ADVANCED RELAY SELECTION SCHEMES FOR
COOPERATIVE COMMUNICATIONS

by

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THE REQUIREMENTS FOR THE DEGREE OF

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Faculty of Applied Sciences

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Abstract

It is widely considered that cooperative diversity, in idealized conditions, can provide dramatic performance improvements in capacity and error probability. But non-ideal phenomena, such as fast time variations of the channel, channel estimation errors (CEE), feedback delay (FD) and co-channel interference, degrade the performance of the cooperative communication networks.

This thesis studies the impact of these phenomena on advanced relay selection techniques in cooperative communication networks. In particular, Chapters 2-4 treat inter-vehicular communication (IVC) networks with fast time variations in the channel. For different relaying protocols, the impact of CEE and FD on the performance of cooperative communication with relay selection is presented in terms of average capacity, average symbol error rate, outage probability, and the achievable diversity order. Chapters 5 and 6 discuss how interference deteriorates the performance of a cognitive secondary network using relay selection in the presence of a primary network which has priority access to the spectrum. The primary and secondary network signals are treated as mutually interfering noise. Transmission powers in the secondary network are constrained such that the primary user can still perform satisfactorily. Based on interference levels, new selection and power control techniques are presented for the secondary network, and the outage probability is determined. Finally Chapter 7 summarizes the research and proposes future works related.
To my beloved parents
and
my adorable fiancée
Acknowledgements

Thank you God for letting me be here and for letting me make it this far.

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He helped me a lot when I needed help, he guided me we I needed guidance, and he did all his best to link me to industry owners and to find me career opportunities. I am really thankful for his supports.

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At last I would like to express my immense gratitude and respect to my lovely parents, family and fiancée who never stopped supporting, encouraging and standing by me during the ups and downs of my life. I dedicate this thesis which is the outcome of my life to the ones to whom I owe every single success in my life.
List of Symbols

$X$ \hspace{1em} Random variable $X$

$x$ \hspace{1em} Represents a vertical vector

$X$ \hspace{1em} Represents a matrix

$E\{\cdot\}$ \hspace{1em} Expected value

$S$ \hspace{1em} Source node

$D$ \hspace{1em} Destination node

$R_m$ \hspace{1em} The $m^{th}$ relay

$CN(0, \sigma^2)$ \hspace{1em} Complex Gaussian random variable with zero mean and variance of $\sigma^2$

$f_X(\cdot)$ \hspace{1em} PDF of random variable $X$

$F_X(\cdot)$ \hspace{1em} CDF of random variable $X$

$h_{ij}$ \hspace{1em} Communication channel between node $i$ and node $j$

$P$ \hspace{1em} Transmitted power in 1 second in Joules/Symbol

$N_0$ \hspace{1em} Receiver noise

$\mathcal{D}(s)$ \hspace{1em} Decoding set

$\{\cdot\}^T$ \hspace{1em} Transposition

$\{\cdot\}^*$ \hspace{1em} Complex conjugation

$\{\cdot\}^H$ \hspace{1em} Transpose complex conjugate

$\| \cdot \|$ \hspace{1em} Absolute value
**List of Acronyms**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>AF</td>
<td>Amplify-and-Forward</td>
</tr>
<tr>
<td>ANC</td>
<td>Analogue Network Coding</td>
</tr>
<tr>
<td>AP-AF</td>
<td>All Participate Amplify-and-Forward</td>
</tr>
<tr>
<td>ASER</td>
<td>Average Symbol Error Rate</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BBO</td>
<td>Binary Biogeography Based Optimization</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>BSS</td>
<td>Blind Source Separation</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Density Function</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>CEE</td>
<td>Channel Estimation Errors</td>
</tr>
<tr>
<td>CR</td>
<td>Cognitive Radio</td>
</tr>
<tr>
<td>CRC</td>
<td>Cyclic Redundancy Codes</td>
</tr>
<tr>
<td>CRN</td>
<td>Cognitive Radio Networks</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>CTS</td>
<td>Clear-to-Send</td>
</tr>
<tr>
<td>DaF</td>
<td>Detect-and-Forward</td>
</tr>
<tr>
<td>DF</td>
<td>Decode-and-Forward</td>
</tr>
<tr>
<td>DPSK</td>
<td>Differential Phase Shift Keying</td>
</tr>
<tr>
<td>DSTBC</td>
<td>Distributed Space Time Block Code</td>
</tr>
<tr>
<td>FD</td>
<td>Feedback Delay</td>
</tr>
<tr>
<td>FPA</td>
<td>Fractional Power Allocation</td>
</tr>
<tr>
<td>FSK</td>
<td>Frequency Shift Keying</td>
</tr>
<tr>
<td>ICA</td>
<td>Independent Component Analysis</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Definition</td>
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<tr>
<td>--------------</td>
<td>-----------------------------------------</td>
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<tr>
<td>ITS</td>
<td>Intelligent Transportation Systems</td>
</tr>
<tr>
<td>IVC</td>
<td>Inter-vehicular Communication</td>
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<tr>
<td>LMSE</td>
<td>Least Mean Square Error</td>
</tr>
<tr>
<td>LoS</td>
<td>Line of Sight</td>
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<tr>
<td>MIMO</td>
<td>Multi-Input-Multi-Output</td>
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<tr>
<td>MRC</td>
<td>Maximum Ratio Combiner</td>
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<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
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<tr>
<td>PAM</td>
<td>Pulse Amplitude Modulation</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<tr>
<td>PDE</td>
<td>Perfect decoding Estimation</td>
</tr>
<tr>
<td>PSK</td>
<td>Phase Shift Keying</td>
</tr>
<tr>
<td>PU</td>
<td>Primary User</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>RTS</td>
<td>Ready-to-Send</td>
</tr>
<tr>
<td>S-AF</td>
<td>Amplify-and-Forward Selection Cooperation</td>
</tr>
<tr>
<td>SC</td>
<td>Selection Combining</td>
</tr>
<tr>
<td>S-DF</td>
<td>Decode-and-Forward Selection Cooperation</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal-to-Noise-and-Interference-Ratio</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SU</td>
<td>Secondary User</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>TWRN</td>
<td>Two Way Relay Network</td>
</tr>
<tr>
<td>ZMUVCG</td>
<td>Zero Mean Unit Variance Complex Gaussian</td>
</tr>
</tbody>
</table>
Contents

Approval ii
Abstract iii
Dedication iv
Acknowledgements v
List of Symbols vii
List of Acronyms viii
Contents x
List of Figures xv

1 Introduction 1
  1.1 Background ............................................................. 1
  1.2 Objectives and Contributions ......................................... 5
    1.2.1 Effect of channel model, Rayleigh fading vs. cascaded Rayleigh fading 6
    1.2.2 Effect of channel estimation errors ................................ 7
    1.2.3 Effect of feedback delay ......................................... 7
    1.2.4 Effect of co-channel interference in cognitive radio networks .... 9
  1.3 Scholarly publications .................................................. 10
2.5 Simulation Results ................................................. 18
2.6 Conclusion ......................................................... 20

3 Relay Selection with Imperfect CSI .................................. 23
3.1 Objective ......................................................... 23
   3.1.1 Related work and contributions ................................. 23
3.2 System Model ..................................................... 24
3.3 Modeling of Channel Estimation Error ............................... 26
3.4 Selection Strategy ................................................ 28
3.5 Average Symbol Error Rate Analysis ................................. 29
3.6 Outage Probability Analysis ....................................... 33
3.7 Average Capacity Analysis ........................................ 34
3.8 Asymptotic Order of Diversity .................................... 36
3.9 Simulation Results ............................................... 37
   3.9.1 ASER analysis ................................................. 37
   3.9.2 Outage probability analysis .................................. 39
   3.9.3 Asymptotic diversity order .................................. 41
   3.9.4 Average capacity ............................................. 42
3.10 Conclusion ....................................................... 44
3.A Deriving the Probability density function of $\gamma_{\text{ub}}$ ............ 44
3.B Proof of Theorem 1 ................................................ 46
3.C ASER Performance for PAM, QAM, and PSK Modulation Schemes .... 47
   3.C.1 Overall picture ................................................. 47
   3.C.2 PAM constellation ............................................. 50
   3.C.3 QAM constellation ............................................. 51
   3.C.4 PSK constellation ............................................. 52

4 Relay Selection with Feedback Delay .................................. 54
4.1 Objective ......................................................... 54
   4.1.1 Related work and contributions ................................. 55
4.2 System Model ..................................................... 56
   4.2.1 Model of channel estimation error .............................. 57
   4.2.2 Feedback delay model ......................................... 59
4.3 Outage Probability Analysis ....................................... 59
1.3.1 Journal papers .................................. 10
1.3.2 Ongoing/Submitted research papers ............... 10
1.3.3 Conference papers ................................ 11

2 Relay Selection for Vehicular Communications ........ 12
  2.1 Objective ......................................... 12
     2.1.1 Related work and contribution .................. 12
  2.2 System Model ...................................... 13
  2.3 Outage Probability ................................ 14
  2.4 Maximum Achievable Diversity Order .............. 17
  2.5 Simulation Results ................................ 18
  2.6 Conclusion ........................................ 20

3 Relay Selection with Imperfect CSI ..................... 23
  3.1 Objective .......................................... 23
     3.1.1 Related work and contributions ............... 23
  3.2 System Model ...................................... 24
  3.3 Modeling of Channel Estimation Error ............ 26
  3.4 Selection Strategy ................................ 28
  3.5 Average Symbol Error Rate Analysis ............. 29
  3.6 Outage Probability Analysis ...................... 33
  3.7 Average Capacity Analysis ......................... 34
  3.8 Asymptotic Order of Diversity .................... 36
  3.9 Simulation Results ................................ 37
     3.9.1 ASER analysis ................................ 37
     3.9.2 Outage probability analysis .................. 39
     3.9.3 Asymptotic diversity order .................. 41
     3.9.4 Average capacity ................................ 42
  3.10 Conclusion ........................................ 44
  3.A Deriving the Probability density function of $r_{\text{stub}}$ .... 44
  3.B Proof of Theorem I ................................ 46
  3.C ASER Performance for PAM, QAM, and PSK Modulation Schems 47
     3.C.1 Overall picture ................................ 47
     3.C.2 PAM constellation ............................. 50
4 Relay Selection with Feedback Delay

4.1 Objective ................................................. 54
  4.1.1 Related work and contributions .......................... 55
4.2 System Model ................................................. 56
  4.2.1 Model of channel estimation error .......................... 57
  4.2.2 Feedback delay model .................................. 59
4.3 Outage Probability Analysis .................................. 59
  4.3.1 Outage probability with feedback delay and channel estimation errors 60
  4.3.2 Probability of decoding set ............................... 61
  4.3.3 Outage probability conditioned on the decoding set $D(s)$ .......... 62
4.4 Average Capacity ............................................. 64
4.5 Average Symbol Error Rate .................................... 67
4.6 Asymptotic Diversity Order ................................... 69
  4.6.1 Diversity order with FD and perfect CSI .................. 70
  4.6.2 Diversity order in the presence of FD and CEE, when $P$ is constant . 71
  4.6.3 Diversity order in the presence of FD and CEE, when $P \to \infty$ .... 71
4.7 Simulation Results ............................................. 71
  4.7.1 Outage probability ........................................ 72
  4.7.2 Average capacity ......................................... 74
  4.7.3 ASER performance ....................................... 74
  4.7.4 Diversity order .......................................... 76
  4.7.5 Training power $P$ ....................................... 76
  4.7.6 Distance ................................................ 76
4.8 Conclusion ................................................... 81
4.A Deriving the Probability Density Function of $\gamma_{m,d}$ ........... 82
4.B ASER Performance for PAM, QAM, and PSK Modulation Schemes .... 83
  4.B.1 Overall picture ......................................... 83
  4.B.2 PAM constellation ....................................... 85
  4.B.3 QAM constellation ....................................... 86
  4.B.4 PSK constellation ....................................... 88
5 Relay Selection in CRNs with interference

5.1 Objective ................................................. 90
   5.1.1 Related work and contribution ................. 90
5.2 System Model ........................................... 92
   5.2.1 Channel estimation ............................... 94
   5.2.2 Transmission power constraints ............... 95
   5.2.3 Relay selection scheme ......................... 96
5.3 Performance Analysis of Cooperative CRNs with Interference Constraints ...... 97
   5.3.1 Outage probability analysis .................... 98
5.4 Discussion .............................................. 99
5.5 Simulation Results .................................... 102
5.6 Conclusion ............................................. 105
5.A Cumulative Density Function, $F_\gamma(x)$ ....... 106
5.B Cumulative Density Function, $F_\zeta(x)$ .......... 107
5.C Derivation of Outage Probability .................. 108

6 Modified Selection Cooperation in CRNs

6.1 Objective ................................................. 110
   6.1.1 Related work and contribution ................. 110
6.2 System Model ........................................... 111
   6.2.1 Channel estimation ............................... 113
   6.2.2 Transmission power constraints ............... 113
   6.2.3 Relay selection scheme ......................... 116
6.3 Outage Probability Analysis ........................... 117
6.4 Simulation Results .................................... 118
   6.4.1 Effect of Relay Position: ....................... 118
   6.4.2 Effect of Primary User Position or primary user power: .... 119
6.5 Conclusion ............................................. 120

7 Conclusion and Future Works

7.1 Conclusion ............................................. 121
7.2 Future Works .......................................... 123

Bibliography .................................................. 127
## List of Figures

1.1 Cooperative diversity. ................................................................. 1  
1.2 AF and DF cooperative protocols. ........................................... 2  
1.3 LMSE channel estimation and relay selection in the first strategy. ........ 3  
1.4 LMSE channel estimation and relay selection in the second strategy. .... 4  
1.5 Selection and feedback delay. $f_s = 20$ KHz, $f_D = 600$ Hz .............. 8  
2.1 Outage probability for equal power scheme .................................... 19  
2.2 Outage probability with power allocation ...................................... 20  
2.3 Outage probability, Rayleigh fading vs. Cascaded Rayleigh fading ....... 21  
2.4 Diversity order ........................................................................... 21  
3.1 LMSE for channel estimation. Principal of orthogonality is described .... 27  
3.2 ASER vs. SNR in the presence of CEE, $M = 2$ ................................. 38  
3.3 ASER vs. SNR in the presence of CEE, $M = 3$ ................................ 38  
3.4 Outage probability vs. SNR in the presence of CEE, $M = 2$ .............. 39  
3.5 Outage probability vs. SNR in the presence of CEE, $M = 3$ .............. 40  
3.6 Outage probability for $M = 2$ relays vs. distance for different SNR values 40  
3.7 ASER vs. SNR, Performance of S-AF and AP-AF Cooperation for $M = 2$ 41  
3.8 Outage probability vs. SNR, Performance of S-AF $M = 3$ ............... 42  
3.9 Average capacity per bandwidth vs. SNR, Relay selection Scenario, $M = 2$ 43  
3.10 Average capacity per bandwidth vs. SNR, S-AF vs. AP-AF ............... 43  
3.11 Minimum distance in the receive constellation ................................. 52  
4.1 System model for decode-and-forward relay selection with delayed feedback 57  
4.2 Signal transmission procedure ..................................................... 60  
4.3 Outage probability vs. SNR in the presence of FD, $M = 4$ ............... 72
4.4 Outage probability vs. SNR in the presence of FD, $M = 2, 3, 4$ ....... 73
4.5 Outage probability vs. SNR in the presence of FD and CEE ........ 73
4.6 Lower bound average capacity vs. SNR ........................................ 74
4.7 ASER vs. SNR in the presence of FD, $M = 3$ ............................... 75
4.8 ASER vs. SNR in the presence of FD/CPE, $M = 2$ ......................... 75
4.9 Diversity order versus SNR in the presence of FD ......................... 77
4.10 Diversity order versus SNR in the presence of FD, $M = 2, 3, 4$ ....... 78
4.11 Diversity order vs. SNR, $\rho_f = 0.9$ and $M = 3$ ......................... 78
4.12 ASER vs. SNR for different values of the training sequence power .... 79
4.13 ASER vs. distance ................................................................. 80
4.14 Probability of decoding set vs. distance ...................................... 81
4.15 $d_{\text{min}}$ in the PSK modulation scheme .................................... 88

5.1 Schematic of the assumed system, primary and secondary networks topology .... 92
5.2 Outage probability for limited threshold limit $I, I = 13$ dB ............. 101
5.3 Outage probability for limited threshold limit $I, I = 30$ dB ............. 101
5.4 Outage probability vs. $P_t, P_o = 30$ dB, $I = 13$ dB, $P_t \to \infty$ and $M = 2$ . 103
5.5 Outage Probability vs. the $P_{UT}$ position, $P_o = 30$ dB, $I/P_o \to \infty$ and $M = 2$ 104
5.6 Outage Probability vs. $P_{UT}$ and $P_{UR}$ positions .......................... 104
5.7 Outage Probability vs. $P_{UT}$ and $P_{UR}$ positions .......................... 105

6.1 Channel estimation and relay selection in two time slots ................. 113
6.2 Outage probability vs. relay position ............................................ 118
6.3 Outage probability vs. $P_{UT}$ positions ........................................ 119
Chapter 1

Introduction

1.1 Background

The promise of high spectral efficiency and capability of providing great capacity improvements in a wireless fading environment, as reported by [1] and [2], has led to widespread interest in multi-input multi-output (MIMO) communications. However, due to size, cost, and/or hardware limitations, a wireless device may not be able to support multiple transmit antennas. Cooperative diversity was proposed as an alternative to MIMO systems.

![Cooperative diversity](image)

Figure 1.1: Cooperative diversity.

It has been demonstrated that cooperative diversity provides an effective way of improving spectral and power efficiency of the wireless networks [3, 4]. The main idea behind cooperative diversity is that in a wireless environment, the signal transmitted by the source \( S \) is overheard by other nodes, which can be defined as “relays” [5] (see Fig. 1.1). The
source and its partners can then jointly process and transmit their information, thereby creating a “virtual antenna array”, although each of them is equipped with only one antenna. It is shown in [6–14] that cooperative diversity networks can achieve a diversity order equal to the number of paths between the source and the destination, however, the need of transmitting the symbols in a time division multiple access (TDMA) fashion limits the improvement in capacity.

There are many protocols provided for cooperation between source, relays and destination, among which amplify-and-forward (AF) and decode-and-forward (DF) absorbed the most attention in research.

![AF and DF cooperative protocols.](image)

The AF protocol transmission of data from source to destination \(D\) is divided into \(M+1\) consecutive time slots. In the first time slot the source node broadcasts its data toward the destination node as well as the relays. The relays receive the transmitted signal, normalize it power-wise\(^1\) and forward the normalized version toward the destination in \(M\) consecutive non overlapping time slots following TDMA transmission [4, 14] (see Fig. 1.2). Generally in DF, in the first time slot the source node transmits its data toward the destination and the relays. The relays decode the received signals from the source node and constitutes a set called the decoding set. The decoding set is defined as the set of the relays which successfully decoded their received signals [5, 7]. DF protocol in general can either be used for coded sequences which is called coded DF, or can be applied for uncoded signals which is termed as uncoded DF. In the coded DF, cyclic redundancy codes (CRC) or other error correction codes [15, 16] are added to the symbols in the source node. Then the relays upon

\(^1\)The normalization is performed by the relay so that it remains in its power constraint.
decoding the received signals from the source node can verify the credibility of their decoded signal by checking with the transmitted known error correction codes. In the uncoded DF the scenario is a bit different. The credibility of detection in the relay nodes in this scheme is verified by defining a threshold SNR. In particular, if the received SNR at the relays exceed a predefined threshold limit the relays are in the decoding set with a high probability, viz., close to 1. From the second time slot, the relays that are in the decoding set transmit their signals toward the destination in $D$ consecutive time slots, where $D$ represents the cardinality of the decoding set.

Existence of practical limitations such as power allocation constraints, synchronization between relays, and bandwidth efficiency makes multiple relay deployment less efficient. In this regard, relay selection was proposed to alleviate the loss in spectral efficiency caused by multiple relay schemes and also to moderate the power allocation constraints [7–9, 11–13].

There are two main selection strategies in the literature. In the first scenario, the relay nodes are in charge of monitoring of their individual source and destination instantaneous CSI [17]. In this strategy, CSI is estimated at the relay nodes for further decisions on which relay is the best node to cooperate. In particular, when the relay nodes overhear a ready-to-send (RTS) packet from the source terminal, they estimate the source link CSI. Then, upon receiving a clear-to-send (CTS) packet from the destination, the corresponding destination link at the relays is estimated. Right after receiving the CTS packet, the relay nodes start a timer which is a function of the instantaneous relay-destination CSI$^2$. The timer of the best

\[ \text{Figure 1.3: LMSE channel estimation and relay selection in the first strategy.} \]

\[ \text{The timer is a function of relay-destination CSI for the DF mode and source-relay-destination CSI} \]
relay expires first, and a flag packet is sent to other relays, informing them to stop their timers as the best relay is selected. Then the relays that receive the flag packet, keep silent and the selected relay participates in communication [17] (see Fig. 1.3).

In the second strategy, mainly for DF, the relay nodes estimate their individual uplink CSI by applying least mean square estimator (LMSE) on the received pilot symbols from the source node. The relays that correctly decode their received symbols from the source, send a flag packet to the destination, announcing that they are ready to participate in cooperation. The destination terminal, on the other hand, estimates the downlink CSI, orders the received SNRs from all relays in the decoding set, and feeds back the index of the best relay that introduces the maximum received SNR via a $\lceil \log_2 M \rceil^+$ bit feedback link, where $\lceil \cdot \rceil^+$ is a “quantize to the upper level” function and $M$ is the number of relays. The selected relay then operates with full power [14] (see Fig. 1.4).

![Figure 1.4: LMSE channel estimation and relay selection in the second strategy.](image)

In both strategies, a very important issue that must be taken into consideration is the selection speed. Communication links among terminals are time varying with a macroscopic rate in the order of Doppler shift, which is inversely proportional to the channel coherence time [17]. Any relay selection scheme must be performed no slower than the channel coherence time, otherwise selection is performed based on the old CSI, while the channel conditions are altered at the time that selection is performed. This might lead to a wrong selection of relay and therefore affect the performance of the system.

(harmonic mean [18]) for the AF protocol.
CHAPTER 1. INTRODUCTION

Recently, there have been considerable research efforts on the performance analysis of cooperative diversity including the derivation of closed-form formulas for the average symbol error rate (ASER) [5, 6, 8, 10–12, 19], the outage probability [7, 9, 10, 13, 17], and the average capacity [7, 10, 11, 20]. In particular [5] studies the performance of both AF and DF cooperative diversity at high SNRs for general channels. [6] analyzes the exact ASER performance of AF protocol in cooperative communication when all the relays participate in transmission. [8] studies the ASER performance of selection cooperation versus all participate cooperation with power allocation. The authors in [10–12] study relay selection in Rayleigh and Nakagami fading channels. [19] studies the performance of cooperative OFDM with multiple relays and relay selection.

Another topic that we investigate in this dissertation is relay selection in the context of cognitive radio networks (CRN). Cognitive radio (CR) [21–23] is an emerging communication paradigm in which wireless systems have the ability to sense the environment and change their transmission or reception parameters to communicate efficiently, avoiding interference with licensed (primary) or unlicensed (secondary) users. An important feature of cognitive radio is to maximize spectrum utilization without causing harmful interference to other users, leading to a higher data rate wireless access in order to meet the increased demand for bandwidth intensive services, while improving energy efficiency.

To control the interference experienced at the primary users (PU), three basic spectrum access paradigms, i.e., overlay, underlay, and interweave, can be utilized [24]. In the overlay paradigm, the cognitive radio users employ advanced coding techniques to control the interference imposed on the primary users, while maximizing spectrum usage for their own communications. In an underlay system, a secondary user (SU) can have full spectrum access if the imposed interference on the primary users’ spectrum is less than a threshold limit. Finally, in the interweaved approach, the secondary users exploit the spectrum holes to communicate freely without disrupting the primary users’ transmissions. We study the performance of relay selection in a cooperative SU network in the vicinity of a PU, taking into account the mutual co-channel interferences in our analysis.

1.2 Objectives and Contributions

The objectives of this research are to propose/extend and analyze advanced relay selection schemes in cooperative communications networks. The overall focus of this research is to
study the performance of relay selection algorithms in the setups with non ideal channel acquisition. The effects of rapid channel variations in inter-vehicular communication, channel estimation errors, delayed feedback links and co-channel interference on relay selection in cooperative networks and new relay selection schemes in cooperative CRNs with co-channel interference are the main subjects that we study in this research. In each topic, we will perform theoretical analysis and use simulations to verify the results. The following is a summary of the main topics and contributions in this thesis:

1.2.1 Effect of channel model, Rayleigh fading vs. cascaded Rayleigh fading

Vehicular ad-hoc networks (VANETs) [25] are a crucial component of the intelligent transportation systems which involve the application of advanced information processing, communications, sensor, and control technologies in an integrated manner to improve the functioning of the transportation system. VANETs have received much attention in recent years with their potential of enormous improvements in the road safety, and elimination of the excessive cost of traffic collisions. Besides safety improvement and traffic management, other applications, such as audio/video streaming, real-time gaming and high-speed internet access are also envisioned. Considering the lack of infrastructure, cooperative diversity [26] (also known as user cooperation) has been recently proposed as an efficient solution to many challenging physical-layer problems in VANETs [27]. User cooperation takes advantage of the broadcast nature of the radio-frequency transmission and creates virtual antennas among the nodes which are willing to share their resources. Dual-hop transmission is a special case of user cooperation where no direct transmission between source and destination is possible. In chapter 2, we consider a dual-hop inter-vehicular communication system where a source vehicle communicates with a destination vehicle via another vehicle acting as a relay node. We assume that a number of vehicle nodes are willing to act as relays, therefore relay selection is performed among them. Relay selection has been studied extensively in the literature, see e.g., [9,12,13,17,28] and the references therein. However, current results are limited to Rayleigh fading channel assumption. This fading model is typically used for cellular systems which involve a highly-elevated stationary base station and a mobile node at street level. On the other hand, in inter-vehicular systems, both transmitter and receiver nodes are in motion with their antennas at lower and comparable elevations. Experimental
and theoretical studies [4, 29] have reported cascaded (double) Rayleigh fading as an appropriate small-scale fading model for inter-vehicular channels. In chapter 2 we will investigate the outage performance and the achievable diversity order of dual-hop transmissions with relay selection over cascaded Rayleigh fading channels.

1.2.2 Effect of channel estimation errors

There exists a rich body of literature on relay selection [7–14]. While several relay selection schemes are proposed for both DF and AF networks, most of these schemes assume the availability of perfect CSI at the network nodes. Although channel estimation for relay communication has also been studied in literature as in [30–32], very few works have analyzed the effect of channel estimation errors (CEE) on the performance of cooperation in the wireless networks [33–35]. In chapter 3 we investigate the impact of CEE on the AF cooperative networks with relay selection. Following the legacy of AF protocol we study a system with one source, one destination and \( M \) potential relays to cooperate with the source in transmitting its data toward the destination. In our system only one relay which is defined as the best relay is selected based on a cost function. In this system acquisition of CSI is established based on an LMSE estimator which indeed exposes the system with CEE. Adopting a model for the errors from [36] we extend detailed analysis to study the effect of CEE on the performance of the target system with relay selection. We analyze the probability of outage in the system, average capacity, ASER, and finally the achievable diversity order of the system in the presence of CEE. We show that under the assumption of constant training power the outage probability and the ASER curves are subjected to an error floor in medium to high SNRs which imparts the diversity order of the system is reduced to zero. Nevertheless, if the training power is increased in parallel with the data SNR then the full diversity order of \( M + 1 \) is obtained. At last, extensive computer simulations corroborate of the provided theory in this chapter.

1.2.3 Effect of feedback delay

While channel estimation is a major concern in the cooperative networks due to issues of designing training sequences, synchronization, and bandwidth efficiency limitations; another problematic phenomenon is the delay in the feedback links. Precise channel estimation is commonly performed in the receiving nodes like relays and destination; however acquiring
the CSI in the transmitter nodes is cumbersome. In this case either a reverse training should be transmitted from the receiver nodes toward the transmitter nodes (relays toward source/destination toward relays and/or source) which calls for installing transceivers at all nodes and a complicated synchronization or the estimated CSI should be fed back via a feedback link toward the transmitter nodes. Exploiting the feedback links however are accompanied with side effects. On one hand channels in the wireless medium are generally time varying especially when the nodes in the network are moving. On the other hand the feedback links impose a delay in the reverse transmission of information due to distance. Therefore, when the feed-back CSI reaches the transmitter nodes with delay, the actual state of channel information is changed due to time varying nature of the channel. This means that the transmitter decides on the transmission based on a wrong information which results in a degradation in the system performance (see Fig. 1.5).

![Figure 1.5: Selection and feedback delay, $f_s = 20 \text{ KHz}$, $f_D = 600 \text{ Hz}$](image)

In chapter 4 we investigate the effect of feedback delay (FD) and CEE on the performance of a decode-and-forward selection cooperation (S-DF) scenario in a cooperative network. In our system a source node cooperates with only one relay out of $M$ potential relays to transmit its data to the destination node via DF protocol. In this thesis we adopt the uncoded DF protocol for our system model. Besides, in the system model we use in chapter 4 only one relay is selected to cooperate with the source node according to a cost function. Therefore the transmission is performed only in two time slots.
In chapter 4 we investigate the joint effect of CEE and FD on the performance of S-DF cooperation in terms of outage probability, ASER and average capacity. We show that the diversity order of the system in the best condition is reduced to 1 in the presence of FD. Simulation results at the end of chapter 4 confirm our interpretation.

1.2.4 Effect of co-channel interference in cognitive radio networks

In chapters 5 and 6 we investigate the outage probability of underlay cognitive radio systems with relay selection. In particular, we consider a secondary multi-relay network operating in the AF mode and only the “best” relay is selected, which satisfies an index of merit in each chapter. In chapter 5 the proposed selection strategy takes into consideration the effect of PU interference solely on the relay nodes. That is, we assume that the secondary multi-relay network is exposed to unwanted interference from a neighboring PU network under the assumption that the interference imposed on the secondary destination node can be mitigated. In chapter 6, we consider the imposed interference on the secondary destination as well. We derive closed-form outage probability expressions and further present a thorough asymptotic diversity order analysis of the underlying scenarios. Simulations corroborate the results and give further insight into the performance of the proposed selection strategies in chapters 5 and 6. In this chapter, we investigate the performance of relay selection in an underlay cognitive radio system in the presence of primary user interference. In particular, we consider a secondary multi-relay network operating in the AF mode. The best relay is then selected to cooperate with the secondary source node according to an index of merit. We adopt a maximum power allocation scheme based on the maximum tolerable interference on the primary user network. We derive a closed-form outage probability expression for the secondary multi-relay network and further presented a thorough asymptotical diversity order analysis. We also study the performance of the system under asymptotic power and interference cases and compare the proposed scheme to the conventional direct transmission and overlay cognitive radio scenarios.

Finally we summarize the thesis with discussing the conclusions and the results from each section in chapter 7. We also present possible future works and potential synergies, energies, and interests in the research in related fields.
1.3 Scholarly publications

The research efforts during my Ph.D. program have resulted in the following scholarly publications. The materials in this thesis present only a portion of the works that have been done during this period. Fortunately, these papers have dragged some attention and are cited more than 55 times in the recent 18 months. The impact of imperfect channel acquisition in relay selection is introduced for the first time to the literature via the works that are presented in this dissertation.

1.3.1 Journal papers


1.3.2 Ongoing/Submitted research papers


1.3.3 Conference papers


Chapter 2

Relay Selection for Vehicular Communications

2.1 Objective

In this chapter, we investigate cooperative diversity with relay selection over cascaded Rayleigh fading channels. In particular, we analyze the performance of a relay selection scheme for cooperative vehicular networks with the DF protocol. Only the “best” relay, which satisfies an index of merit, is selected. We ignore the direct transmission between the source (S) and its destination (D), and assume that the destination has perfect knowledge of the \( S \rightarrow R \) and \( R \rightarrow D \) channel gains. We study the performance of the underlying scheme in terms of outage probability and investigate its achievable diversity order\(^1\). Finally, simulation results are presented to corroborate the analytical results.

2.1.1 Related work and contribution

Recent advances in information and wireless technologies have led to growing interests in the development of the intelligent transportation systems (ITS), which promise enormous improvement in the road safety, and elimination of the excessive cost of traffic collisions. The so-called inter-vehicular communication (IVC), a key component of the ITS architecture, \(^1\)Achievable diversity order is defined as the guaranteed slope of the curves associated to the ASER, or outage probability of a transmission scheme in wireless Rayleigh fading channels at high SNRs. Through this thesis we use the term asymptotic diversity order for the same concept.
involves the application of advanced information processing, communications, sensor, and control technologies in an integrated manner to improve the function of the transportation system. Although the main motivation of IVC is safety improvement and traffic management, other applications, such as audio/video streaming and high-speed internet access, have recently emerged.

Recent research has shown that the communication paradigm of IVC is quite different from conventional point-to-point communication channel [4]. In particular, experimental and theoretical results have shown that in inter-vehicular communications, the cascaded Rayleigh model for the channel is a more realistic representation than single Rayleigh distributed channel and offers more accurate results [4, 29].

In [17], Bletsas et al. have studied relay selection with the DF protocol. In their proposed scheme, the “best” relay is chosen from the set of relays that decoded the information correctly. The outage probability and the ASER expressions of relay selection for the DF protocol have been studied in [12] and [13], respectively. In [9], Beres and Adve have analyzed selection cooperation in a network setting, i.e., multi-source networks. The setups in [9, 12, 13, 17] consider an error-free source-to-relay link, using error detection schemes, e.g., CRC [16].

The works in [9, 12, 13, 17] assume Rayleigh fading environments. This assumption can be justified for cellular scenarios; however, it is not a realistic assumption in IVC networks.

Although there have been considerable research efforts on relay selection over Rayleigh fading channels, to the best of our knowledge, there have not been any reported results that analyze the performance of relay selection in the context of IVC networks. In this chapter, we aim to fill this research gap and investigate relay selection in IVC scenarios over cascaded Rayleigh fading channels with DF relaying.

2.2 System Model

We consider a multi-relay scenario with $M$ relays. We assume that the relay, $R_m$, $m = 1, ..., M$, the source and destination terminals are equipped with single transmit/receive antennas. In our system model, $h_{sm}$ and $h_{md}$ represent the channel coefficients between $S \rightarrow R_m$ and $R_m \rightarrow D$, respectively. Both hops are modeled as the product of two independent complex Gaussian random variables [37], $h_{sm} \Delta h_{1m} h_{2m}$ and $h_{md} \Delta h_{3m} h_{4m}$, where $h_{km}$ is a complex Gaussian random variable with zero mean and variance of $\sigma_{km}^2$, for
CHAPTER 2. RELAY SELECTION FOR VEHICULAR COMMUNICATIONS

\( k = 1, 2, \ldots, 4 \) and \( m = 1, 2, \ldots, M \). Therefore, \(|h_{sm}|\) and \(|h_{md}|\) follow cascaded Rayleigh distribution. We assume that all underlying channels are quasi-static which can be justified for vehicular communication scenarios in rush-hour traffic [37]. We further assume, without loss of generality, that the additive white Gaussian noise (AWGN) terms have zero mean and variance of \( N_o \). Assuming a half duplex constraint\(^2\), the data transmission is performed in two time slots. In the first time slot, the source terminal \( S \) transmits its data to all the potentially available \( M \) relays. Next, all \( M \) relays decode their received signals, and check whether the transmitted signal is decoded correctly or not. This can be done via some ideal CRC codes [9], which are added to the transmitted symbols. We define the decoding set \( D(s) \) as the set of relays that decode the transmitted signal correctly. Clearly, only those relay nodes with a good source-to-relay channel can be in the decoding set \( D(s) \). In the second time slot, the best relay that satisfies an index of merit participates in the transmission and broadcasts its decoded symbols towards the destination. We assume that the source has a power constraint of \( P \) Joules/symbol and similarly each relay node in \( D(s) \) can potentially transmit its information with \( P \) Joules/symbol.

2.3 Outage Probability

Assuming that the communication between the source and the destination targets an end-to-end data rate \( R \), the relay \( R_m \) is in the decoding set \( D(s) \) if the \( S \rightarrow R_m \) link observes an instantaneous capacity per bandwidth \( C_{sm} \), that is above the required rate \( R \)

\[
C_{sm} = \frac{1}{2} \log_2 (1 + \alpha_m) \geq R, \tag{2.1}
\]

where the factor \( 1/2 \) accounts for the two time slot transmission protocol, \( \alpha_m = \frac{P}{N_o} \alpha_{1m} \alpha_{2m} \), where \( \alpha_{km} \equiv |h_{km}|^2 \) for \( k = 1, \ldots, 4 \) and \( m = 1, 2, \ldots, M \).

**Proposition:** The probability that the relay \( R_m \) decodes the transmitted signal incorrectly is given by

\[
Pr(C_{sm} \leq R) = 1 - \sqrt{\frac{4\lambda_1m \lambda_2m (2^2R - 1)}{SNR}} K_1 \left( \sqrt{\frac{4\lambda_1m \lambda_2m (2^2R - 1)}{SNR}} \right), \tag{2.2}
\]

where \( SNR = \frac{P}{N_o} \) is the average SNR at the receiving node.

\(^2\)A node can either transmit or receive in a specific interval under half duplex transmission scheme.
Proof: Noting that \( \alpha_{km} \sim \lambda_{km} \exp(-\lambda_{km} \alpha_{km}) \), for \( k = 1, \ldots, 4 \) and \( m = 1, 2, \ldots, M \), is Rayleigh fading with \( \lambda_{km} = \frac{1}{E[\alpha_{km}]} \); hence, we can calculate the CDF \( F_{\alpha_m}(x) \) as

\[
F_{\alpha_m}(x) = \Pr(\alpha_m \leq x) = \Pr\left( \frac{\alpha_1 \alpha_2}{\operatorname{SNR}} \leq \frac{x}{\operatorname{SNR}} \right) = \int_0^{\infty} F_{\alpha_1 \alpha_2}(x) f_{\alpha_2}(\alpha_2) d\alpha_2
\]

\[
= 1 - \lambda_2 \int_0^{\infty} \exp\left(-\frac{\lambda_1 x}{\operatorname{SNR}} \right) \exp(-\lambda_2 \alpha_2) d\alpha_2
\]

\[
= 1 - \sqrt{\frac{4\lambda_1 \lambda_2 x}{\operatorname{SNR}}} K_1\left(\sqrt{\frac{4\lambda_1 \lambda_2 x}{\operatorname{SNR}}}\right),
\]

where \( K_1(\cdot) \) is the modified Bessel function of the second kind, and we have used the fact that \[38\]

\[
\int_0^{\infty} \exp(-a x - bx) dx = \sqrt{\frac{a}{b}} K_1\left(\sqrt{ab}\right).
\]

The outage probability of \( S \rightarrow R_m \) link is subsequently obtained as

\[
\Pr(C_{sm} \leq R) = \Pr\left( \frac{1}{2} \log_2 (1 + \alpha_m) \leq R \right) = F_{\alpha_m}(2^R - 1),
\]

Using (2.3) at \( x = 2^R - 1 \), we obtain (2.2).

Similarly, noting that \( \beta_m = \frac{\alpha_3 \alpha_4}{\operatorname{SNR}} \), \( F_{\beta_m}(x) \) can be written as

\[
F_{\beta_m}(x) = 1 - \sqrt{\frac{4\lambda_3 \lambda_4 x}{\operatorname{SNR}}} K_1\left(\sqrt{\frac{4\lambda_3 \lambda_4 x}{\operatorname{SNR}}}\right).
\]

In order to find the outage probability, following [39], we define a random variable \( \gamma_m \) which represents the instantaneous SNR received at the destination via the \( m \)th path. Typically, this random variable accounts for the channel gains of both \( S \rightarrow R_m \) and \( R_m \rightarrow D \) links. Obviously, the \( m \)th path would be in outage if any of the hops, \( S \rightarrow R_m \) or \( R_m \rightarrow D \), is in outage. If the \( S \rightarrow R_m \) is in outage, then the relay \( R_m \) is not able to decode its received information. Therefore, no relaying would take place, i.e., the relay \( R_m \) is not in the decoding set \( D(s) \), and the received SNR via the \( m \)th path at the destination would be 0. On the other hand, if the relay \( R_m \) is on, i.e., it belongs to the decoding set \( D(s) \), then the received SNR will depend on the \( R_m \rightarrow D \) link only. In the following, we derive the
probability density function (PDF) of the received SNR via the \( m^{th} \) path. First, the PDF of the received SNR given \( R_m \) is off is

\[
f_{\gamma_m|R_m\text{ is off}}(x) = \delta(x),
\]

(2.6)

where \( \delta(x) \) is the Dirac delta function, and the probability of the event that \( R_m \) is off is \( Pr(C_{sm} \leq R) \). On the other hand, the conditional PDF of the received SNR given \( R_m \) is on is expressed by

\[
f_{\gamma_m|R_m\text{ is on}}(x) = f_{\beta_m}(x),
\]

(2.7)

and this event is occurred with the probability \( 1 - Pr(C_{sm} \leq R) \). Finally, the PDF of the received SNR, \( \gamma_m \), via the \( m^{th} \) path is obtained as [39]

\[
f_{\gamma_m}(x) = Pr(C_{sm} \leq R) \delta(x) + [1 - Pr(C_{sm} \leq R)] f_{\beta_m}(x),
\]

(2.8)

and the cumulative density function (CDF), \( F_{\gamma_m}(x) \), is obtained by integrating (2.8) with respect to \( x \)

\[
F_{\gamma_m}(x) = Pr(C_{sm} \leq R) + [1 - Pr(C_{sm} \leq R)] F_{\beta_m}(x),
\]

(2.9)

where \( F_{\beta_m}(x) \) is given as in (2.5), and subsequently, \( F_{\gamma_m}(x) \) is given by

\[
F_{\gamma_m}(x) = 1 - \frac{4x}{SNR} \prod_{k=1}^{4} \lambda_{km} \times K_1\left(\sqrt{\frac{4\lambda_{1m}\lambda_{2m}x}{SNR}}\right) K_1\left(\sqrt{\frac{4\lambda_{3m}\lambda_{4m}x}{SNR}}\right). \]  

(2.10)

It must be noted that the total received SNR via the selected relay \( R_{m^*} \) at the destination terminal is

\[
\gamma_t = \max_{m \in D(s)} \{\beta_m\},
\]

(2.11)

however, using the random variable \( \gamma_m \), the total received SNR can be written as

\[
\gamma_t = \gamma_{m^*} = \max_{m \in M} \{\gamma_m\}.
\]

(2.12)

Note that the expressions in (2.11) and (2.12) are equal; however, the expression in (2.12) is analytically more tractable. The outage probability of selection cooperation in the DF
mode for inter-vehicular scenarios is then obtained as

\[
Pr(C_{sm^*} \leq R) = Pr\left(\max_{m \in M}\{\gamma_m\} \leq 2^{2R} - 1\right)
\]

\[
= \prod_{m=1}^{M} Pr(\gamma_m \leq 2^{2R} - 1)
\]

\[
= \prod_{m=1}^{M} F_{\gamma_m}(2^{2R} - 1).
\]

(2.13)

Finally, the closed-form outage probability formula is given by

\[
Pr(C_{sm^*} \leq R) = \prod_{m=1}^{M} \left[1 - \frac{4^{(2^{2R} - 1)}}{\text{SNR}} \prod_{k=1}^{4} \lambda_{km} K_1\left(\sqrt{\frac{4\lambda_{1m}\lambda_{2m}(2^{2R} - 1)}{\text{SNR}}}\right) \right.
\]

\[\times K_1\left(\sqrt{\frac{4\lambda_{3m}\lambda_{4m}(2^{2R} - 1)}{\text{SNR}}}\right)\].

(2.14)

2.4 Maximum Achievable Diversity Order

In this section, we analyze the achievable diversity order for an inter-vehicular relay selection scenario with the decode-and-forward protocol, which to the best of our knowledge, has not been studied yet.

In wireless communications, diversity order (gain) is the increase in SNR due to some diversity scheme, or how much the transmission power can be reduced when a diversity scheme is introduced, without a performance loss. Diversity gain is usually expressed in decibel, and sometimes as a power ratio. An example is soft handoff gain. For example for selection cooperation the relay associated to the strongest signal is selected. When the \(M\) relay signals are independent and Rayleigh distributed, the expected diversity gain has been shown to be \(M\), expressed as a power ratio [7, 40].

We show that the maximum achievable diversity order for inter-vehicular relay channels with relay selection is \(M - M \log\left(g(\text{SNR})\right)\), where \(M\) is the number of relays, and \(g(\cdot)\) is a logarithmic function of SNR.

To simplify the analysis, without loss of generality, we assume \(\lambda_{km} = 1\), for \(k = 1, \ldots, 4\), \(m = 1, \ldots, M\), and \(\alpha_{km}\)'s are i.i.d exponential random variables for \(k = 1, 2\) and \(m = 1, \ldots, M\). Considering the i.i.d property of \(\alpha_m\)'s and \(\beta_m\)'s we have \(Pr(C_{sm} \leq R) = \)
\( F_{\alpha m}(R_o) = F_{\beta m}(R_o) \) where \( R_o = 2^{2R} - 1 \). Using (2.9), we have

\[
F_{\gamma m}(R_o) = 2F_{\beta m}(R_o) - F_{\beta m}(R_o) \\
\leq 2F_{\beta m}(R_o) \\
\simeq \frac{8R_o}{\text{SNR}} \log \left( \frac{R_o}{\text{SNR}} \right). \tag{2.15}
\]

where we approximate \( F_{\beta m}(R_o) = 1 - \sqrt{\frac{4R_o}{\text{SNR}}} K_1 \left( \sqrt{\frac{4R_o}{\text{SNR}}} \right) \) by \( -\frac{4R_o}{\text{SNR}} \log \left( \sqrt{\frac{R_o}{\text{SNR}}} \right) \) for \( \frac{R_o}{\text{SNR}} \ll 1 \).

Finally, noting that \( \gamma_m \)'s are i.i.d random variables and using (2.13) and (2.15), we obtain an upper bound on the outage probability of the underlying selection scheme as follows

\[
\mathbb{P}(\mathbb{C}_{sm}^* d \leq R) = \left[ F_{\gamma m}(R_o) \right]^M \\
\leq \left[ \frac{8R_o}{\text{SNR}} \log \left( \frac{R_o}{\text{SNR}} \right) \right]^M. \tag{2.16}
\]

The asymptotical diversity order \( d \) is given by the magnitude of the slope of outage probability against average SNR in a log-log scale [4]:

\[
d = \lim_{\text{SNR} \to \infty} -\frac{\log \left( \mathbb{P}(\mathbb{C}_{sm}^* d \leq R) \right)}{\log \text{SNR}}.
\]

Hence, using (2.16), we can write the asymptotical diversity order of the underlying selection scheme as

\[
d \simeq \lim_{\text{SNR} \to \infty} M - \frac{M \log \left( \log \text{SNR} \right) - \log(R_o)}{\log \text{SNR}}. \tag{2.17}
\]

The second term in (2.17) can be shown to tend zero by a simple l'Hôpital’s rule. Thus, the outage probability of selection cooperation in inter-vehicular scenarios scales no slower than \( \mathcal{O}(\text{SNR}^{-M}) \), and hence, the maximum achievable diversity order is \( M \). This gives us a sense that how much the transmission power can be reduced for achieving the same performance in comparison with a direct transmission when this diversity technique is used.

### 2.5 Simulation Results

We consider two different power modes, namely, fractional power allocation (FPA) and equal power allocation (EPA) modes. In FPA, all participating nodes carry the same power, i.e., \( P = \frac{P_o}{M+1} \), where \( P_o \) is the total power budget. On the other hand, in the EPA scheme,
CHAPTER 2. RELAY SELECTION FOR VEHICULAR COMMUNICATIONS

The total available power is $P_o$ and once a relay is selected all the available power, $P_o$, is divided equally between the source terminal and the selected relay, i.e., $P = \frac{P_o}{2}$ each. Fig. 2.1 compares the outage probability of selection cooperation in the FPA mode with the full cooperation scheme, i.e., all the relays participate in communication. It can be easily deduced from this figure that as the number of relays increases, the outage probability decreases. Furthermore, it is noticed that relay selection in the FPA mode performs worse than the full cooperation scheme. This is because in relay selection only the strongest signal is received at the destination, however in the full cooperation mode all the signals including the strongest signals are received. Fig. 2.1 also compares the simulation results with the analytical results, which are obtained using (2.14). For all considered cases, the analytical expressions closely match with the simulation results.

Fig. 2.2 shows the outage probability of relay selection in the DF mode assuming the EPA scheme. It is clear from Fig. 2.2 that, relay selection outperforms the full cooperation scheme. This is because this time the power of the strongest received signal at the destination is $\frac{P_o}{2} E\{ |h_{m,d}|^2 \}$ when relay selection is exerted in the EPA mode; however, the power received at the destination in full cooperation mode is proportional to $\frac{P_o}{M+1} \sum_{m=1}^{M} E\{ |h_{m,d}|^2 \}$. It is also observed that, as the number of relays increases, the gap between selection cooperation...
Figure 2.2: Outage probability of DF selection and regular cooperation for EPA scheme, $\text{SNR} = \frac{P}{N_o}$ and $M = 2, \ldots, 5$. and full cooperation further increases. For the sake of fair comparison in both scenarios, we assume that in the full cooperation scheme the power of $P = \frac{P}{M+1}$ is allocated to each transmitting node.

In Fig. [2.3], the relay selection scheme in cascaded Rayleigh channels is compared with its counterpart in Rayleigh fading channels. As it is obvious from Fig. [2.3], relay selection in Rayleigh fading channels enjoys a better performance than cascaded Rayleigh channels.

Fig. [2.4] shows the diversity order analysis of selection cooperation in a cascaded Rayleigh fading channel versus its Rayleigh fading counterpart. As shown in the figure as the SNR tends to infinity, the diversity order of both channels models approaches $M$, however, in a Rayleigh fading channel, the full diversity order is achieved in lower SNR values. This can also be seen in Fig. [2.3] where the slope of the Rayleigh fading channel curves are slightly steeper than the corresponding cascaded Rayleigh channel curves at the same SNRs.

2.6 Conclusion

In this chapter, we discussed relay selection in DF vehicular networks, where the channels in each link were modeled as cascaded Rayleigh fading. We derived an exact analytical
Figure 2.3: Outage probability of DF selection cooperation in cascaded Rayleigh and Rayleigh fading channels for $\text{SNR} = \frac{P}{N_0}$ and $M = 2, \ldots, 4$.

Figure 2.4: Diversity order versus SNR for DF selection cooperation in cascaded Rayleigh channels for $\text{SNR} = \frac{P}{N_0}$ and $M = 2, \ldots, 5$. 
expression for the outage probability and proved that the maximum achievable diversity order is equal to the number of relays. This means that for no performance loss we can decrease the utilized transmission power by increasing the number of relays. Finally, we verified our results via computer simulations.
Chapter 3

Relay Selection with Imperfect CSI

3.1 Objective

In this chapter, we investigate the performance of selection cooperation in the presence of imperfect channel estimation. In particular, we consider a cooperative scenario with multiple relays and amplify-and-forward protocol over frequency flat fading channels. In the selection scheme, only the “best” relay which maximizes the effective signal-to-noise ratio (SNR) at the receiver end is selected. We show that channel estimation errors (CEE) degrade the performance of amplify-and-forward selection cooperation (S-AF) drastically. We present lower and upper bounds on the effective SNR and further we provide closed-form expressions for the bounds on average symbol error rate (ASER), outage probability and average capacity per bandwidth of the received signal in the presence CEE. A simulation study is presented to corroborate the analytical results and to demonstrate the performance of relay selection with imperfect channel estimation.

3.1.1 Related work and contributions

The impact of CEE on the ASER performance of distributed space time block coded (DSTBC) systems has been investigated in [33], assuming the amplify-and-forward (AF) protocol for a single relay system. Moreover, Gedik and Uysal [34] have extended the work of [33] to a multi relay cooperation system, assuming again the AF protocol. In [35], the symbol error analysis is investigated for the same scenario as in [34]. The authors approximate the received SNR at the destination node and develop a close form expression for the
ASER of AF multi relay systems in the presence of CEE. In this chapter, we investigate the effect of channel estimation error on the ASER, outage probability and capacity of a cooperative diversity system with relay selection. We first introduce our system model in Sec.3.2. In Sec.3.3 we explain the Gaussian error model introduced by [36] and use it in the selection scenario to obtain the maximum ratio combiner (MRC) output SNR. In Sec.3.5 we study the ASER performance of our system in the case of imperfect channel estimation. We develop upper and lower bound analysis for the general relay case to formulate the relay selection scheme. We then derive closed-form formulas for the bounds on the ASER performance with imperfect channel estimation. Further we analyze the ASER performance of our system in the high SNR regime as introduced in [5,42]. We also study the outage probability and average capacity per bandwidth in Sec.3.6 and Sec.3.7, respectively. Diversity analysis is also provided in Sec.3.8. Simulation results are presented in Sec.3.9, followed by conclusions in Sec.3.10.

3.2 System Model

In this section we consider a system in which a source node $S$ transmits information to a destination node $D$ with the help of the best relay node $R_i^*$, which is selected amongst $M$ available relays such that the maximum possible SNR is received by the destination node via the selected relay path. The transmissions are orthogonal, either through time or frequency division.

In the first data sharing time slot, the source node communicates with the destination as well as the relay nodes. In this phase, the signals received by the destination and each relay are

\begin{align}
  y_{sd} &= \sqrt{P_{sd}}h_{sd}x + n_{sd} \\
  y_{si} &= \sqrt{P_{si}}h_{si}x + n_{si}
\end{align}

where $x$, $y_{sd}$, and $y_{si}$ denote the transmitted signal with unit energy and the signals received at the destination and the $i^{th}$ relay node, respectively. $h_{si}$ and $h_{sd}$ are independent zero mean complex Gaussian channel coefficients of the source-relay and the source-destination channels, which include the effect of fading. $P_{sd}$ and $P_{si}$ represent the average signal energies received at the destination and the relay $i$ terminals, respectively, taking into account the path loss and shadowing effects. $n_{sd}$ and $n_{si}$ are additive white Gaussian noises (AWGN).
in the corresponding channels with the same variance \( N_0 \), i.e., \( n_{sd}, n_{si} \sim \mathcal{CN}(0, N_0) \).

In the next time slot each relay node normalizes its received signal and retransmits it to the destination. For the \( i^{th} \) relay, the normalization factor is \( \sqrt{P_{si}|\hat{h}_{si}|^2 + N_0} \) and the signal transmitted from this relay is [7]

\[
x_i = \frac{\sqrt{P_{si}}h_{si}x + n_{si}}{\sqrt{P_{si}|\hat{h}_{si}|^2 + N_0}}.
\]

(3.3)

Since the relays practically do not have a perfect knowledge about the channel \( h_{si}, i = 1, \ldots, M \), the power normalization procedure in each relay is performed based on the estimate of the corresponding channel. We assume in this chapter that \( |x|^2 = 1 \). Based on (3.3), the signal received by the destination from the \( i^{th} \) relay node is [7]

\[
y_{id} = \frac{\sqrt{P_{id}}h_{id}x + n_{id}}{\sqrt{P_{si}P_{id}}} = \frac{\sqrt{P_{si}}P_{id}}{P_{si}|\hat{h}_{si}|^2 + N_0}h_{si}h_{id}x + \tilde{n}_{id}
\]

(3.4)

\[
y_{id} = \alpha_i h_{si} h_{id} x + \tilde{n}_{id}.
\]

(3.5)

where \( h_{id} \) is the zero mean complex Gaussian channel gain from this node to the destination. \( P_{id} \) is the average signal energy received at the destination via the \( i^{th} \) relay in its corresponding time slot which accounts for the shadowing and path loss effects and \( \alpha_i = \frac{\sqrt{P_{id}P_{id}}}{\sqrt{P_{si}|\hat{h}_{si}|^2 + N_0}} \). \( n_{id} \sim \mathcal{CN}(0, N_0) \) denotes the AWGN of the relay destination channel. \( \tilde{n}_{id} \) is the equivalent noise term in \( y_{id} \). It can easily be shown that, conditioned on the channel realizations, \( \tilde{n}_{id} \sim \mathcal{CN}(0, \omega_i^2 N_0) \), where

\[
\omega_i^2 = 1 + \frac{P_{id}|h_{id}|^2}{P_{si}|\hat{h}_{si}|^2 + N_0}.
\]

(3.6)

Supposing only the \( i^{th} \) relay participates in cooperation we look for the condition under which we can select the best relay, \( R_{i^*} \), by searching \( R_{is} \) for \( i = 1, \ldots, M \). Since each relay only amplifies the signal from the source, the destination is the only place where a soft decision of the information symbol \( x \) is computed. In order to achieve maximum likelihood performance, the signals from both diversity branches (direct \( S \rightarrow D \) branch and the branch via the \( i^{th} \) relay) are combined using a maximum ratio combiner (MRC). Decoding of \( x \) is delayed until the relayed signal containing the information symbol \( x \) is received at the destination. Since the noise power is not the same on the two sub channels, both diversity branches must be weighted by their respective complex fading gain over total noise power.
on that particular branch before the combiner. Thus the information symbol after the MRC would be

\[ \hat{x} = \frac{\hat{h}_sd}{\sqrt{P_{sd}}} y_{sd} + \frac{\alpha_i \hat{h}_sd \hat{h}_id}{\hat{\omega}_i} y_{id}, \]  

(3.7)

where

\[ \hat{\omega}_i = \sqrt{P_{si}|\hat{h}_{si}|^2 + P_{id}|\hat{h}_{id}|^2 + N_0}, \]  

(3.8)

since the destination only uses the estimate of the relay-destination channel.

### 3.3 Modeling of Channel Estimation Error

Let the true channel gain between nodes \( i \) and \( j \) and its estimate be \( h_{ij} \sim \mathcal{CN}(0, \sigma_{h_{ij}}^2) \) and \( \hat{h}_{ij} \sim \mathcal{CN}(0, \sigma_{\hat{h}_{ij}}^2) \), respectively. Therefore the channel absolute value is Rayleigh distributed. Assuming a least mean squares estimator (LMSE), we can consider the channel and the channel estimate related to each other as

\[ h_{ij} = \hat{h}_{ij} + d_{ij} \]  

(3.9)

where \( d_{ij} \) is the zero mean Gaussian estimation error with variance of \( \sigma_{d,ij}^2 \). \( h_{ij} \) and \( \hat{h}_{ij} \) are both zero mean Gaussian random variables with correlation coefficient \( \rho_{h_{ij}\hat{h}_{ij}} = \sigma_{h_{ij}} / \sigma_{\hat{h}_{ij}} \). According to principal of orthogonality, the optimal LMSE yields to an estimation error orthogonal to the channel realization \( h_{ij} \), i.e., \( \mathbb{E}[d_{ij} h_{ij}^*] = 0 \), thus \( \sigma_{h_{ij}}^2 = \sigma_{\hat{h}_{ij}}^2 + \sigma_{d,ij}^2 \). Since \( h_{ij} \) and \( \hat{h}_{ij} \) are jointly Gaussian, conditioned on \( h_{ij} \), the channel estimate \( \hat{h}_{ij} \) is Gaussian with mean \( \rho_{e,ij} h_{ij} \) where \( \rho_{e,ij} = \left( \frac{\sigma_{\hat{h}_{ij}}^2}{\sigma_{h_{ij}}^2} \right) \) and variance \( (1 - \rho_{e,ij}^2) \sigma_{\hat{h}_{ij}}^2 \). We can therefore write \[ \hat{h}_{ij} = \rho_{e,ij} h_{ij} + e_{ij} \]  

where \( e_{ij} \sim \mathcal{CN}(0, \sigma_{E,ij}^2) \) with \( \sigma_{E,ij}^2 = (1 - \rho_{e,ij}) \sigma_{\hat{h}_{ij}}^2 \). We can further write \( \sigma_{d,ij}^2 = (1 - \rho_{e,ij}) \sigma_{h_{ij}}^2 \). Fig. 3.1 provides more geometric insight to the problem. In this chapter, \( \sigma_{d,si}^2, \sigma_{d,di}^2 \) and \( \rho_{e,di}, \rho_{e,sd} \) are the channel estimation error variances and their corresponding \( \rho_e \) factors of the \( S-D \), \( S-R_i \), and \( R_i-D \) links, respectively (we only use the correlation coefficients in the simulation section). Using (3.9), we can rewrite the
Figure 3.1: LMSE for channel estimation. Principal of orthogonality is described.

First term of (3.7) as

\[ D_{sd} = \frac{\hat{h}_{sd}^* \sqrt{P_{sd}}}{N_0} \left( \sqrt{P_{sd}} (\hat{h}_{sd} + d_{sd}) x + n_{sd} \right) = \frac{P_{sd}|\hat{h}_{sd}|^2}{N_0} x + \sqrt{P_{sd}^2 \hat{h}_{sd}^*} \left( \sqrt{P_{sd} d_{sd} x} + n_{sd} \right). \]  

(3.10)

Here, we have inserted (3.9) in (3.1) then the resultant expression is substituted into (3.7). Following [35], the received direct path signal after the MRC can be decomposed into the message part given as \( \frac{P_{sd}|\hat{h}_{sd}|^2}{N_0} \) and the noise part which is given as \( \frac{P_{sd} \hat{h}_{sd}^*}{N_0} (\sqrt{P_{sd} d_{sd} x} + n_{sd}) \).

Finally, regarding the independence of the \( \hat{h}_{sd}, d_{sd} \) and \( n_{sd} \), the effective SNR received from the direct path is given by

\[ \gamma_{sd} = \frac{P_{sd}|\hat{h}_{sd}|^2}{N_0} \left( \frac{1}{1 + \frac{P_{sd}}{N_0} \sigma^2_{d,sd}} \right) = \frac{\hat{\gamma}_{sd}}{1 + \epsilon_{sd}} \]  

(3.11)

where \( \hat{\gamma}_{sd} \) is the estimated received SNR from the direct path and \( \epsilon_{sd} = \frac{P_{sd}}{N_0} \sigma^2_{d,sd} \).

Following the same approach, we can write the second term of (3.7) as

\[ D_{id} = \frac{\alpha_i \hat{h}_{si}^* \hat{h}_{id}^*}{\sigma_i^2 N_0} \left[ \alpha_i (\hat{h}_{si} + d_{si}) (\hat{h}_{id} + d_{id}) x + \tilde{n}_{id} \right]. \]  

(3.12)

We can extract the message part, error part due to channel estimation error, and the noise
part of the received signal as

\[ \mathcal{M} = \frac{\alpha_i^2}{\omega_i N_0} |\hat{h}_{si}|^2 |\hat{h}_{id}|^2 x \]  

(3.13)

\[ \mathcal{D} = \frac{\alpha_i^2}{\omega_i N_0} [ |h_{si}|^2 h^*_id d_id + |h_{id}|^2 h^*_si d_si + h^*_si h^*_id d_id d_id] x \]  

(3.14)

\[ \mathcal{N} = \frac{\alpha_i h^*_si h^*_id}{\omega_i^2 N_0} \hat{n}_{id} \]  

(3.15)

respectively. To obtain the effective SNR expression, we need to find the ratio of the signal power to the overall noise power. Since \( h_{si}, h_{id}, d_{si} \) and \( d_{id} \) are zero mean independent processes, we can write the effective SNR of the selected path as

\[ \gamma_i = \frac{P_{si}|\hat{h}_{si}|^2 P_{id}|\hat{h}_{id}|^2}{N_0^2 \left( \frac{P_{si}|\hat{h}_{si}|^2}{N_0} \left( 1 + \frac{P_{id}}{N_0} \sigma_{d,id}^2 \right) + \frac{P_{id}|\hat{h}_{id}|^2}{N_0} \left( 1 + \frac{P_{si}}{N_0} \sigma_{d,si}^2 \right) + \frac{P_{si} \sigma_{d,si}^2}{N_0} \frac{P_{id} \sigma_{d,id}^2}{N_0} + 1 \right)} \]  

(3.16)

With a simple manipulation, we may write (3.16) as follows

\[ \gamma_i = \frac{\hat{\gamma}_{si} \hat{\gamma}_{id}}{\hat{\gamma}_{si} \lambda_{si} + \hat{\gamma}_{id} \lambda_{id} + \epsilon_{si} \epsilon_{id} + 1} \]  

(3.17)

where \( \lambda_{si} = 1 + \epsilon_{id}, \lambda_{id} = 1 + \epsilon_{si}, \hat{\gamma}_{si} = \frac{P_{si}|\hat{h}_{si}|^2}{N_0}, \hat{\gamma}_{id} = \frac{P_{id}|\hat{h}_{id}|^2}{N_0}, \epsilon_{si} = \frac{P_{si} \sigma_{d,si}^2}{N_0}, \) and \( \epsilon_{id} = \frac{P_{id} \sigma_{d,id}^2}{N_0} \). Following [35], \( 1 + \epsilon_{si} \epsilon_{id} \) can be ignored\(^2\); hence, (3.17) can be approximated as [6,35]

\[ \gamma_i = \frac{\hat{\gamma}_{si} \hat{\gamma}_{id}}{\hat{\gamma}_{si} \lambda_{si} + \hat{\gamma}_{id} \lambda_{id}}. \]  

(3.18)

### 3.4 Selection Strategy

In the selection based amplify-and-forward scheme studied in [5–9,44] the best relay selected by the destination terminal is the one that leads to the maximum total received SNR \( \gamma_r \), which is given by

\[ \gamma_r = \gamma_{sd} + \gamma_i^* \]  

(3.19)

---

\(^1\)Conditioned on \( h_{si} \) and \( h_{id} \) the distribution of (3.14) is approximated as Gaussian. The validity of this approximation has been confirmed by simulation. It should be further noted that the same conclusion has been reported by [35] and the references therein.

\(^2\)For a fixed SNR, i.e., \( \frac{P}{N_0} = 5\)dB, we have noticed that the mean square error MSE between \( \gamma_i \) in (3.17) and (3.18) is \( 10^{-10} \) percent of \( \gamma_i^2 \) for \( \hat{\gamma}_{si} = \hat{\gamma}_{id} = 30\)dB.
where
\[ i^* = \arg \max_i \{ \gamma_i \}, \] (3.20)
for \( i = 1 \ldots M \). \( \gamma_r \), which is the total received SNR at the destination, is the addition of the SNR received from the direct path \( \gamma_{sd} \), and the SNR received via the selected relay path \( \gamma_{i^*} \).

### 3.5 Average Symbol Error Rate Analysis

In this section, we derive an ASER expression for the relay selection scheme. We first introduce the following facts [45]

- **Fact I**: If \( X \) and \( Y \) are two random variables with \( Y = mX \), then
  \[ f_Y(\gamma) = \frac{1}{|m|} f_X \left( \frac{\gamma}{m} \right). \]

- **Fact II**: For two independent random variables \( X \) and \( Y \) if \( Z = \min(X, Y) \), then
  \[ f_Z(\gamma) = f_X(\gamma) + f_Y(\gamma) - f_X(\gamma) F_Y(\gamma) - f_Y(\gamma) F_X(\gamma). \]

- **Fact III**: Let \( Y = \max_i (X_i), \ i = 1, \ldots, M \), where \( X_i \)'s are independent random variables, then
  \[ F_Y(\gamma) = \prod_{i=1}^{M} F_{X_i}(\gamma). \]

It is shown in [5, 6, 8, 10–12, 19] that the symbol error rate (SER) in many digital modulation scenarios associated to white zero mean complex Gaussian receiver noise is given by
\[ k_1 Q \left( \sqrt{k_0 \gamma_r} \right), \]
where
\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp \left( \frac{-t^2}{2} \right) \, dt \]
and the values of \( k_0 \) and \( k_1 \) depend on the modulation scheme. For instance for BPSK modulation \( k_0 = 2 \) and \( k_1 = 1 \). The analysis can be generalized to many types of modulation including \( A \)-ary phase shift keying (A-PSK), \( A \)-ary pulse amplitude modulation (A-PAM) modulation, and \( A \)-ary quadrature amplitude modulation (A-QAM) as well [5]3. To find

---

3For the ASER analysis regarding A-PAM, A-QAM, and A-PSK signals please see Sec. 3.C.
the average symbol error rate, we need to integrate \( k_1 Q \left( \sqrt{k_0 \gamma_r} \right) \) over the probability density function (PDF) of \( \gamma_r \), i.e.,

\[
\bar{P}_e = \int_0^{\infty} k_1 Q \left( \sqrt{k_0 \gamma_r} \right) f_{\gamma_r}(\gamma_r) \, d\gamma_r.
\] (3.21)

Finding the exact expression for \( f_{\gamma_r} \) can be quite cumbersome, therefore, in the following and to simplify analysis, we develop lower and upper bounds on \( \gamma_r \). It is straightforward to show that for two arbitrary independent random variables, the following inequality holds [6]

\[
\frac{1}{2} \min(X, Y) \leq \frac{XY}{X+Y} \leq \min(X, Y).
\] (3.22)

Applying (3.22) to \( \gamma_i \) in (3.18) yields

\[
\gamma_{i,lb} \leq \gamma_i \leq \gamma_{i,ub},
\] (3.23)

where

\[
\gamma_{i,ub} = \frac{1}{\lambda_{si} \lambda_{id}} \min(\hat{\gamma}_{si} \lambda_{si}, \hat{\gamma}_{id} \lambda_{id})
\] (3.24)

\[
\gamma_{i,lb} = \frac{1}{2} \frac{1}{\lambda_{si} \lambda_{id}} \min(\hat{\gamma}_{si} \lambda_{si}, \hat{\gamma}_{id} \lambda_{id})
\] (3.25)

Finally, \( \gamma_r \), can be bounded as

\[
\gamma_{lb} \leq \gamma_r \leq \gamma_{ub},
\] (3.26)

where \( \gamma_{lb} \) and \( \gamma_{ub} \) are the lower bound and the upper bound SNR values defined as

\[
\gamma_{ub} = \frac{\hat{\gamma}_{sd}}{1 + \epsilon_{sd}} + \gamma_{i,ub} \]
(3.27)

\[
\gamma_{lb} = \frac{\hat{\gamma}_{sd}}{1 + \epsilon_{sd}} + \gamma_{i,lb}
\] (3.28)

respectively, where

\[
\gamma_{i,ub} \overset{\Delta}{=} \max_i \{ \gamma_{i,ub} \}
\] (3.29)

\[
\gamma_{i,lb} \overset{\Delta}{=} \max_i \{ \gamma_{i,lb} \}.
\] (3.30)

Using (3.26), (3.27) and (3.28), (3.21) can be bounded as

\[
\bar{P}_{lb} \leq \bar{P}_e \leq \bar{P}_{ub},
\] (3.31)
where \( \bar{P}_{lb} \) and \( \bar{P}_{ub} \) are the lower and the upper bounds for ASER. In this chapter, we limit our discussion to the lower bound analysis. The upper bound analysis can be obtained similarly. Since the ASER is a decreasing function of SNR, a lower bound for 

\[ \int_0^\infty k_1 Q \left( \sqrt{k_0 \gamma_{ub}} \right) f_{\gamma_{ub}} d\gamma_{ub} \]

would simply be 

\[ \int_0^\infty k_1 Q \left( \sqrt{k_0 \gamma_{ub}} \right) f_{\gamma_{ub}} d\gamma_{ub} \]

Since our final goal is to obtain the PDF of \( \gamma_{ub} \) we need to find the PDF of \( \gamma_{i,ub} \), for \( i = 1, \ldots, M \) first. Note that the PDF of \( \hat{\gamma}_{si} \) can be expressed in terms of the average SNR \( \bar{\gamma}_{si} = P_{si} E \{ |h_{si}|^2 \} / N_0 \) as 

\[ f_{\hat{\gamma}_{si}} (\gamma) = \frac{1}{\bar{\gamma}_{si}} \exp \left( -\frac{\gamma}{\bar{\gamma}_{si}} \right) \]

Similar expressions can be found for the PDF of \( \hat{\gamma}_{sd} \) and \( \hat{\gamma}_{id} \) as well. Thus, using Fact I, the PDF of \( \gamma_{i,ub} \) can be written as

\[ f_{\gamma_{i,ub}} (\gamma) = \frac{1}{\beta_i} \exp \left( -\frac{\gamma}{\beta_i} \right), \quad (3.32) \]

where \( \beta_i = \frac{\bar{\gamma}_{si} \bar{\gamma}_{id}}{\lambda_{si} \bar{\gamma}_{si} + \lambda_{id} \bar{\gamma}_{id}} \). Using Fact II and noting that \( \gamma_{i*,ub} = \max_i (\gamma_{i,ub}) \), we obtain

\[ f_{\gamma_{i*,ub}} (\gamma) = \frac{\partial F_{\gamma_{i,ub}} (\gamma)}{\partial \gamma} = \frac{\partial \prod_{i=1}^M F_{\gamma_{i,ub}} (\gamma)}{\partial \gamma} \]

\[ = \sum_{l=1}^M f_{\gamma_{l,ub}} (\gamma) \prod_{i=1}^M F_{\gamma_{i,ub}} (\gamma). \quad (3.35) \]

The closed-form formulation for \( f_{\gamma_{i*,ub}} (\gamma) \) would then be (see Sec. 3.A)

\[ f_{\gamma_{i*,ub}} (\gamma) = \sum_{p=1}^M \sum_{m=1}^{\binom{M}{p}} (-1)^{p+1} [\Psi^p b]_m \exp \left( -\gamma [\Psi^p b]_m \right), \quad (3.36) \]

Consequently \( f_{\gamma_{ub}} \) is given by (see also Sec. 3.A),

\[ f_{\gamma_{ub}} (\gamma) = \sum_{p=1}^M \sum_{m=1}^{\binom{M}{p}} (-1)^{p+1} [\Psi^p b]_m \frac{\exp \left( -\gamma / \bar{\mu} \right) - \exp \left( -\gamma [\Psi^p b]_m \right)}{\bar{\mu} [\Psi^p b]_m - 1} \]

\[ \quad \text{where } \bar{\mu} = \frac{\bar{\gamma}_{sd}}{1 + \epsilon_{sd}} \text{ and } \Psi^p \text{ is a binary permutation matrix with dimension of } \binom{M}{p} \times M \text{ which is defined in Sec. 3.A}, \]

\[ b = \left[ \frac{1}{\beta_1} \frac{1}{\beta_2} \ldots \frac{1}{\beta_M} \right]^T. \quad (3.38) \]

\[ ^4\text{Here we have to note that } \gamma_{i,ub} \text{s are not i.i.d random variables, since } \beta_i \text{s are not analogous for different relay paths.} \]
Having noted that
\[
\bar{P}_{lb} = \int_0^{\infty} k_1 Q\left(\sqrt{k_0 \gamma_{ub}}\right) f_{\gamma_{ub}}(\gamma_{ub}) \, d\gamma_{ub} \tag{3.39}
\]
by substituting (3.37) into (3.39), \(\bar{P}_{lb}\) is given by
\[
\bar{P}_{lb} = k_1 \left(1 - k_1 \sum_{p=1}^{M} \sum_{m=1}^{(M/p)} (-1)^{(p+1)} \frac{[\Psi^p b]_m}{\bar{\mu}[\Psi^p b]_m - 1} \right) 
\times \left[ \bar{\mu} \sqrt{\frac{k_0/2}{\bar{\mu}^{-1} + k_0/2}} - \frac{[\Psi^p b]_{m-1}}{\bar{\mu}[\Psi^p b]_{m+k_0/2}} \right]. \tag{3.40}
\]

Here, we have used the fact [38]
\[
\int_0^{\infty} Q(a_0 \sqrt{\gamma}) \exp\left(-b_0^2 \gamma\right) d\gamma = \frac{1}{2b_0^2} \left(1 - \frac{a_0}{\sqrt{b_0^2 + a_0^2}}\right), \quad b_0 \neq 0. \tag{3.41}
\]

Following the same procedure the upper bound ASER is given by
\[
\bar{P}_{ub} = k_1 \left(1 - k_1 \sum_{p=1}^{M} \sum_{m=1}^{(M/p)} (-1)^{(p+1)} \frac{[\Psi^p b]_m}{\bar{\mu}[\Psi^p b]_m - 1} \right) 
\times \left[ \bar{\mu} \sqrt{\frac{k_0/2}{\bar{\mu}^{-1} + k_0/2}} - \frac{[\Psi^p b]_{m-1}}{\bar{\mu}[\Psi^p b]_{m+k_0/2}} \right]. \tag{3.42}
\]

To gain further insight into the performance of the selection scheme under consideration, we resort to the high SNR analysis. Defining \(\check{\gamma}_{ub} = \gamma_{ub}/\bar{\gamma}_{ub}\), where \(\bar{\gamma}_{ub}\) is the average SNR, and following [5, 42], we can approximate the asymptotic behavior of \(f_{\check{\gamma}_{ub}}\) in the high SNR regime with its Mclaurin series expansion, i.e.,
\[
f_{\check{\gamma}_{ub}} = a_0 \check{\gamma}_{ub}^n + o(\check{\gamma}_{ub}) \tag{3.43}
\]
where \(a_0 = \frac{1}{n!} \frac{\partial^n f_{\gamma_{ub}}}{\partial \gamma_{ub}^n}(0)\), if the derivatives of \(f_{\check{\gamma}_{ub}}\) up to order \(n - 1\) are null and
\[
o(\check{\gamma}_{ub}) = \sum_{t=n+1}^{\infty} \frac{1}{t!} \frac{\partial^t f_{\check{\gamma}_{ub}}}{\partial \check{\gamma}_{ub}^t}(0) \check{\gamma}_{ub}^t.
\]

It can be easily shown that by replacing (3.43) into (3.39), the asymptotic behavior of ASER \((\check{\gamma}_{ub} \to \infty)\) is given by [5]
\[
\bar{P}_{lb} \to \frac{k_1 \prod_{m=1}^{n+1} (2m - 1)}{2(n+1) k_0^{(n+1)}} \frac{1}{n!} \frac{\partial^n f_{\gamma_{ub}}}{\partial \gamma_{ub}^n}(0) \tag{3.44}
\].
where we have used the fact that \( f_{\gamma_{ub}}(\gamma_{ub}) = \frac{1}{\gamma_{ub}} f_{\hat{\gamma}_{ub}}(\frac{\gamma_{ub}}{\gamma_{ub}}) \).

Similarly, the asymptotic upper bound ASER can be found as
\[
\bar{P}_{ub} \to \frac{k_1 \prod_{m=1}^{n+1} (2m-1)}{2(n+1) k_0^{(n+1)}} \frac{1}{n!} \prod_{m=1}^{n+1} \frac{1}{\beta_i} \frac{\partial^n f_{\hat{\gamma}_{ub}}}{\partial \gamma_{ub}^n}(0). \tag{3.45}
\]

**Theorem I:** For all \( n < M \), the values of \( \frac{\partial^n f_{\gamma_{ub}}}{\partial \gamma_{ub}^n}(0) \) and \( \frac{\partial^n f_{\gamma_{lb}}}{\partial \gamma_{lb}^n}(0) \) are zero and for \( n = M \)
\[
\bar{P}_{lb} \to \frac{k_1 \prod_{m=1}^{M+1} (2m-1)}{2(M+1) k_0^{(M+1)}} \frac{1}{\mu} \prod_{i=1}^{M} \frac{1}{\beta_i}, \tag{3.46}
\]
\[
\bar{P}_{ub} \to \frac{k_1 \prod_{m=1}^{M+1} (2m-1)}{(M+1) k_0^{(M+1)}} \frac{1}{\mu} \prod_{i=1}^{M} \frac{1}{\beta_i}. \tag{3.47}
\]

**Proof:** see Sec 3.3B

### 3.6 Outage Probability Analysis

The outage probability \( P_{out} \), which is a valid measure of performance in slowly fading channels, is defined as the probability that the source-destination mutual information \( I_{sd} \) falls below the transmission rate \( R \). Defining
\[
I_{sd} \triangleq \frac{1}{2} \log_2 (1 + \gamma_{sd} + \gamma_{i*}) \leq R, \tag{3.48}
\]
the outage probability is given by
\[
P_{out} = Pr \left( \frac{1}{2} \log_2 \left( 1 + \gamma_r \right) \leq R \right) = Pr \left( \gamma_r \leq 2^{2R} - 1 \right). \tag{3.49}
\]

Substituting the introduced upper bound for \( \gamma_r \) leads to adopt a lower bound on the outage probability, i.e.,
\[
P_{out}^{lb} = F_{\gamma_{ub}} \left( 2^{2R} - 1 \right).
\]

We can simply find \( F_{\gamma_{ub}} \) by integrating (3.37) with respect to \( \gamma \), the result of which is given by
\[
F_{\gamma_{ub}} (\gamma) = \sum_{p=1}^{M} \sum_{m=1}^{\frac{M}{p}} (-1)^{p+1} \left[ 1 + \frac{1}{\mu[\Psi^p b]_{m-1}} \exp (-\gamma[\Psi^p b]_{m}) - \frac{\mu[\Psi^p b]_{m-1}}{\mu[\Psi^p b]_{m-1}} \exp \left( \frac{-\gamma}{\mu} \right) \right]. \tag{3.50}
\]
Following the same procedure for the upper bound outage probability we have

\[ P_{out}^{ub} = F_{\gamma_{lb}} (2^{2R} - 1) \]

where \( F_{\gamma_{lb}} (\gamma) \) is given by

\[
F_{\gamma_{lb}} (\gamma) = \sum_{p=1}^{M} \sum_{m=1}^{p} (-1)^{p+1} \times \left[ 1 + \frac{1}{\mu [2\Psi^p b]_m} \exp (-\gamma [2\Psi^p b]_m) - \frac{\mu [2\Psi^p b]_m}{\mu [2\Psi^p b]_m} \exp \left( \frac{\gamma}{\mu} \right) \right]. \tag{3.51}
\]

### 3.7 Average Capacity Analysis

The limiting information rate (in units of information per unit time) that can be achieved with arbitrarily small error probability is called the channel capacity. The average capacity per time slot for S-AF scheme for the perfect CSI case is defined as \[46\]

\[
\bar{C} = \frac{W}{2} \int_0^{\infty} \log_2 (1 + \gamma_t) f_{\gamma_t} (\gamma_t) \, d\gamma_t \tag{3.52}
\]

in bits per time slot, where \( W \) is the transmitted signal bandwidth, and \( \gamma_t \) is the total received SNR assuming perfect CSI. To obtain a formula for the average capacity with imperfect CSI, we have to consider an information theoretic approach. Regarding (3.10) and (3.12), the total signal in the MRC output at the destination node after two time slots is

\[
y_o = h o x + n_o \tag{3.53}
\]

where \( y_o = [D_{sd} \ D_{sd}]^T, \ h_o = \left[ \frac{P_{sd} |h_{sd}|^2}{N_0} \ M \right]^T \), \( x \) is the transmitted signal and is a scalar, and \( n_o = \left[ \frac{\sqrt{P_{sd} h_{sd}^*}}{N_0} (\sqrt{P_{sd} d_{sd} x} + n_{sd}) \ D + N \right]^T \), where \( M, D \) and \( N \) are given in (3.13), (3.14) and (3.15), respectively. Normalizing the received vector \( y_o \) by the noise variance we arrive at

\[
y = h x + n, \tag{3.54}
\]

where \( y \) and \( h \) are defined as

\[
y = \left[ \frac{D_{sd}}{\sqrt{\text{var}(D_{sd})}} \quad \frac{D_{sd}}{\sqrt{\text{var}(D_{sd})}} \right]^T
\]

\[
h = \left[ \frac{P_{sd} h_{sd}^*}{\sqrt{\text{var}(D_{sd})}} \quad \frac{M}{\sqrt{\text{var}(D_{sd})}} \right]^T \tag{3.55}
\]
and $\mathbf{n} \sim \mathcal{CN}(0, \mathbf{I})$. If the noise vector $\mathbf{n}$ is independent of the input signal $x$, then the capacity of the system in (3.54) is given by

$$C = \arg \max_{f(x)} \mathcal{H}(\mathbf{y}) - \mathcal{H}(\mathbf{n}|x)$$

$$= \arg \max_{f(x)} \mathcal{H}(\mathbf{y}) - \mathcal{H}(\mathbf{n})$$

$$= \frac{W}{2} \log_2 (1 + \mathbf{h}^H \mathbf{h}). \quad (3.58)$$

where $\mathcal{H}(\cdot)$ denotes the differential entropy and $f(x)$ is the PDF of the input signal $x$. In our system, the noise vector $\mathbf{n}$ depends on the input signal $x$, and on the other hand, the noise vector $\mathbf{n}$ is not Gaussian. Therefore, (3.56) will not lead to (3.57) and hence we are not allowed to use the capacity formula in (3.58). Instead, using the following inequality [47]

$$\mathcal{H}(\mathbf{n}|x) \leq \mathcal{H}(\mathbf{n})$$

we can bound the capacity with the worst effect of the additive noise on the capacity of the system [48], which is when the noise vector $\mathbf{n}$ is Gaussian. It has been shown in [49], “Fact I”, that if the transmitted signal and additive noise are uncorrelated, the worst case noise has zero-mean Gaussian distribution with the same power of the additive noise. Furthermore, it can be easily seen from (3.54) that $\mathbb{E}\{\mathbf{n}x|\hat{h}_{sd}, \hat{h}_{si}, \hat{h}_{id}\} = 0$. Therefore, the additive noise $\mathbf{n}$ can be replaced by a Gaussian noise with the power 1, leading to the worst case capacity given by (3.58). By substituting $D_{sd}$ and $D_{id}$ from (3.10) and (3.12) in (3.55) and constituting (3.58), we have

$$\mathbf{h}^H \mathbf{h} = \gamma_r.$$

Therefore, the worst case average capacity is given by

$$\bar{C}_{\text{worst}} = \frac{W}{2} \int_0^\infty \log_2 (1 + \gamma_r) f_{\gamma_r}(\gamma_r) \, d\gamma_r.$$

To obtain the lower bound average capacity $\bar{C}_{lb} \leq \bar{C}_{\text{worst}} \leq \bar{C}$, we substitute $\gamma_r$ with $\gamma_{lb}$ and $f_{\gamma_r}$ with $f_{\gamma_{lb}}(\gamma) = \frac{\partial F_{\gamma_{lb}}(\gamma)}{\partial \gamma}$ and we get to

$$\bar{C}_{lb} = \frac{0.72}{W} \sum_{p=1}^M \sum_{m=1}^{(\frac{M}{p})} (-1)^{(p+1)} \frac{[\Psi^p B]_m}{\mu[\Psi^p B]_m - 1} \left[ \mu \exp \left( \frac{1}{\mu} \right) E_1 \left( \frac{1}{\mu} \right) - \frac{1}{[\Psi^p B]_m} \exp ([\Psi^p B]_m) E_1 ([\Psi^p B]_m) \right]. \quad (3.59)$$
Here, we have used the fact that [38]
\[ \int_0^\infty \exp (-\alpha \gamma) \log_2 (1 + \gamma) d\gamma = -\frac{1.44}{\alpha} \exp (\alpha) E_i (-\alpha) \]  
(3.60)
where
\[ E_i (\gamma) \triangleq \int_\gamma^\infty \frac{1}{x} \exp (-x) dx \]
is the exponential integral function.

### 3.8 Asymptotic Order of Diversity

We shall next analyze the underlying selection cooperation system from the diversity point of view. The diversity order is given by the magnitude of the slope of the outage probability as a function of SNR (on a log-log scale) [4]. Equivalently a system achieving a diversity order \( d(R) \), at transmission rate \( R \), has an error probability that behaves as \( \bar{P}_{ub} (SNR) \propto SNR^{-d(R)} \) at high SNR [4]. Utilizing the high SNR analysis which is indicated in (3.47), the ASER is implicitly proportional to \( \gamma_{sd}, \gamma_{si}, \) and \( \gamma_{id} \). Assuming \( \gamma_{sd} = \alpha_1 \bar{\gamma}_{si} = \alpha_2 \bar{\gamma}_{id} = P_0 \), then we have
\[ \bar{P}_{ub} (P_0) \propto 1 + \epsilon_{sd} P_0 \prod_{i=1}^{M} (\alpha_2 \lambda_{si} + \alpha_1 \lambda_{id}) \]
\[ \propto \frac{1 + \epsilon_{sd}}{P_0^{M+1}} \prod_{i=1}^{M} (\alpha_2 \lambda_{si} + \alpha_1 \lambda_{id}) . \]  
(3.61)
This implies that the diversity order in a selection cooperation scenario is highly affected by the channel estimation error. In the case that the channel estimation error is zero or it reduces by increasing the received SNR, \( \epsilon_{sd}, \lambda_{si}, \lambda_{id} \), for \( i = 1, \ldots, M \), would all be equal to unity at high SNR regimes; hence, \[ \bar{P}_{ub} (P_0) \propto \left( \frac{1}{P_0} \right)^{M+1} . \]  
(3.62)
Thus, the maximum diversity order is achieved in selection cooperation schemes with no channel estimation error or in the cases that the channel estimation error reduces with increasing SNR.
On the other hand, if the channel estimation error is independent of the received SNR\(^5\), noting that 
\[ \epsilon_{sd} = \frac{P_{sd}}{N_0} \sigma^2_{d,sd}, \quad \epsilon_{si} = \frac{P_{si}}{N_0} \sigma^2_{d,si}, \quad \epsilon_{id} = \frac{P_{id}}{N_0} \sigma^2_{d,id}, \]
\[ \lambda_{si} = 1 + \epsilon_{id}, \quad \text{and} \quad \lambda_{id} = 1 + \epsilon_{si}, \]
we can replace \( \epsilon_{sd} \) and similarly \( \lambda_{si} \) and \( \lambda_{id} \) by \( P_0 \), at high SNR; hence, from (3.61) the diversity order of the system with selection cooperation tends to zero as \( P_0 \), approaches infinity. This means that in such a system we expect an error floor in the ASER or outage probability curves at high SNR, in which the performance of the system would not improve by increasing the SNR.

### 3.9 Simulation Results

In this section, we consider relay selection scenarios for \( M = 2, M = 3 \). The transmitted symbols are drawn from the BPSK constellation where we assume \( k_1 = 1 \) and \( k_0 = 2 \) and the received energies are assumed to be \( (P_{sd}, P_{si}, P_{id}) = (1, 0.5, 0.5) \times P_0 \) for all relays. It is assumed that due to simplicity of relays the power budget of each relay cannot exceed half the power budget of the source and once a relay is selected, it operates with full power and the other relays remain silent. For convenience and to get more insight to the core of the problem, we assume the special case, where the channels estimation errors of all links have the same variance, i.e., \( \sigma^2_{d,sd} = \sigma^2_{d,si} = \sigma^2_{d,id} = \sigma^2_d \). The variance of noise components is set to \( N_0 = 1 \) and \( R = 1 \text{ bps/Hz} \). Here, unless stated, we assume that \( \sigma^2_d \) is independent of the received SNR.

#### 3.9.1 ASER analysis

In figs. 3.2 and 3.3, we confirm the validity of the derived expressions for the cases \( M = 2, M = 3 \), respectively. As the figures show, the derived lower bound for the ASER is fairly tight. The high SNR analysis also converges to the analytically derived lower bound ASER curves at high SNR. As it is obvious in these figures, if the channel is estimated with even a very small error, an error floor is observed at high SNR. Increasing \( \rho_e \) yields a better performance in the ASER of the system. Also we can deduce from these figures that selection among a larger number of relays improves the ASER performance of the system.

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\(^5\)It is common to assume that the number of pilot symbols are much less than the number of data symbols in a packet. Therefore by increasing the packet average SNR, the SNR associated to the pilots remains almost unchanged. Thus by increasing the SNR the pilot signals used for channel estimation are not stronger and as the result, channel estimation is not improved.
Figure 3.2: ASER vs. SNR. Relay selection scenario, $M = 2$.

Figure 3.3: ASER vs. SNR. Relay selection scenario, $M = 3$. 
3.9.2 Outage probability analysis

Figure 3.4: Outage probability vs. SNR. Relay selection scenario, $M = 2$.

Figs. 3.4 and 3.5 consider the credibility of our analytic results for $M = 3$, and $M = 4$. In these figures the derived formula for the outage probability is compared with the simulation results. A fair match is observed and the same error floor is seen for the case that the system faces a non-decreasing channel estimation error with the received SNR. It is observed in figs. 3.4 and 3.5 that the selection scheme with larger number of relays outperforms the scenario with less number of relays in terms of outage probability. Also the effect of increasing of $\rho_e$ and thus, decreasing of $\sigma_d^2$ in the improvement of the system outage probability is shown in these figures.

We further study the effect of relay location in selection cooperation. In this scenario the distance between $S$ and $D$ is normalized to one. To consider the effect of path loss, the received energies are modeled as $P_{si} = \frac{0.5P_0}{\tilde{d}}$ and $P_{id} = \frac{0.5P_0}{(1-\tilde{d})}$ where $\tilde{d}$ is the normalized distance between $S$ and $R_i$.

Fig. 3.6 shows the outage probability of a relay selection scenario with $M = 2$, versus $\tilde{d}$. The two relays are located between the source and destination nodes with the same distance of $\tilde{d}$ from the source node. As it is noticed the lower bound analysis curves fairly match the curves obtained from simulation. The lowest outage probability is achieved when the selected relay is in the middle of the path between source and destination.
CHAPTER 3. RELAY SELECTION WITH IMPERFECT CSI

Figure 3.5: Outage probability vs. SNR. Relay selection scenario, $M = 3$.

Figure 3.6: Outage probability for $M = 2$ relays vs. distance for different SNR values.
3.9.3 Asymptotic diversity order

Figs. 3.2 to 3.5 also demonstrate the credibility of our diversity analysis for $M = 2$ and $M = 3$. The slope of the curves associated with $\rho_e = 1$ in these figures imply the diversity orders achieved by the selection cooperation scenario. Diversity order $M + 1$ is seen for the perfect channel estimation scenario. Occurrence of error floor in the curves associated with the imperfect channel estimation accounts for the diversity order of zero at high SNRs which is analytically obtained.

![Figure 3.7: ASER vs SNR. Performance of S-AF and AP-AF Cooperation for $M = 2$.](image)

Fig. 3.7 shows the ASER performance curves for S-AF and all participate amplify-and-forward cooperation (AP-AF), where all the relays participate in cooperation. It is noticed that both scenarios exhibit the same slope, i.e., same diversity order. This means that S-AF as well as AP-AF in the cooperative system achieve full diversity order, i.e, $M + 1$, where $M$ is the number of relays. Although selection degrades the ASER performance of the system, it improves the throughput of the system significantly. Also S-AF cooperation requires less power consumption than the AP-AF cooperation scheme.

In the case that the estimation error variance is decreased by increasing the SNR, $\sigma_e^2 \propto \frac{1}{P}$ where $P$ is the power in which the pilot symbols are received. Fig. 3.8 shows the simulation results for $M = 2$. In this figure the received energies are assumed to be $(P_{sd}, P_{si}, P_{id}) = (1, 0.5, 0.5) \times P_o$, for all relays. For each curve the training sequence received power is set
Figure 3.8: Outage probability vs. SNR. S-AF for \( M = 3 \) when the estimation error variance decreases with increasing the SNR. \( P \) in this figure is assumed to be the training power used for channel estimation. In this case \( \sigma_d^2 \approx 1/P \). When \( P \to \infty \) \( \rho_e = 1 \).

to \( P = P_0 \), \( P = P_0/10 \), \( P = P_0/100 \). Maximum diversity order of 3 is achieved in all cases for \( M = 2 \).

### 3.9.4 Average capacity

Fig. 3.9 shows the capacity of the S-AF scenario when we select the best relay out of \( M = 2 \), relays. A capacity ceiling is observed when the channel is estimated with error. After the effect of the receiver noise is completely diminished, which occurs at a certain SNR, the capacity improvement is insignificant from that point on. This is in fact because increasing the SNR does not reduce the channel estimation error in our scenario. As it can be seen, the capacity of the system is very sensitive to the channel estimation performance. A fair upper bound is observed in our simulations. Furthermore, fig. 3.10 shows a comparison between S-AF and AP-AF cooperation for \( M = 2,3,6, \) and 9. Obviously, S-AF cooperation exhibits a huge average capacity improvement specially for \( m > 2 \). Selection out of larger number of relays would slightly improve the throughput of the system.
Figure 3.9: Average capacity per bandwidth vs. SNR. Relay selection Scenario, $M = 2$.

Figure 3.10: Average capacity per bandwidth vs. SNR. Comparison between S-AF and AP-AF cooperation schemes.
3.10 Conclusion

In this chapter we studied the effect of CEE in cooperative systems with relay selection. We analyzed the ASER and outage probability performance of the system in the presence of estimation error. We further introduced a lower bound on the average capacity and derived a closed form formula for its performance in the presence of imperfect channel estimates. We showed under the assumption of constant training power an error floor was imposed on the ASER and outage performance of the system leading the diversity order to zero. However, if the training power increased with the data SNR then the expected full diversity order of the system was preserved. It is must be noted that the performance of the system always depends on how much training power is specified to channel estimation though. Later, we investigated the performance of the system incorporating the effect of distance to our considerations. We showed that the best performance of the system is achieved when the relays are in the middle of the connecting path between source and destination.

3.A Deriving the Probability density function of $\gamma_{r^*ub}$

Consider a case with three relays, i.e., $M = 3$. From Fact III we have

$$F_{\gamma_{r^*ub}} (\gamma) = \prod_{i=1}^{3} \left( 1 - \exp \left( -\frac{\gamma}{\beta_i} \right) \right).$$
Using (3.32) to (3.35), we have

\[ f_{\gamma_{i,ub}} (\gamma) \]

\[ = \sum_{l=1}^{3} \frac{1}{\beta_l} \exp \left( -\frac{\gamma}{\beta_l} \right) \prod_{i=1, i \neq l}^{3} \left( 1 - \exp \left( -\frac{\gamma}{\beta_{i}} \right) \right) \]

\[ = \frac{1}{\beta_1} \left[ \exp \left( -\frac{\gamma}{\beta_1} \right) - \exp \left( -\gamma \left( \frac{1}{\beta_1} + \frac{1}{\beta_2} \right) \right) - \exp \left( -\gamma \left( \frac{1}{\beta_1} + \frac{1}{\beta_2} + \frac{1}{\beta_3} \right) \right) \right] \]

\[ + \frac{1}{\beta_2} \left[ \exp \left( -\frac{\gamma}{\beta_2} \right) - \exp \left( -\gamma \left( \frac{1}{\beta_2} + \frac{1}{\beta_3} \right) \right) - \exp \left( -\gamma \left( \frac{1}{\beta_2} + \frac{1}{\beta_3} + \frac{1}{\beta_1} \right) \right) \right] \]

\[ + \frac{1}{\beta_3} \left[ \exp \left( -\frac{\gamma}{\beta_3} \right) - \exp \left( -\gamma \left( \frac{1}{\beta_3} + \frac{1}{\beta_1} \right) \right) - \exp \left( -\gamma \left( \frac{1}{\beta_3} + \frac{1}{\beta_1} + \frac{1}{\beta_2} \right) \right) \right] \]

\[ = \frac{1}{\beta_1} \exp \left( -\frac{\gamma}{\beta_1} \right) + \frac{1}{\beta_2} \exp \left( -\frac{\gamma}{\beta_2} \right) + \frac{1}{\beta_3} \exp \left( -\frac{\gamma}{\beta_3} \right) \]

\[ - \frac{1}{\beta_1} \frac{1}{\beta_2} \exp \left( -\gamma \left( \frac{1}{\beta_1} + \frac{1}{\beta_2} \right) \right) \]

\[ - \frac{1}{\beta_1} \frac{1}{\beta_3} \exp \left( -\gamma \left( \frac{1}{\beta_1} + \frac{1}{\beta_3} \right) \right) - \frac{1}{\beta_2} \frac{1}{\beta_3} \exp \left( -\gamma \left( \frac{1}{\beta_2} + \frac{1}{\beta_3} \right) \right) \]

\[ + \frac{1}{\beta_1} \frac{1}{\beta_2} \frac{1}{\beta_3} \exp \left( -\gamma \left( \frac{1}{\beta_1} + \frac{1}{\beta_2} + \frac{1}{\beta_3} \right) \right) \].

(3.63)

Let \( \Psi^p \) be a binary permutation matrix with dimension of \( \binom{M}{p} \times M \). Each row shows one possible \( p \) combination of \( M \) binary bits. In this example

\[ \Psi^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \Psi^2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \Psi^3 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}. \]

We also define \( \mathbf{b} = \begin{bmatrix} \frac{1}{\beta_1} \frac{1}{\beta_2} \frac{1}{\beta_3} \end{bmatrix}^T \) and let \( [\Psi^p \mathbf{b}]_m \) be the \( m^{th} \) row of the vector \( \Psi^p \mathbf{b} \). Using the definitions we just pointed out we can write (3.63) as

\[ f_{\gamma_{i,ub}} (\gamma) = \sum_{p=1}^{3} \sum_{m=1}^{\binom{M}{p}} (-1)^{(p+1)} [\Psi^p \mathbf{b}]_m \exp \left( -\gamma [\Psi^p \mathbf{b}]_m \right). \]

With the same procedure for \( M \) relays, using (3.32) and substituting \( F_{\gamma_{i,ub}} = \sum_{p=1}^{M} \binom{M}{p} \frac{1}{\beta_1} \frac{1}{\beta_2} \frac{1}{\beta_3} \) into (3.35), we obtain

\[ f_{\gamma_{i,ub}} (\gamma) = \sum_{p=1}^{M} \sum_{m=1}^{\binom{M}{p}} (-1)^{(p+1)} [\Psi^p \mathbf{b}]_m \exp \left( -\gamma [\Psi^p \mathbf{b}]_m \right) \]
where \( b \) is defined in (3.38).

Furthermore, it can be easily shown that the PDF of \( \mu = \frac{\gamma_{sd}}{1+\epsilon_{sd}} \) is given by

\[
 f_{\mu}(\gamma) = \frac{1}{\bar{\mu}} \exp\left(\frac{-\gamma}{\bar{\mu}}\right),
\]

where \( \bar{\mu} = \frac{\bar{\gamma}_{sd}}{1+\epsilon_{sd}} \), and the PDF of \( \gamma_{ub} \), can be found as follows

\[
 f_{\gamma_{ub}}(\gamma) = \frac{\partial}{\partial \gamma} \left[ \mu + \gamma_{i,ub} \right] f_{\gamma_{i,ub}}(\gamma_{i,ub}) d\gamma_{i,ub}.
\]

Using \( F_{\mu}(\gamma) = 1 - \exp\left(\frac{-\gamma}{\bar{\mu}}\right) \) and inserting (3.36) in (3.64), we obtain

\[
 f_{\gamma_{ub}}(\gamma) = \sum_{p=1}^{M} \sum_{m=1}^{\lceil M/p \rceil} (-1)^{p+1} \frac{\partial}{\partial \gamma} \int_{0}^{\gamma} \left[ 1 - \exp\left(\frac{-\gamma - \gamma_{i,ub}}{\bar{\mu}}\right) \right] \times [\Psi p b]_m \exp\left(-\gamma_{i,ub}[\Psi p b]_m\right) d\gamma_{i,ub}
\]

\[
 = \sum_{p=1}^{M} \sum_{m=1}^{\lceil M/p \rceil} (-1)^{p+1} \frac{[\Psi p b]_m}{\mu[\Psi p b]_m - 1} \left[ \exp\left(\frac{-\gamma}{\bar{\mu}}\right) - \exp\left(-\gamma[\Psi p b]_m\right) \right].
\]

### 3.B Proof of Theorem I

From the basic principles of the moment generation function (MGF) of \( \gamma_{ub} = \mu + \gamma_{i,ub} \), we have

\[
 M_{\gamma_{ub}}(s) = M_{\mu}(s) M_{\gamma_{i,ub}}(s),
\]

thus using the initial value theorem, the value of \( \frac{\partial^n M_{\gamma_{i,ub}}}{\partial \gamma_{i,ub}^n} (0) \) is given by

\[
 \frac{\partial^n f_{\gamma_{ub}}}{\partial \gamma_{i,ub}^n} (0) = \lim_{s \to \infty} s^{n+1} M_{\gamma_{ub}}(s) = \lim_{s \to \infty} s M_{\mu}(s) s^n M_{\gamma_{i,ub}}(s)
\]

\[
 = f_{\mu}(0) \frac{\partial^{n-1} f_{\gamma_{i,ub}}}{\partial \gamma_{i,ub}^{n-1}}(0).
\]

where

\[
 \frac{\partial^{n-1} f_{\gamma_{i,ub}}}{\partial \gamma_{i,ub}^{n-1}}(0) = \frac{\partial^n F_{\gamma_{i,ub}}}{\partial \gamma_{i,ub}^n}(0).
\]
In light of Fact III, we can simply deduce that for \( n < M \), \( \frac{\partial^n F_{\gamma_{i,u}^{ub}}}{\partial \gamma_{i,u}^{ub}} (0) \) is the summation of some terms in the product form, where each product form contains at least one cumulative density function (CDF) of \( \gamma_{i,u}^{ub} \), at zero. Since \( F_{\gamma_{i,u}^{ub}} (0) = 0 \), we have \( \frac{\partial^n F_{\gamma_{i,u}^{ub}}}{\partial \gamma_{i,u}^{ub}} (0) = 0 \); hence, from (3.68), \( \frac{\partial^n f_{\gamma_{i,u}^{ub}}}{\partial \gamma_{i,u}^{ub}} (0) = 0 \). For \( n = M \), we have

\[
\frac{\partial^{M-1} f_{\gamma_{i,u}^{ub}}}{\partial \gamma_{i,u}^{ub}} (0) = \frac{\partial^{M} F_{\gamma_{i,u}^{ub}}}{\partial \gamma_{i,u}^{ub}} (0)
\]

\[
= M! \prod_{i=1}^{M} f_{\gamma_i}^{ub} (0) + e \left( \gamma_{i,u}^{ub} \right)
\]

(3.69)

where in this equation \( e \left( \gamma_{i,u}^{ub} \right) \) denotes all the other terms. Note that each term in \( e \left( \gamma_{i,u}^{ub} \right) \) has a product form and contains at least one CDF of \( \gamma_{i,u}^{ub} \) at zero and \( e \left( \gamma_{i,u}^{ub} \right) = 0 \). Substituting (3.69) into (3.67) yields

\[
\frac{\partial^{M} f_{\gamma_{i,u}^{ub}}}{\partial \gamma_{i,u}^{ub}} (0) = M! f_{\mu} (0) \prod_{i=1}^{M} f_{\gamma_i}^{ub} (0)
\]

\[
= M! \frac{1}{\mu} \prod_{i=1}^{M} \frac{1}{\beta_i}.
\]

(3.70)

By (3.24), (3.25) and Fact I and following similar steps to above, we obtain

\[
\frac{\partial^{M} f_{\gamma_{i,u}^{lb}}}{\partial \gamma_{i,u}^{lb}} (0) = 2M! \frac{1}{\mu} \prod_{i=1}^{M} \frac{1}{\beta_i}.
\]

(3.71)

Finally by substituting (3.70) and (3.71) into (3.44) and (3.45), we obtain the result in (3.46), which completes the proof.

3.C ASER Performance for PAM, QAM, and PSK Modulation Schemes

3.C.1 Overall picture

In this part we derive closed form formulas for ASER performance of constellations such as \( A \)-ary pulse amplitude modulation (PAM), quadrature amplitude modulation (QAM) and phase shift keying (PSK)\(^6\).

\(^6\)An alternative procedure to find ASER is given in [50].
CHAPTER 3. RELAY SELECTION WITH IMPERFECT CSI

Let’s assume that the transmitted signal which is chosen from a constellation of \(A\) points is denoted by \(x^{(a)}\), and that the distance\(^8\) between two adjacent points in the transmit constellation is \(2\Delta\). Therefore regarding (3.10) the received signal at the destination, in the first time slot and after the MRC can be rewritten as

\[
D_{sd} = \frac{\hat{h}_{sd}^* \sqrt{P_{sd}}}{N_0} \left( \sqrt{P_{sd}} (\hat{h}_{sd} + d_{sd}) x^{(a)} + n_{sd} \right) \\
= \frac{P_{sd} |\hat{h}_{sd}|^2}{N_0} x^{(a)} + \frac{\sqrt{P_{sd} \hat{h}_{sd}^*}}{N_0} \left( \sqrt{P_{sd}} d_{sd} x^{(a)} + n_{sd} \right).
\]

(3.72)

Following the same procedure as in Sec. 3.3 we reach to the effective SNR received from the direct path

\[
\gamma^{(a)}_{sd} = \frac{P_{sd} |\hat{h}_{sd}|^2 |x^{(a)}|^2}{1 + \frac{P_{sd} |x^{(a)}|^2}{N_0} \sigma_{d, sd}^2} = \frac{\hat{\gamma}_{sd}}{|x^{(a)}|^{-2} + \epsilon_{sd}},
\]

(3.73)

where \(\hat{\gamma}_{sd}\) is the estimated received SNR from the direct path. Defining the minimum distance of two adjacent points, in the receive constellation, in the first time slot, at the destination node and after the MRC by \(d_{\min, sd}\), we have

\[
d_{\min, sd} = 2\Delta P_{sd} |\hat{h}_{sd}|^2.
\]

(3.74)

Besides, the noise term variance at the destination node in the first time slot is given by

\[
N_{sd}(a) = P_{sd}^2 |\hat{h}_{sd}|^2 \sigma_{d, sd}^2 |x^{(a)}|^2 + P_{sd} |\hat{h}_{sd}|^2 N_0.
\]

(3.75)

Now we define

\[
\gamma_{sd}^{\text{eff}}(a) = \frac{d_{\min, sd}^2}{2 N_{sd}(a)} = k_0(a) \gamma_{sd}^{(a)}
\]

(3.76)

where

\[
k_0(a) = \frac{2\Delta^2}{|x^{(a)}|^2}.
\]

(3.77)

With a similar approach and noting (3.12) to (3.17) we reach to

\[
\gamma_i = \frac{\hat{\gamma}_{si} \hat{\gamma}_{id}}{\gamma_{si} \lambda_{si} + \gamma_{id} \lambda_{id} + (\epsilon_{si} \epsilon_{id} |x^{(a)}|^2 + |x^{(a)}|^{-2})}
\]

(3.78)

\(x^{(a)}\) can later be replaced by \(x^{(a_1, a_2)}\) for \(a_1, a_2 = 1, \ldots \sqrt{A}\) in QAM modulation.

\(\Delta\)This assumption is only valid for PAM and QAM signals.
where $\lambda_{si} = |x^{(a)}|^2 + \epsilon_{sd}$, $\lambda_{id} = |x^{(a)}|^2 + \epsilon_{si}$, $\hat{\gamma}_{si} = \frac{P |\hat{b}_{si}|^2}{N_0}$, and $\hat{\gamma}_{id} = \frac{P |\hat{b}_{id}|^2}{N_0}$. Again we ignore the last term in the denominator; hence, (3.78) can be approximated as [6, 35]

$$\gamma_{i}^{(a)} = \frac{\hat{\gamma}_{si} \hat{\gamma}_{id}}{\hat{\gamma}_{si} \lambda_{si} + \hat{\gamma}_{id} \lambda_{id}}. \quad (3.79)$$

Defining the minimum distance of two adjacent points, in the receive constellation, in the second time slot, at the destination node and after the MRC by $d_{\min,id}$, we have

$$d_{\min,id} = 2\alpha_{i}^2 \frac{|\hat{h}_{si}|^2 |\hat{h}_{id}|^2}{\Delta_{i}^2 N_0} \quad (3.80)$$

Besides, the noise term variance at the destination node in the second time slot is given by

$$N_{id}(a) = E\{|D|^2\} + E\{|N|^2\}. \quad (3.81)$$

Now we define

$$\gamma_{id}^{\text{eff}}(a) = \frac{d_{\min,id}^2}{2N_{id}(a)} = k_{0}(a) \gamma_{i}^{(a)}. \quad (3.82)$$

Later, the best relay is selected according to

$$\gamma_{r}^{(a)} = \gamma_{sd}^{(a)} + \gamma_{i^{*}}^{(a)}, \quad (3.83)$$

where

$$i^{*} = \arg \max \{\gamma_{i}^{(a)}\}. \quad (3.84)$$

noting that the channels, estimation errors and the noise term in the first and the second time slot are totally independent and since the received signals are passed through the MRC respectively it can be shown that the probability of error between two adjacent points in the transmit constellation at the destination is given by [51]

$$P_e = Q \left( \sqrt{\frac{d_{\min, sd}^2}{2N_{sd}(a)}} + \max_{i} \left\{ \frac{d_{\min,id}^2}{2N_{id}(a)} \right\} \right) \quad (3.85)$$

Defining $\lambda_{sd} = 1 + \epsilon_{sd}$ and regarding the new values of $\lambda_{si}$ and $\lambda_{id}$ and noting that the values of $\hat{\gamma}_{si}$, $\hat{\gamma}_{id}$ and $\hat{\gamma}_{sd}$ do not change, we conclude that all the analysis in Sec. 3.5 remains unchanged except that all the PDFs and SNRs are functions of $|x^{(a)}|^2$. Therefore all the
obtained derivations for $\bar{P}_e$, $\bar{P}_{lb}$ and $\bar{P}_{ub}$ must be averaged over all possible values of $|x^{(a)}|^2$ in the transmit constellation. In particular (3.21) changes to

$$
\bar{P}_e = \frac{1}{A} \sum_{a=1}^{A} \int_{0}^\infty k_1(a)Q\left(\sqrt{k_0(a)\gamma^{(a)}}\right) f_{\gamma^{(a)}}\left(\gamma^{(a)}\right) d\gamma^{(a)}
$$

(3.86)

where $k_0(a)$ and $k_1(a)$ is defined according to the modulation type. Therefore (3.40) and (3.42) can be rewritten as

$$
\bar{P}_{lb} = \frac{1}{A} \sum_{a=1}^{A} \frac{k_1(a)}{2} - \frac{k_1(a)}{2} \sum_{p=1}^{M} \sum_{m=1}^{(M)} (-1)^{(p+1)} \frac{[\Psi^{p}b^{(a)}]_m}{\mu^{(a)}[\Psi^{p}b^{(a)}]_m - 1} \times
$$

$$
\left[ \bar{\mu}^{(a)} \frac{[\Psi^{p}b^{(a)}]_m}{1/\bar{\mu}^{(a)} + k_0(a)/2} - \left[\Psi^{p}b^{(a)}\right]_m^{-1} \frac{[\Psi^{p}b^{(a)}]_m}{1/\mu^{(a)} + k_0(a)/2} \right]
$$

(3.87)

and

$$
\bar{P}_{ub} = \frac{1}{A} \sum_{a=1}^{A} \frac{k_1(a)}{2} - \frac{k_1(a)}{2} \sum_{p=1}^{M} \sum_{m=1}^{(M)} (-1)^{(p+1)} \frac{[\Psi^{p}b^{(a)}]_m}{\mu^{(a)}[\Psi^{p}b^{(a)}]_m - 1} \times
$$

$$
\left[ \bar{\mu}^{(a)} \frac{[\Psi^{p}b^{(a)}]_m}{1/\bar{\mu}^{(a)} + k_0(a)/2} - \left[\Psi^{p}b^{(a)}\right]_m^{-1} \frac{[\Psi^{p}b^{(a)}]_m}{1/\mu^{(a)} + k_0(a)/2} \right].
$$

(3.88)

respectively. Similarly at high SNR regarding Theorem I, the upper and lower bound ASERs change to

$$
\bar{P}_{lb} \rightarrow \frac{1}{A} \sum_{a=1}^{A} \frac{k_1(a)}{2} \prod_{m=1}^{M+1} (2m-1) \mu^{(a)} \prod_{i=1}^{M} \frac{1}{\beta^{(a)}_i}
$$

$$
\bar{P}_{ub} \rightarrow \frac{1}{A} \sum_{a=1}^{A} \frac{k_1(a)}{2} \prod_{m=1}^{M+1} (2m-1) \mu^{(a)} \prod_{i=1}^{M} \frac{1}{\beta^{(a)}_i}.
$$

### 3.C.2 PAM constellation

In this constellation the signal $x$ at the source node is chosen from the constellation points denoted by [51]

$$
x^{(a)} = (2a - 1 - A) \Delta \quad a = 1, \ldots, A
$$
CHAPTER 3. RELAY SELECTION WITH IMPERFECT CSI

where $2\Delta$ is the spacing between transmit constellation points. Assuming that $E\{|x|^2\} = 1$ we have [51]

$$E\{|x|^2\} = \frac{1}{A} \sum_{a=1}^{A} (2a - 1 - A)^2 \Delta^2 = 1$$

which concludes $\Delta = \sqrt{\frac{3}{2(A-1)}}$. For an interior point in the constellation, there are two mutually exclusive ways of making a symbol error whereas only one for the end points, therefore [51]

$$P_e(a) = \begin{cases} 
2Q\left(\sqrt{k_{0,PAM}(a)\gamma_r^{(a)}}\right) & \text{for interior points} \\
Q\left(\sqrt{k_{0,PAM}(a)\gamma_r^{(a)}}\right) & \text{for end points}
\end{cases}$$

(3.89)

where $k_{0,PAM}(a) = \frac{2}{(2a-1-A)^2}$. Therefore in (3.86) to (3.89) for the PAM modulation $k_{1,PAM}(a)$ is either 1 or 2 depending on the location of the constellation point.

3.C.3 QAM constellation

For this constellation the transmitted signal is chosen from $A$ possible transmit constellation points [51]

$$x^{(a_1,a_2)} = (2a_1 - 1 - \sqrt{A})\Delta + j(2a_2 - 1 - \sqrt{A})\Delta \quad a_1, a_2 = 1, \ldots, \sqrt{A}.$$  

$\Delta$ is defined as the half spacing parameter between two adjacent constellation points and in the case $E\{|x|^2\} = 1$ we have [51]

$$E\{|x|^2\} = \frac{\Delta^2}{A} \sum_{a_1,a_2=1}^{\sqrt{A}} (2a_1 - 1 - \sqrt{A})^2 + (2a_2 - 1 - \sqrt{A})^2 = 1$$

which concludes $\Delta = \sqrt{\frac{3}{2(A-1)}}$. Noting that

$$|x^{(a_1,a_2)}|^2 = (2a_1 - 1 - \sqrt{A})^2 \Delta^2 + (2a_2 - 1 - \sqrt{A})^2 \Delta^2$$

the probability of error between two adjacent points in the receive constellation at the destination is given by

$$P_e = Q\left(\sqrt{k_{0,QAM}(a_1,a_2)\gamma_r^{(a)}}\right).$$

(3.90)

where $k_{0,QAM}(a_1,a_2) = \frac{2}{(2a_1-1-A)^2+(2a_2-1-A)^2}$. 

CHAPTER 3. RELAY SELECTION WITH IMPERFECT CSI

The interior points, the points on the corners, and the points on the sides in the QAM constellation contribute differently to the probability of error. The probability of error for different points in the constellation is given by [51]

\[
P_e(a_1, a_2) = \begin{cases} 
4Q\sqrt{k_{0,QAM}(a_1, a_2)\gamma_r^{(a)}} & \text{for interior points} \\
3Q\sqrt{k_{0,QAM}(a_1, a_2)\gamma_r^{(a)}} & \text{for side points} \\
2Q\sqrt{k_{0,QAM}(a_1, a_2)\gamma_r^{(a)}} & \text{for corner points}
\end{cases} \tag{3.91}
\]

Therefore in (3.86) to (3.89) for the QAM modulation \(k_{1,QAM}(a_1, a_2)\) is either 2, 3 or 4 depending on the location of the constellation point.

3.C.4 PSK constellation

For this constellation the transmitted signal is chosen from \(A\) possible unit amplitude transmit constellation points

\[
x^{(a)} = e^{j\frac{2\pi a}{A}} \quad a = 0, \ldots, A - 1.
\]

The minimum distance between two adjacent points in the receive signal constellation from the direct path is given by (see fig. 3.11 (a))

\[
d_{\min, sd} = P_{sd}|\hat{h}_{sd}|^2 \sin\left(\frac{\pi}{A}\right). \tag{3.92}
\]

Figure 3.11: \(d_{\min, sd}\) in the receive PSK constellation (left) and \(d_{\min, id}\) in the receive PSK constellation (right).

\[
d_{\min, sd} = P_{sd}|\hat{h}_{sd}|^2 \sin\left(\frac{\pi}{A}\right). \tag{3.93}
\]
In this case, noting that $|x^{(a)}| = 1$ and that
\[ N_{sd}(a) = P_{sd}|\hat{h}_{sd}|^2\sigma_{d, sd}^2 + P_{sd}|\hat{h}_{sd}|^2N_0 \]
we can define $\gamma_{sd}^{\text{eff}}$ as
\[
\gamma_{sd}^{\text{eff}} = \frac{d_{\text{min}, sd}^2}{2N_{sd}(a)} = k_{0,\text{PSK}} \gamma_{sd}
\]
where $k_{0,\text{PSK}} = \frac{\sin^2(\frac{\pi}{\alpha_1})}{2}$.

Similarly in the second time slot the minimum distance between two adjacent points in the receive constellation is (see fig. 3.11(b))
\[
d_{\text{min, id}} = \frac{\alpha_1^2}{\omega_1^2 N_0}|\hat{h}_{si}|^2|\hat{h}_{id}|^2\sin(\frac{\pi}{A})
\]
Besides, the noise term variance at the destination node in the second time slot is given by
\[ N_{id}(a) = E\{|D|^2\} + E\{|N|^2\}. \]
Again the effective SNR in the $i^{th}$ branch is given by
\[
\gamma_{i}^{\text{eff}} = \frac{d_{\text{min, id}}^2}{2N_{id}} = k_{0,\text{PSK}} \gamma_{i}
\]
and later, regarding (3.85) the probability of error between two adjacent points in the transmit constellation at the destination is given by
\[ P_e = Q\left(\sqrt{k_0(a)\gamma_r}\right) \]
It can be shown that a union bound on the error probability of PSK signals is given by \cite{51}
\[ P_e^r = 2Q\left(\sqrt{k_{0,\text{PSK}}\gamma_r}\right). \]
Therefore $k_{0,\text{PSK}} = 2$ in (3.86) to (3.89) for the PSK modulation $k_{1,\text{PSK}}(a)$ is always 2 disregarding the location of the constellation point.
Chapter 4

Relay Selection with Feedback Delay

4.1 Objective

In chapter 3 we analyzed the effect of CEE, one of the most undesired-performance-degrading phenomena, on the amplify-and-forward cooperative networks with relay selection, S-AF, in terms of outage probability, average capacity and ASER. We stated that in practice availability of perfect CSI at the cooperative network nodes is a dream that comes true with the cost of losing bandwidth and power efficiencies.

In this chapter we analyze the performance of relay selection in the uncoded S-DF, when CEE exist. Additionally we introduce the occurrence of another phenomenon in cooperative networks with relay selection called feedback delay (FD). In practical scenarios, the fading channel gains are unknown and need to be estimated before performing relay selection. In S-DF the best relay is selected at the destination node. The index of the selected relay is then fed back to all potentially available relays. Due to the time varying nature of the fading channels, which is a function of the Doppler shift of the moving terminals, the CSI corresponding to the selected relay is time varying. Consequently, relay selection is performed based on outdated CSI and hence the selection algorithm may not yield the best relay.
4.1.1 Related work and contributions

The setups of [8–10, 12, 13, 17, 20, 52, 53] assume perfect CSI knowledge, however in practical scenarios, the fading gains of communication links are unknown and need to be estimated. Since channel estimation is required for the selection procedures, performance degradation due to CEE is inevitable. In [44, 54, 55], we analyze the effect of CEE on the outage probability, ASER and average capacity of S-AF in details. DSTBC systems are as well exposed to unwanted CEE phenomenon and degradation of their performance in the presence of CEE is inevitable as well. This is proved when Cheng et al. [33] investigated the impact of CEE on the ASER performance of DSTBC systems, assuming the AF mode. Building upon a similar set-up, Gedik and Uysal [31] extend the work of [33] to a system with $M$ relays. In [35], the symbol error rate performance is investigated for the same scenario as in [31].

One of the pioneer papers that proposes a feedback link for a selective diversity communication system is [56]. Skinner and Cavers in [56] propose a feedback scheme in selective diversity systems for the sake of regulating error probability on slowly fading communication channels. They propose and analyze a subchannel selection diversity system when a round-trip delay exists in the system. They prove that the performance of their selective diversity scheme gains 20 to 9 dB of performance improvement depending on the amount of the round trip delay in their system. They also show that fractional delay in their proposed feedback link reduces the promising diversity order of the system to 1.

Although there have been research efforts on conventional MIMO antenna selection with FD and CEE (see for example [57]), only a few isolated results is reported in the context of cooperative communications\(^1\). Suraweera et al. analyze the effect of FD on the performance of a partial relay selection scheme with the AF protocol [60]. In [61], Vicario et al. analyze the outage probability and the achievable diversity order of an opportunistic relay selection scenario with FD.

In this chapter, we investigate the impact of FD and CEE on the performance of a cooperative diversity scheme with relay selection. Assuming imperfect channel estimation and FD, our contributions are summarized as follows:

- We derive exact closed-form formulation for outage probability.

\(^{1}\)At the time we were working on this topic very limited cited works were reported, however at this time a huge amount of research papers are available in the literature, see e.g. [58, 59]
• We derive a lower bound on the average capacity.

• We derive exact ASER and outage probability expressions for S-DF relaying.

• We demonstrate that the asymptotic diversity order\(^2\) is reduced to one with FD which is in line with the results introduced by [61] and reduces to zero in the presence of channel estimation error.

• We present a comprehensive Monte Carlo simulation study to confirm the analytical observations and give insight into system performance.

The rest of the chapter is organized as follows: In Sec. 4.2, we introduce our system setup and the selection strategy. In this section the channel estimation error as well as the delayed feedback link models are illustrated. In Sec. 4.3 and Sec. 4.4 we study the information theoretic performance of the system. In particular, in Sec. 4.3, we investigate the outage performance of the system and develop a closed-form expression for the outage probability and in Sec. 4.4 we introduce a lower bound on the instantaneous capacity and derive a closed-form analytical expression for the average lower bound capacity. In Sec. 4.5 we study the ASER performance of the system in details and derive an exact expression for the ASER. In Sec. 4.6 we analyze the asymptotic order of diversity in S-DF. In particular, we investigate the effect of CEE and FD on the achievable diversity order of the system. Simulation results are presented in Sec. 4.7, and the chapter is concluded in Sec. 4.8.

4.2 System Model

Fig. 4.1 shows the selection cooperation system model studied in this chapter. We consider a multi-relay scenario with \(M\) relays. We assume that the relay \(R_m, m = 1, ..., M\), the source \(S\), and the destination \(D\) are equipped with single transmit and receive antennas. In our system model, we ignore the direct transmission between the source and its destination, due to shadowing. \(h_{sm}\) and \(h_{md}\) represent the channel fading gains between \(S \rightarrow R_m\) and \(R_m \rightarrow D\), respectively.

\(^2\)Achievable diversity order can be obtained from a lower bound analysis which guarantees the diversity performance of the system. Asymptotic diversity order is the high SNR regime diversity order that is seen in the system performance. If the asymptotic diversity order analysis is applied to a lower bound of the system performance, then both terminologies carry the same message.
Assuming a half duplex constraint, the data transmission is performed in two time slots. In the first time slot the source terminal transmits its data to all potentially available $M$ relays. After receiving the source signal via independent channels, all relays, i.e., $R_m$, $m = 1, 2, \ldots M$, decode their received signals and check whether the transmitted signal is decoded correctly or not. We define the decoding set $D(s)$ as the set of all relays that decode the transmitted signal correctly. Clearly, only those relay nodes with a good source to relay channel can be in the decoding set $D(s)$. In the second time slot, the best relay that satisfies an index of merit participates in the transmission and broadcasts its decoded symbol towards the destination.

In this chapter we assume that the source transmits data with average $P_s$ Joules/symbol and similarly each relay node in $D(s)$ can potentially transmit its information with average $P_m$ Joules/symbol, and the receiver noise power is normalized to 1.

### 4.2.1 Model of channel estimation error

All channels are assumed to be independent and non-identically distributed (i.n.i.d), with $h_{ij} \sim \mathcal{CN}(0, \sigma_{h_{ij}}^2)$ where $h_{ij}$ denotes the channel between node $i$ and node $j$. A block fading (quasi-static flat fading) single path scheme is considered for the links. Therefore the channel realizations are assumed to be constant over a block and correlated across blocks. The cross correlation coefficient between the $(n-i_o)^{th}$ and the $n^{th}$ block is $\rho_{f_{ij}} = J_0(2\pi f_{d_{ij}} T_{i_o})$, where $f_{d_{ij}}$ is the Doppler frequency, $J_0(\cdot)$ is the zeroth order modified Bessel function of first kind and $T$ is the block duration. Least mean square error (LMSE) estimator is employed so that the estimation error is orthogonal to the channel estimate.
Let the true channel be $h$, the training sequence with the length $N$ be $x_p$ such that $\|x_p\|^2 = 1$ and the energy allocated to the training symbol be $P$ Joules/Sequence, then $\hat{h}_{ij}$, the estimated channel via LMSE is given by
\[
\hat{h}_{ij} = \rho_{e_{ij}} h_{ij} + d_{ij} \tag{4.1}
\]
where $\rho_{e_{ij}} = \frac{P}{P+1/\sigma_{h_{ij}}^2}$ and $d \sim \mathcal{CN}(0, \sigma_{d_{ij}}^2)$ is the channel estimation error and $\sigma_{d_{ij}}^2 = \rho_{e_{ij}}^2 / P$. Similarly $h_{ij}$ can be modeled as [54]
\[
h_{ij} = \hat{h}_{ij} + u_{ij} \tag{4.2}
\]
where $u_{ij}$ can be interpreted as the channel estimation error and $E\{u_{ij}\hat{h}_{ij}^*\} = 0$ due to principle of orthogonality for optimal estimation. $\hat{h}_{ij}$ and $u_{ij}$ are zero mean Gaussian random variables with variances $\sigma_{\hat{h}_{ij}}^2$ and $\sigma_{u_{ij}}^2$ given by
\[
\sigma_{\hat{h}_{ij}}^2 = \rho_{e_{ij}}^2 \left( \sigma_{h_{ij}}^2 + \frac{1}{P} \right) \tag{4.3a}
\]
\[
\sigma_{u_{ij}}^2 = \frac{1}{P+1/\sigma_{h_{ij}}^2} \tag{4.3b}
\]
respectively. The correlation coefficient between the true channel and the estimated channel is given by
\[
\rho_{h\hat{h}_{ij}} = \frac{E\{h_{ij}\hat{h}_{ij}^*\}}{\sigma_{h_{ij}} \sigma_{\hat{h}_{ij}}} = \frac{\sigma_{\hat{h}_{ij}}}{\sigma_{h_{ij}}} \tag{4.4}
\]
In the first time slot each relay estimates its channel from the source node. Still in the first time slot, each relay transmits training symbols $x_{p_m}$ with $x_{p_m}^T x_{p_i} = \delta_{mi}$ such that the destination terminal estimates the channels from the corresponding relays. It must be noted that transmission from the source node and all the relays should be well synchronized so that the half duplex transmission scheme is followed. The index of the best relay is fed back and the best relay is chosen. In the second time slot the selected relay transmits towards the destination.

\footnote{Following the described pilot assisted channel estimation scenario, the channel estimation is improved by increasing $P$. However, one can assume that the number of training symbols in each frame is sufficiently less than the data symbols, so that increasing the training symbols’ power would not lead to an increase in the average power and hence, the training symbols’ power is independently adjustable, meaning that the channel estimation error is independent of the received data SNR. In this chapter we consider both scenarios in our simulation.}
4.2.2 Feedback delay model

In this chapter, to simplify the notation, we adopt the second strategy mentioned in [1] for relay selection. However, the first strategy obeys the same rules and formulations.

In the second strategy, since the feedback link only transmits the index of the selected relay, a lower feedback bandwidth is required.

Let the estimated channel be \( \hat{h}_{ij} \) and the previously (old) estimated CSI based on which the selection is performed be \( \hat{h}_{ij,sl} \). Since \( \hat{h}_{ij} \) and \( \hat{h}_{ij,sl} \) are both zero mean and jointly Gaussian they can be related as follows [57]

\[
\hat{h}_{ij} = \sigma_{\hat{h}_{ij}} \left( \frac{\rho_{f_{ij}} \hat{h}_{ij,sl}}{\sigma_{\hat{h}_{ij,sl}}} + \sqrt{1 - \rho_{f_{ij}}^2} v_{ij} \right)
\]

where \( v_{ij} \sim \mathcal{CN}(0,1) \). We stress that \( \hat{h}_{ij,sl} \) is the channel which is used for relay selection (using the prior knowledge of the channel), whereas \( \hat{h}_{ij} \) is the channel which is used for decoding (using the current knowledge of the channel). Note that although both \( \hat{h}_{ij} \) and \( \hat{h}_{ij,sl} \) are estimated at the destination, because of the time varying path loss and shadowing effect due to the moving nature of antennas, generally \( \sigma_{\hat{h}_{ij}}^2 \neq \sigma_{\hat{h}_{ij,sl}}^2 \).

4.3 Outage Probability Analysis

In this section we study performance of the our system from the information theory perspective. In the first time slot, the source terminal communicates with the relay terminals by transmitting the signal \( x \) where \( \mathbb{E}\{|x|^2\} = 1 \). Let the received signal at each relay in the first time slot be \( y_{sm,sl} \), \( m = 1, \ldots, M \). To decode the received symbol, each relay estimates the corresponding \( S \rightarrow R_m \) channel and then decodes the received signal using a maximum likelihood (ML) decoder. The received signal at the \( m^{th} \) relay in the first time slot can be written as

\[
\hat{x} = \sqrt{P_s} \hat{h}_{sm,sl}^* y_{sm,sl}
\]

\[
= \sqrt{P_s} \hat{h}_{sm,sl}^* \left( \sqrt{P_s} \hat{h}_{sm,sl} x + n_{sm,sl} \right)
\]

\[
= \sqrt{P_s} \hat{h}_{sm,sl}^* \left[ \sqrt{P_s} \left( \hat{h}_{sm,sl} x + u_{sm,sl} \right) x + n_{sm,sl} \right]
\]

\[
= \sqrt{P_s} \hat{h}_{sm,sl}^* x + \sqrt{P_s} \hat{h}_{sm,sl}^* u_{sm,sl} x + \sqrt{P_s} n_{sm,sl}
\]

\[
\text{message component} \quad \text{error component} \quad \text{noise component}
\]

\[\text{(4.6a) and (4.6b)}\]
where \( n_{sm,sl} \sim \mathcal{CN}(0,1) \) is the relay noise and have substituted (4.2) into (4.6a). Using (4.6b), the received effective SNR at each relay in the first time slot (when selection is performed) is given by

\[
\hat{\gamma}_{sm,sl}^{\text{eff}} = \frac{P_s |\hat{h}_{sm,sl}|^2}{1 + P_s \sigma_{u_{sm,sl}}^2} = P_s \hat{\gamma}_{sm,sl}
\]

(4.7)

where \( \hat{\gamma}_{sm,sl} = \frac{|\hat{h}_{sm,sl}|^2}{1 + P_s \sigma_{u_{sm,sl}}^2} \).

Similarly, still in the first time slot, the destination estimates the \( R_m \rightarrow D \) channels for \( m \in D(s) \) and calculates the effective SNR of each link which is given by

\[
\hat{\gamma}_{md,sl}^{\text{eff}} = \frac{P_m |\hat{h}_{md,sl}|^2}{1 + P_m \sigma_{u_{md,sl}}^2} = P_m \hat{\gamma}_{md,sl}
\]

(4.8)

where \( \hat{\gamma}_{md,sl} = \frac{|\hat{h}_{md,sl}|^2}{1 + P_m \sigma_{u_{md,sl}}^2} \). Here, \( \hat{\gamma}_{sm,sl} \) and \( \hat{\gamma}_{md,sl} \) are exponentially distributed with parameters \( \lambda_{sm,sl} = (1 + P_s \sigma_{u_{sm,sl}}^2)/\sigma_{h_{sm,sl}}^2 \) and \( \lambda_{md,sl} = (1 + P_m \sigma_{u_{md,sl}}^2)/\sigma_{h_{md,sl}}^2 \).

If the relay \( R_m \) is in the decoding set, by sending a flag packet, it signals its capability of participating in cooperation. Then, based on the first time slot channel realizations, the destination selects the relay with the best \( R_m \rightarrow D \) link (see Fig. 4.2)

\[
m^* = \arg \max_{m \in D(s)} \{\hat{\gamma}_{md,sl}\}.
\]

(4.9)

- **Figure 4.2: Signal transmission procedure.**

### 4.3.1 Outage probability with feedback delay and channel estimation errors

Once the best relay is selected by destination, the index of \( R_{m^*} \) is fed back to all relays via a delayed feedback link. This means, at the time the relays receive the index (beginning of the second time slot), the system’s CSI has changed due to the time varying nature of communication links. Let the current \( R_{m^*} \rightarrow D \) channel realization (channel realization in the second time slot), the corresponding estimate and the received SNR from the selected
relay be \( h_{m^*d}, \hat{h}_{m^*d} \) and \( P_{m^*} \hat{\gamma}_{m^*d}, \) respectively. Due to FD, relay selection is done based on the channel coefficients in the first time slot instead of current coefficients, i.e., \( \hat{\gamma}_{m^*d} \) where \( m^* = \arg \max_{i \in D(s)} \{ \hat{\gamma}_{id,sl} \} \) and \( \hat{\gamma}_{md} = \frac{|\hat{h}_{md}|^2}{1 + P_m \sigma_{md}^2} \).

Assuming that the communication between source and destination targets an end-to-end data rate \( R \), the system is in outage if the \( S \rightarrow R_{m^*} \rightarrow D \) link observes an instantaneous capacity per bandwidth\(^4\) \( C' = \frac{1}{2} \log (1 + P_{m^*} \hat{\gamma}_{m^*d}) \) that is below the required rate \( R \), i.e.,

\[
P_{out} = Pr \left[ \frac{1}{2} \log (1 + P_{m^*} \hat{\gamma}_{m^*d}) \leq R \right]
= \sum_{D(s)} Pr[D(s)] Pr \left[ \hat{\gamma}_{m^*d} \leq R_o, \hat{\gamma}_{m^*d,sl} = \max_{m \in D(s)} \{ \hat{\gamma}_{md,sl} \} | D(s) \right]
= \sum_{D(s)} Pr[D(s)] \sum_{m \in D(s)} Pr \left[ \hat{\gamma}_{md} \leq R_o, \hat{\gamma}_{md,sl} \geq \max_{i \in D(s)} \{ \hat{\gamma}_{id,sl} \} | D(s) \right],
\]

\[(4.10)\]

where \( R_o = \frac{2^{2R} - 1}{P_s}. \)

### 4.3.2 Probability of decoding set

The relay \( R_m \) is in the decoding set \( D(s) \) if the \( S \rightarrow R_{m^*} \) link observes an instantaneous capacity per bandwidth \( C_{sm,sl} = \frac{1}{2} \log (1 + P_s \hat{\gamma}_{sm,sl}) \) that is above the required rate \( R \)

\[
C_{sm,sl} = \frac{1}{2} \log (1 + P_s \hat{\gamma}_{sm,sl}) \geq R. \quad (4.12)
\]

Noting that \( \hat{\gamma}_{sm,sl} \) is exponentially distributed, relay \( R_m \) is in the decoding set if \[9\]

\[
Pr[R_m \in D(s)] = Pr[\hat{\gamma}_{sm,sl} \geq R_{o,s}].
= \exp(-\lambda_{sm,sl} R_{o,s})
\]

\[(4.13)\]

where \( R_{o,s} = \frac{2^{2R} - 1}{P_s}. \)

Finally, the probability of selecting a specific decoding set is \[9\]

\[
Pr[D(s)] = \prod_{m \in D(s)} \exp(-\lambda_{sm,sl} R_{o,s}) \prod_{m \notin D(s)} \left[ 1 - \exp(-\lambda_{sm,sl} R_{o,s}) \right].
\]

\[(4.14)\]

\(^4\)Due to CEE, the instantaneous capacity is a lower bound on the true instantaneous capacity. Consequently, the derived outage probability expression is also a lower bound in the presence of channel estimation error. For further details, please refer to Sec. [4.4].
4.3.3  Outage probability conditioned on the decoding set $\mathcal{D}(s)$

Conditioned on the decoding set $\mathcal{D}(s)$ with the old channel realizations (channels in the first time slot), the outage probability with the new CSI (CSI in the second time slot) is

$$ P_{\text{out}}^{\mathcal{D}(s)} = \text{Pr} \left[ \hat{\gamma}_{md} \leq R_o, \hat{\gamma}_{md,sl} \geq \max_{i \in \mathcal{D}(s), i \neq m} \{ \hat{\gamma}_{id,sl} \} \mid \mathcal{D}(s) \right]. $$

(4.15)

Defining

$$ \chi_m \overset{\Delta}{=} \max_{i \in \mathcal{D}(s), i \neq m} \{ \hat{\gamma}_{id,sl} \} $$

and noting that $\chi_m$ and $\hat{\gamma}_{md}$ are independent, the conditioned outage probability on the decoding set is

$$ P_{\text{out}}^{\mathcal{D}(s)} = \int_0^\infty \text{Pr} \left[ \hat{\gamma}_{md} \leq R_o \mid \mathcal{D}(s), \hat{\gamma}_{md,sl} \right] \times \text{Pr} \left[ \hat{\gamma}_{md,sl} \geq \chi_m \mid \mathcal{D}(s), \hat{\gamma}_{md,sl} \right] f_{\hat{\gamma}_{md,sl}}(\hat{\gamma}_{md,sl}) d_{\hat{\gamma}_{md,sl}} $$

$$ = \int_0^\infty F_{\hat{\gamma}_{md}}(R_o) F_{\chi_m}(\hat{\gamma}_{md,sl}) f_{\hat{\gamma}_{md,sl}}(\hat{\gamma}_{md,sl}) d_{\hat{\gamma}_{md,sl}}. $$

(4.16)

Conditioned on $\hat{\gamma}_{md,sl}$ and using (4.5), $\hat{\gamma}_{md}$ has a non-central Chi-square distribution with two degrees of freedom and parameter $\eta_m = c_m \hat{\gamma}_{md,sl}$, where $c_m = \frac{2\rho^2}{1-\rho^2_f m}$. Therefore, we can write $F_{\hat{\gamma}_{md}}(x)$ as [62]

$$ F_{\hat{\gamma}_{md}}(x) = \sum_{k=0}^{\infty} e^{-c_m \lambda_{md,sl} \hat{\gamma}_{md,sl}} \left( c_m \lambda_{md,sl} \hat{\gamma}_{md,sl} \right)^k \frac{\gamma(k + 1, \frac{\lambda_{md,sl}}{2})}{(k!)^2}, $$

(4.17)

where $\gamma(\cdot, \cdot)$ is the incomplete gamma function defined as

$$ \gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt, $$

and $\lambda_{md} = (1 + P_m \sigma^2_{a_{md}}) / \sigma^2_{h_{md}}$. 
Furthermore, \( F_{\chi_m}(x) \) is given by
\[
F_{\chi_m}(x) = \prod_{i \in \mathcal{D}(s) \setminus \{m\}} \left[ 1 - e^{(-\lambda_{id,sl}x)} \right]
= 1 - \sum_{i \in \mathcal{D}(s) \setminus \{m\}} e^{-\lambda_{id,sl}x} + \sum_{i, j \in \mathcal{D}(s) \setminus \{m, j\}} e^{-(\lambda_{id,sl} + \lambda_{jd,sl})x} - \ldots
+ (-1)^{D-1} \sum_{i, j, \ldots, l \in \mathcal{D}(s) \setminus \{m, j, \ldots, l\}} e^{(-\lambda_{id,sl} + \lambda_{jd,sl} + \ldots \lambda_{ld,sl})x}. \tag{4.18}
\]
Substituting (4.17) and (4.18) into (4.16) and noting that 
\[ f_{\gamma_{md,sl}}(x) = \lambda_{md,sl} e^{-\lambda_{md,sl}x} \]
and also using the fact that [38]
\[
\int_{0}^{\infty} x^k \exp(-ax) \, dx = \frac{k!}{a^{k+1}}
\]
we can write the outage probability, conditioned on the decoding set, as in
\[
P_{out}^{\mathcal{D}(s)} = \sum_{k=0}^{\infty} \frac{\lambda_{md,sl}^{k+1} \gamma(k+1, \lambda_{md}R_o/2)}{(k!)^2} \left( \frac{c_m}{2} \right)^k
\times \int_{0}^{\infty} \gamma_{md,sl}^k \left[ e^{-(\lambda_{md,sl} + \lambda_{md,sl} \frac{c_m}{2}) \gamma_{md,sl}} - \sum_{i \in \mathcal{D}(s)} e^{-(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{md,sl} \frac{c_m}{2}) \gamma_{md,sl}} - \ldots
+ \sum_{i, j \in \mathcal{D}(s) \setminus \{m, j\}} e^{-(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{md,sl} \frac{c_m}{2}) \gamma_{md,sl}} - \ldots
+ (-1)^{D-1} \sum_{i, j, \ldots, l \in \mathcal{D}(s) \setminus \{m, j, \ldots, l\}} e^{-(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \ldots \lambda_{ld,sl} + \lambda_{md,sl} \frac{c_m}{2}) \gamma_{md,sl}} \right] \, d\gamma_{md,sl} \tag{4.19}
\]
which can be evaluated as
\[ P_{\text{out}}^{D(s)} = \sum_{k=0}^{\infty} \frac{\gamma(k + 1, \lambda_{md} R_s/2)}{(k!)} \left( \frac{c_m}{2} \right)^k \left[ \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \frac{c_m}{2})^{k+1}} - \sum_{i \in D(s)} \lambda_{md,sl} \frac{\frac{c_m}{2}}{(\lambda_{md,sl} + \frac{c_m}{2})^{k+1}} \right] \]

Finally, substituting (4.14) and (4.20) into (4.11), we obtain

\[ P_{\text{out}} = \sum_{D(s) \in D(s)} \prod_{m \in D(s)} \exp \left( -\lambda_{sm,sl} R_s \right) \prod_{m \in D(s)} \left[ 1 - \exp \left( -\lambda_{sm,sl} R_s \right) \right] \]

\[ \times \sum_{D(s) \in D(s)} \sum_{m \in D(s)} \sum_{k=0}^{\infty} \frac{\gamma(k + 1, \lambda_{md} R_s/2)}{(k!)} \left( \frac{c_m}{2} \right)^k \left[ \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \frac{c_m}{2})^{k+1}} - \sum_{i \in D(s)} \lambda_{md,sl} \frac{\frac{c_m}{2}}{(\lambda_{md,sl} + \frac{c_m}{2})^{k+1}} \right] \]

\[ + (-1)^{D-1} \sum_{i, j, \ldots, l \in D(s)} \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \frac{c_m}{2})^{k+1}} \]  

For the special case where \( P_s = P_m, \lambda = \lambda_{sm,sl} = \lambda_{md,sl} = \lambda_{id,sl} \), and \( c_1 = c_2 = \ldots c_D = c \), (4.21) reduces to

\[ P_{\text{out}} = \sum_{l=1}^{M} \exp \left( -l \lambda R_s \right) [1 - \exp \left( -\lambda R_s \right)]^{M-l} \times l \]

\[ \times \sum_{k=0}^{\infty} \frac{\gamma(k + 1, \lambda R_s/2)}{(k!)} \left( \frac{c}{2} \right)^k \sum_{m=1}^{l-1} \frac{(l-1)}{(m-1)} \]  

4.4 Average Capacity

In this section, we derive an analytical expression for average capacity in the presence of CEE and FD. At the destination in the second time slot and after matched filtering, the
received signal over the selected link is given as
\[
\hat{x} = \sqrt{P_m \hat{h}_{m,d}^*} y_{m,d} \\
= \sqrt{P_m \hat{h}_{m,d}^*} (\sqrt{P_m \hat{h}_{m,d} x + n_{m,d}}) \\
= \sqrt{P_m \hat{h}_{m,d}^*} \left[ \sqrt{P_m (\hat{h}_{m,d} + u_{m,d})} x + n_{m,d} \right] \\
= \frac{P_m |\hat{h}_{m,d}|^2 x + P_m \hat{h}_{m,d}^* u_{m,d} x + \sqrt{P_m \hat{h}_{m,d}^* n_{m,d}}}{\text{message component error component noise component}}.
\] (4.23)

Defining
\[
q \triangleq P_m |\hat{h}_{m,d}|^2 \quad \text{(4.24a)} \\
\tilde{n} \triangleq P_m \hat{h}_{m,d}^* u_{m,d} x + \sqrt{P_m \hat{h}_{m,d}^* n_{m,d}} \quad \text{(4.24b)}
\]

(4.23) can be written as
\[
\hat{x} = qx + \tilde{n}. \quad \text{(4.25)}
\]

The instantaneous capacity conditioned on the decoding set is defined as
\[
C^{D(s)} = \max_{f_x(x)} I(\hat{x}, x) = \max_{f_x(x)} \{\mathcal{H}(\hat{x}) - \mathcal{H}(\hat{x}|x)\} = \max_{f_x(x)} \{\mathcal{H}(\hat{x}) - \mathcal{H}(\tilde{n}|x)\}. \quad \text{(4.26)}
\]

where \(f_x(x)\) is the probability density function (PDF) of the input signal \(x\) and \(\mathcal{H}(\cdot)\) is the differential entropy function. It can be deduced easily from (4.25) that \(\tilde{n}\) and \(x\) are uncorrelated, i.e., \(E{\tilde{n} x^*} = 0\). However, \(\tilde{n}\) and \(x\) are not independent and therefore \(\mathcal{H}(\tilde{n}|x) \neq \mathcal{H}(\tilde{n})\).

This means that the standard procedures for deriving capacity expressions are not applicable to (4.26). Alternatively, noting that conditioning does not increase the entropy, i.e.,
\[
\mathcal{H}(\tilde{n}|x) \leq \mathcal{H}(\tilde{n}),
\]

we can lower bound the capacity as
\[
C^{D(s)} \geq \max_{f_x(x)} \{\mathcal{H}(\hat{x}) - \mathcal{H}(\tilde{n})\}. \quad \text{(4.27)}
\]

It must be noted that the distribution of \(\tilde{n}\) is not Gaussian. However, following [49], Theorem 1, we may assume that \(\tilde{n}\) has zero-mean complex Gaussian distribution with the same variance, which is the worse case distribution. In this case, we can re-write (4.27) as
\[
C^{D(s)} \geq \max_{f_x(x)} \{\mathcal{H}(\hat{x}) - \mathcal{H}(\tilde{n})\} \geq \max_{f_x(x)} \{\mathcal{H}(\hat{x}) - \mathcal{H}(n^\dagger)\}. \quad \text{(4.28)}
\]
where $n^t \sim \mathcal{CN} \left( 0, P_m^2 |\hat{h}_{m^*d}|^2 \sigma_u^2 |\hat{h}_{m^*d}|^2 + P_m |\hat{h}_{m^*d}|^2 \right)$. Maximizing (4.28) with respect to $f_x(x)$ and using (4.25), a lower bound on the average capacity, conditioned on the decoding set, can be written as

$$C_D^{(s)} \geq C_D^{(s) \text{lb}} = \frac{1}{2} \log \left( 1 + \frac{\left| \hat{h}_{m^*d} \right|^2}{P_m |\hat{h}_{m^*d}|^2 \sigma_u^2 |\hat{h}_{m^*d}|^2 + P_m |\hat{h}_{m^*d}|^2} \right)$$

where $\hat{\gamma}_{m^*d} = \frac{|\hat{h}_{m^*d}|^2}{1 + P_m |\hat{h}_{m^*d}|^2 \sigma_u^2 |\hat{h}_{m^*d}|^2}$. Hence, a lower bound on the S-DF capacity can be obtained as

$$C_{\text{lb}} = \sum_{D(s)} C_D^{(s) \text{lb}} \Pr[D(s)].$$

Therefore, the average capacity bound, $\bar{C}_{\text{lb}}$, can be written as

$$\bar{C}_{\text{lb}} = \frac{1}{2} \sum_{D(s)} \Pr[D(s)] \times \int_0^\infty \log \left( 1 + P_{\hat{\gamma}_{m^*d}} |D(s)| (\hat{\gamma}_{m^*d}) d\hat{\gamma}_{m^*d} \right).$$

Substituting (4.40) into (4.31) yields

$$\bar{C}_{\text{lb}} = \frac{1}{2} \ln 2 \sum_{D(s)} \left[ \prod_{m \in D(s)} \exp \left( -\lambda_{sm,sl} R_{o,s} \right) \prod_{m \in D(s)} \left[ 1 - \exp \left( -\lambda_{sm,sl} R_{o,s} \right) \right] \right]$$

$$\times \sum_{m \in D(s)} \left( \sum_{k=0}^{\infty} \sum_{\mu=0}^{k} \frac{(\frac{\mu}{2})^k}{(k-\mu)!} \left[ (-1)^{k-\mu-1} \left( \frac{\lambda_{md,sl}}{2P_m} \right)^{k-\mu} \times \text{Ei} \left( -\frac{\lambda_{md,sl}}{2P_m} \right) + \sum_{t=1}^{k-\mu} (t-1)! \left( -\frac{\lambda_{md,sl}}{2P_m} \right)^{k-\mu-t} \right] \right)$$

$$\times \left[ \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \lambda_{md,sl} \frac{c_m}{2})^{k+1}} - \sum_{i \in D(s)} \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{md,sl} \frac{c_m}{2})^{k+1}} \right]$$

$$+ \sum_{\substack{i,j \in D(s) \ni i \neq j}} \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{md,sl} \frac{c_m}{2})^{k+1}}$$

$$+ \ldots$$

$$+ (-1)^{D-1} \sum_{\substack{i,j, \ldots, l \in D(s) \ni i \neq j, \ldots, l}} \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \ldots \lambda_{ld,sl} + \lambda_{md,sl} \frac{c_m}{2})^{k+1}}. \quad (4.32)$$
CHAPTER 4. RELAY SELECTION WITH FEEDBACK DELAY

Note that in our derivation, we have used the integration formula [38]

\[ \int_0^\infty \ln(1 + ax)x^ke^{-x}dx = \sum_{\mu=0}^{k} \frac{k!}{(k-\mu)!} \left[ \frac{(-1)^{k-\mu-1}e^{1/a}}{a^{k-\mu}} \right] \]

\[ \times \text{Ei} \left( -\frac{1}{a} \right) + \sum_{t=1}^{k-\mu} (t-1)! \left( -\frac{1}{a} \right)^{k-\mu-t} \]

(4.33)

where

\[ \text{Ei}(\gamma) \triangleq \int_{\gamma}^\infty \frac{1}{x} \exp(-x)dx \]

is the exponential integral function.

In the special case where \( P_s = P_m, \lambda = \lambda_{sm,sl} = \lambda_{md,sl} = \lambda_{id,sl}, \) and \( c_1 = c_2 = \ldots c_D = c, \)
the average capacity bound can be obtained by

\[ \bar{C}_{lb} = \frac{1}{2} \ln 2 \sum_{l=1}^{M} \exp(-l\lambda R_{o,s}) \left[ 1 - \exp(-\lambda R_{o,s}) \right]^{M-l} \times \sum_{k=0}^{\infty} \sum_{\mu=0}^{k} \frac{\left( \frac{c}{2} \right)^k}{\lambda^{k+1}(k-\mu)!} \times \left[ \frac{(l-1)}{2} \sum_{m=1}^{l-1} \frac{m-1}{m + \left( \frac{c}{2} \right)^{k+1}} \right]. \]

(4.34)

4.5 Average Symbol Error Rate

In this section, we look into the performance of our system from the communication theory perspective. In particular, we derive a closed form expression for the ASER of S-DF in the presence of CEE and FD for BPSK modulation. The results for other modulation schemes are given in Sec. 4.B. Let the received SNR via the selected path in the second time slot be denoted by \( \hat{\gamma}_{m*,d} = P_m \hat{\tilde{\gamma}}_{m*,d}. \) Then, noting that \(|x|^2 = 1\) the ASER can be written as

\[ \bar{P}_e = \int_0^\infty k_1 Q \left( \sqrt{k_0 P_m \hat{\tilde{\gamma}}_{m*,d}} \right) f_{\hat{\tilde{\gamma}}_{m*,d}}(\hat{\tilde{\gamma}}_{m*,d}) d\hat{\tilde{\gamma}}_{m*,d} \]

(4.35)

where \( k_1 \) and \( k_0 \) depend on the modulation scheme, and \( f_{\hat{\tilde{\gamma}}_{m*,d}} \) is the PDF of \( \hat{\tilde{\gamma}}_{m*,d}, \) which is given by (see Sec. 4.A for details)

\[ f_{\hat{\tilde{\gamma}}_{m*,d}}(x) = \prod_{i=1}^{M} B_i \delta(x) + \sum_{\mathcal{D}(s)} \left[ \prod_{i \in \mathcal{D}(s)} (1 - B_i) \prod_{i \notin \mathcal{D}(s)} B_i \right] f_{\hat{\tilde{\gamma}}_{m*,d}|\mathcal{D}(s)}(x) \]

(4.36)
where \( \delta(x) \) is the delta function and \( B_i = \frac{k_1}{2} \left[ 1 - \sqrt{\frac{k_0 P_s}{k_0 P_s + 2\lambda_{si,sl}}} \right] \). \( \gamma \) is also small. Conversely, when the relays receive low average SNR then the threshold limit is put to use. At high SNRs where \( B_i \) is expected to be very small the threshold limit is also small. Conversely, when the relays receive low average SNR then the threshold limit is let to be loose.

Regarding (4.38) and (4.39), the conditional PDF of the received signal can be then written

\[
B_i = \frac{k_1}{2} \left[ 1 - \sqrt{\frac{k_0 P_s}{k_0 P_s + 2\lambda_{si,sl}}} \right].
\] (4.37)

Here, \( f_{\gamma_{m,sl}^{D(s)}}(x) \) is the conditional PDF of the received signal via the selected path, which is given by

\[
f_{\gamma_{m,sl}^{D(s)}}(x) = \frac{\partial F_{\gamma_{m,sl}^{D(s)}}(x)}{\partial x}.
\] (4.38)

Following the same procedure for obtaining \( P_{\text{out}}^{D(s)} \) we reach to

\[
F_{\gamma_{m,sl}^{D(s)}}(x) = \sum_{k=0}^{\infty} \frac{\gamma(k+1, \lambda_{md} x/2)}{(k!)} \left( \frac{c_m}{2} \right)^k \left[ \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{ld,sl} + \lambda_{md,sl} c_m/2)^{k+1}} \right]
\]

\[
- \sum_{i \in D(s)} \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{ld,sl} + \lambda_{md,sl} c_m/2)^{k+1}} + \sum_{i, j \in D(s)} \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{ld,sl} + \lambda_{md,sl} c_m/2)^{k+1}} - \ldots
\]

\[
+ (-1)^{D-1} \sum_{i, j, \ldots, i \in D(s)} \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{ld,sl} + \lambda_{md,sl} c_m/2)^{k+1}}.
\] (4.39)

Regarding (4.38) and (4.39), the conditional PDF of the received signal can be then written as by

\[
f_{\gamma_{m,sl}^{D(s)}}(x) = \sum_{k=0}^{\infty} \frac{\lambda_{md} e^{(-\lambda_{md} x/2)}}{2k!} \left( \frac{c_m}{2} \right)^k \left[ \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{ld,sl} + \lambda_{md,sl} c_m/2)^{k+1}} \right]
\]

\[
- \sum_{i \in D(s)} \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{ld,sl} + \lambda_{md,sl} c_m/2)^{k+1}} + \sum_{i, j \in D(s)} \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{ld,sl} + \lambda_{md,sl} c_m/2)^{k+1}} - \ldots
\]

\[
+ (-1)^{D-1} \sum_{i, j, \ldots, i \in D(s)} \frac{\lambda_{md,sl}^{k+1}}{(\lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{ld,sl} + \lambda_{md,sl} c_m/2)^{k+1}}.
\] (4.40)

\footnote{We may assume the relay \( R_t \) is in the decoding set if the \( S \rightarrow R_t \) instantaneous SNR in the first time slot is above a predefined threshold limit \( \gamma_t \), then in (4.36) \( B_i = F_{\gamma_{si,sl}^{D(s)}}(\gamma_t) = 1 - e^{(-\lambda_{si,sl} \cdot \gamma_t)} \). This way an adaptive threshold is put to use. At high SNRs where \( B_i \) is expected to be very small the threshold limit \( \gamma_t \) is also small. Conversely, when the relays receive low average SNR then the threshold limit \( \gamma_t \) is let to be loose.}
Finding a closed form formula for the integral in (4.35) is not tractable. However, by substituting (4.36) into (4.35), we obtain

\[
\bar{P}_e = \frac{k_1}{2} \prod_{i=1}^{M} B_i + \sum_{D^{(s)}} \left[ \prod_{i \in D^{(s)}} (1 - B_i) \prod_{i \notin D^{(s)}} B_i \right] \times k_1 \sum_{m \in D^{(s)}} \sum_{n=1}^{\infty} \sum_{k=0}^{\infty} a_n \Gamma \left( k + \frac{n+1}{2} \right) / k! \left( \frac{c_m}{2} \right)^k \\
\times \left( \frac{\lambda_{md}}{2P_{m\theta}} \right)^{(k+1)} \left[ \frac{\lambda_{md,sl} k+1}{\left( \lambda_{md,sl} + \lambda_{md,sl} \frac{c_m}{2} \right)^{k+1}} - \sum_{i \in D^{(s)}} \frac{\lambda_{md,sl} k+1}{\left( \lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{md,sl} \frac{c_m}{2} \right)^{k+1}} \right] \\
+ \sum_{i, j \in D^{(s)}} \frac{\lambda_{md,sl} k+1}{\left( \lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{md,sl} \frac{c_m}{2} \right)^{k+1}} - \cdots
\]

where we have used the approximation [63]

\[
Q(x) \approx e^{-\frac{x^2}{2}} \sum_{n=1}^{n_a} a_n x^{n-1},
\]

where

\[
a_n = \frac{(-1)^{n+1} \left( A \right)^n}{B \sqrt{\pi} (\sqrt{2})^{n+1} n!},
\]

with \( A = 1.98 \) and \( B = 1.135. \)

In the special case where \( P_s = P_m, \lambda = \lambda_{sm,sl} = \lambda_{md,sl} = \lambda_{id,sl} \) and \( c_1 = c_2 = \ldots c_D = c, \) (4.41) reduces to

\[
\bar{P}_e = \frac{k_1}{2} B^M + \sum_{l=1}^{M} \left[ \binom{M}{l} (1 - B)^l B^{M-l} \right] \times k_1 l \\
\times \sum_{k=0}^{\infty} \sum_{n=1}^{n_a} a_n \Gamma \left( k + \frac{n+1}{2} \right) / k! \left( \frac{c}{2} \right)^k \times \frac{\lambda}{2P_{s\theta}} \\
\times \sum_{m=1}^{l} \frac{l-1}{m-1} \frac{1}{(m + \frac{c}{2})^{k+1}}
\]

where \( B = B_1 = \ldots = B_M. \)

### 4.6 Asymptotic Diversity Order

In the previous section, we have derived an exact ASER expression with FD and CEE, which is valid for the entire SNR range. To gain further insights into the system's performance,
we focus here on the asymptotic SNR regime and analyze the asymptotic ASER for S-DF conditioned on the decoding set, i.e., \( \bar{P}_e|D(s) \). For \( P_s, P_m \to \infty \) we can fairly assume that \( P_S = P_m \) and \( \lambda = \lambda_{sm,sl} = \lambda_{md,sl} = \lambda_{id,sl} \) and \( c_1 = c_2 = \ldots c_D = c \).

Using (4.44), we can write the ASER, conditioned on the decoding set, as

\[
\lim_{P_s \to \infty} \bar{P}_e|D(s) = \lim_{P_s \to \infty} k_1 \sum_{k=0}^{\infty} \sum_{n=1}^{n_a} a_n \Gamma \left( k + \left( \frac{n+1}{2} \right) \right) / k! \left( \frac{c}{2} \right)^k
\]

\[
\times \left( \frac{\lambda}{2P_s k_0} \right)^{k+1} \sum_{m=1}^{l} \frac{1}{m+\frac{c}{2}} \frac{1}{(m-1)} \left( l - 1 \right)
\]

\[
= \lim_{P_s \to \infty} \sum_{k=0}^{\infty} \sum_{n=1}^{n_a} g(n,k) \left( \frac{\lambda}{2P_s k_0} \right)^{k+1} \left( \frac{1}{1 + \frac{\lambda}{2P_s k_0} \left( \frac{n+1}{2} \right)} \right)
\]

(4.45)

where \( g(n,k) \) is a constant depending on \( k \) and \( n \).

In the following, we discuss three different scenarios:

4.6.1 Diversity order with FD and perfect CSI

With perfect CSI, we have \( \lambda = 1/\sigma_h^2 \). Therefore, as \( P_s \to \infty \), the dominant term in (4.45) would be the term corresponding to \( k = 0 \) and \( n = 1 \), i.e.

\[
\lim_{P_s \to \infty} \bar{P}_e|D(s) \propto \frac{1}{1 + k'/P_s} \times \frac{1}{P_s}
\]

\[
\propto \frac{1}{P_s + k'} = \mathcal{O}(P_s^{-1})
\]

(4.46)

where \( k' \) is a constant. Thus, in this case, the achievable diversity order in the presence of delayed feedback is 1.

\[6\] In (4.44) the order associated with \( B^M \) is \( M \) when \( \lambda \) is constant and is \( 2M \) when \( \lambda \propto 1/P_s \). The order associated with \( \sum_{l=1}^{M} \left( \begin{array}{c} M \\ l \end{array} \right) (1 - B)^{l} B^{M-l} \) is zero in each case. Therefore, in the case that \( \lambda \) is constant the diversity order is \( \min(M, \mathcal{O}(P_e|D(s))) \) and in the other case the diversity order is \( \min(2M, \mathcal{O}(P_e|D(s))) \).
4.6.2 Diversity order in the presence of FD and CEE, when $P$ is constant

In the presence of channel estimation error, if we assume the training symbols’ power $P$ remains constant\(^7\) while $P_s = P_m \rightarrow \infty$ then $\lambda$ is proportional to $P_s$, and it should be taken into consideration. From (4.45), we have

$$\lim_{P_s \rightarrow \infty} P_{e|D(s)} \propto \lim_{P_s \rightarrow \infty} \sum_{k=0}^{\infty} \sum_{n=1}^{n_a} \left( \frac{\lambda}{P_s} \right)^{k+1} \frac{1}{\frac{k+1}{k+1}} = K$$

and $k^l$ and $K$ are constants. Hence, the asymptotic diversity order in the presence of FD and CEE is 0. This means that the outage probability or the ASE curves are expected to be saturated at high SNR with imperfect CSI when the training power is constant.

4.6.3 Diversity order in the presence of FD and CEE, when $P \rightarrow \infty$

In this case at high SNRs $\sigma^2_{\hat{h}} = \sigma^2_h$ and $P_s \sigma^2_u \rightarrow 1$ therefore in this case the behavior of the system would be similar to the first case and hence the diversity order would be 1.

4.7 Simulation Results

In this section, we investigate the performance of selection cooperation in the presence of CEE and FD through Monte-Carlo simulation. The transmitted symbols are drawn from an antipodal BPSK constellation, which means, $k_1 = 1$ and $k_0 = 2$. The node-to-node channels are assumed to be zero mean independent Gaussian processes, with variance $\sigma^2_{h_{ij}} = 1/(d_{ij}^\alpha)$ and $\sigma^2_{\hat{h}_{ij,sl}} = 1/(d_{ij,sl})^\alpha$. $d_{ij}$ and $d_{ij,sl}$ are the distances between nodes $i$ and $j$ at time $n$ and $n-1$ and $\alpha$ is the order of path loss. In this case, $\hat{h}_{ij}$ and $\hat{h}_{ij,sl}$ are zero mean Gaussian processes with variance defined in (4.3b). The variance of receiver noise is set to 1 for all the relays and the destination (the effect of noise can be included in the transmitter power or in the channel instantaneous power) and $R = 1$ bps/Hz. In the first part of the simulations for the sake of simplicity and to get an overview of the behavior of the system we assume a symmetric scenario where all the channel variances as well as $\rho_e$s and $\rho_f$s are identical. We

\(^7\)This case is occurred when the number of training symbols are sufficiently smaller than the number of data symbols in a frame. Therefore a change in training symbol power would not make an impact on the total sequence power and thus, the training power may be adjusted independently.
also assume $P_s = P_m$ and $P = 1$ Joules/Sequence for the first part. In the second part of the study however, we incorporate the effect of distance and unequal powers in the simulation parameters.

4.7.1 Outage probability

![Figure 4.3: Outage Probability for $M = 4$ and perfect CSI in the presence of delay in the feedback link. In this figure $\rho_e = 1$ and $\rho_f = 0.6, 0.7, \ldots 1$. It is observed that the performance of the system is drastically degraded in the presence of FD.](image)

Fig. 4.3 shows the performance of the system with $M = 4$ relays with FD for $\rho_f = 0.6, 0.7, 0.8, 0.9$ and 1. In Fig. 4.3, we assume perfect CSI, i.e., $\rho_e = 1$. We observe a perfect match between the analytical and simulation results. It can also be deduced from the slope of the curves that, for $M = 4$ and $0 < \rho_f < 1$, the diversity order of the selection scheme is 1. In the case of ideal feedback link, i.e., $\rho_f = 1$, the diversity order of 4 is observed. Fig. 4.4 illustrates the performance of the system for $M = 2, 3, 4$, with $\rho_f = 0.9, 1$. In Fig. 4.4, the same slope for different number of relays is noticed, confirming our analytical observations. For the case of $\rho_f = 1$ and choosing $M = 3, 4$, the diversity orders of 3 and 4 are observed, respectively. In Fig. 4.5, we study the effect of both FD and CEE. An error floor, in the presence of channel estimation error, is noticed, as predicted by (4.47).
Figure 4.4: Outage Probability for $M = 2, 3, 4$ and perfect CSI in the presence of delay in the feedback link. In this figure $\rho_e = 1$ and $\rho_f = 0.9$. It is observed that the performance of the system is drastically degraded in the presence of FD.

Figure 4.5: Outage probability for $M = 2$ in the presence of delay in the feedback link and channel estimation error. In this figure $\rho_f = 0.9$. It is observed that in the case the training sequence power is constant the channel estimation error is not recovered by increasing the SNR and therefore the system is subjected to an error floor. Selection in this case fails to provide diversity order.
4.7.2 Average capacity

![Graph showing average capacity versus SNR for different values of \( \rho_f \) and \( \rho_e \).](image)

Figure 4.6: Lower bound average capacity versus SNR for different values of \( \rho_f \) and \( \rho_e \). A huge gap between the ideal case and the sample cases with channel estimation error exists. Channel estimation error in this case is constant with respect to SNR.

Fig. 4.6 shows the lower bound average capacity in bits per second per Hz per bandwidth versus SNR. It is observed that the presence of CEE result in capacity ceilings in the average capacity curves. It can be also seen that FD aggravates the average capacity performance of the system. However, CEE have a greater effect in worsening the average capacity performance of the system.

4.7.3 ASER performance

Fig. 4.7 shows the ASER in the presence of FD for \( M = 3 \). We assume perfect CSI knowledge. Assuming an ideal feedback link, the full diversity order of 3 is achieved. However, with FD, the diversity order is reduced to 1, as predicted earlier.

Fig. 4.8 illustrates the performance of the ASER in the presence of CEE and FD for \( M = 2 \). It is clear from Fig. 4.8 that CEE reduce the diversity order of the system to zero, confirming our earlier analysis.
Figure 4.7: ASER for $M = 3$ and perfect CSI in the presence of delay in the feedback link. In this figure $\rho_e = 1$, $\rho_f = 0.6, 0.7, \ldots, 1$. Again the ASER performance of the system is heavily under the influence of FD. FD reduces the diversity order of the system to 1.

Figure 4.8: ASER for $M = 2$ in the presence of delay in the feedback link and channel estimation error. In this figure $\rho_f = 0.9$. Again the ASER performance of the system is heavily under the influence of CEE and FD. CEE reduces the diversity order of the system to zero if the training sequence power is not increased with the data SNR. FD reduces the diversity order of the system to 1.
4.7.4 Diversity order

The asymptotical diversity order $d$ is a measure of parsimony in power utilization respected to a direct transmission mode for obtaining the same performance in a Rayleigh fading channel and is given by the magnitude of the slope of ASER against average SNR in a log-log scale [4]:

$$d = \lim_{\text{SNR} \to \infty} - \frac{\log (\bar{P_e} | D(s))}{\log \text{SNR}}. \quad (4.48)$$

In particular the steepest the ASER curve is, the more power we can save to obtain the same performance.

Fig. 4.9 shows the diversity performance of the system in the presence of FD link for different values of $\rho_f$. We assume perfect CSI knowledge. It is observed that the asymptotical diversity order of the system tends to 1 for all $\rho_f$ values. This illustrates the destructive effect of FD and demonstrates that the impact of relay selection in this case is neutralized in terms of diversity order.

Fig. 4.10 illustrates the asymptotical diversity for different number of relays i.e., $M = 2, 3, 4$. It is obvious that at high SNR the diversity order is independent of the number of relays in the system.

Fig. 4.11 depicts the asymptotical diversity order in the presence of CEE. It is noticed that the asymptotical diversity order in this case is reduced to zero, confirming our earlier observations in Figs. 4.5 and 4.8.

4.7.5 Training power $P$

Fig. 4.12 represents the ASER behavior of the system when $P_s = P_m$ and $\rho_f = 1$ for different values of $P$. In all cases $P$ increases with SNR. As it can be seen diversity order is preserved in all cases and is equal to the full diversity order, however the performance of the system degrades as less power is allocated to the training sequence.

4.7.6 Distance

Fig. 4.13 Shows the ASER performance versus distance. In this simulation we assume that the distance between $R_1 \rightarrow R_2$ is the same as $S \rightarrow D$ distance, and is normalized to 1m. We also assume that the relays are located symmetric with respect to $S \rightarrow D$ virtual line.
Figure 4.9: Diversity order versus SNR. $\rho_e = 1$ and $M = 4$. The figure is plotted for different values of $\rho_f$ and is obtained from (4.45). As $\rho_f$ is increased regarding (4.5) the channel mean is increased and therefore the ASER curve fall more rapidly; however on the contrary, increasing SNR decreases $\hat{\gamma}_{md,sl}$ which leads to decrease in the channel mean. Therefore by increasing the SNR the channel would gradually become more Rayleigh than Rician. The peak in the diversity order is observed when increasing $\rho_f$ is no more helpful in keeping the channel Rician.
Figure 4.10: Diversity order versus SNR. $\rho_e = 1$, $\rho_f = 0.9$ and $M = 2, 3, 4$. The figure is obtained from (4.48).

Figure 4.11: Diversity order versus SNR. $\rho_f = 0.9$ and $M = 3$. The figure is plotted for different values of $\rho_e$ and is obtained from (4.48).
Figure 4.12: ASER for different values of $P$, $M = 2$. In this figure it is obvious that increasing the training sequence power pays off for the ASER performance. In this case as can be seen in this figure the full diversity order is still maintained however the ASER performance depends to how much training power do we spend. DF in this case is still impairing. Diversity order in the presence of FD is always 1.
of sight. Therefore the locations of the \( S, D, R_1 \) and \( R_2 \) are \((0, 0), (0, 1m), (d, 0.5m) \) and \((dm, -0.5m)\). We change \( d \) from \(0.1m\) to \(0.9m\) and plot the ASER versus distance for different values of SNR. It must be noted that the path loss exponent for the corresponding channels is chosen to be 4. In some environments, such as buildings, stadiums and other indoor environments, the path loss exponent can reach values in the range of 4 to 6 [64].

As the plot indicates depending to \( P_S \) and \( P_{m^*} \) the best performance is achieved when the relays are located almost in the middle of the \( S \rightarrow D \) path. The more the \( P_s \) power is the more probable the relays are in the decoding set. Therefore the performance of the system depends on the selected relay’s transmission power. With less transmission power the \( R_{m^*} \rightarrow D \) link path loss degrades the performance and therefore to overcome this situation the relays must be close to \( D \). Besides, if \( P_s \leq P_{m^*} \) combating with the path loss in the backward link the relays try to lodge themselves in the decoding set. Therefore the optimal location to settle the relay towers would be in the vicinity of the source node.

The more the relays approach the source node, the more probable they are in the decoding set, however the path loss regarding the \( R \rightarrow D \) link is increased and that degrades the

---

**Figure 4.13:** ASER versus distance. The distance is normalized with respect to source-destination distance. In this figure \( \rho_f = 1 \) and \( P_{t,s} = P_s \), \( P_{t,1} = P_{t,2} = P_s / 2 \), \( M = 2 \). To provide the best ASER performance the relays must be located almost in the middle of the \( S \rightarrow D \) direct path. As depicted here, if the selected relay transmits with less power then it is optimal if the relay is closer to the destination.
CHAPTER 4. RELAY SELECTION WITH FEEDBACK DELAY

system performance. Similarly as the relays approach the destination node the probability of decoding set decreases tremendously. This means that there is hardly a relay in the decoding set and therefore the probability that a relay transmits is decreased although the $R \rightarrow D$ channels’ path loss are negligible. To observe our statement mentioned above we also have simulated the probability of a decoding set consist of two relays versus distance. Fig. 4.14 Shows the $Pr[D(s)]$ with $D = 2$ performance versus distance. Obviously as the distance of the relays with respect to the source node is increased the probability of decoding set in decreased.

![Figure 4.14: $Pr[D(s)]$ versus distance. The distance is normalized with respect to source-destination distance. In this figure $\rho_f = 1$ and $P_{t,s} = P_s$, $M = 2$. $Pr[D(s)]$ in this figure is the probability that both relays are in the decoding set.](image)

\[ P = \text{dB to 10dB} \]

4.8 Conclusion

In this chapter, we discussed relay selection for DF protocol in cooperative networks. We showed that the presence of CEE and FD degraded the performance and also reduced the diversity order of S-DF. In particular in the presence of FD the diversity order reduced to one 1 and in the presence of fixed channel estimation error the diversity order becomes 0 and an error floor is visible in the ASER and outage probability versus SNR curves. Also we studied the case where the training power sequence increased in parallel with the data SNR.
CHAPTER 4. RELAY SELECTION WITH FEEDBACK DELAY

We saw that the diversity loss due to the fixed channel estimation error was recovered by increasing the pilot symbols power. We investigated the impact of path loss on the system performance by incorporating the node-to-node distances to our equations. The optimal location of the relays were found to be in relative to the source and relays’ transmission power. We derived analytic expressions for the outage probability, average symbol error rate and average capacity bound.

4.A Deriving the Probability Density Function of $\gamma_{m\star d}$

Let $\gamma_{m\star d}$ denote the normalized received SNR at the destination terminal over the $S \rightarrow R_{i\star} \rightarrow D$ link. Then, the PDF of $\gamma_{m\star d}$ is written as

\[
f_{\gamma_{m\star d}}(x) = Pr(\text{all relays are off})f_{\gamma_{m\star d} | (\text{all relays are off})}(x) + \sum_{\mathcal{D}(s)} Pr(\text{relays in } \mathcal{D}(s) \text{ are on})f_{\gamma_{m\star d} | (\text{relays in } \mathcal{D}(s) \text{ are on})}(x). \tag{4.49}
\]

The $i^{th}$ relay decodes its received signal erroneously with probability $B_i$ which is given in (4.37). Since all $S \rightarrow R_i$ links are statistically, we have

\[
Pr(\text{all relays are off}) = \prod_{i=1}^{M} B_i. \tag{4.50}
\]

On the other hand, if all the relays are off, then no communication would occur between source and destination terminal. The received SNR at the destination terminal would be zero. Therefore, the conditional PDF can be written as [39]

\[
f_{\gamma_{m\star d} | (\text{all relays are off})}(x) = \delta(x). \tag{4.51}
\]

The probability of decoding set, given that there is at least one relay in $\mathcal{D}(s)$, is given by

\[
Pr(\text{relays in } \mathcal{D}(s) \text{ are on}) = \left[ \prod_{i \in \mathcal{D}(s)} (1 - B_i) \prod_{i \notin \mathcal{D}(s)} B_i \right]. \tag{4.52}
\]

Inserting (4.50), (4.51) and (4.52) in (4.49) yields (4.36).
4.B  ASER Performance for PAM, QAM, and PSK Modulation Schemes

4.B.1  Overall picture

In this part we derive closed form formulas for ASER performance of constellations such as A-ary PAM, QAM and PSK.

Let’s assume that the transmitted signal which is chosen from a constellation of \( A \) points is denoted by \( x(a) \) and that the distance between two adjacent points in the transmit constellation is \( 2\Delta \). Regarding (4.6b) the received signal in the \( m^{th} \) relay after passing through the MRC is given by

\[
\hat{x}(a) = P_s |\hat{h}_{sm,sl}|^2 x(a) + P_s \hat{h}_{sl}^* u_{sm,sl} x(a) + \sqrt{P_s} \hat{h}_{sl}^* n_{sm,sl},
\]

and therefore the instantaneous effective SNR is

\[
\gamma_{sm,sl}^{\text{eff}}(a) = \frac{P_s |\hat{h}_{sm,sl}|^2 |x(a)|^2}{P_s |x(a)|^2 \sigma_{u_{sm,sl}}^2 + 1}.
\]

Besides, the probability of erroneous detection of a symbol with its adjacent constellation point in the \( m^{th} \) relay node in the first time slot is given by [51]

\[
P_e^r(a) = Q \left( \sqrt{\frac{d_{\text{min}}^2}{2N(a)}} \right)
\]

where

\[d_{\text{min}} = 2\Delta P_s |\hat{h}_{sm,sl}|^2\]

is the minimum distance between two adjacent points in the signal constellation after the MRC, and

\[N(a) = P_s^2 |\hat{h}_{sm,sl}|^2 |x(a)|^2 \sigma_{u_{sm,sl}}^2 + P_s |\hat{h}_{sm,sl}|^2.\]

Substituting the values of \( d_{\text{min}} \) and \( N(a) \) in (4.55) the instantaneous symbol error rate at the \( m^{th} \) relay is given by

\[
P_e^r(a) = Q \left( \sqrt{k_0(a) \gamma_{sm,sl}^{\text{eff}}(a)} \right)
\]

\[a\text{An alternative procedure to find ASER is given in [50].}
\[b\text{\( x(a) \) can later be replaced by \( x(a_1, a_2) \) for \( a_1, a_2 = 1, \ldots, \sqrt{A} \) in QAM modulation.}
\[c\text{This assumption is only valid for PAM and QAM signals.} \]
where

$$k_0(a) = \frac{2\Delta^2}{|x(a)|^2}.$$  \hspace{1cm} (4.58)

Similarly regarding (4.8) still in the first time slot the effective SNR calculated by the destination is given by

$$\hat{\gamma}_{md,sl}^{\text{eff}}(a) = \frac{P_m |x(a)|^2 |\hat{h}_{md,sl}|^2}{P_m |x(a)|^2 \sigma_{md,sl}^2 + 1}. \hspace{1cm} (4.59)$$

Again in the second time slot we define

$$\hat{\gamma}_{md}(a) = \frac{P_m |x(a)|^2 |\hat{h}_{md}|^2}{P_m |x(a)|^2 \sigma_{md}^2 + 1}. \hspace{1cm} (4.60)$$

Regarding the definition of $\hat{\gamma}_{sm,sl}^{\text{eff}}(a)$, $\hat{\gamma}_{md,sl}^{\text{eff}}(a)$ and $\hat{\gamma}_{md}(a)$, all are random variables with exponential distribution and with parameters

$$\lambda_{sm,sl} = \frac{P_s |x(a)|^2 \sigma_{sm,sl}^2 + 1}{P_s \sigma_{sm,sl}^2}$$

$$\lambda_{md,sl} = \frac{P_m |x(a)|^2 \Delta^2 \sigma_{md,sl}^2 + 1}{P_m \sigma_{md,sl}^2}$$

and

$$\lambda_{md} = \frac{P_m |x(a)|^2 \Delta^2 \sigma_{md}^2 + 1}{P_m \sigma_{md}^2}$$

respectively.

Based on the first time slot realizations the best relay which contributes to the best $R_m \rightarrow D$ channel is selected, i.e.,

$$m^* = \arg \max_m \{\hat{\gamma}_{md,sl}^{\text{eff}}(a)\} \hspace{1cm} (4.61)$$

therefore

$$\hat{\gamma}_{sm,d}^{\text{eff}}(a) = \frac{P_{m^*} |x(a)|^2 |\hat{h}_{md}|^2}{P_{m^*} |x(a)|^2 \sigma_{md}^2 + 1}. \hspace{1cm} (4.62)$$

Noting the theory we mentioned above and the procedure in Sec.4.3.3 and regarding (4.38) we can easily conclude that

$$f_{\hat{\gamma}_{md}^{\text{eff}}(a)}(\mathcal{D}(s)) = f_{\hat{\gamma}_{md}(a)}(\mathcal{D}(s)).$$
Additionally, the PDF of $\gamma_{\text{eff}}^{\text{m}_d}(a)$ is given by

$$f_{\gamma_{\text{eff}}^{\text{m}_d}}(x) = \prod_{i=1}^{M} B_i \delta(x) + \sum_{D(s)} \prod_{i \in D(s)} (1 - B_i) \prod_{i \notin D(s)} B_i f_{\gamma_{\text{eff}}^{\text{m}_d}}(D(s))(x)$$  \hspace{1cm} (4.63)$$

where $B_i$ is calculated according to the modulation scheme.

In the sequel, we mention the procedures to find $B_i$ and the ASER for each modulation scheme.

### 4.B.2 PAM constellation

In this constellation, the signal $x$ at the source node is chosen from the constellation points denoted by [51]

$$x^{(a)} = (2a - 1 - A)\Delta \quad a = 1, \ldots, A$$

where $2\Delta$ is the spacing between transmit constellation points. Assuming that $E\{|x|^2\} = 1$, we have [51]

$$E\{|x|^2\} = \frac{1}{A} \sum_{a=1}^{A} (2a - 1 - A)^2 \Delta^2 = 1$$

which concludes $\Delta = \sqrt{\frac{3}{A^2 - 1}}$. For an interior point in the constellation, there are two mutually exclusive ways of making a symbol error whereas only one for the end points, therefore [51]

$$P_e^r(a) = \begin{cases} 
2Q \left( \sqrt{k_{0,PAM}(a)} \gamma_{\text{eff}}^{\text{m}_d}(a) \right) & \text{for interior points} \\
Q \left( \sqrt{k_{0,PAM}(a)} \gamma_{\text{eff}}^{\text{m}_d}(a) \right) & \text{for end points}
\end{cases}$$  \hspace{1cm} (4.64)$$

where $k_{0,PAM}(a) = \frac{2}{(2a - 1 - A)^2}$.

Regarding (4.64), the average error probability conditioned on the CSI at the $m^{th}$ relay is given by [51]

$$P_e = \frac{1}{A} \sum_{a=1}^{A} P_e^r(a) = \frac{1}{A} \sum_{a=1}^{A} k_{1,PAM}(a) Q \left( \sqrt{k_{0,PAM}(a)} \gamma_{\text{eff}}^{\text{m}_d}(a) \right)$$  \hspace{1cm} (4.65)$$

where $k_{1,PAM}(a)$ is either 1 or 2 depending on the location of the constellation point.
Based on the theory we improved $B_i$ in (4.63) which is indeed the ASER in the $i^{th}$ relay, is the average of (4.65) over channel realizations and is given by

$$B_i = \frac{1}{A} \sum_{a=1}^{A} \left\{ k_{1,PAM}(a) \mathbb{E} \left[ \int_0^\infty Q \left( \sqrt{k_{0,PAM}(a)\gamma_{si,sl}(a)} \right) f_{\gamma_{si,sl}(a)}(a) \, d\gamma_{si,sl}(a) \right] \right\}$$

$$= \frac{1}{A} \sum_{a=1}^{A} k_{1,PAM}(a) \left[ 1 - \sqrt{\frac{k_{0,PAM}(a)}{k_{0,PAM}(a) + 2\lambda_{si,sl}}} \right]. \quad (4.66)$$

Following up the same procedure for deriving (4.41) we reach to the ASER for PAM signals in the sequel

$$\bar{P}_e = \frac{1}{A} \sum_{a=1}^{A} \left\{ \frac{k_{1,PAM}(a)}{2} \prod_{i=1}^{M} B_i + \sum_{\mathcal{D}(a)} \left[ \prod_{i \in \mathcal{D}(a)} (1 - B_i) \prod_{i \notin \mathcal{D}(a)} B_i \right] \right\}$$

$$\times k_{1,PAM}(a) \sum_{m \in \mathcal{D}(a)} \sum_{k=0}^{\infty} \sum_{n=1}^{\infty} \frac{a_m \Gamma \left( k + \frac{(n+1)}{2} \right)}{n!(2k_{0,PAM}(a))^k} \left( \frac{c_m}{2} \right)^k$$

$$\times \left( \frac{\lambda_{md}}{2k_{0,PAM}(a)} \right)^{(k+1)} \left[ \frac{\lambda_{md,s}^{k+1}}{(\lambda_{md,s} + \lambda_{id,s} + \lambda_{jd,s} + \lambda_{md,sl} + \frac{c_m}{2})^{k+1}} - \sum_{i \in \mathcal{D}(a)} \frac{\lambda_{md,s}^{k+1}}{(\lambda_{md,s} + \lambda_{id,s} + \lambda_{jd,s} + \lambda_{md,sl} + \frac{c_m}{2})^{k+1}} \right.$$  

$$+ \sum_{i,j \in \mathcal{D}(a)} \frac{\lambda_{md,s}^{k+1}}{(\lambda_{md,s} + \lambda_{id,s} + \lambda_{jd,s} + \lambda_{md,sl} + \frac{c_m}{2})^{k+1}}$$

$$+ (-1)^{D-1} \sum_{\substack{i,j,\ldots,l \in \mathcal{D}(a) \atop i \neq j,\ldots,l}} \frac{\lambda_{md,s}^{k+1}}{(\lambda_{md,s} + \lambda_{id,s} + \lambda_{jd,s} + \ldots \lambda_{ld,s} + \lambda_{md,sl} + \frac{c_m}{2})^{k+1}} \right\}. \quad (4.67)$$

### 4.B.3 QAM constellation

For this constellation the transmitted signal is chosen from an $A$ possible transmit constellation points [51]

$$x^{(a_1,a_2)} = (2a_1 - 1 - \sqrt{A})\Delta + j(2a_2 - 1 - \sqrt{A})\Delta \quad a_1, a_2 = 1, \ldots, \sqrt{A}.$$  

$\Delta$ is the half spacing parameter between two adjacent constellation points and in the case $\mathbb{E}\{|x|^2\} = 1$ we have [51]

$$\mathbb{E}\{|x|^2\} = \frac{\Delta^2}{A} \sum_{a_1,a_2=1}^{\sqrt{A}} (2a_1 - 1 - \sqrt{A})^2 + (2a_2 - 1 - \sqrt{A})^2 = 1.$$
which concludes $\Delta = \sqrt{\frac{3}{2(A-1)}}$. Noting that

$$|x^{(a_1,a_2)}|^2 = (2a_1 - 1 - \sqrt{A})^2 \Delta^2 + (2a_2 - 1 - \sqrt{A})^2 \Delta^2$$

and considering (4.57), the probability of error between two adjacent points in the receive constellation in the first time slot at the $m$th relay is given by

$$P_e = Q\left(\sqrt{k_{0,QAM}(a_1,a_2)} \hat{\gamma}_{sm,s}^{\text{eff}}\right).$$

(4.68)

where $k_{0,QAM}(a_1,a_2) = \frac{2}{[(2a_1-1-A)^2+(2a_2-1-A)^2]}$.

The interior points, the points on the corners and the points on the sides in the QAM constellation contribute differently to the probability of error. The probability of error for different points in the constellation is given by [51]

$$P_e(a) = \left\{\begin{array}{ll}
4Q\left(\sqrt{k_{0,QAM}(a_1,a_2)} \hat{\gamma}_{sm,s}^{\text{eff}}\right) & \text{for interior points} \\
3Q\left(\sqrt{k_{0,QAM}(a_1,a_2)} \hat{\gamma}_{sm,s}^{\text{eff}}\right) & \text{for side points} \\
2Q\left(\sqrt{k_{0,QAM}(a_1,a_2)} \hat{\gamma}_{sm,s}^{\text{eff}}\right) & \text{for corner points}
\end{array}\right.$$  (4.69)

From (4.69) we can calculate $B_i$ by

$$B_i = \frac{1}{A} \sum_{a_1,a_2=1}^{\sqrt{A}} \left\{k_{1,QAM}(a_1,a_2) \int_0^\infty Q\left(\sqrt{k_{0,QAM}(a_1,a_2)} \hat{\gamma}_{si,sl}^{\text{eff}}(a_1,a_2)\right)\right. \\
\left. \times f_{\hat{\gamma}_{si,sl}}(a_1,a_2) d\hat{\gamma}_{si,sl}(a_1,a_2)\right\}$$

$$= \frac{1}{A} \sum_{a_1,a_2=1}^{\sqrt{A}} k_{1,QAM}(a_1,a_2) \left[\frac{1}{2} - \sqrt{k_{0,QAM}(a_1,a_2)} + 2k_{0,QAM}(a_1,a_2) + 2\lambda_{si,sl}\right].$$  (4.70)
4.B.4 PSK constellation

For this constellation the transmitted signal is chosen from $A$ possible unit amplitude transmit constellation points

$$x^{(a)} = e^{i2\pi a / A} \quad a = 0, \ldots, A - 1.$$ 

It can be shown that the minimum distance between two adjacent points in the receive signal constellation after the MRC is given by (see Fig. 4.15)
CHAPTER 4. RELAY SELECTION WITH FEEDBACK DELAY

\[ d_{\min} = P_{s}|\hat{h}_{sm,sl}|^2 \sin \left( \frac{\pi}{A} \right). \]  

(4.72)

In this case, noting that \(|x(a)| = 1\) and that \(N(a) = P_{s}\sigma^2_{\text{sm,sl}} + 1\), the probability of error between two adjacent points is given by

\[ P_{e} = Q \left( \sqrt{\frac{d_{\min}^2}{2N(a)}} \right) = Q \left( \sqrt{k_{0,\text{PSK}} \gamma_{\text{eff}}^\text{sm,sl}} \right) \]  

(4.73)

where \(k_{0,\text{PSK}} = \frac{\sin^2(\frac{\pi}{A})}{2}\).

It can be shown that a union bound on the error probability of PSK signals in the first time slot and at the \(m^{th}\) relay can be given by [51]

\[ P_{e} = 2Q \left( \sqrt{k_{0,\text{PSK}} \gamma_{\text{eff}}^\text{sm,sl}} \right). \]  

(4.74)

Finally by setting \(k_{1,\text{PSK}} = 2\) and following the same procedure in the previous sections and noting that

\[ B_i = k_{1,\text{PSK}} \int_0^\infty Q \left( \sqrt{k_{0,\text{PSK}} \gamma_{\text{eff}}^\text{sm,sl}} \right) f_{\gamma_{\text{eff}}^\text{sm,sl}} d\gamma_{\text{eff}}^\text{sm,sl} \]

\[ = \frac{k_{1,\text{PSK}}}{2} \left[ 1 - \sqrt{\frac{k_{0,\text{PSK}}}{k_{0,\text{PSK}} + 2\lambda_{s_i,sl}}} \right] \]  

(4.75)

we reach to the ASER for PSK signals in the sequel

\[ \bar{P}_e = \frac{k_{1,\text{PSK}}}{2} \prod_{i=1}^M B_i + \sum_{D(s) \in D(s)} \prod_{i \in D(s)} (1 - B_i) \prod_{i \in D(s)} B_i \]

\[ \times k_{1,\text{PSK}} \sum_{m \in D(s)} \sum_{k=0}^{\infty} \sum_{n=1}^{\infty} a_m \Gamma \left( k + \frac{n+1}{2} \right) / k! \left( \lambda_{md} \right)^k \left( \lambda_{md} + \frac{c_m}{2} \right)^{k+1} \]

\[ \times \left( \frac{\lambda_{md}}{2k_{0,\text{PSK}}} \right)^{(k+1)} \left[ \frac{\lambda_{md}^{k+1}}{\left( \lambda_{md,sl} + \frac{c_m}{2} \right)^{k+1}} - \sum_{i \in D(s)} \frac{\lambda_{md,sl}^{k+1}}{\left( \lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \frac{c_m}{2} \right)^{k+1}} \right] \]

\[ + \sum_{i, j \in D(s)} \frac{\lambda_{md,sl}^{k+1}}{\left( \lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \frac{c_m}{2} \right)^{k+1}} \]

\[ + (-1)^{D-1} \sum_{i, j, \ldots, l \in D(s)} \frac{\lambda_{md,sl}^{k+1}}{\left( \lambda_{md,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{id,sl} + \lambda_{jd,sl} + \lambda_{ld,sl} + \frac{c_m}{2} \right)^{k+1}} \]  

(4.76)
Chapter 5

Relay Selection in CRNs with interference

5.1 Objective

In this chapter, we investigate the outage probability of underlay cognitive radio, CR, systems with relay selection. In particular, we consider a secondary multi-relay network operating in the AF mode. Amongst all the potential relays only the “best” relay which satisfies an index of merit is selected which participates in cooperation. The proposed selection strategy takes into consideration the effect of primary user, PU, interference. That is, we assume that the secondary user, SU, is exposed to unwanted interference from a neighboring PU network.

5.1.1 Related work and contribution

There have been considerable recent works on interference management in CRNs [60, 65–73]. Cooperative diversity, which has emerged as a promising approach to increase spectral and power efficiencies, network coverage, and reduce outage probability, has been recently studied in the context of CRNs [74–80]. Specifically, to increase the performance of the secondary network without increasing the secondary transmitter power, which imposes additional unwanted interference on the underlay primary network, cooperation in the secondary network became popular. This is mainly because it is proven that with less transmission power, selective cooperation communication can reach to the same performance in comparison to the
conventional direct communication [3].

Several interference mitigation techniques applicable to CRNs have been reported in [65], including spectrum shaping, predistortion filtering, and spread spectrum. In MIMO-CRNs, transmit beamforming [66–70] or precoding [71–73] are used to proactively cancel the interference from CR transmitters to the primary networks. The authors in [81] e.g. choose optimal beamforming vector in a MIMO underlay cooperative network such that to target maximum signal-to-interference-and-noise-ratio (SINR) at the secondary user terminals while keeping the interference and noise leakage to the primary users below a threshold. Luo et al. in [82] analyze the outage performance of relay selection in the underlay CRNs for DF cooperation. However they do not consider the interference effect on the relays. They assume all transmitting nodes transmit with maximum power such that the imposed interference on the primary user network is acceptable. They propose a lower bound on the outage probability performance in their scheme. In [83] the authors present two low-complexity interference aware multiple relay assignment schemes for CRNs. The main objective in assigning multiple relays is to maximize the sum capacity of the CRNs under the constraint of acceptable interference to the primary users. [84] et. al. present a binary biogeography based optimization (BBO) algorithm-based low-complexity interference aware relay assignment scheme with power control for a CRN with one source node, multiple relays and multiple destination nodes. In their work they tackle the problem of assigning relays to cognitive radio secondary users with power limitations and interference constraints. In [75], Zou et al. propose an adaptive relay selection scheme for CRNs. In particular, they analyze the outage probability of a secondary user given a constraint on the outage probability of primary transmissions (i.e., primary outage probability). Suraweera et al. analyze the outage performance of a DF repetition-based relaying scheme with relay selection [79]. They show that under an imperfect spectrum acquisition scenario, the outage probability degrades significantly. Lee and Andrews [74] study the outage probability performance of CRNs with allocating power to the secondary network transmitters based on the interference they impose on the PU by adjusting an interference threshold limit. However, in their work, they do not consider the effect of the PU interference on the SU.

In the AF relay networks, mitigating interference at the relay nodes requires additional complexity, which may not be practically and economically feasible, however the interference at the destination node of the secondary network can be easily mitigated using some blind signal processing techniques [65, 85–87].
In this chapter, taking into account the effect of the PU interference at the SU relay nodes, we analyze the outage probability performance of a S-AF scheme in the underlay cognitive relay networks. We further derive exact outage probability expression under the condition that the amount of interference imposed on the PU user is less than or equal to a threshold limit. Throughout this chapter we assume availability of perfect necessary CSI at all nodes.

5.2 System Model

We consider a scenario where a secondary user network with a source-destination pair, denoted by $SU_S$ and $SU_D$, and multiple relay nodes $R_m$, $m = 1, \ldots, M$, coexists with a primary user network as shown in Fig. 5.1. In our scheme, the source node broadcasts its information to all relays, then the best relay forwards a normalized version of its received signal to the $SU_D$ utilizing the AF protocol. We ignore the direct transmission between the $SU_S$ and the $SU_D$ due to shadowing. We stress that only one relay, which satisfies an index of merit is selected. We assume channel reciprocity for the channel coefficient between node $i$ and node $j$ denoted by $h_{ij}$, i.e., we assume $h_{ij} = h_{ji}$. We also assume that all channels are quasi-static block fading and that the block duration is twice the transmission time of each time slot. This means that all the channels in the PU and SU are assumed to be static during a full data transmission in the SU network. In our system model, the PU network
CHAPTER 5. RELAY SELECTION IN CRNS WITH INTERFERENCE

is consist of a primary user transmitter $PU_T$ and a primary user receiver $PU_R$, and shares the same frequency band with the SU network. For simplicity, we assume a direct point-to-point transmission between the $PU_T$ and the $PU_R$. We model the transmitted signal from the $PU_T$ as a single source of interference imposed on the relays of the SU network. Generally the noise at the secondary receiver may contain interference from the primary transmitter as well, thus in general the noise term at the secondary user may be non-white and non-Gaussian. In the case the noise term is not Gaussian the analysis provided here is a lower bound for the actual outage probability.

In the first time slot, the received signal, at relay $R_m$, is given by

$$y_m = \sqrt{P_s} h_{sm} x + \sqrt{P_t} h_{tm} x' + n_m$$

(5.1)

where $y_m$ is the received signal at the $m^{th}$ relay, $h_{sm} \sim \mathcal{CN}(0, 1/\lambda_{sm})$ and $h_{tm} \sim \mathcal{CN}(0, 1/\lambda_{tm})$ denote the $SU_S \rightarrow R_m$ and $PU_T \rightarrow R_m$ channel coefficients, respectively. The transmitted information symbols from the $SU_S$ and the $PU_T$ are assumed to be drawn from independent long enough zero mean unit variance Gaussian codebooks and are denoted by $x$ and $x'$, respectively. $n_m \sim \mathcal{CN}(0, 1)$ denotes the noise at the $m^{th}$ relay. Having noted that $E\{|x|^2\} = E\{|x'|^2\} = 1$, then $P_s$ and $P_t$ denote the average signal energies transmitted by the $SU_S$ and the $PU_T$, respectively.

In the second time slot, each relay normalizes its received signal by

$$G_m = \sqrt{\frac{P_m}{E\{|y_m|^2\}}} = \sqrt{\frac{P_m}{P_s|h_{sm}|^2 + P_t|h_{tm}|^2 + 1}}$$

(5.2)

and forwards a scaled version to the $SU_D$. The received signal at the $SU_D$ is then given by

$$y_d = G_m h_{md} y_m + n_d$$

(5.3)

where $P_m$ is the average energy transmitted from the $m^{th}$ relay, $h_{md} \sim \mathcal{CN}(0, 1/\lambda_{md})$ is the relay-destination channel coefficient and $n_d$ is defined as

$$n_d = n_{do} + P_t h_{td} x'$$

(5.4)

where $n_{do} \sim \mathcal{N}(0, 1)$ is the destination noise and $h_{td}$ is the $PU_T \rightarrow SU_D$ channel. Since $x'$ is drawn from a Gaussian codebook, conditioned on $h_{td}$, $P_t h_{td} x'$ can be assumed as a Gaussian
interference at the destination which keeps $n_d$ still Gaussian [74, 88]. Therefore the authors in [74] ignore the effect of the PU signal on the SU by adding the interference terms to the receiver noise terms in the SU network nodes. However, in their work they model the noise at all SU terminals as unit variance complex Gaussian random variables which disarms the authors in analyzing the effect of the PU noise on the SU network. In this chapter however, we also incorporate the effect of the PU interference on the SU relay nodes, noting that at the SU-D one can use blind techniques such as independent component analysis (ICA) for non-Gaussian random variables [89], blind source separation (BSS) techniques for Gaussian signals [85, 86] or other methods [90] to mitigate the interference from the PU at the SU-D. Therefore for simplicity throughout this chapter we assume that the PU interference at the SU can be mitigated and that $n_d \sim \mathcal{N}(0, 1)$.

By substituting (5.1) into (5.3), the received instantaneous SINR at the SU-D is then given by

$$\gamma_{D_m} = \frac{\gamma_{sm}\gamma_{md}}{\gamma_{sm} + \gamma_{md} + 1 + \gamma_{md}\gamma_{tm} + \gamma_{tm}}$$

(5.5)

where $\gamma_{sm} = P_s|h_{sm}|^2$, $\gamma_{md} = P_m|h_{md}|^2$ and $\gamma_{tm} = P_t|h_{tm}|^2$.

In (5.5), the effect of PU interference appears as an extra term in the denominator.

5.2.1 Channel estimation

In our scenario we assume both primary and secondary networks specify a fraction of the transmission packet to sending training symbols with known average transmission energy $P$. At the beginning of the first time slot, meaning at the training interval, the PU transmits $x'_p^T = [x'_{p1}, x'_{p2}, \ldots, x'_{pN}]$, where simultaneously the SU transmits $x_p^T = [x_{p1}, x_{p2}, \ldots, x_{pN}]$ as pilot symbols. The symbols are designed so that $x_p^T x'_p = 0$. In the training interval of the first time slot the $m^{th}$ relay and the PU-R receive

$$y_m = Ph_{sm}x_p + Ph_{tm}x'_p + n_m$$

(5.6)

and

$$y_r = Ph_{sr}x_p + Ph_{tr}x'_p + n_r$$

(5.7)

respectively, where $n_m, n_r \sim \mathcal{N}(0, I_N)$. According to (5.6) and (5.7) by considering the orthogonality of $x_p$ and $x'_p$, the channels $h_{sm}$, $h_{tm}$, $h_{sr}$ and $h_{tr}$ can be estimated simultaneously.

\[\text{1}\text{We consider the total impact of the PU on the SU in chapter 6.}\]
at the relay nodes and the $PU_R$ as

\[
\hat{h}_{sm} = \frac{x_p^T y_m}{P} = h_{sm} + \frac{x_p^T n_m}{P}
\]

\[
\hat{h}_{tm} = \frac{x_p^T y_m}{P} = h_{tm} + \frac{x_p^T n_m}{P}
\]

\[
\hat{h}_{sr} = \frac{x_p^T y_r}{P} = h_{sr} + \frac{x_p^T n_r}{P}
\]

\[
\hat{h}_{tr} = \frac{x_p^T y_r}{P} = h_{tr} + \frac{x_p^T n_r}{P}.
\]

By increasing the SNR one can make sure that the estimation error is negligible.

Similarly at the beginning of the second time slot the destination sends pilot signals and therefore assuming channel reciprocity the relays receive

\[
y_m = P h_{md} x_p + n_m.
\] (5.9)

According to (5.9), the channel $h_{md}$ is estimated at the $m^{th}$ relay

\[
\hat{h}_{md} = \frac{x_p^T y_m}{P} = h_{md} + \frac{x_p^T n_m}{P}.
\]

After $h_{md}$ is estimated at each relay, the relays transmit pilots towards the $PU_R$ for estimation of $h_{mr}$. The training sequences are transmitted either simultaneously, by transmitting orthogonal pilots in a code division multiple access (CDMA) fashion, or consecutively, via TDMA transmission. It is then the $PU_R$’s role to make decisions and inform the $SU_S$ and the $R_m$ about the allowed transmission powers $P_s$ and $P_m$ via a feedback link.

Having estimated the relative channels at each time slot, the system proceeds to the data transmission interval afterwards.

### 5.2.2 Transmission power constraints

The presence of PU interference is very undesired in the SU network and its negative impact on the probability of outage in the SU is trivial. The interference imposed on the $SU_D$ node by the $PU_T$ can be mitigated using advanced signal processing techniques [86, 89, 90]. As for the interference imposed on the relay nodes however, reduction of the interference is desired on the one hand, and on the other hand the relay nodes are limited to be simple with minimal computational capabilities. This technological shortcoming in the case of AF relaying calls for a minimal signal processing load on the relays, meaning that if there is
interference present at the relays then it may be very hard to to deal with it. Therefore this chapter aims to investigate the performance of S-AF in underlay CRNs when PU interference is present at the relay nodes while assuming that the interference at the destination node is handleable.

Mutually the interference that the SU imposes on the PU is also disastrous. Although by using signal processing and coding techniques it is possible to minimize the interference, the importance of power management in the SU leading to minimal devastating consequences on the PU can never be neglected. In this section we study the power management in the SU network such that the PU senses not more than a threshold limit of interference.

Defining an interference threshold $\bar{I}$, which is the maximum tolerable interference level at which the PU, transmitting with power $P_t$, can still maintain reliable communication, we confine the transmitted powers of the $SU_S$ and $R_m$ to $P_s \leq \frac{\bar{I}}{|h_{sr}|^2}$ and $P_m \leq \frac{\bar{I}}{|h_{mr}|^2}$, respectively. $h_{sr} \sim \mathcal{CN}(0,1/\lambda_{sr})$ and $h_{mr} \sim \mathcal{CN}(0,1/\lambda_{mr})$ are the respective channels from the source and the $m^{th}$ relay to the primary user receiver. Furthermore, assuming the maximum transmission power constraints $P_s \leq P_o$ and $P_m \leq P_o'$, where $P_o$ and $P_o'$ are the maximum allowable transmission powers of the $SU_S$ and $R_m$, respectively, the transmission power constraints of the $SU_S$ and $R_m$ become [60, 80, 91]

$$P_s = \min \left\{ \frac{\bar{I}}{|h_{sr}|^2}, P_o \right\}$$

$$P_m = \min \left\{ \frac{\bar{I}}{|h_{mr}|^2}, P_o' \right\}.$$  \hfill (5.10a)

In the system we study, each node operates either in full transmission power ($P_s$ or $P_m$) or remains silent.

In the training interval in the first time slot $h_{sr}$ is estimated at the $PU_R$ and if $\frac{\bar{I}}{|h_{sr}|^2} < P_o$ the channel is quantized and fed back to the source for its use in (5.10a), otherwise the $PU_R$ only transmits a flag confirming that the $SU_S$ should use $P_o$ Joules per symbol for data transmission. The same scenario is applicable to the relay nodes and (5.10a) in the second time slot.

### 5.2.3 Relay selection scheme

In conventional *interference free* relay selection scenarios, the best relay, which achieves the maximum received SNR at $SU_D$, is selected according to the following criterion [7–9, 14]

$$m^* = \arg \max \left\{ \gamma_{D_m}^c \right\}$$ \hfill (5.11)
where
\[ \gamma_{Dm}^c = \frac{P_o|h_{sm}|^2P_o'|h_{md}|^2}{P_o|h_{sm}|^2 + P_o'|h_{md}|^2 + 1}. \] (5.12)

However, assuming the presence of interference at the relay nodes the optimal selection criterion for our scenario is given by
\[ m^* = \arg \max_m \{\gamma_{Dm}\}, \] (5.13)
where
\[ \gamma_{Dm} = \frac{\gamma_{sm}\gamma_{md}}{(\gamma_{md} + 1)(\gamma_{tm} + 1) + \gamma_{sm}} \]
\[ \simeq \frac{\gamma_{sm}(\gamma_{md} + 1)}{\gamma_{md} + \gamma_{tm} + 1}, \]
which can be written as [9]
\[ \gamma_{Dm} \simeq \min \left( \frac{\gamma_{sm}}{\gamma_{tm} + 1}, \gamma_{md} \right). \] (5.14)

As mentioned in Sec. 5.2.2 each relay knows its forward and backward channel as well as all the allowable data transmission powers at the end of the training period in the second time slot. In this strategy, CSI is estimated at the relay nodes for further decisions on which relay is the best node to cooperate. In particular, when the relay nodes overhear an RTS packet form the source terminal, they estimate the source link CSI, i.e., \( h_{sm} \). Then, upon receiving a CTS packet from the destination, the corresponding destination link at the relays, i.e., \( h_{md} \), are estimated. Right after receiving the CTS packet, the relay nodes receive instructions for powers \( P_s \) and \( P_m \) and they calculate \( \gamma_{Dm} \), inversely proportional to what, a timer is ignited. The timer of the best relay expires first, and a flag packet is sent to other relays, informing them to stop their timers as the best relay is selected. Then the relays that receive the flag packet, keep silent and the selected relay participates in communication [17].

### 5.3 Performance Analysis of Cooperative CRNs with Interference Constraints

In this section, we present outage probability analysis for the relay transmission scheme S-AF with interference constraints. Also, we present a thorough discussion of the system
performance under different scenarios and further compare the performance with the conventional interference free relay networks.

5.3.1 Outage probability analysis

Assuming that the communication between the $SU_S$ and the $SU_D$ targets an end-to-end data rate $R$, the probability of an outage in the system can be written as

$$P_{out} = Pr \left( \frac{1}{2} \log_2 (1 + \gamma_{D,m'}) \leq R \right)$$

$$= F_{\gamma_{D,m'}} (2^{2R} - 1) . \quad (5.15)$$

**Proposition 1:** Defining $\gamma = P|h|^2$ where $P = \min(\frac{\tilde{I}}{|h|^2}, P)$, $h \sim CN(0,1/\lambda)$, $\tilde{h} \sim CN(0,1/\tilde{\lambda})$, and $\tilde{P}$ is a constant, the cumulative density function (CDF) of $\gamma$ is given by

$$F_{\gamma}(x) = 1 + \exp \left( -\frac{\lambda x}{\tilde{P}} \right) \left( \frac{\exp \left( -\frac{\tilde{\lambda} I}{\tilde{P}} \right)}{\frac{\lambda x}{\tilde{P}} + 1} - 1 \right). \quad (5.16)$$

**Proof:** See Sec. [5.A](#).

**Proposition 2:** Defining $\tilde{\gamma} = \tilde{P}|\tilde{h}|^2$ where $\tilde{P}$ is a constant and $\tilde{h} \sim CN(0,1/\tilde{\lambda})$, the CDF of $\zeta = \frac{\gamma}{\gamma + 1}$ is

$$F_{\zeta}(x) = 1 + \frac{\exp \left( -\frac{\lambda x}{\tilde{P}} \right)}{1 + \frac{\lambda x}{\tilde{P}}} \left[ \exp \left( -\frac{\tilde{\lambda} \tilde{I}}{\tilde{P}} \right) - 1 \right] - \frac{\lambda x}{\tilde{P}} \exp \left( \frac{\tilde{\lambda} \tilde{I}}{\tilde{P}} \left( \frac{\lambda x}{\tilde{P}} + 1 \right) \right)$$

$$E_1 \left( \left( \frac{\lambda x}{\tilde{P}} + \frac{\tilde{\lambda}}{\tilde{P}} \right) \left( \frac{\tilde{\lambda} \tilde{I}}{\lambda x} + 1 \right) \right) \quad (5.17)$$

where

$$E_1(x) \overset{\Delta}{=} - \int_{-\infty}^{\infty} \frac{1}{\gamma} \exp(-\gamma) d\gamma .$$

**Proof:** See Sec. [5.B](#).

**Proposition 3:** The outage probability of S-AF, in the presence of PU interference and
secondary user power constraints is given by

\[ P_{\text{out}} = \prod_{m=1}^{M} 1 - \frac{\exp(-R_0 \left( \frac{\lambda_{sd} \bar{I}}{P_o} + \frac{\lambda_{md} R_o}{\lambda_{tm} P_o} \right))}{1 + \frac{\lambda_{sd} P_t R_o}{\lambda_{tm} P_o} \exp(-\lambda_{sr} \bar{I}) - 1} \left( \frac{\exp\left( \frac{-\lambda_{mr} \bar{I}}{P_o} \right)}{\lambda_{md} R_o + 1} - 1 \right) \]

\[ + \frac{\lambda_{tm} \lambda_{sr} \bar{I}}{\lambda_{sm} R_o P_t} \exp\left( \frac{\lambda_{tm} \left( \frac{\lambda_{sr} \bar{I}}{\lambda_{sm} R_o} + 1 \right) - \lambda_{md} R_o}{P_o'} - 1 \right) \]

\[ E_1 \left( \left( \frac{\lambda_{sm} R_o}{P_o} + \frac{\lambda_{tm}}{P_t} \left( \frac{\lambda_{sr} \bar{I}}{\lambda_{sm} R_o} + 1 \right) \right) \right) \] (5.18)

where \( R_o = 2^{2R} - 1 \).

**Proof:** See Sec. 5.C.

### 5.4 Discussion

In this section, we investigate the effect of \( P_o, \bar{I} \), and the primary user power \( P_t \) on the system performance. Furthermore, we discuss the achievable diversity order under various conditions. For simplicity, we assume that \( P_o' = P_o \).

**Unlimited available power \( (P_o \to \infty) \) with interference threshold \( \bar{I} \)**

Let \( P_o \ll \bar{I} \), then the average power allocated to the \( SU_S \) and the selected relay is determined by \( P_s = P_m = E\{ \min(P_o, \bar{I}) \} \), which is \( P_o \), when \( P_o \ll \bar{I} \). This in turn improves the outage performance as \( P_o \) increases. However, as \( P_o \) exceeds the threshold limit, the diversity order reduces to zero, irrespective of the number of relays or the threshold \( \bar{I} \). Mathematically, the asymptotical diversity order \( d \) with respect to \( P_o \) is given by the magnitude of the slope of outage probability curve against average \( P_o \) in a log-log scale [4]:

\[ d = \lim_{P_o \to \infty} \frac{\log P_{\text{out}}}{\log P_o} \]
Assuming \( P_o = P'_o \to \infty \), and noting that \( R, \bar{I}, P_t, \lambda_{md} = \lambda_{sr} = \lambda_{sm} = \lambda_{tm} = \lambda \), are all constants and using Proposition 3 we have

\[
\begin{align*}
d &= \lim_{P_o \to \infty} - \log \left( \left\{ 1 - \frac{\bar{I}}{P_o R_o} \exp \left( \frac{\bar{I}}{P_t} \left( \frac{\lambda}{\lambda P_o} + 1 \right) \right) E_1 \left( \frac{\lambda}{P_t} \left( \frac{\lambda}{\lambda P_o} + 1 \right) \right) \right\}^M \right) \\
&= \lim_{P_o \to \infty} - \frac{\log(K^M)}{\log(P_o)} \\
&= 0.
\end{align*}
\]

(5.19)

Hence, the diversity order of the system with respect to \( P_o \) is zero in this case.

**Unlimited available power with no interference threshold** (\( \bar{I} \gg P_o \) and \( P_0 \to \infty \))

In this case the full diversity order, which is dependent on the number of participating relays, is expected. To illustrate, let \( P_o = P'_o \) and \( \lambda_{sm} = \lambda_{md} = \lambda \), then, at high \( P_o \) values, the received SINR can be approximated by

\[
\gamma_{D_m^*} = \max_m \min(\gamma_{sm}, \gamma_{md})
\]

where \( \gamma_{sm} \) and \( \gamma_{md} \) are approximated by \( P_o |h_{sm}|^2 \) and \( P_o |h_{md}|^2 \), respectively, and are exponentially distributed by the parameter \( \lambda \). With simple mathematical manipulations, the outage probability can be then approximated by

\[
\begin{align*}
P_{out} &= \left[ 1 - \exp \left( - \frac{2\lambda R_o}{P_o} \right) \right]^M \\
&\approx \frac{1}{P_o^M}
\end{align*}
\]

(5.20)

which indicates that the diversity, in this case, is \( M \) as expected.

**Primary user power \( P_t \)**

It is quite trivial that as the primary user increases its power the performance of the cooperative network is aggravated, since the interference imposed on the network due to the primary user increases. Thus, from (5.14), an increase in the value of \( \gamma_{tm} \) causes a decrease in the system SINR which results in an increase in the system outage probability. Therefore the outage probability of the system is directly affected by the primary user power.
Figure 5.2: Outage Probability for $\bar{I} = 13dB$ when $P_o \to \infty$ and $M = 2$, $P_t = 10dB$. An error floor in the depicted SNR range is seen when the threshold is a small amount.

Figure 5.3: Outage Probability for $\bar{I} = 30dB$ when $P_o \to \infty$ and $M = 2$ and $P_t = 10dB$. 
5.5 Simulation Results

In this section, we study the performance of a cognitive relay network in terms of outage probability for the scenarios in the sequel:

- **Scenario I**: Direct communication SU$_S$ and SU$_D$.
- **Scenario II**: Conventional cooperative system.
- **Scenario III**: S-AF overlay cognitive cooperative system (interference free).
- **Scenario IV**: S-AF underlay cognitive cooperative system with power constraint and interference assumption.

We consider a simple case where SU$_S$, SU$_D$, and two relay nodes $R_i (i = 1, 2)$, are located at the coordinates $(0, 0, 0)$, $(d, d, 0)$, $(0, d/2, 0)$ and $(d/2, 0, 0)$, respectively. The PU$_T$ is located at the coordinate $(0, 0, d_p^t)$ and the PU$_R$ is located at $(0, 0, -d_p^r)$. We assume $P_o = P'_o$ and the system threshold rate $R$ is set to 1 bps/Hz.

In Fig. 5.2, we investigate the performance of the underlying scheme assuming $P_t = 10dB$, $\bar{I} = 13dB$, $d = 1$ and $d_p^t = d_p^r = 2$. As this figure clearly demonstrates, for Scenario IV, the underlay S-AF scheme suffers from an error floor, which is due to the power constraint on the system. On the other hand, Scenarios II and III do not consider any power constraints; hence, they do not suffer from error floor and are able to achieve a full diversity order. The dashed line also describes the true SINR given by (5.5) and approximated by (5.14). It is clear that the approximation is a fair match with the true curve.

In Fig. 5.3, we assume $\bar{I} = 30dB$ and the range of $P_o$ is from $0 - 30dB$. As it can be easily seen, the performance of the underlay and overlay cognitive relay networks is similar in the high SNR region, both achieving a diversity order of 2 as predicted earlier. Figs 5.2 and 5.3 also demonstrate the benefits of user cooperation over direction transmission.

Fig. 5.4 investigates the behavior of the system under the assumptions made in Sec. 5.4. Here, we plot the performance of $P_{out}$ vs. $P_t$. We consider $P_o = 30dB$ and $\bar{I} = 13dB$. It is observed that the performance of the direct transmission is independent of $P_t$, which is quite expected, as we have assumed that the interference imposed on the SU$_D$ by the PU$_T$ is negligible and can be mitigated using advanced signal processing techniques. On the other hand, it is observed that for the underlay S-AF scheme, the system performance degrades as $P_t$ increases. This is mainly due to the fact that the relay nodes are not able to mitigate the excessive interference imposed on the relays. Furthermore, it is demonstrated in Fig. 5.4...
that, at low values of $P_t$, the conventional S-AF and the S-AF overlay schemes provide similar performance, which is mainly due to the low interference. Fig. 5.5 investigates the effect of displacement of the $PU_T$ on the performance of the system. We assume that the distance $d_{PT}$, which represents the distance between the $PU_T$ and the secondary network, takes the values 0.5, 1, and 2, respectively. It is clear that the performance of the secondary networks degrades for small $d_{PT}$ values. Furthermore, it is also seen that the performance of the direct transmission for this case remains unchanged.

Figs. 5.6 and 5.7 shows the effect of primary user transmitter and receiver displacement, respectively. Fig. 5.6 demonstrates that increasing $P_t$ and decreasing $d_{PT}$ reduces the performance in the underlay and overlay cognitive radio schemes; however, the performance of conventional S-AF and the direct transmission schemes remains unchanged.

Fig. 5.7 demonstrates that the overlay S-AF scheme remain unchanged due to variations in the position of the $PU_R$, since, in the overlay scheme, no power constraints are imposed upon secondary user. It is also interesting to note that the performance of the direct transmission is affected by the $PU_R$ displacement.
Figure 5.5: Outage Probability vs. the $PU_T$ position, $P_o = 30dB, \bar{I}/P_o \rightarrow \infty$ and $M = 2$.

Figure 5.6: Outage Probability vs. the $PU_T$ position, $P_o = 30dB, \bar{I} = 13dB, P_t \rightarrow \infty$ and $M = 2$. The closer the $PU_T$ is to the secondary network the worst the outage performance of the secondary network is.
Figure 5.7: Outage Probability vs. the $PU_R$ position, $P_o = 30dB, \bar{I} = 13dB, P_l \rightarrow \infty$ and $M = 2$. The farthest the $PU_R$ is the more power the $SU_S$ and $R_m^*$ can use and therefore the better the outage performance of the secondary network is.

5.6 Conclusion

In this chapter, we investigated the performance of relay selection in an underlay cognitive radio system in the presence of primary user interference. Particularly, we considered a secondary multi-relay network operating in the AF mode. The best relay is then selected to cooperate with the secondary source node according to an index of merit. We adopted a maximum power allocation scheme based on the maximum tolerable interference on the primary user network. We derived a closed-form outage probability expression for the secondary multi-relay network and further presented a thorough asymptotical diversity order analysis. We also studied the performance of the system under asymptotic power and interference cases and compared the proposed scheme to the conventional direct transmission and overlay cognitive radio scenarios.

We learned that if the PU transmission power is increased then the outage probability of the SU is drastically aggravated. The same scenario is observed when the $PU_T$ is close to the SU network. Path loss in this case acts against the SU desired performance. The closer the $PU_T$ to SU network is, the more the interference imposed on the SU is sensed and the higher the probability of outage would go.
We also observed that if the \( PU_R \) is close to the SU network, this time the interference from the SU is problematic and must be managed. In this case the PU reduces the sensed interference by sending controlling signals to the SU ordering the SU transmitting nodes to reduce their transmission power, this itself would lead to a leap in the probability of outage in the SU network.

Finally we presented numerical analysis to corroborate the analytic results and to have further insight into the performance of the proposed selection scheme.

5.A Cumulative Density Function, \( F_\gamma(x) \)

In this section, we derive the analytical expression of \( F_\gamma(x) \). Noting that \( h \sim CN(0,1/\lambda) \) and \( \hat{h} \sim CN(0,1/\hat{\lambda}) \), and denoting \( |h|^2 \) and \( |\hat{h}|^2 \) by \( \alpha \) and \( \hat{\alpha} \), the CDF of \( \alpha \) and \( \hat{\alpha} \) follow exponential distributions with \( F_\alpha(x) = 1 - e^{-\lambda x} \) and \( F_{\hat{\alpha}}(x) = 1 - e^{-\hat{\lambda} x} \), respectively. The random variable \( P = \min(\bar{\alpha}, \hat{\alpha}) \) can be written as

\[
P = \begin{cases} 
\tilde{P} & \hat{\alpha} \leq \frac{I}{P} \\
\frac{\tilde{I}}{\alpha} & \hat{\alpha} \geq \frac{I}{P}
\end{cases}.
\]  

(5.21)

Based on (5.21), we have

\[
Pr(P \leq x) = Pr\left( P \leq x, \hat{\alpha} \leq \frac{I}{P} \right) + Pr\left( \frac{\tilde{I}}{\alpha} \leq x, \hat{\alpha} \geq \frac{I}{P} \right).
\]

Thus, for \( x \leq \tilde{P} \) we can write

\[
Pr\left( P \leq x, \hat{\alpha} \leq \frac{I}{P} \right) = Pr(P \leq x) Pr\left( \hat{\alpha} \leq \frac{I}{P} \right) = 0
\]

(5.22)

and

\[
Pr\left( \frac{\tilde{I}}{\alpha} \leq x, \hat{\alpha} \geq \frac{I}{P} \right) = Pr\left( \hat{\alpha} \geq \frac{\tilde{I}}{x}, \hat{\alpha} \geq \frac{I}{P} \right)
\]

\[
= Pr\left( \hat{\alpha} \geq \frac{\tilde{I}}{x} \right) \cdot \left[ 1 - F_{\hat{\alpha}}\left( \frac{\tilde{I}}{x} \right) \right]
\]

(5.23)

Similarly, for \( x \geq \tilde{P} \)

\[
Pr\left( P \leq x, \hat{\alpha} \leq \frac{I}{P} \right) = Pr(P \leq x) Pr\left( \hat{\alpha} \leq \frac{I}{P} \right) = 1 \cdot F_{\alpha}\left( \frac{\tilde{I}}{P} \right)
\]
and
\[ P_r\left(\frac{\bar{I}}{\hat{\alpha}} \leq x, \hat{\alpha} \geq \frac{\bar{I}}{\bar{P}}\right) = P_r\left(\hat{\alpha} \geq \frac{\bar{I}}{x}, \hat{\alpha} \geq \frac{\bar{I}}{\bar{P}}\right) = \frac{1}{1-F_{\hat{\alpha}}\left(\frac{\bar{I}}{\bar{P}}\right)} \] (5.24)

From (5.22) to (5.24), the CDF of the random variable \( P \) is given by
\[ F_P(x) = \begin{cases} 1 - F_{\hat{\alpha}}\left(\frac{\bar{I}}{x}\right), & x \leq \bar{P} \\ 1, & x \geq \bar{P} \end{cases} \] (5.25)

Finally, \( F_{\gamma}(x) \) can be given as
\[ P_r(\gamma \leq x) = \int_{0}^{\infty} P_r\left(P \leq \frac{x}{\alpha}\right) f_\alpha(\alpha) \, d\alpha = \int_{0}^{\frac{x}{\bar{P}}} f_\alpha(\alpha) \, d\alpha + \int_{\frac{x}{\bar{P}}}^{\infty} \left[1 - F_{\hat{\alpha}}\left(\frac{\alpha\bar{I}}{x}\right)\right] f_\alpha(\alpha) \, d\alpha = \int_{0}^{\frac{x}{\bar{P}}} \lambda \exp(-\lambda \alpha) \, d\alpha + \int_{\frac{x}{\bar{P}}}^{\infty} \lambda \exp\left(-\alpha\left(\frac{\bar{I}}{x} + \lambda\right)\right) \, d\alpha = 1 + \exp\left(-\frac{\lambda x}{\bar{P}}\right) \left(\frac{\exp\left(-\frac{\lambda x}{\bar{P}}\right)}{\frac{\lambda x}{\bar{P}} + 1} - 1\right) \]
which completes the proof for proposition 1.

5.B Cumulative Density Function, \( F_\zeta(x) \)

Regarding the definition of \( \gamma \) and \( \hat{\gamma} \) and noting that the distribution of \( \gamma \) is given in Sec. 5.A and \( F_{\hat{\gamma}}(x) = 1 - \exp\left(-\frac{\lambda x}{\bar{P}}\right) \), the probability density function (PDF) of \( \kappa \) which is defined as
\[ \kappa \equiv \hat{\gamma} + 1 \]
is given by $f_{\kappa}(x) = \tilde{\lambda} \tilde{\rho}_{\kappa \gamma} \exp(-\tilde{\lambda} \kappa x) u(x - 1)$, where $u(\cdot)$ is the unit step function. To find $F_{\zeta}(x)$, we can write

$$F_{\zeta}(x) = \Pr\left( \frac{\zeta}{\kappa} \leq x \right)$$

$$= \int_{0}^{\infty} \Pr\left( \frac{\zeta}{\kappa} \leq x \kappa \right) f_{\kappa}(\kappa) \, d\kappa$$

$$= \frac{\tilde{\lambda}}{\tilde{P}} \int_{1}^{\infty} \left[ 1 + \exp\left( -\frac{\lambda_{\kappa} x}{\tilde{P}} \left( \exp\left( \frac{\tilde{\lambda}}{\tilde{P}} \right) - 1 \right) \right) \right] \exp\left( -\frac{\tilde{\lambda} (\kappa - 1)}{\tilde{P}} \right) \, d\kappa$$

$$= 1 + \frac{\tilde{\lambda}}{\tilde{P}} \exp\left( \frac{\tilde{\lambda}}{\tilde{P}} \right) \int_{1}^{\infty} \exp\left( -\frac{\tilde{\kappa} (\tilde{\rho}_{\kappa \gamma} + \tilde{\lambda})}{\tilde{P} \lambda x} \right) \, d\kappa$$

$$- \int_{1}^{\infty} \frac{\tilde{\lambda} \tilde{\rho}_{\kappa \gamma} \exp\left( -\frac{\tilde{\lambda}}{\tilde{P}} \right)}{\lambda x} \exp\left( -\frac{\tilde{\lambda} \tilde{\rho}_{\kappa \gamma} + \tilde{\lambda}}{\tilde{P} \lambda x} \right) \right) \exp\left( -\kappa \left( \frac{\tilde{\rho}_{\kappa \gamma}}{\tilde{P} \lambda x} + \frac{\tilde{\lambda}}{\tilde{P}} \right) \right) \, d\kappa$$

(5.26)

after some mathematical manipulations and using the fact that [38]

$$\int_{1}^{\infty} e^{-\mu z} \, dz = e^{\beta \mu} E_{1}(\mu \beta + \mu)$$

for the second integral in (5.26), we arrive at

$$F_{\zeta}(x) = 1 + \frac{\exp\left( \frac{-\lambda x}{\tilde{P}} \right)}{1 + \frac{\lambda_{\rho_{\kappa \gamma}}}{\tilde{P} \lambda P}} \left[ \exp\left( -\frac{\tilde{\lambda}}{\tilde{P}} \right) - 1 \right]$$

$$- \frac{\tilde{\lambda} \tilde{\rho}_{\kappa \gamma} \exp\left( \frac{\tilde{\lambda}}{\tilde{P}} \right)}{\lambda x \tilde{P}} \left( \frac{\tilde{\rho}_{\kappa \gamma}}{\lambda x} + 1 \right) \) E_{1}\left( \left( \frac{\lambda x}{\tilde{P}} + \frac{\tilde{\lambda}}{\tilde{P}} \right) \left( \frac{\tilde{\rho}_{\kappa \gamma}}{\lambda x} + 1 \right) \right)$$

(5.27)

and that finishes the proof to the proposition 2.

### 5.C Derivation of Outage Probability

To find the outage probability $P_{\text{out}}$, using (5.15) and noting that $\gamma_{D_{m^*}} = \max_{m} \left\{ \min \left( \frac{\gamma_{sm}}{\lambda_{tm} + 1}, \gamma_{md} \right) \right\}$, our aim is to find the CDF of $\gamma_{D_{m^*}}$. Utilizing proposition 1 and noting that $\gamma_{sm} = P_{s} |h_{sm}|^2$, by setting $\lambda_{\gamma} = \lambda_{sm}, \tilde{\lambda} = \lambda_{sr}$ and $\tilde{P} = P_{o}$, $F_{\gamma_{sm}}(x)$ is given as

$$F_{\gamma_{sm}}(x) = 1 + \exp\left( -\frac{\lambda_{sm} x}{P_{o}} \right) \left( \frac{\exp\left( -\frac{\lambda_{sr} L}{P_{o}} \right)}{\lambda_{sm} x + 1} - 1 \right)$$

(5.28)
CHAPTER 5. RELAY SELECTION IN CRNS WITH INTERFERENCE

Using proposition 2 and by defining

\[ \zeta_m \triangleq \frac{\gamma_{sm}}{\gamma_{tm} + 1} \]

and

\[ \gamma = \gamma_{sm}, \quad \tilde{\gamma} = \gamma_{tm}, \quad \tilde{\lambda} = \lambda_{tm} \]

we have

\[ F_{\zeta_m}(x) = 1 + \exp\left(\frac{-\lambda_{tm}}{1 + \frac{\lambda_{tm} P_t}{P_o}}\right) \left[ \exp\left(\frac{-\lambda_{sr} \tilde{I}}{P_o}\right) - 1 \right] \]

\[ -\frac{\lambda_{tm} \lambda_{sr} \tilde{I}}{\lambda_{sm} x P_t} \exp\left(\frac{\lambda_{tm}}{P_t} \left(\frac{\lambda_{sr} \tilde{I}}{\lambda_{sm} x} + 1\right)\right) E_1\left(\left(\frac{\lambda_{tm}}{P_t} + \frac{\lambda_{tm}}{P_t}\right) \left(\frac{\lambda_{sr} \tilde{I}}{\lambda_{sm} x} + 1\right)\right). \] (5.30)

Again, from (5.16) by setting \( \gamma = \gamma_{md}, \ \tilde{P} = P'_o, \ \lambda = \lambda_{md} \) and \( \tilde{\lambda} = \lambda_{mr} \), we have

\[ F_{\gamma_{md}}(x) = 1 + \exp\left(-\frac{\lambda_{md}}{P'_o} x\right) \left(\frac{\exp\left(-\frac{\lambda_{md}}{P'_o} \tilde{I}\right)}{\lambda_{md} \tilde{I} + 1} - 1\right). \] (5.31)

Finally, noting that

\[ \gamma_{D_m} = \min(\zeta_m, \gamma_{md}) \]

\( F_{\gamma_m}(x) \) is given by [45]

\[ F_{\gamma_{D_m}}(x) = 1 - (1 - F_{\gamma_{md}}(x)) (1 - F_{\zeta_m}(x)) \]

\[ = F_{\gamma_{md}}(x) + F_{\zeta_m}(x) - F_{\gamma_{md}}(x) F_{\zeta_m}(x) \] (5.32)

and, consequently, using the independence of \( \gamma_{D_m} \) 's, we have

\[ F_{\gamma_{D_m^*}}(x) = \prod_{m=1}^{M} \left[ F_{\gamma_{md}}(x) + F_{\zeta_m}(x) - F_{\gamma_{md}}(x) F_{\zeta_m}(x) \right]. \] (5.33)

The outage probability of the system is given by \( F_{\gamma_{D_m^*}}(R_o) \) which concludes the proof of proposition 3.
Chapter 6

Modified Selection Cooperation in CRNs

6.1 Objective

In chapter 5 we studied the impact of the primary user interference on the secondary user network. We mentioned there that the presence of the PU interference has a devastating impact on the SU probability of outage.

In this chapter we study the outage performance of a secondary cooperative network in the vicinity of a primary network by incorporating the effect of the PU in the outage analysis. Particularly we study the effect of the PU interference on the relay nodes in the SU network as well as the $SU_D$. Then by taking into account the impact of interference at all nodes in the SU we present a modified relay selection scheme that achieves the minimal outage probability. We also propose a power allocation scheme for the SU such that the contribution of the interference imposed on the PU is minimal. We propose an analytic approximate of the outage probability of the SU, afterwards by utilizing computer aided simulations we justify the proposed theory.

6.1.1 Related work and contribution

The authors in [82] study the probability of outage in the cooperative underlay cognitive radio networks for the DF cooperation. They restrict the maximum transmission power of all the transmitting nodes such that they do not impose interference on the PU network.
They propose a lower bound on the outage probability performance in their scheme. In [92] we analyze the probability of outage in an AF selection cooperation scheme considering an instantaneous threshold limit on the transmission powers in the SU network. Besides, the effect of primary network interference on the secondary network is also studied. In [92] however only the interference imposed on the relay nodes is taken into consideration where the interference at the secondary user destination the $SU_D$ is ignored and is assumed to be separable by some blind signal processing techniques [85]. In this chapter we present a scheme in which the maximum allowed transmission powers in the SU is calculated in the primary network and fed back to the secondary network via a low bit feedback link. Furthermore the effect of the PU interference is fully investigated on the SU. In particular interference imposed on the relays as well as the destination node in the SU network is taken into consideration. Finally along with proposing a closed form formula for the outage performance of the SU network, we study the behavior of both primary and secondary networks via computer simulations.

6.2 System Model

In this chapter we consider a secondary network source $S$ that communicates to the secondary network destination $D$ via $M$ possible relay candidates $R_m$, $m = 1, \ldots, M$, following the AF cooperative transmission. We assume that there is no line of sight (LoS) between the secondary user source ($SU_S$) and the secondary user destination ($SU_D$) due to shadowing. In the vicinity of the SU network, we consider a PU network consist of a primary user transmitter ($PU_T$) communicating with a primary user receiver ($PU_R$). All the transmitter-receiver nodes are assumed to be single antenna and all the channels are frequency flat and Rayleigh fading with $h \sim N(0, 1/\lambda)$, where $\lambda$ accounts for shadowing effect and the path loss$^1$. Both networks are assumed to be utilizing the same frequency band and hence there is a mutual interference imposed on each network on behalf of the other one. The interference from the primary network is received by the relays’ antenna as well as the destination’s. The PU transmits uncorrelated data symbols denoted by $x'$ towards the $PU_R$ directly. The data symbols are chosen from a long zero mean unit variance complex Gaussian (ZMUVC) codebook with average energy of $P_t$ Joules/symbol. The received signal at the $PU_R$ is given

$^1$The subscript is dropped for simplification of notation.
by
\[ y_r = \sqrt{P_t} h_{tr} x' + I_{SU \rightarrow PU} + n_r, \quad (6.1) \]
where \( h_{tr} \sim \mathcal{N}(0, 1/\lambda_{tr}) \), \( I_{SU \rightarrow PU} \) is the interference imposed on the PU on behalf of the SU and \( n_r \sim \mathcal{N}(0, 1) \) is the PU noise\(^2\).

The cooperative transmission in the secondary network is performed in two consecutive time slots. In the first time slot the PU transmits data symbols denoted by \( x \) drawn from a long ZMUVC codeword towards the relays. The received signal at the \( m^{th} \) relay node is
\[ y_m = \sqrt{P_s} h_{sm} x + \sqrt{P_t} h_{tm} x' + n_m, \quad (6.2) \]
where \( P_s \) is the PU average transmission energy per symbol, \( h_{sm} \sim \mathcal{N}(0, 1/\lambda_{sm}) \) is the \( S \rightarrow R_m \) channel, \( h_{tm} \sim \mathcal{N}(0, 1/\lambda_{tm}) \) and \( n_m \sim \mathcal{N}(0, 1) \) is the receiver noise at the \( m^{th} \) relay node.

In the second time slot, each relay normalizes its received signal by
\[ G_m = \sqrt{\frac{P_m}{P_s |h_{sm}|^2 + P_t |h_{tm}|^2 + 1}} \quad (6.3) \]
and forwards a scaled version to the SU\(_D\). The received signal at the SU\(_D\) is then given by
\[ y_d = G_m h_{md} y_m + P_t h_{td} x' + n_d \quad (6.4) \]
where \( P_m \) is the average energy per symbol transmitted from the \( m^{th} \) relay. The variables \( h_{md} \sim \mathcal{CN}(0, 1/\lambda_{md}) \) and \( h_{td} \sim \mathcal{N}(0, 1/\lambda_{td}) \) are the \( R_m \rightarrow D \) and \( PU_T \rightarrow D \) channel coefficients and \( n_d \sim \mathcal{N}(0, 1) \) is the destination receiver noise.

By substituting (6.2) and (6.3) into (6.4), the received instantaneous SINR at the SU\(_D\) is then given by
\[ \gamma_{PU_T} = \left( \gamma_{sm} + \gamma_{md} + 1 \right) + \gamma_{tm} \left( \gamma_{sm} + \gamma_{td} + 1 \right) \quad (6.5) \]
where \( \gamma_{sm} = P_s |h_{sm}|^2 \), \( \gamma_{md} = P_m |h_{md}|^2 \), \( \gamma_{tm} = P_t |h_{tm}|^2 \), \( \gamma_{td} = P_t |h_{td}|^2 \) and \( I_{PU \rightarrow SU} \) is the interference imposed on the SU from the PU\(_T\).

In (6.5), the effect of the PU\(_T\) interference appears as an extra term in the denominator.

\(^2\)The noise variance is normalized to 1 since it is possible to consider the effect of the receiver noise variance in the total received SNR.
6.2.1 Channel estimation

Data $P_U$ and $SU_S$ transmit orthogonal pilot symbols

$$x_{p,i}^T x_{p,j} = 0$$

$h_{ir}, h_{tm}, h_{td}, h_{sr}, h_{sm}$ are all estimated

Time Slot I

Time Slot II

Data

$US_D$ transmits pilot

$x_p, d = x_i + h_{td} x_j$

$x_i^T x_j = \delta_{ij}$

$h_{dm} = h_{md}$ is estimated

$R_m$ transmits pilot

$h_{mr}$ is estimated

Relay Selection

Data

$P_m$ and $P_s$ is determined

Figure 6.1: Channel estimation and relay selection in two time slots.

All the pilot transmitting nodes specify total of $N$ training symbols to channel estimation at the beginning of each time slot. These training sequences are designed to be orthogonal in time, i.e., $x_{p,i}^T x_{p,j} = \delta_{ij}$, where $x_p$ is the $i^{th}$ training sequence. In the first time slot $h_{ir}$ and $h_{sr}$ are both estimated at the $PU_R$ using the concept of orthogonality. Simultaneously, $h_{sm}$ and $h_{tm}$ are estimated at $R_m$ and $h_{td}$ is estimated at the $SU_D$. Later, at the beginning of the second time slot the destination sends a coded version of training symbols and $h_{td}$ which was previously estimated toward the relays$^3$ and hence, $h_{dm}$ and $h_{td}$ are retrieved at the same time. Assuming channel reciprocity $h_{dm} = h_{md}$. $h_{mr}$s are estimated at the $PU_R$ following either a TDMA pilot transmission or via sending simultaneous orthogonal training sequences.

6.2.2 Transmission power constraints

In a primary-secondary network interaction the reliable communication of the PU is of higher priority. Therefore, the secondary network should be designed such that the amount of interference that is imposed on the PU is acceptable. A good measure for the amount of acceptable interference is the outage probability of primary network. Particularly it is desired that the primary network faces an upper bound probability of outage such that in the worst case scenario retrieving data in the $PU_R$ is achievable. In [74, 92] the authors propose a threshold limit $\bar{I}$, for the acceptable interference imposed on the $PU_R$. Then they confine the transmitted powers of the $SU_S$ and $R_m$ to $P_s \leq \frac{I}{|h_{sr}|^2}$ and $P_m \leq \frac{I}{|h_{mr}|^2}$, respectively. $h_{sr} \sim \mathcal{CN}(0, 1/\lambda_{sr})$ and $h_{mr} \sim \mathcal{CN}(0, 1/\lambda_{mr})$ are the respective channels from the source and the $m^{th}$ relay to the primary user receiver. Furthermore, assuming the maximum transmission power constraints $P_s \leq P_o$ and $P_m \leq P'_o$, where $P_o$ and $P'_o$ are the maximum allowable transmission powers of the $SU_S$ and $R_m$. They adopt transmission

$$^3x_{p,d} = x_i + h_{td} x_j.$$
power constraints
\[ P_s = \min \left\{ \frac{\bar{I}}{|h_{sr}|^2}, P_o \right\} \]
for the $SU_S$ and
\[ P_m = \min \left\{ \frac{\bar{I}}{|h_{mr}|^2}, P_o' \right\} \]
for $R_m$. However, the threshold limit $\bar{I}$ is considered to be constant, when in reality this interference tolerance threshold is depends to the instantaneous state of the $h_{tr}$ channel. Something that the authors in [74, 92] neglect.

Besides, to find the appropriate transmission power the secondary network should be aware of $h_{mr}$ and $h_{sr}$. Obtaining these channels either requires transmission of training symbols from the $PU_R$ (assuming the $PU_R$ is equipped with transmit antennas as well) and perform channel estimation at the $SU_S$ (assuming the $SU_S$ is equipped with receive antenna as well) and $R_m$, or estimating $h_{sr}$ and $h_{mr}$ at the $PU_R$ and feeding back channel estimates to the $SU_S$ and $R_m$, respectively. In both cases a large amount of data due to training symbols and/or channel estimates in the feedback links is transmitted. If practically feasible this would reduce the bandwidth efficiency of the system and make the synchronization of $PU-SU$ a headache.

To overcome this problem we propose a new index of merit through which the transmission powers in the SU networks can be achieved. Referring to (6.1) the interference imposed on the $PU_R$ in the first and the second time slots are respectively given by

\[ I_{SU \rightarrow PU, 1} = \sqrt{P_s h_{sr}} \]

and

\[ I_{SU \rightarrow PU, 2} = \sqrt{P_m h_{mr}}. \]

Therefore, the received SINR at the $PU_R$ in the first and the second time slots would be $\gamma_{r,1} = \frac{P_s h_{sr}}{\lambda_{ad+1}}$ and $\gamma_{r,2} = \frac{P_m h_{mr}}{\gamma_{ad+1}}$, respectively.

**Property 1:** Defining $\bar{\gamma} = \bar{P} |\tilde{h}|^2$ where $\tilde{h} \sim \mathcal{N}(0, 1/\lambda)$, and $\bar{\gamma} = \bar{P} |\tilde{h}|^2$ where $\tilde{h} \sim \mathcal{N}(0, 1/\lambda)$, the outage probability of $\gamma_o = \frac{\bar{\gamma}}{\bar{\gamma}+1}$ is given by

\[ P_{out} = 1 - \frac{\exp\left(-\lambda R_o / \bar{P}\right)}{1 + \lambda R_o / \bar{P}} \]
where $R_o = 2^{2R} - 1$ and $R$ is the desired data rate in the primary network in bps/Hz.

**Proof:** The outage probability is given by

$$P_{out} = Pr \left( \log_2 \left( 1 + \frac{\hat{\gamma}}{\hat{\gamma} + 1} \right) \leq R \right)$$

$$= \int_0^\infty Pr \left( \hat{\gamma} \leq zR_o \right) f_z(z)dz$$

(6.6)

where $z \triangleq \hat{\gamma} + 1$, $f_z(z) = \frac{\hat{\lambda}}{\hat{T}} \exp \left( -\frac{\hat{\lambda}}{\hat{T}} (z - 1) \right) u(z - 1)$ and $u(\cdot)$ is the unit step function.

Inserting $f_z(z)$ in (6.6) Property I is proved.

Based on the result obtained from Property I the PU outage probabilities in the first and second time slots are given by

$$P_{out,1} = 1 - \frac{\exp(-\lambda_{tr}R_o/P_t)}{1 + (\lambda_{tr}P_sR_o)/(\lambda_{sr}P_t)}$$

and

$$P_{out,2} = 1 - \frac{\exp(-\lambda_{tr}R_o/P_t)}{1 + (\lambda_{tr}P_mR_o)/(\lambda_{mr}P_t)}$$

respectively. When there is no interference imposed on the PU the interference free outage probability of the PU is given by

$$P_{out}^{IF} = 1 - \exp(-\lambda_{tr}R_o/P_t)$$

which is the best probable outage performance of the PU.

Based on the shadowing effect, the path loss and the transmission power used in the PU, the PU sketches a redline on the acceptable probability of outage in the primary network by setting an upper bound $\bar{I} \geq P_{out}^{IF}$ for the probability of acceptable outage, more than which the system would not communicate reliably. In this regard in the first time slot $P_{out,1}$ must be less than $\bar{I}$ which leads to

$$P_s \leq \left( \frac{\exp(-\lambda_{tr}R_o)}{1 - \bar{I}} - 1 \right) \frac{\lambda_{sr}P_t}{\lambda_{tr}R_o}$$

(6.7)

where $[\cdot]^-$ means quantization to the closest smaller quantization level.

Similarly in the second time slot

$$P_m^{*} \leq \left( \frac{\exp(-\lambda_{tr}R_o)}{1 - \bar{I}} - 1 \right) \frac{\lambda_{mr}P_t}{\lambda_{tr}R_o}$$

(6.8)

Finally the secondary allowable transmission powers are assessed in the $PU_R$. Assuming the SU can transmit only in $\mathcal{K}$ distinct power levels, using a total number of $\log_2(\mathcal{K})$ bits, the

\[^4\text{we assume } \mathcal{K} \text{ is a power of 2.}\]
power level at which the SU or $R_m^*$ should use is transmitted towards their corresponding node at the beginning of each time slot right after channel estimation. These transmissions could be performed via feedback links which operate in another narrow frequency band.

By using this technique at each time slot the PU needs to send only $\log_2(K)$ bits of information via a feedback link wherein the schemes proposed by [74,92] the channel estimates as well as the threshold limit $\bar{I}$ should be fed back to the SU and $R_m^*$ which require much more feedback link bandwidth.

### 6.2.3 Relay selection scheme

In the second time slot when all the required channel estimates are available at relays the $PU_R$ feeds back the power the SU and each relay must use in a $(M+1)\log_2(K)$ bit data stream. Then each relay $R_m$ calculates $\gamma_{D_m}$ and forms a timer inversely proportional to $\gamma_{D_m}$. While counting down, the timer that expires sooner transmits a flag to all other relays, commanding them to remain silent in the second time slot. Finally, the selected relay transmits its data towards the destination node at the end of the second time slot. (see Fig. [6.1]).

The optimal selection criterion for our considered scenario is given by

$$m^* = \arg \max_m \{\gamma_{D_m}\}$$

where

$$\gamma_{D_m} = \frac{\gamma_{sm}\gamma_{md}}{\gamma_{sm}(\gamma_{td} + 1) + \gamma_{md}(\gamma_{tm} + 1) + (\gamma_{td} + 1)(\gamma_{tm} + 1)} \approx \frac{\gamma_{sm}}{(\gamma_{tm} + 1)} \cdot \frac{\gamma_{md}}{(\gamma_{td} + 1)}.$$

which can be written as [9]

$$\gamma_{D_m} \approx \min\left(\frac{\gamma_{sm}}{\gamma_{tm} + 1}, \frac{\gamma_{md}}{\gamma_{td} + 1}\right). \quad (6.9)$$

In the following section we derive a closed form formula for the outage probability of the secondary network considering that

$$m^* = \arg \max_m \min\left(\frac{\gamma_{sm}}{\gamma_{tm} + 1}, \frac{\gamma_{md}}{\gamma_{td} + 1}\right). \quad (6.10)$$
6.3 Outage Probability Analysis

In this section we derive a closed form formula for the outage probability of the SU network based on the selection criterion in (6.10).

**property II:** The outage probability of the SU in the presence of the PU interference is given by (6.11),

$$
\Pr_{out} = 1 - \sum_{i_1=1}^{M} \frac{\alpha_{i_1} e^{-\beta_{i_1} R_0}}{1 + \frac{P_t \beta_{i_1} R_0}{\lambda_{md}}} + \sum_{i_2=i_1+1}^{M-1} \sum_{i_3=i_2+1}^{M} \frac{\alpha_{i_2} \alpha_{i_3} e^{-(\beta_{i_2} + \beta_{i_3}) R_0}}{1 + \frac{P_t (\beta_{i_2} + \beta_{i_3}) R_0}{\lambda_{md}}} 
- \sum_{i_4=1}^{M-2} \sum_{i_5=i_4+1}^{M-1} \sum_{i_6=i_5+1}^{M} \frac{\alpha_{i_4} \alpha_{i_5} \alpha_{i_6} e^{-(\beta_{i_4} + \beta_{i_5} + \beta_{i_6}) R_0}}{1 + \frac{P_t (\beta_{i_4} + \beta_{i_5} + \beta_{i_6}) R_0}{\lambda_{md}}} + \ldots
$$

where $\alpha_i = \frac{e^{-(\lambda s R_0)/(P_s)}}{1 + (\lambda s P_t R_0)/(P_s \lambda_{ii})}$ and $\beta_i = \frac{\lambda d}{P_t}$.

**proof:** From Property I we conclude that the cumulative distribution function (CDF) of $\gamma_{1,m} = \frac{\gamma_{sm}}{(\gamma_{tm}+1)}$ is given by

$$
F_{\gamma_{1,m}}(\gamma) = 1 - \frac{e^{-(\lambda s P_t \gamma)} / P_s}{1 + (\lambda s P_t \gamma) / (P_s \lambda_{tm})}.
$$

Furthermore for $\gamma_{2,m} = \frac{\gamma_{md}}{(\gamma_{tm}+1)}$ conditioned on $\gamma_{td}$ we have

$$
F_{\gamma_{2,d}|\gamma_{td}}(\gamma) = 1 - \exp \left( -\lambda_{md} t\gamma / P_m \right)
$$

where $t = \gamma_{td}+1$. Finally the CDF of $\gamma_{D_m}|\gamma_{tm} = \max_{m} \min(\gamma_{1,m}, \gamma_{2,m}|\gamma_{tm})$, is given by

$$
F_{\gamma_{D_m}|\gamma_{tm}}(\gamma) = \prod_{m=1}^{M} \left[ 1 - \left( 1 - F_{\gamma_{1,m}}(\gamma) \right) \left( 1 - F_{\gamma_{2,m}}(\gamma) \right) \right]
= \prod_{m=1}^{M} \left[ 1 - \frac{e^{-(\lambda s P_t \gamma) / P_s} e^{-(\lambda_{md} t\gamma / P_m)}}{1 + (\lambda s P_t \gamma) / (P_s \lambda_{tm})} \right]
$$

(6.11)

Finally noting that $f_t(t) = \frac{\lambda d}{P_t} e^{-\frac{\lambda d (t-1)}{P_t}} u(t-1)$ the outage probability is given by

$$
\Pr_{out} = F_{\gamma_{D_m}}(R_0)
= \frac{\lambda d}{P_t} \int_{1}^{\infty} \prod_{m=1}^{M} \left[ 1 - \alpha_m e^{-\beta_m t R_0} \right] \frac{\lambda d (t-1)}{P_t} e^{-(\lambda d (t-1)) / P_t} dt.
$$

(6.12)

After some mathematical manipulations (6.12) reaches to (6.11).
6.4 Simulation Results

To study the proposed scheme, we assume the $SU_S$ located at $(-1/2, 0)$ communicates with the $SU_D$ located at $(1/2, 0)$ via two candidate relays located at $(0, d)$ and $(0, -d)$, respectively. The coordinates present a point in the Cartesian coordinate plan and the distances are normalized with respect to the distance between the $SU_S$ and the $SU_D$. The $PU_T$ and $PU_D$ are respectively positioned at $(-.25, 10)$ and $(.25, 10)$. Path loss is assumed to obey a forth order power law. We assume a four bit feedback link for transmitting the information about the allowed transmission powers from the PU to the SU.

6.4.1 Effect of Relay Position:

Fig. 6.2 studies the effect of relay distance on the outage probability of the SU where $P_t = 20 dB$, the $PU_T$ is located at $(0, 100)$ and the $PU_R$ is located at $(0, 99.5)$. This will guarantee that the $SU_S$ and $R_m^*$ can transmit with full power, since the outage probability of the PU network is approximately $2^{-11}$. As the plot reveals the best position for the selected relay is when it is located almost in the middle of the path from source to destination. If the relay power is exactly the same as the source power this point would be exactly in the
middle. If the relay power is less than the source power then the relay is better to be located closer to the destination. This is because with the source power more the signal can travel a longer distance while combating with the path loss. Therefore for the best performance the relay should be located close to the destination. On the contrary, when the relay power is more than the source power because of the same relay it is optimal if the relay is located close the source terminal. If the relay distance to the destination is increased it is as if its transmission power is decreased and thus the same behavior is expected.

6.4.2 Effect of Primary User Position or primary user power:

Fig. 6.3 at the top shows the outage probability of the SU under the proposed power con-

straints versus the PU distance to the origin. The positive distance means the PU is approaching the origin and the negative distance means it is receding away from the origin. In this simulation the coordinates of the PU and the PU are (. − 25, 10) and (.25, 10), respectively. The primary network approaches the origin and then recedes away from the origin toward (−.25, −10) and (.25, 10). It can be seen that right at the origin the probability of outage is maximized. This is because on one hand the PU is getting close to the relays and destination imposing stronger interference and on the other hand the closeness of the SU and relay nodes to the PU forces the transmitting nodes in the SU network
to reduce their power and hence the $S_U$ probability of outage would increase. The bottom plot in Fig. 6.3 the $P_s$ and $P_{m^*}$ versus the PU distance to the origin. It is shown that the minimum transmission power are applied when the PU network is close to the SU network. Finally note that increase in $P_t$ is as if the PU distance to the SU is decreased. In both cases similar result is observed.

### 6.5 Conclusion

In this chapter we studied the effect of the PU interference on the SU probability of outage. We considered the imposed interference on behalf of the PU network on the relays and the $SU_D$ node. Furthermore by considering a more realistic scenario we found the proper transmission powers in the SU network such that the outage probability in the PU network is within an acceptable range. We studied the impact of the $PU_T$ power $P_t$ and also its distance to the SU on the outage probability of the SU network. We demonstrated that when the PU network gets close to the SU network the outage in the SU network will be highly probable. The two fold reason for the leap in the probability of outage is that on the one hand the decrease in the distance means a lot more interference on the SU, and on the other hand the SU should reduce its transmission power so that the interference imposed on the PU would not get higher than the tolerable limit. We also found that based on the powers each transmitting node in the SU use, the best place for the relays to achieve the minimal probability of outage in the SU network is in the middle of the path between the $SU_S$ and the $SU_D$. 
Chapter 7

Conclusion and Future Works

7.1 Conclusion

In this thesis we studied the non ideal phenomena that degrade the ideal performance of cooperative communication networks with relay selection. We investigated the amount of degradation in the performance of wireless communication networks with selection cooperation in the presence of undesirable phenomena such as fast channel variation, CEE, FD, and co-channel interference in terms of outage probability, ASER and average capacity.

In particular in chapter 2 we investigated the effect of fast channel variations in vehicular networks. We mentioned that the communication paradigm of IVC is quite different from conventional point-to-point communication channels. In particular, the communication environment in an inter-vehicular network setting can be highly volatile due to the highly dynamic variation in channel quality, and that the cascaded Rayleigh model for the channel is a more realistic representation than single Rayleigh distributed channel and offers more accurate results. We analyzed the outage performance of a multi relay system with relay selection and showed that selection provides full diversity order. In comparison with the conventional Rayleigh fading channels we found out that cascaded Rayleigh channels in practice have a worse performance in terms of outage probability.

In chapter 3 we studied the effect of CEE in cooperative systems with relay selection. We analyzed the ASER and outage probability performance of the system in the presence of estimation error. Furthermore we introduced a lower bound on the average capacity and derived a closed form formula for its performance in the presence of imperfect channel estimates. We showed under the assumption of constant training power an error floor was
imposed on the ASER and outage performance of the system leading the diversity order to zero. However, if the training power increased with the data SNR then the expected full diversity order of the system was preserved. It is must be noted that the performance of the system always depends on how much training power is specified to channel estimation though. Later, we investigated the performance of the system incorporating the effect of distance to our considerations. We showed that the best performance of the system is achieved when the relays are in the middle of the connecting path between source and destination.

In chapter 4 we discussed relay selection for DF protocol in cooperative networks. We showed that the presence of CEE and FD degraded the performance and also reduced the diversity order of S-DF. In particular in the presence of FD the diversity order reduced to 1, and in the presence of fixed channel estimation error the diversity order becomes 0, and an error floor is visible in the ASER and outage probability versus SNR curves. Also we studied the case where the training power sequence increased in parallel with the data SNR. We saw that the diversity loss due to the fixed channel estimation error was recovered by increasing the pilot symbols power. We investigated the impact of path loss on the system performance by incorporating the node-to-node distances to our equations. The optimal location of the relays were found to be in relative to the source and relays’ transmission power. We derived analytic expressions for the outage probability, average symbol error rate and average capacity bound.

Then we moved on towards incorporating the effect of co-channel interference in cooperative networks with relay selection in the context of cognitive radio where managing co-channel interference between the PU and the SU is of critical importance. Particularly in chapter 5 we investigated the performance of relay selection in an underlay cognitive radio system in the presence of primary user interference. Particularly, we considered a secondary multi-relay network operating in the AF mode. The best relay is then selected to cooperate with the secondary source node according to an index of merit. We adopted a maximum power allocation scheme based on the maximum tolerable interference on the primary user network. We derived a closed-form outage probability expression for the secondary multi-relay network and further presented a thorough asymptotical diversity order analysis. We also studied the performance of the system under asymptotic power and interference cases and compared the proposed scheme to the conventional direct transmission and overlay cognitive radio scenarios.
CHAPTER 7. CONCLUSION AND FUTURE WORKS

We learned that if the PU transmission power is increased then the outage probability of the SU is drastically aggravated. The same scenario is observed when the $PU_T$ is close to the SU network. Path loss in this case acts against the SU desired performance. The closer the $PU_T$ to SU network is, the more the interference imposed on the SU is sensed and the higher the probability of outage would go.

We also observed that if the $PU_R$ is close to the SU network, this time the interference from the SU is problematic and must be managed. In this case the PU reduces the sensed interference by sending controlling signals to the SU ordering the SU transmitting nodes to reduce their transmission power, this itself would lead to a leap in the probability of outage in the SU network.

Finally in chapter 6 we studied the effect of the PU interference on the SU probability of outage. We considered the imposed interference on behalf of the PU network on the relays and the $SU_D$ node. Furthermore by considering a more realistic scenario we found the proper transmission powers in the SU network such that the outage probability in the PU network is within an acceptable range. We studied the impact of the $PU_T$ power $P_t$ and also its distance to the SU on the outage probability of the SU network. We demonstrated that when the PU network gets close to the SU network the outage in the SU network becomes highly probable. The two fold reason for the leap in the probability of outage is that on the one hand the decrease in the distance means a lot more interference on the SU, and on the other hand the SU should reduce its transmission power so that the interference imposed on the PU would not get higher than the tolerable limit. We also found that based on the powers each transmitting node in the SU use, the best place for the relays to achieve the minimal probability of outage in the SU network is in the middle of the path between the $SU_S$ and the $SU_D$.

7.2 Future Works

- Noncoherent Relay selection in cooperative networks

We note that having a perfect knowledge of CSI is on the one hand critical and on the other hand practically impossible. This has sparked interests in the development of blind techniques in which relay selection is performed without a perfect knowledge of CSI. Chyi in [93] has proposed a method for noncoherent detection for FSK selection combining (SC) scenarios based on energy detection. Farhadi and Beaulieu have
extended the work of [93] to a cooperative scenario with AF relaying [94]. In their scenario, all the relays amplify their signal received from the source antenna and forward them towards the destination terminal. At the destination node, the best relay which receives maximum energy is selected. The transmitted signal is also decoded using maximum energy detection at the destination node. The same procedure is proposed by [95] for the simpler case where decode-and-forward (DF) cooperation and selection combining at the receiver are used. The proposed schemes focus on selection combining at the receiver; however, none of them consider relay selection in cooperative networks. This is mainly because for relay selection, having a knowledge about the relays-to-destination CSI or at least energy is critical, and hard to obtain. Song and Letaief in [96] have proposed a delayed non-coherent relaying scheme to achieve space diversity without requiring CSI at the relays. Specifically, each relay non-coherently amplifies and forwards its received signals with different delays such that a virtual multipath channel is created between the source and destination. Recently the authors of [97] have proposed a differential modulation scheme for relay selection in detect-and-forward (DaF) one way cooperative networks. Using differential modulation, they propose a differential selection scheme which without considering a full knowledge of CSI selects the best relay. Also, Song et al. in [98] have proposed a selection scheme for AF analog network coding (ANC) scheme in bidirectional relay channels using differential modulation. In [99] the authors have investigated a two-phase protocol for noncoherent two-way relay communications. Their scheme incorporates single-threshold noncoherent energy detection and a digital network coding process at the relay node. Energy detection is optimized for both phases of the protocol, and a threshold that exhibits no error floors is derived in closed form. In their work they assume single relay cooperation.

In line with this dissertation in focusing on the ambiguities in the CSI in cooperative networks, we can investigate if it is possible to eliminate the necessity of CSI acquisition in the nodes that are cooperatively communicating with each other. In this regard, a noncoherent receiver design and a relay selection strategy for cooperative networks can be proposed. In this scenario, either for relay selection or for data detection, pilot assisted channel estimation is relinquished. Also no feedback link is needed for CSI awareness at the transmitter nodes. FSK modulation and DF relaying protocol as the transmission-cooperation scheme is proposed and relay selection is performed based
on the max-min criterion on the received instantaneous signals at the relay nodes.

- **Joint Selection Beamforming in MIMO cooperative networks:**
  The 5th generation mobile network (5G) research prognosticates emerging of in-group cooperative relays or cellular repeaters that by utilizing macro-diversity and beamforming techniques are able to provide higher bit rates to a larger portion of the cell. In this regard cooperative communication protocols that are able to fortify the ability of transmitting higher bit rates are highly demanding and therefore, advanced relay selection techniques that are proven to be capacity increasing can never be neglected in the future. Furthermore the joint beamforming-selection techniques in cooperative communication can even be more promising in terms of achieving higher data rates. Utilizing MIMO antenna systems in the context of cooperative communication and taking advantage of MIMO diversity along with cooperative diversity can be a response to the demand of higher data rates. In this regard a MIMO source cooperates with multiple MIMO relays to transmit its data towards a MIMO destination. Beside selecting the best MIMO relay to cooperate with the source node, choosing the best beamformer to optimally transmit the source data towards the destination is a hot topic. In this context, on the one hand extending joint beamforming-selection techniques in cooperative networks is optimistically promising. On the other hand since a necessary condition for an optimal beamforming is availability of CSI in the transmitter side (for transmit beamforming) a slight imperfection in the channel estimate can degrade the performance of the system drastically. Also the availability of CSI at the transmitter requires either a reverse pilot transmission from relays and/or destination to the source terminal or availability of feedback links from the mentioned nodes to the source node. In either case the bandwidth efficiency of the system decreases and thus the proficiency of the provided techniques can be questioned. A good way to moderate this issue is to use as much fewer feedback bits as possible to provide the transmitter, with information about the current CSI of all links in the system so that the transmitter can decide on which relay to select or what beamformer to use. Alternatively the beamformer can be calculated at the receiver, mapped to a predefined common codebook between the transmitter and the receiver, be assigned an index and be informed to the transmitter via a low bit feedback link. However this can be only achieved if only the codebook index of the beamformer is fed back. Finding an optimal
codebook for all the possible choices of beamformers in a cooperative communication system can be a hot future research.

- **Large-Scale Multiple-Input Multiple-Output Wireless Systems**\(^1\):
  The advantages of MIMO technology are well established. Nevertheless, there is fast-growing interest in large-scale multiple antenna systems (tens to hundreds of low-power antennas) which offer advantages over conventional (a few antennas only) MIMO, including cooperative networks. These advantages are the familiar higher data rates, increased link reliability, and power savings at the base stations. The latter benefit is significant for energy consumption with an expected ten-fold decrease in the required transmit power. The cost of being able to implement such new technology is the development of a significant amount of new engineering science that will fuel much future research.

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Bibliography


