein examines the idea that a great deal of what we do is just socially and biologically a primitive way of acting and responding. We just ‘act’, and this action does not need us to have a theoretical structure to enable it. Robinson (2004) notes the connection to the work of Thomas Reid, who discussed the notion of a natural
language consisting of sounds and facial expressions and gestures. Wittgenstein
discusses the idea of pain (PI §315), where the natural language, as seen in a child,
would be grimacing and crying (PI §310). Our spoken language is developed on top of
this natural language as a way to build a shared reality. Through acculturation, reports of
pain come to replace the earlier expressions of pain that signalled the presence of pain.

Vygotsky, like Wittgenstein and Reid, suggests that language is developed following
actions and is “determined by a historical-cultural process” (Vygotsky, 1986 p. 94).

*Thought is born through words. A word devoid of thought is a dead thing
[...] The connection between thought and word, however, is neither
preformed nor constant. It emerges in the course of development, and itself
evolves. [...] The intent here is not to detract from the value of the word...
[But] the word was not the beginning - action was there first; it is the end of
development, crowning the deed.*

For Wittgenstein, the problem of knowledge, and by extension ‘understanding’, now
becomes part of a language game. Knowledge inherently resides in a community, not the
individual. If this is the case then it is the sharing of knowledge that maintains it, and the
sharing of knowledge is done through conversation. Within conversation our natural
language is seen through our bodies, in the form of expressions and gesturing. The
traditional way of the ‘mental’ is rejected and replaced by an essential discursive model.

**Going on**

Given that Wittgenstein’s purpose is primarily to force us to question how we think of the
terms we use, there is a suggestion of how ‘understanding’ can be gleaned. Wittgenstein
writes:

*Thus what I wanted to say was: when he suddenly knew how to go on,
when he understood the principle, then possibly he had a special
experience--and if he is asked: “What was it? What took place when you
suddenly grasped the principle?” perhaps he will describe it much as we
described it above--but for us it is the circumstances under which he had
such an experience that justify him in saying in such a case that he
understands, that he knows how to go on (§153).*
The notion of being able to ‘go on’ I take to be a key component which indicates a student’s understanding. Wittgenstein develops this notion further in later adding:

> The words “Now I know how to go on” were correctly used when he thought of the formula: that is, given such circumstances as that he had learnt algebra, had used such formulae before.--But that does not mean that his statement is only short for a description of all the circumstances which constitute the scene for our language-game.--Think how we learn to use the expressions “Now I know how to go on”, “Now I can go on” and others; in what family of language-games we learn their use. We can also imagine the case where nothing at all occurred in B’s mind except that he suddenly said “Now I know how to go on”--perhaps with a feeling of relief; and that he did in fact go on working out the series without using the formula. And in this case too we should say--in certain circumstances--that he did know how to go on. (§179)

> This is how these words are used. It would be quite misleading, in this last case, for instance, to call the words a “description of a mental state”.--One might rather call them a “signal”; and we judge whether it was rightly employed by what he goes on to do. (§180)

Wittgenstein’s idea of ‘going on’ might be taken as an important indicator, or signal, of a student’s understanding. This ‘understanding’ is seen to include a range of examples including, as in §179, the form Skemp (1987) identified as ‘instrumental’. The notion also has flexibility in allowing for a careful study of the extent to which the student ‘goes on to do’. In this way understanding is seen to be an action rather than an attempt to make it fit to a model or to identify some ‘state’ or schema existing within the brain. As Rorty (1979) illustrated, we can allow a neurological explanation of knowledge, a sensory input that causes the firing of a combination of neurons, the same combination firing again rekindles a similar ‘memory’. I would suggest that the structural theories, such as that of Hiebert and Carpenter (1992), have value as a way of describing understanding from an external view; of categorising the process according to some agreed upon, if that is even possible, criteria. It is doubtful that the model matches any actual process occurring
within the student, which must necessarily be both more flexible, dynamic, and integral of body and environment than any model can meaningfully describe.

How, then, do we move from a Wittgensteinian perspective which is aimed at breaking down ideas rather than developing new ones, to something of use in an educational perspective. If the fly is indeed out of the bottle, how are we to keep track of it? In the next chapter I will consider some ideas of how this might be accomplished, examining the work of Davis (2007) related to listening; Fleener, Carter, & Reeder (2007) related to language games in the classroom; and Sfard’s theory of commognition (2008). I will then move on to relate these notions to how I feel conversation and gesture can be used as important tools to recognise understanding in the classroom.

**Summary so far**

In this chapter, I have discussed four theories of understanding and illustrated how difficulties arise based largely on a lack of common agreement on what understanding is. Each theory therefore tends to reflect the view of the author as much as it tries to give an accurate picture of the students’ understanding. I see these theories as being descriptive of the process educators expect to see in a student, rather than relating to what may be actually occurring. In addition, the theories seem limited to examining what is considered as understanding in particular ways. By situating understanding as a process in the mind, it remains unexaminable except on an inferred basis. In addition, by giving structure to a mental process, we are essentially claiming an a priori status for optimal understanding. Even attempts at a more dynamic approach, such as the Pirie-Kieren model, suggest an ultimate structure of understanding exists, applicable to all students and in all situations. In contrast, by situating understanding in the language of the user, and in particular in conversation, the notion of understanding becomes more visible and more dynamic. By developing the approach of Wittgenstein in seeing understanding as an ability to ‘go on’, an avenue to examine understanding is opened.

In the next chapter I connect this idea of understanding, as the ability to ‘go on’, to the Sfard’s (2008) framework of commognition, and Davis’s (1996) distinction between
conversation and discussion. Understanding – and here, mathematical understanding - is then seen as the activity of ‘going on conversationally’.

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Chapter 4

Theoretical Framework and Literature Review

*Mathematics education begins and proceeds in language, it advances and stumbles because of language, and its outcomes are often assessed in language. Durkin and Shire (1991, p. 3)*

In the last chapter I considered the idea of ‘understanding’. I began by examining a selection of theories of understanding that, although by no means exhaustive, illustrate common ways to interpret ‘understanding’. I drew out commonalities between these theories with a view to illustrating why I feel that they have limited ability to capture the meaning of the term. In particular, I pointed out how the common aspect of placing understanding as a structure in the mind complicates the issue, and is more representative of an external viewer’s description and categorisation of the process of understanding, rather than a reflection of what is taking place. I further discussed how, by developing a Wittgensteinian definition of understanding as ‘the ability to go on’, understanding can be thought of as not in the mind, but in the actions of the student. In this chapter I draw upon and extend these ideas to form the basis for my theoretical framework. I begin by re-examining and building on Wittgenstein’s definition of understanding through the framework of commognition offered by Sfard, before looking in more detail at the work on conversation and Conversation Analysis. As part of the communication framework, I then examine how ideas relating to gesture analysis and posture analysis can develop this framework further. My intent is to provide a basis upon which to interpret the communication between students which I present in Chapters 6 through 9.
Commognition

In this section, I examine the ideas of Anna Sfard as they relate to this research. In particular, I use the ideas of *Thinking as Communicating* (Sfard, 2008) as a framework for my study in conjunction with Conversation Analysis and gesture analysis.

Overview

Sfard’s work has many ties to the ideas of Wittgenstein, whom she references frequently in her book on this topic. In particular, she echoes Wittgenstein’s views when she ponders if issues such as the ‘mind-body problem’ or ‘transfer of learning’ might not just be ‘a matter of lexical ambiguities’ (p. xv). A key idea, for me, is her discussion of the Metaphor of Object (pp. 42-49). This is a tendency to give intangible things the sense of being an object, and to then treat it in the same way as a ‘real’ object; manipulating it and using it in conjunction with other objects. Sfard discusses such metaphors as concept and learning disability in this way after exploring the example of number in more detail. She notes that most people seem to think of number as an object, as if a number could be found lying around. The process of objectification removes any metaphoric link and makes an idea seem real. This objectification follows a two-step process, beginning with reification – turning something into an object (concretising it), and followed by alienation – wherein the object is turned impersonal. Numbers are then the reification of the process of counting, so that children go from 3 balls plus 5 balls give 8 balls, to the statement $3 + 5 = 8$ through a gradual process of reification. The word number becomes used as both the signifier and the signified – a confusion waiting to happen.

Sfard posits the possibility of a ‘discourse’ (a term she uses to refer to mathematical talk (Sfard, 2009) in a less formal way than a Discourse Analyst might) that makes no reference to such objects as concepts, learning disabilities, or abstraction. What we are in fact seeing is people in action. It is only observers, when describing people in action, who tend to reify impressions and begin the process of objectification. The observer sees the action of thinking and wishes to formalise it, leading to a reification of the word, and ultimately that “knowledge is a kind of material” (p. 49) and the human mind is a container. Learning becomes ‘gaining knowledge’; thinking becomes moving around ‘entities’ of knowledge in the mind. Concepts, ideas, schemas, notions – all become
smaller or larger units of knowledge. The issue is that this then leads towards equally problematic associated terms such as ‘construction’ and ‘acquisition’ of knowledge. Further to this, with reference to Wittgenstein, arguments over the use of these objects become part of the discourse on education as though they were clearly defined real objects. As Sfard notes, the discourse on thinking itself becomes shaped by the reifications in the image of a discourse on material objects.

Sfard sees a lack of operationality as the beginning of a researcher’s problems, leading to the confusing metaphors that arise from the practice of incorporating terms borrowed from other walks of life. The resulting lack of clarity, for example, leads to the use of quantitative assessment of students, with a belief that a numerical output has more validity than what students have to say. Sfard’s move, following Wittgenstein’s thinking, is to operationalise the discourse with an examination of words such as thinking, learning, and communicating. For Sfard, thinking is operationally defined as “the individualised version of interpersonal communication” (p. 85), with the notion being that the individual plays the role of all interlocutors. Sfard goes further by introducing the term commognition as a combination of communication and cognition, with the intent being to view them as “two facets of the same phenomenon” (p. xvii). In this sense then, thinking can be thought of having a conversation with oneself in the case where such thinking is open to discovery. Important to this is the view that communication is the precursor to individual thinking, a view in keeping with the ideas of thinkers such as Vygotsky.

Sfard also takes a nondualist approach to talk in avoiding mental objects such as schema, in keeping with the ideas I expounded upon in Chapter 3; only empirically observable phenomena are dealt with. In this light, Sfard views a concept as an observable “word or other signifier together with its discursive use” (p. 296). In a similar vein, Sfard refers back to Wittgenstein in writing that “meaning is neither a thing in the world nor a private entity in one’s mind: it (is) an aspect of human discursive activity and, as such, (is) public and fully investigable” (p. 73). Sfard takes pains to stress that such activities are not windows to a private universe, but exist in their own right. This position is separated from behaviourist accusations by stressing that “the complexity of different forms of human doing is neither modular nor describable with a finite set of rules” (p. 74). Actions and ‘forms of life’ are inextricably intertwined, and as messy as they are public
and investigable. By operationalising talk without reference to private entities, the discourse on human doings can be disobjectified.

Disengaging entirely from objectification is neither practical nor desirable, as Sfard notes. The process is an important part of generalizing and of “increasing the effectiveness of communication” (p. 52). Utterances become more flexible and applicable. Objectifying something allows us to capture a moment of an action and refer to it again later, and in this way to accumulate achievements. The problem occurs when this process is extended to human activity – humans constantly change yet reification typecasts by capturing a moment:

In particular when the effectiveness of learning is seen as determined by such personal givens as potentials, givens, gifts, or disabilities, failure is likely to perpetuate failure and success is only too likely to beget success.

(p. 56)

Anecdotally, I illustrate this with a grade 12 student I taught who typically performs at a ‘C’ level in assessments, declaring that she was ‘not good at math’. In contradiction, she had performed well in past external mathematics contests. She decided to participate in the 2013 Canadian Senior Mathematics contest (which has no guessable multiple choice component), saying that she ‘always does well in contests’, and achieved the top mark in the school. She saw these as two separate areas, ‘class’ and ‘contest’, and perpetuated different approaches and responses in each area. Because she has ‘named’ the areas they have become objects and each object then influences the way she thinks about her performance in the future. Referring to Sfard again, defining collapses the discursive hierarchy, an ontological collapse that can produce “illusionary dilemmas” and “phony dichotomies” (p. 57). A practical example might be in labelling students with learning disabilities in schools (in British Columbia) so that they can become ‘designated’. The student then becomes an object to which funding and educational aides can be attached. There are clear benefits in so doing, but there are also dangers in collapsing the ‘disability’ into a ‘category’ with defined boundaries that may not fit the particular child exactly.
If knowledge is considered as a discursive action then it behaves quite differently from object-like information. Sfard wants us to think in terms of difference rather than sameness, in the same way as a child, and to do this requires disobjectification of the discourse on thinking. In this way, à la Wittgenstein, we might avoid linguistic traps inherent in the language. When we consider the cultural dependency of language, and the high cultural mix in modern schools, then the point is even more salient. With this in mind Sfard examines several 'quandaries' in education, with the intent of 'dissolving them' over the course of her work. Of particular interest to this research, is Sfard's Quandary of Understanding.

**The Quandary of Understanding**

As discussed in Chapter 3, understanding is often described as though it were a state. A state implies stability and invariance but Sfard cites research illustrating how someone can seem to have good understanding in one situation only to be at a loss in what seems to address the same 'concept' in a different situation. If it is not a state, and is not constant, then it is not diagnosable, despite our constant attempts to do so. Sfard points out that the oft used idea that to understand is the same as to “grasp the meaning” (p. 61), leads to a causal relationship between conceptual understanding and procedural proficiency when this in fact might not be the case, especially in the area of transfer. Sfard then turns to Wittgenstein’s attempt to replace ‘grasping the meaning’ with the ‘ability to go on’ (Wittgenstein, 1967), of being capable of deciding on a new step after each step already made (Sfard 2009, p. 62). Sfard points out a difficulty with this, using the example of the mathematician Paul Hamos, who was able to ‘go on’ but still felt he did not ‘understand’. For Sfard, it seems Wittgenstein’s definition lacks an intuitive sense of understanding, although I would point out that Hamos’s criteria for judging his understanding is likely to have been quite demanding. This may in fact be a case similar to Socrates’ demand (see Ch. 3) for a reductionist understanding to a level few could achieve. Sfard writes:

> *It is clear that in judging their own understanding people are using different criteria than when trying to assess understanding of others.* (p. 75)
An option Sfard chooses is to use the definition of understanding that is most helpful but which also offers the dialogical approach that, instead of talking about what people have, the talk is focused on what people do. Trying to describe the way a person understands is preferable to talking about their understanding. As Wittgenstein stressed, there are many linguistic pitfalls one can fall into because the language games are too complex to yield deterministic definitions. Sfard’s answer is to define understanding within the practice of commognition as:

*An interpretive term used by discursants to assess their own or interlocutors’ ability to follow a given strand of communication; the commognitive researcher, rather than assessing participants’ understanding, is interested in the interplay of the participants’ first- and third-person talk about understanding and their object-level discursive activity.* (p. 302)

This needs to be unpacked further, and I do so first by referencing and interpreting an example from the work of Roth and Radford (2011). The authors offer a transcript of a problem solving session involving a group of students who are trying to determine a number pattern: “Mario asks Thérèse what she is doing, and the latter suggests… ‘just copy me’… Aurélie, in a plaintive intonation, suggest(s), ‘We have no idea what you are doing’…(later)… ‘I don’t understand and I will never understand’” (Roth & Radford, 2011, pp. 47-49).

Roth and Radford are, in this example, examining the negative emotional valuation in a task as part of a cultural–historical analysis of learning and teaching in mathematics. As I address later when considering prosody, the emotional connection to cognition is an important consideration in any conversation. I use the example here, however, to illustrate how understanding is being viewed from the first- and third-person perspective as well as in Wittgenstein’s discursive activity of being able to ‘go on’. Mario and Aurélie are able to see Thérèse ‘going on’ with the problem (after earlier saying “now I understand”). Thérèse can see a path forward and this leads her to suggest the she understands; they all seemingly recognise Thérèse’s ability to ‘go on’ as an indication of this understanding. Thérèse offers the others a chance to copy her work but, while this might be seen as an example of ‘going on without understanding’, Aurélie recognises
that she and Mario have no idea what Thérèse is doing; she recognises that in order to understand she needs to be able to go on by herself. In the same way, Paul Hamos was able to mechanically follow steps but (from his more advanced perspective) felt uncomfortable in doing so, sensing that he would flounder in an unexpected situation. To claim understanding the individual needs to be able to project this forward; to sense that they could ‘go on’ in other, perhaps dissimilar situations. Roth and Radford interpret this as the student having a need to see the object/motive of their actions, and discuss the students’ frustrations in not being able to do so. While we, as teachers, may only have the actual talk of the student to guide us to examine his/her understanding, the individual has a more intuitive sense of how they feel they are able to perform in the future. It seems important to point out here, as Roth and Radford suggest, that this sense of ‘not understanding’ can be an emotional response based upon the frustration caused by the cognitive conflicts generated in the problem. Indeed, as Sfard (2008) points out, “complaints about a lack of understanding” may be more common than a claim of understanding” (p. 272).

My interpretation of the episode related by Roth and Radford is that while Sfard is suggesting that a student’s understanding can be gleaned from their talk about their understanding, this talk is inextricably tied into the emotional state of the student in working with the problem. This emotion may be just related to the problem at hand, but is more likely to be a relic of previous attempts at problem solving in this area as well as the problem at hand. Belief in being ‘good at something’ propels the individual to persevere and develop the understanding as s/he goes on further into the problem. As teachers, from a dialogical standpoint, we can only view the students’ actions in their talk as a means of developing understanding through that talk while being aware that the student may not share that sense of understanding. Knowing where a solution is going may give a sense of understanding even though the student does not complete the process; being faced with an unfamiliar routine may leave the student unsure of where to go, feeling frustrated (as in Roth and Radford’s example) and desperate for help. As Sfard notes:

*Commognitive conflict, which is sometimes indispensable for learning to begin, is also likely to produce an initial sense of incomprehension. Turning incomprehension into a stepping stone rather than a hurdle to learning is often a matter of appropriate handling of the interaction. Those who wish to*
help people learn, therefore, need to be able to identify situations in which the student is likely to claim incomprehension, and those who want to be able to improve teaching need to be able to tell why teachers’ claims about student understanding are often different from students’ own. (p. 272)

From this basis I infer a need for the classroom teacher to be able to have a better sense of the interactions within students’ talk so that s/he is able to know when to intervene or whether to take a more distant role. Taking a more general definition of conversation than I do in this research, Sfard (1998) concludes that “conversation does seem to have great potential as a mode of learning; yet, on the other hand, only certain types of conversation are likely to bring this potential to fruition” (p. 50). She considers that “the more we are aware of the discursive processes that constitute our mathematical activity, the better chance we have of attaining appropriate control of these processes; the better our control, the more effective our students’ learning” (p. 50). The role of the teacher is paramount if the task is not to be turned into a waste of time. This leads to one of the questions this research is designed to examine, namely: what is the role of the teacher in overseeing classroom talk? The particular lens I wish to apply to this question involves recognising the development of understanding within the talk as conversation.

**Conversation and Understanding**

Previous studies investigating how collaborative activity can help promote mathematical learning focus on its different aspects. McCrone (2005), for example, notes that collaboration “allow(s) students […] to consolidate their thinking by putting their ideas into words, and hence, to build a deeper understanding of key concepts” (p. 111). Francisco (2103) concludes that collaborative activity helps to promote mathematical understanding by providing opportunities to build on one another’s ideas. This view is in keeping with views that collaborative talk allows for a reflective stance on mathematical reasoning (Manouchehri & Enderson, 1999). A shift to viewing mathematical thinking as a “dynamic process, emerging and unfolding moment by moment in the classroom” (Martin, Towers, & Pirie, 2006, p. 150) can also be seen in the work of Mercer (2004), who writes: “The nature of human activity is that knowledge is shared and people jointly construct understandings of shared experience” (p. 138). This construction, I suggest, requires that students establish a conversation.
As discussed in an earlier chapter, in this research I adopt the distinction between a conversation and a discussion as articulated by Davis (1996), building on the work of Gadamer. In this distinction, a conversation is seen as an open-minded exchange of ideas, while in a discussion the move is towards the articulation of pre-formulated ideas. The distinction is important because, as Davis notes, "we tend to have classroom discussions rather than conversations" (p. 39). In a discussion the goal of the participants is often to maintain and defend their views rather than to achieve a consensus. This can lead to the teacher using the discussion to attempt to transmit information rather than build understanding, often in the form of the teacher initiation–student response–teacher evaluation (IRE) sequence. Freire (1992), for example, describes the “fundamentally narrative character of schools” (p. 71), using a banking metaphor wherein teachers ‘deposit’ knowledge to the students. The more the student allows themselves to be filled, the better the student they are seen to be. Davis writes about discussions not ‘threatening horizons’ while citing Gadamer’s view that the conversation can lead to a “fusion of horizons” (p. 26). Conversations are fluid and oriented towards deepening understanding of the issue at hand. As Gadamer (1975) writes, true conversation is:

> a process of two people understanding each other. Thus it is a characteristic of every true conversation that each opens himself to the other person, truly accepts his point of view as worthy of consideration and gets inside the other to such extent that he understands not a particular individual, but what he says. (p. 347)

Gadamer’s approach has been said to focus more on human understanding than abstract knowledge. This understanding is not built from the transmission of meanings between individuals but is a creative understanding occurring in the conversation (White, 1994). The emphasis, for me, is on the notion of the understanding occurring in the conversation. Understanding is not brought to the conversation to be shared, but is instead developed as part of the talk-in-interaction of the conversation. This is not to say that individuals cannot develop their own understanding, which may be seen as ‘self-talk’ in reference to the ideas of Sfard concerning commognition. However, the conversation has the potential to develop further understanding through the interaction of the participants if they are engaged with the process. This may be because there is more going on in the conversation than the exchange of words.
For William James (1890), conversation is a perceptual process as well as an exchange of ideas. It is a sensual experience going beyond speaking and listening to include perception and conception. All the senses become involved, including emotion. In fact, James suggests that rationalism can be intrusive on the conversational experience. Importantly, the context of the conversation affects what is heard and perceived in the conversation. Non-verbal communication purportedly accounts for the bulk of communication in classroom talk (e.g. Mehrabian, 2009), suggesting there is as much an element of showing in talking as there is telling, whether the speaker intends this or not. Body language, intonation, and prosody are all important contributors to the message conveyed. In more formal discussion, much of the affect is lost to the more analytical nature of what is being said, and the more guarded, protective, stances of the participants. In a discussion, participants have a preformed opinion which does not allow change. Baker (1995), in researching subjects’ observations on being actively engaged, noted that in a conversation you listen to hear what is being said and speak with the other person as a consideration in mind. A comment from a participant in Baker’s study illustrates Davis’s (1996) notion that the participants “shape the direction of the talking” in the conversation” (p. 28). Baker (1995) writes:

The more I am able to actually hear what is being said, the easier it is to find my own voice. The more I tend to react to what is being said, the more I tend to just be making noise (p. 206).

The sense here is that simply responding to questions without being actively engaged caused the respondent to say things which she did not actually understand. A connection here can be made to the Topaze effect (Brousseau, 1997), wherein a teacher tries to prompt a student into giving an answer but fails, and looking for some kind of understanding keeps lowering the bar. The student tries to say anything related to the questioning until the teacher finds something acceptable. In this situation the answer is being given in the question for the student to repeat, but the student is really just ‘making noise’. The key point I am making here is that being able to ‘go on’ in a conversation is different from doing so in a discussion. The lack of engagement in the discussion is what gives the speaker the sense of ‘not understanding’.
A conversation, then, is “more than just an intertwining of separate voices” (Davis, 1996, p. 40). There is a connection here to the idea of something co-emerging (Varela, 1999) from the process, and Gadamer (1994) has written on the idea of the conversation as a triad between the interlocutors and the topic. A conversation is seen to have a spirit of its own and “the more genuine a conversation is, the less its conduct lies within the will of either partner”. No one knows in advance what will “come out” of a conversation: “Understanding or its failure is like an event that happens to us” (Gadamer, 1994, p. 383).

Conversations lack the ‘I speak, you speak’ structure of most discussions; they are filled with interruptions, pauses, incomplete thought, exclamations, laughter. The tone is more “what we think” (Davis, 1996, p. 118). Davis adds, “The conversation, enabled by our capacities to listen, is a meeting of (embodied) minds”. (p. 42). A true conversation, then, sets its own path and we cannot know what will emerge from it until it is over.

Conversation is an important aspect of social interaction, but we are also in continuous conversation with ourselves. Conversation can be seen as a means to develop our meanings and understandings as we integrate our experiences with those of others, or attempt to integrate events with our personal perceptions. A conversation may be conceived as a movement towards a new understanding. To Wittgenstein (1967) this involved a “change in aspect” (PI xi (p. 489)), of seeing things in a different way, or ‘seeing-as’. If interlocutors engage with each other and enquire into possibilities, the differences shared can be catalysts of change. If a conversation is successful, it builds towards the possibility of such changes. What, though, determines a successful conversation in the mathematics classroom?

**Success in Conversation**

Attention needs to be given to the unfolding conversational enquiry and to the context or space that holds the conversation if it is to promote learning. Davis poses the question “Who is listening to whom?” (Davis, 1996, p. 34) Rather than seeking to promote conversation we are often in competing monologues and more attention needs to be given to active listening. But what are we listening for? Sfard poses the question “What is the mechanism of mathematical thinking which makes verbalisation of mathematical
ideas beneficial to the whole process?” to a panel of mathematics education researchers (Sfard, Nesher, Streefland, Cobb, & Mason, 1998 p. 41). Her question, though, is framed from a cognitivist stance and stands more as a challenge than an enquiry. At issue is whether mathematical thinking can be considered a ‘mechanism’ that can be adequately described. If we instead equate knowledge with conversation, as Sfard cites Rorty (1979) and Foucault (1972) as doing, then what is listened for is quite different. In this case, as Sfard points out, Rorty is using a broad definition of conversation which includes all forms of communication, including textbooks. She doubts the metaphorical equation ‘knowledge = conversation’ to be the right type of argument. Sfard also casts doubts on the Interactionist argument towards a ‘community of learning’, pointing out that the “The motif of isolation and lack of meaningful communication with others returns time and again in mathematicians’ accounts of their own practice” (p. 42), even raising the suggestion that communication may distract a researcher in their thinking. Sfard’s question to the panel - “Do you think that mathematics can and should be learned through conversation (in the sense of interactive oral exchange)?” (p. 42), I find somewhat misleading, however, in the use of the term ‘learned’. For me the term implies moving between states of knowledge whereas I prefer to think in terms of developing understanding, or knowing. Learning mathematics seems a different proposition than developing mathematics. While learning may occur, conversation may best be viewed as developing an understanding, in particular a mutual understanding, of what the students believe they know.

The response to Sfard’s questions to the panel makes for interesting reading and I draw on some of these ideas in summary. There is a difference between natural language and mathematical language and the cognitivist, as Nesher points out, may be listening for the correct usage of mathematical language as an indication of understanding. However, because a student can use the correct terms does not necessarily mean the student has an understanding. As discussed above, the words could be ‘mere noise’. Even using the term in the right context might not be enough, as this too might be something ‘learned’ without understanding. For example, I have had students say they have used a spirit level to determine the plumbness of a stand, but when pressed on this they were unable to go on and explain exactly what a ‘spirit level’ was. It would seem more informative to listen for a student’s response to such terms than listen to their usage. In other words, to
listen to how the conversation develops around such usage. Nesher sees the ability of students to explain their methods as a beneficial conversation, provided they know what makes up a good mathematical argument. This approach, however, seems more of a discussion in that the student is not necessarily responding conversationally but could be repeating a process, like an actor learning lines. While explaining ideas to others has merit, it would seem to me that it would only be an effective practice if the parties are open to taking these explanations apart in conversation, and are able to go on in response to questioning or to work with them in a new situation.

Steefland illustrates his response with historical and classroom examples, the gist of which points to the need for the individual to communicate with others, following productive self-communication, in order to fully develop their ideas. In the subsequent conversation – “if it is nourished by the constructive input of all its participants” (p. 46) a mutual understanding can be developed to the benefit of all involved. Cobb, in turn, stresses the particular nature of the students’ participation in the conversation. Talk alone should not be the measure, but rather the students’ attempts to articulate their activity interpretations. Cobb is in favour of judicious input from the teacher (or a selected student) to ensure the conversation stays on track, perhaps echoing the concern expressed by the final panelist, John Mason, regarding degeneration to ‘chat’. When should the teacher intervene? How does the teacher know when best to intervene and when to stay away? The presence of a teacher changes the dynamic of a group and can stifle the conversation if it is in flow. If Cobb is correct (speaking from his social constructivist stance) then a teacher needs clues as to the progress of the group, and this is an area I shall return to address later.

Mason points out that personal communication is vital to effective task communication in a non-authoritarian environment, while recognising that it can be overdone. Mason sees a conjecturing environment to be fundamental to productive mathematical conversation – a shared struggle to find ways to express ideas and convince others. He also casts doubts on the social constructivist-based argument that students will somehow learn through developing a productive talk amongst themselves. He notes this “as specious as the behaviourist-based argument that students will learn mathematics by being conditioned or trained to employ techniques and algorithms correctly on typical test items” (p. 49). Mason also sees a cycle in that individual and collective work can prepare
the ground for hearing what an expert has to say, which in turn prepares the ground for
individual and collective work. To me, this would be a process of all participants
maintaining an open mind to each other’s ideas; a series of on-going conversations with
time for individual reflection. A micro-picture of this can be seen in larger group
conversations which allow an individual time to step back from the group to self-
communicate. Strong evidence of this process may be seen if the individual rejoins the
group and is able to make an immediate contribution; weaker evidence may be seen
from private gesturing or posture changes even without an added contribution.

Sfard, in summary, concludes that “conversation does seem to have great potential as a
mode of learning; yet, on the other hand, only certain types of conversation are likely to
bring this potential to fruition” (p. 50). Recognition of these ‘certain types’ would seem to
link back to the separation between the conversation and the discussion. While Sfard is
using a broad interpretation of the term ‘conversation’, my inference is that the
‘conversation’ that lacks potential is really a discussion, or worse, a ‘chat’. An important
ability for a teacher to develop is to be able to recognise the differences in classroom
talk, and in moving to this end it is useful to examine further Wittgenstein’s ideas about
language games. Attention needs to be given to the unfolding conversational enquiry and
to the context or space that holds the conversation if it is to promote learning. I will return
to examine Sfard’s ideas in this context further at a later point.

Language Games

Throughout this research, I have been informed by Davis’s (1997) interpretive approach
and by Sfard’s (2001,2008) communication framework, in conjunction with the thinking of
Wittgenstein. A similar approach is adopted by Fleener et al. (2004), who view
Wittgenstein’s language-games approach as offering a synthesis of the work of Davis
and Sfard. Fleener et al. focus on Wittgenstein’s notion that meaning resides in social
interactions. The authors recognise the difficulty in denying that words have meaning by
raising the question: without meaning as reference, how can social meanings occur?
They ask “how do human beings use language to negotiate meaning and share
understandings within a social systems framework?” (Fleener, Carter, & Reeder, 2004,
p. 446). Their answer to this lies with Wittgenstein’s notion of the language game, as
outlined by Sfard (2001, 2008), and introduced in Chapter 3. Sfard focuses on
communication, and contends that as we learn to play the language games, meaning is generated and understanding occurs. Fleener et al. focus on Wittgenstein’s idea that “to imagine a language means to imagine a form of life”, and note that “language games as a form of life […] can powerfully connect students with the discourse–domain of a particular discipline and can help them view their world differently as the language games evolve” (p. 447). The authors are emphatic that teachers are trying to share with students a form of life – that of experiencing their world as mathematical. If the students’ play in the language game is deficient, then their meanings are disjoint, they are not seeing the world as mathematical. To achieve this, as discussed above, the student requires a change in aspect, to change the way they communicate about their world. In other words, the student needs to change his or her language game in order to participate in the mathematical culture. A student who is unable to make this change is described as aspect-blind by Wittgenstein. Observing how the student plays in the language game gives the potential to reveal more about the nature of a student’s actions in ‘going on’. As important, examining the language games of the classroom can reveal much about the milieu in which the students are asked to learn, in the sense of how used students are to expressing themselves.

Playing with, at, and in

To extend their process, Fleener et al. focus on the interpretations of Wittgenstein by Genova (1995), who emphasises Wittgenstein’s notion of ‘playing’ in the language games. Genova develops three senses of ‘play’ which Fleener et al. use as tools to analyse classroom talk and teacher listening. This ties into the notion of ‘teaching from the sidelines’ and, as such, may offer extra layers of analysis for this research.

When students ‘play-with’ language they ‘suppose’, stretching and extending their meanings through analogy. New meanings may emerge when students take a ‘what-if’ approach as the game continues. ‘Playing-with’ language games tend to be generative as students share personal interpretations, extend ideas, and contextualize meanings. In the ‘playing-with’ language game, listening is an important aspect, especially if the listening is done from a hermeneutic perspective, as outlined by Davis (1997). Such listening is participatory rather than evaluative. Space is given to exploring rather than being funneled to converge on a pre-conceived meaning. The location of meaning is
within the game, and a teacher's role is to be a participant in the exploration rather than a director. Meanings are not developed or understanding shown if the teacher attempts to channel the process.

When students “play-at language games” (Genova, 1995, p.124), they are using the appropriate language and procedures, but the meanings are perceived as external. Fleener et al. write that “understanding is often hidden rather than pursued” (p. 450), but I do not believe this is what Wittgenstein would say since this would internalise understanding; there is simply no understanding shown. In this game, the student is often trying to simply respond in a manner they think the teacher is expecting, often resulting in a comical guessing game until the correct term is found. (Such an process can result in the ‘Topaze effect’ (Brousseau, 1997). In ‘playing-at’ games the students are “playing a role, but not necessarily trying to make sense of the words or correct their own confusions” (Fleener, Carter, & Reeder, 2004, p. 450)

Genova uses the term ‘playing-in’ language games to liken the game to that of acting in a play wherein a script is being followed. Genova uses this term to describe how Wittgenstein uses such analogies as fly-bottles to make his point. The difficulty, Genova points out, “is not to have those who are looking over one’s shoulder misinterpret what one is doing” (Genova, 1995, p 124). The correct language is being used, but the way it is being used may not be perspicuous at first telling. The measure of success is that all players are able to go on; clarity is found not by analysis but by amplification and repetition. I believe that Fleener et al. somewhat misinterpret Genova with regard to this type of play. They liken it to actors repeating words of a play without knowing what the words mean, but this to me is better described by ‘playing-at’ games. They give the example of students who, correcting their own homework papers, may mark a problem as wrong because the form of the answer does not match the provided answer (Fleener, Carter, & Reeder, 2004, p. 451). While this is reminiscent of an actor simply following a script, I do not feel that this is Greeno’s intent while describing Wittgenstein in this way. For Wittgenstein’s ‘playing-in’ language, my interpretation is that the actor has control of the play and is using the script and the play to demonstrate meaning and develop understanding as the play unfolds.
I would agree with Fleener et al. (2004) that understanding can be developed by ‘playing with’ language games and thwarted rather than encourage by ‘playing-at’ language games, but it may be that, although not immediately obvious, understanding is being demonstrated in ‘playing-in’ games. It also seems apparent, as Fleener et al. suggest, that parallel play, where students engage in simultaneous yet individual games, is possible in ‘playing-at’ and ‘playing-in’ games. In such parallel games, it may be difficult for the teacher to know when each game is being played, and listening for clues in phrases such as “Is this how you want us to do this?” is important from a teacher’s perspective.

With the distinction between classroom talk, conversation, and discussion made earlier, and a sense of how a consideration of language games can support recognition of the difference between these terms, I now turn to how Conversation Analysis can be used to examine these aspects further.

**Conversation Analysis (CA)**

I begin with a brief background to CA in order to help situate the process and also make a connection to the latter ideas of Wittgenstein. CA was developed in the 1960’s through the collaboration of Harvey Sacks, Emanuel Schegloff, and Gail Jefferson with a view to examining the underlying social organisation that makes social interaction possible. Prior to CA, the dominant sociological approach to the analysis of action (theories of social order taking into account the actions and reactions of individuals) was that of the Harvard sociologist Talcott Parsons. Parson’s model combined cultural, personality, and social systems, but viewed cultural values as playing only a minor role. Mutual understanding and shared meaning were considered the outcome of pre-existing common knowledge with shared norms of conduct. For Parsons, the detailed analysis of social interaction was seen as less significant than analysing a macro social system. As a result, the analysis of language and meaning was deemed part of a linguistics study. Within linguistics, however, the influence of Chomsky was effecting a separation between language and meaning (Chomsky, 1957) and this separation was furthered by Saussure’s definition of the scope of linguistics (Saussure, 1959). The result was that “the relevance of talk-in-action fell between disciplinary boundaries” (Goodwin &
Behaviour within interaction was seen as background noise obscuring the ideal structures linguists sought and some speech-act theorists went so far as to base their analysis on isolated sentences stripped of any social context, in contrast to the original theory. Perlocution, an act performed by saying something rather than in saying something (Austin, 1962) became marginalised as a speech act, thereby losing a bridge between action and interaction. The dominant paradigms of the time were thus antithetical to an analysis of social interaction (Goodwin & Heritage, 1990).

The insufficient attention paid to areas such as practical reasoning and common-sense knowledge in enabling interlocutors to grasp each other’s intentions – and ultimately achieve mutual understandings – began to be questioned (Garfinkel, 1967). Goodwin & Heritage (1990) describe how Garfinkel based his work, in which he demonstrated mutual understanding to be highly contingent and revisable, on the writings of Schutz (1962), who was converging with the later ideas of Wittgenstein in terms of meaning being created in the context rather than by the words used. For Schutz, understanding was perpetually renewed and without guarantee; it was not a structure an individual held or necessarily retained. Understanding was seen as being informed by the presumption of ‘the general thesis of reciprocity of perspectives’, which assumes that experiences of others are “identical for all practical purposes” (Goodwin & Heritage, 1990, p. 286). This leads to ideas of ‘common-sense knowledge’ and a sense that analysts needed to include mundane events in their studies. Garfinkel’s methods showed that individuals particularise their interactions to social settings and that the resulting actions redetermine the sense of the current context. Garfinkel, along with Erving Goffman, who considered that conversational interaction represented an institutional order sui generis (Heritage, 1997), had set the groundwork for CA to emerge.

**Sacks and Schegloff**

Sacks and Schegloff were both students under of Erving Goffman and as such were also in contact with Garfinkel and his ideas.

*The discipline of conversation analysis essentially emerged as a fusion of the interactive and phenomenological/ethnomethodological traditions. Within this fusion, interactional materials would be used to investigate the*
Sacks reportedly became interested in the organization of conversation through his work at a suicide counselling hotline (Pomerantz & Fehr, 1997). He wanted to know if the seriousness of the caller could be determined from their conversational organization. In a sense, he wanted to be able to read the caller from the way they engaged in the conversation in order to gauge their risk level. A key part of his findings was whether or not the caller was willing to give their name at a conversationally appropriate point. A serious suicide risk caller would bypass this point by initiating a repair sequence— a short series of turns which served to solve some other problem and so avoid revealing their name. He concluded that there are norms associated with conversations concerning when particular events should happen. Sachs realization was that verbal interaction has an organization. This is seen in Conversation turn-taking where responses between interlocutors often occur in pairs and where there was an expectation of a certain response. Previously it was considered that language was simply a medium to pass on thoughts; Sachs realized it also had a social structure. Sacks, along with Schegloff and Jefferson then looked at other areas of conversation and realized that turns were systematically designed objects which performed activities in interaction. The goal of their analysis was then to determine the nature of these objects. For example, they found that turns appear in sequences so that a conversation has an introduction sequence followed by a definite marker to begin the core sequence. The word ‘okay’ is often used as a marker between sequences and is thus a useful device to initiate the end of the core sequence and move to the pre-closing sequence. It was conceived that there is an institutionalised set of interactional rules, procedures, and conventions that provide the framework for interactions in a particular context. Examining a conversation for this subset requires an analysis of the social context and mutual knowledge of the participants. Paul ten Have describes CA’s basic analytic strategy as: “Take what people are doing, that is saying, not-saying, saying something in a particular manner, at a particular moment, etc. and try to find out the kind of problem for which this doing might be a solution” (ten Have, 2007, p. 16).
I take from this that a conversation has a sense of determination based upon the way the interlocutors manage its progress through their social interactions and sense of mutual understanding. To put this into the context of this research, I suggest that the early work on CA has ties to the ideas of Wittgenstein, as developed in Chapter 3. Not directly perhaps, but in the sense that meaning is generated in the context of the conversation and not uniquely by the words uttered. Sacks emphasises that there is no predetermined structure to a conversation, but nevertheless conversations exhibit an organization that can be analysed. From this organization the analyst is able to make interpretations and inferences that the language itself does not directly say. It is this discernable organization that may give clues to whether students are engaged in a conversation and whether the conversation is of the ‘certain type’ that Sfard alluded to above. The conversation will not ‘go on’ unless there is a shared sense of meaning and connection as well as a willingness to communicate openly.

**Aspects of CA**

CA never treats sentences as isolated, self-contained or invented. Sentences (the abstract entities) and utterances (the stream of speech produced by a speaker) are understood as forms of action situated within specific contexts. Utterances are analysed from the talk, or actions, that emerge from them, meaning that the analyst is also looking ahead to the outcomes in order to interpret the full implication of an utterance. The interactional sequence was the analytical innovation that formed the basis of CA; each conversational action defines the situation to which subsequent talk is oriented. The adjacency pair is the elementary form of sequence developed by Sacks and Schegloff, wherein the first pair part requires the production of a reciprocal action, or second pair part, at the first opportunity. The sequence is normatively organised in the sense that it is required from a social aspect, rather than necessarily associative. For example, the more formal greeting “How do you do?” has the acceptable response “How do you do?” rather than requiring an actual answer to the question. A lack of response to the greeting is noticeable and creates immediate tension. Not providing the ‘preferred response’ (Grice, 1975) is a way a speaker can influence the conduct of their interlocutors and Gricean implicature, discussed further below, is an important aspect of CA. The adjacency pair is more than a just a structure however, it is the way participants hold one another accountable to produce coherent courses of action. Each participant must be able to
analyse the other’s actions in order to give the appropriate response, meaning that interpretations of what is being said are intertwined. For the conversation to develop it is necessary that participants are able to incorporate preceding events into what they are saying, and for this to have meaning to the interlocutors. This then provides a basis to judge all parties understanding of what has transpired (Goodwin & Heritage, 1990).

While most social scientists at that time considered utterances to be made up of relatively few orderly products, Sacks took the approach that there is order at all points (Jefferson, 1984). Nothing in an utterance is considered too trivial to overlook. For example, the way overlapping occurs in speech has implications. In some cases a speaker may drop out of a turn and then recycle it when the current speaker has finished. At other times, a speaker may drop out and not take up a new turn, or offer only a minimal recipientship and exhibit attention to the overlapping talk before getting back to his/her own overlapped talk (Jefferson, 1984, p. 3). Alternatively, the interrupted party may produce an acknowledgement token and follow that with a shift in topic. These tokens can consist of objects such as ‘yeah’, ‘Hmm hm’ etc. Jefferson argued that ‘yeah’ exhibits a shift from recipiency to speakership, while ‘hmm hm’ exhibits passive recipiency, giving the current speaker a signal to continue. It is important to note that the cause for these differences cannot be determined from the words spoken at the time, but from the actions that follow.

Sacks demonstrated that turns and sequences exhibit institutional context. An area of focus has been on opening and closing encounters, on how information is requested, delivered and received, and on particular descriptive terms related to the institution (Sacks, 1995). How students start to collaboratively work on a given problem is then of particular interest to this research. The developing turn organization has the potential to inform us regarding any mutual understanding that is being formed. It would seem that, unless there is some belief in a mutual understanding, a conversation lacks the foundation upon which to develop. It is important here to stress the sequential nature of turns. It is not just that turns are connected but that responses are conditionally relevant to utterances in prior turns. This conditional relationship is referred to as being normative. If sequential turns at talk are absent then there is usually an attempt to re-establish a connection, known as next-turn proof procedure. In this procedure, the speaker tries to determine if the recipient shows their understanding or interpretation of the utterance. If
this fails, a lack of mutual understanding is implied. Comprehension can, however, be worked out over the course of several turns (Mazur, 2004). It should be added that an individual’s understanding may be masked by an unwillingness to engage in conversation with another while at the same time being capable of engaging in self-talk.

**Turn Taking**

The seminal CA work of Sacks, Schegloff, and Jefferson articulated three basic facts: turn taking occurs; one speaker tends to speak at a time; and turns are taken with as little overlap as possible. The turn form, turn content, and turn length are all of interest in the research. Turns between teachers and students have more boundaries than those between students as it is unusual to find overlapping speech in such a case (Mazur, 2004). Turns themselves are made up of two components, namely turn constructions and turn distribution. A turn construction is then made up of turn units that may consist of just a single word. What is important is that the unit has projectability, meaning that the speaker can project the type of unit it will be before it is over. The unit also contains transition-relevant points, which make it possible for transition between speakers. These features allow for meaningful overlap between interlocutors as the listener can pick up on the projection of the speaker’s unit and be aware when transition can take place without interruption being impolite.

Turn distribution refers to how a speaker is designated a turn at talk. Sacks outlined three simple rules which seem to govern turn distribution in a conversation, but he was emphatic that these were not prescriptive rules and could more easily be referred to as ‘practices’. 1) The current speaker may designate the next speaker; if not any speaker can self-select; if no other speaker is selected, the first speaker may continue. 2) Whatever the result of rule 1, the rule reappears at the next transition-relevant place. If the listener misreads the projection, then they must undergo a repair sequence, which means they must engage in one or more turns that address any affront to the previous speaker. Schegloff (1992) points out that projection is a *possible* transition point rather than an *actual* transition point because in natural conversation the optimum condition is for as little time to elapse as possible between turns. This requires that the respondent is able to read the situation quickly and this can lead to errors.
Examining student talk in terms of the turn-taking sequencing is a fundamental aspect of CA and therefore a key aspect of what this research will be examining. My claim is that if there is no sign of clear turn-taking then the students are not engaged in a conversation. Whether or not this realization affects the progress of the talk, and by extension the progress in the problem, is therefore a key point of interest. CA is not restricted to a study of turn-taking, however, and there are other important aspects which are also investigated to provide information. In the following section, I will outline some other aspects which have relevance to this research.

Other Aspects of CA

Prosody

A conversation is also indicated when the participants attempt to engage one another in an informative and relevant way (Hutchby & Wooffitt, 2008). An aspect of this is seen in the cooperative overlap in which one speaker talks at the same time as another, demonstrating an interest in the conversation. (I emphasise the use of the word cooperative here, as part of a conversation rather than a discussion, where the overlap/interruption may be more ‘hostile’.) Another important aspect of speech is prosody, which can include variation in syllable length, loudness and pitch. Intonation also plays a role in all forms of talk as it expresses both textual and interpersonal meanings. Intonation and rhythm, especially the pitch contour of speech, figure prominently in the information system (Halliday, 2009).

In CA, an important part of the markup system includes indicators of intonation and loudness as well as pauses, both within a word and between words. An aim of the conversation analyst is to include all information possible within the transcript in order to be aware of any pattern that emerges. Although not a CA person, a key point of the linguist Michael Halliday’s position is that intonation plays a similar role for speech as grammar does for the written word but, I would add, has a more embodied aspect. Written grammar is more abstract and is a learned skill, while vocal intonation is can be an aspect of emotion, such as a rising tone in anger. Choices of tone play a significant role in realizing interpersonal meanings associated with mood and modality and play the role of foregrounding new information. Tone sequences create textual structures for the interlocutor. For example, in relation to English, a non-falling tone creates an expectation
that some further information is coming. Tones that start high and drop are usually declarative, while tones that start low and end high are usually interrogative. A flat tone throughout an utterance with a sharp rise at a chosen element is a negating proposition for that element: “A level tone can be informational while fall-rise seems determinate but isn’t, and rise-fall seems not determinate but is” (Halliday, 2009, p. 192).

The prosody of a syllable, word, utterance, or phrase can, shape the sense of meaning within that unit. Wittgenstein suggests that when people communicate with each other, they may have to choose between a private language and a common language. The rules of a private language may not be the same as the rules of a common language. The meaning of words in a private language may not be the same as the meaning of words in a common language. People may need a common language in order to share an understanding of the meaning of words but I suggest that prosody can be used to attach meaning to words even when words are not used in their normal context. For example, the single word ‘cool’ may have a dictionary definition, but can take several meanings, depending on context and intonation.

In addition the aspects of CA discussed above, an awareness of affect in a social situation is important. In order to participate in a conversation in an engaged manner, a condition stressed as vital to the success of the conversation, the participants must feel comfortable with that engagement. Several features have been suggested as having importance in this area, and the key work of Goffman (1972) and Grice (1975) will be discussed next.

**Politeness**

Tatsis & Rowland (2006) discuss the Politeness theory of Brown and Levinson (1987) based on the work of Goffman (1972). This includes Goffman’s notion of face, which is the “social value a person effectively claims for himself by the line others assume he has taken during a particular contact” (p. 5). Face is further categorized in terms of positive face, for social approval, and negative face, expressing the need for freedom of action. Tatsis and Rowland (p. 258) elaborate on face threatening acts (FTA) and face-saving acts (FSA), as well as providing taxonomy of strategies to deal with these situations. Another important part of any student talk is the commonplace redressive action ‘face-work’, used to indicate that no face threat is intended. Initially, when a speaker tries to
organize his/her thinking and lacks information, vague language is used in order to protect positive face. Such phrases as ‘What if..’ can serve to redress positive face to prevent any offense being taken, while negative face is redressed by ensuring that suggestions are not seen as giving direction. This is particularly important in successful group work where power struggles can be an underlying influence. A big part of conversation is devoted to face-work and Goffman looks to finding an order that transcends the individual culture being studied. Goffman’s study of interactions focuses not on the individual but within acts between the individual and others in a social setting (Goffman, 1972). He looks for a minimal model of social interaction applicable in a general sense rather than being characterised by the behaviours of individuals.

Individual behaviour in a group setting is not prescribed, but Goffman (1972) suggests that there are only a few options available to an individual in any encounter. The choice an individual makes is dependent on their immediate and long term goals. An interesting aspect of this is the idea that an individual can have a difference ‘face’ when interacting with different groups of people. Having the right face for the given situation gives the person confidence and assurance; when in the wrong face, the person can feel judged or threatened. Goffman also sees face as being transferable, so that an individual can ‘give face’ to another. This allows the other to take “a better line than he might otherwise have been able to take” (p. 9). In a classroom context, this might mean supporting a friend’s position even knowing it to be wrong, or giving them a way out of having made a mistake (such as a teacher might do). One aspect of conflict in face work is that it generally requires an audience. Goffman points out that “the dilemma is not the enactment of the incident, but the confusion over what face to use to handle it” (p. 26).

In terms of this research, the idea of face is an important aspect when students are randomly being grouped and asked to discuss a mathematics problem. Within the confines of a class of students, notions of ability and social acceptance are quickly established. Students may have a face of being ‘weak’, or ‘disinterested’ in mathematics and so strive to maintain this face through their comments and the level of their perceived engagement. Students who project strength in mathematics may be unwilling to accept comments from other students, taking this as a threat to their face. More often, in my experience, students who are labelled as being strong in mathematics at an early age can be reluctant to take risks in class as any failure is seen as an affront to face:.
“Embarrassment is caused by a failure of expectations: Social obligations are not sustained” (Goffman, 1972, p. 105). This can result in stunting their growth in this area. As Goffman writes: “Whatever his position in society, the person insulates himself by blindness, half-truths, illusions, and rationalizations. He makes an ‘adjustment’ by convincing himself, with the tactful support of his intimate circle, that he is what he wants to be”. (p. 43)

The point I make here is that a student can act in a quite different manner when grouped with a different set of peers; either being more reserved than normal, or being more confident than usual. In a related area, another aspect of conversation was studied by Paul Grice, resulting in what are referred to as the ‘Maxims of the Cooperative Principle’.

While Grice uses ‘conversation’ in the broader sense of ‘talk’, his work has relevance here. Grice presented his maxims as guidelines for successful communication, but they can also be viewed as presumptions a listener makes about an utterance. My suggestion here is that a conversation in the meaning I use – an open-minded exchange of views – may only be sustainable if Grice’s maxims are upheld. In this regard I feel they are important to consider.

**Conversation Implicature**

Conversation implicature (Grice, 1975) examines the relation between what people say in an utterance, and what they actually mean. Grice refers to this as: “a certain subclass of nonconventional implicatures” (p. 45) It is based on the belief that talk exchanges are characteristically cooperative efforts; the interlocutors generally want to make sense of what each other are saying in order to move the conversation forward. To avoid having to be explicit in every detail, the speaker often leaves out information which the listener must infer. An example Grice gives is:

\[A: \text{I am out of Petrol.}\]

\[B: \text{There is a garage around the corner.}\]

B implicates, to the best of his knowledge, that the garage (gas station) is open and has petrol (gas) to sell. For this not to be the case would be a violation of the cooperative principle in the sense that B would be misleading A. Grice outlined four ‘maxims’ of cooperation that a speaker must not violate if cooperation is to occur.
The Maxims of the Cooperative Principle (CP)

- **Quality**: Speak only what you believe is true; do not say that for which you lack evidence.

- **Quantity**: Be as informative as is required; do not make your contribution more informative than is required.

- **Manner**: Be brief, orderly, and unambiguous; avoid obscurity of expression.

- **Relevance**: Be relevant to the matter in hand

For example, ‘Well’ can be a hedge on Grice’s maxim of Quality and can often precede a contribution the user knows adds nothing to the conversation. As Rowland observes “the participants care about the mathematics, but they also care about themselves, their feelings and those of their partners in the conversation” (Rowland, 2000, p. 125). The listener must assume that the speaker is trying to communicate (being cooperative), speaking truthfully, relevantly, and informatively. The listener will try to make adjustments to make the utterance conform, and the speaker exploits this to make the exchange workable within the context of the situation. If a student is working with a teacher and says “I really didn’t get it”, there is an implication based on the relationship that help is being requested. If the student is working with a partner on a problem and says “I really didn’t get it”, the implication of help may depend on the prosody of the utterance and familiarity with the person being addressed.

It is again important to stress that, as with the ‘rules’ discussed earlier, these maxims are not prescriptive. They are empirical practices and are not hard and fast; they are used unknowingly by the majority of people. It should also be noted that the maxims can be deliberately flouted, as in the use of sarcasm or deliberate vagueness. Once again it is the context and future unfolding of the conversation that gives these expressions meaning.

In terms of this research, I see CA as a lens to provide an insight to the interaction between the students. In a similar way, Sikveland and Ogden (2012) use CA to examine understanding as ‘a temporally-bound achievement accomplished through (and embedded in) turns at talk’ (p. 167). This analysis may be important in helping to
determine whether a conversation struggles due to the interaction between its participants rather than the level of difficulty of the mathematics. Flouting of one or more of the maxims established by Grice may cause a breakdown in a conversation, something that becomes apparent in any social situation once you become aware of the maxims. Strict adherents to CA will point to a different focus in this research than may be typically found when CA is used. To reiterate, I am not using CA to discover the underlying social organization through which the social interaction in the classroom is made possible. As noted earlier, I am taking the basic ideas of CA - turn taking, adjacency pairs etc. – as being well established. My premise is that these are signs that conversation is taking place, that there is a mutually beneficial exchange of ideas occurring within the talk of the students. When these signs are absent, the conversation has broken down. What I am interested in investigation in this way, is how this notion of a conversation can be seen to correlate with progress in the task at hand. If the students are demonstrating learning in the way Sfard (2001, 2008) suggests, this is seen as students changing their discourse about the topic. In addition, it might be expected that there is a movement towards finding a mutually agreed upon solution to a given problem.

Having outlined the core ideas of CA that have informed this research, I now pursue the idea of further connections to gesture and body language in more detail.

**Gesture and Body Language**

When students engage in mathematical problem solving in a group situation, there is a clear need for good communication to occur within the group if all participants are to gain from the collective experience. In everyday talk, gestures have been considered to be an integral part of communication (e.g. Kendon, 2004; McNeil, 2005, Sikveland & Ogden, 2012) and linked to speech in a semantic and temporal way, while body language plays a part in any group talk (Goffman, 1972). Goffman refers to expressive cues which we use as part of the communication process, illustrated by Vertegaal et al., who make a strong link between the amount of eye contact people give and receive to their degree of participation in group communications (Vertegaal, van der Veer, & Vons, 2000). In addition, Hastings describes how certain eye movements may be associated with particular kinds of thinking (Hastings, 2006). For the majority of people, forming a mental
image seems to be embodied by looking up and to the right, while dealing with feelings seem to be embodied by looking down and to the right. Looking up and to the left is usually connected to remembering an image, while looking down to the left is often associated with self-talk and negative imagery.

Roth (2000), for example, describes a conversation as gestures and talk, adding that the gestures and words only take on specific meaning in the interaction. As such, Roth sees thinking being shifted into the world before the listener rather than confined ‘in the head’ (p. 368). Radford (2009) supports this position, noting that “Thinking does not occur solely in the head but also in and through a sophisticated semiotic coordination of speech, body, gestures, symbols and tools” (p.111). Sfard (2009) observes that combining speech and gesture brings about an “obvious synergetic effect” (p.193), adding that gestures to be “crucial to the effectiveness of mathematical communication […] to ensure that all the interlocutors speak about the same mathematical object” (p. 197). Sfard’s position is worth further consideration, given the discussion of her commognitive framework earlier in this chapter.

Sfard (2009) is particular about the need to define the terms she uses, something she notes as often lacking in other research. She uses the definition of language as “denoting a communication–mediating symbolic system with rules for creating permissible elements (meaningful expressions) from those previously constructed” (p. 193), while gesture is defined as “a body movement fulfilling communicational function” (p. 194). Defined in this way, language and gestures are seen as categorically different, and so not combinable. Instead, Sfard restricts the connection to verbal acts only – the utterance (an act executed by emitting a series of sounds, the communicational effectiveness of which originates, among others, in how these sounds are related one to another), where utterance and gesture may combine into a communicational act. As a final point in this regards, Sfard stresses that

*gestures […] do not owe their communicational effectiveness to their being members of a symbolic system with syntax, but rather to our spontaneous ability, grounded in our cultural experience, to relate certain body movements to certain familiar things in the world (p. 194).*
These definitions may be related to Kendon’s Continuum, as developed by David McNeil (1992, 2005), wherein various forms of gesture are placed on a continuum ranging from the completely unintentional gesticulation to the formalised sign languages such as ASL. Within this spectrum lie emblems, which are informal but intentional gestures which carry a known cultural meaning (Table 4.1), and pantomimes which are deliberate gestures used to illustrate an action or activity.

<table>
<thead>
<tr>
<th>Gesticulation</th>
<th>Pantomime</th>
<th>Emblem</th>
<th>Sign Language</th>
</tr>
</thead>
</table>

**Table 4-1 Gesture continuum and gesture space**

While McNeil originally defined gesticulation as “idiosyncratic spontaneous movements of the hand and arms accompanying speech” (McNeil, 1992, p. 38), he revised this to
include any part of the body as being able to gesticulate (McNeil, 2005). At times when
the hands are occupied, for example, gesticulation can occur with head motions and
facial gestures. Gesticulations have been categorised into four broad areas (McNeil,
1992), but there is some acknowledged overlap in how these perform and multifunctional
gestures are commonly seen.

**Gesticulation types**

I will refer again to these types in Chapter 5, on the methodology used in this research,
but a brief description will help what follows:

**Iconic**

Iconic gestures represent an actual action or object.

**Metaphoric**

Metaphoric gestures represent an abstract idea.

**Dietic**

Dietic gestures point to or at something.

**Beat**

Beat gestures carry no meaning and are often timed with prosodic peaks in speech.
Beat gestures can be associated with emphasis or an emotional state. A fast staccato
beat can indicate an agitated state, while slow deliberate beats may indicate
determination.

Of further interest is McNeil’s acknowledgement of performative gestures (McNeil, 2005)
which enact what they perform (Müller, 2004). Thus a rolling ball gesture enacts the
motion but may still be considered spontaneous. Although McNeil does not place
performative gestures on his continua, (for he now introduces more than one continuum)
it seems logical that, in occurring spontaneously, they should occur in the region after
gesticulation but before the more deliberate emblem or pantomime. Left unmentioned by
McNeil is the act of extending a gesture using a tool – the position of such an action on
the continuum would logically depend on how conscious the decision to use the tool was.
My sense is that a deliberate choice of a tool, such as a computer interface tool, moves into the performative gesture region of the continua. Gesticulations are further defined as being “nonmorphemic, as not being realised through a system of phonological form constraints, and having no potential for syntactic combination with other gestures” (McNeil, 2005, p. 7). While it may be suggested that gestures can be combined to give added meaning, to do so deliberately moves the speaker along the continuum towards the pantomime. The gesticulation tends to be a stand-alone gesture, although may be repeated in a form of emphasis. In other words, gesticulations cannot be reduced to simpler forms, are not shaped by any patterns of speech, and follow no fixed rules; as such they lack linguistic properties. This implies that speech and gesticulations/performatives combine into a system of mutual support, with each modality performing its own function. This seems in keeping with the definition of Sfard (above).

McNeil (2005) also describes the gesticulation as having a global feature, implying that the meaning of the gesticulation is contained ‘at once’ within the gesture as a whole; the gesticulation cannot be reduced to its parts to provide new meanings. If this is compared to an utterance, the meaning of the whole is built by the collective meanings of each word, and this is described as being segmented. There is a reductionist property to an utterance which is not present in a gesticulation. This ties in with the observation that an utterance does not need to be a complete sentence in order to be meaningful to an interlocutor. The build-up of meaning can be inferred by the listener who can then determine a marker for turn-interchange, as discussed earlier. The nature of a gesticulation is that it may last a very short time, especially if the gesture is facial. As Sfard (2009) notes, this may require a fine-grained analysis to the order of tenths-of-a-second. While utterances and gestures are not always conjoined and each one of them can constitute a communicational act on its own, for the purpose of this research my primary interest lies in how gesticulations, rather than the more deliberate emblems or pantomimes, and speech combine to produce a communicative act.

Co-speech Gesticulations

While co-speech gesticulations may not be intentionally designed to convey part of the message, there is evidence that the nature of a speaker’s gestures change if they are not speaking directly to another person (e.g. Bavelas and Chovil, 2008), such as when using
a telephone. The nature of the gesture also appears to change given the context of the any talk; gestures may be less precise when there is a greater common ground between interlocutors (Gerwing and Bavelas, 2004), or more iconic in situations which are more lexically ambiguous. Roth (2000) points out that talk and gesture are especially co-present during communicative acts in science activities, and to this I would add group problem-solving activities in the mathematics classroom.

Other researchers (e.g. Goodwin, 2000) have examined the role of gesture on the sequential organization of conversation. Clark and Wilkes-Gibbs (1986) argue that interlocutors in a conversation create meaning jointly, with the aim of creating mutual understanding, a process referred to as grounding (Clark and Brennan, 1991). The process is considered to be in constant need of attention since, at best, the interlocutors can only believe that they have understood what each other meant. Such a belief, however, may be sufficient to allow the talk to productively continue based on the situation. Roth (2000), for example, notes that deictic gestures 'point' out some aspect of the context and therefore make it salient against everything else, which becomes background. Roth stresses that such gestures are combined with an utterance to only provide resources for the listener's interpretation (p. 1684). The impression, then, of students working together on a problem, is one of a continuous need to repair meaning and make connections to each other, where by meaning I refer to Wittgenstein (1967) and the idea of how words are used.

A difficulty, as Sfard points out, is in avoiding word usage that reflects a dualist viewpoint when describing the use of gestures. If communication is “already an act of thinking [...] then thinking can take any form, including gesturing” (Sfard, 2009, p. 195). In this regard, it cannot be said that gestures 'express' or 'package' meaning, or 'express ideas' etc., but that they, with utterances, are the “building blocks of commognitive process” (p. 195). Taking Sfard's definition of a mathematical object as “a signifier together with its realization tree” (p. 197), and accepting that one of the basic conditions for effective mathematical communication is a common realisation of the focal nouns, then gestures can be seen as a way to assist with that realisation. Sfard gives examples, cited from the work of Edwards (2009), in which students were reported to have realized the noun fraction with "gestures referring to ‘cutting,’ ‘splitting,’ and ‘slices’" (p. 198). Roth and Thom (2009) also contend that gestures realize words, although by words I would add
that they help realise the meaning being developed alongside the use of words. Sfard ties this into embodiment by adding:

*By using gestures to realize words, we create a bodily counterpart of what is being talked about. The gestural procedures would often be automated; sometimes, they would be remembered by our bodies much better than the words are remembered by our minds.* (p. 195)

Or, as Lüdeking (1990) writes, “When someone expresses embarrassment by his gestures, these gestures do not denote an independently existing embarrassment but rather embody it” (p. 224). Wittgenstein (1967) illustrates this idea using the aroma of coffee; something impossible to fully describe but recognisable in a gesture because of its embodiment of the emotion that goes with it. However, the gesture needs a context to situate it before it can become meaningful. As an aspect of embodiment, meaningful language must also contain verbal gestures in the form of tone, rhythm and emotion. It would seem that the rhythm of the utterance and the associated feelings can become part of the meaning developed and may even be incorporated into a new gesture.

Where views of cognition focus on situated action in an embodied sense, the cognitive system evolves to support actions in specific situations, including social interactions (Barsalou, 2008). The interactions between perception, actions, and other agents are stressed during goal achievement, such as might be seen during group work in the mathematics classroom. Of particular interest is how interlocutors ‘connect’ with one another in such interactions. Simulation theories (e.g. Goldman 2006) refer to mirror neuron circuits to suggest that, in order to recognise an interlocutor’s actions, the perceived action is simulated in one’s own motor system. In particular, the response is strongest to the goal of the action, rather than the action itself (Barsalou, 2008). We infer what someone is going to do by simulating it ourselves. Moreover, the idea extends to emotions (e.g. Gallese et al., 2007), so that in understanding how others feel we experience that emotion ourselves, i.e. understanding is in the action of feeling it at that time. By extension, then, when we are engaged with others in social interaction it seems that one aspect of the interaction should be such simulation to the point that interlocutor’s mimic each other’s actions, including gestures. This can be in an immediate form or repetition of past embodied actions. For example, as mentioned earlier, Wittgenstein’s
students could often be recognised by the gestures they used, which echoed the characteristics of their mentor (Edmonds & Eidinow, 2001). Anecdotally, perhaps due to my perceived accent, students of mine often tell me that they ‘hear’ particular topics explained in my voice when they recall them.

My intent here is not to go deeply into theories of embodied (or grounded) cognition, but to raise the idea that evidence of mimicry in terms of gesture and language may be another sign that students are engaged in useful conversation.

Mimicry (gesture echoing)

Wilder (1968) makes a case for the distinguishing feature of man from other animals to be found in the way man uses symbols. Wilder introduced the idea of symbolic reflex and linked this to the action of students simply repeating their teachers’ actions; or symbolic initiative when they are able to diverge, or move on, from this echoing behaviour. Wilder posits that only man is capable of symbolic initiative. The suggestion is that students begin by engaging in symbolic reflex but then, if they are capable and taught in the right way, they can move to symbolic initiative. It may be that subconscious echoing of demonstrated behaviour is an important part of developing a common communication process.

The occurrence of mimicry, or echoing, in co-speech gesturing has been recently examined by Kimbara (2008) and in face-to-face communication by Holler & Wilkin (2011). Holler and Wilkin found that mimicry does occur in such interaction and concluded that “mimicked gestures play an important role in creating mutually shared understanding” (p. 148). In this situation, the authors had participants describe figures on cards to each other and found that mimicked gestures appear to function in a number of ways, such as constituting an integrated whole with the verbal utterance or by carrying most of the communicational burden. They concluded that “the mimicked gestures appear to facilitate the mutual understanding of the particular aspect that was being referred to” (p. 143). Holler and Wilkin also found that mimicked gestures were used to express acceptance, suggesting that such gestures were an important part of the conversational organization, even when such acceptance was not expressed verbally. Non-verbal gestures were also found to be important in signalling incremental
understanding, something the authors paraphrased as “I am following what you are saying in an effort to reach shared understanding with you” (p. 145). Holler and Wilkin suggest that “The finding that interlocutors pick up on each other’s gestures during dialogue is further evidence that co-speech gestures do indeed communicate information to recipients” (p. 149). I would argue with their phrasing however, which suggests a transfer of information. In keeping with a participationist rather than an acquisitionist stance (Sfard, 2006), this view is better expressed by Roth (2000) who notes, following Goodwin, that “the human body maintains an essential rationality and provides others with the interpretive resources they need for building common ground and mutual intelligibility” (p. 1685). He adds that such gesturing provides resources to collaborative thinking-through processes by affording the production of public accounts. Talk in a mathematics classroom may be considered similar to that in a science classroom, which Roth sees as being “highly elliptic and indexical” (p. 1687). Such talk may only be rendered comprehensible with the help of gestures that stand in iconic relation to the representations at hand.

A limitation of many studies, however, is that they are focussed on objects that one party is attempting to describe to another. There is usually something very tangible that the speaker has seen and is trying to describe (e.g. for Holler and Wilkin it is abstract shapes with figure like qualities). A similar limitation can be seen in the work of McNeil (1992, 2005), wherein participants are asked to recall scenes from a cartoon they have watched. Students working in a classroom are generally describing or talking about mathematics that is not a recollection of an action but rather an ongoing action. Some of the actions involved may be hard for a student to put an image to in quite such a dynamic way as McNeil’s subjects. As a result, it might be expected that the gestures can often be more subtle, especially in the early stages of working together. In the case of mathematical problem solving, the participants in the talk are trying to create a solution without one member having a privileged informational position (such as would occur if a teacher was present). In addition, any power relations within the group may lead to a particular student being granted a dominant starting position. In this event, mimicked gestures may be an attempt by a student to reflect the mannerisms of his/her interlocutor with the aim of acceptance.
**Speech–Gesture coordination**

Further studies indicate that gesturing is used to lighten the cognitive load while thinking (Goldin-Meadow, Nusbaum, Kelly, & Wagner, 2001), not only allowing the speaker to use more resources to access memory but also to play a role in shaping the speaker’s cognitive state. The issue I would have with such views is in thinking of memory and cognition as ‘states’ that can be ‘accessed’ or ‘constructed’. Such a viewpoint goes against the dialogical viewpoint discussed earlier. Goldin-Meadow et al. claim that by not gesturing, the student is not using their full cognitive capacity and does not perform as well as when they do. Further, gestures can predict how students go about solving a problem (Alibali, Bassok, Solomon, Syc, & Goldin-Meadow, 1999). In keeping with the earlier discussion, Goldin-Meadow et al. (2001) write “gesture and speech form an integrated [...] and synergistic system” (p. 521); but add that “gesture then has the potential to display thoughts that are not conveyed in the speech”. The difficulty with this statement is its implication that the gesture is representational of the thought, which has occurred ‘in the head’, rather than being part of the thought. The gesture is the communicational act if there is no utterance to accompany it.

**Summary so far**

The essence of this chapter has been to develop the idea of seeing understanding as the ability to ‘go on in conversation’. Furthermore, a conversation is seen as incorporating gesture and body language as an integral part of the process. Viewed in this way, understanding is now placed ‘in the open’ rather than being a process hidden within the mind which can only be inferred by examining the static results of intermediary processes. Other theories of understanding tend to be reflective, attributing features to understanding based on suggested structures which are thought to be constructed by the individual. The constantly shifting nature of an individual’s understanding makes these theories seem specious, or at least incomplete. By adopting Wittgenstein's position of not thinking of understanding as a thing but as a dynamic process, and incorporating the ideas of Sfard, Davis, and Fleener, Carter and Stacy, then it is ‘understanding as a result of action' that the classroom teacher needs to look for and support. Being a teacher on the sideline while students are working gives an opportunity to look for and listen to the
engagement of students working on a task. My intent in the next few chapters is to observe students in action in order to determine if there is a correlation between the activities seen in conversation-gesture and developing understanding of the student. To do so I will be using the tools of Conversation Analysis and gesture analysis as outlined in this chapter. Throughout Chapters 6-9, where I present the results of this research, I emphasize where students are able to ‘go on conversationally’ and link this to progress in the problem. In drawing out the pattern occurrence of this across several examples, the intent is to show how ‘going on conversationally’ may be construed as mathematical understanding.
Chapter 5

Methodology

In this chapter, I outline the methods used in the main study of this research. With a clearer idea of the limitations and possibilities of working in an active classroom, I was able to modify my methods so as to capture quality recordings with minimal noticeable impact on the students. I also had a more directed purpose in engaging with the research questions as outlined in Chapter 2, but I still wanted to give myself flexibility to adapt to new findings. I will begin this chapter by giving a general description of the setting of the study before outlining the procedure used to capture recordings in more detail. I will then outline the technical side of transcribing the recordings and tie this process in with the general concepts of conversation analysis (CA) and gesture analysis. The combined process I shall refer to as conversation–gesture analysis (CGA), wherein is also incorporated posture analysis and observed body language. A similar line of research is followed by Sikveland and Ogden (2012) in examining how shared understanding is reached through a sequence of turns and combinations of speech and gesture. My aim was to include any observable behaviour under the CGA umbrella with a view to determining what aspects could help inform this study. There are several research studies that incorporate a multi-modal approach, so I begin with my reasons for taking a CGA approach as opposed to one of those.

Other Multimodal Approaches

Jewitt (2013) describes multimodality as “an inter-disciplinary approach drawn from social semiotics that understands communication and representation as more than language […] provid(ing) concepts, methods and a framework for the collection and analysis of visual, aural, embodied, and spatial aspects of interaction” (p. 250). While it might be said that any research involving a combination of modes, such as talk and gesture, is multimodal, the emphasis of most research in this area has been on
Discourse\textsuperscript{6}. As mentioned in the Introduction chapter, the focus of Discourse Analysis is traditionally more on the particular linguistic features of the classroom talk rather than looking for patterns in the way the conversation is organised. For example, Discourse studies have covered areas such as lexical bundles (Herbel-Eisenman & Wagner, 2010); the influence of discourse on mathematical thinking (White, 2003); and discursive positioning and emotion (Evans, Morgan, & Tsatsaroni, 2006).

The common use of classroom video now makes a multimodal approach more attractive and viable, and this is increasingly being made easier by computer analysis tools such as optical flow (which follows the pattern of motion) and edge detection (which detects points in an image where the brightness changes), (e.g. O'Halloran, Tan, Smith, & Podlasov, 2011).

Not all research using a keyword of multimodality offers a distinct framework (e.g. (Arzarello, Paola, Robutt, & Sabena, 2009), but may focus on a ‘suitable model’, in their case the semiotic bundle (see also Radford, 2003). Such methods examine the use of signs in the mathematics classroom, focusing on a particular use of language and gesture. Systemic-functional social semiotics (Halliday & Matthiessen, 2004) provides a unified theoretical framework for examining how these multiple resources combine in mathematics education. The framework has a linguistics base, for example, to study “the way in which language may serve as a crucial window for researchers […] to develop knowledge about uses of language within mathematical practices that may be helpful for teaching and learning” (Morgan, 2006, pp. 221-222).

The intent of CGA is to look for broader patterns of conversational organization rather than the specific use of language designed to inform the researcher about the way talk and gesture are used in a given setting. For example, the particular gesture or talk used in an individual working session is not a focus of this research, rather I am interested in

\textsuperscript{6} Ryve (2011) reports on a general lack of consistency regarding the use of the term discourse, referring to general classroom talk and Discourse as a method of study. Ryve also notes a lack of a clear theoretical framework in many of the 108 articles analysed.
the way gestures and talk are used across a diverse set of situations by the students in order to establish a shared understanding.

**The Setting**

The study was focused on two grade 5 classes over the period of their school year in a Canadian school. The school is located in a city east of Vancouver, BC, and consists of a wide range of cultural backgrounds typical of the area as a whole. Immigration to the area from many parts of the world is an ongoing process and this produces a range of English language skills in the school. All the students in this study had a working knowledge of English, but some were clearly more fluent than others. Students were observed to converse freely outside of the classroom (and often within it) about various social issues typical of grade 5 students. Such observations helped give an indication as to whether any of the students in the study found conversing to be a challenge in a non-instructional setting. The school and classroom are considered to be ‘safe environments’ in which to learn, meaning that were no obvious barriers to student participation, or nervousness from the students about participating in class. Classrooms are encouraged to be places where students examine their thinking and, as such, the activities captured are not presented in an atypical way to the students. The classroom is bright, with one wall being a bank of windows, and desks are typically arranged in clusters of four. The room is colourful, with posters and student work adorning the walls. Lessons often spilled out into the corridor or common spaces around the school and student groups were comfortable with being sent to work in small alcoves in order to work quietly or to have more space. The classroom teacher, Avio, had a generally relaxed manner and was able to elicit confident responses from the students when he addressed them. The students (n = 32) demonstrated a wide range of attitudes to their work, from showing a very motivated approach, to indications of attention difficulties. No students, however, were
designated as having learning disorders\textsuperscript{7}. I would consider the students to be generally polite and respectful. I did not observe any overt occurrences of classroom angst or misbehaviour during the times I was present.

Recordings of classroom activities were carried out from September through to June on a weekly basis. In total, over 150 recordings were made during this time, ranging in length from short clips to longer, full-class recordings (see Appendix for further detail).

Recordings were also made of groups of students following the task completion, either when presenting to the class or when engaged in a full class discussion. Individual class presentations were also recorded. Classroom contact time was a 45 minute lesson used as part of the classroom teacher's normal mathematics programme. With a few exceptions the classroom teacher, Avio, generally selected the lesson activities and taught the lesson, with my role being that of an observer. I did talk with Avio to offer some suggestions as to suitable problems and points of focus, but my intention was more to observe the interactions in a classroom setting rather than to direct them, in order to avoid setting up situations which would possibly create what I was looking for, rather than allowing these situations to occur as naturally as possible. In particular, I wanted to avoid the approach I have noticed in many studies where a set stimulus is given in order to produce the conditions seen as suitable for the study. For example, McNeil (1992) investigates gestures by first showing a subject a cartoon, and then asks them to describe aspects of what they have seen. This practice ensures that gesturing occurs, given that people tend to gesture more when describing something they have seen to another person. In a similar manner, Goldin-Meadows (2003) relies heavily on students completing a given equation in order to examine what she refers to as ‘gesture mismatches’. These are valid techniques which have the advantage of allowing the researcher to examine a particular aspect of gesturing but, while I recognised the danger in having too loose a structure for the study, I wanted to see if there were characteristics

\textsuperscript{7} Being designated as a special needs student is a formal ministry process requiring assessment and the development of an individual education plan. Designated students have access to additional support. There were only a few such students in the school where the research was carried out.
common to any type of classroom talk. I did not want the interactions to be reflective of the problem type or situation.

Finally, these particular classes were selected because the mathematics lessons corresponded to a time when I was able to attend. At the same time, Avio showed an interest in the study and was more than willing to accommodate me in his classroom. In this regard, selection was by convenience rather than by design, but this again added to the sense of ‘it could be any class on any day’ that I wanted to work with.

**Organising the Students and Collecting the Data**

In my pre-study, I had used a single video camera and attempted to record the group work which was established following the teacher’s introduction to a lesson. This did not prove a viable option as, in order to capture the group dynamics the camera needed to be set back, which then made recording of sound problematic. While the human ear seems able to tune in to individuals speaking in the classroom, the camera’s sound recording makes this much more problematic. To help alleviate this problem, I decided to use a more than one camera and a sound recorder placed centrally within the group. Students very quickly learned to ignore the recording devices for the most part, although obviously they were aware of the fact that they were being recorded. To help reduce any further intrusion into the process, I set the cameras up on tripods and left them to record the students without my close presence. I typically then moved around other groups in the room in a similar manner to the classroom teacher, but taking field notes about what I observed at the same time. The video clips were then downloaded and examined using the software *ExpressScribe* (see fig. 5.1), which allowed the clips to be slowed down and played frame by frame if necessary. I found that by slowing down the speech it was much easier to pick up what the students were doing and to transcribe the audio. The clips could also be controlled with a foot pedal so that sections could be easily rewound and played again. Because several clips can be loaded into the menu it was also easy to switch back and forth between the audio recording and the visual recordings so as to allow any difficult-to-hear clips to be listened to from a different location, and any hidden gestures to be viewed from an alternate angle.
In order to help the process further, after consultation and at Avio’s suggestion, several of the groups were moved outside of the classroom. This was a common practice even when I was not present and so was not introducing anything unusual to the students. Isolating the groups reduced the background noise of the recordings considerably, as well as allowing me to set a camera further back from the group. This allowed for group movement to remain generally within the camera shot even though the camera was not being actively controlled. With these adaptations I found I was able to make what I consider accurate transcriptions of the group work, while at the same time allowing me to clearly capture a high percentage of any gestures made. I felt confident that I was developing a good database of recordings of high enough quality to allow me to begin my analysis.

Table 5-1 ExpressScribe, the transcription software used in this research

In the classroom sessions, all material was relevant to the learning outcomes expressed in the British Columbian grade 5 curriculum, but were always ‘problem based’ rather than instructional. Avio had generally covered the content in which the problems were situated, but in some cases he presented unique situations. This process was no different from the normal activities established in the classroom. The typical format of each lesson was that Avio would introduce the task, either orally or through a handout.
The students were placed in groups for which the makeup was varied each time. Avio consulted with my records in order to ensure that every student in the class had the opportunity to be recorded, and that groups were constantly varied in order to examine some different group dynamics. Groups varied between two and five students; sometimes the genders were mixed, and at other times they were the same; groupings were not made on perceived ability.

Students were then told to silently read through the task and given a brief time to think about the problem. Students were instructed not to write anything down at this point. After some consultation with Avio, we decided that having pen and paper often seemed to be a source of distraction for the students and so he would often tell them not to have these with them until given permission. It seemed that when these items were allowed, students tended to spend time writing such things as their name during the time we wanted them to be thinking about the problem. In some cases, we felt that a student might be doing this as a way to avoid thinking, along with other distracting activities such as organising a pencil case, etc. At other times, the students were asked to stand rather than sit down at this stage of the process. This seemed to help the students focus on the task in hand, especially as the class was at the end of the day. The intent was always to give the students every opportunity to think about the assigned problem from the moment it was assigned.

The students were then asked to discuss how they thought the problem should be solved. They were encouraged to think of more than one way to approach the problem. When Avio felt the students were ready, they were allowed to retrieve pen and paper to work on the problem, or to use white boards or manipulatives as appropriate. As the lesson drew to a close, Avio would generally bring the students together to discuss their findings and draw some conclusions. This was an opportunity for the students to offer their ideas to the whole class and share in each other’s findings. These group sessions were also recorded and used as part of the analysis. In these recordings the intent was to observe how students discussed the work they had done in addressing the task, and what results they would be willing to share.
Sorting the Data

Each class was generally recorded in the same time slot at the beginning of the week. Occasionally, the business of the school would alter this day. In some lessons, there was the opportunity to record both group work and whole class interaction, as well as individual presentations. My habit was then to make an initial raw (no markup) transcription of these recordings during the week before the next visit to the class. I would keep a note of the participants and also a copy of any artifact produced in the session. These artifacts were used to help clarify something the student had said as well as what they may have been referring to. I would also make some notes on points that seemed to stand out, as well as a general description of the recording. This helped in allowing me to go back and locate a previous occurrence of something that seemed familiar.

Transcription

My initial task lay in sorting through all the recordings in order to determine which ones to focus on, with a concern that selection bias not be a factor. To begin with, recordings in which the audio quality was insufficient to clearly determine the speech were discarded. These occurred mostly at the beginning of the year before the best technique was found to make clear recordings. Moving the group to a quieter location also helped in this regard, but occasionally passers-by would stop and talk too close to the camera, or some distraction would occur, spoiling the recording. Similarly, given that one focus of this research was on gestures, recordings in which one or more students moved out of the camera view for a significant time were also discarded. Another focus was an attempt to capture, as much as possible, the natural talk between the students; as such, recordings in which the students were clearly interacting with the camera were also discarded. Camera interaction was a factor at the start of the school year, but the students rapidly became accustomed to the presence of the camera, meaning that few recordings needed to be discarded after the first few weeks of the year. While I do not claim that students were not aware that they were being recorded, and therefore that their actions were completely unaffected, there were several occasions in which the recording of the students’ behaviours were seen to change when the classroom teacher moved over to the group. As an example, I offer the following anecdote to illustrate how the camera,
having become a familiar item to the students, became essentially invisible to them: a pair of students were working outside of the classroom and the recording showed them to be completely off-task until the (substitute) teacher returned, at which time they began to make statements as if they had been fully engaged. When the teacher left, they returned to their socialising as if unaware that they were being recorded.

During the recordings, I moved around other groups in the class and tried to keep an eye on the recorded students from afar, making field notes related to anything that stood out. I recognise here that some CA analysts eschew the use of field notes, considering that they are “an inappropriate source for conducting Conversation Analysis because they are inherently selective” (Psathas & Anderson, 1990, p. 76). Because this study is a broader analysis than just of conversation, and because a broad aim of this study was to connect what a teacher might observe from a distance to that which is occurring in the detail of the conversation, some notes from this perspective seemed justifiable. These notes also included actions by groups who were not being recorded if they did something that was noticeable. For the most part this focused on large gestures being made or posture similarities and differences within and between the groups.

Prior to making a transcription, I watched the recording through with the intent of looking for any general features that immediately stood out, again making notes. At the same time, I made an attempt to initially classify recordings as in one of three broad categories: groups apparently making no progress in the assigned problem, groups that seemed to make some progress before running out of time or ideas, and groups that seemed to have moved towards a solution they were satisfied with (not necessarily the same as expected by the teacher). I then later examined the recordings in each group to see if there were any common features which arose. I tried to be aware of group dynamics (some groupings never lead to successful work habits and Avio made subtle adjustments to keep certain students apart, as is common in most classrooms), the quality of the question (some questions were clearly outside of the capacity of the students and produced no more than confusion or frustration), and the mood of the class (for example some recordings were discarded after it became apparent that a particular event in the school calendar was having a detrimental effect on the students working habits, such as Halloween).
Creating the final detailed transcription was an elaborate process that went through several steps. Once I had the recordings grouped in the manner described above, I went through again and checked the transcription without detailed markup. This allowed me the opportunity to examine the recording carefully to get a closer sense of the interactions and the features of the students-at-talk. Following this, I went through the recordings again and narrowed down the selection further to those recordings that stood out as exemplifying different things in the process. This step was necessarily subjective and was re-done on several occasions and included some discussions with Avio to ensure my observations seemed valid. I felt it important to get a sense of what characteristics the teacher of the class was noticing about the students, without him having seen the video transcriptions. I wondered if these features also came through in my more detailed analysis, and what possible basis could be identified for them. I also wanted to note Avio’s sense of which students were successful in their studies, and which students seemed to be struggling in this area.

**Conversation–Gesture Analysis Markup**

My next step was to go through the recordings in more detail using the tools of Conversation Analysis. All tapes were marked up based on the transcription notation system outlined by Psathas (1995), which I describe in greater detail below. A sense of the necessary repeated process of methodical listening/viewing can be felt in the following extract:

*One thinks about what one is hearing/viewing, tries to pick up and remember the sounds, the actions, the pauses, etc. so that the hearing/viewing can be recalled later. How the parties are interacting, how they speak and move, what the actual sequences are, etc. are noted. Hearing/viewing the interaction as it occurs on the tape gives the analyst an intimate familiarity with its details in the (real time) temporal flow of actual sequences. The analyst experiences the interactional events as actual occurrences, with sensitivity to their meanings as these emerge and are displayed by and for the persons engaged in the interaction. (Psathas and Anderson, 1990, p. 77)*
The result of this process is that I became very familiar with each recording, yet found something new with each pass. The listening became integral to the process of analysing the session, and the transcription developed in such a way as to capture the session as closely as possible. Even so, I still found it necessary to return to the original recordings at times in order review some new detail that seemed to come through from comparing transcripts of different sessions. This served to reinforce the idea that the transcription is still just a version of the data, and not the data in itself (Psathas & Anderson, 1990).

After adding the appropriate conversation markup, I watched the recording again to look for gesturing and other salient features. In my earlier pre-study, I had included the gesture markup with the conversation markup, being careful to indicate the temporal location of each gesture from its starting stage, through its stroke stage, and ending up with its completion. While this level of detail was instructive in showing how a gesture is temporally linked to the accompanying language (McNeil, 1992), I felt it added an unnecessary level of complexity to these transcriptions and so chose to simplify the process. For the reproduced transcriptions I present in the next chapter, the gesture location is indicated and illustrated separately and in further detail.

In order to break down the process further, I initially focused on the opening exchanges of the groups, as suggested by my introductory study. My initial impression had been that how a group started was a snapshot which often reflected the whole group talk, and that key features demonstrated there would be repeated throughout the recording. From a broad base of these transcriptions, I then narrowed the selection down further in order to examine how the group continued to work on the problem. My selection was based on trying to isolate a few good examples of the process, taken from each of the earlier groupings based on observed progress. In this way, I hoped to confirm or refute any thoughts I held from my earlier observations. In each case, I tried to hold true to a central tenet of Conversation Analysis, that grounds for any interpretations must rest on evidence in the talk itself. The students themselves must demonstrate specific orientations toward the interpretations being proposed for there to be sufficient grounds for assuming that such interpretations are significant.

I also recognise that this secondary stage of the research was analysed with the ideas around understanding – as prompted by the initial study described in Chapter 2 and
developed through the framework established in Chapters 3 and 4 – in mind; that a student’s understanding is developed in a conversational space rather than being a reflection of a student’s state of mind. In addition, we can learn something about this understanding by examining the features of a student’s conversation, and any gestures (including body language) they incorporate into this conversation. I cannot therefore claim to have a completely objective viewpoint in my analysis and I am aware that my own interaction with the interpretation of the recordings is inevitable. I have tried to keep an open mind, and be as open to change as possible throughout this process, but I do not consider that it is possible to be completely independent. Even in using mark-ups to annotate a transcription there is an element of interpretation, a choice of what to include or not to include. One of the reasons I wanted to use CA, and its potentially distracting mark-up system, was to include as much detail as possible about the recording. In doing so, I feel that other people viewing the transcription are getting as close as possible to the original talk. In many transcriptions I have seen, I have wondered about how the writer’s choice of punctuation has subtly altered the intended meaning of the speakers, and how much our own interpretation of that punctuation causes a further drift from the original. My goal was to find a balance between readability and fidelity in the transcriptions.

In viewing the transcripts further, I tried to make an analysis of what I saw on common-sense grounds, trying to examine utterances to gain a sense of what they were adding to the group talk. I tried to determine how the utterances seemed to interact so that sequences could be observed in the manner of related responses. To do so, I needed to be clear on what the problem was intended to elicit from the students, as well as the direction the students seemed to be headed. I did not want to be judgemental as I was more interested in the organization of interactions than the end result, but I felt it was important to have a sense of the mathematical development in relation to what was expected. These expectations often accompanied the problem itself in the source material (e.g., when a problem was taken from the NCTM database). In this way, I was also looking at how the group was progressing with the problem, either in alignment with the intended outcome or in a different direction. This was important because a group might be able to hold a conversation about a problem without developing anything
mathematically, and this would then serve to inform me further about the relationship between group conversations and the development of mathematical understanding.

A basic presumption in CA is that procedures are adequate for the problem in hand rather than being pre-specified (ten Have, 1990). The intention is, as suggested by ten Have, that in making the transcription, I am being forced to attend to details that might escape the ordinary listener. Once made, the transcriptions then provide more detail to check my analysis at a later date when the original episode may be forgotten. By the same token, the transcriptions provide other readers more information than might ordinarily recorded talk. Even so, every nuance of the data cannot be recorded and so there is still a concern regarding selective analysis that I was consciously aware of. To help this process further, I was grateful to be able to present my data at conferences within my own graduate community as well as outside of it. In addition, by having a large corpus of very detailed transcripts, I felt a practical compromise between precision and reliability could be reached in the selected simplified transcripts I provide in my data analysis chapter, in the manner suggested by Heritage (1984).

Using markup symbols to transcribe the data

Schegloff (1968) opened one of the first published papers on CA by writing “My object in this paper is to show the raw data of everyday conversational interaction can be subjected to rigorous analysis” (Schegloff, 1968 p. 1075). This is, in fact, not my intention. I take it as given that CA has been around for long enough, and is respected enough as a technique, that Schegloff’s intent has been largely fulfilled. My intention in this research is to use this process to examine the corpus I have collected in a similar manner to that taken by Schegloff, who began by looking at his data set (500 telephone calls to a disaster centre) for patterns. Once he had found such a pattern (e.g. the answerer of the call always speaks first) he looked for exceptions in order to shed further light on the rule. For example, in an earlier piece of research which led to this thesis, I analysed students collectively solving mathematics problems via email (Wells, 2011). In this I observed that whomever initiated the correspondence typically did not offer a solution to the problem assigned. This was generally an ‘overture’ or invitation to collaborate. The recipient of the email would then offer the first suggestion towards a solution. In the isolated case when the initiator started by suggesting a solution to the
problem, the electronic ‘conversation’ did not start as productively. It seemed in this case that the concept of ‘face’ had been violated and that repair work was necessary before the communication could be as collaborative as in the other examples. These were the sort of patterns I set myself to look for as I began my detailed research.

It is worth pointing out here that it is not just the talk that is listened to and transcribed; any sounds that are made are included. For example, marked in-breaths or sighs can be significant in communication. Such sounds – along with changes in body language – can serve as indicators for a turn at talk (Psathas & Anderson, 1990). In-breaths are transcribed using ‘.hh’ (more h’s are added to indicate the duration of the in-breath), while out-breaths use the ‘h’ mark but without the preceding period. In other situations non-verbal sounds can indicate agreement/disagreement or simply indicate that a listener is still engaged.

Perhaps as important as these sounds is the lack of them. Silences are an important aspect of conversation and are indicated using brackets containing either periods to indicate a micro-pause, or a time in seconds. For example (.) would indicate a very brief pause, while (..) is slightly longer, and (0.5) indicates a half-second pause. While adding a (.) might seem redundant, it is important to realise that speakers do not tend to pause between words in the way that spaces are used in text. Even the smallest pause is apparent to the listener, and longer pauses may be taken as an invitation, or an opportunity, to claim a turn at talk.

Overlapping speech is also considered important in a conversation. Such speech may occur as a result of a user trying to take a turn at talk from the user due to disagreement, and also as a way to ‘finish the sentence’ with the user as an indication of agreement (it may also indicate that the listener was not actually listening to the speaker). In my transcriptions I have adopted the underline as an indicator of overlap as I find that to be a less cluttered method than adding brackets as is conventionally done. This is important for me as I am also trying to indicate gestures in the transcript. As an example:

Sally: I would like to go

Fred: I have an idea
Is, I feel, more readable than

Sally: I would like to go

Fred: I have an idea

and so I adopt this approach whenever possible. I generally try to space the utterances so that the overlaps align, but this convention can sometimes lead to text that is challenging to follow and so is not always adopted.

Additional markups are important as indicators of prosody, or the rhythm and intonation of the utterance. An example might be:

D: *I didn't really get it* ↑

As a recap on the markup described earlier, the symbol o is used to indicate and bracket the utterance where the student is speaking softly. In contrast, utterances made more loudly are indicated by the use of uppercase letters. The :: symbol is used to indicate that the word is being stretched. More :: symbols can be added to indicate a longer stretch.

The ↑ is used as an indicator of a rising pitch, which I use rather than the question mark, which I find distracting and presumptuous in that it suggests a question which may not be there, as is seen in some transcriptions. The utterance above is seen to end in a rising pitch which, while generally accompanying a question, may also indicate an interrogative tone, or simply a student’s manner of speech and occur part-way through a word, as in thirty, which rises in pitch in the middle of the word and stays high. In the same manner thirty indicates the pitch dropping at the end of the word.

The > symbol is used to indicate speech is speeding up, while the < symbol indicates it is slowing down. For example:

yeah (...) <that could work.> (1.5) It might have been:

The < > here indicates that the student spoke ‘that could work’ slowly following a brief but noticeable pause. This is followed by a measurable pause of 1.5 seconds before the student continues to speak at her normal pace. The word ‘been’ is stretched out significantly, as indicated by the four ‘::’ symbols.
Two other markups that are frequently used are italics to indicate an emphasis (deciding on an emphasis is often aided by an accompanying gesture), and the ‘＝’ sign to indicate when two utterances are not separated by any discernible pause. I summarise these symbols in a table in the results chapter, but I always kept a copy of these on paper as I went through this process to help both in the transcription and in re-reading the transcribed text.

| I repeat the table of markup symbols below for the readers convenience, but also to highlight my switch from using [] to indicate overlap, to instead using underline, as explained above. brackets a quietly spoken utterance. | Capitals indicate LOUder sounds |
| : indicates a stretching of the sound. Repeated use indicates longer stretching | > brackets faster speech< |
| = indicates there is no discernible pause | Underline indicates overlapping speech |
| (, ) (..) indicates a short or slightly longer pause | (4.1) indicates a pause in seconds |
| ↑ ↓ represents rise or fall in pitch | *Italic* indicates emphasis |

**Table 5-2 A brief guide to Markup symbols**

**Next Positioning**

A further aspect of the coding I applied to the transcripts was to add codes to signify something in the utterance that seemed significant. These may refer to gestures, such as [FG] to represent the presence of a facial gesture, such as a look of puzzlement; or [P] to indicate that one student was pressing for further explanation. My thinking here was to indicate actions which could then be examined as features of the talk and which might be examined across the grain of several transcripts. In addition, while the adjacency-pair framework was an early feature of the work of both Sacks and Schegloff, this organises a somewhat restricted range of conversational actions. The concept of ‘next positioning’ broadens the idea of the adjacency pair so that an utterance, as an action, may “project,
but not strictly require, one among a range of possible next actions” (Goodwin & Heritage, 1990, p. 288). An example given is the use of an acknowledgement token, which can project the continuation of another speaker’s talk without requiring it. In using codes to indicate such actions, my intention is to open up larger ranges of talk for analysis as connected and interactive groupings. The idea is to help see if actions in the conversation are being shaped by the context of previous actions, as well as in shaping any future actions. In doing so, the organization of the conversation could then be viewed in a more dynamic way; what is being developed in the conversation would be unique to that situation and might not be readily re-created in a future conversation. What I was looking for as I examined the recordings was a sense of whether the conversation ‘had a life of its own’, and what this may be implying. The idea of ‘next positioning’ may be an important aspect of this as it stems from the turn-taking nature of a conversation, as studied by Sacks, Scheglof and Jefferson (1974). In this, participants are seen to make inferences about emergent meaning from the developing organization of existing talk. How students position themselves to respond to an interlocutor’s turn at talk may be important in indicating how they are developing understanding through the exchange. For this reason, my intention was to examine not just the student’s talk, but the organization of it and how students positioned themselves within it so that they might be able to continue the conversation. It may be that someone is a ‘speaker’, even in their silence. The social nature of the conversation, if the talk can be seen to be interactive, may then reflect students projecting meaning on each other as well as developing it themselves. By coding for interactions as well as conversation, I hoped to be able to see if this organization was accessible in classroom mathematical talk.

**Coding for Gestures**

Adding gesture markup to an already crowded transcript adds a further challenge. Ideally, every gesture from the merest facial expression (often referred to as micro-expressions) to the broadest body movement would be included. Being selective in what to include reduces the number of markups in the transcript, but at the same time it goes against a core idea of CA in that everything should be included and these details then be studied for the presence of any pattern or salient feature. Being selective brings in potential bias before the process is begun. My solution to this problem was to first create a large scale transcript which contained all the markup I could add; I then set about
examining these in order to determine what stood out both as a feature within the clip and then as a common feature across the collection of videos. For reproduction in this document, and bearing in mind the limitations of the page size, I have then focussed on those features which stood out when reproducing the transcripts here. I recognise this as a compromise which may be suggestive of cherry-picking data, but I took care not to ignore data that contradicted anything I concluded (or perhaps more importantly to ignore any lack of evidence to support my conclusions in non-included clips). Rather, anything left out was because it did not seem to add anything to, or subtract from, the research. The only way to avoid this would be to include the original clips in their raw form. As an example, consider the following utterance:

“Well first I thought we could try doing six, six hundred and twelve minus, well we know six times two umm equals twelve, so twelve…”

Transcribed as:

<table>
<thead>
<tr>
<th></th>
<th>well first I thought we could try doing six (..) six (.) hundred (1.2) and tew:le (2.1) minus</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(1.9) well we know six times (..) tw:o umm (1.5) equals twelve (1.6) s::o (1)↓twelve (0.8)</td>
</tr>
<tr>
<td></td>
<td>/*****/</td>
</tr>
</tbody>
</table>

Simone makes a gesture with her mouth, tightening her lips to one side.

Simone makes a facial gesture as she provides her initial thinking. Specific facial gestures tied to talk are indicated in the transcript using the /***/ notation. This is a personal gesture which helps to give a sense of her activity as she speaks. However, the pauses in her speech give a similar sense of hesitancy and the gesture is not seen as informing about any conversational organization. For this reason the reference to this
gesture is omitted. Gestures which are deemed to be of interest are then marked and treated in a separate table. Comments regarding body position are included in the transcript between ((brackets)). In the case of body gestures where there is a clear prestrike action and a recovery following the action, the gesture is marked as [~~/***/-.-.], where the number of included symbols increases with the temporal duration of the gesture. The term ‘hold’ may also be included to indicate a gesture being frozen during a passage of talk.

I considered that gestures would be an important markup to add because if both speakers and listeners are active in building a turn at talk, then any actions or non-actions can lead to modifications in how the speaker performs; “such a perspective integrates the body into the analysis of talk and action” (Goodwin & Heritage, 1990, p. 293). By including body motions in my analysis, I am then hoping to include how the students are interacting with each other and their environment. Any conversation arising may also be a function of the body language and interaction of the participants and I felt it important to record any information that may help in this regard. I was interested in how the speaker’s actions might provide positions for a listener, and how the listener’s actions might support or weaken the actions of the speaker. For example, how head movements, facial gestures, posture, overlaps, intonation, etc. can affect how a speaker continues his/her turn at talk, or how the interlocutor responds.

In order to assist this analysis, the following table lists codes that I use to indicate actions which commonly arose in the recorded sessions. These actions and codes were not predetermined but rather arose from observation and were added after several viewings and after transcript markups and gesture markups had been added. They are not exhaustive of actions but rather represent those actions seen often enough to seem important. The codes are added in the last column of the transcript.
<table>
<thead>
<tr>
<th>[RS] Read the question, and then stop talking.</th>
<th>[AP] Adjacent pair</th>
<th>[CT] Change track</th>
</tr>
</thead>
<tbody>
<tr>
<td>[RC] Read, then comment immediately</td>
<td>[I] Interruption causing turn change</td>
<td>[O] Overlap</td>
</tr>
<tr>
<td>[BG] Body gesture, such as a shrug.</td>
<td>[IO] Interrupt–overlap which does not stop the first speaker</td>
<td>[D] Distraction</td>
</tr>
<tr>
<td>[FG] Facial gesture, such as a look of puzzlement, raised eyebrow, smile, surprise, etc.</td>
<td>[R] Repeat</td>
<td>[SC] Sentence completion (indicating listening)</td>
</tr>
</tbody>
</table>

**Table 5-3 Transcript codes**

An important aspect of this research was in allowing the data to speak back to the research and suggest new approaches. One such discovery was the evidence of gesture and posture echoing (Chapter 9). For part of this analysis, I selected twenty random recordings in order to perform a count on how often such echoing was observable. To select these recordings, I used a random number generator from www.random.org.

<table>
<thead>
<tr>
<th>Randomly generated selection of 20 video clips using Random.org</th>
</tr>
</thead>
<tbody>
<tr>
<td>76 115 58 87 36</td>
</tr>
<tr>
<td>147 47 139 125 9</td>
</tr>
<tr>
<td>130 72 98 18 74</td>
</tr>
<tr>
<td>22 63 132 105 54</td>
</tr>
</tbody>
</table>

When going through the transcripts the occurrence of gesture echoing became apparent as an important aspect of understanding (see chapter 9). To facilitate further analysis a random number generator was used to select twenty recordings to examine in more detail (see Appendix for further detail).

**Table 5-4 Randomly generated video clips**

Each of these selected recordings was then examined for evidence of both posture and gesture echoes. To be cautious, I only counted a gesture echo if the echo of one student was repeated by an interlocutor within two turns of talk, and if the gesture was identical. Posture echoing had to occur at the same time.
The following four chapters present examples of the data collected and an analysis of these as outlined above. Given that not all of the recorded data could be presented, my intention is to present transcripts which represent typical examples of certain occurrences observed. I also present a broader analysis of the data examining trends seen across several clips. In doing so I selected images of a similar nature to illustrate a point of observation.
Chapter 6

Results and Analysis: Starting

In this chapter, I will be examining the transcripts of the recordings made over the course of this research. The study was focused on two grade 5 classes over the period of their school year and an analysis of the subsequent filmed talk of groups of students as they worked on problem-solving activities set by their teacher. As mentioned in Chapter 5, 150 recordings were made during this time, ranging in length from short clips to longer, full class recordings. Some recordings were of individual students, while up to five students were recorded in group sessions. The following selection of transcripts is used to illustrate trends observed within these recordings. The CGA approach has a focus of looking for patterns and similarities across a large number of recordings rather than drawing out particular details from a few such recordings. The first of these patterns was that the sessions had three stages which stood out, and I have divided these transcripts into three chapters in order to illustrate these: the Introductory stage, the Development Stage, and the Extension Stage. Further sub-division might be possible within individual examples, but no consistent pattern was seen across all the examples to justify this, or suggest any advantage in doing so. I then examine each of these stages in turn, using a number of clips, in order to compare and contrast the features of these stages. I focus, initially, on six clips, which I then reduce to three in the final two stages.

The Introductory stage

The importance of the introductory stage in the students’ attempt to solve the problems assigned to them was apparent in a general overview of all of the clips. While there is no definite indicator to separate where the introductory stage ends, there was a clear sense of when students moved from setting out the parameters under which they were working to starting to make progress on the solution process. Perhaps more significantly, some
groups did not move beyond this stage and I discuss some reasons for this in this section. What does seem apparent from the body of recordings is that the introductory stage is important in setting up the dynamic for the group work. This is then an important time for a teacher as what s/he notices about the first few minutes of group interaction can be an indicator of whether or not the group will be productive. This can be a time when there are many distractions for both teacher and student, but taking a moment to settle the class to engage in a focused start to their group talk and then actively teaching from the sideline has the potential to indicate that immediate intervention is required in order to help foster a learning environment.

Where groups did move beyond the introduction stage, I have chosen to break the transcript at what seemed a recognisable developmental point, as I indicate in each case. The six cases presented in detail were selected to illustrate different aspects of the introductory stage which could be seen in the whole corpus of data recorded. This is a finer analysis of the three broad categories to illustrate similarities and differences. Within this finer analysis the intent is to illustrate how that which may be construed as understanding of the instructions for the problem, or then as mathematical understanding of the problem itself, correlates to ‘going on conversationally’.

As discussed in more detail in Chapter 5, for the transcripts in this section I have separated the gesture markup from the conversation markup in order to reduce the visual load. This allows the incorporation of coding markers to indicate gestures and other significant actions with a view to correlating these over several transcripts. Any gesturing within the session is then featured after the main body of the transcript. A summary guide to markup symbols and transcript codes can be found in Chapter 5. My interpretation of the transcripts and the arguments I make from these interpretations depend very strongly on the extra information carried by the marking up. While this marking up can make the transcript initially more difficult to read, I believe it is important because if carries more information about the conversational aspects of the talk than do typical transcripts.

The first example I offer illustrates my observation of an important feature of the introductory stage: establishing the interactive space via the opening turn. This example is interesting for it being atypical, and therefore standing out. In almost all cases recorded the students begin the problem by one student reading the question and then pausing to
allow the other(s) to offer input. This seems in keeping with Grice’s (1975) maxims that there should an orderly start to the process (maxim of Manner). In particular, this seems evident when the first speaker, Simone, opens the talk. Simone may also flout an aspect of Grice’s maxim of quantity, in that she offers more information than is required. Simone starts to read the question and then pauses, a signal usually implying that her interlocutor, Eric, should continue to read the rest of the problem; that is the typical opening action-pair. As Eric attempts to take his turn, however, Simone over-talks him and then rushes to suggest an opening strategy. In the few cases where this speak-then-suggest action occurred in my recordings, it resulted in uncooperative responses from the other group member(s). It seems likely that this opening strategy is taken as a threat to the interlocutor’s face (Goffman, 1967), in violation of politeness norms (Brown & Levinson, 1978), resulting in some kind of negative response. It seems that this then affects the rest of the talk, in that responses are more combative than cooperative. There is no obvious sense that the response is intentionally so, but it does seem to be the case.

I will begin by establishing the problem Simone and Eric were working on before examining the session in more detail.

1. Simone and Eric and the 612 problem

*Question:* “Anna came across an interesting puzzle, something times something equals 612. What might be the missing numbers?”

This problem was given in November of the school year. The students had been working on multiplication but had not yet been introduced to division. The intention of the problem was to have the students exposed to an open-ended question where several results were possible. In addition, the problem was a departure from the standard ‘n x m’ problems in multiplication. Two-digit multiplication had been covered in accordance to the BC curriculum, and both students had been able to demonstrate that they were capable of doing so, if not always reliably. The class pairings were made so that students who had demonstrated similar abilities to perform standard two-digit multiplication would work together, so that one student would not dominate the interaction. These students had worked together before and to all indications had an amiable relationship. By this stage in the year, the students were familiar with their classroom teacher’s directives on problem
solving in groups: that they start by reading the question aloud and discussing how they think they would solve the problem before they start writing. Nevertheless, the teacher always chose to remind the students of the procedure and encourage them to consider their ideas first.

This problem itself is interesting in that it does not immediately suggest a ‘procedure’ for the students to follow. They must interpret the question as asking them to discover the factors of 612, but at this stage they are lacking any familiarity with the term for it to be a natural interpretation. They have been exposed to the idea that numbers that make up a product are called factors, but the usage of the term is not formally introduced until grade 6 in the BC curriculum. As such, it was expected that the first hurdle the students would need to address would be the vagueness of having two unknowns. The equals sign, coupled with the word ‘times’, should prompt the students to recognise a processing action, which needs to be extracted from the number sentence. The absence of terms producing the given result requires that the students develop an understanding of the equals sign, because now they have to recognize that there is a relationship between the missing numbers represented by ‘something’ and ‘something’. There is a conception of ‘equalness’ in that the outcome of the process must equal 612. In this regard, 612 becomes the object of the problem, and needs to be identified as such. My expectation, from the ‘sound’ of the question, was that the groups might start the problem by identifying 6 x 102 as a solution, and then build from there.

In order to give an easier initial read of the transcript of the recording, I present it first without markup, using the first edit from the initial viewing. The extra information conveyed to the reader by the marked up version which follows should then be clear by comparison. For privacy reasons I have editing the students faces where appropriate. In shots where the students are not recognisable, I have kept the original picture. In the earlier transcripts (Chapter 1), I chose to blur the students’ faces, but with this more detailed analysis I have opted to use a line drawing which I believe obscures recognition but still allows expressions to be seen.
153

Table 6-1 Transcript  video #1 Simone & Eric and the 612 problem unmarked

<table>
<thead>
<tr>
<th></th>
<th>S:</th>
<th>Anna came across this interesting ↓ puzzle (.) something times something equals six-hundred and twelve (2.0) ((head down))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>13.5 E:</td>
<td>&quot;what (.) might (.) be (.) the missing numbers&quot; [O]</td>
</tr>
<tr>
<td>3</td>
<td>S:</td>
<td>the missing numbers (0.7) We↓:ill (0.8) so (.) &gt; how many solutions can you find (.) show all your thinking plus explanation&lt; (.)</td>
</tr>
<tr>
<td>4</td>
<td>21.6 S:</td>
<td>well first I thought we could try doing six (..) six (.) hundred (1.2) and two:ive (2.1) min↓us (1.9) &gt; well we know&lt; six times (..) two:um (1.5) equals twelve (1.6) s::o (1.1) twelve (0.8) [RC]</td>
</tr>
</tbody>
</table>
| 5 | 42.0 E: | wait so six hundred twelve minus (..) twelve equals six hundred (3.1) why are we /**/.

((glance)) [I] [FG][P] |
| 5b | S | now what twelve /**/.

((Glance)) [FG] |
| 6 | E | minus ing to cu six hundred and twelve |
| 6b | S: | you'll see and then [IO][PA] |
| 7 | 52.8 S: | what times (3.5) now what times something equals six hundred† (1.8) ((looks up at E, E glances at S then looks down)) [Q] |
| 8 | 1:01.1 E: | errrr b b b |
| 9 | 1:03.5 S: | /****hold you should know this↑ (0.7) ((smiling)) [Q][FG] |
As I discussed earlier, this is an atypical start to group talk. In almost all cases, recorded groups begin the problem by one student reading the question and then pausing to allow the other(s) to offer input. Simone violates the norm by her actions in lines 1-4. She starts
to read the question and then pauses, a signal usually implying that Eric should continue to read the rest of the problem. As he attempts to do so, however, Simone over-talks him and then rushes to suggest an opening strategy. Eric’s first reaction to this is to interrupt Simone and press her on her opening statement. This in itself is rare at this stage in the talk, and is particularly unusual for Eric, a normally quiet-spoken boy. Simone offers no repair and avoids Eric’s press; instead she further challenges him with a question (7). This junction in the exchange also marks the first time Simone looks up at Eric, other than a quick glance in line 5 when Eric interrupted her (see Table 6.3 below). The exchange does not come across as cooperative, and neither student supports the other’s utterances. Although there are turns at talk, I would classify this more as discussion than as conversation. The only conversational adjacent pairings come in lines 19-23, and this section might be classified as a proto-conversation, but this is quickly cut-off as Simone calls on the teacher for help rather than expressing her question to Eric. Throughout the exchange, neither student seems able to answer the other’s questions, or show confidence in what the other is saying.

<table>
<thead>
<tr>
<th>9</th>
<th>“You should know this”: Simone looks up at Eric and smiles as she asks him the question. It is done in a friendly manner, without any confrontation or condescension in her tone, but may be interpreted as an affront to Eric’s positive face (that of self-esteem).</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>‘Should’ is a modal auxiliary verb, expressing expectation, and may seem to be an implicit performative, calling for an action on Eric’s part. However, Simone’s tone and previous turn at talk lacks a sense of illocutionary force - in that there does not seem to be a clear intention behind what she is expecting of Eric; it may be more of a hedge tactic to stall for time or maintain her own face.</td>
</tr>
<tr>
<td>12</td>
<td>Eric’s response incorporates direct pointing at Simone. He correctly answers her question, but she makes a mistake when trying to correct him (13). Eric corrects her back, and she is forced to acknowledge that she made a mistake; she responds by smiling and quickly looking down, writing on her paper.</td>
</tr>
</tbody>
</table>
Eric seems to have repaired the affront to his face with a very forceful stabbing gesture towards Simone, who is no longer looking at him.

Table 6-3 Gestures per line - video #1 Simone & Eric

It is not entirely clear, in line 4, that Simone actually intended to subtract the 12 from 612; despite Eric’s interpretation and interruption in line 5. Simone seems to recognise a division of six as a start to the problem, as might be expected, but she is unable to clearly articulate the process at this stage. She recognises twelve as 6 x 2 but her broken statements do not give a clear purpose. She is focused on her paper rather than engaging with Eric, her posture is insular – head on folded arms – and her utterance slips to a form of ‘thinking aloud’. Simone’s extended use of ‘so’ is typically a filler used to maintain turn time, but does not seem to be in this case. Eric’s press (5) does not prompt an explanation from Simone other than a dismissive ‘you’ll see’ (6). Simone’s return question, even though eventually answered by Eric (12), does not enable the students to ‘go on’ with the problem, when in fact it might have had the students’ made a better connection with each other. To this point, their exchanges have not been cooperative and there does not seem to be an understanding developing. Simone has recognised ‘six’ as a divisor of twelve and Eric has identified six as a factor of six hundred. They talk around the connection in frames 22-24, but are unable to progress. Instead of supporting one another’s ideas, it seems as though when one student is on the verge of a breakthrough the other one interrupts in a negative way, as with Simone’s interruption in frame 21, when she claims to have ‘got it’ after Eric’s (apparently empty) “maybe” offering.

Simone closes this first passage (line 18) by dropping her head to her arms, a gesture suggesting frustration, but may also just be a dramatic gesture (see Table 6.4) following the impasse since there is no body language in the following section which further suggests frustration.
Simone’s body language may be indicative of frustration, of her not being able to ‘go on’ with the problem at this point. She does seem frustrated in the rest of the talk, however, so this may be more a dramatic signal to Eric than something born out of her emotions.

Simone claims to have “got it”, but this seems to be an empty gesture in response to Eric’s intake of breath followed by “maybe” being repeated as if he is on the verge of a breakthrough.

Eric beats out the repeat of the phrase “a hundred times six” using his fist on the word ‘hundred’ and his finger on ‘six’.

For some reason, Simone ‘shushes’ Eric as he thinks aloud. Although it might be conjectured that she does not want other students to hear him, there is no way to know that, or any indication from the talk to support that view.

Eric holds up a flat palm as he offers “one times twelve”. Simone’s facial gesture as Eric speaks is not inclusive, she seems skeptical.

Table 6-4 Gestures per line - video #1 Simone & Eric

Simone resets her stance (line 19) and repeats the question. Eric responds by repeating the question, but he then intakes breath and repeats “maybe” several times. Although it is not clear that he has made a breakthrough, Simone duplicates the intake of breath and claims to have “got it”. Eric immediately responds by pointing at her again, repeating the previous offering of “a hundred times six”, but done so with a space between his words as if emphasising what is to come. He beats out ‘a hundred times six’ but his voice trails away and it is evident they have both made empty gestures. Simone’s head drops to her arm again (line 22) as Eric tries to add more; this gesture seems to be related to her way of signalling the end of an exchange. Their apparent ‘ah ha’ moments seem to be a
further example of their lack of cooperation, in that they use them to compete rather than build upon each other’s ideas.

Eric repeats what they have already stated (line 24) but Simone, while writing, makes a shushing gesture. The intent of this is unclear; she may want some quiet time to think, but she also glances over her shoulder after making the gesture, as if afraid they are being over-heard. Following Simone’s gesture, she decides to seek extra help from the teacher and raises her hand, looking around again to locate where the teacher is. The teacher is meanwhile busy with another group so she turns back to address Eric, keeping her arm raised. She addresses Eric quickly, suggesting she has made a decision about something, and seems to have the idea of using addition to complete the problem, presumably 6x100 + something multiplying to 12 (perhaps 6x2, her original thought in line 4). She is mixing up addition and multiplication (line 25) but Eric seems to be sufficiently tuned in to suggest 1x12. He offers this holding his hand up to Simone, palm forward as if asking her to pause (line 26). Simone questions Eric in a curt tone and with what seems to be a skeptical look regarding Eric’s thinking; they are still confronting rather than collaborating, and Eric is unable to offer a meaningful response.

There seems to be no point in the exchange where the two students engage in any real conversation. Both students are making statements which do not directly build on each other, although are related. The students’ gestures are forceful but in a confrontational way and do not support their words. They are communicating, as indicated by a sharing of the initial idea, but I would suggest that the lack of conversation correlates to a lack of any understanding being demonstrated; they are unable to go beyond Simone’s initial idea.

To summarise, what I believe this example demonstrates is how the opening exchange sets the tone for any future talk about the problem. In particular it seems to affect the confidence of the individuals and their feeling of working in a supportive environment. These are important aspects which need to be established if a conversation is to instantiate. Without the dynamics of such a conversational space it does not seem that a shared understanding can develop.
In order to support my suggestion that a conversational space is needed to build understanding, my next examples illustrate how the opening phase is more usually regulated to generate that space. The first example illustrates an instantiation in a situation where the interlocutors do not seem to be initially comfortable with each other. Following this, for contrast, the second example is of an opening interaction between two students who are comfortable working together. Both of these next examples are of students working on a problem called 'Design Time', a situation facilitated in this case by dividing students into groups of three, with one student acting as the camera operator. This method makes the camera a more obvious participant in the process, but was a technique used on occasion in order to capture several groups at once.


This problem was set with the intention of combining spatial awareness with multiplication. In order to make progress the students must first develop a mutual understanding of the terminology used in the question. I use this problem as an example, not because it is challenging mathematically, but because it demonstrated clear stages of progress. The students had more difficulty in understanding the intent of the question than was expected, which led to a longer introductory stage than normal. In addition, I recorded 5 groups working on this and so any differences or similarities between the groups could also be analysed. An interesting feature of this was how different groups adopted different gestures to represent length and width, but once introduced the gesture remained consistent within the group.

Question: The desks in your classroom have always had the same shape. A new shape is being considered which might be better for students to work on.

The new desk will be:

- Rectangular
- Twice as wide as the original desk (the side facing you)
- Half as long as the original

Without taking any measurements, discuss what effect these changes will have on:
1. The area of the desk?
2. The perimeter of the desk?

**Problem 6.4 Design Time**

Nadia and Alex are sitting in the corridor, on a window ledge, outside of the classroom. They are given the question on a sheet of paper but are told only to discuss it at this stage. JinSu is holding the camera and recording the interaction between Nadia and Alex. At a significant point in the interaction, Nadia takes the camera from JinSu and turns it on her. In this case, the camera is playing an active role in the process and Nadia’s actions may indicate her awareness of the ‘teacher role’ it represents. Such interaction was not seen to occur when the camera was set on self-record and is an indication of the effect on group talk a teacher can have. Having presented several marked-up transcripts now, I present the following transcript only in marked-up form.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A: okay (.) let me read the question ((They read silently to themselves for 16.1s))</td>
<td>[RS]</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>18.7 N: Well (1.1) ↓ I mean (..) r↑eady (3)</td>
<td>[BG]</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>23.5 A: okay (2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>25.4 N: ((clears throat)) Well (0.5) If it’s recta:ngular it co:uld change or it could not (..) beca:use &gt;its already a rectangle↑&lt; (1.5) and then (0.8) on the last two no matter what it the perimeter and the area will change (..) rather get higher or lo↓wer (0.5)</td>
<td>[AG]</td>
<td></td>
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</tbody>
</table>

**Table 6-5 Transcript video #2 Nadia and Alex and the Design Time problem markup**

This opening illustrates how Nadia negotiates the opening stage of the problem in contrast to Simone from the first example. Where Simone seemed to violate an expected aspect of politeness by not being inclusive of Eric from the start, Nadia’s opening is interesting in that (line 2) she almost makes the same error, but redresses herself before any damage is done. She uses ‘well’ as a marker to introduce her turn, but then checks herself and pauses to ensure that Alex is comfortable for her to start. In this way, the talk seems to be established, without a power struggle, as being a common working space.
Nadia and Alex sit side-by-side but turned away. They do not seem to be sharing a common space at this time.

Nadia elongates the word ‘rectangular’ while moving her pen-holding hand through an arc; at the same time she looks towards Alex’s paper. She continues to use her hand to beat out her words.

Table 6-6 Gestures per line (corresponding to Table 6.5) – video #2 Nadia and Alex

Nadia initiates the talk vocally and also leans slightly towards her partner (line 2). Because Alex is turned away they cannot make eye contact, but Nadia looks towards Alex’s paper as she speaks. She is using vocal gestures to help put across her meaning; emphasising the word ‘rectangular’ by elongating it, and speaking more quickly when she explains her meaning “it’s already a rectangle” after drawing out ‘because’ to indicate she is about to do so. She raises the pitch of ‘higher’ and drops it on ‘lower’, all the time using her pen to conduct the flow of words. It later becomes apparent that she has mistaken the three bullet points in the question as separate options rather than as all being applied together. This is apparent from line 11 and 13 where she visualizes the rectangle as becoming a square. Nadia continues to try to think through the shape and becomes distracted by the practicality of the desk as an object to work on (13). Perhaps as a way to buy time to think quietly, Nadia passes the question to JinSu for ideas.

Alex’s response to Nadia is distracted; it seems he is still working through the meaning of the question. His response (line 5) is an adjacent pair to Nadia’s line 4, but his sentence is broken by hesitant spaces and he elongates the ‘as’, prompting Nadia to engage in sentence completion. Alex’s fast response may indicate that he was not searching for the word as much as thinking it through, perhaps indicated by the elongated ‘mean’. There is further overlap and Alex turns squarer to Nadia, pointing to the figure on her sheet as he speaks (line 7). This exchange is brief, terminating in line 10 as Alex elongates ‘so’ to nothing. In line 13, Nadia has shifted her focus to the practical aspects of a different desk shape. She uses vocal gestures to emphasize key words and rising intonation on ‘space’ (line 2) and ‘messier’, indicating suggestions, but in speaking she also recognizes more
information in the question and this allows her to be more certain of the new desk shape. This new confidence is suggested in the falling intonation of the third occurrence of ‘space’. Alex’s response is still non-committal (line 14), as suggested by his flat tone and his movement away from Nicole back to his original position.

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<tbody>
<tr>
<td>5</td>
<td>43.5</td>
<td>A: so like (((clear throat)) the (0.5) width would be (.) twice as:::=</td>
<td>[AP]</td>
</tr>
<tr>
<td>6</td>
<td>50.1</td>
<td>N: Wi::de=</td>
<td>[SC][VG]</td>
</tr>
<tr>
<td>7</td>
<td>50.4</td>
<td>A: =wide (.) does that me::an like two times the (.) that width</td>
<td>[AP][AG]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>/<del>/</del>/****/ over there↑↑↑↑↑↑</td>
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<tr>
<td>8</td>
<td>54.7</td>
<td>N: the width so it’d be like (..) there and then that again (2.7)</td>
<td>[O] [PG]</td>
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<tr>
<td></td>
<td></td>
<td>/<del>/</del>/**** hold /-.-./</td>
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<tr>
<td></td>
<td></td>
<td>((repeated pointing gestures occur in the next few lines))</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>A: or like</td>
<td>[IO]</td>
</tr>
<tr>
<td>10</td>
<td>1:00.8</td>
<td>A: So::: (3.3)</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1:04.4</td>
<td>N: and then (.) half as lo::ng the original (.) would be its &gt;shortened&lt; so it would be an almost perfect square (..) it would be like stopped right there (2)</td>
<td>[PG]</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>A: the next thing</td>
<td>[IO]</td>
</tr>
<tr>
<td>13</td>
<td>N: and (0.5) the (1.2) what (0.5) so just ask what effect</td>
<td>[HG] [VG]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>these changes would have on (.) well if you make a (.) it</td>
<td></td>
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<tr>
<td></td>
<td>half as lo::ng as the original and would be smaller and</td>
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<tr>
<td></td>
<td>would limit your working spa↑ce (..) and twice as wi::de</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>you would have no:: working spa↑ce but (..) er or (..) it</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>could also get umm (..) messie↑r (..) and now it see &gt;it</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>says the side facing&lt; so it would be that getting twice as</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>long so it would be really wi::de (..) so it would be huge</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>and it would take up more class sp↓ce</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1:48.8</td>
<td>A: Yeah it could yeah</td>
<td>[P]</td>
</tr>
<tr>
<td>15</td>
<td>1:49.1</td>
<td>N: and then for the regular (..) rectangular it depends if the</td>
<td></td>
</tr>
<tr>
<td></td>
<td>rectangle gets smaller or la:rg↑er (1.6) JinSu (..) do you</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>have anything to add (2.4)</td>
<td></td>
<td></td>
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</tbody>
</table>

Table 6-7 Transcript video #2 – Nadia and Alex and the Design Time problem
Alex turns to engage with Nadia, even pointing to her sheet. She in turn leans back and towards him while replying. Their gesture space is seen to close when they are communicating with each other, and then expand again when this brief exchange ends.

Nadia continues to use her pen to beat out her words. Such a motion often seems to be often linked with a confident approach in emphasizing ideas, but in this case she seems to be picking out important aspects of the question.

Table 6-8 Gestures per line – video #2 Nadia and Alex

Nadia is still thinking about the question bullets as separate items (line 15), picking out what she sees as important points but without yet tying them together. She seems to abruptly change tack, making a general statement before shifting the focus to JinSu, the camera operator. This approach could be seen as a form of hedging in that she is giving herself more time to think about the question without being ‘on camera’. If so, this may also illustrate how the more direct presence of the camera is affecting the students’ turns at talk in this example.

Table 6-9 Transcript video #2 Nadia, Alex & JinSu; the Design Time problem

JinSu is clearly unprepared for this turn of events and looks at the sheet with side eyes. She is trying to assimilate the question in silence, but Nadia presses her further. JinSu hedges by elongating her words, but she seems to pick up the intent of the question quickly, her eyes narrow and her voice speeds up as she sees that the rectangle is “twice as larger half as wide” (line 18). Off camera, Nadia picks up on the word ‘will’ as defining the new size to contain all three parameters. She is more relaxed (using Alex’s name) as...
she now seems to understand the question and is able to ‘go on’ with it. The tone of the talk becomes more relaxed and becomes interlinked, as indicated by the adjacent pairs and overlapping speech. All three seem to develop a common understanding that the stated changes have the same effect as just rotating the table. The burst of overlapping and adjacent pairing in this section, along with the frequent use of ‘yeah’ is indicative of a conversation arising between them.

<p>| | | | |</p>
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<tr>
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<tbody>
<tr>
<td>24</td>
<td>A:</td>
<td>ohh half as long and just chop the whole thing in half=</td>
<td>[O]</td>
</tr>
<tr>
<td>25</td>
<td>J:</td>
<td>yeah=</td>
<td>[O]</td>
</tr>
<tr>
<td>26</td>
<td>N:</td>
<td>no its just gonna flip so like that (..) that's all its gonna do</td>
<td>[AP]</td>
</tr>
<tr>
<td>27</td>
<td>J:</td>
<td>yeah that would make sense like that and it'll flip (.) over=</td>
<td>[AP/AG]</td>
</tr>
<tr>
<td>28</td>
<td>N:</td>
<td>=no its just gonna flip so like that (..) that's all its gonna do</td>
<td>[O]</td>
</tr>
<tr>
<td>29</td>
<td>A:</td>
<td>=ohh yeah</td>
<td>[O]</td>
</tr>
</tbody>
</table>

Table 6-10 Transcript video #2 Nadia, Alex & JinSu; the Design Time problem

JinSu gestures to show Nadia and Alex how the rectangle could be rotated.

Nadia and Alex stand and address each other directly.

Nadia’s gestures are now made using her full arm, this increased expressiveness is often seen when a student becomes more confident.

Table 6-11 Gestures per line - video #2 Nadia, Alex & JinSu

Alex is still holding the camera on JinSu, and she gestures as she talks about the table ‘flipping’. Her gestures are made high in front of her face (Table 6.11) and in a positive manner usually associated with confidence throughout the videos in this research. If the thought was not originally hers, the gesture suggests that she has picked up on Nadia’s meaning in the way that Nadia intended.
As JinSu takes back the camera, we see that Nadia and Alex have risen to their feet and are facing each other (Table 6.11). Their body language is now more relaxed and they use bigger gestures which incorporate Nadia’s use of a nearby container to illustrate a point. Their personal space has been shared and the earlier stiffness of their posture has been removed. There are fewer and shorter pauses in their speech, and little gap between their turns at talk. I would suggest that they have come to a mutual understanding of the question at this point. This is indicated by their confidence to be able to ‘go on’ with the problem, as seen in Nadia’s comment “we can deal with that” (line 35), and Alex’s “now we can do it” (line 37).

| 30 | N: | so that’s all it’s really doing |
| 31 | 3:31.2 | A: oh it’s gonna be the same thing ((J takes the camera back, N and A are now standing)) |
| | | [AP] |
| 32 | N: | = yeah so its gonna sTay the sa:me (. ) its just gonna like swi:tch si:des |
| | | [AP] |
| 33 | 3:38.0 | A: yeah (0.5) |
| 34 | 3:39.1 | N: now it makes sense |
| 35 | 3:39.9 | A: we can deal with that (1) it would still be the same |
| | | [AP] |
| 36 | 3:43.8 | N: yes (..) so say (.) if we (.) cut this in half (..) then that straight out (.) its going to be the same size |
| | | [AG] |
| 37 | 3:50.8 | A: yes (1) good now we can do it. |

Table 6-12 Transcript video #2 Nadia, Alex & JinSu; the Design Time problem

This recording demonstrates two key features noticed in this research. Nadia demonstrates that as she has grown more confident in what she is doing – her gesture space has expanded and she is using larger, full-arm gestures. In addition, the interlocutors speak more quickly, overlap their utterances without losing meaning, and engage in more adjacent pairings. These are the features of conversation that I suggest are indicative of the students having a sense of shared understanding. It may not, of course, necessarily mean that they actually do. However, it seems that they need this sense of understanding if they are to progress further with a problem. In recordings where these effects were not seen, such as in other examples I will show, a lack of these – what I would call conversation signposts – was a feature of unproductive activity.


In episode 2 above, the students demonstrate two phases to their problem solving. These phases may be described, in Wittgensteinian terms, as concerning two different
language games. In the first game, the students are playing with the language related to
the meaning of the question rather than with the language of the mathematics of the
question. If the classroom teacher has a specific intent in assigning a question then the
question needs to be framed in such a way that the students at that grade level can have
access to the intended game, that of the mathematics. This is generally described as
interpreting the question. In a poorly designed question, the majority of the class may
either be unable to gain access to the mathematics, or may spend too long trying to gain
access for the desired outcomes to unfold. In some cases this may be intentional, but in
the Design Time question the intent was that the students move quickly into the stage of
measurement, multiplication, and to see the effect of changing the shape on area and
perimeter. Episode 2 illustrated how interpretation of the problem started slowly and, it
seems, mostly internally as characterised by extended silences from both Nadia and
Alex. If Alex’s input is removed from lines 1 through 17, Nadia’s comments could just as
well be read as her thinking aloud, or having a conversation with herself. The apparent
lack of co-development in the early stage may be likened more to a discussion from
Alex’s point of view, as he is not indicating any signs of building or changing his
understanding. Their conversation with each other developed once a breakthrough had
been made.

I have selected the following episode to illustrate how the same process can occur
through conversation between the group members. In this episode there are few pauses
between the utterances, many examples of adjacent pairs in the turn-taking process, and
extensive use of gesturing. The three girls repeat the interpretation they each have until
they seem satisfied that they share a mutual understanding of what the question is
asking. The whole process takes about the same time as episode 2 above, but it is more
transparent to the observer and clearer that each girl is satisfied with the final outcome.
Instead of pointing to an image on their sheet, Sarah and Mangeet use full arm gestures
and the sheet-as-a-tool to help make their point clear. Although we do not see Jo, the
camera operator, we can infer that she is gesturing because the camera moves from side
to side and up and down as she talks.
Once again, the opening exchange is characterised by a student reading the question and then pausing to allow the others to give input. This seems necessary to establish turn-taking and give interlocutors an equal standing in the talk.

Mangeet initialises the talk and, in keeping with the idea of politeness and the maxim of quantity, pauses for Sarah to make an initial contribution, but receives only minimal response. Sarah’s posture suggests that she is thinking about the question (it is stiff with her head cocked to one side while looking at her paper). Mangeet presses Sarah to confirm her suggestion (line 3), using a questioning tone as a way to prompt. She establishes eye contact and makes a broad arm gesture to make clear her meaning.
Sarah takes the cue to contribute, overlapping then speaking slowly to emphasise that
she sees width and length differently. She stretches the word ‘right’, which acts as turn-
switching offer. Her look towards the ceiling is a gesture commonly attributed to thinking.
Mangeet accepts her turn with a simple agreement, effectively passing the turn back to
Sarah, who uses vocal gestures to emphasize ‘twice’ and ‘long’ by stretching the vowels,
and ‘half’ by speeding up. She supports her speech with large gestures of her arms. As
Mangeet responds, Sarah starts an overlap, but stops when Mangeet does not relinquish
her turn immediately. The turns are short but relevant and build up their understanding of
the problem. The frequent adjacent pairings and overlaps are strong indicators that the
girls are having a conversation about the meaning of the problem.

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<th>Table 6-15 Gestures per line – video #3 Sarah, Mangeet &amp; Jo</th>
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<td>6-7</td>
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<td>9-11</td>
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There are short pauses between their words, but no extended silence. The girls are able
to ‘go on’ with their conversation while at the same time there is physical evidence
through their gestures (Table 6.15) that they are building a mutual understanding of the requirements.

The transcript continues to the three-minute mark in a similar manner before all three girls are comfortable with the meaning of the question. Jo has been convinced that height is not an issue and both Sarah and Mangeet use their paper to identify the same sides as width and length, with the paper oriented the same way. Jo agrees to the papers held out before her. In this episode, there is evidence that the girls are able to develop their understanding through their shared conversation. They are not making independent statements, but rather build on each other’s contribution through a mixture of adjacent turns and gestures in a shared gesture space. Their large gestures suggest they are confident in making their statements, which may reflect the comfort level they have in working together as much as their own statements. This contrasts with episode 2, in which the gestures remained minimal until there were signs of greater confidence in the meanings themselves. In this episode, the girls seem to be gesturing with the language on a larger scale, while in episode 2 Nadia makes the larger gestures as her confidence seems to grow. This episode illustrates what was seen in general throughout the research, a connection between confidence and the dynamic nature of the students’ gesturing which often accompanied a conversational like organization of the associated talk. This suggests that the students had a sense that they were sharing a mutual understanding of the problem at that point in time. The next episode is given to illustrate that this may not be the mathematical understanding which was the intent of the problem.

4. Adrianne and Iona and the Design Time problem

If gesture size is indicative of the confidence of the speaker, and conversation is indicating a growing understanding in the exchange, then the following episode illustrates that this confidence or understanding may not be the intended outcome of the teacher in setting the problem. The problem is still ‘Design Time’, but the talk does not move into a mathematical space in the initial stage. In this case, I will start by examining the gestures, as might be seen from afar:
Both girls are making large, confident gestures which seem accurately to reflect the changes suggested for the desk, i.e. making it longer facing the student. Iona (seated to the right in each panel) makes very expansive gestures, filling the space in front of them. The focus of their talk, however, is on the practicality of such a design rather than its mathematical properties. I include a short excerpt here to illustrate:

| 4 | I: | Some of the effect on the area of the desk would be (.) umm (.) well you’d have more space umm you’d have less space umm to write on | [AG] |
| 5 | A: | yeah, because it would be this big and this space ( ) so its like (?) and only half the paper | [AP] [AG] |
| 6 | I: | yeah but it would get this but it would get bigger this way | [AP] [O] [AG] |
| 7 | A: | yeah so if you were right (.) if you used your right hand it would be easier | [HG][CT] |
| 8 | I: | but then again (.) what are you gonna do with all this space (.) that isn’t a practical (.) at all like (.) change to desks | [AG] |
| 9 | A: | We::ll if (.) they kept it the way umm the length across then it would be perfect because you could like put two people on the desk or one person could have all their (.) pens and stuff then the (.) workbooks could be there and like >two people could< share like textbooks and the other person could have their space (1) | [AP] [HG] |
| 10 | I: | but then again (.) umm the desks now can still fit your workbooks and things like that and lots of like your pencil box and you still have (.) plenty of room to write and to keep your water bottle | [AP] |

The girls started with Adrianne reading out the problem and then pausing until Iona made the first contribution. When considering the area of the table, however, it can be seen that they are seeing area as a workspace rather than a mathematical object. They are
concerned about placement of books, pens, and water bottles. The talk has characteristics of a conversation, with several adjacent pairs, and they keep their turn-taking orderly and in compliance with Grice’s maxims (if you consider what they count as being relevant as acceptable). They are able to ‘go on’ in their work, building on each other, and demonstrate a mutual understanding of the new shape of the table, even folding their papers in half to form the correct new shape by taking the original shape of a single piece to indicate the starting desk.

My point in using this episode is to illustrate that just because the students can have a conversation and demonstrate confidence in their gestures, they may not mean that they are building an understanding of the mathematics of the problem in the way the teacher (or the problem) may have intended. At the same time, I believe this episode does illustrate the characteristics of a shared understanding in a more general case. The two girls are in agreement about the desk, which is to say that each seems to believe that they are seeing the problem in the same way (again, this does not mean that they actually do). The confidence, the gestures, and the conversational organization seem to be key indicators of this shared belief. These would seem to be features a teacher can pick up on, even from afar. Clearly, this is not enough if a lesson is to be productive in the sense of the teacher’s expectations, even though it may be productive in ways the teacher did not expect. In order to build on this platform, further evidence of mathematical understanding must be found, and it is to this I turn to next.

Many problems, using different approaches, addressed the idea of changing the parameters of a shape in order to determine how the shape’s properties change. Design Time was one which illustrated the difficulties students can have in interpreting a question in terms of the mathematics. In the next episode, I examine more of the mathematical content covered in a problem that is based on a similar idea to Design Time. In this example, I will highlight how the actual mathematics in group work needs to be attended to carefully by the teacher.

5. Aaron, Bashir, and Chan: Don’t fence me in

This problem was set by the classroom teacher as part of a ‘Problem of the Week’ task. It was given in January as a way of reviewing and reinforcing the concepts covered earlier
in the year. The problem was taken from the University of Waterloo’s website as part of the ‘Measurement Strand’. The problem choice was influenced by its similarity to a problem on the previous year’s FSA 7 (The BC Ministry of Educations Foundation Skills Assessment that is expected to be written by all grade 7 students in the province, and tests for ‘grade level’ skills in English and Mathematics from grades 5 & 6), and was seen as a target standard for the class to eventually reach.

Part of Jan’s yard is fenced to make a pen for her dog. The pen measures 10 metres wide by 20 metres long.

a) If Jan increases the length of the pen by 5 metres, how much does the area of the fenced part increase?

b) Jan wants to enlarge the pen as she has adopted a second dog. She plans to move one side of the pen outwards to increase the area by 40 square metres. What is the least number of metres of additional fencing Jan will need to enclose the enlarged pen if she re-uses all the existing fencing?

Table 6-18 Don’t fence me in problem

To solve part a) of this problem, students needed to first interpret the story to extract the mathematics requested. They needed to be able to recognize a process connecting area, length, and width. A picture was provided to assist with the meaning of the terms wide and long by labelling the sides of the pen with the corresponding values, avoiding some of the issues seen with the Design Time problem discussed above, but it was still necessary that the students knew the connection between long and length. The students had to make some conceptual connections prior to any mathematical process in that they needed to be able to visualize the effect of increasing length, recognizing that two sides are changed to form an elongated shape which is still rectangular. The associated problem notes indicate that the students need to recognise the situation, have the skill to use the process, and be aware of the independence of the factors. In terms of this research, I was looking to see if these requirements would be made apparent in the students’ potential conversation.

The first sequence reiterates how, in this case, a group of three students negotiate the opening balance of power in the group. Here the students read the problem together
while in other cases the students may take turns to read the problem one line at a time. Chan’s opening statement “so let’s read it first” is very much a reflection of the classroom teacher’s general instruction given throughout the year. He uses an open arm gesture in the form of an invitation as he speaks. Even when Aaron states what the question requires, Chan insists that they read it together and they do so; Ahmed starts but Chan completes the statement as the others’ voices trail away.

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<tbody>
<tr>
<td>1</td>
<td>C: so let’s read it first (with an open arm gesture like an invitation)</td>
<td>[AG] [TC]</td>
</tr>
<tr>
<td>2</td>
<td>A: it asks for the area</td>
<td>[O]</td>
</tr>
<tr>
<td>3</td>
<td>C: so let’s read it first=</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>A: If Jan increases the length of the pen</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>B: increases the length of the pen by 5 metres</td>
<td>[O]</td>
</tr>
<tr>
<td>6</td>
<td>C: increases the length of the pen by 5 metres how much (...) does the area (...) fenced part have (0.7) INCR↑rise (1.4)</td>
<td>[O] [HG] [RS]</td>
</tr>
<tr>
<td>7</td>
<td>15.4 A: so (...) umm (...)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>B: I think that=</td>
<td>[I]</td>
</tr>
<tr>
<td>9</td>
<td>17.2 A: =&gt;you have to first&lt; find out how many metres does it have right no::w (0.8) in the area err (.)</td>
<td>[I]</td>
</tr>
<tr>
<td>10</td>
<td>C: okay guys ((The gesture is an emblem, indicating ‘stop’))</td>
<td>[IO] [AG]</td>
</tr>
<tr>
<td>11</td>
<td>B: yes so there’s</td>
<td>[IO]</td>
</tr>
</tbody>
</table>

Table 6-19 Transcript video #5 Aaron, Bashir & Chan: Don’t fence me in

Aaron announces his intention to take a turn using the marker ‘so’ but pauses long enough, despite using a second marker umm to indicate he still wants his turn, that Bashir initiates a turn of his own with the phrase “I think that” (line 8). Aaron immediately responds by interrupting Bashir to retake his turn, speaking quickly in order to do so. Chan may recognise this as non-cooperational (a violation of the manner maxim in not being orderly) and speaks over Aaron in an attempt to retain control of the talk (line 10, see also Table 6.19). Aaron ignores the attempt and continues, but as soon as he pauses with the elongation of ‘now’ (line 9), Bashir attempts to reclaim his turn at talk (line 11). Although Bashir’s interruption overlay (line 11) is ignored by Aaron, when Aaron pauses again, despite using the filler err, Bashir takes the opportunity to continue (line 12). It is worth focusing on the gestures at this point as they, along with the talk, seem to point to the dynamics of the group’s interaction.
There is a sense in the opening of one student, Chan, trying to corral the group into working together while the other two students adopt a more isolated approach. Both Aaron and Bashir adopt a closed posture, arms tight to their body and forearms on the table. They are focused on their paper and do not look to the other members of the group. Chan, in contrast, is using large arm container gestures as though trying to pull them together.

| 10 | Chan uses the recognisable emblem gesture with both palms making a downward motion, as a signal to stop, or perhaps calm down. Aaron and Bashir both look down at their paper. There has been no eye contact between them yet. |
| 12 | Bashir gestures by first changing his posture to be more upright, then drawing his arms to his body and rotating them down to the page. As he does so he briefly glances up towards Chan (frame 3) and then down. It is as though he is claiming the gesture space. |
| 14 - 15 | Bashir is using a container gesture (framing the rectangle) while talking of width and length. Chan points into Bashir’s gesture space to describe the middle as the area. Bashir then folds in his container. |

Table 6-20 Gestures per line - video #5 Aaron, Bashir & Chan

When Bashir makes his first significant contribution to the talk (line 12), he begins by changing his posture to being more upright and then drawing in his arms before rotating
them back down to the table. The effect is that of claiming the gesture space and thereby the turn at talk (Table 6.19). Bashir’s gesture (line 14-15 Table 6.19) is then a small hand-framed container gesture to locate the fence object in space. It is of note that Chan then reaches over to include himself into Bashir’s gesture space (overlay in line 14) in what seems to be a deliberate attempt to be inclusive.

The effect is that Aaron cedes the floor to Bashir, but he is quick to interrupt again when Bashir pauses (line 12). He does, however, keep his eyes on Bashir as he speaks and the interruption is not an attempt to regain a turn at talk. This time Bashir acknowledges Aaron, and while he is overlapped by Chan, Chan’s overlap reinforces rather than challenges Bashir. The mood seems to have changed to be more cooperative from this point.

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Transcript</th>
<th>Transcript Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>B:</td>
<td>22.1</td>
<td>[BG] [HG]</td>
</tr>
<tr>
<td>13</td>
<td>A:</td>
<td>28.7</td>
<td>[I] [AP]</td>
</tr>
<tr>
<td>14</td>
<td>B:</td>
<td>30.4</td>
<td>[AP] [HG]</td>
</tr>
<tr>
<td>14b</td>
<td>C</td>
<td></td>
<td>[IO] [HG]</td>
</tr>
<tr>
<td>14c</td>
<td>B</td>
<td></td>
<td>[AP] [HG]</td>
</tr>
<tr>
<td>14d</td>
<td>C</td>
<td></td>
<td>[AP] [IO]</td>
</tr>
<tr>
<td>15</td>
<td>C</td>
<td></td>
<td>[I][HG]</td>
</tr>
<tr>
<td>16</td>
<td>A:</td>
<td>37.5</td>
<td>[I][HG]</td>
</tr>
<tr>
<td>17</td>
<td>B:</td>
<td>42.6</td>
<td>[AP] [SC] [HG]</td>
</tr>
<tr>
<td>18</td>
<td>C</td>
<td></td>
<td>[AP][IO]</td>
</tr>
<tr>
<td>19</td>
<td>A:</td>
<td>47.6</td>
<td>[I] [AP]</td>
</tr>
<tr>
<td>20</td>
<td>C</td>
<td></td>
<td>[AP] [SC] [HG]</td>
</tr>
<tr>
<td>21</td>
<td>A:</td>
<td>50.1</td>
<td>[CT][H]</td>
</tr>
</tbody>
</table>
When Aaron says "it does not change anything" (line 13), the implication may be that it does not generally matter which side you determine as length and width for a rectangle, the area is always the product and is commutative. In this problem, however, the distinction does matter, and the group has not recognised that at this point. Aaron points out that the picture indicates the length as 20m (line 16) and Bashir reiterates that (line 17) by noting that the extra 5m is to be added to the twenty metre side. Chan's overlap with Bashir indicates they are in agreement. The section from lines 15-20 can be seen as a string of adjacent pairs with overlapping and supporting statements. This is indicative of a conversation as the students are open to changing what they have said. A mutual agreement on what is changing is developing, but this does not mean that they have understood the implications of this change yet. They are still playing with the language of the question.

In line 21, Aaron brings up the idea of perimeter, which is a change in direction for the conversation. This may indicate that, while he shares an understanding of the rectangle’s shape and size, he is not thinking about the area properties in the same way Bashir and Chan are. The fact that the gestures Aaron is making on the page are linear movements of his hand, compared to the container gesture of Bashir, which include a contribution from Chan, might support the idea that Aaron is thinking on different lines. In addition, Aaron’s utterance (line 19) in referring to the area calculation is hard to decipher. Bashir again speaks during Aaron’s pause and Chan overlaps with a return to area as a
multiplication. There is no indication that these responses are in relation to the use of perimeter, or just retained from before. Aaron’s response is just filler, which may indicate that he is not focusing on Chan’s area calculation (line 24). Bashir, on the other hand quietly presses the multiplication question (line 25) and Chan duly responds with the answer (line 26). Aaron’s response to this is of surprise (line 27), as indicated by his body language, which is to look up and smile, leaving a slight pause before saying “you are so correct”. Interestingly, for the first time, Bashir looks up from his paper and looks directly at Aaron. He seems a little surprised at first, but then smiles. I conjecture here that Aaron, thinking in terms of perimeter, is slightly surprised by Chan’s value. This view is supported by the next section of talk (Table 6.21).

Chan’s gestures have been quite expressive throughout, and may reflect his confidence in dealing with area and the notion of multiplication. He leans forward throughout the talk, closing the gesture space, and his hands are active whenever he speaks. Bashir has sat quite formally and looked at his paper, but his hands have been active in the air, suggesting that he is thinking spatially; clearly he is listening, but not in the usual way one would expect in a conversation. It may be that he is part of the talk and yet not immersed in it. In contrast, Aaron’s gesturing has been limited to moving his pen along the paper to follow the length and width, or beating out his words (line 16). While the group’s talk may have moved to a conversational level around the rectangle’s properties, their gesturing, due to its different forms, does not support the idea of collaboration. It seems that Aaron may be operating on a separate level and so this may actually be a mixture of discussion and a conversation, despite the conversational markers discussed earlier.
Aaron’s gestures are linear on his sheet. Bashir’s gaze is projected downwards for the majority of the exchange.

Chan leans forward in his seat to close the gesture space. As he does the calculation, he uses his fingers, clasped above his head, to beat with the count.

Aaron’s response to Chan’s calculation seems like a surprise to both Bashir, who looks up for the first time, and Chan, who stops moving his fingers momentarily before humming a victory tune.

**Table 6-22 Gestures per line - video #5 Aaron, Bashir & Chan**

The last section of the talk is given in Table 6.23, and in this the divergence in the students’ thinking seems apparent. Aaron brings up a change to the width by 5 (line 31), even though the question does not require this. He makes the erroneous claim that if the width is now 15m and the length remained as 20m then the area would still be the same, 250m$^2$. His statement is supported quickly by Chan, suggesting that he has not thought it through. Bashir adds in a flat tone (line 33) that “whatever you do it does not really change anything”; it is possible he is thinking about a fixed rectangle having its width and length switched, since this was the essence of the earlier talk. Chan’s follow-up comment supports the idea that he and Bashir are thinking this way, as he comments (with a high gesturing of his hands in the air, see Table 6.24) that twisting and turning the shape does not do anything, only shrinking it (line 35). Aaron’s comment in line 37 support the idea that he is thinking in terms of the linear property of perimeter, and there is some confusion with area, even though he is using the terms separately and calculates the value as 60 (line 36). It may be that because he has noticed that the perimeter does not
change, that is his reasoning for the area being the same. Noticed in class, this would have been a good teaching moment; as it stands, it reflects a weakness in the question in not offering an avenue to explore such a misconception.

<table>
<thead>
<tr>
<th></th>
<th>Time (s)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A:</td>
<td>1:12.9</td>
<td>Yes but first (1.1)</td>
<td></td>
</tr>
<tr>
<td>B:</td>
<td></td>
<td>yeah</td>
<td></td>
</tr>
<tr>
<td>A:</td>
<td>1:15.4</td>
<td>And those have to change a bit is this ahh this becomes (. ) fifteen its still umm two hundred n fifty</td>
<td>[HG] [HG]</td>
</tr>
<tr>
<td>C:</td>
<td></td>
<td>yep (0.5)</td>
<td></td>
</tr>
<tr>
<td>B:</td>
<td>1:21.4</td>
<td>and so whatever you do it doesn't really change anything</td>
<td>[AP]</td>
</tr>
<tr>
<td>A:</td>
<td></td>
<td>yeah</td>
<td></td>
</tr>
<tr>
<td>C:</td>
<td>1:23.4</td>
<td>If you twist’n turn it (.) anythin (..) except (.) if you shrink it it’ll do something</td>
<td>[AP] [HG] [AG]</td>
</tr>
<tr>
<td>A:</td>
<td>1:28.9</td>
<td>(1.9) and err umm (1.7)) and er::r (..) a Peri↑me (.) perimeter is umm (1.0) sixt↓y (2.6)</td>
<td></td>
</tr>
<tr>
<td>C:</td>
<td></td>
<td>or expand it it’ll do something</td>
<td></td>
</tr>
<tr>
<td>B:</td>
<td>1:38.6</td>
<td>“twenty”</td>
<td></td>
</tr>
<tr>
<td>A:</td>
<td>1:38.8</td>
<td>yeah sixty because umm ten plus twenty equals thirty (.) and there’s the other side which is (.) thirty plus thirty equals (.) sixty</td>
<td></td>
</tr>
<tr>
<td>C:</td>
<td></td>
<td>yep (..) sixty</td>
<td>[AP] [FG]</td>
</tr>
<tr>
<td>B:</td>
<td></td>
<td>Yep</td>
<td></td>
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</tbody>
</table>

Table 6-23 Transcript video #5 Aaron, Bashir & Chan: Don’t fence me in

There is a significant pause after Aaron calculates the perimeter. Chan looks at him with a furrowed brow, suggestive of being in thought. He could be using the time to calculate the perimeter himself, since he confirms the value, but the speed at which he calculated the area may suggest that there are other thoughts. I would suggest that he is confused by Aaron’s switch to perimeter, but that cannot be confirmed. Bashir’s face remained passive in this interaction. Another point of interest was Aaron’s hand gestures, when speaking of perimeter, became more expansive. He used his hands and forearms to support his speech rather than just his pen as he had previously done. This may suggest that he is more comfortable in dealing with perimeter.
Aaron’s hand gestures become more expansive as he talks about perimeter, perhaps suggesting a greater degree of comfort in this topic.

Chan uses his hands extensively when describing how you can **twist** (his actions indicate a forward rotation), and **turn** (his hands indicate a sideways rotation), but not **shrink** (his hands come together), or **expand** (his hands move apart).

Chan looks puzzled by Aaron’s calculation of perimeter; he takes longer to confirm the calculation than his previous rate of calculations would suggest he should be able to.

<table>
<thead>
<tr>
<th>Table 6-24 Gestures per line - video #5 Aaron, Bashir &amp; Chan</th>
</tr>
</thead>
<tbody>
<tr>
<td>The purpose of including this episode is to bring out the important point that it is possible for some students to participate in the talk but not the conversation that others may be having. In this case, Chan and Bashir seem to be talking about the area while Aaron is focused on the perimeter. I would also point out that the pronoun ‘we’ only occurs three times in the transcript, and those occur during lines 12-16, a conversation in which they establish a commonly agreed principle about which side to increase in length. At other times, Bashir and Aaron use the general form of ‘you’, which is less inclusive. When Chan is trying to bring the group together at the start he uses ‘let’s’; thereafter he confines his utterances as statements in either support or response. As the talk continues, the difference in the way they are thinking seems to be more apparent from their responses to one another. Comments and calculations are met with polite agreement rather than thoughtful consideration. The result is that errors in thinking are allowed to stand unchallenged. The implication of this could be that the students do not have a shared understanding of where they are with regards to the problem. This leads</td>
</tr>
</tbody>
</table>
to the question of whether or not they are able to ‘go on’ with their attempt to find a
solution. I will return to this question at a later point.

While there is some suggestion as evidenced by a passive but present power struggle
throughout the above episode, the following extract is included as comparison. In this
episode the negotiation of turns is completed more politely and raises the question as to
whether this leads to a greater collective understanding of the problem.

6. Richard, Peter, and Jill and The 612 problem

This episode is another group working on the problem discussed in episode 1 above. In
this case, the group is more functional in the way they work together. In contrast to
episode 1, they open with one student reading and then pausing for input. It also
illustrates how three students seem to share a common understanding.

The students, Richard, Peter, and Jill, sit around a circular table in the library.

<table>
<thead>
<tr>
<th></th>
<th>R:</th>
<th>P:</th>
<th>J:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I’ll read it</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Okay</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Common factors (.). Anna came across this interesting problem (1.3) a (.). what might the missing numbers (0.6) be (0.4) how many solutions can you find (0.9) show all your thinking and explanations (1.0)</td>
<td>[RS]</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>so something times something equals six hundred and twelve</td>
<td>[AP]</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>twelve</td>
<td>[O]</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>“so it must be something (..) we might want”</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>well I th:ink I know that it might be a two digit number (1.2) wait (.)</td>
<td>[I]</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>yes (.) it’s a two digit number (..) or three ((nodding))</td>
<td>[FG]</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>two or three</td>
<td>[O] [AP]</td>
<td></td>
</tr>
</tbody>
</table>

Table 6-25 Transcript video #6 Richard, Peter, and Jill : 612

The initial turns are short, as each student provides input. The marker ‘so’ is again a
common way to establish a turn (lines 4 and 6), and overlapping occurs where Jill says
something known (line 5/6). Early eye contact is established between Richard and Jill.
Peter is a soft speaker and Richard turns to look at him as he speaks, but interrupts
when Peter pauses. Note that Peter starts by using a collaborative we (line 6) but
Richard, after using well as his interrupt marker, uses a possessive ‘I’ (line 7). He turns
back to look at Jill and places a heavy emphasis on think, indicating a hedge, his
intention made uncertain by his use of ‘it’ instead of ‘one of’ in referring to a factor. His
posture is leaning forward and ‘business like’, while Jill leans back in her chair and
Richard sits up straight. (The clear difference in their posture has relevance which I
discuss further in Chapter 9). As Richard pauses, Peter fills the gap by agreeing with him
but Richard uses wait to suggest he has more to say (line 7). Jill agrees with Richard but
Peter overlaps that a three digit number is also possible, (line 9), to which Jill also
agrees.

The significance of this opening salvo is that all three students have used the exchange
to establish their equal participation in the talk. All have made a contribution, however
slight, which is directed to the group. Each student has made eye contact with the others.
There have been no hand/arm gestures to this point, reflecting a common theme in this
research that such gestures are generally more evident when a student is giving an
explanation than when making a statement.

The body language of each student sitting around the table is different. Richard sits forward; his focus is on
Jill who returns his eye contact. Peter’s eyes are flicking between
Richard, Jill and his paper. Peter’s
body language is stiff, but not
exclusionary, suggesting his soft
tone reflects his personality rather
than confidence.

Table 6-26 Gestures per line - video #6 Richard, Peter, & Jill

In this task, it is necessary for the students to first understand the implication of the
statement “something times something” as requiring two numbers which have a product
of 612, and it is interesting that Richard’s opening mathematical statement is that ‘it’ must
be a two digit number. Jill and Peter agree, even extending ‘it’ to a possible three digits.
There seems to be an unspoken agreement about what ‘it’ is at this early stage, and this
shapes the way they are thinking about the problem. Continuing:
In line 11, Richard has a realisation that a product is required. He has now switched to a more inclusive ‘we’, which he uses five times. In his use of ‘break’, he is implying factors found through division, but his final statement shows that his thinking around doing that is to guess the factors using multiplication, providing an example to illustrate. Richard begins this line by raising his hand, an attention gaining gesture which serves the same purpose as the markers such as ‘so’ and ‘well’. Although his use of ‘ah’ also serves as such a marker, there is more of an affected connection to the exclamation ‘ah’ then the more deliberate use of ‘so’ and ‘well’. In response, Peter uses ‘actually’ to mark his turn (line 12), and he brings his hands to his temples as he does so, another embodied gesture suggesting thinking. He then brings his arm down to point to Richard’s paper. Peter’s utterance is an adjacent pairing but fails to add anything new to Richard’s comment, and in fact may have stalled it. Peter speaks in a staccato way, with bursts of hurried speech followed by pauses; this has the effect of detracting from what he is saying. Both Richard and Jill turn away from their direct eye contact halfway through line 12. There is a sense here that Peter is violating the maxim of quality in his turn at talk and that Richard and Jill’s response to this is a loss of attention.
Richard announces his term with a raised hand linked to his ‘ah’ exclamation. This seems like a reactive gesture to his idea, an example of the gesture being embodied.

Peter makes a more cerebral gesture in connection with his marker ‘actually’ as he claims his turn. It gives the impression that he is about to add something profound to Richard’s idea.

He completes the gesture by pointing to Richard’s paper. Both Richard and Jill give Peter their attention, but their attention drifts as Peter speaks.

Table 6-28 Gestures per line - video #6 Richard, Peter, & Jill

A key reason for the teacher selecting this question was to foster the connection between multiplication and division, so this is an important point for the students to pick up on. Unfortunately, they are unable to ‘go on’ with this and the thought is left hanging. Each student produces filler sounds (lines 13-15), perhaps indicating their continued presence in the talk, before Richard tries to reset the thought process back to his two digit idea (line 16). Richard uses ‘we’ again to include the others and both Peter and Jill respond to this inclusion (lines 17 and 19).

| 22 | 1:24.3 | R: | oh / have a strategy (. ) / have a strateg↑y | [Ⅰ] |
| 23 |       | P: | yeah↑                                      |     |
| 24 |       | J: | yep                                        |     |
| 25 | 1:27.9 | R: | so we can like do the lattice metho↑d (. ) but when we do it we go (. ) s:ix (. ) to twleve into two (. ) and then we figure out (. ) like what could equal the tw: o↑ and the number that it would ma↑ke (. ) and then times↓ it |     |
| 26 | 1:38.1 | P: | good idea ((accompanied by a flick of the forefinger)) | [HГ] |
| 27 |       | J: | ye↑ah                                      |     |

Table 6-29 Transcript video #6 Richard, Peter, and Jill : 612
Peter (line 21) mimics an approach commonly stressed by the classroom teacher, to think of a strategy, and to look for a pattern. As he does, he starts to flick through his binder and asks for time to do so. This suggestion seems to have prompted Richard into action again (line 22), but notice when he declares this, by interrupting Peter, he repeats his claim with a rising intonation. The effect is to ask for permission from the others to continue and both of them pick up on this and give their assent, Peter questioningly, as if to say “do you really”, and Jill flatly in acceptance of the idea (lines 23 and 24). Richard’s next utterance is linked to his first suggestion (line 11), but now has the authority of a known method; the lattice. His proposal is not easy to parse, but Peter responds with a flick of a raised forefinger, suggesting that he was able to follow the proposal. Likewise, Jill’s rising tone yeah implies that she has also followed Richard’s idea.

As noted earlier, this excerpt is included as an illustration of a cooperative opening to a problem session between three people. Even though they have yet to make any real progress mathematically, there has been a tone of acceptance established in which the students seem confident in offering suggestions. This offers a good point to follow the introductory stage of problem solving into the middle part of a problem session, what I shall refer to as the developmental stage. I categorise this step as being entered once a group has a workable plan they then choose to develop in order to find a solution. Not all groups reach this stage. Excerpt 1 above is such an example, where the students were unable to settle on a workable strategy without the guidance of the classroom teacher. Before moving on to the developmental stage, however, it may be helpful to summarise the important aspects from the episodes discussed so far. In particular, the summary addresses the question of how each episode informs us as examples from the whole corpus of work studied. I have selected cases which I feel illustrate the more general range of openings seen in the greater body of the research, covering over 150 recordings.

8 The lattice method is a standard multiplication method commonly taught in schools at this level. The students were familiar with it and it was a popular method of multiplying, although they were aware of other methods too.
Summary so far – the Introductory Stage

My first example was chosen to illustrate how students generally follow the Maxim of Quantity (Grice, 1975) and politeness (Brown & Levinson, 1978) when working together. This is particularly evident in the introductory stage where the violation of these maxims seems to prevent an orderly start to the session. It seems apparent that by the time students have reached grade 5 they are well enough established in the interactions of conversation to carry this through into the mathematics classroom. It is also apparent that an aspect of the Maxim of Quantity is adhered to in that students’ utterances tend to supply just what information is felt as necessary for the talk to continue, and give other group members the opportunity to make their contributions. In the last example, it was clear that Peter's drawn-out explanation quickly caused the other students to lose focus, a result of a lack of quality in what he was saying as well as the time he was taking to say it. These maxims relate to emotional aspects of a conversation and are important factors which underlie how a conversation develops. This may also point to a reason why some students who have the ability to make a contribution to a group can find it difficult to do so in a way that is acceptable to the other group members.

My second example was used to illustrate how students typically start a group problem solving session and how they can manipulate the introductory stage in order to make progress in a situation where there is no obvious immediate cooperation. This approach seems to particularly relate to students who like to have some time to think through a problem by themselves first, something a group setting can make problematic.

In the third example, I chose an exchange that typified a more gregarious approach and one that I feel illustrates how a group can use a conversation to build a shared understanding of the problem. While the second group built on ideas that seemed to be largely formed individually then put together, the third group did so in a more interactive way, talking through each idea as it arose. The point here is to show that the conversation can be focussed internally or externally and while the former is harder to monitor, there is still a stage where an external conversation is used to indicate progress. An important feature of this example was also the gesturing used by the students as part of their cooperation. While the gesturing was more restrained in the second example, it was readily apparent when the students chose to share their ideas.
I selected the fourth example to illustrate how the conversational approach can be seen in a situation where the focus of the conversation is not on the desired mathematics. A group may seem to be making excellent progress in terms of the conversational organization and their use of bold gestures, but can be off-track in relation to the outcomes desired from the given problem.

In the fifth episode, my intent is to illustrate how not all members of a group may be engaged in a conversation but can instead be talking along a different line. It may be that two or more members of a group are conversing while one (or more) is not. In this case, the group can gloss over mistakes because they are not all listening to each other. The organization of the conversation is lacking during parts of the interaction and it can become a conversation/discussion hybrid.

The final example illustrates how members of a group can build a platform of cooperation by establishing a healthy conversational space, allowing them to put forward and examine ideas that may take them forward, even if their initial ideas stagnate. It is this example I examine further in the next chapter as the students move into the development stage of their work.
Chapter 7

Results and Analysis: Development

To explore the development stage of the problem solving, I will be using three excerpts, two which are a continuation from the introductory stage, and one which is new. The first example carries on from the last example used in the introductory stage.

The Development Stage

*Episode 7b. A continuation of Richard, Peter, and Jill and The 612 problem*

Richard has put forward the suggestion that factors of 612 might be found if they apply the lattice method of multiplication backwards in order to guess one of the factors by starting with the second factor as 2. He is seeing division as a process of doing multiplication in reverse. Richard’s explanation may lack clarity (Table 6.28, line 25), but the gist of it does seem to get across to Peter and they are able to work together from the beginning. It takes Jill a little longer to pick up on what Richard and Peter are doing, but then they all contribute to the process. In order to make better sense of what they are saying, I will begin this section by illustrating their process, as indicated by their notes at the end of the session. Because they left several half completed or crossed-out diagrams, it is easier to put these together as shown in Table 7.1.

The three students spent some time drawing out a lattice on each of their papers. Richard started by constructing the lattice shown in the first row of Table 7.1:
Richard’s visible initial lattice. Jill reproduced this as she was watching Richard as he drew the lattice. Peter seemed to draw the same lattice without looking at Richard but pointed out the need for 612 along the bottom, indicating that his had it in place.

The group’s first attempt was to try to reconstruct the problem in this way by filling in numbers that added to the bottom row. There was confusion in what they were multiplying, how to account for multiplication by zero, and how to find the values that could replace the threes.

Table 7-1 Lattice method (a) - video #6 Richard, Peter, & Jill

Jill did not contribute to this process until the very end, but her body language, leaning in and watching, suggested that she was at least trying to follow what Richard and Peter were doing. She did, however, make a significant comment (for me) that went unheard by the boys when Richard first inserted the three that would ultimately unravel the process (line 34b below). Richard led the process at the early stage, setting out the values shown in line 2 of Table 7.1. He placed a 1 and a 2 in the bottom right square and zeroes above so that these diagonals would add to the appropriate numbers on the bottom row. He then placed a 0 and a 3 in the top row, presumably thinking (but perhaps subconsciously) that $6\div2=3$ and that the 3 could not go in the box below because it would affect the diagonal sum to 1. This move confused Peter, however, and he questioned Richard on the validity of this, raising the question that something times zero would then need to be three. Richard tried to argue his point, but unconvincingly. Richard was unable to suggest what values could be placed in the boxes to replace the two threes; he was unable to ‘go on’ with his process at this point.

With a sense of what the students were trying to do, a more careful analysis of the organization of their talk can now be undertaken.

<table>
<thead>
<tr>
<th>Time</th>
<th>Transcript</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>R: okay, we know that this is going to be zero two (1)</td>
</tr>
<tr>
<td>29</td>
<td>P: ye:ah (1.5) so:: yeah (1) shouldn’t we put the number in first [AP]</td>
</tr>
<tr>
<td>30</td>
<td>R: we don’t know what number it is though [AP]</td>
</tr>
<tr>
<td>31</td>
<td>P: no (.) our umm answer [AP]</td>
</tr>
<tr>
<td>32</td>
<td>R: ((leans back and looks at the page)) yeah (5) okay so we know that’s going to be equal to (.) zero plus one equals [AP]</td>
</tr>
</tbody>
</table>

189
one (..) so

33 P ((gesturing to the page and tracing out a diagonal line))
this is going to be (.) you know what I mean [AP][AG]

34 3:50 R: yeah (.) so t:wo (.) that’s going to be z:ero↑ umm we can so thre::e choose three at the to↑p ((looks to Peters page)) at this top (..) and the zero (.) that equals (.) that (0.5) [AP]

34b 3:53 J: three (..) the↑re↑

34c R: >so first of all< now what we’re going to figure out is (..) what is::: what the numbers could be=

35 4:09 P: =oh that’s at the top [AP]

36 4:10 R: so it was it was like (.) what would the numbers be (..) to give you the answε↓er [AP]

37 4:14 P: yeah so::: = ((looking at his page, eyes narrowed)) [FG]

38 4:17 R: =so (..) it would be (.) so ((hand hovering over the page)) [HG]

39 4:21 P: but (.) how does anything times zero equal thre↓e [PI][FG]

40 4:25 R: "anything times zero equals three" well (.) this is supposed to be a thirty ((runs his finger along the page where the three is; Peter sits back in his chair and smiles)) it’s supposed to be the number thirty

41 4:39 P: ((leans over to Richard’s page)) "let me see" (2) you erase that ((tapping the page)) [AP][BG]

42 R: oh (..) zero three [AP]

43 4:42 P: yeah (.) I know but wait (..) yet but anything ((sigh)) (..) so >if we go by the lattice method< then a↑nything times three would equal nothin↑g ((he spreads his hands on nothing in a questioning manner)) anything times zero equals nothing righ↑t

44 4:59 R: I know (.) that’s why you put a zero up here ‘cos there’s no thousand number ((Peter raises his forearm with one finger extended, as if to make a point)) [AP][AG]

45 5:03 P: but .hh.hh ((leans forward)) so (..) [VG]

46 R: so what wo:uld >if you had< two things what would be (.) so:: (3) what could you times to get thre↓e

47 5:20 P: three ti::mes oh wo↑r (2) err something does equal th↑irty (2) its three (..) thirty so (..) it could be ten times three< [AP][AG]

48 5:38 R: ten times three [AP]

49 P: but (.) that’s <impossible> because it has to be (..) a one digit number [AP][FG]

50 5:45 R: yeah but that ((pointing to the page, possibly at the 1 on the bottom row of the lattice)) would be ten and there would be three ((Peter is holding up his hands and counting with his fingers)) [AP][AG]

Table 7-2 Transcript video #6 Richard, Peter, and Jill : 612

This section is marked by a nearly continuous passage of adjacent pairs in the turn taking between Richard and Peter. They adhere to Grice’s maxim regarding quantity, as
their utterances tend to be just long enough to make a meaningful contribution, while offering ample opportunity for the others to contribute. Other than two pairs of filler lines (lines 37 and 38 & lines 45 and 46), each utterance carries the talk forward. The passage is marked by the absence of overlapping utterances or completed sentences, suggesting that each student is finishing his thought and its response is being considered based on what was said rather than any preconceived ideas. The effect is like placing building blocks onto a base. As such, it differs from the more usual chaotic talk, where students interrupt and talk over each other as their ideas flow. In addition, there is little by the way of large gesturing in this section. This is in keeping with evidence from other clips that large gestures tend to be used by a student when (confidently) explaining an idea rather than forming or developing one. Gestures observed in this section tend to be confined to body posture and facial expression (lines 39 and 49). Vocal gestures, in the form of sighs (lines 43 and 45), are also used to indicate involvement in the thought process. This conversational organization seems to reflect Richard leading the conversation with his idea, which has been endorsed by Peter. They are struggling to build an understanding together, to be able to ‘go on’ with the solution as they each see it. Peter’s role in this is in trying to follow Richard’s idea, but in an engaged and challenging way. The organization of their talk suggests that they are working with the same idea in mind, based upon their constant deictic gestures to the page and their responses to each other’s turns at talk.

At this point, it seems that Richard and Peter are playing in the language game framed by the use of the lattice method in order to solve this problem. By this I mean that they are not trying to establish the process (playing with), or simply using the terms without meaning (playing at), but are actively engaged in the process. Up to line 50, the game has constrained their thinking to a particular process and its adaptation to their aims. While the idea is reasonable, they cannot develop sufficient understanding of the method in order to work it backwards.

Jill is silent throughout this section and does not participate in the conversation between Richard and Peter. It may be that she did not follow Richard’s reasoning when he described his proposed method (line 25), and is unable to ‘go on’ with his process when he is unable to himself. At this point, may seem that she is unable or unwilling to participate in the language game established by Peter and Richard. However, her quiet
contribution in line 34b is of significance. An important aspect of Conversation Analysis is the need to look forward and backward in a transcript in order to verify any suggestions, and it will be seen that Jill’s apparent passiveness is indicative of her being unable to follow the boys’ talk. Another indicator is her body language, for her posture indicates that she remains focused on what is being said, frequently changing position to see what is being written (see Table 7.3 line 44).

Other interesting points to note in the organization of the conversation include the extensive use of we throughout this section, and the verbalised self-talk and questioning that occurs. It is as though the students are sharing their thinking, even to the point where Peter states “you know what I mean” (line 33) and Richard agrees. Richard’s statement: “>so first of all< now what we’re going to figure out is (...) what is::: what the numbers could be” draws out ‘so first of all’ (line 34c), emphasizes ‘now’, and uses the inclusive ‘we’re’. Put together, the effect is of collaborative thinking as they try to work towards finding a solution.

Peter is troubled by the implication that multiplication by zero is giving the value of three. He makes the point as a statement rather than a question, as indicated by the falling tone at the end of his utterance (line 39). He also emphasises ‘anything’, suggesting that he can see no place for this breach of a known rule. Richard seems concerned and repeats the problem, but not silently, just softly (line 40). He seems willing to accommodate this clearly troublesome statement into his thinking, and makes a suggestion regarding place value for the three. Peter’s response is interesting in that he does not challenge the statement but instead leans back in his chair and smiles, but it is not a smile of humour, rather, I would interpret it as an indication that he does not accept Richard’s suggestion. He presses his point (line 43) and Richard tries again to respond, again using place value to suggest a possible way out. Peter’s vocal gestures and movement suggest that he is not convinced but is unable to press the point further and they move to consider possible products to give thirty, presumably in response to Richard’s suggestion (line 47). Peter is able to quickly see problems in that approach, emphasising ‘impossible’ by drawing it out (line 29).
As Richard goes through populating the lattice, Jill quietly questions the positioning of the three at the top. Her query goes unnoticed by Richard or Peter, but is significant in what is to follow.

“Well this is supposed to be the number thirty” Peter leans back and ‘smiles’ in a non-mirthful way at Richard’s suggestion that positioning the top three as a thirty will make any difference to his objection.

“let me see” Peter needs to close the space between himself and Peter so that they are working on the same diagram now. Their interaction has moved from an individual approach to a common one, both verbally and bodily. Peter is trying to follow Richard’s argument.

“I know, but wait” Peter makes a sagging motion with his shoulders; he is clearly troubled by the position of the three and the fact that it implies multiplication by zero to have a value. This may be the objection Jill was aware of, but did not vocalise, or she may just be wondering why Richard chose to use a three. Jill made no audible sound in support of Richard’s concerns.
“Anything times three equals nothing right?”. Peter presses his point. Jill’s body posture indicates that she is engaged in the conversation, despite her lack of contribution to it.

Table 7-3 Gestures per line - video #6 Richard, Peter, & Jill

At this point in the talking, Richard has a realization that they could be using division. It does not seem that he immediately thinks of using division to find the second factor of 612, given that they are working with 2 as the first factor; more likely he is recognising the way to get the missing number is through division. This breakthrough in the conversation allows them to ‘go on’ again and they make progress towards a second realisation that they can simply divide 612 by 2 to find the number they are looking for. Presumably, because they are still in the mindset of using the lattice method, they complete that process as a way to confirm their answer.

51 5:00 P: that would be three er six (1.5) nine (4.5) we could try to work bAckwa↑rds (1.5) (slaps the desk) divi↑sio↓n

52 5:16.5 R: ye↑ah ((points to Peter)) division (2) course (..) long division ((they all start some calculations)) six one two divided b::y⁰ [AP] [HG] [BG]

53 5:28.5 P: what’s it divide by though (( leans back in his chair, drops shoulders)) [BG]

54 5:29.4 R: I don’t know (2.5) [AP]

55 5:32.5 P: what is the diviso::r (2)

56 5:36.0 R: we need to know at least one number to do the multiplication (3.5) so if you had a number here and you times it (..) it would need to be (..) like maybe three (0.5) this could probably (..) like maybe (..) one (.) no that wouldn’t w:ork⁰ [CT]

57 5:55.1 P: =n::o ummmmm divided b:y ((holds up a hand and looks at [HG]
his fingers)) so in general let’s just try do it a (.) a few different wa:ys (.) what we have to figu

<p>| | | |</p>
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<tbody>
<tr>
<td>58</td>
<td>6:06.2</td>
<td>R: W→ait (.) I got it (.) we make an array with like with ci:rc→les and then we figure out what equals six hundred twelve [HG] [CT]</td>
</tr>
<tr>
<td>59</td>
<td>6:16.0</td>
<td>P: &gt;yeah but still&lt; (.) if it wants six hundred and twe:ve (.) ((points to page)) then that would be a lot of dots ((Jill nods)) [HG] [AP]</td>
</tr>
<tr>
<td>60</td>
<td>6:29.4</td>
<td>R: yeah yeah but what’s half of six hundred twe↑lve [AP]</td>
</tr>
<tr>
<td>61</td>
<td>6:33.5</td>
<td>P: &lt;six hundred and twelve&gt;, what’s half of th:at= [AP]</td>
</tr>
<tr>
<td>62</td>
<td>6:34.2</td>
<td>J: three hundred and six [AP]</td>
</tr>
<tr>
<td>63</td>
<td>6:36.8</td>
<td>P: three hundred and six [AP]</td>
</tr>
<tr>
<td>64</td>
<td>6:38.1</td>
<td>R: yeah (.) three hundred six (1) so if we did three hu:ndred and si:x ((there is a distraction as students walk by)) [AP]</td>
</tr>
<tr>
<td>65</td>
<td>6:55.1</td>
<td>P: so that times tw:o equals (4) six (.) twelve (3.5) so th:en (2) it could be this (1) so this might be (1.5) an answer because (1) three hundred and six times 2 equals (0.5) we’d like it to be exa:ct [AP][HG]</td>
</tr>
<tr>
<td>66</td>
<td>7:20.0</td>
<td>R: This equals six hundred twenty two right here ((he leans over to point at Jill’s page)) (...) wait you did three hundred six times two equals six hundred twelve (...) so was it six hundred twelve [HG]</td>
</tr>
<tr>
<td>67</td>
<td></td>
<td>P: how’s that= [FG]</td>
</tr>
<tr>
<td>68</td>
<td></td>
<td>R: =I don’t know how it’s possible [AP]</td>
</tr>
<tr>
<td>69</td>
<td>7:26.5</td>
<td>P: ((he leans over and explains how to complete the calculation to get 612, I do not reproduce the transcript for that here.))</td>
</tr>
<tr>
<td>70</td>
<td>7:48.2</td>
<td>R: so that might work ((he throws up his arms)) there we go (he smiles)) well you solved that pretty fast ((to Jill)(0.5) okay s:o then we have a possible one (1) what must the missing numbers be ((writes down)) we have that (.) now what else can we fi↑nd</td>
</tr>
<tr>
<td>71</td>
<td></td>
<td>J: We still haven’t done that one ((she nods towards Richard)) [BG]</td>
</tr>
<tr>
<td>72</td>
<td>7:55.1</td>
<td>P: yeah (.) anyways let’s just finish off this lattice method</td>
</tr>
</tbody>
</table>

Table 7-4 Transcript video #6 Richard, Peter, and Jill : 612

This section is again characterised by short utterances which build on each other until there is a stumbling point, and then build again around the difficulty. In this way, the conversation moves the problem forward. There is an interesting and changing balance between who is leading the conversation, resulting in a push–pull effect that allows them to make progress. They are developing an understanding of the process as the conversation progresses by playing in and with the language game of division. To support this notion, in line 55, where Peter vocalises what is the divisor, there is no intonation change in his utterance, making it more of a statement than a question; something he knows related to division. Since he has already asked the question ‘what’s it divided by though’ he is repeating himself but now using the correct terminology; he is
playing with the language. Richard’s response shows that he is still working with multiplication (line 56), but Peter pulls him back to the idea of division (line 57). Richard’s comments can be seen to be general statements rather than specific to the problem, as indicated by his use of the word ‘you’ (line 56), which does not refer to Peter, but to a general ‘other’. In this way it seems that Richard and Peter are now playing with the language as they seek to move the problem forward. Richard’s suggestion to use circles, returns to playing in the language and provides the spark for him to see division as partition (line 58) instead of the reverse of multiplication. Peter points out the impracticality of drawing 612 dots (line 59) and Jill, demonstrating her continued involvement in listening, nods in support. The actions of Richard are changing the thinking of Peter and of Peter’s actions in return. This not only has the effect of changing Richard’s own actions, but of producing new actions from Peter, which cause further changes in the actions of Richard. There is a recursiveness about this process of building a shared understanding of the problem.

As noted earlier, Jill’s silence should not be taken for a lack of participation, as it seems clear that she is engaged, if unwilling to contribute at this stage. This is supported by her swift answer to Richard’s query (line 62) and then again in calculating the reverse multiplication (line 70) – which interestingly they all try to do using the standard multiplication algorithm rather than using the half-formed lattice. This may be related to the earlier notion of the students playing with the language of the problem in initiating a solution; choosing to use a classroom driven strategy rather than a method they were perhaps more familiar with from grade 4 (i.e. multiplication by a single digit). When they have convinced themselves that the division was correct by checking with multiplication, they choose to close the circle of their thinking and complete the multiplication using the lattice they had started (see Table 7.5). Jill’s quick response to the calculation at the end may even suggest that she was already aware of 306 and 2 as factors.
The group eventually arrived at the following solution, but completed it as a check once Peter realised it could be more easily completed by division.

![Lattice method (b) - video #6 Richard, Peter, & Jill](image)

Peter then suggests that $306 \times 2$ might be an answer (line 65), meaning an answer to the original question. He does not get a chance to explain what he means by the phrase 'like it to be exact' because Peter realises his calculation is different. Richard has calculated $306 \times 2 = 622$ and looks to Jill for a second opinion. Peter patiently explains where he went wrong before acknowledging Jill’s speed of work (line 70). Again, there are few large gestures in this section. Peter uses his body to express his difficulties and leans over to work with Richard when explaining. Richard uses small gestures of his hand for the most part, only becoming more animated when the group makes a breakthrough (lines 52 and 58). This absence of large gestures may be a reflection of the continued building nature of the solution as opposed to any student trying to explain something conceptual to the others. Peter’s explanation to Richard is detail oriented and carried out with a series of small pointing gestures.

Peter marks the end of this developmental stage with the emphatic use of now (line 70) and the group is ready to look for further factors of 612.

52  Yeah! Richard becomes animated on the occurrence of when he feels there is a breakthrough; otherwise his gesturing is confined to small hand movements and he tends to focus on his paper. Other than one motion to point to Jill’s calculation (line 69), he points to his own paper when gesturing. Over the extent of this research, this type of minimal gesturing seems to occur mostly when a student is following a process.
Peter's gestures occur when he is describing a concern or when he is doing a calculation. His focus often seems to be directed inwards or towards Richard. When he points to a paper, it is Richard’s. Over the extent of this research, this type of gesturing seems to indicate a student thinking about something with being overly confident about it. When the confidence grows, so does the physical size of the gesture.

Jill’s quietness appears to be more shyness than a lack of confidence. Her body posture often stiffens up when Richard says something unclear or pauses uncertainly.

Table 7-6 Gestures per line - video #6 Richard, Peter, & Jill

The group has successfully reached a point from which they can ‘go on’. They appear to now have a mutual understanding of how to get a pair of factors, and may have developed some connections between multiplication and division in this process.

I will return to this group when discussing the final stage of problem solving in a later section. Before doing so, however, I will examine the developmental stage of episode 5, the ‘Don’t fence me in’ problem with Aaron, Bashir, and Chan. Recall that, in contrast to the collaborative working of Richard, Peter, and Jill, this group were exhibiting some problems in working together in the introductory stage and were unable to establish a conversation between all members of the group. The question is then how this affects their development of a possible solution.

**Episode 5b. Aaron, Bashir, and Chan: Don’t fence me in**

‘Don’t fence me in’ (episode 5) was a problem about changing the area of an enclosure by adding 5m to its length. In the introductory stage the students negotiated the meanings in the question in order to make some decisions about the problem. In contrast to episode six, the students followed more individual processes and lacked any conversational flow. At the end of the introductory stage, there was a consensus about which side had increased, but the group overlooked Aaron’s erroneous claim that the same area would arise if the other side (the 10m side) was increased instead. The following extract, which follows from episode 5, is interesting because it illustrates one student dominating the talk, reducing it to a discussion rather than a conversation. Any
sense of mutual understanding is questionable and this is highlighted by Bashir’s struggle to follow Aaron, as well as Chan’s comments about Aaron’s individual approach (line 39).

| 1  | 14.0 | C: S:O↑ (0.8) |
| 2  | 15.2 | A: Okay so the first multiply ten (1) times ten twenty-five because that |
| 3  | B: We have to do um the twenty (.) ((head nod to side)) Plus five |
| 4  | A: yeah (.) twenty plus five that’s what I did (2) |
| 5  | 25.8 | C: so ten plus five equals fifteen:en |
| 6  | A: >wait no not ten plus five<= |
| 7  | C: =>Twenty plus< five= |
| 8  | 30.5 | A: =it is alright it doesn’t matter |
| 9  | C: .hh .hh (brief look up to Aaron)) (1) that’s it (.) ((shake of the wrist)) <i like doing twenty-five side> (2.5) |
| 10 | 35.5 | A: Equals twenty-fi::ve so and then |
| 11 | 39.0 | C: Both sides twenty-five no↑w (1.5) wait (.) |
| 11b| A: and then u::m |
| 11c| C: am I thinking correctly= |
| 12 | 45.9 | A: =and then you u:m um ten multiplied ten (1.5) times twenty-five (..) you just umm clo umm just take off the:: >zero first< (0.5) then just and just do one times |
| 12b| 59.0 | C: yes take off |
| 13 | A: twenty-five and add the zero to the end to the end of that |
| 13b| C: two fifty |
| 13c| A: minus u::m (1.8) two-fifty er minus the original one which is ten times (.) twenty (1) two-hundred and then |
| 14 | 1:13 | C: wait wait wait (.) <slow down> (..) <slow down aaron> slo::w d:o:wn (4) why are we qui<et↑ (0.5) |

Table 7-7 Transcript video #5b Aaron, Bashir, and Chan; Don’t fence me in

Aaron’s comments are a continuous outpouring of his thoughts, in the manner of egocentric speech, barely slowed by Chan’s overlaps. These overlaps are separate thoughts rather than complimentary to Aaron’s utterances. Chan’s facial expression (line 14) suggests he has lost track of Aaron’s ideas and he finally reaches across the space between them to break the flow of Aaron’s words. He uses his hands like an inverted container as if to quell the outpouring of words (see Table 7.10). Aaron does not speak quickly, but he continues with head down and with minimal interaction while Bashir continues to look down at his own sheet. Aaron’s response is to stop talking rather than
to deal with Chan’s interrupt; when Chan queries his silence, Aaron simply continues as before.

<table>
<thead>
<tr>
<th>Time</th>
<th>A:</th>
<th>C:</th>
<th>16b A:</th>
<th>16c C:</th>
</tr>
</thead>
<tbody>
<tr>
<td>15:25.5</td>
<td>&quot;so er twenty times ten equals two hundred and fifty times er two hundred and fifty minus (head shake) two hundred equals fifty so that’s how many it will have&quot;</td>
<td>&quot;sixty is the (..) (slight shake of the head)&quot;</td>
<td>&quot;so that’s how many increase in the area (.)&quot;</td>
<td>&quot;like (..) from that’s (moves his hand in a circle over the page)&quot;</td>
</tr>
<tr>
<td>138.2</td>
<td>&quot;that’s what it’s asking ((A leans back; B looks at A’s paper))&quot;</td>
<td>&quot;S::o that’s a (referring to part a)&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:49.6</td>
<td>&quot;so that’s just fifty for (1) do you get that ((looks to Bashir))&quot;</td>
<td>&quot;[I]&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:54.6</td>
<td>&quot;do that ((looks to A smiles and shakes his head))&quot;</td>
<td>&quot;[HG]&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:56.8</td>
<td>&quot;check’ (1) aaron AAron you have to ch:eck (1) ((Aaron leans over to look at Bashir’s paper)) be:ca::use&quot;</td>
<td>&quot;[TC]&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2:03.0</td>
<td>&quot;fine &lt;(.) fifty plus two hundred equals two hundred and fifty&quot;</td>
<td>&quot;That works (3) hh ((smiles and turns away))&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2:06.8</td>
<td>&quot;That works (3) hh ((smiles and turns away))&quot;</td>
<td>&quot;[BG] [FG]&quot;</td>
<td></td>
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Table 7-8 Transcript video #5b Aaron, Bashir, and Chan; Don’t fence me in

When Aaron is done, he leans back in his chair as a signal. With Aaron having cleared the space above his paper, Bashir seems to feel comfortable looking across at it. Perhaps aware of the gesture, Aaron cuts off Chan and asks Bashir if he ‘got that’ (line 19). Bashir looks at him and smiles. The humour may be being used to indicate a general difficulty in following Aaron. He shakes his head to indicate his confusion. It is not clear if Chan has managed to follow Aaron, despite his flat statement of acknowledgement in line 18, but his use of the teacher comment (line 21) requiring a check suggests that he did not. This does not imply that Chan could not calculate the answer himself, but probably that he was unable to follow Aaron. When Aaron supplies a brief response to the check, Chan raises his eyebrows, turns away, and laughs briefly. This gesture seems to imply that Aaron has dismissed his concern rather than answered it (lines 22-23). It would seem that Chan was looking for a clearer explanation of what Aaron was doing and his response may have an air of sarcasm (line 23), particularly when combined with his body gesture. It could also be interpreted as being surprised at Aaron’s quick
response, or using laughter as a face saving measure (see Table 7.10). In any of these interpretations, it seems clear that the students are not working together.

Aaron tries to point out to Bashir what he needs to do by leaning over to Bashir’s paper. This action may be contrasted with Richard’s gesture in episode 5b, in which Richard moves over to Jill in order to find the correct answer. Bashir’s role in this case is more passive because he is not engaged in the talk. In Richard’s conversational talk, he was empowered to seek Jill’s help. In Aaron’s discussion, in which ideas are being transmitted rather than worked on collaboratively, there is no connection being made for Bashir. Even when Bashir tries to add his thoughts, Aaron ignores them and continues (lines 25 and 32). Aaron also speaks over Chan’s overlays without recognition (line 28). Chan’s use of the word ‘guess’ in line 31 indicates that he remains unconvinced by Aaron’s help for Bashir, and perhaps even for himself.

Chan interrupts Aaron more forcefully (line 34). Perhaps sensing that Aaron is still not helping, Chan interrupts Aaron’s gesture space over Bashir’s work by placing his hand into the gap between Aaron and Bashir, and suggests that Bashir “just write the answer” (line 35). Bashir tries again to ask for help (line 37), but Aaron continues to simply ‘talk’. Finally, Chan stops the talk around part a), asking more pointedly why Aaron is working

<table>
<thead>
<tr>
<th>Line</th>
<th>Time</th>
<th>User</th>
<th>Action/Comment</th>
<th>Markers</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>2:10.0</td>
<td>A</td>
<td>err you have to do ten times twenty first ((to Bashir))</td>
<td>[HG]</td>
</tr>
<tr>
<td>25</td>
<td>2:13.4</td>
<td>B</td>
<td>“then I (. ) then just”</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>2:15.0</td>
<td>A</td>
<td>=ten times twenty i::s</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>2:16.2</td>
<td>C</td>
<td>two hundred=</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>2:17.1</td>
<td>A</td>
<td>=two hundred and then two hundred plus fifty equals</td>
<td>[O]</td>
</tr>
<tr>
<td>29</td>
<td></td>
<td>C</td>
<td>“two hundred (. )” NO</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>2:19.6</td>
<td>A</td>
<td>two hundred umm fifty minus two hundred “equals fifty”</td>
<td>[B]</td>
</tr>
<tr>
<td>31</td>
<td>2:23.8</td>
<td>C</td>
<td>I guess that works</td>
<td>[BG]</td>
</tr>
<tr>
<td>32</td>
<td>2:22.2</td>
<td>B</td>
<td>You missed=</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>2:26.6</td>
<td>A</td>
<td>=No this one’s right ((points to Bashir’s work)) but you just have to (. ) so this one’s two hundred (. ) right↑ (2) then you subract (2) yeah you can do it this way too two hundred (2)</td>
<td>[HG]</td>
</tr>
<tr>
<td>34</td>
<td>2:36.6</td>
<td>C</td>
<td>whAtT</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>2:43.2</td>
<td>C</td>
<td>JUst write the answer to the question . hh . hh</td>
<td>[HG]</td>
</tr>
<tr>
<td>36</td>
<td>2:45.7</td>
<td>A</td>
<td>so it depends if you reduce by fifty the area</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>2:48.5</td>
<td>B</td>
<td>“why’s it two hundred fift↑ y”</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>2:49.3</td>
<td>A</td>
<td>okay ( . ) that’s when you then</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>2:51.2</td>
<td>C</td>
<td>aaron (0.5) aaron (. ) why are you doing it all by yourself now . hh it’s like its copying . hh . hh</td>
<td>[VG]</td>
</tr>
</tbody>
</table>

Table 7-9 Transcript video #5b Aaron, Bashir, and Chan; Don’t fence me in

Chan interrupts Aaron more forcefully (line 34). Perhaps sensing that Aaron is still not helping, Chan interrupts Aaron’s gesture space over Bashir’s work by placing his hand into the gap between Aaron and Bashir, and suggests that Bashir “just write the answer” (line 35). Bashir tries again to ask for help (line 37), but Aaron continues to simply ‘talk’. Finally, Chan stops the talk around part a), asking more pointedly why Aaron is working
by himself. As Chan says, what Bashir is doing is no more than copying. Interestingly, this comment is made without a physical gesture. Chan has shown that he has been quite demonstrative throughout the session (see Table 7.10), so the lack of physical gesture here may be significant, perhaps dismissive or disengaging. Chan does give a slight laugh, which may be interpreted as a vocal gesture intended as redressive face action; allowing him to make his point without affronting Aaron.

| 14 | Chan gestures to Aaron to slow down by cupping his hands over Aaron's work space. |
|  |
| Bashir looks at Aaron's work as Aaron describes the answer. Aaron makes no gestures, and his voice is flat. When Aaron has finished he asks Bashir if he 'got that'. Now Bashir looks him in the eye, and shakes his head. He is smiling (line 20). |
| Chan uses a flat hand in a cutting gesture to interrupt Aaron’s explanation (line 15). Chan’s response is to Aaron’s explanation of his own work. His gesture is an emblem typically used to implore explanation (line 21). When Aaron continues the check for his work, Chan turns away and laughs at its brevity (line 23). |
Aaron leans across to help Bashir by pointing to Bashir’s work (line 33). Contrast this with episode 6b in which Richard reaches over to Jill for her help.

Chan interrupts this working space but only to tell him to “just write the answer” (line 35).

Table 7-10 Gestures - video #5b Aaron, Bashir, and Chan; Don’t fence me in

This episode shows how a group can reach an answer non-conversationally by allowing one student to dominate the process. Aaron’s own misconceptions about area (further reinforced in line 8) remain unaddressed, and it is doubtful at this point if Bashir has participated in any developed mutual understanding. Chan’s own understanding is still questionable; it is unlikely that he has followed Aaron’s thinking, but he may have reached some form of understanding by himself, since he seems to accept the answer of fifty reached by Aaron. Aaron, to the exclusion of the rest of his group, seems to be playing in his own language game. At this point Aaron moved the group on to part b) of the question without reacting to Chan’s ‘individual’ comment. I shall return to look at their progress later in dealing with the third ‘extension’ stages of the videos. In the extension stage, I will be looking for more evidence to support the findings suggested in the introductory and developmental stages of the problem.

The next episode, however, is another example of students working in the developmental stage. In contrast to the previous example there is extensive use of physical gesturing as the students discuss and develop their solution to the problem. The episode also illustrates how confident students take their turn by overlapping and interrupting their interlocutors, but in an acceptable and inclusive way.

7. Sally, Maggie, and Chris and the Ferryboat problem.

The Ferryboat problem is a common problem involving moving objects across a river in a particular order or taking certain restrictions into account. This was given as a ‘problem of the week’ by the classroom teacher and selected with the purpose of stimulating students to think backwards from a total while examining several possibilities. The exercise was chosen to support work on remainders in division as well as to enhance the link between
multiplication and division. The problem is taken from the University of Waterloo’s Centre for Education in Mathematics and Computing website.

Problem

*A trip across the river by the Pictou Ferryboat is full if there are either 10 cars on board, or 6 trucks (it never carries cars and trucks at the same time). One day, the ferryboat made five trips, and was full on each trip. If it carried a total of 42 vehicles across the river, how many cars were transported?*

In this problem the students need to recognize that the total number of vehicles must be the sum of a multiple of 10 and a multiple of 6. It might be expected that the students would examine the remainders after dividing 42 by multiples of 10 (or 6) and determine which of those was a multiple of 6 (or 10). Students should also be aware of the need to use whole numbers while being careful to satisfy all of the stated conditions in the question. A systematic approach to determining the possible outcomes would probably be a desirable outcome for most teachers, and is detailed in the solution provided with the problem.

Prior to this episode, the students initiated the problem together with Sally and Maggie sharing turns to read it aloud. The students were asked not to write anything down until they thought they had both a strategy and possible solution to work with. In keeping with the common practice noted in Chapter 6, there was a pause after the reader finished the question. The ensuing conversation led to a short introductory stage in which they clarified the setting of the problem and their initial interpretation of it. All three students participated in an exchange of ideas as to what was being asked. The following transcription (Table 7.11) picks up after this stage with Maggie’s response to Chris’s question:
Table 7-11 Transcript video #7 Sally, Maggie, and Chris: The Ferryboat problem

Table 7-12 Gestures - video #7 Sally, Maggie, and Chris: The Ferryboat problem

Maggie is apparently sorting through the problem as she speaks (line 2) and it is interesting that she describes herself as doing so ‘in my head’ at the same time as she is verbalising her thoughts to the group. This allows her to put forward her ideas in response to Chris’s question even though she has not yet fully formed them. This would suggest that she is confident in her method even before she has an exact answer. To support this idea she uses extensive and expansive body gestures from a standing position (Table 7.12). These gestures move towards pantomime in the sense that she seems to be miming out the action of the ferryboat more than referring to the values with specific hand gestures or beats. Supporting this is her vocal emphasis on the word ‘time’ rather than the values per trip. This suggests that she is not thinking about the values themselves, but rather as trips they represent – how many times six can be subtracted from forty-two in order to leave an answer she can ‘go on’ with. Line 2 is also delivered with a slight staccato style, similar to the way Richard spoke in episode 6 when it seemed he was thinking as he was talking. A similar effect of broken speech can also be seen when a student struggles to explain something, particularly if pressed, but in those cases there is often very little gesture accompanied with the speech, either bodily or vocally. In these cases, gesturing may be an indication of an active inner thinking. I suggest that Maggie is building an understanding of the problem through an externalised inner-
conversation; she is thinking as she is communicating. This can be compared with Aaron’s vocalising in the previous episode which is in question as an attempt to communicate. Maggie is dominating the group’s space physically as she plays in the language game framed by the terminology of multiplication and division.

Chris indicates that he is attentive to Maggie by quickly picking up on the limitation of her stated process (line 3). He presses her on her point of subtracting six, not because he doesn’t see why it is six that she subtracts, but why she only does it once. He uses ‘just once’ to make this clear.

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<tbody>
<tr>
<td>3</td>
<td>1:44.3</td>
<td>C: But that’s just one [AP] [P]</td>
</tr>
<tr>
<td>4</td>
<td>1:45.6</td>
<td>M: Yeah but then I still have &lt; like um (.) and then I realised that I have to put thirty there because there’s six there (.) so then thirty cars and twelve trucks [AP] [AG] [PR]</td>
</tr>
<tr>
<td>5</td>
<td>1:54.0</td>
<td>C: Oh yeah (0.5) there’s cars on board right you mean you could keep subtracting the (1) [AP] [O] [PG]</td>
</tr>
<tr>
<td>6</td>
<td>1:00.2</td>
<td>M: Okay because what I was thinking is you could do three so it would have (.) one (.) two (.) three right= [AP] [AG]</td>
</tr>
</tbody>
</table>

Table 7-13 Transcript video #7 Sally, Maggie, and Chris: The Ferryboat problem

In response, Maggie emphasizes ‘yeah’ and draws it out in recognising Chris’s concern, then speeds up her next words to indicate she has a response to his press. Here, (line 4) she begins in the first person, ‘I still have’, but then switches to the past tense in using ‘realised’. In line 2, Maggie used present tense combined with the general form of ‘you’ to begin her description, switching to first person after explaining that she was thinking as she was speaking. This suggests that her ‘outward thinking process’ has ended and that she has been able to draw her ideas together. By being able to ‘go on’ in this way, she has developed an understanding of the problem which she is now sharing. Again, she gestures broadly but with gestures which depict the action of moving the cars rather than the arithmetic of doing so. She uses a container gesture to suggest that she is considering groups of vehicles (see Table 7.14). To support this viewpoint, she makes no indication of how she reached the answer of thirty, but emphasises it as a result simply because there is six there. Chris is left to infer how Maggie reached her answer and his overlap (line 5) suggests that he has had some breakthrough in realising the role of the cars. His emphasis on ‘board’ may be an indication that he is starting to develop an
understanding of what Maggie is saying in terms of blocks of cars and trucks in motion. He sees the need to subtract (emphasising and drawing out the word), but then stalls. He is unable to ‘go on’ with the flow of his thinking at this point, which may just be a temporary thing as he may have switched to a more economical internal thinking, but this indicates he has not quite developed the same understanding as Maggie yet. He is using a more general ‘you’, which may in fact be particular to Maggie at this point. His gesturing is restricted to pointing at the question, but in a beat manner which may again be reflective of his thinking.

Maggie takes the opportunity to further clarify her understanding by ‘going on’ to explain where the thirty came from (line 6). She uses a marker ‘okay’ following Chris’s pause. There is an element of the manner maxim here (orderliness), in that she feels she has given Chris enough time but now wants to add more. There is also a brief pause after her marker and this may be a redressive face action to give Chris an opportunity to continue. Again, Maggie speaks in terms of trips rather than calculations. She only implies that she is now working with trips of ten cars and her emphasis, again using bold gestures to enact the trips, is on how she reached thirty by adding units of ten rather than by dividing thirty by ten.

| Line 4: Maggie uses a container gesture when she speaks of moving vehicles, suggesting that she is thinking in terms of groupings. | Line 5: Chris uses his pen to point to the question but a beating gesture when saying subtracting. |

Table 7-14 Gestures - video #7 Sally, Maggie, and Chris: The Ferryboat problem

Maggie’s drawn out use of the term ‘right’ is an offer to Chris to take a turn back. She has added more of her thinking and adheres to the quantity maxim that it be provided in a manageable chunk. Chris takes his turn and demonstrates that he is now able to ‘go on’ further (line 7).
Out of context, line 7 would appear to make very little sense. Chris’s statements are seemingly disjointed as he shifts from a reference to twelve, presumably meaning two ferries of six trucks, and the need to leave zero cars behind – perhaps a reference to having a whole number of vehicles transported. It would seem that he also is thinking as he is talking, revealing disjointed thoughts as he sifts through the information. In this research, short bursts of speech separated by pauses appear to be associated with utterances that help develop the solution, while drawn out words are often seen in connection to less helpful utterances. Chris reaches out for a pen as he picks up on subtracting from 42, but Sally stops him. Sally’s first utterance in this stage is in playing the role of the teacher, telling Chris that he cannot write yet (line 8). Her emphasis is heavily on ‘write’, as the teacher had informed them that they were only to talk. What Sally does, however, is close the working space between herself and Chris and focus on what he is pointing to. In the background, Maggie is gesturing as Chris speaks (Table 7.14), which may indicate that she is still thinking about the problem. Chris himself fills in Sally’s inferred demand and goes back to verbalising his thinking (line 9), again using broken phrases, but gradually working towards a clearer statement of what he meant by the ‘twelve’ in line 7. When Chris pauses, his turn has taken a typical turn length and

<table>
<thead>
<tr>
<th>Transcription</th>
<th>Time (s)</th>
<th>Speaker</th>
<th>Utterance</th>
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</thead>
<tbody>
<tr>
<td>7 C: =what you so that's just twelve that'll leave it like (..) ze:ro cars left over rig↑ht (0.8) &gt;you know what I mean&lt; (.) zero cars in the vehicles no vehicles left over so (..) yeah so yo↑u (.) so you can put forty-two minus</td>
<td>1:06.4</td>
<td>C:</td>
<td>[AP] [HG]</td>
</tr>
<tr>
<td>8 S: &gt;No you can't&lt; write yet=</td>
<td>1:18.9</td>
<td>S:</td>
<td>[I] [TC]</td>
</tr>
<tr>
<td>9 C: =oh yeah just discuss it so (.) like (.) say (2) okay so you wanna make it whole nu↓mbers (..) the number is forty-two vehic↑les ri↑ght (.) and then (.) let the tru↑cks go back and forth twi↑ce (. ) like get the tru:cks twice across the river so that's twelve (.) tru↑cks (..)</td>
<td>1:20.7</td>
<td>C:</td>
<td>[BG] [HG]</td>
</tr>
<tr>
<td>10 M: and then you can do cars</td>
<td>1:41.1</td>
<td>M:</td>
<td>[AP]</td>
</tr>
<tr>
<td>11 C: and then that'll be thi↑rty &gt;and then three&lt; times the ca↓rs (0.9)</td>
<td>1:42.2</td>
<td>C:</td>
<td>[AP]</td>
</tr>
<tr>
<td>12 M: ye↑ah (1.5)</td>
<td>1:46.1</td>
<td>M:</td>
<td></td>
</tr>
<tr>
<td>13 C: oke[y] (2)</td>
<td>1:49.2</td>
<td>C:</td>
<td></td>
</tr>
<tr>
<td>14 M: oh (.) n↓o(0.5) &gt;I was just&lt;thinking about like if you know that six time six equals thirty-six then if you added ten the↑n you would have forty six and not forty-t↓wo (..) so that wouldn’t work (1) sally can you explain it</td>
<td>1:52.3</td>
<td>M:</td>
<td>[BG] [TC] [Q]</td>
</tr>
</tbody>
</table>

Table 7-15 Transcript video #7 Sally, Maggie, and Chris: The Ferryboat problem
Maggie takes the opportunity to add to his thinking (line 10), but she is interrupted before she can finish as Chris has determined the remainder of vehicles after the twelve trucks have been accounted for. He has worked through to the answer Maggie offered earlier on his own terms. It seems that he has been able to develop a mutual understanding with Maggie over the proffered answer.

The exchange to this point has been a stream of adjacency pairs which, even though Maggie appears to have determined the answer without help, has led to Chris also being able to ‘go on’ and complete the calculation. It should be pointed out that at this stage, while there is evidence to suggest that he now understands what Maggie did to get her answer, there is no evidence that he has an understanding of the intended concept of the problem. It does seem that he is able to play in the language of the problem, however, and has been actively engaged in working through the solution. The phrasing Chris is using is interesting in that he seems to be speaking in instructions to a general audience; saying let the trucks go back and forth in the manner a teacher might explain a problem. It is almost as if he is thinking it through in the teacher’s voice. This may be indicative of trying to follow a process. Maggie acknowledges his efforts with a rising intonation in ‘yeah’, and Chris responds with a quiet ‘okay’, a suggestion of satisfaction in completing this part.

There is a brief pause, but Maggie indicates that she has been thinking through this pause by revealing that she was considering other options for a solution. She begins the utterance with a shrug and an arm gesture which includes rolling her hand outwards. This gesture may be viewed as redressive in nature, an early indicator that what she is about to say does not work out. What line 14 does, however, is to establish that she is now considering doing the problem from the other direction. She is taking a multiple of six as opposed to starting from forty-two and subtracting groups of six. She is then trying to add 10 in order to reach forty-two. Because she determines this answer to be forty-six, it seems clear that she is avoiding subtraction. She recognises that this is incorrect and then surprisingly changes direction in the conversation by asking Sally if she could explain the problem. This may be an attempt to buy time in order to keep thinking about the problem, but she may also wish to make sure that Sally can represent the group if she is called upon in the plenary session, something the teacher stresses all group members should be able to do. The shift to question Sally does afford an opportunity to
get a sense of what Sally has picked up from the conversation from which she has, in this stage, excluded herself.

<table>
<thead>
<tr>
<th>Line 7: Chris makes a sweeping gesture on saying zero cars.</th>
<th>Lines 8-9 Sally leans in to where Chris is pointing and Maggie gestures in the background.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 9: Go back and forth. Chris moves his hands from side to side.</td>
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<tr>
<td>Line 10: Then you can do cars. Maggie gestures a movement from side to side.</td>
<td></td>
</tr>
<tr>
<td>Line 14: Maggie begins her description of her alternative idea with a shrug, rolling her wrist outwards.</td>
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Table 7-16 Gestures - video #7 Sally, Maggie, and Chris: The Ferryboat problem
Sally begins by affirming that she could before standing up and looking ahead rather than towards either Chris or Maggie. She begins with a louder marker ‘so’ and speaks as though presenting (line 15). Her first utterance is a restatement of the question with drawn out emphasis on the main points, stating ‘vehicles’, ‘trucks’, and time in the fashion of highlighting a page. Interestingly, she puts particular emphasis on the word ‘it’, a single word seemingly used to represent the movements of the ferry. The lack of any pauses in this first section again makes it seem like the start of a presentation, but as she starts to describe the process, pauses appear. The sentence structure may represent her memory of what Chris and Maggie have said and she pauses long enough after describing the trucks crossing that Maggie feels the need to help out (line 16). Chris now interrupts Maggie (line 17), apparently taking on the teacher role for himself in pointing out that they all “need to be able to explain it”. In this, he may be responding to Sally’s comment in line 8, preventing him from writing. Sally is able to interrupt herself and complete the calculation. There is no attempt to explain the thinking behind this, however, so it is not clear if she is remembering the answer or if she has developed a mutual understanding of the problem with her peers. Her gesturing, however, does support the idea that she is following the movement of the vehicles as she speaks (Table 7.18), while her earlier posture while Chris and Maggie were speaking indicates that she was engaged with what they were saying.
Sally stands when asked to explain. She begins by pointing to her notes then looks ahead when recalling the solution. She speaks as though presenting a remembered speech, but her gesturing is high and confident and matches the movement of the ferry.

**Table 7-18 Gestures - video #7 Sally, Maggie, and Chris: The Ferryboat problem**

For Sally, having an understanding of Maggie’s and Chris’s solution may be quite different from understanding the problem. It may be that Sally is merely playing at the language game in this section; using the terms without understanding. In order to ascertain this, further evidence is necessary. This evidence may be found in the third stage of a problem, the extension stage, and I will move onto this next chapter. In this third stage, I will be examining evidence to demonstrate any mutual understandings a group has developed, enabling them to ‘go on’ further. I am aware that any suggestions made in this regard cannot claim to indicate what a group, or a student within that group, actually knows. There is also a danger of assuming a ‘deficit model’, by implying that if a student cannot progress further then that would mean they do not share in the mutual understanding of the group, or that the group did not build a mutual understanding. At best, I claim that if a group, or group member, can ‘go on’ further then this is evidence to support the idea that there is an understanding present which they can build on. Another pitfall to be avoided in this stage, I feel, is what Davis (1996) refers to as ‘evaluative listening’, wherein a particular solution is sought. I tried to make a conscious effort to be open to listening for a solution/process rather than the solution/process as I examined the episodes in detail.
Summary so far – the Development Stage

The examples chosen to represent the development stage focussed on illustrating how a good conversational space established in the introduction stage serves as a foundation for successful development. While these are but three examples selected, they represent the typical observation found. It was apparent that progress in the task and a sense of a developed and mutual understanding, generally accompanies the type of talk I have identified as conversational. Conversational talk is characterised by adjacent pairs, overlaps, and/or the completion of each other’s utterances in a mutually supportive way. It is also clear that gesturing should be seen as an integral part of conversation. Groups tend to flounder in their progress when there is a lack of conversational talk. While the conversation may be a feature of cooperative talk which is a by-product of understanding, my sense is that it is a crucial part of the development of understanding by helping to provide an organizational platform through which a mutual growth occurs. In cases such as episode 5, that of Aaron, Bashir, and Chan as followed above, the disconnect of the students presents a barrier to the development; by not being able to communicate well, the students seem to be unable to make collective progress. The contrasting episode of Richard, Peter and Jill shows how a group of students can help each other to be successful and how their talk is conversational when progress is being made.

The significance of the development stage is that it illustrates how the conversation can move from general talk about the problem to also incorporate the mathematics. In this way, the students are developing understanding as they are able to ‘go on’ with the problem conversationally and move towards a solution. Whereas the episodes in Chapter 6 were about the problem, the episodes in this chapter relate to the development of the mathematics. In both cases, progress and conversation go together. The indications from these results are that the classroom teacher has a tool, in terms of observing the interaction between the students, to help monitor the group’s developing understanding. There is evidence that not only does conversational organization, but also gesture and posture indicate that the students are making collective progress in developing an understanding of the problem, and I will give more specific examples of this in Chapter 9. This was also seen in the previous chapter, but the results discussed in this chapter suggest that these indicators can be used to highlight progress in the actual mathematics.
the students are producing. This is, I consider, an important development from results discussed in the previous chapter.

In the next chapter, I examine how students interact as they are able to complete what they consider to be a solution to the problem they have been given. I am looking to see if the students in a group demonstrate a common understanding of the mathematics they have been asked to work on. Once again I pose the question of whether a classroom teacher can be informed about the students’ progress and understanding by observing how the students interact conversationally and through gesture and posture. In this case my intention is to examine how the students conclude their talk in the sense of how they feel they have addressed to original problem.
Chapter 8

Results and Analysis: Extension

In the previous two chapters, I have reported on how important establishing a conversational space seems to be in developing a common understanding of the problem in the introductory stage, and in making mathematical progress within the problem. My intent in this chapter is to pursue this idea to determine if an established conversation space allows the students to ‘go on’ to develop the mathematics further.

The Extension Stage

I identify the extension stage of the problem as occurring after students feel that they have reached a solution to the problem assigned. At this point they may choose to work on an extension question, be pressed by a teacher to explain their work, or are asked to present their findings to the class. At this stage, I am less concerned with the organization of any conversation that is happening, except when this illustrates a point, but focus more on where the conversation has led the students. I will therefore not reproduce detailed transcripts, which can be lengthy, but simplified versions of sections of the talk.

I begin by continuing the episode discussed at the end of the previous section in order to support the suggestions made following that more detailed analysis. Recapping, there was strong evidence of a conversation between Maggie and Chris leading to evidence of the development of a mutual understanding between them. The status of Sally’s understanding was less clear; while she was able to recite the solution obtained by Chris and Maggie, no evidence was available in that section to support or deny a conclusion that she had reached an understanding of the process behind the problem. This is an example of an episode I feel is worth following further in order to provide supporting
evidence for what has been suggested in Chapter 7. This is in accordance with an important aspect of Conversation Analysis, that unfolding events be used to provide support for any observations based on earlier utterances. If the researcher suggests a meaning for an utterance, there should be evidence to support this later in the session.

7b. Sally, Maggie, and Chris and the Ferryboat problem.

Following Maggie’s suggestion of a possible alternative solution, as outlined in Chapter 7, Chris quickly suggested that another possible answer could be that, because $6 \times 7 = 42$, there could be seven trips carrying only trucks. Maggie at first agreed to this but then recognised that the question specified only 5 trips were made, disqualifying Chris’s answer. This proposal may also relate back to Chris’s comment in line 7 (Table 7.16) in which he made a reference to zero cars, which on first reading seemed to refer to the use of whole numbers. Any such interpretation must be made with caution and, as a researcher, I look for evidence of meaning in a temporally flexible recording; the interlocutors, in contrast, must make their interpretations in real time. The broken speech of a student thinking through a problem can be confusing as the statements may be fragments of expressed thoughts, especially if the student is thinking ahead of his or her speech – which is to suggest that the student’s self-talk is occurring while he or she is trying to externally communicate. For a conversation to progress in this manner, the interlocutors must be making several inferences based on their own partial understanding at that point in the conversation. How much of an utterance is used to help build understanding is unknowable, but the need to process and incorporate these inferences may be why a maxim of quantity is subconsciously adhered to in a conversation.

Following Chris’s suggestion, the group went quiet and a little distracted. The classroom teacher moved over to question the students:

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<tr>
<td>1</td>
<td>T:</td>
<td>so (.) do you think you have your answer?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>C:</td>
<td>&gt;you would have&lt; 3 groups of 10 cars (.) and umm 2 groups (.) of 6 trucks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>T:</td>
<td>so is there more than one way to do it? What are the possible combinations?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>M:</td>
<td>well there’s other ways to do it (.) cuz we (.) that’s what we’re trying to figure out right now</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>C:</td>
<td>you could bring over twelve trucks first (.) and then thirty cars (..) oh yeah that’s no different at all (2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 8-1 Transcript video #7 Sally, Maggie, and Chris: The Ferryboat problem

Chris’s use of ‘groups’ instead of ‘trips’ (line 2) suggests a connection to the language of multiplication used in the classroom, one of the goals of the question. The teacher also has another purpose in mind - he wants the students to consider more than one possibility and not stop at their first ‘answer’. His questioning is therefore directed towards this goal. Maggie responds that there are other ways to do it (line 4) but places a heavy emphasis on ‘ways’. Taken with line 7, in which she observes that the question is ‘specified so much’, there is a suggestion that she recognises that there can be no other solution. Her rising intonation on ‘specified’ points to this as being a key part of the question and its solution. It is this specification that takes away an open-ended response to the problem, and she appears to have recognised that. Pressing further, the teacher asked them how they know that is the only answer (line 8). Of interest first in Chris’s reply (line 10) is the addition of the word ‘combinations’ to the language used by the students; prior to the teacher’s use of this word it had not been part of the language game of the students in their own talk. The teacher, by entering the talk, has changed the game. A second important point is Chris’s statement that you know “in your mind” that the answer is correct. Where the teacher may be pushing for a systematic description of the possibilities, it seems that Chris and Maggie are able to discard ones which do not work without the need to be as systematic as the teacher would like. Sally’s response (line 9) does little more than restate the question, and provides no further evidence that she has reached a mutual understanding with Chris or Maggie.

At this point the teacher tried another approach:
This new question (Table 8.2 line 1) was posed by the teacher to test the students’ ability to sort out the possible combinations, and the response is immediate. Maggie and then Chris both determine the three possible ways within a few seconds, supporting the idea that they have developed their understanding of the problem through their talk and are indeed able to do the calculations ‘in their mind’ (i.e. self-talk). They have been able to ‘go on’ to adapt to a new situation based upon the same thinking. Sally seems to take a more random approach by guessing a combination and then trying to work through it by adding groups of six. She shows signs of knowing the process needed to find a possible answer, but of not adopting the faster way of performing the calculation using multiplication or division. Maggie and Chris make no attempt to explain this to her and their focus drifts. This may be a result of the teacher being present, removing any obligation on the others to help, as was seen earlier when Maggie had asked Sally to explain. The teacher gives Sally time to do the calculation and this may have been a good time to work further with Sally. Sally, however, makes a jump to suggesting the use of a chart (line 6), perhaps the first step in a systematic approach, and this may have been enough to prevent any further intervention. The teacher’s follow-up question, of whether it matters which one you do first, may reflect his ‘thinking ahead’ while Sally was attempting the calculation, but Sally is quick to realise the intent of the question, dismissing it. Interestingly, Maggie is able to ‘go on’ with a deeper connection in making the analogy to the commutative properties of a rectangle. On reflection afterwards, the teacher saw his interaction with Sally as an opportunity missed; an example of listening
for a response rather than listening to a student. The opportunity was there to press Sally further and perhaps draw her into a conversation.

The teacher’s final move was to suggest the removal of a restriction, so that the problem could be more open-ended. The choice of the group was to remove the restriction on cars and trucks travelling separately rather than on the number of trips. This led to further discussion, elements of which I shall return to later in a discussion centred on student presentation of their work.

What the above episode is chosen to illustrate was that the students who were engaged in the conversation, Maggie and Chris, were able to ‘go on’ to both explain and extend their work to encompass changes offered by the teacher. This supports the idea that they, at that time, had developed an understanding of the problem and how to solve it. The speed at which they solved the extension question also strongly suggests that they were using the properties of multiplication to complete the calculations. There is no evidence to support or refute their use of division, but there is evidence that Chris was using number facts, such as $6 \times 7 = 42$ to help him (at one point singing a mnemonic to illustrate this – “six times seven is forty-two, now you know it’s true”). While Maggie’s speed at finding the solution at the start of the problem may suggest that she was capable of performing the operations required to solve this question, she demonstrates in several places an internal communication that is helping develop her understanding of the material. In a similar way, Chris seems to also develop his understanding through the course of the conversation with Maggie. Again, his ability to modify the process to quickly find answers to the extended question supports the idea that he has the ability to ‘go on’ in this problem. Sally, while being able to follow and play at the language game developed by Maggie and Chris, does not demonstrate evidence of being able to ‘go on’ and further develop her work. There is nothing to suggest that she has developed a good understanding of the problem. This is significant if it is tied to her lack of active participation in the conversation of the problem. Sally’s need to add groups of six in order to find a solution to the extension question demonstrates that, while she may follow what Maggie and Chris did, she has not been able to ‘go on’ to the same level. There is no certainty in this statement, but I use it in the sense of being a falsification test on connecting conversation with understanding.
In this next episode, I examine further episode 6b with a view to provide evidence in support of the idea that students engaged in conversation demonstrate an understanding in the sense that they are able to ‘go on’ to a further stage.

6c. Richard, Peter, and Jill and The 612 problem

At the end of their development stage, the group were able to determine a possible pair of factors of 612 by dividing 612 by 2. As in episode 7, above, two students had driven the conversation which led to evidence of the development of a mutual understanding of the process. In episode 7, Maggie and Chris had participated in the conversation while Sally had remained quiet. Further analysis, in episode 7b above, suggested that Maggie and Chris were able to ‘go on’ to develop their understanding, applying it to new situations. In episode 6b, Richard and Peter had built the conversation which led to a mutual understanding of how to proceed, while Jill had remained essentially quiet through the process. In episode 7b, there was no evidence to suggest that Sally was able to ‘go on’ to work with the extending questions. In this episode, I examine whether the same analysis can be applied to Jill, and if not why that might be. As before, I focus on the result of their ongoing discussions and pick out incidents which can be illuminating.

To begin, Peter summarises what they have found and re-states the question (Table 8.3 line 1). Richard’s response is not helpful but Peter continues by extending the thinking which led to their first solution, this time suggesting dividing by two again (line 3). It is Jill who responds to his question while Richard is seemingly multiplying by two (line 4). Peter, who was looking for something, simply restates the answer without a questioning tone, suggesting that he has not done the calculation but is accepting Jill’s answer.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1</td>
<td>P:</td>
</tr>
<tr>
<td>2</td>
<td>R:</td>
</tr>
<tr>
<td>3</td>
<td>P:</td>
</tr>
<tr>
<td>4</td>
<td>R:</td>
</tr>
<tr>
<td>5</td>
<td>J:</td>
</tr>
<tr>
<td>6</td>
<td>P:</td>
</tr>
<tr>
<td>7</td>
<td>J:</td>
</tr>
<tr>
<td>8</td>
<td>P:</td>
</tr>
</tbody>
</table>

Table 8-3 Transcript video #6 Richard, Peter, and Jill and 612
Jill quietly states the answer directly to Peter. Peter responds by shifting his body position to face Jill more. They maintain this alignment as the talk develops.

Peter does his calculations without writing anything down, but uses his fingers as counting guides.

### Table 8-4 Gestures per line - video #6 Richard, Peter, & Jill and 612

The body language of the students undergoes a change at this point in the talk. While Richard is still focussed primarily on doing calculations on his sheet, Peter and Jill are now more focused on each other. Peter does not write anything down and it seems Jill writes down her answers without obvious calculations. While Richard appears to be doing calculations his utterances do not drive the problem forward. To give a better sense of Richard’s talk as an externalised self-talk, I have isolated the major utterances from Richard during this phase in Table 8.5. I have not included his verbalised and softly spoken calculations and excluded comments from Peter which Richard essentially passed over. I choose the term ‘pass over’ rather than ‘ignore’ because Richard acknowledged Peter’s turns at talk with facial or body gestures but did not include them in his process.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>wait no (.) that's</td>
</tr>
<tr>
<td>2</td>
<td>okay (.) I'll try that (.) a hundred and fifty-three times three</td>
</tr>
<tr>
<td>3</td>
<td>yeah (.) I wonder what the other ones are</td>
</tr>
<tr>
<td>4</td>
<td>°&lt;Three hundred and six&gt;° (.) &gt;hang on&lt; (.) I'll do it (2) three hundred (0.5) and six (1) times two (.) times two equa::ls</td>
</tr>
<tr>
<td>5</td>
<td>if we can split it three times (.) it probably wo↑uld</td>
</tr>
<tr>
<td>6</td>
<td>what about fi↑ve (.)</td>
</tr>
</tbody>
</table>
The above table suggests that Richard’s contribution to the talk is no longer offering anything significant, being made up of agreements and offers to calculate for the most part. Only line 6 is making a suggestion. By comparison, in the development stage, Richard was clearly part of the conversation which helped Peter move forward in the solution to the problem. Richard and Peter were playing with and in the language game in developing their understanding. In this stage, it is Jill and Peter who are able to play in the language game Peter and Richard developed in order to ‘go on’ to find the other answers. While Richard’s utterances suggest that he is aware of the process required, his skillset does not seem developed enough to allow him to generate any further answers. Jill, who did not seem to contribute in the development stage, is clearly capable of performing the necessary calculations more proficiently than either Richard or Peter.

Later in the discussion (Table 8.7), Peter ‘goes on’ to suggest finding further answers by dividing by three (line 1), and complete the calculation himself (line 4). Jill simply states another possible answer as 153 x 4 (line 3) which she found, when explained to the classroom teacher at a later point, by dividing 306 by two again. In this way, Jill has shown that she could ‘go on’ from the group’s previous answer. Peter, likewise,
demonstrates that he can ‘go on’ to determine divisibility by 5 (line 8). In this, we are party to his thinking process as he articulates the rule for determining divisibility by running through the pattern of numbers ending with 0 or 5. His emphatic ‘No’ at the end is matched by his body language which is that of success, indicating that the word ‘no’ is referring to his decision that 612 is not divisible by 5. Jill looks at him and is shaking her head as he concludes ‘no’, but she is smiling in agreement. To support this, Peter references the last two digits of 612, and now Jill nods.

<table>
<thead>
<tr>
<th></th>
<th>J:</th>
<th>it could also be one (.) cos six hundred and twelve times one ((shrugs))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>P:</td>
<td>yeah (.) good (.) so (.) there’s a chain (.) it’s divisible by one and two and three and four=</td>
</tr>
<tr>
<td>3</td>
<td>R:</td>
<td>= but not five</td>
</tr>
<tr>
<td>4</td>
<td>P:</td>
<td>not five (.) what about six</td>
</tr>
<tr>
<td>5</td>
<td>R:</td>
<td>I’ll try</td>
</tr>
<tr>
<td>6</td>
<td>P:</td>
<td>but wa†it (.) six and twelve are eighteen (.) and eighteen is divisible by six so (.)</td>
</tr>
<tr>
<td>7</td>
<td>R:</td>
<td>yeah</td>
</tr>
<tr>
<td>8</td>
<td>J:</td>
<td><em>if it’s divisible by tw:o (.) and divisible by three</em></td>
</tr>
<tr>
<td>9</td>
<td>P:</td>
<td>that’s i†t (.) six into six goes one and six into twelve is 2 (.) so yes</td>
</tr>
<tr>
<td>10</td>
<td>R:</td>
<td>so six times (.) I’ll check i†t</td>
</tr>
<tr>
<td>11</td>
<td>J:</td>
<td>six times a hundred and two</td>
</tr>
<tr>
<td>12</td>
<td>P:</td>
<td>six times ((looks up the uses his fingers)) (1) yeah (.) ye†ah</td>
</tr>
<tr>
<td>13</td>
<td>J:</td>
<td>&gt;I DIVided it by two and then divided it by three more&lt; ((she beats out her calculation by nodding her head)) and got a hundred and two ((smiles))</td>
</tr>
<tr>
<td>14</td>
<td>P:</td>
<td>cool</td>
</tr>
</tbody>
</table>

Table 8-7 Transcript video #6 Richard, Peter, and Jill and 612

Moving forward again, Table 8.7 shows Peter and Jill playing further in the language game and developing the solution set. Jill’s confidence to join in with the talk has seemingly increased, and she contributes more to this stage. Line 1 suggests that she is now actively looking for pairs of factors to give 612 (see also line 11). In line 13, she forcibly takes a turn by speaking louder and faster to explain how she knew 6 was a factor of 612 in a different way from Peter (line 9), who had passed over her earlier utterance (line 8) as he was doing his own calculation. Jill and Peter are building their solution set with a series of interlaced utterances rather than adjacent pairs. There is little by way of overlap and gestures are minimal. Is this a conversation? It may be considered as such, in the same way as there was conversation between Richard and Peter in the development stage. The utterances can be considered as standalone, but at the same
time they do build on each other and the interlocutors are responding to each other’s utterances.

How then, does this differ from episode 7b? I suggest that the seed of Jill’s successful entry into the extension stage could be seen in her limited but relevant contributions in the development stage. The bulk of the conversation in the development stage between Peter and Richard was in sorting out the process by which they could find the answer. Ultimately their conversation which led to the final method covered ground which was not used in that method. When asked by the classroom teacher what their strategy was, Richard replied that “we divided 612 by 2 and then found what multiplied 2 to give 612. Then we did the others.” There was no mention of the earlier attempts using the lattice method. Jill demonstrated that she followed what Richard was trying to do with the lattice – questioning his positioning of the three in doing so (Table 7.3 line 34b), demonstrated active body language throughout, and demonstrated that she could do the calculation at the end of the conversation once Peter and Richard had settled on dividing 612 by 2 (Table 7.4 line 62). In comparison to Sally, who seemed to be playing at the language game when asked to contribute (Table 6.17 line 15), Jill appears to have been silently playing in the language game with Peter and Richard during the development stage, and actively playing in the language game with Peter in the extension stage. As a result, Jill is able to ‘go on’ in this stage and demonstrates that she has developed an understanding of the problem. Peter is also able to ‘go on’ and build upon his earlier work and demonstrates that he has developed an understanding of the problem. Again, it is important to stress that this understanding may not be the same understanding as what the problem designer (the intended curriculum) or the classroom teacher was looking for (the implemented curriculum).

As a final episode in this section, I examine how episodes 7b and 6c compare with the continuation of episode 5b, the ‘Don’t fence me in’ problem. This episode was characterised by a lack of cooperation between the students and it is interesting to follow these students further to see if their inability to create a conversational space does indeed limit their ultimate progress.
5c. Aaron, Bashir, and Chan: Don’t fence me in

In the developmental stage of this problem, the group of boys demonstrated little conversational activity and the talk was dominated by statements from Aaron, attempts to clarify and engage from Chan, and apparent confusion in Bashir’s case. The talk was more of a discussion because the students’ utterances did not build upon each other in a mutually supportive way. Aaron seemed to be making statements which took nothing from the comments of Chan or Bashir, and while this did not prevent him from coming to the expected answer for part a), he did so while yet to resolve the issue about the effect on area of changing the side lengths. It was noted that Aaron seemed to be playing in his own language game up to this point. What I question in this next stage is whether the group can make progress in addressing an extension of the problem, or if this is hampered by their lack of conversation to this point. The extension to the problem reads:

b) Jan wants to enlarge the pen as she has adopted a second dog. She plans to move one side of the pen outwards to increase the area by 40 square metres. What is the least number of metres of additional fencing Jan will need to enclose the enlarged pen if she re-uses all the existing fencing?

The intended outcome of this problem is that the students investigate a problem with multiple stages and an alternative solution. The problem is not open ended as it specifies that only one side of the pen is increased, and this is a point the students need to recognise. In this part, the students are given the area and so must recognise how to find the increased length to give the extension, a process involving division once they recognise that one side of the new rectangle is constant. The problem then introduces the idea of perimeter in asking for the amount of additional fencing, and the students have to recognise that the change in perimeter is being asked for. Finally, the problem requires optimization of the answer, so both methods must be performed and compared.

A strong motivation for the classroom teacher to choose this problem was in the need for the students to work together to organise how to solve the problem in an efficient way.

<table>
<thead>
<tr>
<th></th>
<th>A:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>so we’ll have two hundred umm square (.) metres as the side (0.5) so it if she wants to expand it</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>C:</td>
<td>aaron (.) why are you doing</td>
</tr>
<tr>
<td>3</td>
<td>A:</td>
<td>so that’s twenty right here (2) and then</td>
</tr>
<tr>
<td>4</td>
<td>C:</td>
<td>twenty there (.) ten</td>
</tr>
</tbody>
</table>
Chan begins the episode by reading the question and Aaron responds in the same manner as in the development stage; by stating what he thinks is a solution (Table 8.8). Chan again challenges Aaron regarding his lack of teamwork and this time there is no laughter to act as redressive face work. Chan has to repeat his challenge (lines 2, 6, and 8), even using an abrupt cough to interrupt Aaron’s flow. Aaron, however, ignores Chan. Bashir forcibly interrupts (line 10) by raising his voice, speaking quickly, and invading Aaron’s space, and Aaron responds to Bashir’s request as opposed to Chan’s challenge. As Aaron starts to explain though, Bashir sinks back to his chair and his smile goes. Chan looks down at his hands and sighs (see Table 8.9).

<table>
<thead>
<tr>
<th>5</th>
<th>A:</th>
<th>&gt;we can take out the ten then this would be&lt; fo:ur (1.5) and this wou wait (.) er this would be si::x (.) and this would be umm twenty-four (.) that works (.) yeah so umm expand it twenty into twenty-four because 4 umm (.) we have four here</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>C:</td>
<td>((cough)) why are you doing everything again</td>
</tr>
<tr>
<td>7</td>
<td>A:</td>
<td>and you er add four here (.) four metres is um:m forty square metres you umm keep moving (..) down (..) so (..)</td>
</tr>
<tr>
<td>8</td>
<td>C:</td>
<td>aaron why are you doing everything again? aaron (1)</td>
</tr>
<tr>
<td>9</td>
<td>A:</td>
<td>and then umm=</td>
</tr>
<tr>
<td>10</td>
<td>B =&gt;</td>
<td>Wait can you try and explain&lt; what you’re trying to do now</td>
</tr>
<tr>
<td>11</td>
<td>A:</td>
<td>&gt;so I’m trying to exp:and this place&lt; (.) by four er four metres (..) and umm &gt;to make this space smaller&lt; to (.) this would only be six now (1) and this will be ((Chan sighs and looks down, Bashir moves back to his space)) tw↑enty-four</td>
</tr>
</tbody>
</table>

Table 8-8 Transcript video #5c Aaron, Bashir, and Chan: Don’t fence me in

“Wait can you try and explain”. Bashir interrupts Aaron by asking him to explain and moving into his work space. Chan had previously failed to interrupt Aaron’s flow of talk.
Bashir moves back as Aaron explains, his focus seemingly lost; Chan sighs and focuses on his hands. Their body language suggests that Aaron’s explanation is not helping.

Table 8-9 Gestures - video #5c Aaron, Bashir, and Chan: Don’t fence me in

The group continues in this way, with Bashir trying to get Aaron to explain his process to him and Chan trying to interpret what Aaron is saying. Neither Chan nor Bashir are adding anything new to the exchange and Aaron makes two further attempts to explain to Bashir, neither of which is helpful. Chan gets increasingly frustrated and the classroom teacher moves over to speak to the group after they had been working on the problem for a total of eleven minutes. In this next section, the teacher tries to determine if the group as a whole knows what they are doing (Table 8.10).

<table>
<thead>
<tr>
<th></th>
<th>T:</th>
<th>So, have you got and answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>A:</td>
<td>yes ((nods))</td>
</tr>
<tr>
<td>3</td>
<td>B:</td>
<td>yeah (.) sort of</td>
</tr>
<tr>
<td>4</td>
<td>T:</td>
<td>okay, Bashir, do you understand what you did?</td>
</tr>
<tr>
<td>5</td>
<td>B:</td>
<td>well (.) yeah umm so we had to expand the fence but and so</td>
</tr>
<tr>
<td>6</td>
<td>A:</td>
<td>&gt;so you have to expand the&lt;</td>
</tr>
<tr>
<td>7</td>
<td>T:</td>
<td>No no, let Bashir</td>
</tr>
<tr>
<td>8</td>
<td>B:</td>
<td>we had to expand it at the least we could (.) umm and (.) we had (.) to and we had well we (..) and it says we what is the least numbers of metres of additional fencing Jan will need to enclose (.) to enlarge the pen (.) so (.) umm so she umm (..) would add four (.) four square four extra square feet to umm (..) to expand the width</td>
</tr>
<tr>
<td>9</td>
<td>T:</td>
<td>how did you come up with that?</td>
</tr>
<tr>
<td>10</td>
<td>B:</td>
<td>(3) we::il</td>
</tr>
<tr>
<td>11</td>
<td>C:</td>
<td>we didn’t really come up=</td>
</tr>
<tr>
<td>12</td>
<td>T:</td>
<td>=let him think (5) so you’re telling me the answer but you haven’t told me what your thinking was behind that (9) so where did the 4 come from? (5) ((Bashir looks to Aaron)) so you let him supply you with the answer then?</td>
</tr>
<tr>
<td>13</td>
<td>B:</td>
<td>I was asking him how (.) how did you get that↑t but he’s just saying add the additional fencing umm (..) four (2)</td>
</tr>
<tr>
<td>14</td>
<td>T:</td>
<td>Chan, can you tell us?</td>
</tr>
<tr>
<td>15</td>
<td>C:</td>
<td>&lt;Aaron wa:s .hh basically doing most of the work&gt; .hh</td>
</tr>
<tr>
<td>16</td>
<td>T:</td>
<td>That’s not what we want is it? So can you tell us, did you understand what</td>
</tr>
</tbody>
</table>
The teacher, picking up on Bashir’s hesitant ‘sort of’ (Table 8.10 line 3), presses Bashir for an explanation. Aaron tries to take over the explanation (line 6), but the teacher insists that Bashir do so. Bashir starts again (line 8) but his utterance is full of fillers and he hedges by restating the question before making a confused final statement. His manner is hesitant and his gesturing, to support what he is saying, is a container gesture used to frame the pen in space above the page (see Table 8.11 frames 1-3); this is demonstrated when he moves his hands apart when saying “expand the width”. He also taps the page when giving information. After his description he folds his hands together, suggesting closure.

Bashir has stated an increased width of ‘four’, but referring to it in square feet reflects his lack of understanding of what he is doing. Because he has not built up an understanding in conversation with Aaron, all he can do is to repeat Aaron’s answer. The teacher presses further and Bashir simply looks down at the page. After three seconds of silence, Bashir offers another drawn out filler before Chan comes to his rescue (line 11). During the silence, Chan was gesturing to Aaron by pointing a flat hand towards him, and looking directly at him with an expression suggestive of expectation or blame. Finally,
Chan points out that “we didn’t really come up with (the answer)”; the ‘we’ in this case does not seem to include Aaron. The teacher still wants to allow Bashir time to make some suggestions as to process and his own thinking, rather than go with Chan’s assertion alone. Bashir is unable to offer anything and eventually turns to look at Aaron in a gesture of conceding Chan’s claim (line 12). Chan comments that he was “following along” (line 17), but at no point does he offer any suggestion to explain what he was following. While he did seem to be following Aaron in the discussion, and agree with him at times, he has nothing to offer by way of explanation as a result. Chan’s participation in the discussion was seemingly not effective in his coming to an understanding. It seems clear that he was not playing in the language game of the problem with Aaron and as such did not develop an understanding, even though he may have been able to follow part of what Aaron was saying. The drop in tone at the end of line 17, followed by a nervous laugh, accompanies a stiffening of his gesturing in pressing down on the table. It seems clear that Chan is also uncomfortable in being pressed.

Aaron interrupts to claim that he did try to explain, which to him is true, but then goes into an example of his explanation which makes it clear why the Bashir and Chan had difficulty following him (line 19). He is making discursional statements rather than giving an explanation; he does not explain why he has added 4m to the width, even when pressed by the teacher to do so. When pressed further (line 22), his explanation breaks down. In attaining the correct answer, Aaron may have simply ‘seen’ that the required increase in area of forty could be easily calculated with the factors of 4 and 10. Recognising the length as 10m would then have prompted an increase in width of 4m. However, if this answer was ‘seen’ rather than thought through in terms of the properties of the rectangle, there would have been little internal conversation to help him build an understanding of what was happening. Without that understanding he is unable to ‘go on’ (at this point in time) with the process, either in explaining the answer or relating the answer correctly to the perimeter. The question itself may be partly at fault in offering an easily recognizable mental calculation, so that an answer was available without a depth of thought. In a functional group setting this may not have been an issue, but instead provided an easy entry point for conversation about the problem. In some other groups in the class this was the case for this problem, and in such cases the students were able to articulate more readily what was happening. I suggest that it is the lack of conversational
organization that reflects this group’s lack of success in developing a meaningful understanding of the problem.

Table 8-11 Gestures - video #5c Aaron, Bashir, and Chan: Don’t fence me in

**Summary so far – the Extension Stage**

In giving these three examples of groups in the extension stage of problem solving, I have tried to show how the students’ understanding may be more clearly demonstrated and how this relates back to the conversational organization of the earlier stages. I have selected these examples as exemplars of the many episodes when a similar effect was seen. In these selected cases, the outcome was clearer but not exceptional. Students who failed to establish a conversational space seemed unable to develop a solution to a given problem that would enable them to ‘go on’ to extend their thinking to variations in
the questions or to related questions. When a conversational space was established there was frequent evidence to support the idea that the students were able to develop mutual understanding and move towards a solution. This was further supported by their frequent ability to extend their thinking further. Whether or not the conversational activity can be linked actively to a developing understanding, it seems clear that it at least accompanies it. In this regard, the results support the notion that a teacher can view conversation as being an indicator of developing understanding.

In the next chapter, I wish to highlight two significant observations that arose from watching the recordings. While not outlined in the original lines of research, I consider these results important to include as they arose from observing the recorded data. Both of these situations bear directly on the research questions regarding a teacher’s ability to observe the progress of students working in a group from a distance so as not to disturb the working of the group. I illustrate how the use of gesture can be shaped by the confidence of the student when speaking. I have referred to this feature several times in the discussion above and I now highlight some cases in presentations where this is readily apparent. The use of broad kinesthetic gestures by a student can be linked to the confidence a student has in offering their solutions, and supports the work of Gerofsky (2008) in this area. In addition, I highlight situations in which gesture and posture mimicry is seen in conjunction with progress in solving a problem.
Chapter 9

Observing Students in Action

In this chapter, I present some features of student talk that stood out from watching students in action. These were not anticipated or looked for but emerged as a pattern the more recordings I watched. In the sense that these observations grew from the data, they are perhaps more exciting than those features I was interested in before I began this research. The connection between the dynamics of gesture and the student’s progress in a problem became evident at an early stage, and this is the subject of the first section of this chapter. That the students seem to also illustrate mimicry (echoing) in their gesture and posture, the focus of the second section of this chapter, increasingly stood out once I became attuned to it. Both of these sets of results have significance in the way a teacher can actively observe students at work; neither depends on the students’ talk and both can be observed ‘at a distance’.

Students Presenting their Work

I begin by looking at the gestures of students in a classroom discussion at the beginning of the grade 5 unit on area. The classroom teacher has asked the students if they can say what area is. He has posed the question to the class and these responses are from students who are willing to offer an answer. The first example (Table 9.1) is from the student who raises her hand first to respond to the question. She is sitting on her desk in a slouched position, but her hands and arms immediately begin to make large gestures that match her response. She makes no significant facial gestures and looks at the teacher as she responds, only once glancing away and up to her right as she pauses slightly before saying the word ‘object’. Even though she is seated, her gestures fill the peripheral gesture space in front of her.
The (..) object (.) like a circle
The utterance begins by pointing one hand above the other when saying 'object', before forming a small shape higher in space, and completing the image by using the index fingers of both hands to draw out a large circle in the air from eye level down to her finger tops meeting again at waist level. The gesture moves into the left and right periphery.

…and then area (..) is the whole thing (..) like (.) the inside
He gesture drops to the lower periphery and then the hands are spread out and wave in the space before her created by the earlier circle, effectively filling the peripheral space.

Table 9-1 Gestures – Addressing the class while describing area

The second student to raise her hand offers a similar explanation but extends the description to include the term ‘perimeter’. Again she holds her arms high and makes large gestures, creating the sense of a physical soda can in the air in front of her which she rotates and manipulates (Table 9. 2). In speaking she uses no hesitations of any significance and no fillers to break up the flow of her talk. She looks at the teacher while speaking except once looking at her hands when she rotates them to represent the bottom of the can. At one point she uses the expression “I guess” with the guess drawn out, but this may reflect concern over her description rather than what she is saying. It is interesting to note that the majority of the other students in the class did not turn to watch her gestures, seeming to vaguely focus on the front of the class while (perhaps) listening only. This suggests that she is not using the gestures as an intentional form of additional communication, but that they are an integral part of her thinking.
If you have like a pop-can the bottom is like a little circle that sorta stands out that would be the perimeter
Her hands shape out the object in space as a can and she points to its base. She then draws a circle in the air with one hand with the other hand seems to hold the object up. While McNeil’s gesture space is essentially two-dimensional, it is clear that the speaker is also filling a very three-dimensional gesture space.

And the inside that is like closer to the can I guess is like the area
The hand that ‘held’ the object now rotates towards her and she uses the other hand to hold the position of the bottom of the can in place. She then pushes her hands together to depict the area of the base.

Table 9-2 Presentation gestures

In both of the above examples, the students made gestures with elevated arms in the space between waist and chin level. The gestures were dynamic and were connected to the meaning of what they were saying, representing the physical object they were talking about and illustrating a manipulation or aspect of the object that may be hard for them to describe in words alone. It would seem from this that for other students to access the full meaning of these utterances, they would benefit from focusing on the speaker. Developing a classroom culture wherein the class focuses their attention visually on a speaker as well as audibly may therefore have benefits in terms of student-to-student learning. By extension, this implies the same should be true when students listen to a teacher.

In a third example from the same class, the teacher asked another student to re-voice what the first two students had stated. This was perhaps due to being aware, in a teacherly moment, of a lack of attention from the selected student. In this case, the student had not elected herself to speak and it is of interest to note that she spoke with minimal hand gestures to begin with (Table 9.3). As she continued, however, her gestures grew until, at the end, she gestured as boldly as the previous students. She
gave a sense of meaning to area using her gestures, even though she was not consistent with her use of metre and square metre. She animated her explanation with relation to the size of a mouse, even making a mouse like gesture as she did so. She ended up using her whole upper body to gesture the explanation. At the same time, her focus shifted from a vague spot above the teacher to looking directly at the teacher.

Well (..) you can have the two (.) like a square centimetre and umm (.) a square metre (..) and the square metre is like umm (..)

To begin she starts with a slouching posture and her utterance comprises fillers and pauses. She makes slight beat gestures with her hands. She speaks slowly.

if you have a farmland or something and you want to calculate acreage you can be like I have a thousand square metres but then if you were imagining like (..)

The student’s speech speeds up and there are no fillers or pauses; her arms rise and she grasps a piece of space as she says ‘acereage’. Her eyes do not track her gestures but focus on the teacher.

a mouse it’s like three or a quarter of a metre (.) you would say it’s like 5cm. So it’s like you can measure an area within stuff (.) so (..) yeah

She makes a pantomime hand gesture on saying mouse then raises her hands again to form a container gesture which shrinks horizontally on saying “a quarter of a metre”. Her arm makes a slicing gesture when she says ‘within’.

**Table 9-3 Presentation gestures**

Students seem to use broad and bold gestures when they are confident that they have an understanding of what they are saying. The more confident they seem to be then the
more dynamically they gesture. Such gesturing can evolve into an almost pantomime-like gesture (see Table 9.4, panels m and n). Students who lack confidence in what they are saying, as suggested by hesitant or unclear utterances, generally have stiff body language and hold their hands together when they speak. I conjecture that this may even be a subconscious mechanism to prevent their gestures from revealing something. This may also involve putting their hands into their pockets, hooking them into their clothing, or using a pencil or some other object (see Table 9.4 lines c, f and j). Some students seem to also hold onto their legs if they are standing (see Table 9.4 line g). In cases where a student who initially lacks confidence but then appears to gain that confidence through class feedback during a presentation, their gesturing changes. Table 9.4 lines f) and g) illustrate such cases; in f) the student’s arms shift from his pockets to gesturing in the low peripheral space before his waist, to finally the upper peripheral space below his chin; in g) the student shifts from a slouching position with arms fixed to her side to one where she is gesturing in the centre space before her. By observing students in situations such as shown in Table 9.4 line 1, and also images a) to e) when they are also being recorded, the connection between the confidence the student has in their utterance and their gesturing seems firmly established. By inference, then, the important idea I wish to develop here, is that by watching students from afar, even without recording their utterances, a conjecture can be made about the confidence of their utterances. By extension it may be claimed that the student feels that they have a better understanding. In terms of what has been written here, I would say that they are able to ‘go on’ further with their explanation, and are able to recognise this. Such recognition increases their confidence and results in a more dynamic gesturing. There is also a case for a teacher to emphasise their own gesturing when addressing a class. In doing so they may encourage gesturing by exemplifying its use, even if done so indirectly.

It should be stressed, however, that that students’ confidence in their understanding does not necessarily mean that they are able to ‘go on’ in a way the teacher might be looking for, but they may be receptive to engaging in a conversation which can help in that regard. Examples are shown in Table 9.4 j) to n) where the students were captured working away from the camera and engaged in dynamic gesturing, either in the seated or the standing position. When these students were approached by the classroom teacher and pressed to explain their work, they were found to be confident in their own
understanding of the problem and willing to share this with the teacher. In recordings made in this research where a student was observed presenting, there was only one example where a student gestured in a confident and dynamic way when there seemed to be no connection between what she was saying and the expected outcomes of the problem and at the same time was unwilling to talk about this further with the teacher. In all other cases where the student gestured dynamically but did not reach the desired outcome, there were clear links the teacher could connect to, even if this was not always the case if the teacher was focused on listening for rather than listening to.

1. Even when seated, students still gesture. Here two students engage in a conversation about the problem they are working on.

a) & b) When presenting at the board, confident students gesture to numbers in a dynamic way and use their arms to follow the motion of their words in an action similar to a conductor as they talk.

c) The student lacking confidence uses few gestures and typically holds their hands together (a pen often serves as the link). Body gestures are not dynamic and may involve swaying or fidgeting.
Students standing and discussing problems often make very large gestures involving their whole upper body. Expressive gestures are typically held high and start centred on the body.

f) illustrates how a student who starts off an explanation lacking in confidence is often stiff in posture and hold their hands/arms fixed, in this case by hooking them in this pockets. As his confidence grows he frees his hands and they gesture in a low space around his waist. As his confidence grows further, his gesturing rises higher and is body language is more dynamic.

Another example of starting out in a non-confident way but, with class encouragement, ending positively. The students arms start inactively and are held to her side; she has slumped shoulders. After she grows in confidence (following positive feedback from her peers and the classroom teacher), she stands up straighter and uses her arms to gesture. Her arms are held in the centre gesture space between her waist and chin even when she does not gesture. Even though she holds a pen it does not link her hands as in c).
<table>
<thead>
<tr>
<th>h)</th>
<th>Describing an image in the board from afar, the student surrounds it with his hands and draws it forwards in cupped hands. Even if a student is away from the object they describe, the confident student will reach out and include it.</th>
</tr>
</thead>
<tbody>
<tr>
<td>i)</td>
<td>In this example, the student tries a few times to answer a question. There are a lot of pauses and fillers but no gestures. She gives up. She keeps her hands linked and close to her chest.</td>
</tr>
<tr>
<td>j) Observed from a distance, the student is using a stretching gesture.</td>
<td></td>
</tr>
<tr>
<td>k)</td>
<td>Using the ideas from watching students presenting and working in groups monitored by the camera, an inference can be made that students observed from a distance who use dynamic gesturing are also confident about the ideas they are expressing. A larger container gesture, such as seen in k) is commonly used, as is the gesture seen in l) which often accompanies reference to smaller amounts. Seated gestures tend to be more confined but can still stretch outside of the centre into the peripheral area.</td>
</tr>
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</table>
In m) and n), very dynamic gesturing is observed when the students are standing. Checking the students’ progress confirmed that they were making headway with the problem they were working on.

**Table 9-4 Presentation gestures**

These results tie in with, and extend, the findings of Gerofsky (2008), who reported a finding between gesture type and a propensity to engage in school mathematics after observing students gesturing at the board in relation to graphing. My findings offer further evidence to support this view while extending these observations to general classroom activities. There seems to be a correlation between the confidence level of a student and the dynamics of their gesturing. Students who are more confident of their ideas and engaged in the thinking tend to use more and larger gestures; but there may also be a reciprocal effect in that increased gesturing could be helping to increase confidence. If this was the case, then there would be a natural feedback effect which a teacher could look to support. I pointed out in an earlier chapter that having students stand rather than sit when discussing a problem seemed beneficial to the process, and this might tie into the idea of dynamic feedback, both in gesture and in the spoken form of a conversation. If this was in fact the case then encouraging students to move as well as speak could help some students in their development.

In the next section, I examine another feature that stood out from the research, that of gesture and/or posture mimicry.

**Mimicry (gesture echoing)**

In this final section, I draw attention to the students’ use of mimicry, or gesture echoing, in both their gesture and posture as they work together in a group setting. Although the term ‘mimicry’ is used in the literature, it carries a sense of being a deliberate copy,
perhaps even in a derogatory way. The use of the term ‘echo’ seems more appropriate in this regard\(^9\) and so I will shift to using this term from this point.

Table 9.5 illustrates a conversation between Gina and Susan. The problem concerns the change in area of a desk reduced to half its length but doubled in width.

\[
\begin{array}{c}
\text{“You’re taking it in half (..) and then …. doubling one side, right?”}
\end{array}
\]

\[
\begin{array}{c}
\text{“You take some of it off and you add it to the other side (0.5)”}
\end{array}
\]

Table 9-5 Echoed gestures when describing the same process

This example matches several recorded in this lesson and is of interest because, while the gestures used differed between groups, there was evidence of gesture echoing between interlocutors when the students were able to make progress. In examples where the students were unable to make progress, there was no clear evidence of gesture matching. In this example Gina initiated by describing the desk using large gestures. Susan, in her adjacent turn, echoed the dynamic gesturing of Gina in describing the table.

\(^9\) My thanks go to David Pimm for suggesting this term.
Table 9.6 shows another example of gesture echoing between two girls working in a group on a problem where they were asked to estimate the size of a bag required to hold a million dollars in $100 dollar notes. Panel 2 shows one girl, Jasmine, making an initial gesture which is then echoed by Gina (panel 3) as they engaged in conversation. As the conversation develops, Jasmine moved gradually closer to Gina until their gesture space became shared. They continued to echo each other’s gestures as they did so. During this time, the conversation was rich, and led to a clear progression in the problem’s solution.

Table 9.6 also shows the group engaging in posture echoing. The three girls adopted an almost identical posture once they started to work on the problem together.

The three girls in this group adopt a common posture while speaking as seen in panel 1. The initial gesture of the student in panel 2 is echoed by the student in panel 3. The boy in the group can only gain the others’ attention when he mimics their posture.

The first student closes the space between herself and her interlocutor as their conversation develops. Both their posture and their gestures are the same.

Table 9-6 Gesture and Posture echoing

The male member of the group, Jason, seemed to be shut out by this common posture and found it very difficult to gain attention (panel 1) until he adopted a similar posture (panel 3). A male‒female dynamic or other social situation, may account for this early barrier to Jason’s inclusion, and he may not be aware of his own change in posture.
during the process, but in order to participate he appears to need to connect through posture first.

The group shown in Table 9.7 also showed signs of gestural echoing, but in this case it was rare. Panel 5 illustrates the only clear echoed gesture, a cutting motion used in conjunction with talk of division. A common deictic gesture, as shown in panel 3, seemed to serve the similar purpose of connecting the group while talking. While there were other gestures which were repeated by different members of the group, such as the spread fingers shown by the girl on the left side of panel 5, these may or may not be echoed gestures since they occurred more than two turns after the initial gesture.

A second example of posture echoing is illustrated in Table 9.7. Panels 1 and 2 show three of the group have adopted a pose while the fourth student has become disengaged, initially standing while the others leaned, and then a different student sitting while the others stood. Throughout this problem session, the group came together in this way, either in pairs, as a threesome, or all together whenever they were successfully sharing something about the problem (as indicated by the transcript). The common posture varied, as shown between panel 1 and 2, but was generally shared by the members of the group. There were occasions when a student stepped back from this shared gesture space, as illustrated in panel 4. This was followed by a return to the group posture, usually when the student felt they had something to share, or had given up on an idea.

Another example of posture echoing. The postures change but those students who are engaged seem to adopt a common stance. Panel 3 shows echoing of a deictic gesture as the students work around a sheet.
In panel 4 the student has momentarily detached from the group and gestures to himself. On return he makes a cutting gesture which is echoed by his interlocutor.

Table 9-7 Gesture and Posture echoing

For further analysis of this effect, I selected twenty recordings at random (see table 5.4) and reviewed them paying specific attention to echoing. In these recordings I counted twenty-one clear incidents of gesture echoing where students echoed a given gesture exactly within two turns at talk. In four of the twenty recordings, no clear gesture echoing was observed. Only two recordings demonstrated no posture or gesture echoing and in both of these recordings the students made little progress with the problem. In all cases, gesture echoing accompanied conversational adjacent pairs rather than an isolated utterance. Groups generally demonstrated several adoptions of posture echoing and, in all but one case, this coincided with on-task work and resulted in progress with the problem. Gesture echoing tended to be associated with actions, such as the description of shapes or objects, or mathematical operations such as divide, increase and counting. Very little echoing was associated with student activities centred on calculating. In seven of the recordings, the students were standing, and in these recordings, gesture echoing was seen in six cases. These tended to involve a larger gesture space than when the students were seated. There was only one case involving three students mimicking gestures in succession. Generally, only pairs of students echoed gestures whereas posture mimicking tended to involve more members of the group.

Overall, echoed gestures clearly occurred but were not seen to be used extensively while students were working on the mathematical processes. Gesture mimicking was predominantly used, and seemed important, in establishing the situation in which the mathematics was framed. When gesture mimicking was observed as related to the actual mathematics, the gestures were seen to represent ‘cutting’ (as in division), ‘framing’ (as in framing a shape such as a circle), ‘counting’ (particularly the action of skip counting...
using a bouncing motion) and a ‘this-and-that’ gesture where the flat hand was rotated at the wrist in a back and forth motion (as in referring to two cases). The predominant gesture seen during discussion about mathematical processes was deictic, with students pointing to the pages being working on. While these gestures often looked similar, there is not enough evidence to suggest mimicking, given the limited variations of pointing. Table 9.3, panel 3, illustrates this type of gesture.

These results indicate that posture imitation is an important part of group work. When students were working productively on a problem, or exploring an idea together, they tended to imitate each other’s posture, whether standing or sitting. These common postures shifted throughout the working session and demonstrated enough variation to indicate that it was not merely coincidental. When a student opted out of the common posture they rarely added to the thinking of the group, or their attempted contribution was less well-received. In some cases it appeared that a student removed themselves from the group so that they could think through a situation independently as in these cases the student self-gestured (Table 9.3 panel 4) before re-joining the group. In just over half of such cases, the students made a positive contribution to the group. In other situations, a student moved out of the group and showed no signs of thinking independently about the problem (i.e. using some kind of self-gesturing or facial expression); in none of these cases did the student return to offer anything new.

The study suggests that echoed gestures can play a role in creating a mutually shared understanding of the situation within which the problem is set. The echoed gestures may help to coordinate a mathematical process amongst the group so that mathematical actions are seen to be agreed upon. This communication of acceptance in a process has been seen as a core step in the process of reaching a shared understanding in classroom talk (Clark and Wilkes-Gibb, 1986). While gesture–mimicking may not be significant in advancing the mathematical process itself, it may be seen by the interlocutors as an acceptance that the speaker is understood and seen to be making progress. Gesture echoing is part of the collaborative process but relies on the belief of the interlocutors that they have interpreted each other’s intent in the same way. It must also be noted that such gesturing may be subject to interpersonal relationship issues. Students with a strong rapport with each other may be more likely to mimic gestures.
Summary so far

In this chapter, I have highlighted two specific areas of interest which arose during observations on students in a group setting. I find these to be particularly significant in that they are features which are observable by a teacher without the need to disturb the group process. In the first section, I drew attention to the dynamics of the gestures students used, initially from observations when students presented to the class, but then also by watching students working together. Arising from these observations came a second realization regarding the mimicking nature of both gesture and posture.

Analysis of the recordings of student work provides evidence that students echo each other’s posture when being collaborative, and also mimic each other’s gestures as a means to establish a common process. As such, echoed gestures may play an important part in helping to establish a shared understanding amongst the interlocutors and assist in progression of the collaborative effort. Given this possibility, there is an opportunity for teachers’ observing from afar to recognise good opportunities to intervene in order to best facilitate the group’s progress. When a group is seen to mimic each other’s posture or gestures then this may be an indication to stay away from the group and allow them to continue to develop their ideas. If there is no evidence of such echoing then that may indicate a good time to offer support to the group.
Chapter 10

Summary and Conclusions

Overview

In Chapters 2 and 3, I outlined how I have come to think of both knowledge and understanding and how these terms fit into and extend the theoretical framework upon which the research is based. I outlined how many existing theories of knowledge and understanding seem too structured and rely on mental structures which cannot in practice be seen. The variety of terms and descriptions used for these structures create both uncertainties in this supposed process while at the same time try to outline it precisely. I also discussed how the terms ‘knowledge’ and ‘understanding’ seem to be used interchangeably and with a sense that they are ‘taken as known’ by both writer and reader. Further, they are often used loosely and with seemingly different intents within the same discussion. Before I could use such terms in this research, I determined a need to operationalise them in a practical way. I do not entirely reject previous theories but I view them as tools useful in a reflective sense which can be helpful in categorising a past static event. My belief is that understanding is a dynamic activity which can be fleeting and needs to be supported. Conversation modelling and judicious teacher intervention are ways through which this support may be offered. Consequently, I used the definitions that an individual’s knowledge is their on-going interpretation of the accumulation of experiences and resources that represent their life at any given time. It is a source for actions. How the individual goes on from this platform conversationally, to build on this knowledge, represents their understanding. I further developed this to include gesture and prosody as integral aspects of conversation.

In Chapter 3, I laid the foundation of my definitions following the later work of Wittgenstein, who proposed the ideas that meaning is a developed in the use of
language and that understanding is the ability to ‘go on’. In Chapter 4, I addressed concerns regarding these ideas and extended Wittgenstein’s definition by combining Sfard’s framework of commognition and Davis’s views on conversation. Going into the research stage of my work, I felt I had a working definition of both knowledge and understanding that I could investigate. Considering knowledge as a source of action and understanding as the ability to ‘go on conversationally’ allowed me to investigate these as ‘visible/audible’ aspects. Noticing a connection between students experiencing progress in their work and the way they engaged in conversation and incorporated gesture, supports the idea of knowledge and understanding being viewed in this way. My pilot study also enabled me to experiment with the data collection process and ultimately end up with a corpus of data that could be analysed. I used the process of Conversation Analysis to examine and code the data and extended this to also take careful note of gestures and prosody used by the students. The results of this analysis were presented in Chapters 6 through 9. I therefore view CGA as an active technique to view an active process with a view to supporting its development.

My thesis grew from my pilot research, following which I posed my research question. I turn to address this question next.

In Response to the Research Question

At the outset of this research, I stated my over-arching research question as: are there key moments in the talk, i.e. from the organisation of that talk, which we can observe, that inform us about the development of a student’s understanding?

At this point, I can return to my question with a better understanding of both the query and how to respond to it. In terms of this research, I would say that I am able to ‘go on’ to address this question conversationally. One important conclusion I have come to is that there is no definitive answer to my question. Any ‘moment’ that can be focussed on requires interpretation and any assessment is therefore subjective. Instead, what I can offer guidelines which I believe can be used to inform the classroom teacher, or any observer. While there has been a great deal of research into discourse, collaborative work, and gesture independently (see chapter 4), less work has been done in combining
these topics. This research adds to and extends studies (e.g. Sikveland & Ogden, 2012; Mercer, 1996; Logan, Lowrie, & Diezmann, 2014) which utilise a combined approach. This process I have referred to as Conversation–Gesture Analysis (CGA). An exhaustive CA analysis of talk is clearly not practical on a daily basis, CGA is intended a summary of what can be useful and practical to observe.

The first key moment I would point to is the opening of the talk. If the talk did not begin in the cooperative way indicated in chapter 6, the session did not seem to develop mathematically. Either the students were unable to develop a way to solve the problem, or there was no sense that the students had changed their discourse about the mathematics in the problem. Such a change in discourse is an indicator Sfard (2001) uses as a sign of learning. Students engaging in an atypical start to the session were unable to ‘go on’ to change their discourse in this way. In this regard, it can be claimed that they did not demonstrate shared understanding as they were unable to ‘go on’ in their collective talk. Students who opened the session by establishing good grounds for a conversation, in the sense of creating a mutually supportive atmosphere in keeping with the ideas of politeness (Brown & Levinson, 1978) and cooperation (Grice, 1975), were more likely to engage in a mutually supportive exchange of ideas and move to the actual mathematics of the problem. This is in keeping with previous studies that found students ‘proceed in perfectly reasonable and productive ways’ (Confrey, 1991, p. 111), but highlights the conversational nature of such ways which seem to lead to success.

A second key observation was in the manner in which the students’ gestures changed during the session. When students’ gestures increased in size, this was observed to coincide with an improved vocalization of their thinking. Students demonstrating large gestures were, at the same time, able to ‘go on’ with their ideas. In keeping with the premise of this research, they could then be said to be demonstrating understanding. It must be emphasised, however, that this can only be taken as their understanding. We can only assume that a mutual understanding has developed and that this is supporting a change in their discourse about the mathematics. This change in the dynamics of the gesturing supports research by Gerofsky (2008), who reports on gesture diagnosis and intervention, and Winter, Perlman, and Matlock, (2013), who note that gesturing size depends on local context, culture, and the ongoing discourse.
A third key point to look for, relates to how the students are interacting with each other while participating in the task assigned. As discussed in chapter 7, identifying a conversation between the students is a sign that they are working together to build a mutual understanding. If students are engaged in talk which is conversational, which will typically be accompanied by gestures, understanding is being developed. This is in keeping with the idea of ‘exploratory talk’ (Mercer, 1999, 2004), in which talk is mutually supportive when seeking to address the task in hand. Even students engaging in self-talk (thinking from Sfard’s perspective) about the mathematics involved can typically be seen to gesture. In such cases, the gesturing was small and typically involved the hands and face rather than the whole body.

A fourth moment to be aware of is a shift from the development stage of the problem to the extension stage. As outlined in chapter 8, this stage was typically indicated by the students posing or responding to ‘what if’ type questions. This is an example of a conjecturing atmosphere referred to by Mason as necessary for learning to occur (Sfard, Nesher, Streefland, Cobb, & Mason, 1998). If there are signs of a developing conversation in the extension stage, then the students are demonstrating a clear ability to ‘go on’.

In lieu of the extension stage being reached in an acceptable time frame, a plenary discussion about the problem that allows students an opportunity to demonstrate their thinking is preferred. During this period, students are asked to talk about their use of mathematics in attempting to address the task given. If they are able to articulate this process, and respond to questions, then I take this to be an indication of realising understanding. Such a view ties in with Vygotsky’s ideas of a social phase of learning while extending this to the idea that understanding is developed further if the exchange is conversational. Zolkower and Shreyar (2007) describe a similar process they refer to as ‘thinking aloud’, where guidance by the teacher is seen as an important aspect of the conversation. This step also ties in with the idea of providing students opportunities to critically re-examine how they have made mathematical claims in order to build their understanding (Francisco, 2012).

To expand on these ideas, I next address the sub-questions posed in chapter 1. In responding to the questions, which led to the main research question, I use the term ‘us’
to refer to any classroom observer, but in particular the classroom teacher. By extension, I would hope that any other interested party could also benefit from recognising this as an area of interest. I would further extend ‘us’ to include the interlocutors within a conversation, and this may be an important part of the learning process.

**Addressing the research sub-questions**

In Chapter 1, prior to establishing the research question addressed above, I developed three sub-questions:

How can the way a student opens the group talk inform us about the way the student initiates his/her thinking? Is there an organised approach to this process?

In keeping with a basic tenet of CA that there must be sufficient evidence in order to make a claim, there was no clear indicator observed which could be reliably used to answer this question. Any suggestion about initial thinking can only be inferred from the resulting activities and so would be speculative. However, this research does indicate that the more organised the group talk is in the introductory stage, then the more likely the group are to make progress with the task. Classrooms in which conversation is promoted and the teacher is active in listening for conversation and watching for gesture could support the students in initiating their thinking. This ties in with earlier research (Wells, 2009) promoting an orderly start to a problem solving session. In this, students were required to think quietly about the problem for about one minute before sharing, without access to pen or paper, their possible solution methods. This also supports research in fostering student talk by having teachers explicitly spend time to raise students’ awareness of how they talk together (Mercer 1999).

A related question was able to be more clearly addressed in this research.

**Extensions to the first sub-question**

Related to this, a further question might be: Does the way a student starts group talk tell us all we need to know about how it will continue?

It was very evident that students in group work opened their talk in a specific way, and that this way indicated if a group would be dysfunctional. In all cases in this research, the students began with one or more of the group members reading out the problem. In
groups where the last reader did not pause for a sufficient time to allow another student to make an opening contribution, the group did not communicate well. For the reader of the problem to immediately offer a solution seemed to be an affront to the other group members. This may be interpreted as a loss of ‘face’, as outlined by Goffman (1972) and developed by Brown and Levinson (1975). The result is that the early talk tends to be confrontational rather than supportive. Without the supporting organization of conversation, the talk does not seem to lead the students forward. I would note here that this feature might be considered as a reflection of the particular classroom setup of the teacher, Avio, who encouraged students to interact in a certain way. Fostering social politeness was one of Avio’s intentional goals. However, in observing other classes of my own outside of this study, both at the high school and post-secondary level, a similar start to problem solving was observed. This was true even on occasions where I was covering a class for another teacher and so had no previous contact with how the students worked. While further research would be necessary to support this claim, it seems very likely that this is a common marker for success in early group talk.

A further related question I posed in Chapter 1 may also be addressed: **Does the beginnings of group talk tell us more about the student than it does about their understanding of a concept?**

To answer this question about the beginnings of group talk, it is necessary to look back at how a particular student engages in the talk at later stages. It is evident that some students are less active in the group talk than others, but this does not mean that they are not developing mathematical understanding. Using Sfard’s ideas about thinking as a form of self-communication (Sfard, 2008), then it is possible that the student may be involved in the conversation from an internal point of view. Evidence was presented in Chapter 4 specific to a student who made few, but pertinent, contributions to the conversation. In this case, body language in the form of gesture and posture could be observed as indicators of engagement. While internal conversations cannot be commented on, such things as posture echoing can be used as an indicator that the student is in a position to ‘go on’ with their thinking and contribute to the conversation at a later stage or if pressed by the teacher or another student. The frequent presence of small gesturing which accompanied such ‘withdraw and return’ behaviour can further support the idea that gesture is an integral part of such conversation. This gesturing can
include facial gestures and noticeable posture changes. Interpreting such actions is not practical, but recognizing them can be helpful to the teacher on the sideline.

The second sub-question posed in Chapter 1 will now be addressed:

**How can the ‘conversational aspect’ of group talk (or lack thereof) inform us about a student’s developing understanding?**

A feature of group talk that was apparent was that students’ talk consisted of episodes of conversation and episodes of discussion. The data suggests that it was the episodes of conversation, when students would interact and utterances could be seen as sets of adjacent pairs that drove the interaction forward, that were most closely associated with development of the problem. Intervals of discussion, when students made statements they did not support or which were not accepted by the group had less chance of moving the problem solution forward. It may be that the discussion arose from a particular student’s internal conversation, but unless that student then engaged conversationally with the groups a mutual or shared understanding could not be formed. The apparent lack of mutual or shared understandings was a feature of dysfunctional groups. The presence of conversations and a sense of shared understanding were seen as a feature of functioning groups. These findings support research in this area which concludes that shared understanding emerges when one person’s ideas are taken up and elaborated on by others in the group (Martin, 2006; Francisco, 2012).

**Extensions to the second sub-question**

A further question might be: In terms of determining the students’ understanding or ‘readiness to continue’, is it sufficient to pose a topic and observe how the students initially addresses it? In other words, is it even necessary to have the students’ complete problems?

If understanding is accepted to be manifested in the ‘going-on’ process of the task, then reaching a specified answer does not affect any mathematical understanding. In many observed cases, group work was brought to a halt before problems were completed in order that a whole-class discussion could take place. Interrupted groups were as likely to contribute to these discussions as those who had completed their work. The caveat for this seemed to be that they should at least be able to reach the development stage of the
problem. This is of relevance to classrooms employing differentiated learning or open-ended approaches in that any time allotted for any task should be long enough for students to get into the development stage but need not require that students complete the problem, provided there is an opportunity for students to talk about their process before the task is closed. Open-ended problems would seem to fall into this category. Students should be able to demonstrate that they are able to go on with the problem, so asking a randomly chosen person in a randomly selected group – and establishing an expectation of doing so – to present his or her work in an interactive way with the class would be a way to reinforce and build upon understanding developed in the group stage. These results tie in with research that shows that long-term retention is improved when events are verbalising at the time they occur (Goswam & Bryant, 2007). At the same time, the teacher should be aware that this may differ from the shared understanding that was developed in the group conversation. This process also allows students time to cycle through the learning again, an important part of building understanding (Martin, Towers, & Pirie, 2006).

Further to this, the following question can be addressed: **What does this tell us about the students’ understanding? Can they ‘carry on’ from that point and develop their understanding further, or is the moment fleeting?**

As noted earlier, mathematical understanding can be a lasting effect or can be fleeting. In viewing understanding as a dynamic process illustrated by the student’s ability to ‘go on’ conversationally, it may be that a particular mathematical understanding has a limit which is reached when the conversation ceases. If a problem is revisited then a new conversation must be initiated which will likely be different from the previous one. In this way the mathematical understanding of the student will also be different. It need not be the case that one mathematical understanding is wrong and the other correct. The understanding is therefore seen as fluid. In traditional theories of understanding, it is unclear why, if an internal structure has been established, a student who demonstrates mathematical understanding one day seems to have a different mathematical understanding upon review, as discussed in Chapter 3. In this research, it was seen that a student could offer a conversationally rich explanation in group talk but be less articulate when called to explain their work to the class (or vice versa). The results
indicated that confidence was a factor in this, and that peer and teacher support was beneficial, as indicated by increased gesturing size.

If insight is seen as an action stemming from knowledge which then allows a student to ‘go on’ in a conversation, then an explanation may be offered for the temporary nature of understanding. If the mathematical understanding in the conversation is fed back as the student’s knowledge, then a potential for further action is established. Since this knowledge only offers a potential for action, the resulting utterance can be different each time. In contrast, a mental structure would suggest the same, or a similar, utterance should occur each time. From Sfard’s participationist perspective, learning is about changing the students’ discourse. Developing understanding as conversation can then support this change in discourse by shaping it. If the conversation in which the mathematical understanding was developed is rich enough then there is more chance that this change can occur. If the group cannot sustain the conversation long enough then there is less chance of this occurring. Re-focusing the coactions of the group in the class conversation can further support this, as noted above.

In the middle section of the talk, it was observed that students can ‘draw into a conversation’ as particular points coincide with their knowledge. The following question relates to this action by a particular student.

**How the talk terminates was another area of interest: Are the students able to reach a common, or shared, point of understanding?**

The research results discussed in Chapter 6 indicates that problem solving goes through three loosely defined stages. There is evidence of an introductory stage in which students establish the meaning and expectations of the problem. Some strategy may also be selected from perhaps several suggested at this point. Some groups are unable to form enough of a conversation, and so reach a presumed shared understanding (either of the problem or its mathematics), to progress to the next stage. I emphasise here that students can only presume a shared understanding based on the responses from their partner(s). A lack of conversation makes it harder for group members to determine if this understanding has been reached. In this way, conversation (which includes gesture) can also be what drives the problem solution forward. Rather than just see this understanding as leading to a solution it also seems evident that the group talk
consists of a series of understanding events, or episodes. The conversation must then also exist on a larger scale in tying these episodes together in order to progress towards a solution. To be successful in this, it also seems evident that the students need to hold a shared mathematical understanding with their interlocutors. This mathematical understanding may be quite different but can be taken as shared and allows the process to continue. Evidence that the mathematical understanding is not shared can then cause the conversation to break down, requiring repair work to be done. If one student is able to ‘go on’ in their thinking then repair work may exist in the form of an explanation, and this explanation is evidenced by an increased and further animated gesturing. As noted above, it is the process of engaging in a conversation about the problem that establishes the mathematical understanding and should therefore be seen as the goal of the problem.

If group talk reaches a conclusion then there is evidence of a final conversation, as this is then the students’ way of indicating that they have a shared mathematical understanding of the result. In many cases this then results in the students moving to a more social aspect, believing that they are done. In a few cases (at this level) a student questioned this conclusion further and generated further discussion. If the group talk was unable to reach a conclusion and the session generally did not end with a conversation. In this case there was often evidence that the group were not interacting as seen in their posture and gesturing.

The third sub-question posed will now be addressed:

How can the nature of a student’s gesturing/body language help inform us about a student’s understanding?

There is good evidence in this research that gesturing and body language must be considered as an important part of the communication process. In each episode discussed in Chapters 6 through 8, the gesturing of the student was seen to alter in relation to the organization of the talk. As noted, hand/arm gesturing was most dynamic when a student was explaining their idea or describing an event. The more confident a student seemed to be then the larger gesture space they occupied. This supports the work of Gerofsky (2008, 2010). The change in gesturing was even more evident when a student was standing and their hands were free, for example when explaining their work.
to the whole class. If students held an object, such as a board marker, the gesturing was reduced. When students were working mathematically, i.e. actually on the problem, their gesturing was greatly reduced.

If we accept the idea that understanding is the ability to ‘go on’ conversationally, then any written calculations may just be illustrations of knowledge in that they lack a conversational organization. The mathematical understanding may arise before and after such actions when the student converses internally, what can be viewed as the thinking process according to Sfard (2009), or externally with another student or teacher. This may also offer a reason why it is difficult to explain a process and perform it at the same time. In addition to hand/arm gesturing, vocal gesturing was also evident in the conversations between the students. The intonation and tone variations of the conversation seem to provide important clues to the listener in order to help them reach a shared mathematical understanding of the solution being generated. Eye contact and posture echoing were examples of how group members indicated to each other that they were, or believed themselves to be, sharing an understanding. In Chapter 9, it was also discussed how gesture echoing was another indicator that students were working on a shared understanding (which may be mathematical or about the problem situation).

**Extensions to the third sub-question**

I have noted in several places that students who are more confident of their ideas and engaged in the thinking tend to use more and larger gestures. I also put forward the idea there may be a reciprocal effect to this so that an increase in gesturing could be helping a student’s confidence. This would point to a natural feedback effect which a teacher could look for and support. When students were able to stand and move around when discussing a problem, they seemed to benefit from being able to do so as required. This can tie into the idea of dynamic feedback, both in terms of bodily motion and in the spoken form of a conversation. If this was in fact the case, then providing students with a flexible workspace could help some students in their development.

Further to this question, the following point can be addressed: **Are these points indicated by a change in the way the student gestures or in their body language? Can we observe this from a distance?**
An important observation of this research for me was the occurrence of posture and gesture echoing, as discussed in Chapter 9. A further important aspect of gesturing that was immediately noticeable was in the way students’ gestures changed when they were explaining their thinking to the group or class. Students working in unison generally displayed evidence of posture echoing. Students within the group who were not displaying this echoing were either disengaged or had stepped back to consider the problem privately. Evidence of these states could be inferred by self-gesturing the students displayed when thinking further about the problem, and/or their gestures when they returned to the group. These results support previous research in this area (Kimbara, 2008; Holler & Wilkin, 2011), while extending the findings to classroom activity.

If a student had a seemingly important idea then they would quickly seek to establish a turn at talk, even interrupting if necessary. Such interruptions were generally seen as acceptable behaviour by the group. Having established the turn at talk, the student then demonstrated gesturing that was more animated and which occupied a larger gesture space. The gestures, particularly facial and body gestures, of the interlocutors gave evidence of whether the new idea was acceptable. If accepted, the idea generated conversation.

**Teaching from the sidelines**

Teaching from the sidelines is a suggested approach to group collaborative work based on the findings of this research.

The intent of this research was to suggest indicators that a teacher could look for in a classroom setting which relate to the understanding of a student. Recording and using the tools of conversation–gesture analysis, as done in this research, is clearly impractical. The results from this research do, however, suggest that there are indicators that a teacher can look for in real-time in order to help recognise developing understanding amongst students in the classroom. Even if the notion of understanding as going on conversationally is rejected, I believe these markers are still valid in helping the teacher determine how to manage group talk in the classroom. The concept of teaching from the sidelines is that the teacher stands far enough away from a group so as to
minimise his/her effect on that group. At the same time, the teacher can be aware of the progress of the group by looking at and listening to the group members.

My pilot study highlighted how it was very possible for me to stand back from any group in the room and yet tune into the group talk. This process seemed far more difficult when listening to a recording. It seems evident that I was taking in more information than just the sound when I was in the classroom, but also that I was able to do this from across the room. Through this research I have now become aware of looking for the gestures and postures of the students as they interact with one another, when before this may have been done subconsciously. It is possible to recognise, for example, when students are engaged in mutual activity from any posture and/or gesture echoing. A dysfunctional group – one that may be helped by intervention – is apparent by a lack of such echoing. An individual who is not engaged in a larger group is similarly noticeable.

By listening carefully to how groups begin to solve a problem, a teacher may choose to intervene quickly if the student who reads the problem impolitely violates the turn-taking aspect which characterises the introductory stage of the group work. Similarly, flouting any of Grice’s maxims of quality, quantity, relevance, and manner, or that of politeness, can stall the functioning of the group. Working with students who routinely flout these maxims might be considered as a way to better integrate them into the class. It may be the case that students are expected to know how to function in a group setting, and while this is a skill many have developed, there is work that can be done to improve this. Conversation may be a skill teachers need to teach, example, and foster in their classrooms if they are to have success in non-traditional ways of teaching.

Students who are functioning cooperatively in a group are best left alone as the casual ‘dropping in’ of a teacher may break the pattern of conversation and disrupt the understanding process. Conversely, it is important that a group be monitored so that they remain on-task and develop a mathematical understanding that meets the intended outcome of the problem (or extends beyond it or beside it in a productive manner). Learning how to listen to and watch for productive indicators of success would be a useful habit of mind for teachers to develop. Similarly, giving students the opportunity to come together and converse about what they have achieved, even for incomplete work, should also be seen as an important part of this process. Having an expectation that
anyone can be called upon to offer an explanation may also be important here as a means to prepare students to speak before their peers.

**Suggestions and further research**

If a teacher is to be successful in fostering mathematical understanding in the classroom through group talk then the following suggestions, arising from the results of this research, may be adopted. These are based on markers that can act as indicators of a developing mathematical understanding which can add to a student's knowledge.

a) Position groups so that it is possible to move around the room and eavesdrop on them without being obvious in doing so.

b) Listen to the talk in an active way. In particular, listen for evidence of conversational organization as indicated by adjacent pairs, overlapping speech, carry-ons (where there is little to distinguish students’ turns at talk), and tonal changes. Listen for examples of playing at rather than playing in the language game of the problem. Students playing at the game may be parroting words without demonstrating mathematical understanding.

c) Be aware of how the group talk begins. Listen for violations of the maxims. Be prepared to take aside and coach students who repeatedly violate these maxims. If a group appears to be making no progress this may be an indicator that a maxim has been violated and that the students are in conflict, even though they may not be aware of this conflict themselves.

d) Watch for posture echoing and gesture echoing as good indicators that the group is functioning well. Posture ‘drop-out’, when a student disengages from the group or becomes isolated from the group, is something that should be watched for. A remedy might be as simple as moving into the sightline of such a student. If the student does not return to the group posture quickly then an intervention may be called for. The student can also be watched for evidence of ‘self-gesturing’ – usually in the form of small hand movements – which is a good indicator that the student is still engaged but is involved in an internal conversation in order to clarify his/her thinking.
e) Notice the gestures of the students. Close gesturing tends to be associated with calculations, internally or externally, while large gesturing is associated with explanations or descriptions. The bigger the gesture space the student uses then the more confident he/she is of what they are saying. Many mathematical gestures related to numbers are deictic in nature; while those related to shape and space tend to be more representative, either iconic, metaphoric, or a combination of both. Beat gestures are often used to indicate a number, either in itself or as a counting process, but also to establish a rhythm. Container gestures are commonly used when there is no clear shape to a thought, but the speaker is trying to grasp something as an idea and give it form.

f) Foster conversational skills with students. Encourage an open exchange of thoughts.

g) Leave ample time at the end of group work to allow for some students to explain what has been discovered about the problem other than just giving a method to find the solution. Select students to speak rather than only taking volunteers. Foster a conversational attitude to this session.

By incorporating these suggestions into the regular classroom, and recognising a student’s understanding as their ability to ‘go on conversationally’, a teacher is in a better position to look for such understanding when students are engaged classroom talk. This approach places mathematical understanding into the conversational space rather than seeing it as an internal process. Not all such conversation is externally vocalised, but being aware of body language and gesturing may be an indicator of when a student is thinking as an internal conversation related to the mathematics. Importantly, being more aware of the interaction between the students and their participation in the talk can give the teacher guidance in when to best intervene to support the students.
Bibliography


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Appendix.

Data Collection Summary

Recordings were made on a regular basis as outlined in chapter 5. The following table provides further information on these recordings.

Number of recordings used: 150
Total time of recordings: 26.5 hours
Total time of raw transcriptions: 20.2 hours
Total time of marked up transcriptions: 9.1 hours
Recordings of individual students presenting: 41
Recordings of two student group: 29
Recordings of three student group: 30
Recordings of four student group: 21
Recordings of five student group: 6
Recordings of class conversation: 22
Marked up recordings of individual students presenting: 7
Marked up recordings of two student group: 20
Marked up recordings of three student group: 20
Marked up recordings of four student group: 15
Marked up recordings of five student group: 4
Marked up recordings of class conversations group: 4