Characterization and Interpretation of Polythermal Structure in Two Subarctic Glaciers

by

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Abstract

I use ice-penetrating radar to probe the thermal structure in two small glaciers in the Saint Elias Range, southwestern Yukon. I develop processing workflows to separate bed and englacial reflections in radar and use these to build maps of both bed topography and englacial scattering. Comparison with borehole data shows that englacial scattering occurs in ice at the freezing point. The pattern in thermal structure suggests that the observed regime is dominated by accumulation zone processes. I develop a numerical model to simulate steady and time-dependent thermal regimes in glaciers. Diagnostic simulations support the hypothesis that meltwater entrapment is a critical control on the observed structure. Sensitivity tests suggest a climate sensitivity such that thinning and retreat of the near-surface aquifer may dramatically alter the thermal structure. Prognostic simulations illustrate scenarios in which these polythermal glaciers may cool as climate warms in the future.

Keywords: glacier; polythermal; radar; kluane; enthalpy; climate
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Chapter 1

Introduction

1.1 Motivation

Alpine glaciers are an important component of Earth’s cryosphere. Along with seasonal snow they are a significant contributor of freshwater in drainages around the world (Barnett et al., 2005; Neal et al., 2010). Furthermore, the melting of glaciers is known to make a measurable contribution to sea level rise (Berthier et al., 2010; Bahr and Radić, 2012). Glaciers and ice caps worldwide are measured to be losing mass rapidly (e.g. Kaser et al., 2006). Changes in dynamics directly affect changes in alpine glaciers and are relevant to the environmental (e.g. hydrological, climatic, geomorphological) role glaciers will play in the future. Therefore, a comprehensive understanding of glacier dynamics worthwhile.

Glacier thermal state is a piece of this understanding. The temperature and water content of glacier ice are known to be first-order controls on its viscosity (Cuffey and Paterson, 2010, Ch. 3). Because heat is produced by ice as it undergoes deformation, thermal state and mechanical behaviour are closely linked. In addition, thermal structure influences hydrology (Irvine-Fynn et al., 2011) and likely plays a role in the behaviour of some surging glaciers (Clarke et al., 1984; Fowler et al., 2001). Glaciers have been observed with a variety of thermal regimes (Hutter et al., 1988; Irvine-Fynn et al., 2011), and thermal structure depends to a degree on local climate (Pettersson, 2004). This thesis is motivated by the need for both field data and conceptual models of how thermal structure develops in glaciers in the Saint Elias Mountains of Yukon Canada, which among glaciers on the Gulf of Alaska, are disproportionally represented in terms of freshwater contribution and sea level rise (Arendt et al., 2002).
1.2 Glacier deformation and thermal structure

The deformation of ice is nonlinear (Orowan, 1949), and the relation of stress to strain (deformation) rate can be represented with a power-law rheology in the form

$$\dot{\epsilon} = A \sigma^n,$$  \hspace{1cm} (1.1)

where $\epsilon$ indicates uniaxial strain rate and $\sigma$ refers to uniaxial stress (Glen, 1955). The exponent $n$ describes the nonlinearity of the rheology, and $A$ is the flow coefficient. Laboratory experiments show that there are several deformation mechanisms that occur under different strain and stress regimes, and imply a value of the stress exponent $n$ between 1.9 to 4.0 (Goldsby and Kohlstedt, 2001). Closure rates of glacier cavities (Nye, 1953) and borehole inclinometry from glaciers (Harper et al., 2001) provide field observations that fit well with the approximation $n = 3$ for the flow characteristics of natural ice under glaciologically-relevant stress regimes. An experiment in which the outputs of a glacier flow model are compared to surface velocity observations in East Antarctica finds $n = 3.5$ to give the best fit (Cuffey and Kavanaugh, 2011).

The thermal regime of glaciers has been defined in terms of basal temperatures (e.g. Clarke et al., 1984; Fowler et al., 2001), as well as in terms of the bulk ice temperature (e.g. Blatter and Hutter, 1991; Pettersson, 2004). In the former terminology, “warm” (temperate) glaciers are those that reach the melting point at the bed, while cold glaciers are frozen at the bed (Fowler et al., 2001). In the bulk ice definition, glaciers are temperate only if they consist of a mixture of ice at the melting point and some fraction of water (excluding the surface boundary layer affected by annual temperature fluctuations), and cold if the volume of ice is entirely sub-freezing. Polythermal glaciers consist of a mixture of cold and temperate ice distributed throughout their volume (Blatter and Hutter, 1991). The pattern of cold and temperate ice in polythermal glaciers can vary broadly as a function of climate, glacier dynamics, and geometry (e.g Pettersson, 2004).

Pettersson (2004) describes several possible patterns of cold and temperate ice in polythermal glaciers. Under cold conditions such as in the high Arctic, glaciers may be primarily cold ice, but temperate at or near the base due to strain heating (Figure 1.1a). Little to no meltwater is formed in the upper glacier during the ablation season, so temperate ice is not advected through the glacier. This pattern is observed in John Evans Glacier in the Canadian High Arctic (Wohlleben et al., 2009). McCall Glacier in the Alaskan Brooks Range exhibits a similar struc-
Figure 1.1: Hypothetical distributions of cold and temperate ice in polythermal glaciers, after Blatter and Hutter (1991) and Pettersson (2004). Temperate ice is indicated by the black swatches. Possible causes for each distribution are discussed in the text.

ture (Trabant et al., 1975; Rabus and Echelmeyer, 1997). Although meltwater freezing warms the accumulation area ice, it remains sub-freezing because the heat released is less than or equal to the heat lost during the winter. Ice in the upper layers of the ablation region is colder still, but McCall Glacier contains a temperate layer near the base (Trabant et al., 1975; Rabus and Echelmeyer, 1997).

In another scenario (Figure 1.1b), the upper part of the accumulation area may be cold, but the lower region may become warm enough during the ablation season to form temperate ice (Pettersson, 2004). The temperate region may be a localized mid-glacier patch as cold ice originating above subsequently resurfaces in the ablation zone (Figure 1.1b), or it may dominate the ablation zone of the glacier.

In a reversed geometry, the upper glacier may be temperate due to latent heat released by meltwater entrapped in firn in the accumulation zone, while the lower glacier is cold because meltwater runs off efficiently (Figure 1.1c–d) (Pettersson, 2004). This pattern is common in
Svalbard and on the lee side of Scandinavian mountains, leading Aschwanden et al. (2012) to term this regime “Scandinavian”, as opposed to the “Canadian” pattern described above. The cold surface layer in the ablation zone of Storglaciären in Sweden is well-documented (Holmlund and Eriksson, 1989; Pettersson et al., 2003, 2007; Gusmeroli et al., 2012).

1.2.1 Factors influenced by thermal structure

The understanding of how temperature and water content influence glaciers has developed substantially over the history of glaciology (Blatter et al., 2010). With the advent of increased computational power, interest in modelling ice masses with nontrivial temperature distributions increased. This led to the first thermomechanically-coupled ice flow models in the late 1970s and 1980s (e.g. Jenssen, 1977; Oerlemans, 1982; Budd and Smith, 1983; Huybrechts and Oerlemans, 1988). Thermomechanically-coupled models account for differences in ice behaviour at different temperatures. Numerous such models exist today (Payne et al., 2000; Pattyn et al., 2008).

Ice deformation  Temperate ice deforms more rapidly under a given stress than cold, sub-freezing ice, meaning that variations in ice temperature influence the movement of glaciers. Physically, the flow-law coefficient $A$ is one determinant of ice viscosity, and is predominantly a function of temperature $T$, pressure $P$, and water content $\omega$. For temperatures spanning terrestrial ranges, $A(T, \omega)$ varies more than three orders of magnitude (Cuffey and Paterson, 2010). The effects of $T$ and $\omega$ on $A$ are often described using an Arrhenius law of the form

$$A(T, \omega) = A_0(\omega, \vartheta) \exp\left(-\frac{Q(T) + VP}{RT}\right),$$

(1.2)

where $A_0(\omega, \vartheta)$ is a scaling prefactor, $R$ is the gas constant, $Q$ is creep activation energy, and $V$ is the activation volume for creep. The variable $\vartheta$ is used here to represent a variety of additional controls on viscosity, including impurity content, strain history, grain size, and stress (Treverrow et al., 2012). From laboratory observations it is known that the sensitivity of ice to temperature within 10°C of melting is even greater than this relationship predicts with a constant activation energy $Q$, implying $Q = Q(T)$ (Cuffey and Paterson, 2010). The combined effects of diffusion and grain boundary slip on deformation rate may be responsible for this temperature dependency (Goldsby and Kohlstedt, 2001). Although ice viscosity was once thought to be
strongly pressure-dependent (e.g. Demorest, 1938), the pressure effect due to $V$ is small and can be neglected (Cuffey and Paterson, 2010).

In addition to the temperature and pressure effects, variations in water and impurity content also alter the flow-law coefficient. The sole experimental enquiries into the water effect have been conducted by Duval (1977), in which it was found that variations in the flow law coefficient could span a factor of three by raising water content by 0.8%. Impurities may either soften or stiffen ice, depending on concentration, grain size, and the stress and temperature regimes (Cuffey and Paterson, 2010). Likely impurities to be found in glacier ice include sodium chloride, sulphuric acid, and hydrochloric acid. A well-known example of impurity contrast in natural ice exists across the Wisconsinian-Holocene boundary in Antarctic ice. Here, impurity content correlates with both smaller grain sizes, deformation-enhancing ice fabrics, and larger deformation rates (Cuffey and Paterson, 2010).

Both temperature and water content effects in the flow law may be important\(^1\), because $A$ expressed in (1.2) is sensitive to both. The influence of the flow-law coefficient on glacier deformation can be explored in a simple case as follows, in which the flow dynamics are represented by a stress balance within a laterally infinite\(^2\) slab of uniform thickness on an inclined plane. Deformation is assumed to be exclusively bed-parallel (laminar), and the internal stress balance within the ice is

$$\tau_{xz} = \rho g (H - z) \sin \alpha,$$

where $\rho$ is density, $g$ is gravitational acceleration, $H$ is sheet thickness, and $\alpha$ is basal slope. The geometry that gives rise to (1.3) is illustrated in Figure 1.2.

The simple version of the flow law, valid for the conditions described above, is

$$\dot{\epsilon}_{xz} = A(T, \omega) \tau_{xz}^*.$$  \hspace{1cm} (1.4)

Combining Equations 1.3 and 1.4, and integrating from the slab base to an arbitrary elevation

---

\(^1\)Impurity concentration is ignored for reasons expressed later, however for the simple analysis of viscosity effects to follow, the cause of viscosity variations is irrelevant.

\(^2\)There are no wall stresses, and velocity depends only on the vertical coordinate $z$. 

Figure 1.2: Simplified force balance for an infinite sheet of uniform thickness. The driving stress is assumed to be $\tau_{xz}$, which depends on the normal stress $\rho g (H - z)$ and the surface inclination angle $\alpha$ (Cuffey and Paterson, 2010). Illustrative velocity vectors similar to those that can be calculated from Equation 1.5 are shown in grey.

\[ u(z) = 2 \int_0^z \dot{\epsilon}_{xz} dz = \frac{2A (\rho g \sin \alpha)^n}{n + 1} (H^{n+1} - (H - z)^{n+1}) + u(0) \]  

(1.5)

where $u(0)$ represents the basal sliding. Neglecting sliding, both the surface flow velocity and the vertical velocity gradient will scale linearly with $A$ for a fixed geometry. If two idealized glaciers are observed to have the same surface velocity, but $A$ is double in one compared to the other, the less viscous glacier will be 16% thinner (for $n = 3$). A second integration over the entire slab thickness gives the ice flux $q$:

\[ q = \int_0^H u(z) dz = \frac{2A (\rho g \sin \alpha)^n}{n + 1} \int_0^H (H^{n+1} - (H - z)^{n+1}) dz + \int_0^H u(0) dz \]

\[ = \frac{2A (\rho g \sin \alpha)^n}{n + 1} \left( H^{n+2} - \frac{H^{n+2}}{n + 2} \right) + u(0) H \]

\[ = \frac{2A (\rho g H \sin \alpha)^n}{n + 2} H^2 + u(0) H. \]  

(1.6)

Comparison of the two idealized glaciers considered above (identical surface velocities, $A$ varying by a factor of two, and thickness adjusted accordingly) yields a discharge in the thinner, less...
viscous (higher $A$) of the two glaciers related to that in the more viscous (lower $A$) by

$$\frac{q_2}{q_1} = \sqrt{\frac{8}{2^{n/2}}}.$$  

Therefore, the flux $q_2$ of the less viscous, thinner, faster flowing glacier is 68% greater than that in the other, $q_1$.

**Meltwater drainage**  Ice temperature is also an important influence on the evolution of meltwater drainage systems in glaciers. Conventional wisdom has in the past suggested that the drainage from glaciers significantly below the melting point should be predominantly supraglacial (Paterson, 1994; Hodgkins, 1997), although this view has been challenged (e.g. Schroeder, 2007; Gulley et al., 2009). Should it be true that cold ice inhibits water penetration to the bed, then basal lubrication should be affected. The accuracy of this conceptual model is not entirely clear (e.g. Bælum and Benn, 2011), but it seems to be the case that the thermal characteristics of ice are important to hydrology (e.g. Irvine-Fynn et al., 2011). The cold tongue of Glacier de Tête Rousse, France, allows basal water to pool in the mid-glacier (Vincent et al., 2012). Hydrofracture may permit water to penetrate cold ice and reach the bed sufficiently quickly to prevent refreezing (van der Veen, 2007). Recent work has documented examples of this hydrofracture process on both glaciers (Boon and Sharp, 2003) and ice sheets (Das et al., 2008).

### 1.2.2 Controls on thermal structure

**Climate**  In polar ice masses, the temperature below the seasonally-affected layer is traditionally regarded as an indicator of average annual surface temperature (Cuffey and Paterson, 2010). This is valid when surface temperature is primarily below freezing, so that the limiting effect caused by the inability of the ice surface to rise above the melting point is minimized, and so that energy is not sequestered into a liquid fraction within the ice. The conditions above are not necessarily realized for many glaciers, for which air temperatures during the ablation season may frequently rise above the melting point.

Alpine glaciers are worthy of attention in the context of a warming climate for several reasons. Compared to the major ice sheets of Greenland and Antarctica, the contribution of alpine glaciers to sea level rise is likely to be realized in the relatively near future (Meier et al., 2007). Ice in alpine glaciers has a relatively short residence time, increasing their sensitivity to climate
fluctuations. Observations indicate that the vast majority of land-terminating glaciers are in re-
treat (Kaser et al., 2006). Studies focused on glaciers in Alaska and the Canadian northwest
have found that these glaciers contribute disproportionately to the total mass loss by alpine
.glaciers at present, and that by extension, they are having an immediate effect on sea level
(Arendt et al., 2002; Berthier et al., 2010). Dynamics play an important role in controlling how
.glaciers respond to changes in climate and mass balance. A critical way in which this occurs
involves the thermal structure of the glacier because of the dependence of ice rheology on
temperature and water content.

Furthermore, knowledge of a glacier’s thermal structure provides a window into past climate
and recent glacier kinematics. Whether or not a glacier is in thermal equilibrium with its environ-
ment may be inferred from ice temperatures (e.g. Wohlenben et al., 2009; Rippin et al., 2011).
Differences in observed temperature from what would be predicted from simple models inform
our understanding of glacier heat flow and provide clues as to what processes are important.
Internal temperatures can also reflect local climate changes (Rabus and Echelmeyer, 2002;
Gusmeroli et al., 2012), albeit, a most likely nonlinear one.

**Melt refreezing** A potentially important source of glacier heat is the melt season heat input
at the glacier surface. Solar heating is not likely to directly affect the heat distribution below
an annual surface layer. Under realistic conditions, a surface boundary layer roughly 10–15 m
thick is likely to be affected by the surface energy balance, which includes turbulent heat fluxes
and solar radiation (e.g. Pettersson et al., 2007; Wheler and Flowers, 2011). Heating ice from
a boundary cannot raise the internal temperature beyond the melting point under any circum-
stance, and in practise will not lead to temperate ice inside a glacier (Fowler, 1984; Blatter and
Hutter, 1991). Crevasses may increase the amount of solar radiation absorbed as well as the
depth that heating from solar radiation penetrates (Pfeffer and Bretherton, 1987; Cathles et al.,
2011), but the problem of creating extensive regions of temperate ice remains.

Instead, surface melting in the accumulation zone can contribute to englacial thermal struc-
ture by forming liquid water that percolates into the snow pack, firn, or conduits such as
crevasses. Some may be entrapped and remain liquid (Lliboutry, 1976; Pettersson et al., 2004),
while the rest refreezes, releasing latent heat (Paterson, 1971; Rabus and Echelmeyer, 1997).
The rate of heat release by refreezing water is given by

\[ \frac{\partial Q}{\partial t} = -\lambda_f \frac{\partial m}{\partial t} \]  

(1.7)

in which \( \lambda_f \) is the latent heat of fusion, and \( m \) is the mass of liquid water, which diminishes over time as it freezes. This heat warms the surrounding snow, firn, and ice, creating a temperate mixture of ice and water that advects downstream as the glacier flows (Aschwanden and Blatter, 2005).

**Dissipative heating** The plastic creep of ice yields heat because of the work performed to cause deformation (Cuffey and Paterson, 2010). Work is the scalar product of force and displacement \( d \), so for a small volume within the ice, the work \( W \) done is by definition

\[ W = \sigma_{ij} \ell^2 d, \]  

(1.8)

where \( \sigma_{ij} \) is the tensor of the applied stress and \( \ell \) is the dimension of the deforming volume. This can be stated in terms of strain by substituting \( \epsilon = d/\ell \). For a volume of ice \( V = \ell^3 \), work is performed (and heat is generated) at the rate

\[ \frac{\partial W}{\partial t} = \frac{\partial Q}{\partial t} = V \dot{\epsilon}_{ij} \tau_{ij}, \]  

(1.9)

where \( \tau_{ij} \) is the deviatoric stress tensor, equal to \( \sigma_{ij} - \sigma_M \), with \( \sigma_M \) as the mean hydrostatic pressure. Also known as strain heating, this heat source is most significant where deformation rates (the velocity gradient) are the largest. As implied by the simplified glacier flow model (1.5), this is largest where \( z \) is small, i.e. near the base of the glacier (e.g. Robin, 1955). This conclusion is supported by more complex flow models (e.g. Price et al., 2007) and by borehole observations (Harper et al., 2001), indicating that it is robust, and that strain heating will be largest near the glacier base. Similar arguments can be used to show that strain heating is larger near the valley walls of an alpine glacier than near the flow line (Nye et al., 1952), or at the margins of an ice stream, where shear rates are high (Truffer and Echelmeyer, 2003).

**Geothermal heating** Similar to surface heating, the application of heat at the basal boundary cannot, in isolation, explain the existence of temperate ice in the glacier interior (Fowler, 1984).
For a cold-based glacier, the geothermal flux will contribute to the temperature gradient near the base of the glacier. Once the glacier base becomes temperate, however, geothermal heating ceases to play a role. This is because the basal temperature becomes pinned at the pressure melting point (Robin, 1955), and water at the base is presumed to join a subglacial hydrological network. Additional geothermal heat transferred to the basal ice serves only to cause melting, moving the basal boundary upward rather than contributing to englacial enthalpy. The resulting basal ablation may alter the basal heat distribution.

**Firn compaction** The compaction of low density snow and firn into higher density ice yields small amounts of heat (Cuffey and Paterson, 2010). This is because the volume change associated with densification is caused by hydrostatic pressure within the firn column, which means that work is performed according to a statement of the First Law of Thermodynamics, \( Q = P \Delta V \), where \( Q \) is heat, \( P \) is pressure, and \( \Delta V \) is a volume change. The derivation of heat sourced from firn compaction is similar to that for dissipative heating. Following Cuffey and Paterson (2010), all but the \( \epsilon_{zz} \) components of the strain tensor are taken to be negligible based on the assumption that firn compresses primarily downward, and the following calculation ignores the full strain representation. If the upper level of firn in the accumulation area has a density gradient \( \frac{\partial \rho}{\partial z} \), then as a condition for mass conservation,

\[
\frac{w}{V} \frac{\partial \rho}{\partial z} = \frac{\partial \rho}{\partial t},
\]

(1.10)

where \( w \) is the vertical advection rate. The density of a fixed mass of firn material is \( \rho = m/V(t) \). Differentiating density with respect to time,

\[
\frac{\partial \rho}{\partial t} = -\frac{m}{V(t)} \frac{\partial V}{\partial t},
\]

(1.11)

for which \( m \) is required to be a constant with time. Substituting Equation 1.11 into 1.10 and rearranging to place the volume derivative on the left,

\[
\frac{\partial V}{\partial t} = -\frac{wV^2}{m} \frac{\partial \rho}{\partial z}.
\]

(1.12)
Finally, substituting in the definition of $\rho$ and combining with the First Law of Thermodynamics,

$$\frac{\partial Q}{\partial t} = -\frac{P_{Vw}}{\rho} \frac{\partial \rho}{\partial z}.$$  \hspace{1cm} (1.13)

This provides a relation for the rate that heat is added to a volume of firn as it undergoes densification under hydrostatic pressure. The source of the heat is the gravitational potential energy that is relinquished as the firn column collapses in thickness.

1.3 Field site

The Saint Elias Mountains (southwestern Yukon, northwestern British Columbia, and southeastern Alaska) contain a wide variety of glacier types and morphologies, encompassing high elevation ice fields, outlet glaciers, piedmont glaciers, high elevation plateaus, and small valley glaciers. This variety exists due to the steep gradients in temperature and precipitation that occur across the range (Clarke and Holdsworth, 2002). The mountain complex incorporates the highest peaks in Canada, including Mount Logan (5959 m) and Mount St. Elias (5489 m). To the southwest, the Pacific Ocean acts as a moisture source, as well as a temperature mediator. Peaks such as the aforementioned define a topographical divide that causes the climate to become more continental to the north (Clarke and Holdsworth, 2002).

A shift in glacier thermal and morphological characteristics accompanies the mountainous transition from a maritime to a continental climate. Low elevation glaciers on the coastal side of the range crest tend to be temperate, while those either at high elevations or on the continental side are more likely to be subpolar in nature (Clarke and Holdsworth, 2002). A relatively large proportion of the St. Elias Range glaciers are known to surge, or undergo episodic fast flow and advance (Post, 1969; Meier and Post, 1969; Clarke et al., 1986). An understanding of the factors that control glacier surges is still developing (e.g. Kamb et al., 1985; Clarke et al., 1986; Lingle and Fatland, 2003), however glacier geometry and thermal structure both appear to be correlated with surge history (Jiskoot et al., 2000). The concentration of surge-type glaciers in the St. Elias Range is among the highest in North America (Post, 1969).
Figure 1.3: Location map for the St. Elias Mountains, with insets depicting North and South Glacier field sites.
Both the prevalence and variability of glaciers in the St. Elias Range make it a good location for glaciological study. Because of their size, glaciers in Alaska and the Yukon are worth studying to better understand topics such as how dynamics develop within large ice masses, and how large glaciers react to spatial and temporal gradients in climate. It is well-known that the effects of climate change are predicted to be amplified in high-latitude regions (e.g. Manabe and Stouffer, 1980; Moritz et al., 2002; Holland and Bitz, 2003). Although measurements vary, glaciers in northwestern North America are estimated to contribute significantly to global eustatic sea level rise (Arendt et al., 2002; Berthier et al., 2010). Future mass loss by glaciers worldwide is expected to be an important component of sea level rise through the 21st century and beyond (Radić and Hock, 2011), drawing a direct connection between glaciological study and climate change impacts. Observing how glaciers in the St. Elias Range respond to changing climate may also be useful for understanding how glaciers will respond elsewhere.

Two unnamed glaciers on the continental side of the St. Elias Range comprise the study area for this thesis (Figure 1.3). Both glaciers are at similar altitudes, stretching from 2000–3000 m above sea level. Located near the Arctic Institute’s Kluane Lake Research Station, these glaciers have been the subject of previous published and unpublished studies (e.g. De Paoli and Flowers, 2009; MacDougall and Flowers, 2010; Flowers et al., 2011; Wheler and Flowers, 2011; MacDougall et al., 2011) that focus primarily on glacier dynamics and mass balance modelling. On the basis of Clarke and Holdsworth (2002), these glaciers are expected to be polythermal, comprising a mixture of temperate and sub-freezing ice.

The first glacier (“South Glacier”) is 5 km long and has a predominantly southward-facing aspect. The glacier bends roughly 1 km from the valley head, such that the accumulation area faces northeast. Roughly 3 km down-glacier, a small tributary joins the main body from the west. The ice below this is heavily debris-covered at the glacier margin from a medial moraine, as well as rock falling from the western valley wall. Slightly down glacier, another tributary joins from the eastern side of the main glacier. The confluence is debris-covered. It is possible that this lobe has contributed a significant volume of ice in the past. It has been suggested that South Glacier is a surge-type glacier (De Paoli and Flowers, 2009), based on historical aerial photos that show that the glacier terminus underwent both major retreats and advances during the 20th century. At present, the terminus has retreated relatively far (∼1.5 km) up the valley. A year-round meteorological station called “MidMet” serves as an informal landmark located 1.6 km from the terminus. A second meteorological station called “HighMet” is 3.3 km from the
terminus. These two stations divide the glacier roughly into thirds. Annual pole displacements in the middle third of the glacier are typically in the range of 20–30 m a\(^{-1}\). There is strong evidence that much of the surface displacements in the middle portion of the glacier are accommodated by basal sliding (De Paoli and Flowers, 2009; Flowers et al., 2011). The lowermost kilometre of the modern glacier is presently stagnant. Significant summer speed-ups in flow velocity are observed. The present equilibrium line altitude is roughly 2550 m above sea level (Wheler and Flowers, 2011), which corresponds to 3.1 km from the terminus. Roughly 60% of the glacier centreline length is below the equilibrium line. Net balance is roughly \(-3.0\) m a\(^{-1}\) near the terminus and 0.5 m a\(^{-1}\) in the upper accumulation zone (Flowers et al., 2011).

The northern glacier (“North Glacier”) is about 7.5 km long, faces northwest, and has not been known to surge. The geometry of this glacier appears simpler than that of South Glacier; it fills an elongate valley without any major bifurcations, convergences with other glaciers, or noteworthy changes in aspect. There is a gap in the glacier-left valley wall, from which a small distributary glacier flows. Velocity data are more sparse both spatially and temporally than they are for South Glacier, and uncertainties are higher. Poles in the mid-glacier and upper glacier appear to move at rates in the range of 8–16 m a\(^{-1}\) (unpublished data). The eastern limit of the North Glacier accumulation zone shares an ice-covered divide with the neighbouring valley glacier. The precise location of the ice divide over the saddle to the neighbouring glacier is unknown.

1.4 Research objectives

The objectives of this thesis can be divided into two components: (1) a data collection-based investigation of thermal structure in South Glacier and North Glacier, and (2) a numerical modelling-based analysis of the observed structure. The first component involves collecting ice-penetrating radar data to supplement previously-collected surveys (Chapter 3). Analysis of these data will require the development of techniques for interpreting and consolidating the data into useful maps of glacier geometry and thermal structure (Chapter 4).

The second component is meant to complement the first by using a numerical model of heat flow within glaciers to explore the dynamics and boundary conditions necessary to describe the observations. The englacial energy balance model will be coupled to an existing flow dynamics code, yielding a fully thermomechanically-coupled model (Chapter 6). Together, these two
components are intended to lead to a fuller understanding of how polythermal structure develops within glaciers in general (Chapter 7), and South Glacier and North Glacier in particular (Chapter 8).
Chapter 2

Radar Background

An important field technique used in glaciology mapping using ice-penetrating radar. This tool is useful because it permits imaging of subglacial and englacial features. This chapter describes the important concepts behind the application of radar to common glaciological problems, and reviews frequently-used methodologies for interpreting radar data.

2.1 Physical basis

Ground-penetrating radar uses radio-wave reflections to image the subsurface. Although often applied to soils and shallow geological boundaries, radar techniques can be used effectively on glaciers because cold ice is relatively permeable to electromagnetic waves at frequencies in the megahertz range (Daniels, 2004). Temperate ice attenuates radio waves in commonly-used frequencies more strongly because of the dependence of wave transmission properties on temperature and, more importantly, the presence of water (Bogorodsky et al., 1985). Nonetheless, Watts and England (1976) show that by using low frequencies, radar is capable of successfully sounding temperate glaciers.

2.1.1 Dielectric properties of ice

The rate at which an electromagnetic wave travels depends on the dielectric properties of the medium. Dielectric permittivity ($\epsilon$) consists of a real translational part $\epsilon'$ and an imaginary dissipative part $\epsilon''$ (Bogorodsky et al., 1985):

$$\epsilon = \epsilon_0(\epsilon' - i\epsilon'').$$

(2.1)
The permittivity of free space ($\epsilon_0$) serves as a reference for relative permittivity.

Real permittivity is weakly a function of temperature, and strongly a function of frequency (Fujita et al., 2000; Petrenko and Whitworth, 2002). For very low frequencies (on the order of 100 kHz and lower), the polar nature of the water molecule dominates real permittivity (Bogorodsky et al., 1985). This occurs because there is some degree of molecular reorientation within the ice crystal lattice under the influence of an electrical field. The molecular reorientation requires thermal activation of the lattice, weakening the electrical field. At higher frequencies (up to the microwave range), molecular reorientation has a smaller effect (Petrenko and Whitworth, 2002). Polarization within the crystal lattice still occurs at high frequencies, but takes the form of a rapid redistribution of electrons (Petrenko and Whitworth, 2002). This process requires much less energy and the relative permittivity of ice in the megahertz range is low (Petrenko and Whitworth, 2002). The limiting relative permittivity at high frequencies is denoted as $\epsilon_\infty$, while the “static” relative permittivity at small frequencies is $\epsilon_s$. The transition point $f_c$ below which Debye relaxation is dominant is the characteristic frequency (Bogorodsky et al., 1985). Glaciologically-useful frequencies are higher than the characteristic frequency of ice, which is around 8 kHz at 0°C (Bogorodsky et al., 1985). Temperature also affects permittivity, but only by about 2% over the range −75°C to 0°C (Bogorodsky et al., 1985). For the relevant frequencies, it is reasonable to consider that the real permittivity of ice to be a constant near the limiting value (Bogorodsky et al., 1985; Petrenko and Whitworth, 2002; Fujita et al., 2000, e.g.):

$$\epsilon' \rightarrow \epsilon_\infty \approx 3.2.$$  \hfill (2.2)

The loss tangent $\tan \delta$ is a measure used to describe the dissipation of electromagnetic waves in a dielectric material. Bogorodsky et al. (1985) approximate the loss tangent as

$$\tan \delta = \frac{\epsilon''}{\epsilon'}.$$ \hfill (2.3)

The imaginary permittivity is an important control on the loss tangent, such that when imaginary permittivity is low, the medium is less lossy. Imaginary permittivity peaks at the characteristic frequency before dropping off (Bogorodsky et al., 1985). Although data are sparse, imaginary permittivity appears to reach a minimum around 1 GHz, before rising again into the microwave range (Fujita et al., 2000). The loss tangent for a megahertz-range wave propagating through ice is small, which accounts for the relative transparency of ice to radar. Where the imaginary
component of the permittivity is small, the velocity \( v \) of an electromagnetic wave is given by

\[
v = \frac{c}{\sqrt{\mu \epsilon}},
\]

(2.4)

where \( c \) is the speed of a wave in a vacuum and \( \mu \) is the relative magnetic susceptibility of the medium (Daniels, 2004). In ice, this \( v \) been measured to be in the range \( 1.68 - 1.70 \times 10^8 \) m s\(^{-1}\) (Fujita et al., 2000).

Reflection of electromagnetic wave energy occurs at boundaries between media that have different dielectric properties. Defining the impedance of a material as

\[
\eta = \sqrt{\frac{\mu}{\epsilon}},
\]

(2.5)

the power reflection coefficient is (Daniels, 2004)

\[
R = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}.
\]

(2.6)

In nonmagnetic materials like ice, \( \mu = 1 \) (Bogorodsky et al., 1985; Daniels, 2004), so (2.6) becomes

\[
R = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_2} + \sqrt{\epsilon_1}} = \frac{W - 1}{W + 1},
\]

(2.7)

(2.8)

where

\[
W \equiv \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_2}}.
\]

The power reflection coefficient \( R \) describes how waves are expected to reflect from interfaces between ice (\( \epsilon' \approx 3.2 \)), bedrock materials (\( \epsilon' \approx 4-16 \)), and water bodies (\( \epsilon' \approx 85 \)) (Daniels, 2004). When the reflecting material has a higher relative permittivity than the transmitting material, the sign of \( R \) will be negative, and the polarity of the reflected wave will be inverted. This can be used to infer the properties of subglacial reflectors (e.g. Arcone et al., 1995; Arcone and Yankielun, 2000; Bradford and Harper, 2005).

In contrast to ice, molecular reorganization plays a major role in water at glaciologically-
useful frequencies (Daniels, 2004). Therefore, water has a very high relative real permittivity compared to ice ($\epsilon'_w \approx 85$). The presence of small (relative to wavelength) water inclusions in ice alters the bulk permittivity, and by (2.4), the propagation velocity (Navarro et al., 2005). In some studies, this effect has been leveraged to make water content measurements (e.g. Macheret et al., 1993; Moore et al., 1999; Macheret and Glazovsky, 2000; Murray et al., 2000; Gusmeroli et al., 2010). When water inclusions are large, contrasts with the electrical properties of the ice make them detectable by radar (Jacobel and Anderson, 1987; Hamran, 2004).

### 2.1.2 Radar measurements

Measuring ice thickness is the most common use of radar in glaciological contexts (Bogorodsky et al., 1985). Both airborne and ground-based surveys have been used since the late 1950s to map the ice-bed interface by directing electromagnetic waves into the ice and measuring the delay until the arrival of the bed reflection (Bogorodsky et al., 1985; Arcone et al., 1995). Frequencies from 5–1000 MHz are commonly used for this purpose (Hubbard and Glasser, 2005). Radar data collected for bed sounding can be viewed in the context of a single sounding at one location, sometimes called an A-scope or A-scan, or as a two-dimensional array of data across multiple locations or antenna offsets, variously called a B-scope(scan), a wiggle plot, a Z-scope(scan), or a radargram (Bogorodsky et al., 1985; Daniels, 2004; Hubbard and Glasser, 2005).

Two obstacles to making accurate interpretations of radar data are noise and clutter. Noise is the random error introduced into a radar signal by environmental and instrumental factors. Because it is random, it can be reduced by averaging multiple measurements (Daniels, 2004). Clutter is non-random interference that obscures the desired signal, and cannot be reduced by simple averaging (Daniels, 2004). Subsurface multiple-reflections, valley wall reflections, or englacial water may cause clutter in a radar signal.

### 2.2 Radar applied to the study of glacier thermal structure

Radar can be applied to the study of glacier thermal structure. Two important techniques are (1) measurement of bed reflectivity and (2) imaging of internal reflection horizons. Both techniques are based on the detection of water.
2.2.1 Bed reflectivity

It is frequently hypothesized that variations in bed reflection power can be used to draw conclusions about conditions at the bed (e.g. Bentley et al., 1998; Gades et al., 2000; Jacobel et al., 2009; Pattyn et al., 2009). This technique has been applied to Antarctic ice streams. One challenge of applying this method is properly correcting the received bed reflection strength to varying ice attenuation rates. Bentley et al. (1998) infer that the bed beneath Ice Streams B and C (Whillans Ice Stream and Kamb Ice Stream, respectively) is unfrozen, while the ridge in between has a frozen bed. To correct for depth, they assume a reflected power in one part of Kamb Ice Stream, and calculate an attenuation rate based on received bed power and ice thickness. Gades et al. (2000) use the relative reflectivity of the bed beneath neighbouring Siple Dome and the Siple Ice Stream margin to make predictions about the existence of layers of water or saturated till. They introduce bed reflection power (BRP) as proportional to the sum of squares of received radar amplitude within a window around the bed reflection wavelet, normalized to the length of the window. In notation suited to the discrete nature of the data, they write,

\[ BRP \propto \frac{1}{2(t_2 - t_1)} \sum_{i=t_1}^{t_2} V_i^2, \]  

(2.9)

where \( t_1 \) and \( t_2 \) are the window boundaries, and \( V \) is the signal amplitude. In terms of sample numbers \( n_1 \) and \( n_2 \),

\[ BRP \propto \frac{1}{2(n_2 - n_1 + 1)} \sum_{i=n_1}^{n_2} V_i^2. \]

In order to correct for the englacial attenuation of radar wavelets, Gades et al. (2000) divide the measured BRP by a function that scales exponentially with travel-time. This exponential function is calculated based on the observed travel-time-to-reflection power relationship. An assumption implicit in this method and the one used by Bentley et al. (1998) is that englacial attenuation properties do not vary spatially. However, MacGregor et al. (2007) find that differences in ice temperature and impurity concentration can cause radar attenuation rates to vary widely in space.

Jacobel et al. (2009) use similar methods on Kamb Ice Stream, correcting echo intensities for depth based on an empirical method and then attempting to interpret the resulting relative intensities. In contrast to Bentley et al. (1998) and Gades et al. (2000), they recalibrate their attenuation function for different locations. They find that brightly-reflecting areas of Kamb Ice
Stream correlate with areas where water has been found in boreholes, and that dimmer areas are restricted to a slow-flowing spot around a bedrock rise. They conclude that bed reflectivity is a proxy for basal conditions.

Similar work has been performed on alpine glaciers. Copland and Sharp (2001) studied John Evans Glacier in the Canadian high Arctic. They use BRP similar to that of Gades et al. (2000), corrected for depth using an exponential gain curve. With this, Copland and Sharp (2001) attempt to infer basal hydrological conditions. On the basis of variations in BRP as well as internal reflecting power (IRP) in the lower ablation area, they predict a small region of temperate ice in the lower glacier, a somewhat larger region over which the bed is at the pressure melting point, and a region of cold-based ice in the accumulation zone. In this manner, Copland and Sharp (2001) map thermal structure consistent with a cold-climate glacier (Chapter 1). In the warm-based ablation zone, they observe a correlation between areas of high BRP and areas where a simple hydraulic potential-based flow-routing model predicts subglacial water to collect.

More recently, Pattyn et al. (2009) present radar data from McCall Glacier in northern Alaska and calculate BRP adopting the method of Gades et al. (2000) and Copland and Sharp (2001). They apply a uniform exponential depth correction, calculated by fitting a line to the BRP collected along the glacier centreline. Englacial reflections are minor throughout most of McCall Glacier. The connection between BRP and the presence of water on McCall Glacier is not clear, although there does appear to be some correlation between the zones where sliding is observed and where uncorrected BRP is high. This correlation is not as strong when comparing the depth-corrected BRP. Pattyn et al. (2009) also calculate BRP along one kilometre of the glacier’s length in both 2003 and 2005, and show that the two profiles resemble each other, implying a temporal stability to bed reflectivity.

Despite the widespread use of BRP calculations for diagnosing bed properties, Matsuoka (2011) argues that important properties of englacial attenuation are frequently disregarded. Based on a simple model of temperature-dependent attenuation, Matsuoka (2011) shows that expected variations in both geothermal fluxes and accumulation rates can cause large changes in BRP. To address this, it is recommended that care be taken to estimate spatially variable attenuation rates before making bed condition diagnoses.
2.2.2 Internal reflections

For depth sounding, internal reflections may represent clutter because they obscure the bed reflection. Internal reflecting horizons may be mistakenly identified as the bed horizon itself (Dowdeswell et al., 1984), leading to flawed ice thickness calculations. However, englacial reflections also contain information about the internal structure (e.g. Murray et al., 1997), hydrology (e.g. Jacobel and Raymond, 1984), and thermal regimes of glaciers (e.g. Björnsson et al., 1996; Pettersson et al., 2003).

Because the temperature control on ice permittivity is low (Bogorodsky et al., 1985), detecting thermal boundaries depends on the dielectric contrast between ice and water (Dowdeswell et al., 1984). Radar scattering is often interpreted to be an indicator of thermal structure (e.g. Hagen and Sætrang, 1991; Ødegård et al., 1997; Pettersson, 2005). Typically, this scattering is manifested as a dense pattern of diffraction hyperbolae or as diffuse volumes of weak reflections (Björnsson et al., 1996; Ødegård et al., 1997; Murray et al., 1997; Rippin et al., 2011). Temperate ice contains a water fraction of up to a few percent (Lliboutry, 1976; Pettersson et al., 2003), and in large enough concentrations this water causes observable englacial reflections (Bamber, 1988; Pettersson, 2005; Bradford et al., 2009). Other causes of englacial reflections, aside from water, include moraine or landslide debris, ash layers, and variations in ice acidity (Dowdeswell et al., 1984; Matsuoka et al., 2002). Contextual clues are necessary for distinguishing different sources of reflections (e.g. Murray et al., 1997).

2.2.3 Previous contributions

The observed correlation between englacial water content and radar-scattering invites a theory relating the two. Rayleigh scattering theory requires that the dominant scatterer dimension be much smaller than the wavelength of the reflected wave. For ice-penetrating radar this is not necessarily valid, because reasonable englacial targets may be as large as decimetre-scale (Harper and Humphrey, 1995; Pettersson, 2005), while typical wavelengths are on the order of meters (e.g. Ødegård et al., 1997). Citing evidence for englacial scattering from 60 MHz airborne radar surveys flown over Spitsbergen in 1983, Bamber (1988) uses Mie scattering theory from optics to describe the radar cross-section of englacial bodies. Mie scattering theory is more general than Raleigh scattering (Watts and England, 1976). As applied to radar scatter, Mie scattering solutions rely on an assumption of independent spherical scatterers, in support for
which there is limited evidence\(^1\) (Harper and Humphrey, 1995). Importantly, Mie scattering theory as outlined by Bamber (1988) and Pettersson (2005) predicts very little backscattering from water bodies with radii smaller than one-tenth the radar wavelength. Mie scattering alone precludes water-derived scattering from being observed with low-frequency radar (e.g. Hagen and Saetrang, 1991; Bradford and Harper, 2005; Brown et al., 2009), so other mechanisms must be involved.

Pettersson (2005) presents experimental results that compare the scattering efficiency of englacial reflectors, using five different antennas spanning a frequency range of 155–1150 MHz. The two lower frequency antennas are Yagi-type, while the three others are log-periodic. All five antennas were used on a transect 400 m down-glacier from the equilibrium line of Storglaciären in Sweden. Four antennas with centre frequencies from 345 MHz to 1150 MHz identified the cold-temperate transition surface (CTS) at a similar depth, while a 155 MHz antenna identified the CTS up to 10 meters deeper. Pettersson (2005) hypothesizes that the varying frequency-dependent scattering efficiencies of Mie reflectors explains the results. In some areas, the 155 MHz antenna detects the CTS at the same depth as the higher frequency antennas. This inconsistency is attributed to variations in the gradient of scatterer diameter with depth. The four higher frequency antennas have theoretical minimum scatterer sensitivity ranging from 1.2–5.0 cm, from lowest to highest frequency. Due to the agreement of results at higher frequencies, Pettersson (2005) infers that upper CTS reflectors are in the 0.05–0.1 meter range in the Storglaciären transect. Furthermore, the close agreement found using all five radar antennas in parts of the transect suggests that CTS scatterer size varies spatially, and that locally the dominant scatterer size at the transition may be larger than 0.1 meters. There is some observational evidence for englacial voids on this scale (Harper and Humphrey, 1995; Fountain and Walder, 1998).

The above discussion implies that radar frequencies below several hundred megahertz may be unsuited for detecting thermal structure. Despite this, relatively low frequencies have been successfully used to infer the existence of temperate ice in polythermal glaciers. Hagen and Saetrang (1991) employed an 8 MHz radar system to detect closely spaced reflections within several Svalbard glaciers. They note that the englacial scattering that they observe corresponds well with the measured zero-degree isotherm on one the glacier, where data exist. Although

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\(^1\)Should deviation from the independent scattering criterion be important, then the returned radar energy might be expected to be larger than that predicted by Mie scattering solutions. The scatterer geometry also has a strong effect on the result (Pettersson, 2005).
they believe that the reflections that they observe represent englacial water, they suggest that a higher frequency radar system would provide greater sensitivity to scattering in exchange for poorer depth penetration. Jania et al. (1996) note on Hansbreen that low frequency internal radar horizons described previously overestimate the thickness of the upper cold layer, as verified by borehole instrumentation.

Brown et al. (2009) tested radar using frequencies from 5 MHz to 100 MHz on Bench Glacier in Alaska, and found a nearly identical scattering horizon at every frequency. Surveys at 5 and 10 MHz used a Narod-type pulse transmitter (Narod and Clarke, 1994), while those at 25 MHz and above used a Sensors and Software 1000 V PE100A transmitter. At 5 MHz, the Mie scattering theory of Bamber (1988) and Pettersson (2005) would suggest that englacial scatterers are roughly 3 meters in size. The boundary detected by all radar configurations probably does not correspond to a transition from cold to temperate ice, because Bench Glacier is expected to be wholly temperate based on boreholes that do not refreeze. Water content varies spatially in Bench Glacier, and the region occupied by non-scattering ice is similar to that where water content is predicted to be low based on radar wave velocities (Bradford and Harper, 2005).

Gusmeroli et al. (2012) repeat cold layer thickness measurements on Storglaciären originally performed by Holmlund and Eriksson (1989) and Pettersson et al. (2003). The newer measurements are made with a 100 MHz radar instead of the 345 MHz radar in the 1989 and 2003 studies. Gusmeroli et al. (2012) find that their data compare well with direct temperature measurements from boreholes. The Storglaciären cold layer is expected to be thinning based on the previous two radar datasets, with which the new data are consistent. Gusmeroli et al. (2012) speculate that operational differences between their pulse-type radar and the continuous-wave stepped-frequency radar in the previous studies may explain their success using lower frequencies than would be expected to work according to Pettersson (2005).

A final note regarding radar sensitivity to englacial reflections pertains to reflector geometry. Matsuoka et al. (2007) show that englacial reflection strength can vary depending on polarization plane (i.e. antenna orientation), consistent with a preferred orientation for englacial conduits. When the reflecting body is elongated, its radar signature becomes strongly dependent on radar polarization. This effect is different for water-filled and air-filled conduits. Water-filled conduits return echoes most strongly when parallel to the dipole antennas, whereas air-filled conduits respond more strongly when perpendicular to the antennas (Matsuoka et al., 2007).
2.3 Survey types

Common-offset (CO) and common-midpoint (CMP) surveys are two important survey designs (Hubbard and Glasser, 2005). Common-offset surveys provide plan-view (horizontal) resolution over an area (Daniels, 2004), while common-midpoint sounding is used to estimate radar wave velocity versus depth (Annan, 2004; Hubbard and Glasser, 2005).

2.3.1 Common-offset surveys

The common-offset survey is a frequently used field survey configuration for ice-penetrating radar. In this survey geometry, the receiver and transmitter are moved in concert (Figure 2.1), so that the spacing between them remains constant (Daniels, 2004; Hubbard and Glasser, 2005). As the radar system is moved over transects on the glacier, data are acquired as traces. A collection of traces along a profile defines a radar line. In the idealized case of a broad and flat reflecting surface, the reflection at each trace comes from immediately below the midpoint between the transmitter and receiver (the nadir point). In practise, the dominant radar reflection is likely to come from some position other than the nadir point of the radar system (Welch et al., 1998; Hubbard and Glasser, 2005). This problem can be illustrated in the case of a dipping planar reflector. The energy radiated by a radar system is not focused, and the shortest reflecting path from the transmitting to the receiving antennas reflects off of a point upslope of the nadir point. This same effect causes point reflectors to appear as hyperbolae (Daniels, 2004; Hubbard and Glasser, 2005). In the case of a moving radar system over a bed with locally-uniform slope, the apparent slope $\phi$ of a continuous surface will be related to the true slope $\theta$ by

$$\tan \phi = \sin \theta.$$  \hspace{1cm} (2.10)

If variation in the reflector depth is constrained to the plane of the radar line, results from neighbouring soundings can be used to invert for the true geometry. The orientation of the radar transects affects the nature of off-nadir reflections. In a generalized glacial valley depth variation is assumed to be largest across valley, and off-nadir reflections are more likely to derive from directions orthogonal to the valley plunge direction (Bauder et al., 2003).

To avoid spatial aliasing, the spacing between consecutive radar observations along the line should be no greater than $\lambda/4$, where $\lambda$ is the wavelength of the radar wave (Welch et al.,

\footnote{See Appendix A.}
Figure 2.1: Common-offset survey geometry. The transmitter (Tx) and receiver (Rx) are moved together across a profile. Measurements are taken in multiple locations (subscripts 1–3), and the antenna spacing remain constant.

1998). The same requirement also applies to spacing between adjacent lines. Such a dense grid of radar traverses is usually impractical, and this requirement is often relaxed (e.g. Bauder et al., 2003).

2.3.2 Common-midpoint surveys

Some knowledge of wave propagation velocities through the substrate is necessary to interpret data collected during common-offset surveys; velocity cannot in general be determined from the data directly\(^3\). Typically, values in the range $1.68 - 1.70 \times 10^8$ m s\(^{-1}\) are assumed, however velocity can be determined directly by using one of two survey types (Figure 2.2): the wide angle reflection and refraction survey and the common-midpoint survey (Hubbard and Glasser, 2005). Both operate by increasing the relative distance between the transmitter and receiver and compiling observations with multiple offsets (Annan, 2004). The distinction between these two survey types is that the former moves only one of the two antennas and requires the assumption of a planar reflector because the survey midpoint shifts, whereas the latter holds the survey midpoint in place by moving the two antennas symmetrically. The wide angle reflection and refraction survey inherits its name from the analogous seismic method, in which refraction of seismic waves through multiple depth strata may occur. The common-midpoint survey is a better choice where the reflector may be sloping (Hubbard and Glasser, 2005).

As the offset between the transmitter and receiver antennas increases, the ratio of the distance travelled by the directly-transmitted wave (air wave) and the reflected wave shrinks. Provided that either the triggering time or the air wave velocity are known, the wave propagation

\(^3\)If a point reflector is imaged in the radar line, wave velocity can be determined from the diffraction hyperbola (e.g. Moore et al., 1999; Rippin et al., 2011)
velocity through ice can be determined (Hubbard and Glasser, 2005). Eisen et al. (2002) use dielectric profiling to independently determine electromagnetic wave propagation velocity and show that common-midpoint surveys provide a good measurement of velocity.

2.4 Review of processing methods

2.4.1 Bed reflection filtering

Much of the analysis that can be done using radar data requires accurate determination of bed and englacial reflection arrival times. The arrival time of the bed reflection wavelet may not be clear from raw data due to numerous sources of noise and attenuation that affect a radio wave moving through a glacier. The electromagnetic wave power decreases as a result of transmission from the radar antenna into the ice, geometric spreading as the wave front moves outwards from the antenna, target scattering, and material attenuation (Daniels, 2004). The signal recorded at the receiving antenna may also be subject to random or nonrandom influences, such as cosmic interference, instrument drift, or other radio transmissions (e.g. Murray et al., 1997). Noise may be reduced by averaging traces at acquisition time (stacking), but post-processing is often carried out prior to radar interpretation (e.g. Arcone et al., 1995; Murray...
et al., 1997; Bauder et al., 2003). The goal of signal processing is to maximize sought-after components within the signal while minimizing noise and clutter (Daniels, 2004). In the case of depth-sounding, this means enhancing the bed reflection while discarding englacial features and random noise, and other radar clutter unrelated to the bed. While mapping englacial structures, the appropriate processing strategies are different.

Gain control serves to correct for natural sources of wave power attenuation, such as geometric expansion and transmission loss (e.g. Eisen et al., 2009). Like many tools used to process radar data, gain control filters are adapted from analogues used in seismic data processing. Because of these natural losses, reflectors that occur later within a radar trace return less energy than those earlier within the trace. The distortion resulting from a gain control filter attempts to either restore the physically correct power to later parts of the radar trace, or to exaggerate deeper regions to assist in interpretation. Both linear and nonlinear filters are possible. As an example of linear gain control, Claerbout (1985) suggests multiplying a time-series of seismic reflection data by $t^2$ and gives a physical justification. Transmission loss is low for radar waves in ice and distances are relatively short, so a factor of $t$ may be better for radar (A. Calvert, personal communication). Nonlinear alternative techniques may employ exponential time functions, power laws, or automatic gain control algorithms that normalize the integrated power within a time window to some constant (Claerbout, 1985; Murray et al., 1997; Bauder et al., 2003).

Filters that operate based on frequency criteria are used extensively within radar processing (e.g. Murray et al., 2000; Gades et al., 2000; Matsuoka et al., 2007). These filters may be used to remove particular frequency components. Relevant categories of reducing filters are those that are applied in the time-domain, and those that are applied in the frequency domain (Smith, 1997). Time-domain filters, such as simple moving averages, are effective for removing high-frequency noise, or, by spectral inversion, very low frequency drift. In the latter application, they are often termed “dewow” or “detrend” filters within the radar literature (e.g. Eisen et al., 2009; Gusmeroli et al., 2010). Moving average filters are considered to be time-domain filters because they have optimal or very good capacity for removing (or preserving) noise while retaining (or discarding) changes in signal amplitude as a function of time. In contrast, they have slow or irregular behaviour in the frequency domain, making them poor for separating signals by frequency (Smith, 1997).

Frequency-domain filters are more effective for frequency separation. Such filters can either
be applied in Fourier space using a Fast Fourier Transform, or with an equivalent convolution in the time-domain (Smith, 1997). A simple convolution kernel for a frequency domain filter can be designed by transforming a Fourier space step function into the time domain. This results in a sinc function, \( y(t) = \sin \frac{t}{t} \). For practical reasons, the infinite-domain sinc function is truncated and tapered by multiplication against a windowing function, the width \( w \) of which depends on the required transition bandwidth (Smith, 1997). The Blackman windowing function is frequently used, because it provides good attenuation outside of the desired frequency range (stopband). This function is

\[
b(t) = \frac{1 - \alpha}{2} - \frac{1}{2} \cos \left( \frac{2\pi t}{w} \right) + \frac{\alpha}{2} \cos \left( \frac{4\pi t}{w} \right),
\]

where \( t \) is time within window width \( w \) and \( \alpha \) typically equals 0.16 (Smith, 1997). The resulting kernel provides a “windowed-sinc” filter. When convolved with a signal, this filter has a better frequency-domain response, at the expense of introducing ringing artifacts in the time-domain.

A relative of the windowed-sinc filter, the Butterworth filter, is widely used in radioglaciology (e.g. Murray et al., 2000; Gades et al., 2000). The primary advantage of the Butterworth filter over a windowed sinc filter is computational speed (Smith, 1997).

### 2.4.2 Migration

In general, migration is a set of related techniques used to convert the time axis of radar data into a depth axis (Claerbout, 1976). The term migration is often used to specifically refer to an approximate solution of the wave equation to reconstruct subsurface geometries. Migration is commonly-used within seismology and the petroleum industry (Claerbout, 1985), and also has applications in glacier radar studies (e.g. Harrison, 1970; Rabus and Echelmeyer, 1997; Bauder et al., 2003; Bradford and Harper, 2005).

Radar data can be visualized by displaying adjacent traces as columns. With unmigrated data, radargrams can be misleading because they represent the timing of reflection arrivals at the receiving antennas, rather than the actual reflector geometry. If the reflecting surfaces are flat and of effectively infinite width, then the vertical axis of the radargram is equivalent to a space axis multiplied by a scaling factor based on wave velocity. In the more likely case that the reflectors are neither flat nor infinite, off-nadir reflections are visible in the radargram. This affects the unmigrated radargram in two primary ways. In the first way, hyperbolae form around point reflectors and other laterally discontinuous reflectors. These reflections are often referred
to in the literature as diffraction hyperbolae (e.g. Rippin et al., 2011). The second effect is to change the apparent slope of dipping reflectors, such that

$$\tan \phi = \sin \theta$$

(2.12) for apparent dip angle $\phi$ and true dip angle $\theta$, as given by the well-known migrator's equation$^4$ (Stolt, 1978; Scales, 1995; Upadhyay, 2004). This apparent dip effect can cause U-shaped glacier valleys to appear V-shaped in raw data (Bauder et al., 2003). In valley glaciers, steep subglacial surfaces increase the likelihood of significantly off-nadir reflections (Welch et al., 1998).

Migration corrects for these effects by collapsing hyperbolae around their reflectors and restoring correct dip angles. Numerous techniques for migration exist within geophysics, each with a set of trade-offs in complexity, assumptions, and performance. Rabus and Echelmeyer (1997) employ a simple visual scheme that relies on the fact that possible surfaces from which a wave may reflect form an ellipse with the transmitting and receiving antennas at the foci. After generating ellipses for every point along a survey line, the bed surface is taken to be the envelope of the overlapping ellipses. Pattyn et al. (2009) find that this migration technique improves the self-consistency of their ice-thickness measurements. On the other hand, Bauder et al. (2003) report that this method is potentially misleading, because having too few data points or terrain that is too steep causes the ellipse envelope to underestimate the actual ice thickness, judged from borehole lengths. They find that a phase-shift migration scheme achieves better results.

Kirchoff migration is a numerical scheme that sums reflected power along predicted diffraction paths. It is simple to understand and apply, and easily adapted to complicated subsurface velocity structures. Kirchoff migration fails when the density of reflectors is high enough to alter the bulk permittivity of the ice. Additionally, Kirchoff migration may not constructively sum diffraction patterns in the case of complex geometries (Arcone et al., 1995). This latter tendency may result in the inadvertent attenuation of reflections.

Finite difference methods based on an exploding reflector model have been successfully used (e.g. Claerbout, 1971; Claerbout and Doherty, 1972; Claerbout, 1976; Stolt, 1978). In the exploding reflector model, every point on the surface of a subsurface reflector emits a wave

$^4$See Appendix A.
pattern simultaneously at time zero. This wave obeys the Huygens principle, and expands in all directions. The finite differencing technique steps the observed pattern backwards in time based on the scalar wave equation to recover the initial geometry (Claerbout, 1976). Stolt (1978) extended this method by transforming the horizontal spatial dimension into the Fourier domain. This modification improves the dispersion characteristics of the migration when reflectors dip steeply (Stolt, 1978).

Frequency-wavenumber (F-K) migration, also known as Stolt migration (Yilmaz and Doherty, 2001), solves the scalar wave equation in a two-dimensional frequency-wavenumber domain (Stolt, 1978). In the frequency-wavenumber domain the solution to the wave equation becomes a multiplication by a scalar function (Stolt, 1978). The transformed data are then restored to the spatial domain with inverse transforms. This method is relatively fast, and works well with steeply-dipping reflectors. Varying velocity structures are difficult to account for within the migration itself, and are instead typically dealt with by stretching the domain and assuming constant velocity (Claerbout, 1985).

In order to achieve the best result possible, Welch et al. (1998) use two-pass migration to approximate a three-dimensional migration. They show that the maximum horizontal resolution obtainable by a single trace is defined by the first Fresnel zone, which is the area on a flat surface that contributes to a radar reflection at a given frequency. Migration is used in this context to focus the image beyond the limit imposed by the Fresnel zone, reaching a depth-independent maximum resolution of $\lambda/2$ for the frequency-wavenumber migration used, where $\lambda$ is wavelength (Welch et al., 1998).

### 2.4.3 Processing for bed detection

The processing decisions made for radar data depend on characteristics of the data and the questions to be answered. For this reason, there is no standard processing suite used prior to ice radar interpretation. Several examples help to clarify what level of processing is typically used in different studies.

To process bed reflection data, it is common to apply a dewowing filter, gain control, and a passband filter, the precise characteristics of which vary from study to study depending on the nature of the data and the software available (e.g. Murray et al., 1997; Bauder et al., 2003; Murray et al., 2007). For their 100 MHz radar data collected with a commercial radar system, Murray et al. (1997) first remove duplicated traces and interpolate over missed traces, after
which they apply a dewowing filter. Next, they use a piecewise gain control algorithm that consists of an automatic gain control filter prior to 1000 ns\(^5\) in each trace, and a linear correction afterwards. Finally, they use a relatively wide bandpass filter centred around their nominal frequency. Bed slope corrections are applied using both Kirchoff and frequency-wavenumber migration routines. Bauder et al. (2003) use a simpler set of processing steps to calculate ice thickness of an alpine valley glacier. They use automatic gain control, followed by unspecified band pass filtering to eliminate noise. Finally, they migrate their data by solving the downward continuation problem in the Fourier domain. Murray et al. (2007) use similar steps in a slightly different order. They apply frequency-wavenumber migration immediately after the dewowing filter, after which they use a much lower frequency bandpass filter, with a flat pass band between 2 and 10 MHz for mixed 50 MHz and 100 MHz data. They then use an instantaneous frequency transformation to highlight low frequency signal components.

2.4.4 Englacial reflection processing

Internal reflectors are chosen for enhancement less frequently than bed reflectors. Older radio-echo sounding radar systems used for measuring the thickness of continental ice sheets are difficult to apply to englacial reflections because of scattering problems and poor vertical resolution. Monopulse-style radar systems offer greater potential for englacial imaging compared to older radio-echo sounding systems (Arcone et al., 1995; Murray et al., 1997).

The techniques used to identify englacial scattering are somewhat poorly-developed. Multiple authors have described englacial scattering as overlapping hyperbolae or as dense collections of point scatterers (e.g. Bamber, 1988; Hagen and Sætrang, 1991; Rippin et al., 2011). Murray et al. (2007) note that in some areas, scattering is associated with discrete hyperbolae, but not in others, and they assume that where hyperbolae are visible, water body sizes are larger.

The literature on processing radar data in order to enhance englacial features is less prolific than that for basal surface detection. Band-pass filtering remains a potential tool for its ability to attenuate frequencies associated with random noise or system drift. In addition to this, gain control techniques are helpful, just as for bed reflection applications. Because the backscattering caused by englacial targets like water at the cold-temperate surface usually lacks independent diffraction patterns, migration is not an obvious tool to use (Eisen et al., 2009), however it has

\(^5\)A 1000 ns window is roughly equivalent to 80 meters depth.
been employed in some studies (Murray et al., 2007). Gusmeroli et al. (2012) found that migration did not change the interpreted depth of the cold-temperate surface observed in radar data, and did not generally apply it. Murray et al. (2007) describe a processing workflow in which they simply apply a constant gain correction function following use of a dewow filter and migration, foregoing any further bandpass filtering.

2.5 Depth calculation

Interpretation of radar data follows raw data processing (if any), and often involves ice thickness calculations and the development of bed topography maps (e.g. Herzfeld et al., 1993; Flowers and Clarke, 1999; Bauder et al., 2003). This gives rise to two additional steps: the identification of event wavelets for travel-time and depth calculations, and the interpolation of these results across regions without radar soundings.

2.5.1 Event picking and phase analysis

An idealized radar echogram for ice thickness sounding consists of two direct coupling waves, an air wave and a ground wave, and a basal reflection. The air wave is the first of the direct coupling waves to reach the receiver, travelling at $3.0 \times 10^8$ m s$^{-1}$. The ground wave represents the radio wave that travels through the ice, snow, or firn directly from the transmitter to the receiver, and is of opposite polarity to that of the air wave (Arcone et al., 1995). It is expected to arrive later than the air wave because the speed of wave propagation through the surface is generally slower. Internal reflections may occur when radar waves encounter reflectors before the bed. The basal reflection consists of energy that travels downward from the transmitter, reflects at the dielectric boundary represented by the interface between the basal ice and subglacial material, and returns upward to the receiver.

In practice, various other effects play a role in creating the waveforms visible on a radar trace. The transmitter–receiver separation may be sufficiently small such that the time difference between the air wave and ground wave arrivals is too short to prevent interference between for the two waves. Additionally, the air wave creates a subsurface head wave, and the ground wave introduces an inhomogeneous surface wave (Annan, 1973). These add clutter to the direct coupling waves. Internal reflections may make it difficult to identify the bed reflection.

For each radar sounding incorporated into a bed map, the arrival time of the bed reflection
wavelet must be determined (picked) (e.g. Gades et al., 2000). The arrival time of the directly-transmitted air wave (direct coupling) may also be measured for use as a timing reference. A reference point on each wavelet must be chosen for picking. A common choice is the “first break” in energy for both the direct coupling and the reflected wave (e.g. Murray et al., 2000; Stuart et al., 2003; Rippin et al., 2011) (Figure 2.3). An advantage of this choice is that it reduces uncertainty related to polarity reversals when scattering is low by identifying only on an increase in signal amplitude, rather than an expected wavelet polarity.

Figure 2.3: Illustration of picking the first break of event wavelets. DC refers to the direct coupling, which is the superposition of the air wave and ground wave. Because radio waves travel faster through air than ice, the first break of the DC wave is synonymous with that of the air wave. Data shown here were collected on South Glacier in May 2011 using 10 MHz antennas (Chapter 3).

Phase analysis can be used as a guide to identify the bed reflection\(^6\). As illustrated by equation (2.7), when the dielectric permittivity of the reflecting material is much larger than the transmitting material, the reflection coefficient will be negative, corresponding to the reflected wave having a polarity opposite that of the incident wavelet (Arcone et al., 1995). Because of the low real permittivity of ice, virtually all likely reflecting surfaces will have this relationship\(^7\) (Daniels, 2004), so the reflected wave can be expected to exhibit a polarity reversal. The data presented by Narod and Clarke (1994) show that the polarity of the direct ground wave is reliably opposite to that of the direct air wave, so the polarity of the air wave can be used as a reference for that of reflected waves. A reflection with the same polarity as the air wave results from a low-to-high dielectric boundary (Arcone et al., 1995). The polarity of the return wave should be inverted relative to the down-going wave and the ground wave, and the same as the air wave. Returns with polarity in opposition to this expectation may suggest an empty cavity.

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\(^6\)Using wavelet polarity as a tool in radar analysis has been called both phase analysis (e.g. Arcone et al., 1995; Arcone, 1996; Murray et al., 1997; Moore et al., 1999) and polarity analysis (e.g. Moorman and Michel, 2000; Bælum and Benn, 2011).

\(^7\)Dry rock interfaces may be dielectrically similar to ice (Daniels, 2004). In this case, the polarity of returned wave might not reverse, and the amplitude would be much weaker.
2.5.2 Kriging interpolation

Spatial interpolation of radar observations falls into a broad class of geospatial interpolation problems. It is often useful to make predictions about variables in locations were there are no data (Cressie, 1993). For many problems, it intuitively makes sense to guess a value that is similar to neighbouring values. There are a number of schemes that can be used to accomplish this, including nearest-neighbour estimation, linear interpolation, and spline models. One technique that has been extensively used within the earth sciences is kriging (Cressie, 1993). Kriging was originally developed to predict the distribution of mineral deposits, but has been successfully applied to a broad range of spatial interpolation problems (Carr, 1995).

Kriging is a statistical interpolation scheme that predicts the value of a dependent variable at a point in space by weighting a set of nearby observations based on their distance from the prediction location and a variogram (Carr, 1995). A variogram is a distance-dependent model of variance in a spatially-dependent variable. An empirical variogram represents the measured data, typically by plotting variance as a function of distance (lag). From this, a mathematical model variogram that captures the important characteristics of the empirical variogram is constructed (Kitanidis, 1997). These characteristics include the rate of increasing variance with distance, the maximum variance (sill), and anisotropy in the observations. The variogram is modelled using mathematical functions, such as Gaussian or spherical functions (Kitanidis, 1997). A positive variance at zero lag, called a nugget effect, can be added to represent measurement error as white noise (Cressie, 1993). Without a nugget effect, kriging is an exact interpolator (Cressie, 1993). Different model variograms can be used to represent the variance as lag increases in different directions. For example, Flowers and Clarke (1999) use eight separate variograms to model ice thickness data from Trapridge Glacier, Canada.

The kriging prediction is made by computing weights that minimize kriging variance and applying them to the observed data (Cressie, 1993; Kitanidis, 1997). Kriging variance is the product of the optimized weights and the variogram. Then, the kriging problem can be expressed as a system of linear equations, for which the solution is a vector of weights (Kitanidis, 1997; Cressie, 1993). The weights are computed using Lagrange multipliers (Cressie, 1993). For computational efficiency (Cressie, 1993), the kriging prediction is usually made using a small number of observations from a limited search radius (e.g. Herzfeld et al., 1993; De Paoli, 2009).
Several varieties of kriging are available, including simple kriging, ordinary kriging, and universal kriging (Cressie, 1993; Kitanidis, 1997). Simple kriging makes the assumption that the data have a zero mean. Ordinary kriging relaxes the assumption to a constant mean. Universal kriging relaxes this further to assume that the data mean is a linear combination of known functions (Cressie, 1993). This permits kriging to be applied to data with a spatial trend.

Despite the statistical basis for kriging, there are several criticisms (e.g. Philip and Watson, 1986; Journel, 1986) and caveats related to the application of the technique. Any interpolation scheme will be sensitive to qualitative decisions about which data are retained or thrown out. Kriging also depends on the choice of parameters relating to search strategy, degrees of anisotropy, model variogram construction, and stationarity. Should assumptions about any of these be untrue, the interpolation will suffer. For example, Herzfeld et al. (1993) investigate the influence imposed by changing various kriging parameters while developing a bed map of Storglaciären. They find that choices relating to the local search window are of particular importance, although in their study, the search radius terminates significantly before the sill of their model variogram for computational efficiency.

Some of these choices may be arbitrary or subjective. Selecting a smaller search radius is permissible if the limited search neighbourhood statistically approximates the full dataset (Cressie, 1993). Flowers and Clarke (1999) report that their interpolation is insensitive to small changes in kriging parameters, but that it is important for the variogram model to be generally representative of the data.
Chapter 3

Field Data

In this chapter the data and data acquisition methodologies are described. These data include both ice-penetrating radar (GPR) data collected over four years on South Glacier and North Glacier, as well as borehole temperature profiles measurements made using temperature acquisition cables (TACs). Chapter 4 describes data processing techniques.

3.1 Radar

3.1.1 Instrumentation

Data were collected using a radar system comprising an impulse-type transmitter in the design of Narod and Clarke (1994) and a hardware digitizer used as part of a receiver system (Figure 3.1). The digitizer is connected to a netbook computer, allowing the collection and storage of data as described by Mingo and Flowers (2010). The antennas are resistively-loaded dipoles with 4-6 resistors. Antennas with nominal centre frequencies of 10 MHz, 35 MHz, and 50 MHz were used. Some antennas were supplied by Icefield Instruments Inc., while I built additional sets. Antennas are based on the design described by Wu and King (1965) and Shen and King (1965), and were constructed following the assumptions of Jones (1987). Specifically, resistors are chosen to fit an impedance profile $Z(x)$ defined by

$$Z(x) = \frac{R_0}{h - x}, \tag{3.1}$$

where $x$ is position along an antenna arm with length $h$, and $R_0$ is a parameter that describes both the intrinsic impedance of the propagating medium, as well as characteristics of the current
carried along the antenna. Antenna length \( h \) is set to be one-quarter the wavelength of the desired radar wave. Following Jones (1987), \( R_0 \) is assumed to be 300 \( \Omega \), although this may not be optimal for all frequencies or environments. Antennas that I built use four resistors per arm at 82\( \Omega \), 120\( \Omega \), 200\( \Omega \) and 620\( \Omega \) positioned according to (3.1). Antennas assembled by Icefield Instruments Inc. use 5–6 resistors per arm, however the performance difference relative to a 4 resistor antenna is negligible (B. Narod, personal communication).

\[ \text{Figure 3.1: Schematic diagram of the radar system, divided in transmitter and receiver components.} \]

<table>
<thead>
<tr>
<th>Component</th>
<th>Type</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitter</td>
<td>Narod-style impulse</td>
<td>Delivering 1.1 kV pulses at 512 Hz</td>
</tr>
<tr>
<td>Digitizer (2008, 2009)</td>
<td>NI-5133</td>
<td>50 MHz bandwidth, 100 MHz sampling rate</td>
</tr>
<tr>
<td>Digitizer (2011)</td>
<td>PicoScope 4227</td>
<td>100 MHz bandwidth, 250 MHz sampling rate</td>
</tr>
<tr>
<td>Netbook</td>
<td>ASUS EeePC 900</td>
<td>Connected to digitizer and GPS by USB</td>
</tr>
<tr>
<td></td>
<td>ASUS EeePC 901</td>
<td></td>
</tr>
<tr>
<td>On-board GPS</td>
<td>Rikaline 6017</td>
<td>Manufacturer-reported accuracy of &lt;10 m</td>
</tr>
<tr>
<td>Hand-held GPS (2011)</td>
<td>Magellan Triton 300</td>
<td>Manufacturer-reported accuracy of 3–5 m</td>
</tr>
</tbody>
</table>

\[ \text{Table 3.1: Hardware components of the ice-penetrating radar system.} \]

Power is supplied from a combination of 12 V lead-acid batteries and 14.4 V lithium-ion batteries\(^1\). During operation, the transmitter repeatedly feeds voltage impulses into the transmitting antennas, while the receiver records averaged (“stacked”) observations at regular time intervals. This averaging operation is intended to improve signal-to-noise ratio (Daniels, 2004). The digitizer component of the receiver is set to automatically record a single (unstacked) interval of

\(^1\)Prior to 2011, only lead-acid batteries were used
data whenever it detects a waveform that satisfies a triggering requirement. This triggering requirement is a threshold voltage chosen during instrument operation such that the direct coupling wave triggers a recording, but other reflections do not. This allows all data to have the same time reference (the air wave) so that they can be stacked and saved.

Prior to 2011, the transmitter was pulled on a sled while the receiver was carried by an operator. In 2011, the radar configuration was modified such that two separate sleds carry the transmitter and receiver with the antennas strung between them. The sleds are pulled by operators travelling on foot. A third configuration was tested in the spring of 2011, in which the receiver and transmitter are mounted on a single sled with the receiving and transmitting antennas alongside. Because of unrelated antenna problems stemming from incorrectly chosen resistors, this configuration was never used to collect any of the data described.

A small GPS receiver designed for use in vehicles is mounted alongside the receiver (Figure 3.1b). The geographical position is recorded every time a radar observation is stored. Although elevation is stored alongside horizontal position, for all data analysis elevations are taken from digital surface elevation models for South Glacier and North Glacier based on higher quality kinematic GPS data.

### 3.1.2 Data acquisition

In April–May 2008, 2009 10 MHz radar data were collected on both South Glacier and North Glacier. More data were collected at 10 MHz in May 2011. Surveys with higher frequency antennas were planned for May 2011, but problems with the antennas prevented their use. After resolving these problems, 35 MHz and 50 MHz data were collected in July–August 2011, in addition to more 10 MHz data in particular locations. The full dataset from 2008–2011 covers large portions of both glaciers with data collected at 10 MHz, as well as the lower half of South Glacier with data collected at higher frequencies. A small but dense grid of radar data were collected at 35 and 50 MHz on North Glacier surrounding a borehole temperature cable installed in August 2011.

In addition to the data described above, a small radar sounding dataset (<600 traces) was collected in 2006 using a different set of instruments. These data were included in the interpolated bed map by De Paoli (2009). Rather than using a hardware digitizer to collect tightly-spaced radar traces, these data were collected with an oscilloscope at more widely-spaced intervals. The wide spacing makes it more difficult to correlate reflections between traces, re-
resulting in significant excursions between different reflectors.

### 3.1.3 Common-offset surveys

Common-offset surveys comprise most of the radar data and have been performed both across and along glacier flow (Figure 3.2, 3.3). The data are organized into lines and traces. Radar surveys are made up of lines, which are segments composed of consecutive traces. Lines typically represent transects across some portion of the glacier. Traces are soundings that are stacked at the time of acquisition, and represent the radar returns from a single location. As a convention for describing common-offset radar lines, longitudinal lines are defined as those with a general trend within 45° of the direction of the flow-parallel centreline at the point nearest the line’s centroid. When the trend deviates from the centreline direction by more than 45° degrees, it is referred to as a transverse line.

Traces are recorded at a constant frequency, typically chosen to be in the range of 0.2–0.33 Hz. The spatial resolution of sampling depends on the speed at which the survey moved. In 2008, the spatial resolution is roughly one trace per 8 m travelled. In 2009, sampling is more dense, and is roughly one trace per 5 m travelled. In 2011, sampling resolution is approximately one trace recorded per 4–5 m.

<table>
<thead>
<tr>
<th>Year</th>
<th>Glacier</th>
<th>Lines</th>
<th>Total distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>South</td>
<td>46</td>
<td>30.1 km</td>
</tr>
<tr>
<td>2009</td>
<td>South</td>
<td>19</td>
<td>11.4 km</td>
</tr>
<tr>
<td>2011</td>
<td>South</td>
<td>103</td>
<td>56.6 km</td>
</tr>
<tr>
<td>2008</td>
<td>North</td>
<td>12</td>
<td>13.8 km</td>
</tr>
<tr>
<td>2009</td>
<td>North</td>
<td>17</td>
<td>13.9 km</td>
</tr>
<tr>
<td>2011</td>
<td>North</td>
<td>40</td>
<td>29.5 km</td>
</tr>
</tbody>
</table>

Table 3.2: Summary of common-offset data collected between 2008 and 2011. To reduce the effect of GPS noise on distance calculations, positions have been smoothed with respect to time.

Three sets of repeated 10 MHz radar lines were collected on May 6th, July 10th, and July 25th of 2011 across three transects on South Glacier, referred to as the upper, middle, and lower lines (Figure B.1). The upper line crosses the glacier roughly 1.6 km from the terminus, below the MidMet meteorological station. The middle line crosses below this, in the vicinity of

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2In subsequent diagrams, longitudinal lines are plotted with the down-glacier end on the left, and the up-glacier end on the right. Transverse lines are plotted as if facing down-glacier (with flow vectors pointing into the page).

---
Figure 3.2: Map of South Glacier radar data. Small black dots indicate common-offset radar traces collected in the period 2008–2011 at (a) 10 MHz (b) 35 MHz, and (c) 50 MHz. The locations of common-midpoint surveys and instrumented boreholes are labelled in (a).
Figure 3.3: Map of North Glacier radar data. Black dots indicate common-offset radar traces collected in the period 2009–2011. The acquisition locations of data collected using 10 MHz antennas are represented in (a), while (b) shows an enlarged view of the more limited dataset collected at higher frequencies in the mid-glacier region.
Figure 3.4: Example of unprocessed common-offset data, collected from South Glacier in May 2011 using 10 MHz antennas. The vertical dimension is raw digitizer time. The horizontal dimension is trace number. The bright horizontal bar at 400 ns represents the direct coupling between receiver and transmitter. The strong irregular line below is the bed reflection. The weaker hyperbolae above the bed reflection are the signals from englacial reflectors. The occasional periods of redundant traces occur when the radar system continues to collect data while stationary (most obvious for traces 0-80).

an ablation stake known as H02 and roughly 1.3 km from the terminus. The lower line crosses the glacier at the lowest elevation, near an ablation stake known as L02 and 0.7 km from the terminus. This lower line was not repeated during the July 10th survey because of problems related to the on-board GPS unit. For all repeated surveys, the 10 MHz antennas were used, the antenna spacing was nominally 12 m, and the receiver was configured so as to not clip high-amplitude peaks from the direct coupling waves. Because of seasonally-changing terrain and limited navigational precision, the radar lines are not identical. Based on the saved GPS locations, repeated lines are typically within 15 meters of each other, however they sometimes differ by more than 30 m.

Numerous GPS problems afflict the common-offset surveys. The first nine radar lines collected in 2008 contain GPS data inadvertently collected in “static mode,” meaning that the position reported by the GPS unit does not change unless the instrument detects movement above a large threshold\(^3\). These lines were discarded from the dataset, and the area represented by these nine lines from 2008 was covered again in 2009. The static GPS problem reappeared in 2011 in the spring, and again in the summer. These data were corrected using methods described in Section 4.2. In summer of 2011, GPS tracks were recorded using a hand-held GPS unit (Table 3.1) in addition to the on-board radar GPS for redundancy.

\(^3\)For similarly-problematic data collected in 2011, this threshold is 25 m. For the 2008 data, the threshold does not appear to be consistent.
3.1.4 Common-midpoint surveys

In addition to common-offset surveys, two common-midpoint surveys were conducted on South Glacier in July 2011, hereafter referred to as CMP01 and CMP02. Each survey consists of a single line, which contains a number of stacked soundings. CMP01 is located slightly west of the MidMet meteorological station, while CMP02 is slightly up-glacier and above the confluence with the western tributary. These sites were chosen because the glacier has a low dip angle in these areas which simplifies data analysis and because the expected thermal structure differs at each location based on scattering in common-offset data. CMP01 overlies both radar-transparent and radar-scattering ice, while ice beneath CMP02 exhibits radar-scattering through the entire ice thickness. Prior knowledge of ice thicknesses in these areas suggest that the bed is roughly 80 m below the surface at CMP01 and 70 m below the surface at CMP02.

![Common midpoint radargrams CMP01 (a) and CMP02 (b), collected from South Glacier at the locations indicated in Figure 3.2. The linear features that arrive first represent the air wave (AW) and ground wave (GW), respectively, and are delayed by a constant rate as antenna offset increases. The hyperbolic features that appear further down represent englacial and basal reflectors.](image)

For each survey, the initial 10 MHz radar traces were recorded with an antenna offset of 8 meters. Subsequent offsets increase by 4 meter intervals in CMP01 and 2 meter intervals in CMP02. Triggering parameters occasionally needed to be adjusted because of the increasing antenna offset. After experimenting with the triggering parameters until triggering occurred reliably at each offset, several traces were recorded.

The common-midpoint survey data are shown in Figure 3.5, with traces for each offset.
averaged together. Midpoint locations are shown in Figure 3.2. The air wave and ground wave components of the direct coupling, which overlap at small offsets, can be seen to separate into distinct wavelets at large offsets. In both of the common-midpoint surveys, a prominent reflector interpreted to represent the glacier bed is visible between 70 and 80 meters depth. Weak englacial reflectors are visible in CMP01, while significant scattering in a lower layer in the glacier and multiple englacial reflectors are imaged in CMP02.

3.2 Ice temperature

3.2.1 Instrumentation

Temperature acquisition cables (TACs) were constructed by Beaded Stream LLC with various customized lengths and sensor spacings and subsequently installed on both glaciers in the period 2008–2011. The sensors are individually addressable from the upper end of the cable through a 1-Wire® bus accessed from a serial interface. This interface permits communication with the cables either by programmable Campbell Scientific CR1000 data loggers, or by a hand-held computer supplied by Beaded Stream LLC. Using CR1000 data loggers provides time series rather than a single measurement, because the data logger can be installed in an enclosure and left in the field. The hand-held data collector is portable, and can be used during site visits and for debugging the programs installed on the CR1000s.

The 1-Wire® sensors that report temperatures along the lengths of the temperature cables have a manufactured accuracy to within 0.5°C (Maxim Integrated Products, 2008). Subsequent calibration improves the accuracy to 0.05°C (B. Shumaker, personal communication). Accounting for instrument drift and other sources of error, Beaded Stream LLC estimate temperature measurements to be accurate within 0.1°C.

Each cable has a hard-coded serial number chosen by the manufacturer. For clarity, cables are referred to here using a naming scheme that incorporates the glacier name, a location keyword, the cable length, and the year installed (Table 3.4). For example, the 15 meter cable installed in 2011 near MidMet on South Glacier is referred to as SG-MIDMET15M-2011.

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4South Glacier is SG and North Glacier is NG.
3.2.2 Installation

The Simon Fraser University Glaciology Group installed two TACs in South Glacier in 2008. Both were connected to Campbell CR1000 data loggers. One installation (SG-UPPER100M-2008), located in the mid-glacier region of South Glacier about 2.2 km from the terminus, failed to save a time series due to a programming error. Data collected in September 2008, nearly three months after installation, indicate that the ice is temperate below a cold surface layer five meters thick. The cable failed entirely at some point before the next visit, and in 2011 it was observed that the area of installation had become heavily crevassed. The second cable (SG-MIDMET15M-2008) was installed near the MidMet meteorological station for surface energy balance applications.

The information provided by these two cables is of limited use for evaluating thermal structure, because SG-UPPER100M-2008 is installed in a crevassed area that appears complicated on radar, while SG-MIDMET15M-2008 was intended for calculating subsurface heat fluxes and installed in a 13 m borehole that likely did not penetrate below the seasonally-affected active
Table 3.3: Locations of the five temperature acquisition cables mentioned in the text. Locations use the WGS 84 reference spheroid and UTM eastings and northings are given using UTM Grid 7 North coordinate system.

<table>
<thead>
<tr>
<th>Cable name</th>
<th>Longitude</th>
<th>Latitude</th>
<th>Easting</th>
<th>Northing</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG-MIDMET15M-2008</td>
<td>139° 7' 21.06″ W</td>
<td>60° 48' 58.19″ N</td>
<td>602122</td>
<td>6743772</td>
</tr>
<tr>
<td>SG-UPPER100M-2008</td>
<td>139° 7' 29.65″ W</td>
<td>60° 49' 18.87″ N</td>
<td>601974</td>
<td>6744408</td>
</tr>
<tr>
<td>SG-MIDMET15M-2011</td>
<td>139° 7' 20.00″ W</td>
<td>60° 48'58.120″ N</td>
<td>602123</td>
<td>6743770</td>
</tr>
<tr>
<td>SG-UPPER85M-2011</td>
<td>139° 7' 11.40″ W</td>
<td>60° 49' 19.49″ N</td>
<td>602249</td>
<td>6744435</td>
</tr>
<tr>
<td>NG-MID75M-2011</td>
<td>139° 9' 13.60″ W</td>
<td>60° 54' 44.97″ N</td>
<td>600120</td>
<td>6754450</td>
</tr>
</tbody>
</table>

layer. In 2011, I chose a location on each glacier in which I expected a vertical profile to reveal polythermal structure based on radar scattering. Cables were obtained with sensor spacings intended to provide good resolution around the expected cold-temperate surface. The South Glacier TAC (SG-UPPER85M-2011) has eight temperature sensors. When it was installed in July 2011, the lowermost sensor was 81.5 m deep into the ice. This cable was connected to a CR1000 data logger. The borehole in which the cable is installed is also instrumented with a water pressure sensor, however the heat generated by this sensor should not be enough to have an effect on the temperature measurements. The North Glacier TAC (NG-MID75M-2011) has seven sensors, and the lowermost reached a depth of 70 m when installed in August 2011. The ice thickness in this location is about 110 m. This borehole is not shared with any other instruments, and no data logger is connected to this TAC. The absence of an installed data logger requires that measurements be made using a hand-held data collector during site visits.

A second South Glacier TAC (SG-MIDMET15M-2011) with one meter sensor spacing replaces the older and no-longer-functioning TAC SG-MIDMET15M-2008 at the MidMet station. The details of sensor positions for all TACs are given in Table 3.4. Coordinates of installation locations are provided in Table 3.3.

TACs SG-MIDMET85M-2011 and NG-MID75M-2011 provide important results useful for validating thermal structure interpretations based on the radar data. SG-MIDMET85M-2011 was installed on July 16th, 2011, and recordings commenced the next day. On August 11th, 2011 the CR1000 connected to SG-MIDMET85M-2011 ceased recording data. When visited on September 4th, 2011, it was not clear what had caused the failure. Wires leading to the CR1000 appeared to be secure and matched to the correct ports. Nevertheless, after disconnecting and reconnecting the wires leading to the data logger and recompiling the recording program, the
TAC resumed normal operation. The temperatures had changed very little over the three weeks since the previous valid measurement, indicating that the ice in the vicinity of the borehole into which the cable was installed had nearly re-equilibrated to surrounding ice temperatures. A time series illustrating the first two-and-a-half months of temperature logging is shown in Figure 3.7.

On North Glacier, data from TAC NG-MID75M-2011 were successfully recorded at the installation time on August 1st, 2011, and again during an autumn visit, on September 8th, 2011 (Figure 3.8). Based on the equilibration time observed at South Glacier, the borehole at North Glacier is expected to have adjusted to ice temperature, and the September measurement is taken to be representative of surrounding ice temperature.

![Figure 3.7: Temperatures (°C) as measured by TAC SG-UPPER85M-2011 during the first seven weeks after installation. The period between August 11th, 2011 and September 4th, 2011 is linearly-interpolated for the reasons described in the text. After roughly three weeks of equilibration, the temperatures are steady.](image-url)
Figure 3.8: Temperatures (°C) as measured by TAC NG-MID70M-2011 in August and September 2011. Similar to SG-UPPER85M-2011, the initially warm measurements cool over time, revealing a subfreezing upper layer. The full ice thickness is roughly 110 m.
<table>
<thead>
<tr>
<th>Cable name</th>
<th>Year installed</th>
<th>Length (m)</th>
<th>Sensor spacing (m)</th>
<th>Sensor installation depths (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG-MIDMET15M-2008</td>
<td>2008</td>
<td>15</td>
<td>1.0</td>
<td>0, 0, 0.2, 1.2, 2.2, 3.2, 4.2, 5.2, 6.2, 7.2, 8.2, 9.2, 10.2, 11.2, 12.2</td>
</tr>
<tr>
<td>SG-MIDMET15M-2011</td>
<td>2011</td>
<td>15</td>
<td>1.0</td>
<td>0, 0, 0.6, 1.6, 2.6, 3.6, 4.6, 5.6, 6.6, 7.6, 8.6, 9.6, 10.6, 11.6, 12.6, 13.6</td>
</tr>
<tr>
<td>SG-UPPER100M-2008</td>
<td>2008</td>
<td>100</td>
<td>5.0</td>
<td>0, 0, 0, 0, 0, 0.5, 5.5, 10.5, 15.5, 20.5, 25.5, 30.5, 35.5, 40.5, 45.5, 50.5, 55.5, 60.5, 65.5, 70.5, 75.5</td>
</tr>
<tr>
<td>SG-UPPER85M-2011</td>
<td>2011</td>
<td>85</td>
<td>Irregular</td>
<td>16.5, 21.5, 26.5, 36.5, 46.5, 56.5, 71.5, 81.5</td>
</tr>
<tr>
<td>NG-MID75M-2011</td>
<td>2011</td>
<td>75</td>
<td>Irregular</td>
<td>15.0, 25.0, 30.0, 35.0, 40.0, 45.0, 55.0, 65.0, 70.0</td>
</tr>
</tbody>
</table>

Table 3.4: All temperature acquisition cables mentioned in the text are summarized here. The depths reported are the installation depths, and vary as the glacier surface ablates. Where depth is reported as 0 m, the sensor is on the glacier surface. Cable locations are given by Table 3.3.
3.3 Additional data sources

Glacier thickness data exist from 76 boreholes in the middle region of South Glacier, between 1.6 km and 2.3 km from the terminus. These data were collected by the University of British Columbia Glaciology Research Group of Dr. Christian Schoof in the period 2008–2011. These data serve as useful validation points for ice thicknesses estimated from radar surveys. Boreholes may deviate from vertical by small amounts, causing slight over-estimations of ice thickness. This effect is not likely to be large because of the trigonometry of small angles. In other cases, boreholes may terminate above the glacier bed. Samples of water extracted from the boreholes are used to try to determine the likelihood that the borehole extends to the bed.

Glacier surface DEMs are used for a number of purposes, including applying topographical corrections to radar lines and developing maps of bed topography based on surface elevation and ice thickness. The DEMs used were constructed by De Paoli (2009) from unpublished data using geodetic-quality kinematic GPS data from both South Glacier and North Glacier. The surface elevation data used for South Glacier are identical to that described by De Paoli (2009), and incorporate recent (2006, 2007) GPS data from most parts of the glacier. Unsurveyed regions include a heavily crevassed zone in the upper glacier 4.2 km from the terminus and a crevassed region 2.8 km from the terminus, as well as the two side tributaries. Surface elevation data from North Glacier were collected in 2007, and provide good coverage of the lower half of the glacier with limited coverage of the upper half. Specifically, the uppermost 600 m of the glacier are unsurveyed, and a roughly 900 m by 500 m region on the glacier-right margin beginning 4.3 km from the terminus is unsurveyed. The GPS data were interpolated using ordinary kriging and combined with contours of the surrounding topography digitized from 1:50,000 scale mapsheets (De Paoli, 2009). In peripheral ice-covered regions where there are no GPS data, the North Glacier DEM relies more heavily on the older topographical mapsheets. These are expected to have lower accuracy compared to the recent GPS data.
Chapter 4

Field Data Processing Methodology

I have used several techniques to organize, clean, and interpret the available data. The two primary data sources are radar surveys and temperature cable measurements. The radar dataset is comprised of common-midpoint and common-offset survey types, which are processed separately. Common-midpoint survey data will be discussed first, because the results that it yields are useful for processing and interpretation of other radar data. Common-offset data comprise the bulk of the radar dataset, and will be described first as applied to bed sounding, using a methodology similar to that of De Paoli (2009). Next, these methods will be extended for englacial reflection processing. Finally, calibration data are applied to some of the temperature cable measurements to compensate for a programming error that prevented proper data correction during acquisition.

4.1 Common-midpoint radar processing

In the common-midpoint surveys discussed here, the air wave velocity is assumed to be $3 \times 10^8$ m s$^{-1}$ and used to back-calculate triggering times based on the air wave arrival time. Linear gain control is applied ad hoc to permit accurate event picking. Event wavelets expected to represent the ground wave direct coupling, a prominent and consistently-identifiable englacial reflector, and the bed are selected using a first-break picking methodology (e.g. Murray et al., 2000; Stuart et al., 2003; Rippin et al., 2011). The travel-times are averaged at each offset and plotted as travel-time versus offset. From these data, the propagation velocity through ice can be determined.

Defining the antenna spacing as $x$ and the reflector depth as $z$, the path distance $tv$ (time
velocity) of a bed-reflected wave as it travels from transmitter to receiver is

\[ tv = 2 \sqrt{z^2 + \frac{x^2}{2}}. \]  (4.1)

This is the model for the common-midpoint hyperbola expected for a reflector at depth \( z \), with ice propagation velocity \( v \). Antenna offset \( x \) is prescribed and travel-time \( t \) is measured. With multiple independent offsets \( x \), velocity \( v \) and depth \( z \) can be solved for.

The sensitivity of travel-time \( t \) as a function of velocity is not the same for every antenna offset. The longer the travel path through the ice, the more sensitive the experiment is to the propagation velocity. Measurements of travel-time are likely to contain some error, so it’s useful to know how different measurements should be weighted. Taking a derivative,

\[ \frac{dt}{dv} = -\frac{2}{v^2} \sqrt{z^2 + \frac{x^2}{2}}. \]  (4.2)

For a planar reflector, depth \( z \) is constant. Velocity \( v \) is assumed constant across all measurements. The sensitivity of the measured travel-time increases with antenna offset, and an optimization of (4.1) to data should weight measurements with large antenna spacing more.

I optimize (4.1) by minimizing the difference between the predicted and observed travel-times, scaling the objective function during the optimization to account for changing sensitivity. Equation (4.2) ensures that residuals are penalized more at greater offsets, because travel-time observations are better indicators of propagation velocity the longer the travelled distance. The signal-to-noise ratio is better at large offsets. The sensitivity of estimated velocity to error is proportional to the distance the radio wave travels through the ice.

Because the depth to reflector \( z \) may be on the same scale as offset \( x \), it cannot be ignored, and must either be assumed or incorporated into the optimization. For shallow reflectors, the weighting function scales linearly with offset, but for deep reflectors, the relationship is hyperbolic. Empirically, the curve-fitting results are largely insensitive to the \( z \)-term in the objective function, so it is reasonable to guess approximate values based on reflected wave travel times and an assumed propagation velocity.

I use curve-fitting with the downhill simplex scheme (Nelder and Mead, 1965; Weise, 2009) to minimize the weighted difference between the observed arrival times and those predicted by the model reflection hyperbola. The downhill simplex scheme is chosen because of its simplicity.
and robustness for simple functions (Press et al., 1986; McKinnon, 1998; Lagarias et al., 1998). I use the implementation in the Scipy scientific programming software (Jones et al., 2001-2011). The shape of the best-fitting curve yields both the velocity and a verification of reflector depth.

4.2 General common-offset data preprocessing

As in Chapter 3, common-offset data are split into a hierarchy ranging from survey to trace. A survey comprises multiple lines, which are roughly equivalent to individual traverses across some section of glacier. Each line in comprised of traces, which is a vector of induced antenna voltages evenly sampled with time. Because stacking is performed at the hardware level, the traces described here are averages of many (usually 100) individual recordings. Each trace is associated with various metadata, the most relevant of which are the GPS coordinates.

Several preprocessing steps are undertaken before interpreting common-offset data. These steps simplify further analysis, remove artifacts, and correct various technical problems that exist within the data. First, the longitude-latitude coordinates returned by the GPS receiver are projected on to a UTM grid. In some cases in the 2011 data, the GPS reports positions in “static mode,” meaning that the reported values only change when the GPS receiver moves more than 25 m. Because this is a relatively large distance on the scale of the radar surveys conducted, these need to be corrected. For data collected in Spring 2011, this is done with a simple linear interpolation of points between the “static” positions, based on the recording timestamp. For transects that cross the glacier in nearly straight lines at constant speed, this is a good approximation. Precision regarding small or rapid changes in speed and direction is lost.

During the July 2011 field campaign, tracks from a hand-held GPS were recorded and saved simultaneously with the radar surveys. For data afflicted with the static mode GPS problem, the coordinates from the on-board GPS have been replaced with those from the hand-held GPS by correlating the GPS timestamps. The hand-held GPS coordinates have been linearly interpolated to higher temporal resolution in order to improve the accuracy and precision of the correlation. As a condition for correlation, the two timestamps are not permitted to differ by more than 15 seconds, and in practice the time deltas are much smaller.

On rare occasions, a radar trace containing only very low magnitude static exists within a line, likely indicating that the receiver failed to trigger on the direct coupling. These traces are
more common in the 50 MHz data, because triggering is more challenging at higher frequencies. These are removed based on a heuristic that compares the cumulative squared voltage within a window early in the trace to a threshold value. When there is no direct coupling, this sum is much smaller than at a normal observation. I have found that setting a threshold of 400 $\mu$V summed over the first 100 samples (400 ns) correctly identifies the problem traces with the PicoScope 4227 digitizer.

Radar traces are recorded at a constant time interval, so redundant traces resulting from the system remaining stationary must be dealt with. This is accomplished by averaging groups of traces that are within a small threshold horizontal distance of each other. This step performs better when the GPS data have been smoothed slightly to diminish spurious variations in position. For this smoothing, a simple five-sample boxcar moving average is used. The horizontal position threshold is then chosen to be 3.0 m, based on the apparent practical accuracy of the smoothed GPS coordinates. As a result of this step, clusters of redundant radar data are combined into representative traces.

Accurate two-dimensional migration requires that radar positions be in straight segments. Therefore, the radar dataset must be coerced into an approximately piecewise-linear geometry. As surveyed, doglegs and direction changes often exist within individual radar lines. Automatic splitting identifies sections within the radar line where the direction changes, so that migration can be applied individually to each segment. Most radar traverses have zero to two major direction changes and are divided into one to three segments.

### 4.3 Ice thickness mapping

#### 4.3.1 Filtering and migration

Effective processing workflows varied slightly depending on the antennas used. For the 10 MHz dataset, which represents the majority of the data, gain control is only necessary when the ice is very thick or englacial scattering is strong. In practice gain control is usually unnecessary for 10 MHz data where the ice thickness is less than 100 m. A typical band-pass filter used for this dataset has a pass band of 8-25 MHz, and transition bandwidths of 5-10 MHz. To observe the bed clearly in the 35 MHz and 50 MHz datasets, gain control is usually necessary. For these data, a band-pass filter with a pass band of 15-45 MHz works better.

I used a migration routine that implements Stolt's frequency-wavenumber method described
in Chapter 2. This method requires an assumption of regular spacing between observations, which is invalid for the raw data. As discussed in Chapter 3, the average spatial resolution varies from approximately one trace per 4–8 m depending on the survey. Sampling intervals are irregular within each survey because recordings are made on a regular time interval and surveys conducted on foot do not travel at a constant speed. As a result, the data used for migration have been linearly-interpolated across the horizontal (space) dimension. This is valid as long as the distance over which the underlying topography or scattering depth changes is large relative to the sampling interval. If the return wavelet is displaced by a half wavelength from one point to another, destructive interference may occur. For a 35 MHz return wavelet and a reflector inclination of 30°, this will happen over distances of about 4 m. At 10 MHz, this distance increases to 15 m. Data collected are dense enough that this is not troublesome.

Valley-transverse data at all frequencies are migrated, but migration is generally not useful for longitudinal radar lines. For longitudinal lines, off-nadir reflections often come from directions outside the vertical plane of the radar line, so information is unavailable from neighbouring radar traces. Migrating transverse lines and not longitudinal lines sometimes causes poor cross-over consistency where the two line types intersect. This is particularly problematic on North Glacier, where subglacial valley walls are steeper. Transverse line ice thickness estimates are better than longitudinal line estimates (Bauder et al., 2003).

<table>
<thead>
<tr>
<th>Nominal Frequency</th>
<th>Filters and processing</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 MHz</td>
<td>Time-proportional gain control if necessary</td>
</tr>
<tr>
<td></td>
<td>Bandpass from 8 MHz to 25 MHz with transition bandwidths of 5 MHz and 10 MHz</td>
</tr>
<tr>
<td></td>
<td>Migration if necessary</td>
</tr>
<tr>
<td>35 MHz</td>
<td>Time-proportional gain control</td>
</tr>
<tr>
<td></td>
<td>Bandpass from 15 MHz to 45 MHz with transition bandwidths of 10 MHz</td>
</tr>
<tr>
<td></td>
<td>Migration if necessary</td>
</tr>
<tr>
<td>50 MHz</td>
<td>Time-proportional gain control</td>
</tr>
<tr>
<td></td>
<td>Bandpass from 35 MHz to 45 MHz with transition bandwidths of 10 MHz</td>
</tr>
<tr>
<td></td>
<td>Migration if necessary</td>
</tr>
</tbody>
</table>

Table 4.1: Description of radar bed processing methodology for each nominal antenna frequency.
4.3.2 Event picking

The time of the wavelet first break is used as a reference for event-picking (e.g. Murray et al., 2000; Stuart et al., 2003; Rippin et al., 2011), where events represent either the antenna direct coupling or the bed reflection. Picking is performed both manually and by ad hoc automated picking algorithms. A number of experimental picking routines have been tested, all attempting to identify meaningful wavelets based on timing, voltage, and voltage derivatives. All automatically-picked waves are inspected visually and manually adjusted where necessary.

Two-way travel time \( t \) is calculated from the picked bed arrival time \( t_2 \) and direct coupling arrival time \( t_1 \) with the relation

\[
t = t_2 - \left( t_1 - \frac{x}{c} \right),
\]

where \( x \) is antenna spacing and the air wave speed is assumed to be close to the speed of light in a vacuum \( c \). The function

\[
h = \sqrt{\frac{t^2 v^2}{4} - \frac{x^2}{4}},
\]

gives ice thickness in terms of \( t, x, \) and propagation velocity through ice \( v \). Combining 4.3 with 4.4 yields a function for depth

\[
h = \sqrt{\frac{t_2 cx}{2} - \frac{t_1 cx}{2} + \frac{t_2^2 c^2}{4} + \frac{t_1^2 c^2}{4} - \frac{t_2 t_1 c^2}{2}}.
\]

4.3.3 Picking uncertainty

The certainty with which reflections can be identified varies widely across the glacier. This variability is often predictable; radargram clarity decreases in the accumulation zone and in other regions containing numerous crevasses. In an attempt to quantify this uncertainty, each picked bed reflection arrival is given a quality rating similar to that described by Flowers and Clarke (1999) and adopted by De Paoli (2009). The ratings are primarily indications of the clarity and spatial coherence of reflections as seen in radargrams, and secondarily of the prominence of the picked reflection as seen in the trace considered alone. To receive a high quality rating, an event must be recognizable as the sought-after source (e.g., a bed reflection must be clearly a bed reflection, and not englacial clutter), and bear some resemblance to events in nearby radar traces. Quality ratings are a measurement of belief, and necessarily subjective. A description of the guidelines used to assign ratings is shown in Table 4.2. The reflection quality ratings
are useful for limiting subsequent analysis to data with high perceived accuracy, as well as for estimated measurement error at each point. I assume direct coupling events to be unambiguous in all data to the degree that assigning quality ratings is not necessary. Where there is any doubt in identifying the direct coupling arrival, the trace is discarded.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Nearly perfect reflections without any multiple reflections or ambiguity</td>
</tr>
<tr>
<td>4</td>
<td>Very good reflections with minor uncertainty; possibly with minor local diffraction patterns or steep slopes</td>
</tr>
<tr>
<td>3</td>
<td>Significant ambiguity or problematic elements, but the bed can be identified with reasonable confidence</td>
</tr>
<tr>
<td>2</td>
<td>Substantial difficulty in identifying the bed reflection, such that the picked event is something of a guess</td>
</tr>
<tr>
<td>1</td>
<td>Extreme uncertainty about picked event</td>
</tr>
</tbody>
</table>

Table 4.2: Bed reflection quality ratings for subjective assessment of picking accuracy

After filtering the dataset with a threshold quality rating, each rating $r_i$ is converted to an approximate estimate of scalar measurement error $\epsilon_{mi}$ in ice thickness. After some experimentation, the empirical function

$$\epsilon_{mi}(r_i) = \sqrt{\frac{50}{r_i^2}}$$

(4.6)

has been chosen to provide a reasonable estimate of the expected picking error in meters. This provides a quantitative (if still subjective) metric by which more weight is given to higher-quality observations in the interpolation to follow. An alternative method is to perform a preliminary gridding of the data and explicitly weight the average value within every grid cell by quality. In contrast to this latter technique, the method adopted reduces the number of parameters that must be chosen by eliminating arbitrary grid size and distance-weighting functions, and allows error estimation to be treated within a geostatistical context.
4.4 Englacial scattering layer processing

Englacial scattering is widely expected to be related to thermal structure (Hagen and Sætrang, 1991; Ødegård et al., 1997; Pettersson, 2005; Rippin et al., 2011). In designing a filtering methodology for englacial reflections, it is important to construct the filters in such a way as to avoid corrupting the data with artifacts from the processing itself. Preliminary inspection of the data reveals that englacial scattering consists of interfering point scatterer hyperbolae. Chaotic small amplitude oscillations in the received signal polarity cannot be easily traced to individual point scatterers. This description of englacial reflections is consistent with the scattering observed by others (see Chapter 2), and should be retained by the filter. To provide guidelines in choosing useful bandpass characteristics, wavelet transforms (e.g. Torrence and Compo, 1998) have been used to inspect the time-evolution of characteristic radar traces in the frequency domain. By first choosing traces in which scattering is easily distinguishable from other reflections and noise, the expected signal characteristics can be estimated. The mid-glacier region of North Glacier provides a good setting, because there exist prominent lobes of scattering at depth (Figure 4.1a).

4.4.1 10 MHz antennas

Beginning with radar data collected using 10 MHz antennas, I compute wavelet transforms with a Morlet mother wavelet using Fortran code made available by Torrence and Compo (1998) (Figure 4.1b-c). In one spectrogram (b), illustrating a transformed trace taken from a location with no apparent scatter, the two regions with significant power correspond to the direct coupling waves and the bed reflection. The bed reflection is centred at about 10-15 MHz, and has little energy above the background level above roughly 25 MHz. In a second spectrogram (c), taken from a region with obvious scattering, there is a broad region of elevated power at frequencies slightly higher than those corresponding to the bed reflection. Power over this portion of trace is higher than background levels at frequencies up to 55 MHz, so this band will be retained in the design of the englacial scattering layer filter.

I use a highpass filter with a cut-off at 50 MHz and a transition bandwidth from the stopband to the passband of 4 MHz. The effect of this is to reduce the bed reflection, while retaining nearly all of the englacial scattering. Compared to applying the opposite technique for the purpose of bed reflection filtering, this does not appear to provide the same amount of improvement. This is
Figure 4.1: Large regions of local englacial scattering are visible in mid-glacier radar transects from North Glacier, shown in (a). The wavelet transform in (b) shows a trace where the bed reflection is the only major feature after the direct coupling. Lighter shades indicate regions with more energy at the receiving antenna. In (c), englacial scattering is prominent, and takes the form of power in the spectrum at high frequencies between the direct coupling and bed reflection. The radar locations chosen for the wavelet transforms (b) and (c) are annotated in (a). The result of the frequency-domain filtering and contrast adjustment are shown in (d). Relative power in the spectrograms (b) and (c) is presented on a log scale.
because (1) the englacial scattering is very weak compared to the bed reflection, and so more
difficult to retain exclusively, and (2) the bed reflection rarely obscures englacial scattering,
whereas the reverse is frequently true.

A final step, which enhances contrast and simplifies interpretation is to apply a shading
scale that increases brightness exponentially with the signal amplitude. Using a positive scal-
ing exponent less than 1 enhances smaller-magnitude features such as englacial scattering,
relative to high-magnitude features such as the bed reflection and direct coupling waves. An
example is shown in Figure 4.1d.

The conclusions drawn from interpreting Figure 4.1 are very similar to those from the original
10 MHz radargram, because the scattering regions were already clear prior to filtering. In other
cases, similar englacial features only become obvious after filtering and contrast enhancement.
In Figure 4.2, the effect of applying the same processing steps to another radar line from North
Glacier is shown. In the raw data, scattering is not as obvious, and its full extent is difficult to
demarcate. In the processed result, the extent of scattering is easier to delineate, and regions
that may have been missed before are now obvious.

Also to be seen in Figure 4.2c is that some of the englacial reflections in the raw data are
arranged in columnar patterns, suggesting that they represent crevasses or incised channels.
This structure is preserved in the processed result, so that structural features can still be distin-
guished and identified within the glacier.

### 4.4.2 Higher frequency antennas

The methodology outlined above works well for data collected using the 10 MHz antennas. The
processing methodology for higher-frequency radar data collected in 2011 is similar, incorpo-
rating band-pass filters to separate englacial scattering from bed features. The higher levels
of random noise in the 35 and 50 MHz data sometimes make it difficult to use the brightness
scaling technique described above. One solution would be to apply a moving average filter to
reduce the noise before the brightness scaling, but this also effectively attenuates the englacial
scatter. The technique adopted is to filter the absolute value of the signal with a 20 ns boxcar
moving average. This attenuates sparsely-distributed noise in each trace, but sums signals in
areas with dense scatter. The filter kernel width, which spans waves with frequencies of 50 MHz
or greater, is chosen based on characteristics of the scattering. Figure 4.3 shows an example
of this methodology applied to field data.
Figure 4.2: Results of applying the englacial scattering filter to a longitudinal line segment from the 2008 10 MHz survey on North Glacier. In (a), scattering is visible in the raw radargram to the right of the vertical bar, with some evidence of weak scattering elsewhere. Vertical structures in some of the scattering likely indicate crevasses. In (b), both the scattering obvious in (a) as well as additional scattering at depth elsewhere are clearly visible. The vertical features in (a) are still recognizable as such in (b). Radargram (c) shows the original unfiltered data, but with the contrast increased. All of the scattering visible is (b) is visible here too, but retained noise and clutter make the data more difficult to interpret.
Figure 4.3: Results of applying the englacial scattering filter to a transverse line segment from the 2011 50 MHz survey on North Glacier. In (a), some scattering is visible. In (b), both the scattering obvious in (a) as well as additional, weaker scattering are clearly visible. Radargram (c) shows the original unfiltered data, but with the contrast increased. Similar to Figure 4.2c, noise makes the raw data difficult to interpret.
4.4.3 Mapping of englacial scattering

Whereas the ice thickness mapping discussed in 4.3 relied on manually or automatically picking a pair of wavelets representing the direct coupling and the reflection returned from a single subglacial surface, picking englacial arrivals is not as straight-forward. Picking englacial reflections using a methodology based on that for bed reflections is difficult, because interference prevents the expression of a consistent archetypical wavelet. In some places, the scattering is so dense that interference between adjacent scatterers makes it difficult to visually identify constituent hyperbolae. Previous efforts to measure the first-arrival time of englacial reflections representing the cold-temperate surface have sought to identify a sharp increase in relative amplitude for averaged traces (Ødegård et al., 1997; Pettersson, 2005). This technique discards sign information, increasing the effect of noise. The reliance on a sharp amplitude increase may cause false negatives when englacial scattering is weak but nevertheless perceptible on the radargram.

In both raw and processed forms, englacial scattering in a single radar trace is difficult to separate into categories that may represent thermal structure versus other englacial features such as crevasses. When multiple traces are viewed in the form of a radargram, tentative associations are possible based on spatial geometry (Murray et al., 1997; Arcone and Yankielun, 2000). Columnar structures are hypothesized to represent crevasses (e.g. Zamora et al., 2007), and distinct point hyperbolae may represent crevasses (e.g. Peters et al., 2007), englacial channels or large water pockets (Arcone et al., 1995). Large diffuse volumes of weak and chaotic scattering are here taken to represent regions of high water content, as in previous studies (e.g. Björnsson et al., 1996; Ødegård et al., 1997; Murray et al., 1997; Rippin et al., 2011). Sharp boundaries are often difficult to identify in zones of englacial scattering. In practise, scattering zones often appear to fade in with depth. It is easier to identify a consistent boundary between “transparent” ice and scattering zones when inspecting groups of traces arranged in a radargram than it is from single traces.

For these reasons, boundaries of englacial scattering are delineated directly on the radargrams. Inspection of radargrams indicates that, when both are present, transparent zones generally overlie scattering zones. Only the upper boundaries of the scattering zones are delineated. Unless multiple well-separated and individually reflecting layers of englacial scatterers can be unambiguously identified in a vertical stack, it is not justifiable to attempt to trace bound-
aries below this. At this level of processing, interference between scatterers and the potential for multiple-reflection events obscures vertical relationships between scatterers of similar magnitude.

The resulting estimates should be interpreted as broad-stroke representations of the englacial scattering layer depth. Differences in scatterer size, concentration, and orientation may affect scattering strength (Bamber, 1988; Pettersson, 2005; Matsuoka et al., 2007). A comparison of results obtained at different frequencies is discussed in Chapter 5.

After tracing the upper onset of each scattering zone, geographical positions are obtained by joining them with the GPS coordinates recorded during data acquisition. The horizontal extent of observed englacial scattering is outlined, and gridded with regular 20-meter square cells with grid registration identical to those in the ice thickness and bed models. The radar-transparent layer thickness is used as the interpolation variable, with radar-scattering layer thickness derived by combining ice-thickness data and assuming that scattering continues to the bed (discussed in 5.4).

### 4.5 Kriging interpolation

As in Flowers and Clarke (1999), interpolation by kriging is used to predict gridded ice thickness from the irregularly-spaced radar data. Kriging is also used to estimate the spatial distribution of the scattering surface depth. Model variograms are developed under the assumption that the empirical variogram can be approximated by a linear combination of spherical functions having the form

\[
\gamma(h) = \begin{cases} 
\left(1 - \frac{3}{2} \frac{h}{\alpha} + \frac{1}{2} \frac{h^3}{\alpha^3}\right) \sigma^2 & \text{for } 0 \leq h \leq \alpha, \\
\sigma^2 & \text{for } h > \alpha,
\end{cases}
\]

or Gaussian functions with the form

\[
\gamma(h) = \sigma^2 \left(1 - e^{-\left(\frac{h}{\alpha}\right)^2}\right),
\]

where \(h\) is lag (distance), \(\alpha\) is the onset of the variogram sill, and \(\sigma^2\) is the variance predicted at the sill. Spherical functions are commonly used within geostatistics (Kitanidis, 1997) and have been found to perform well for two-dimensional as well as three-dimensional problems in geology and soil science (McBratney and Webster, 1986). Flowers and Clarke (1999) found that
for ice thickness data, spherical functions performed better than Gaussian functions. Modelled variograms consisting of both spherical and Gaussian functions are tested using the data at hand.

For ice thickness, independent empirical variograms are estimated for each glacier in the general along-flow axis and across-flow axis directions, so that major anisotropy in the variance is accounted for. The model variograms are simultaneously fit to empirical variograms in both directions considered. For scattering layer depth, no anisotropy is assumed. Fitting is done using a nonlinear truncated Newton method (Nash, 1984; Jones et al., 2001-2011). Fit is optimized using residuals weighted by inverse lag, such that the resulting model variogram captures the variance of points nearest the origin most accurately. This is reasonable because points nearest the variogram origin are likely to be the most important when making a prediction at an unknown location (Kitanidis, 1997). In the case of ice thickness modelling, the squared measurement error (4.3.3) from each observation is added to the variogram as a nugget effect so that differences in picking uncertainty are represented in the final thickness estimate. A constant nugget effect chosen to fit the data is used for interpolating scattering depth.

During the kriging operation, only data taken from within a prescribed buffer around the prediction location are included, based on the observation that points beyond this have little correlation to the true value at the prediction location. For South Glacier, a 1 km radius is used. Because North Glacier is sparsely-sampled (particularly in the uppermost 1.5 km), a 2 km radius is used to avoid sampling artifacts from having too few samples. In sparsely-sampled regions, there are high expected errors. The boundaries of the glacier are seeded with zero-depth point measurements for thickness interpolation. This procedure ensures that the thickness of the ice trends realistically to zero at the glacier margins, rather than ending in vertical cliffs. No such assumption is made for scattering depth.

Ordinary and simple kriging require that data be stationary, meaning that they have no spatial trend. Rather than detrend the data before kriging to remove drift (e.g. De Paoli, 2009), maps are computed using universal kriging, as described by Cressie (1993) and Kitanidis (1997). Ice thickness is assumed to be approximately quadratic, as might be the case of ice in a U-shaped valley that is thick in the centre and thin at the margins. A linear trend function is assumed for scattering depths. The geostatistical package gstat (Pebesma, 2004) is used to implement the kriging operation.

Within gstat, direction-dependence is implemented by introducing an anisotropy parameter.
\( \beta \) (Pebesma, 2004). The variogram model is specified as a combination of functions that are allowed to change as direction deviates from a specified major axis. The function range parameter \( \alpha \) in the direction orthogonal to the major axis is modified by a factor of \( \beta \). The sill onset forms an ellipse with major axis \( 2\alpha \) and minor axis \( 2\alpha\beta \). In the isotropic case, the sill onset forms a circle with radius \( \alpha \).

In some locations near the glacier margin, interpolation by kriging results in ice thicknesses that are smaller than zero. This occurs most frequently where data density is poor. Marginal ice zones are not precisely mapped, however negative ice thickness are unphysical and introduce problems in later calculations. For this reason, non-negative ice thickness is enforced by requiring that ice thickness be larger than or equal to zero.

### 4.6 Temperature data correction

A programming error caused some of the temperature cable data collected by CR1000 dataloggers in 2011 to be saved in uncalibrated form, reducing the accuracy from an estimated 0.1°C to 0.5°C. I have retroactively applied the missing calibration corrections to improve accuracy in the same manner that the correction normally occurs during data acquisition (B. Shumaker, personal communication). Beaded Stream LLC supplies a list of calibration parameters for each cable that they provide. These include an offset \( \delta \) and two values, \( t_z \) and \( \alpha \), for each 1-Wire® sensor. The corrected temperature \( T_{\text{corr}} \) is calculated from the measured temperature \( T_{\text{meas}} \) as

\[
T_{\text{corr}} = -\delta + T_{\text{meas}} - \alpha (T_{\text{meas}} - t_z)^2.
\]

Only data collected from July–August 2011 using the CR1000 data loggers were not corrected at the time of acquisition and require this modification. Raw data retrieved using the hand-held data collector are corrected at the time of acquisition.
Chapter 5

Field Data Results

The data (Chapter 3) are processed using appropriate methodologies (Chapter 4) and conclusions derived from examining the data are presented here. These results include observations about the effects of processing on the data, the consistency of the derived data, and the applicability of the data to the goals outlined in Chapter 1. Interpolation results and observations about bed and englacial thermal structure are addressed. Repeat radar lines are described in Appendix B.

5.1 Radar wave velocity

The common-midpoint surveys performed on South Glacier in 2011 provide important validation of the propagation speed of radar waves in ice. Depth calculations that use data derived from common-offset surveys are sensitive to this propagation velocity. These experiments demon-

<table>
<thead>
<tr>
<th>Survey</th>
<th>Picked Event</th>
<th>Velocity (m s(^{-1}))</th>
<th>Depth (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMP01</td>
<td>Ground wave</td>
<td>$1.72 \times 10^8$</td>
<td>0.0</td>
</tr>
<tr>
<td>CMP01</td>
<td>Internal reflection</td>
<td>$2.23 \times 10^8$</td>
<td>78</td>
</tr>
<tr>
<td>CMP01</td>
<td>Bed reflection</td>
<td>$1.76 \times 10^8$</td>
<td>77</td>
</tr>
<tr>
<td>CMP02</td>
<td>Ground wave</td>
<td>$1.69 \times 10^8$</td>
<td>3.8</td>
</tr>
<tr>
<td>CMP02</td>
<td>Internal reflection</td>
<td>$1.65 \times 10^8$</td>
<td>53</td>
</tr>
<tr>
<td>CMP02</td>
<td>Bed reflection</td>
<td>$1.63 \times 10^8$</td>
<td>69</td>
</tr>
</tbody>
</table>

Table 5.1: Summary of the results from two CMP surveys performed on South Glacier in 2011. With the exception of the result from the CMP01 internal reflector, all values are close to the expected velocity and ice thickness. The 3.8 m depth of the CMP02 ground wave is not thought to be significant.
strate that radar wave velocity differs slightly in the two CMP survey locations, but that the commonly used value of $1.68 \times 10^8$ m s$^{-1}$ is close to the measured velocities. It is now assumed that radar wave velocities in other locations are likewise similar.

In common-offset surveys conducted using our methods, the proximity of the radar antennas does not permit the air wave and ground wave components of the direct coupling to be distinguished from each other. In contrast, the common-midpoint survey data are collected over a range, extending to large antenna offsets. These large offsets allow the two wavelets to separate as a consequence of their different propagation speeds. In the results presented here the ground wave is phase-shifted relative to the air wave, similar to the results obtained by Narod and Clarke (1994).

For each common-midpoint survey, hyperbolae are fit to the ground wave, an internal reflection, and the bed reflection, yielding three separate velocity estimates. There are two common-
midpoint surveys (CMP01 and CMP02), giving six velocity estimates in total. Two apply to the near surface, two apply to an arbitrary upper layer within the ice column, and two apply to the full ice column (Figure 5.1). The results of the hyperbola-fitting are listed in Table 5.1.

5.1.1 Results

Five of the six results match expectations for ice velocity and reflector depth reasonably well. Together, they constrain ice velocity to be in the range \(1.63 \pm 1.76 \times 10^8\) m s\(^{-1}\), bracketing the commonly-used value of \(1.68 \times 10^8\) m s\(^{-1}\). The direct coupling ground wave velocity measurements come closest, with measured velocities of \(1.72 \times 10^8\) m s\(^{-1}\) and \(1.69 \times 10^8\) m s\(^{-1}\) at CMP01 and CMP02, respectively. These results support the use of the commonly-used value for general-purpose sounding, and indicate that it should be accurate to within a few percent.

Compared to the other results, the CMP01 englacial reflector stands out. The picked wavelets for this reflector are much weaker than for the other datasets because the CMP01 lacks good internal reflections for picking. When the CMP01 internal reflector data are fit to a model hyperbola, they scatter more widely around the model than the other results. The best-fit hyperbola suggests a reflector depth greater than that inferred from the corresponding bed reflection dataset and simple sounding calculations. The derived bed depth is consistent with expectations from common-offset surveys using a typical radar velocity. This depth inconsistency as well as the improbably large velocity associated with the solution suggests that the result does not reflect the true glacier characteristics at CMP01.

Bradford and Harper (2005) have analyzed the likelihood of englacial reflectors to reveal accurate velocity estimates in glaciers, and found that accuracy declines when the reflector is offset from the survey midpoint. The CMP01 survey takes place in a portion of South Glacier observed to be mostly non-scattering and with only a few englacial reflectors. It is possible that the internal reflector is not directly beneath the survey as assumed, which would cause the results to be flawed. The same possibility exists for CMP02, but the results are plausible, and the location over which CMP02 took place has been observed to have a more regular scattering pattern, consistent with a planar scattering layer.

To test the above hypothesis, I generated modelled hyperbolae predicted for an imagined reflector at 62 m depth and an aerial (surface) reflector 106 m orthogonal to the survey geometry. The aerial reflection does not correspond to a particular feature or valley wall, but is chosen to match the travel-time from the observed reflection. Both modelled hyperbolae fail to
match the observed data well (Figure 5.2). The curvature of the englacial reflection hyperbola is very similar to that of the bed reflection hyperbola. It is unlikely that echoes from previously transmitted radar waves are being detected because of the slow repeat rate of the transmitter (512 Hz).

5.2 Mapping of ice thickness and bed elevation

One goal is to integrate the newly-acquired radar data into maps of ice thickness and bed topography, similar to Flowers and Clarke (1999) and De Paoli (2009). After performing reflection picking and depth calculations as described in Chapter 4, I test the self-consistency of the radar data through examination of intersections between multiple lines. I identify lines in which off-nadir reflections cause inaccurate sounding measurements, and remove them from the dataset. I then apply universal kriging interpolation to generate predictions of ice thickness and bed topography over South Glacier and North Glacier.

5.2.1 Crossover depth consistency

To evaluate the consistency of the calculated bed depths, it is useful to evaluate their agreement where they intersect. In the upper regions of both glaciers, picking uncertainty is high because
Figure 5.3: Histograms showing the consistency of calculated ice thicknesses between intersecting lines on both (a) South Glacier and (b) North Glacier. The majority of intersections agree to within a few meters. The longer tail of the North Glacier histogram indicates that intersection consistency is poorer there.

of weak bed reflections and strong scattering. On North Glacier, migrated transverse lines imply depths that are consistently greater than longitudinal lines, because off-nadir reflections in the longitudinal lines cannot be corrected through two-dimensional migration. As a result, several longitudinal lines are excluded from the dataset used for generating a North Glacier bed map. This does not appear to be as prolific a problem on South Glacier, likely because the cross-glacier bed angles are often shallower. A single longitudinal line from South Glacier near the terminus is removed because depths inferred across its length agree poorly with crossing lines at several locations.

Within the remaining data, the inferred depths of different radar lines at intersections are generally similar, and the average absolute difference is less than 0.4 m for South Glacier, and 1.8 m for North Glacier. The cross-over consistency is significantly better for South Glacier, where it is greater than 5% of the average ice thickness in 8 locations out of 312 and greater than 10% in 4 locations. For North Glacier, it is greater than 5% in 36 locations out of 170 and greater than 10% in 14 locations. The observed discrepancies are small at the majority of line intersections (Figure 5.3).
5.2.2 Ice thickness interpolation

I have constructed a bed map using data with a quality rating of 2 or better (Table 4.2). For South Glacier, depths derived from 2006 data compare well with depths obtained in 2008 and 2009 in the lower glacier, but poorly above 2.5 kilometres from the glacier terminus. The picked 2006 radar data from this region frequently differ from newer data by over 100 m. In the upper glacier, 3.4 km from the terminus, the 2006 data are often picked with depths less than 50 m, while the 2008 and later data indicate that this is the deepest part of the glacier, with depths in the range of 160–200 m. In many cases, the traces collected in 2006 were truncated at times that would not have included the bed reflection, which should have arrived later. It appears that the depths indicated by the 2006 data correspond to prominent englacial reflectors in the newer data. The sample density is very high in the parts of the glacier covered by the majority of the 2006 survey. In the areas of the upper glacier where data from the 2006 survey comprise a large fraction of the entire dataset, they agree poorly with other data collected in other years. Confidence in the newer data is higher, and the older data have not been incorporated into the bed map.

![Ice thickness semivariograms](image)

Figure 5.4: Ice thickness semivariograms for South (a,b) and North (c,d) Glaciers in the cross glacier (a,c) and along glacier (b,d) directions. Empirical variograms are indicated by crosses, while black lines are model variograms that attempt to fit the data. Anisotropic models are used for both glaciers. Spherical functions are used for South Glacier, while Gaussian models are used for North Glacier. Variogram fitting heavily weights small lags (4.5).

Model variogram parameters are given in Table 5.2. Empirical and model variograms are
Table 5.2: Model variogram parameters fit to ice thickness data from South Glacier and North Glacier. Each model is a combination of two functions for the along glacier and cross glacier directions, respectively. The parameters for the cross-glacier function are denoted with a prime. The sill is the maximum variance. $\alpha$ is range, or lag at which the sill is reached. $\beta$ is the anisotropy parameter. See Chapter 4.

shown in Figure 5.4. The model variograms reproduce the empirical variograms very well at small lags. Because small lags are where the variance is the lowest, this is where the kriging interpolation is most sensitive to the variogram (Kitanidis, 1997). The pronounced U-shape of North Glacier manifests itself in the cross-glacier variogram by variance that decreases with increasing lag from 500 m to 800 m.

Lateral cross-sections of the South Glacier bed have a rectangular cross-section in the mid-glacier region, however there are major bumps in several locations across the glacier. Compared to a previous bed map for South Glacier based on the 2006 and 2008 data sets (De Paoli, 2009), the new bed map defines a deeper glacier in the lower accumulation area. This is due to the inclusion of new data from the 2009 and 2011 surveys, as well the exclusion of the 2006 survey. New filtering techniques make it possible to image the bed at greater depths by suppressing englacial scattering and enhancing deeper reflectors (Figure 5.7).

A small but well-sampled patch 3.1–3.4 km above the terminus and about 280 m across is predicted to be deeper than 175 m. The maximum ice thickness at South Glacier is now predicted to be nearly 200 metres. New radar lines in the 2011 data set also indicate that the western tributary is deeper than previously assumed, with ice thicknesses that are locally in excess of 100 m. The eastern tributary remains unsurveyed and ice thickness is unknown there. Excluding the eastern tributary, the mean interpolated thickness of South Glacier is 64 m.

The bed topography of North Glacier is fairly simple in comparison to South Glacier, with cross-sectional shape varying little over the glacier’s length. Lateral cross-sections are classically U-shaped, with the thalweg generally slightly to the southwest (glacier-left) of the glacier.
Figure 5.5: Ice thickness maps from South Glacier (a) and North Glacier (b), based on 10 MHz data from 2008–2011. Checkered areas indicate regions where data are too sparse or nonexistent.
Figure 5.6: Variance (2.5.2) calculated during the interpolation of the ice thickness maps shown on the previous page. The variance for North Glacier is much larger than that for South Glacier, because the experimental variogram requires a variance model with a much higher sill. The stripes occur because kriging variance is lowest near the surveyed lines.
Figure 5.7: Interpretations of 2006 radar survey data suggest a shallower bed than subsequent surveys. This radargram from May 2011 is constructed from data collected using 10 MHz antennas. The depth identified with the bed is the clearest reflector in the unprocessed data (a). In the processed data (b), a deeper reflection is interpreted as the bed.

centreline. The deepest portions of the glacier cover a long strip in the upper ablation zone. Despite the larger area of North Glacier, the greatest ice thickness is less than that of South Glacier, around 180 m (Figure 5.8). The mean interpolated thickness of North Glacier is 77 m.

The interpolated ice thicknesses derived from radar can be directly compared to borehole length measurements collected by the University of British Columbia Glaciology Research Group of Dr. Christian Schoof (unpublished data). There is a good match between borehole lengths and the ice thickness interpolations, however small discrepancies are observed (Figure 5.9). The average thickness estimate derived from radar surveys is 5.6% smaller than the corresponding borehole length. The average disagreement between radar prediction and borehole measurement is 6.4%. Of the radar predictions, 81% fall within a 10% error envelope. A histogram of the ratio between the radar-derived thicknesses and borehole lengths is shown in Figure 5.9a. The boreholes only cover the mid-glacier area between 1.6 km and 2.3 km from the terminus (Figure 5.9 inset). This region has moderate ice thicknesses (45–90 m) and is densely-sampled in radar surveys. Comparing borehole lengths to the nearest uninterpolated radar measurement yields similar results and statistics.
Figure 5.8: Ice thickness distributions for (a) South Glacier and (b) North Glacier. Solid bars show thickness frequencies for each glacier, while the outlines show the thickness distribution of the other glacier for comparison. The eastern tributary of South Glacier has been excluded from these statistics.

Figure 5.9: Comparison between radar-derived interpolated ice thickness and borehole length measurements on South Glacier. (a) Histogram shows the distribution ratios of radar-derived thickness estimates to borehole measurements. The ratios cluster just below 1, indicating an underestimation by the radar method. (b) Radar predictions plotted against borehole lengths. The solid line shows a 1:1 relationship. The dotted lines show the 10% error envelope. The inset map shows the borehole locations, which are limited to the mid-glacier region of South Glacier.
There are a number of potential explanations for the observed discrepancies. Boreholes are not perfectly vertical, and the lowest portions in particular may tend toward one side or another during drilling. The borehole may terminate early, such as when a debris layer is encountered, or it may continue along the bed for some distance. If the glacier bed is soft, the drill may penetrate beyond the interface that would reflect a radio wave. If the glacier bed is rough on a scale smaller than that resolvable by radar, the boreholes may terminate in depressions below the reflecting depth. The boreholes represent point measurements, while the radar estimate is an interpolated average of nearby measurements. Off-nadir reflections from outside of the plane of a radar survey cannot be corrected through migration, giving the illusion of a thinner glacier. Most of these effects will tend to cause radar predictions of ice thickness to be smaller than borehole lengths, consistent with what is observed.

Using existing surface digital elevation models (Chapter 3), I have generated maps of bed topography by subtracting the thickness predictions described above (Figure 5.10) from the surface topography. This result reinforces inferences from the ice-thickness maps: South Glacier has a more rectangular cross-section with a rough and irregular bed, and North Glacier has a smoother, more classical U-shaped bed. South Glacier has a bowl-like over-deepening roughly 50 m deep 3.3 km from the terminus. The bed of North Glacier has no similarly over-deepened region.

5.2.3 Bed mapping accuracy

There are numerous sources of error and uncertainty in the preceding results. This section attempts to quantify and address the more important sources in order to assess the impact they may have on interpretation of the results.

The digitizer used in 2008 and 2009 has a sampling rate of 100 MHz, while that used in 2011 has a sampling rate of 250 MHz. With a wave propagation speed of \(1.68 \times 10^8\) m s\(^{-1}\) in ice, this implies maximum two-way vertical resolutions of 0.84 m and 0.34 m for the 100 MHz and 250 MHz digitizers, respectively. However, the wavelengths at 10 MHz, 35 MHz, and 50 MHz nominal frequencies are much longer than this timing resolution would require, so digitizer sample rate is not a limiting factor (Mingo and Flowers, 2010). Assuming that resolution is limited to one-quarter of the wavelength, then resolution is no better than 4.2 m, 1.2 m, and 0.84 m, respectively. As described in Chapter 4, the self-assessed event picking error is higher still, and ranges from one to several meters. Therefore, picking accuracy, which has been incorporated
Figure 5.10: Bed topography calculated from subtracting the ice thickness from the surface topography. The contour interval is 30 m. The prominent terrace at the uppermost limit of North Glacier is an artifact stemming from the lack of data and high uncertainty in the area (Figure 5.6) Checkered areas indicate regions where data are too sparse or nonexistent.
into the interpolation of ice thickness should dominate the bed mapping error.

Georeferencing of radar traces depends on GPS accuracy, which is expected to range from 3–10 m, depending on whether real-time Wide Angle Augmentation System (WAAS) corrections are available. This corresponds to a worst case depth error of 2.7 m where bed slope is 15°, considered reasonable for the glacier bed. Where GPS coordinates have been corrected from “static mode” raw data, accuracy is expected to be diminished, but well within the same order of magnitude.

The on-board GPS receiver is mounted on the radar receiver sled, rather than at the midpoint between the two sleds. For this reason, the observed travel times are shifted backwards along radar survey line relative to the true position. This shift is approximately one-half the antenna offset (∼5–10 m), which is below the final grid node spacing (20 m) used for interpolation.

Translation from hand-held GPS locations to radar position is likely to be accurate to within a few meters in most cases. Where on-board GPS coordinates are replaced by hand-held GPS coordinates, the mean time difference between the trace observation and the GPS timestamp is 1.8 s. The average movement speed in the corrected data is 0.7 m s\(^{-1}\), so substituting the hand-held GPS coordinates typically contributes an error of roughly 1 m.

The expected accuracy of interpolated ice thickness predictions depends on both the quality of the data and the density of observations in the local neighbourhood. Variance can be estimated at any point using the modelled variograms developed during the interpolation process (Figure 5.6). The expected deviations from the predicted value range from a few meters near densely sampled areas to over 10 m in poorly sample regions. Regions in the upper glacier have relatively low expected prediction accuracy, due to both sparse sampling, and high picking uncertainty as a result of thick, strongly-scattering ice.

### 5.3 Mapping of the scattering layer

I have identified englacial scattering and developed scattering maps in the manner described in 4.4.3. Englacial scattering is a prominent feature in a number of radargrams that I hypothesize to be related to glacier thermal structure. The scattering maps serve as a preliminary estimate of the englacial thermal energy distribution in South Glacier and North Glacier.
5.3.1 Scattering interpretation

An attempt has been made to identify englacial scattering consistent with thermal structure, shown in Figures 5.11–5.12. The depth distribution scattering identified as being consistent with high water content changes throughout the glacier. In some areas it's confined to isolated pockets near the bed, while in other regions it comprises the entire vertical ice section. Features such as crevasses (Figures 5.11a,e, 5.12c) and surface streams (Figure 5.12d) are avoided where possible. As described in 4.4.3, only the upper surface of scattering is delineated, so vertical structure below this surface (Figure 5.11d) cannot be resolved. Additional radargrams displaying englacial scattering are in Appendix E.

5.3.2 Comparison with temperature cables

For the purpose of comparing the observed englacial scattering with in-situ temperature measurements, radargrams are laid out alongside temperature-depth profiles. Temperatures are corrected for pressure\(^1\). Two temperature acquisition cables (TACs) were installed in boreholes that reach englacial scattering ice. These TACs and nearby radar lines are mapped in Figure 5.13. A correlation between where the scattering begins and where the ice reaches the pressure-melting point can be observed, as shown in Figures 5.14, 5.15, and Appendix D. These support the hypothesis that englacial scattering in the processed radargrams appears where there is temperate ice.

On South Glacier, a 50 MHz radar line crosses over the location of the borehole in which SG-UPPER85M-2011 was installed. Radar lines using 10 MHz and 35 MHz antennas come within 35 m and 95 m, respectively. Both higher-frequency lines are from the 2011 survey, while the 10 MHz line was surveyed in 2008. In the 10 MHz and 35 MHz radar lines, a localized cloud of englacial reflections is visible above the depth of the pressure melting point determined from the temperature data. The observed depth where ice reaches the pressure melting point instead corresponds reasonably well to the depth of the englacial scattering visible to either side of the local cloud (Figure 5.14). In the case of the 50 MHz line, which is the only radar line to have crossed immediately above the borehole location, the depth of scattering corresponds closely with the pressure melting point. Above this point, temperatures are below the pressure melting point, reaching a minimum of \(-2^\circ C\) at a depth of 16.5 m (at the uppermost sensor). Below the

\(^1\)Potential temperature \(\theta_T\) is defined to be the temperature relative to the freezing point of water with a correction to atmospheric pressure \(T_m\), i.e. \(\theta_T = T + P \left(\frac{\partial T_m}{\partial P}\right)\) for ice pressure \(P\).
Figure 5.11: Englacial scatter interpretation, with raw radargrams on the left and filtered radargrams on the right. The dashed line indicates the upper boundary of englacial scattering.
Figure 5.12: Englacial scatter interpretation, with raw radargrams on the left and filtered radargrams on the right. The dashed line indicates the upper boundary of englacial scattering.
Figure 5.13: Locations of (a) South Glacier lines displayed in Figure 5.14 and (b) North Glacier lines displayed in Figure 5.15. The locations of the temperature cables are given by black dots. The 35 MHz and 50 MHz lines on North Glacier share the same tracks. All data are from 2011 except for the 10 MHz line on South Glacier, which is from 2008.

upper scattering surface, the ice is at the melting point.

On North Glacier, a dense network of radar lines conducted around the NG-MID75M-2011 borehole in August 2011 ensures that 35 MHz and 50 MHz observations were made at and around the borehole location (Figure 5.13). Additionally, a 10 MHz line collected in May 2011 comes within 35 m of the borehole location. Comparing the three different frequencies used for these radar lines, it appears that all antennas indicate englacial scattering to begin at roughly the same depth in the processed radargrams. The main difference between different frequencies is that the higher frequency antennas have smaller depth-penetration than the 10 MHz antennas.

The TAC length on North Glacier is just sufficient to reach of the upper surface of englacial scattering seen in the radar data at every frequency (Figure 5.15). The lowest sensor on the TAC indicates the presence of temperate ice at this level.

5.3.3 Temperature cable accuracy

The manufacturer-supplied accuracy for the digital sensors used in the TACs is 0.5°C. The manufacturer of the TACs performs additional calibration steps that are purported to improve accuracy by up to a factor of 10 (B. Shumaker, personal communication). Accuracy is con-
Figure 5.14: Comparisons between TAC measurements and radar observations on South Glacier at (a) 10 MHz, (b) 35 MHz, and (c) 50 MHz. The SG-UPPER85M-2011 borehole is indicated by the thick black bar. Only the 50 MHz line crosses immediately over the borehole, whereas the 10 MHz and 35 MHz lines pass within 35 m and 95 m, respectively. A map showing the locations of these lines is shown in Figure 5.13a.
Figure 5.15: Comparisons between TAC measurements and glacier-transverse radar observations on North Glacier, at (a) 10 MHz, (b) 35 MHz, and (c) 50 MHz. The NG-MID75M-2011 borehole is indicated by the thick black bar. The 35 MHz and 50 MHz lines cross immediately over the borehole, while the 10 MHz line passes within 35 m down-glacier of the borehole. A map showing the locations of these lines is shown in Figure 5.13b.
servatively estimated to be 0.1°C (Chapter 3), which appears to be reasonable based on temperatures recorded as the cables rested in unfrozen boreholes. Still water in recently-drilled boreholes is assumed to be very near the pressure-melting point. At installation time, TAC SG-UPPER85M-2011 sensors recorded a mean potential temperature of 0.004°C, with a standard deviation of 0.041°C (Figure 3.7). Therefore, the temperate recorded in the water-filled borehole is consistent with the known melting temperature. TAC NG-MID75M-2011 sensors recorded a mean potential temperature of 0.033°C, with a standard deviation of 0.025°C (Figure 3.8). The temperature recordings furthest from the pressure-melting point on each cable were off by 0.08°C and 0.10°C, respectively. The other cables were calibrated in the same way, but installation-time readings are not available. Based on the temperature gradient observed at the cold-temperate surface by the two long cables installed in 2011, this error estimate translates to a CTS depth error of 2–3 m. This is much less than the sensor spacing, and therefore neglected.

5.3.4 Scattering layer depth interpolation

Having found that the available data support the hypothesis that englacial scattering is thermally-significant, I have interpolated the observed scattering surface depth over South Glacier and North Glacier. The interpolation is restricted to the approximate regions of both glaciers where scattering is observed. I use isotropic variograms (Figure 5.16), because anisotropy is far less pronounced in the empirical variograms for scattering depth than in the variograms for ice thicknesses.

![Figure 5.16: Radar-scattering layer depth semivariograms for (a) South Glacier and (b) North Glacier. Empirical variograms are indicated by crosses, while black lines are model variograms that attempt to fit the data. Isotropic models with spherical functions are used for both glaciers.](image)

Scattering layer thickness is calculated by subtracting the scattering layer depth from the ice
thickness calculated in 5.2.2. In addition to maps of scattering layer thickness (Figure 5.18) and kriging variance (Figure 5.20), I have generated a map of scattering layer thickness normalized to ice thickness (Figure 5.19). This normalized metric of scattering thickness better represents regions where both scattering layer thickness and ice thickness is small.

### 5.3.5 Scattering layer interpretation

Scattering maps (Figures 5.18 and 5.19) show that both glaciers share a general pattern in which scattering is observed at shallow depths in the upper reaches of the glacier, and at progressively greater depths further down glacier. In addition to this general geometry, there are notable scattering features in both glaciers that have been confirmed by direct comparison with the uninterpolated radargrams. In South Glacier, scattering depth becomes heterogeneous in the mid-glacier region (Figure 5.19a), with large swaths of transparent ice observed between masses that scatter radar waves (Figure 5.11). In the lower ablation zone, the upper boundary of scattering hovers around 30 m depth before disappearing 700 m above the terminus. There is some lateral asymmetry in the mid-glacier from 800 m to 1.7 km from the terminus (Figure 5.17). Here, the primary volume of scattering ice within the western (glacier-right) part of the glacier, with transparent ice to the east.

In North Glacier there are lenses of transparent ice that exist in the accumulation zone, particularly near the broad valley to the east that leads to a neighbouring glacier. In the ablation zone, the depth of scattering develops a distinctive trough of radar-transparent ice, just left of
Figure 5.18: Scattering layer thickness in meters for South Glacier (a) and North Glacier (b), based on all data from 2008–2011. Checkered regions indicate areas where no data was collected. The scattering layer is defined as the fraction of the ice column between the bed and the upper scattering surface.
Figure 5.19: Scattering layer thickness, normalized to total ice thickness for South Glacier (a) and North Glacier (b), based on all data from 2008–2011. Checkered regions indicate areas where no data was collected. The scattering layer is defined as the fraction of the ice column between the bed and the upper scattering surface.
Figure 5.20: Kriging variance (2.5.2) calculated during interpolation of scattering depth. The stripes occur because kriging variance is lowest on the surveyed lines.
the glacier centreline. Within this trough, scattering sometimes pinches out at the bed, but persists within a lobes on either side of the valley (Figure 5.12a). Except for isolated pockets that may represent the presence of crevasses or moulins, scattering is not observed below the glacier midpoint.

The strength of englacial scattering does appear to vary with radar line geometry. Nearby and intersecting lines with different orientations sometimes indicate different englacial scattering structure (e.g., Figure 5.14, where (a) and (b) would yield a different interpretation than (c)). Where this has been observed, it applies to regions of scattering hypothesized to be directly related to scattering from crevasses or conduits rather than thermal structure.

Figure 5.21: Scattering transition between upper and lower reaches of (a) South Glacier and (b) North Glacier. In South Glacier the scattering depth transitions from being virtually zero in the upper glacier (traces 0–100) to being most of the glacier thickness (traces 100–200). Note that the surveyed line has a 90° direction change part of the way along its length. For North Glacier, crevasses obscure interpretation of scattering layer depth between traces 50–160.

The longitudinal transparent-to-scattering transition, particularly in North Glacier, is some-
times abrupt (Figure 5.21). The steep transition results from structures resembling and interpreted as crevasses in the upper glacier (Figure 5.21a). Crevasses are prevalent in the accumulation zones of both glaciers. Although the scattering layer has been demarcated with this in mind, it is possible that the interpretations in this areas overestimates the extent of ice with high water content.

From the maps of scattering structure combined with temperature measurements, both glaciers are interpreted to have water-rich layers at depth. Note that these interpretations apply to the englacial ice, and are not intended to be extended to the bed conditions. The water-rich layer grows thinner down-glacier. The majority or entirety ice in the upper glacier regions may be temperate. Ice in the tongue of both glaciers is either below the pressure melting point or at the pressure melting point with very low water content. The two glaciers are different in their middle regions, in which thermal structure is heterogeneous. The radar-transparent upper layers and negative temperatures recorded by TACs on both glaciers suggest that heat lost through the upper surface causes ice in these regions to be cold.

5.3.6 Scattering layer mapping accuracy

Mapping scattering layer depth is likely to be substantially less accurate than mapping ice thickness, due to both the ambiguity in identifying diffuse englacial reflectors, the lower-precision picking techniques used, the confounding influence of non-thermogenic englacial features, and the potential for reflection anisotropy. Overlapping lines picked for englacial scattering are typically in good agreement with each other to within 10 m, however in some cases the agreement is much less, as can be seen from Figure 5.14, in which crevasses likely play a role. Similar to the ice thickness mapping estimates, scattering layer mapping suffers from sparse sample in the upper portion of the glaciers’ accumulation areas. Variance maps (Figure 5.20) are calculated in the same way as for ice thickness interpolations (5.2.3), however this does not include picking uncertainty.

5.4 Buried zones of radar-transparent ice

In both South Glacier and North Glacier, there are anomalous radar-transparent regions visible at depth, in which internal reflecting power becomes very low (Figure 5.22). These have been observed using 10 MHz antennas. Higher frequency antennas were not deployed in these re-
Figure 5.22: Anomalous regions of radar-transparent ice at depth, observed in South Glacier in (a) 2008 and (b) 2011 using 10 MHz antennas. Inset maps show the locations represented by the radar data.

regions. The extent of these regions is not represented in the mapping described above, because scattering often exists at shallow depths, and englacial scattering thickness is estimated by marking the upper surface of radar-scattering. As depth increases, radar reflection intensities may decrease even after simple gain corrections are applied because of changes in attenuation rates, but this does not explain lateral differences in scattering strength where the ice thickness changes relatively little.

The prevalence of regions characterized by a radar-scattering layer of ice underlain by radar-transparent ice is uncertain. The upper glacier regions of both glaciers, where these features have been observed, are relatively poorly-sampled compared to the mid and lower glacier regions.

There are a number of hypotheses that might explain the existence of deep non-scattering regions. If the ice were locally cold, then no englacial water would exist to cause reflections. Alternatively, the ice might be temperate, yet non-scattering as observed by Brown et al. (2009). This might occur if water quantities are very low despite the ice reaching the melting point, in which case the non-scattering regions still represent regions of elevated glacier enthalpy. A third possibility is that water production does occur, however some efficient but radar-invisible drainage system keeps englacial water quantities low.
Chapter 6

Modelling Thermal Structure

This chapter reviews methodologies that have been applied to modelling thermal structure. Early models focused on the temperature distributions within glaciers and ice sheets (e.g. Robin, 1955; Dahl-Jensen, 1989). Recently developed models have considered polythermal ice, accounting for the possibility of non-zero water content within ice at the melting temperature (e.g. Greve, 1997a; Aschwanden et al., 2012). Below, I describe the theory and implementation of a two-dimensional polythermal model of glacier thermal structure based on an enthalpy-gradient method in detail.

6.1 History of thermal model development

A significant early attempt to quantitatively describe the temperatures in a large ice mass is presented by Robin (1955). Imagining the geometry of an ice sheet similar to Antarctica to be radially symmetric with a surface elevation described by a quadratic function, Robin (1955) calculates the temperature at the center of the ice sheet, based on heat diffusion with constant diffusivity described by

\[ \frac{\partial \theta}{\partial t} = \kappa \nabla^2 \theta, \]  \hspace{1cm} (6.1)

where \( \kappa \) is thermal diffusivity. The problem of ice sheet temperature (\( \theta \)) is solved in the vertical coordinate alone, reducing the problem to a single dimension and allowing (6.1) to be simplified to

\[ \frac{\partial \theta}{\partial t} = \kappa_d \frac{\partial^2 \theta}{\partial z^2} \]  \hspace{1cm} (6.2)
Surface accumulation at the rate \( \dot{b} \) causes vertical advection in the model domain according to

\[
\frac{\partial z}{\partial t} = \frac{z}{h} \dot{b}.
\]  

(6.3)

By substituting 6.3 into 6.2, the steady equation for temperature at the ice divide is

\[
-\frac{\partial \theta}{\partial z} \frac{h}{H} \dot{b} = \kappa_d \frac{\partial^2 \theta}{\partial z^2}.
\]  

(6.4)

Robin (1955) uses analytical techniques to solve (6.4). The geothermal flux at the bed enters the problem as a basal boundary condition and causes the calculated temperature to rise with depth at the center of the ice sheet.

In order to calculate temperatures outward from the ice center, Robin (1955) treats advection of interior ice as a source term in equation (6.4). Strain heating (the release of potential energy as the ice flows) based on Nye (1951) is added to the basal temperature gradient, because ice deformation should be concentrated near the base (Robin, 1955). In the solution of the model equations, the advection of cold ice from the ice sheet center has a strong influence on the thermal structure elsewhere. The vertical temperature profile develops a concave structure in which it is warmer near the base and the surface due to basal heat sources and rising surface temperatures with decreasing elevation, respectively. In the center of the ice column, advection of ice from the interior causes colder temperatures. The overall structure of the temperature profile predicted by Robin (1955) captures that of the ice sheet as known from contemporary borehole measurements (Blatter et al., 2010), although discrepancies exist particularly regarding near-surface temperature gradients. Robin (1955) suggests that nonstationary climate may play an important role modifying the temperature profiles of some locations on the Greenland ice sheet.

Analytical ice sheet temperature modelling is explored further by Weertman (1968), who compares an extended version of the above model to more complete borehole temperature profiles from Camp Century, Greenland. This study revisits the role of horizontal advection and strain dissipation, and experiments with varying the accumulation rate and horizontal temperature gradients. Failing to closely reproduce the observed profile, Weertman (1968) concludes that temporal variation in the surface conditions must indeed play an important role, as previously suggested by Robin (1955).

Improvements to the approximate incorporation of advection into models of thermal struc-
ture (such as the constant source assumption used by Robin (1955)) generally require numerical methods (Hindmarsh, 1999). More detailed treatments of glacier and ice sheet thermal structure have accompanied advances in computational power and techniques over the last several decades (Blatter et al., 2010). Following the description of a three-dimensional numerical ice-sheet model by Mahaffy (1976), Jenssen (1977) introduced a thermomechanically-coupled model. In thermomechanically-coupled models, the nonlinearity between ice temperature and ice velocity that is introduced from the viscosity term is included. The model of Jenssen (1977) uses an explicit-in-time method, solving for the internal flow field of the Greenland ice sheet and the temperature evolution in succession. With every timestep, the computation addresses velocities, temperature changes, and mass continuity. A later model used by Dahl-Jensen (1989) to calculate steady temperatures along a flowband uses an implicit procedure to solve for velocity and temperature.

Similar models have also been developed and applied to small ice masses such as mountain glaciers. Like the above models, a temperature model used by Pettersson et al. (2007) to predict the thickness of the cold layer on Storglaciären disregards water content. Pettersson et al. (2007) use a simple one-dimensional model to study the sensitivity of the Storglaciären cold layer thickness to ice kinematics and climate. The model domain stretches to fit a predefined number of intervals within the thickness of the cold layer. The ice surface defines the upper boundary, and the cold-temperate transition depth defines the lower boundary. Heat moves within the model by either conduction or advection as the upper boundary ablates and the lower boundary migrates downward. The surface temperature is a Dirichlet boundary condition that varies seasonally. The basal boundary condition representing the base of the cold layer is a Cauchy boundary in temperature, with the derivative specified by the local water content at the boundary, kinematics, and thermodynamic properties of the ice:

\[
\left( \frac{\partial \theta}{\partial z} \right)_{\text{CTS}} = \frac{L_f \omega}{\kappa d c_p} (a_m - w_{\text{CTS}}),
\]

where \( L_f \) is the latent heat of fusion, \( \omega \) is the water content at the CTS, \( c_p \) is the heat capacity of ice, \( a_m \) is the vertical migration rate of the CTS, and \( w_{\text{CTS}} \) is the ice emergence velocity at the CTS (Pettersson et al., 2007).

Using their model, Pettersson et al. (2007) perform sensitivity tests by calculating steady state cold layer thickness for a range emergence rates, cold-temperate transition surface water
contents, and fixed surface temperatures. They observe a pronounced sensitivity to emergence rates, with surface temperature playing a role where emergence rates are low. The importance of the emergence rate implies that heat transfer by advection is a significant component of the energy balance. In comparison with data measurements of the cold layer thickness, Pettersson et al. (2007) find a reasonable correlation with their model predictions. Water content at the temperate layer surface, although measured in scattered locations, represents the largest unknown. The data that exist do not appear to correlate well with variations in cold layer thickness. Although no information regarding temporal changes in temperate ice water content is available, the authors conclude that recent observed thinning of the Storglaciären cold layer is consistent with a roughly 1 K increase in the fixed surface temperature.

Temperate ice is nearly isothermal, which has strong repercussions for heat flow. In order to model polythermal ice masses that contain temperate ice, substantially different physics are required for at least a portion of the model domain. Two approaches have been applied to deal with the problem of temperate ice: one in which the model domain is split into cold and temperate sub-domains, and one in which a so-called “enthalpy-gradient method” is applied (Aschwanden and Blatter, 2009). The former category includes the SICOPOLIS model for Greenland and Antarctica (Greve, 1997a,b). The latter method is at this time primarily represented by the work of Aschwanden and Blatter (2009) and Aschwanden et al. (2012). Sub-domain models must track the freezing front that separates cold and temperate ice and splits the model domain into one in which temperature is the primary state variable, and one in which water content is the primary state variable. Enthalpy-gradient methods reformulate the thermodynamic problems in terms of enthalpy (more accurately, internal energy), allowing cold and temperate ice to be handled on a single grid or mesh. Compared to sub-domain-based polythermal models, enthalpy methods simplify the implementation of data structures to represent complex thermal structures. Compared to models that only consider cold ice, polythermal ice models of both varieties more faithfully conserve energy because energy is not lost when temperature rises to the melting point (Aschwanden et al., 2012).

6.1.1 Modelling small glaciers

Fully thermomechanically-coupled models have been applied to small glaciers in order to investigate the processes that control their thermal structure and dynamics. A cold-ice thermomechanical model described by Pattyn (2002) does not account for temperate ice in polythermal
glaciers and therefore does not fully conserve energy, however it remains useful for the study of mostly-cold high Arctic glaciers. Delcourt et al. (2008) apply this model to McCall Glacier and perform several experiments regarding glacier evolution since the 1950s, at which time the glacier had a geometry similar to that during its most recent advance at the end of the 19th century. They find that temperate ice at the base of the modern glacier is the result of higher heat production when the glacier was larger.

In a similar study on a different glacier, Wohlleben et al. (2009) modelled the thermal structure of John Evans Glacier with the hypothesis that thermal disequilibrium in light of changing climate might also explain the temperate basal ice there. In contrast to the previous result, Wohlleben et al. (2009) found that thermal disequilibrium was not a strong factor controlling the thermal regime, but that extremely high rates of heat dissipation associated with the basal drainage were instead sufficient to maintain a temperate base.

6.2 Development of a two-dimensional model

In order to better understand the thermal structure interpreted from radar-based field studies, I develop a two-dimensional model of glacier thermal structure based on an enthalpy-gradient method. The use of a polythermal-ice model is important because of the large volumes of temperate ice inferred to exist beneath the accumulation zone and upper ablation zone. A summary of the model design precedes a description of the heat source equations that comprise the full model. Finally, the numerical methods used for the solution of the model are presented. Modelling strategy and results are given in Chapters 7 and 8.

6.2.1 Enthalpy-gradient formulation

The enthalpy-gradient method for modelling thermal evolution is an alternative to temperature-based methods. As described previously, the enthalpy method's advantages include energy conservation for temperate ice, as well as simplicity of implementation because only a single state variable is required and because complex polythermal geometries are easily handled.

The starting point for the enthalpy-gradient formulation is Fourier's law of heat conductivity

$$q = k \nabla \theta_c. \quad (6.6)$$
where $q$ is the heat flux and $\theta_c$ is the cold ice temperature. As above, $k$ is heat conductivity. Equation (6.6) is not useful in temperate ice.

In pure ice, both thermal conductivity and the heat capacity are functions of temperature:

\[ k = 9.828 e^{-0.0057T} \]  \hspace{1cm} (6.7)

\[ c_p = 152.5 + 7.122T \]  \hspace{1cm} (6.8)

for $T$ in Kelvin (Cuffey and Paterson, 2010, Ch. 9). In snow, thermal conductivity appears from empirical investigations to be approximately a function of density. A relationship suggested by Sturm et al. (1997) gives snow conductivity as a function of density as

\[ k_{\text{eff}} = 0.138 - 1.01 \times 10^{-3} \rho + 3.233 \times 10^{-6} \rho^2 \]  \hspace{1cm} (6.9)

in units of W m$^{-1}$ K$^{-1}$. Conductivity $k$ in cold ice is related to diffusivity $\kappa_c$ by the specific heat ($c_p$) according to

\[ \kappa_c = \frac{k}{\rho c_p} \]  \hspace{1cm} (6.10)

A relationship for moisture flux ($q_m$) in temperate ice has been suggested by Greve (1997a) as

\[ q_m = q_s + q_l. \]  \hspace{1cm} (6.11)

The sensible heat flux ($q_s$) is a function of temperature gradients, and in temperate ice is a result of the pressure influence on the melting temperature. The latent heat flux ($q_l$) represents the mass flux of englacial water. Enthalpy ($H$) is defined as

\[ H = u + \frac{P}{\rho} \]  \hspace{1cm} (6.12)

for internal energy ($u$), pressure ($P$), and density ($\rho$) (Aris, 1989, Ch. 11). If ice is assumed to be incompressible and to experience no volume change when heated (Aschwanden and Blatter, 2009), then the gradient in enthalpy is identical to the gradient in internal energy

\[ \nabla H = \nabla u. \]  \hspace{1cm} (6.13)
The internal energy of pure ice is a function of temperature and water content, such that

\[ u = c_p (T - T_m) + L_f \omega + u_0 \]  

(6.14)

where \( T_m \) is the local melting temperature, \( \omega \) is the water content, \( c_p \) is the specific heat of ice, \( L_f \) is the latent heat of fusion, and \( u_0 \) is a constant. The melting temperature is a function of ice pressure, such that \( T_m = \theta_m - (\partial T / \partial P)P \), where \( \theta_m \) is the potential melting temperature (273.16 K). I assume that pressure \( P \) is close to hydrostatic. Differentiation of (6.14) and substitution with (6.13) results in

\[ \nabla H = \begin{cases} 
  c_p (\nabla \theta - \nabla \theta_m) & \text{for } T < T_m \\
  L_f \nabla \omega & \text{for } T = T_m 
\end{cases} \]  

(6.15)

The inverse relationships, temperature and water content gradients as functions of enthalpy (internal energy), are easily obtained.

Transport balance requires that change in a quantity equal the gradient of the inward fluxes. The general equation describing such conservation is

\[ \frac{\partial \rho \phi}{\partial t} = \nabla \cdot (\kappa \nabla \phi - \rho \mathbf{u} \phi) + Q \]  

(6.16)

where \( \phi \) is a conserved quantity, \( \mathbf{u} \) is a velocity vector, \( \kappa \) is a diffusivity, and \( Q \) is a source term (Patankar, 1980). In the above, the first term on the right hand side represents the diffusive flux, while the second term represents the advective flux in a velocity field. It follows from (6.6), (6.11), (6.15), and (6.16) that

\[ \frac{\partial \rho H}{\partial t} = \begin{cases} 
  \nabla \cdot (\kappa \phi \nabla H) + \rho \nabla \cdot (\mathbf{u} H) + Q & \text{for } T < T_m \\
  \nabla \cdot q_m + \rho \nabla \cdot (\mathbf{u} H) + Q & \text{for } T = T_m
\end{cases} \]  

(6.17)

The moisture flux \( q_m \) may be considered negligible (Greve, 1997a), or it may be replaced by an explicit flux function. Aschwanden et al. (2012) choose

\[ q_m = \kappa_t \nabla H, \]  

(6.18)

where \( \kappa_t \) is the temperate ice diffusivity, giving the moisture flux a convenient form identical
to (6.6). *Aschwanden and Blatter* (2009) choose $\kappa_t$ to be an order of magnitude smaller than $\kappa_c$, which they report is small enough to be negligible in their test scenario. I take a similar approach, choosing $\kappa_t$ to be two orders of magnitude smaller than $\kappa_c$ based on experimentation. The final form of the governing equation is

\[
\rho \frac{\partial H}{\partial t} = \nabla \cdot (\kappa_c \nabla H) + \nabla \cdot (\rho u H) + Q. \tag{6.19}
\]

This is an advection diffusion equation that describes thermal evolution as a function of the spatial distribution of enthalpy and mass flux.

### 6.2.2 Source terms

The source term $Q$ in (6.19) is modelled as the sum of three individual heat sources

\[
Q = Q_b + Q_{\text{str}} + Q_m, \tag{6.20}
\]

where $Q_b$ is basal heating, $Q_{\text{str}}$ is strain heating, and $Q_m$ is the heat associated with meltwater entrapment and potential refreezing. The basal term $Q_b$ is a combination of frictional heat dissipation from basal sliding and water flow and a term that represents geothermal flux ($Q_{\text{geo}}$). Heating from firn compaction (described in Chapter 1) is small and not accounted for.

I prescribe geothermal fluxes to be constant in both time and space. With basal shear stress $\tau_b$ and basal sliding rate $u_b$, the basal heating term is represented as

\[
Q_b = u_b \tau_b + Q_{\text{geo}}. \tag{6.21}
\]

Following (*Cuffey and Paterson*, 2010, Ch. 9), the strain heating term is

\[
Q_{\text{str}} = 2\tau \dot{\epsilon}, \tag{6.22}
\]

with deviatoric stress $\tau$ and strain rate $\dot{\epsilon}$. The two-dimensional implementation of the thermal model does not represent stresses and deformation rates in the flowband-orthogonal direction.
(y), so I parameterize these following Pimentel et al. (2010). Assuming no slip at the valley wall,

\[ \dot{\epsilon}_{xy} \approx -\frac{u_x}{2W} \]  

(6.23)

\[ \tau_{xy} = A^{-\frac{1}{n}} \epsilon_{xy}^{\frac{3}{n}} \]  

(6.24)

for longitudinal velocity \( u_x \), valley half-width \( W \), and flow-law exponent \( n \).

The surface heating term \( Q_m \) is calculated by assuming that meltwater generation is related only to the difference between the surface air temperature \( T_s \) and the melting temperature \( T_m \) by means of a constant degree-day factor \( f_{dd} \), such that

\[ Q_m = (1 - r) \rho w h_{aq} f_{dd} L_f [\min (T_s - T_m, 0)] , \]  

(6.25)

where \( L_f \) is the latent heat of fusion for water. Within the accumulation zone, a fraction \( r \) of the annual surface melt is removed as runoff while the firn captures the remaining meltwater and stores it in a near surface aquifer, similar to the method proposed by Reeh (1991). The near surface aquifer is restricted to the accumulation zone and its thickness \( h_{aq} \) is assumed constant in time. This parameter physically represents the thickness in the upper glacier column that water may penetrate by percolation. Because of refreezing and the formation of superimposed ice lenses, the near surface aquifer thickness may be thinner than the total firn thickness. It is reasonable to expect that the aquifer thickness may vary spatially. For example, the near surface aquifer may be thicker at high elevations resulting in a tapered shape. Alternatively, colder temperatures may cause faster refreezing and reduce high elevation aquifer porosity.

In light of uncertainties regarding the spatial variability of the near surface aquifer, I make the minimal assumption that near surface aquifer thickness is constant in space.

Water captured in the near surface aquifer is stored until it either freezes or exceeds a drainage threshold \( \omega_{aq} \), above which all water is assumed to contribute to runoff and is removed. The threshold in the englacial aquifer \( \omega_{eng} \) is treated similarly (following Greve (1997a)) but is much lower than in the near-surface aquifer to account for lower porosity in ice compared to the surface layer. Field observations have found water content frequently varies with depth (e.g. Murray et al., 2000), suggesting that the maximum water content may depend on the strain and recrystallization history of the ice (Lliboutry, 1976). No heat is added at the surface in the glacier ablation zone, as all meltwater is assumed to be removed by the end of the melt.
season as supraglacial runoff or through crevasses and moulins and therefore unavailable for refreezing or entrapment. No heat exchange between a cryo-hydrologic system and the ice, as proposed by Phillips et al. (2010), is represented.

6.2.3 Boundary conditions

The basal boundary condition for (6.19) is a Neumann boundary with a gradient representative of the prescribed bedrock thermal gradient and local heat generation \( Q_b \) such that

\[
\left. \frac{\partial H}{\partial z} \right|_{z=0} = Q_b \frac{c_p}{k}.
\]

(6.26)

The upper boundary condition for (6.19) is a Dirichlet-type boundary pinned to match either the air temperature or the ice melting point, whichever is lower. The snow and firn layer are modelled as a low density layer at the surface, where diffusivity is calculated according to (6.9). The vertical density profile in the accumulation zone is assumed to vary according to (Schytt, 1958)

\[
\rho(z) = \rho_i - (\rho_i - \rho_s) e^{-Cz}.
\]

(6.27)

A single choice of \( C \) is used for the entire accumulation zone. In the ablation zone, the grid nodes within a transient snow layer of specified thickness \( h_{\text{sd}}(t) \) have density, \( \rho = \rho_s \). I have found the enthalpy difference between an ablation zone column in which density near the surface is directly varied in this way and that in a column with the accumulation and ablation of a snow layer explicitly accounted for to be acceptably small (<0.35 K equivalent) for the present purposes.

6.2.4 Numerical solution

Equation (6.19) steps forward in time on a two-dimensional structured grid that is irregularly-spaced on the vertical axis \( z \) and regularly-spaced on the horizontal axis \( x \) (Figure 6.1). The model domain is transformed such that the glacier bed is at \( z = 0 \) regardless of its elevation and the glacier surface is at the uppermost node, similar to the vertical transformation applied by Hindmarsh and Hutter (1988). For calculating source terms, a transformation back to real elevation coordinates is used. The grid spacing in the \( z \)-dimension is finely resolved at the surface and basal boundaries and coarser in the glacier interior. A specified number of cells
Figure 6.1: The structured model domain for the finite difference scheme. The uppermost cell height is 0.5 m. Cell height increases linearly for the upper 10 layers and then stays at a constant thickness that depends on ice thickness until reaching the base. The lowermost two cells are thinner to improve the modelled accuracy of basal heat flux $Q_b$.

...increase in thickness downward at a constant rate from a minimum thickness at the surface. The cell thickness remains constant in the glacier interior, while the bottom two layers are half the thickness of the interior cells.

Because of the small thickness-to-length ratio of glaciers, I neglect horizontal diffusion by rewriting the first term on the right-hand side of (6.19) as

$$\nabla H \approx \frac{\partial H}{\partial z}. \tag{6.28}$$

This “shallow energy approximation” corresponds to an identical simplification made by Aschwanden et al. (2012) while implementing an enthalpy gradient method for use in PISM$^1$.

Finite differences are an easily implemented and frequently used method for solving partial differential equations. For the numerical scheme, I split the right-hand side of (6.19) into two parts, representing diffusive processes and advective processes in the first and second terms, respectively. Applying the approximation (6.28), the diffusive component can be written as

$$\frac{\rho}{\partial t} \Bigr. \bigg|_{\text{diff}} = \frac{\partial}{\partial z} \left( \kappa_c \frac{\partial H}{\partial z} \right). \tag{6.29}$$

$^1$Parallel Ice Sheet Model, see Bueler et al. (2007); Bueler and Brown (2009)
The explicit finite-difference representation of (6.29) is unstable for large timesteps or high diffusivities (Patankar, 1980). Implicit schemes have better stability characteristics, so I use the implicit Crank-Nicolson scheme (Crank and Nicolson, 1947) adapted for variable diffusivity ($\kappa$) and variable grid cell size ($\Delta z$), given by

$$
\rho \left( v_{j+1}^{t+1} - v_j^t \right) = \frac{\Delta t}{2} \left( \frac{\kappa_{j+\frac{1}{2}} d_j^t}{\Delta z_*} v_{j+1}^t - \frac{\kappa_{j-\frac{1}{2}} d_j^t}{\Delta z_*} v_j^t \right) \\
+ \frac{\Delta t}{2} \left( \frac{\kappa_{j+\frac{1}{2}} d_{j+1}^{t+1}}{\Delta z_*} v_{j+1}^{t+1} - \frac{\kappa_{j-\frac{1}{2}} d_{j-1}^{t+1}}{\Delta z_*} v_j^{t+1} \right) 
$$

(6.30)

where $v$ is the numerical approximation of a conserved quantity (enthalpy, in this case) and $\Delta t$ is the timestep. The value $v$ is evaluated at grid nodes, while $\kappa$ and $\Delta z$ are properties of grid cells. In the notation above, superscripts ($t-1, t, t+1$) denote values at times prior to, at, and subsequent to the time being calculated, respectively. Subscripts ($j-1, j, j+1$) indicate the analogous relationships in space (before, at, and next to the node being calculated). The subscript $*$ denotes the mean of cell-averaged values at a node, such that

$$
\Delta z_* = \frac{2}{\Delta z_{j-\frac{1}{2}} + \Delta z_{j+\frac{1}{2}}}. 
$$

(6.31)

Dirichlet boundary conditions are easily implemented by prescribing the boundary value in $v$ and modifying (6.30) such that $\left( v_{j+1}^{t+1} - v_j^t \right) = \Delta v$. As the surface boundary is a Dirichlet condition that depends on the air temperature, the ice melting temperature, and surface water content, I require that

$$
v_{nz}^t = H_{nz} 
$$

(6.32)

where $n_z$ is the number of grid nodes and $H_{nz}$ is the surface enthalpy. Neumann boundary conditions require that the spatial derivative of $v$ at a boundary is equal to a prescribed value ($\eta$). This is implemented by introducing ghost values ($v'$) to the Neumann boundaries. Based on a centered difference approximation, these ghost values must be

$$
v' = \begin{cases} 
  v_2 - 2\Delta z\eta & \text{at } z = z_b \\
  v_{n_z-1} + 2\Delta z\eta & \text{at } z = z_s 
\end{cases} 
$$

(6.33)
in order to satisfy the Neumann boundary values.\footnote{Note that in the above notation, the first node at \( z = z_b \) bears the subscript 1 and the final node at \( z = z_s \) bears the subscript \( n_z \). These correspond to the basal and surface nodes, respectively.} In the finite difference scheme (6.30), instances of the ghost values \( v' \) can be replaced by these expressions to implement Neumann boundary conditions.

The system of equations defined by (6.30) can be arranged as a linear matrix problem

\[
LV^{t+1} = RV^t
\]  

(6.34)

where \( V^t, V^{t+1} \) are vectors of \( v \) at the present and future timesteps, respectively. The right hand side contains known values, and can be explicitly multiplied. The system (6.34) is then solved for \( V^{t+1} \). Because \( L \) is a tridiagonal banded matrix, it is possible to use sparse matrix methods, however for the small problem sizes involved, this has not in practice proven to be computationally faster.

Several minor simplifications are introduced at this point. Although thermal conductivity (6.7) and heat capacity (6.8) in cold ice are functions of temperature, this dependence is relatively weak. Over the temperature range 260–273 K, conductivity and heat capacity vary by 8% and 4% respectively. This weak temperature-dependence has not been incorporated into the model, and values measured for ice at the melting temperature have been used instead (c.f. Aschwanden et al., 2012). Furthermore, at enthalpies very near the melting point, the transition in diffusivity (\( \kappa_c \to \kappa_t \)) is represented by a smooth function. This occurs over an enthalpy range of 110 J, which is equivalent to 0.025 K below the melting point and 0.008% water content above.

To test the implementation of the Crank-Nicolson scheme (6.30), I have compared the resulting numerical solutions to an analytical solution (Farlow, 1993, Ch. 2). The results show a good match between the analytical and numerical solutions over a range of grid choices (Figure 6.2).

The advective part of (6.19) can be written in the form

\[
\rho \frac{\partial H}{\partial t}_{\text{adv}} = -\rho \mathbf{u} \nabla H.
\]  

(6.35)

Although horizontal gradients in enthalpy are typically small compared to vertical gradients, the horizontal component of the velocity field can be large. This means that no shallow approximation is valid, because in contrast to (6.29), the solution is not clearly dominated by fluxes in one
Figure 6.2: Analytical solution to the diffusion equation (6.29) with constant diffusivity (a) and the difference (b) compared to the numerical solution. Note the logarithmic vertical scale in (b). The space step is 0.02. The grey lines denote different times, with an interval of 0.1. The dashed line in (a) represents the initial value $v_0$.

dimension over another.

Within an advective flow, the solution to $v^{i+1}$ at a position $x_j$ depends on the character of $v^i$ at and upstream of $x_j$ (LeVeque, 1992). The explicit one-dimensional first-order upwind scheme exploits this property through the approximation:

$$v_j^{n+1} - v_j^n = \begin{cases} -u(v_j^n - v_{j-1}^n) & \text{if } u > 0 \\ -u(v_{j+1}^n - v_j^n) & \text{if } u < 0 \end{cases}$$

(6.36)

The solution to (6.36) is highly-diffusive (Tsui, 1991), meaning that regions where the derivative of $v$ is large tend to become smooth over time. Several higher-order upwind schemes address this diffusivity, such as this second-order upwind scheme:

$$v_j^{n+1} - v_j^n = \begin{cases} -u\left(\frac{3}{2}v_j^n - 2v_{j-1}^n - \frac{1}{2}v_{j-2}^n\right) & \text{if } u > 0 \\ -u\left(\frac{1}{2}v_{j+2}^n - 2v_{j+1}^n - \frac{3}{2}v_j^n\right) & \text{if } u < 0 \end{cases}$$

(6.37)

Even when stable, the scheme above develops oscillatory artifacts near steep gradients.

It has been shown\(^3\) that all linear numerical methods of greater than first-order accuracy suffer from non-monotonicity (LeVeque, 1992, Ch. 16). Second-order and higher schemes such as (6.37) are equivalent to (6.36) plus a correction term. One solution is to use a scheme that applies the correction term from a higher-order scheme such as (6.37) to a monotonicity-

---

\(^3\)This is Godunov’s Theorem (Godunov, 1959)
preserving scheme such as (6.36) but that gradually attenuates the correction in the neighbourhood steep gradients. This is the strategy behind “high-resolution methods” (LeVeque, 1992, Ch. 16), and has been adopted for solving (6.35).

In order to apply a high-resolution method, it is necessary to select a function for scaling the high-order correction term. Such a function is called a flux-limiter (LeVeque, 1992). Van Leer’s flux-limiter is reliable (Sweby, 1984). The limiter is defined as

\[
\phi(r) = \frac{|r| + r}{|r| + 1},
\]

where \( r \) is a smoothness parameter

\[
r = \frac{v^n_j - v^{n-1}_{j-1}}{v^n_{j+1} - v^n_j}.
\]

Then, the flux-limited scheme \( f_{\text{FL}}(V) \) is related to the low-order scheme \( f_L(V) \) and the high order scheme \( f_H(V) \) by

\[
f_{\text{FL}}(V) = f_L(V) + \phi(r) [f_H(V) - f_L(V)]
\]

Where \( \phi(r) = 1 \), the result is equivalent to the second-order upwind scheme (6.37). Where \( \phi(r) = 0 \), the result is equivalent to the first-order upwind scheme (6.36). It is possible that \( \phi(r) > 1 \), which amplifies the higher-order correction.

A validation exercise compares the results of the three schemes described above with the
analytical solution in a trivial case with constant velocity in one-dimension (Figure 6.3). The result from the finite differencing schemes is shown after 1000 timesteps. The sharp gradients in the analytical solution cause problems for all numerical schemes. The flux-limited scheme provides a compromise between the higher spatial accuracy and lower diffusion associated with the second-order upwind scheme, and the monotonicity of the first-order upwind scheme.

6.2.5 Ice dynamics

Stress and velocity fields are computed externally from the thermal model. The momentum balance in an incompressible fluid is given by (Greve and Blatter, 2009, Ch. 3):

$$\rho \frac{dv}{dt} = \nabla \tau - \nabla P + f.$$  \hspace{1cm} (6.41)

The acceleration term (left) in a glacier is small and can be neglected (Greve and Blatter, 2009, Ch. 5). The body force $f$ is dominated by the force of gravity $\rho g$. These modifications yield

$$0 = \nabla \tau - \nabla P + \rho g.$$  \hspace{1cm} (6.42)

Scaling arguments can be used to show that in a fluid with a low aspect ratio ($H/L$) such as a glacier or ice sheet, the horizontal variation in the vertical shear stresses $\partial \tau_{xz}/\partial x$ and $\partial \tau_{yz}/\partial y$ are small (Blatter, 1995). Their removal yields the first-order shallow ice approximation, or “Blatter-type” model, which is expanded as

$$0 = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \frac{\partial \tau_{yz}}{\partial z} - \nabla P + \rho g.$$  \hspace{1cm} (6.43)

Further simplification can be achieved by assuming that only the vertical derivative of the horizontal shear stresses is important and that the pressure gradient is hydrostatic, resulting in the zeroth-order shallow ice approximation:

$$\frac{\partial \tau_{xz}}{\partial z} = \frac{\partial P}{\partial x}, \quad \frac{\partial \tau_{yz}}{\partial z} = \frac{\partial P}{\partial y}.$$  \hspace{1cm} (6.44)

Stress and velocity fields computed using a “Blatter-type” first-order approximation (FOA) for ice flow described by Pimentel et al. (2010) and the zeroth-order shallow ice approximation.
(SIA) are available for coupling with the thermal model. To reduce the stress balance to a two-dimensional flowband, the \(\partial / \partial y\) terms from (6.43) and (6.44) are discarded and the lateral deformation rate \(\dot{\epsilon}_{xy}\) is parameterized using equation (6.23). Glen’s flow law

\[
\dot{\epsilon}_{ij} = A \tau^{n-1}_E \tau_{ij}
\]  

(6.45)

defines the rheology necessary to relate ice deformation rates to the stress field. In the FOA model (6.43), an iteration scheme between velocity and viscosity is required for the solution (e.g. Pimentel et al., 2010). The zeroth-order approximation (6.44) can be solved directly.

In the case of the FOA, sliding is included in the form of a Coulomb friction law (Schoof, 2005; Gagliardini et al., 2007; Pimentel and Flowers, 2010). I have prescribed basal water pressures in terms of flotation fractions in order to reproduce observed surface velocities as in Flowers et al. (2011).

### 6.2.6 Flow-law coefficient

The flow-law coefficient \(A\) in (6.45) provides the coupling path from the thermal model (6.19) and the ice dynamics models. It is based on the enthalpy field at every flow mechanics timestep. In cold ice, the temperature-dependent flow-law coefficient is computed following the recommendation of (Cuffey and Paterson, 2010, Ch. 3) as

\[
A = A_0 \exp \left( -\frac{Q_c}{R} \left( \frac{1}{T(H) + \Delta T_m} - \frac{1}{263 + \Delta T_m} \right) \right)
\]

(6.46)

\[
\Delta T_m = \frac{\partial T_m}{\partial P} P
\]

\[
T(H) = \frac{H}{\rho c_p} + T_0
\]

where \(\Delta T_m\) represents the pressure correction for the melting temperature and \(T(H)\) is the temperature of cold ice as a function of enthalpy. \(T_0\) is an arbitrary reference temperature, below which enthalpy is represented within the model as being negative.

Because of the large masses of temperate ice that may exist in polythermal glaciers, the additional softening associated with non-zero water content may be important. In temperate ice, I try multiplying the flow-law coefficient by an enthalpy-dependent enhancement factor,

\[
A_e = e_w(H)A
\]

(6.47)

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using a linear function $e_w(H)$ based on results of Duval (1977) that indicate that $A_w$ is roughly tripled with a 1% water content. This approximately spans the range of observed strain enhancement in temperate glaciers (Cuffey and Paterson, 2010), so I do not extrapolate further.

6.2.7 Model algorithm

The previous section establish methods to solve the diffusive and advective parts of (6.19) and introduce the theory used to calculate stress and velocity fields. The model algorithm for computing thermal evolution over one timestep ($\Delta t$) starting at time $t$ proceeds as follows:

1. Adjust the surface boundary condition and surface density in the ablation zone for time $t$. Using an external ice dynamics model (described below), calculate the flow velocity field $u$ consistent with mass continuity and ice rheology.

2. Calculate the enthalpy change following advection (6.40) from $t \to t + \frac{1}{2}\Delta t$, and update $H$. Calculate the enthalpy change from diffusion (6.30) from $t \to t + \Delta t$, and update $H$. Calculate the enthalpy change from advection (6.40) from $t + \frac{1}{2}\Delta t \to t + \Delta t$, and update $H$.

3. Calculate the source term $Q$ derived from strain heating, basal heat fluxes, and meltwater entrapment. Integrate $Q$ over $\Delta t$ and add it to $H$.

4. Check for water contents that exceed $\omega_{\text{eng}}$ or $\omega_{\text{aq}}$ and apply the drainage function. To avoid instabilities where $\Delta z$ is very small, enforce an enthalpy in ice less than 5 m thick that is in equilibrium with the surface temperature.

Timesteps for the thermal model are limited by the requirement that seasonal changes in $Q_m$ (6.25) be resolved. At every timestep, the thermal loop above is run, and the resulting enthalpy is used to update the flow-law coefficient for the external flow model. The stress and velocity fields are taken as the flow model output, and fed back to the thermal loop for the next timestep. This scheme is explicit in time, and has been proven to be convergent in the case of the shallow ice approximation and a cold ice energy balance (Bueler et al., 2007). Explicit and semi-implicit timestepping schemes iterate in time rather than at each timestep, and are widely used in the glacier and climate modelling community when time-dependent solutions are required. Other iteration techniques lead to faster convergence for steady solutions (e.g. Dahl-Jensen, 1989), but have not been implemented as a part of this model.
Time-stepping between the flow mechanics and thermal model components is synchronous in the SIA model and asynchronous in the FOA model. The increased computational requirements of the FOA make synchronous coupling impractical on the sub-seasonal timesteps required by the thermal model, so instead I choose a half year timestep for the flow mechanics. When ice geometry is allowed to evolve in prognostic models, the glacier surface elevation changes with each timestep based on the mass continuity equation.
Chapter 7

Simulating a Synthetic Glacier

Before applying the model described in Chapter 6 to South Glacier and North Glacier, I have explored the controls on thermal structure within synthetic glaciers in order to develop general hypotheses about the processes relevant to glacier thermal regimes. This chapter is organized into three experiments. These experiments investigate (1) the primary controls on glacier thermal structure, (2) the sensitivity of thermal structure to environmental and model parameters, and (3) changes in thermal structures accompanying rising air temperatures.

7.1 Modelling strategy

I use a simple glacier geometry to isolate the influence of individual environmental and internal variables on thermal structure. Simple glacier geometries also help preserve generality by avoiding effects introduced by irregularities in the prescribed surface and bed topography that might be unique to individual glaciers. I represent the bed as a low-order polynomial function (Fig. 7.1) that is very coarsely similar to the bed of North Glacier:

\[ z_b(x) = 5303x^4 - 13046x^3 + 11093x^2 - 4458x + 1126. \]  

Net balance is approximated as a linear function of ice surface elevation, with a prescribed equilibrium line altitude \( z_{ELA} \) and balance gradient \( \partial \dot{b}_n / \partial z \). I cap the maximum annual accu-
mulation at \( \dot{b}_{\text{max}} \), giving the annual balance function a piecewise-linear shape:

\[
\dot{b}(z) = \begin{cases} 
  z(\partial \dot{b}_n / \partial z) - z_{\text{ELA}}(\partial \dot{b}_n / \partial z) & \text{if } z < z_{\text{max}} \\
  \dot{b}_{\text{max}} & \text{if } z \geq z_{\text{max}}.
\end{cases}
\] (7.2)

where \( z_{\text{max}} = \dot{b}_{\text{max}} / (\partial \dot{b}_n / \partial z) + z_{\text{ELA}} \). Accumulation is not addressed directly but rather is implicitly assumed to compensate for the melt calculated in Eq. (6.25) such that the sum matches the prescribed net balance. In other words, accumulation is not calculated independently, but is determined as the difference between the net balance (7.2) and the melting calculated from a degree day model.

I design a pair of steady-state reference models incorporating all of the heat sources described above. The first, a largely temperate reference model (REFT), arises from the parameter values given in Table 7.1. The second (REFC) is a colder version of the first produced by shifting the air temperature (\( T_{\text{ma}} \)) down by \(-1.5 \) K. The REFT model glacier is polythermal (Fig. 7.1a), with a distribution of temperate ice that is similar to the type “C” configuration illustrated by Blatter and Hutter (1991). Ice within the accumulation zone of the glacier is largely temperate, while a surface layer of cold ice develops in the ablation area. The bed at the terminus is cold. Similar thermal structure has been observed in Svalbard (Dowdeswell et al., 1984), in Scandinavia (Holmlund and Eriksson, 1989), and on the continental side of the Saint Elias Mountains in Yukon, Canada (Chapter 5). The thermal structure of this reference model is heavily influenced by meltwater entrapment in the accumulation zone. By comparison, meltwater entrapment plays a more limited role in the REFC model (Fig. 7.1b). Lower surface temperatures decrease the quantity meltwater production in the accumulation zone, thus reducing the amount of heat generated at the surface. The REFC model exhibits a type “D” thermal structure as identified by Blatter and Hutter (1991), with a temperate zone in the lower ice column. As in REFT, REFC is frozen to the bed at the terminus.

### 7.1.1 Experiment 1: Heat source contributions

In order to investigate the relative importance of different heat sources, I begin with the REFT model and individually remove the contributions from strain heating \( Q_{\text{str}} \), meltwater entrapment \( Q_m \), and basal heating \( Q_b \) before recomputing steady-state thermal structure. I test the effect of allowing glacier geometry to evolve in response to changes in ice viscosity governed by Eqs.
Table 7.1: Physical constants and modelling parameters used for diagnostic model runs. *See text.

<table>
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<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
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<tr>
<td>(\kappa_c)</td>
<td>Cold ice diffusivity</td>
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<td>kg m(^{-1}) s(^{-1})</td>
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<td>(\kappa_t)</td>
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<td>kg m(^{-3})</td>
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<td>J kg(^{-1})</td>
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<td>K Pa(^{-1})</td>
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<td>m</td>
</tr>
<tr>
<td>(A_0)</td>
<td>Flow prefactor</td>
<td>3.5( \times 10^{-25})</td>
<td>s(^{-1}) Pa(^{-3})</td>
</tr>
<tr>
<td>(n)</td>
<td>Glen’s flow-law exponent</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>(Q_c)</td>
<td>Creep activation energy</td>
<td>115e3</td>
<td>J mol(^{-1})</td>
</tr>
<tr>
<td>(R)</td>
<td>Ideal gas constant</td>
<td>8.314</td>
<td>J K(^{-1}) mol(^{-1})</td>
</tr>
<tr>
<td>(g)</td>
<td>Gravitational constant</td>
<td>-9.81</td>
<td>m s(^{-2})</td>
</tr>
<tr>
<td>(\partial b_n/\partial z)</td>
<td>Mass balance gradient</td>
<td>4( \times 10^{-3})</td>
<td>m a(^{-1}) m(^{-1})</td>
</tr>
<tr>
<td>(\dot{b}_{\text{max}})</td>
<td>Maximum mass balance</td>
<td>1.5</td>
<td>m a(^{-1})</td>
</tr>
<tr>
<td>(z_{\text{ELA}})</td>
<td>ELA</td>
<td>650</td>
<td>m</td>
</tr>
<tr>
<td>(\alpha_s)</td>
<td>Snow depth amplitude</td>
<td>1.75</td>
<td>m</td>
</tr>
<tr>
<td>(\phi_s)</td>
<td>Snow depth phase shift</td>
<td>0</td>
<td>days</td>
</tr>
<tr>
<td>(h_{s_{\text{avg}}})</td>
<td>Mean snow depth</td>
<td>1.4</td>
<td>m</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Annual air temperature amplitude</td>
<td>9.38</td>
<td>K</td>
</tr>
<tr>
<td>(T_{\text{ma}})</td>
<td>Mean air temperature at z = 0</td>
<td>1.0</td>
<td>K</td>
</tr>
<tr>
<td>(\partial T/\partial z)</td>
<td>Atmospheric lapse rate</td>
<td>-0.0065</td>
<td>K m(^{-1})</td>
</tr>
<tr>
<td>(\Delta T_H)</td>
<td>Thermal model timestep</td>
<td>20–60</td>
<td>days</td>
</tr>
<tr>
<td>(\Delta T_M)</td>
<td>SIA dynamics model timestep</td>
<td>36.5</td>
<td>days</td>
</tr>
<tr>
<td>(\Delta T_{\text{FOA}})</td>
<td>FOA dynamics model timestep</td>
<td>182.5</td>
<td>days</td>
</tr>
<tr>
<td>(n_z)</td>
<td>Vertical nodes (SIA)</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>(n_{x_{\text{FOA}}})</td>
<td>Vertical nodes (FOA)</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>(n_x)</td>
<td>Horizontal nodes</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>(x_{\text{max}})</td>
<td>Domain length</td>
<td>9</td>
<td>km</td>
</tr>
<tr>
<td>(W)</td>
<td>Glacier half-width</td>
<td>800</td>
<td>m</td>
</tr>
</tbody>
</table>
Figure 7.1: Distribution of cold and temperate ice in reference models REFT and REFC with the shallow ice approximation (a,b) and the first-order approximation (c,d) for ice dynamics. Prescribed mean air temperature $T_{ma}$ is lowered by 1.5 K in REFT to obtain REFC. For REFC, the glacier ice surface is held fixed at the REFT geometry.

(6.46) and (6.47), and compare the results to simulations with a fixed glacier geometry.

The appropriateness of holding the surface geometry fixed depends on the degree by which thermal structure alters ice fluidity in Eqs. (6.46, 6.47). Because of the large differences in temperate ice volume in the REFT and REFC models, I examine the effect that flow-coefficient parametrization has on both.

7.1.2 Experiment 2: Parameter sensitivity

I vary selected parameters in order to explore the sensitivity of steady-state thermal structure to environmental conditions. Each parameter in Table 7.2 is first adjusted independently using the REFT model as the control. I perform tests using both the temperature-dependent flow-law coefficient $A$ and the enhanced flow-law coefficient $A_e$. In reality, the parameters in Table 7.2 are not independent, but considering them as such yields information about the environmental variables controlling thermal structure. Recognizing that some of these variables are correlated, I vary air temperature ($T$), equilibrium line altitude ($z_{ELA}$), and near-surface aquifer thickness ($h_{aq}$) together in order to explore the effects of more realistic forcing regimes. To simplify the interpretation of the results, ice geometry is held fixed.

I use the results of the sensitivity tests to draw preliminary conclusions about how glacier
Table 7.2: Model parameters varied in sensitivity tests.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Reference value</th>
<th>Test range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_b$</td>
<td>Basal heat flux</td>
<td>55</td>
<td>0–1000</td>
<td>mW m$^{-2}$</td>
</tr>
<tr>
<td>$f_{dd}$</td>
<td>Degree-day factor</td>
<td>4.0</td>
<td>1.0–7.0</td>
<td>mm K$^{-1}$ day$^{-1}$</td>
</tr>
<tr>
<td>$r$</td>
<td>Runoff fraction</td>
<td>0.5</td>
<td>0.2–0.8</td>
<td>—</td>
</tr>
<tr>
<td>$\omega_{eng}$</td>
<td>Max ice water content</td>
<td>1.0</td>
<td>0–5</td>
<td>%</td>
</tr>
<tr>
<td>$\omega_{aq}$</td>
<td>Max aquifer water content</td>
<td>10.0</td>
<td>1–15</td>
<td>%</td>
</tr>
<tr>
<td>$h_{aq}$</td>
<td>Aquifer thickness</td>
<td>3.0</td>
<td>0.5–6.0</td>
<td>m</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>Air temperature offset</td>
<td>0.0</td>
<td>0.0–7.0</td>
<td>K</td>
</tr>
<tr>
<td>$z_{ELA}$</td>
<td>Equilibrium line altitude</td>
<td>650</td>
<td>450–800</td>
<td>m</td>
</tr>
<tr>
<td>$C_u$</td>
<td>Advection multiplier</td>
<td>1.0</td>
<td>0.2–2.0</td>
<td>—</td>
</tr>
</tbody>
</table>

thermal structure may evolve in a changing environment. To estimate how near-surface aquifer thickness and equilibrium line altitude might co-evolve with air temperature, I make use of balance sensitivities:

$$\Delta b_n \approx \frac{\partial b_n}{\partial T} \Delta T,$$  \hspace{1cm} (7.3)

where $b_n$ is the net balance and $\frac{\partial b_n}{\partial T}$ can be estimated from field data (e.g. de Woul and Hock, 2005; Oerlemans et al., 2005). Equation (7.3) defines how equilibrium line altitude varies with changes in air temperature. For Experiments 2 and 3, I introduce the assumption that the near-surface aquifer thickness is related to net balance closely enough that net balance sensitivity estimates are applicable and equivalent to aquifer thickness sensitivities:

$$\frac{\partial h_{aq}}{\partial T} \equiv \frac{\partial b_n}{\partial T}.$$  \hspace{1cm} (7.4)

Because glacier geometry is held fixed in Experiment 2, the results of the sensitivity tests do not represent physically consistent thermal regimes and are not directly representative of future thermal structures. To ensure that this simplification does not critically influence the results, all sensitivity tests are therefore repeated with a freely evolving ice surface.
7.1.3 Experiment 3: Prognostic modelling

Transient feedbacks between variables such as mean annual air temperature and accumulation zone extent can be expected, but are not represented in the experiments above. In order to capture such feedbacks and make realistic projections of thermal structure, glacier geometry must be permitted to evolve. I perform prognostic simulations with a range of transient climate forcing scenarios. These scenarios are distinguished by the extent to which the winter balance offsets increasing summer ablation. The initial conditions are the REFT and REFC models. I prescribe an average annual air temperature that increases linearly by 2.5 K over 100 a and then stabilizes. For each model timestep, the near-surface aquifer thickness and equilibrium line altitude are adjusted to track the prescribed temperature according to Eqs. (7.3, 7.4). A fraction of the ablation response to changing temperature is assumed to be offset by changing winter balance. Nineteen possible winter balance responses to warming that offset ablation by fractions spanning 5%–95% are tested.

7.1.4 Parameter values and ranges

The parameter ranges given in Table 7.2 are chosen to span the spectrum of interesting and physically-meaningful model behaviour. The geothermal flux \( Q_{\text{geo}} \) component of \( Q_b \) is poorly constrained in many mountainous regions as well as below the major ice sheets. I take \( Q_{\text{geo}} = 55 \text{ mW m}^{-2} \) as a reference value broadly representative of continental heat flux (Blackwell and Richards, 2004) and set the minimum value of \( Q_b \) to this value of \( Q_{\text{geo}} \). The maximum value tested for \( Q_b \) (1000 mW m\(^{-2}\)) is larger than recent estimates of maximum continental heat fluxes by a factor of about five (Davies and Davies, 2010). Additional heat derived from frictional heating or dissipation from subglacial drainage has been inferred to increase the basal heating term by a factor of ten, to roughly 540 mW m\(^{-2}\) (Clarke et al., 1984).

The degree-day factor \( f_{\text{dd}} \) provides a convenient method by which to estimate the summer mass balance based on surface air temperature. The value of the degree-day factor depends to a large extent on the way in which incoming energy is partitioned between different energy balance components (Hock, 2003). Hock (2003) compiled degree-day factors derived for snow at glacierized sites ranging from 2.7 to 11.6 mm d\(^{-1}\) K\(^{-1}\). Values for ice are typically larger, but are not used here; in our model the degree-day factor is only used to calculate meltwater entrapment (Eq. 6.25) in the accumulation zone, where snow cover is assumed to be perennial.
The range chosen (Table 7.2) spans the commonly reported values tabulated by Hock (2003).

Run-off fractions provide a convenient means of estimating internal accumulation (e.g. Reeh, 1991), although comparisons with more developed methods find this approach to have limited skill in predicting the thickness of superimposed ice (Wright et al., 2007). The value $r = 0.4$ has been reported for Greenland near the run-off limit (Braithwaite et al., 1994), but this should vary substantially depending on firn thickness, firn temperature, and summer mass balance. With a reference value of $r = 0.5$, I alter the run-off fraction between 0.2–0.8 in order to evaluate the contribution of water entrapment.

Englacial water content is poorly constrained by observation, with recent results from under 1% (Pettersson et al., 2004), up to several percent (Macheret and Glazovsky, 2000). Although our reference model enforces immediate drainage for water content ($\omega_{\text{eng}}$) above of 1%, it is likely that the permeability and drainage properties of ice vary spatially, for example as suggested by Lliboutry (1976). Firn porosity has been reported by Fountain (1989) for South Cascade Glacier to be 0.15 with 61% saturation. I assume that the properties of the near-surface aquifer are similar to that of the firn and use a near-surface maximum water content $\omega_{\text{aq}}=10\%$ as a reference value.

A suitable choice for what I call near-surface aquifer thickness ($h_{\text{aq}}$) depends on climatology. Braithwaite et al. (1994) report a percolation depth of 2–4 m on Greenland, while Fountain (1989) estimates the aquifer thickness on South Cascade Glacier in Washington to be 1.25 m. Firn water in Storglaciären resides in a layer up to 5 m thick, while on Aletschgletscher in Switzerland the firn aquifer is 7 m thick (Schneider, 1999; Jansson et al., 2003). I choose $h_{\text{aq}} = 3$ m as a reference value, and test over the range 0.5–6.0 m.

Heat flow within glaciers has been described as advection-dominated (characterized by high Péclet numbers) (Aschwanden and Blatter, 2009), but the relative importance of heat transfer by advection compared to diffusion varies widely from glacier to glacier. I explore the role played by advection in governing thermal structure by adjusting advection rates with a multiplicative factor $C_u$. This illustrates the extent to which heat flow in the reference glacier is advection-dominated. It can also be used to investigate the implications of changing flow velocities that result from dynamic behaviour not explicitly considered in the fixed-geometry experiments.
7.1.5 Evaluation metric

To make quantitative comparisons between different simulations, I use two metrics to describe the modelled glacier thermal structure: (1) equivalent temperature difference relative to a given reference model and (2) temperate ice fraction. The former converts the enthalpy difference between two models into an equivalent temperature field (in Kelvin):

$$\Delta K' = \frac{\Delta H}{c_p}.$$  (7.5)

This is useful for comparing experiments in which the glacier geometries are identical. The second metric is a simple area fraction of temperate ice along the modelled flowband. Where applicable, both metrics are used.

7.2 Experimental results

7.2.1 Experiment 1: Heating source contributions

With the temperature-dependent flow-law coefficient $A$ (Eq. 6.46), the modelled enthalpy without strain heating ($Q_{str} = 0$) is smaller relative to the reference run by an equivalent difference of up to 1.8 K. With the enhanced flow-law coefficient $A_e$ (Eq. 6.47), this difference is slightly larger, up to 2.3 K (Fig. 7.2a). The temperate fraction and mean equivalent temperature difference over the entire flowband domain are presented in Table 7.3.

Even in the upper half of the ice column where stresses and deformation rates are low, equivalent temperatures are cooler in general when strain heating is neglected (Fig 7.2c). Lower deformation rates at depth lead to lower flow velocities in the upper ice column. Lower velocities reduce heat advection from the accumulation zone source $Q_m$ derived from meltwater entrainment. This effect exists whether using flow-law coefficient $A$ or $A_e$ but is greater in the latter because viscosity becomes a function of water content as well as temperature.

The omission of meltwater entrainment ($Q_m = 0$) causes a large change in the modelled thermal structure (Fig. 7.2b). The resulting distribution of temperate ice is similar to the REFC model, in which meltwater entrainment has been physically reduced by lowering surface temperatures. The large mass of temperate ice in Fig. 7.1a becomes limited to the deepest parts of the glacier ablation zone, and the bulk of the ice remains cold. The temperate ice distribution is most similar to the type “D” structure in Blatter and Hutter (1991). In parts of the accumulation
Figure 7.2: Distributions of enthalpy and equivalent temperature difference ($\Delta K'$) for REFT with and without strain heating $Q_{str}$ (a,c) and meltwater entrapment $Q_m$ (b,d). Model runs use flow-law coefficient $A$ as in Eq. (6.46). The dashed lines in (a,b) denote the cold-temperate transition. Darker areas in (c,d) indicate the largest differences in ice enthalpy relative to the REFT reference model. Shaded contours in (c,d) show the equivalent temperature difference for cold ice ($\Delta K'$) in units of Kelvin (Eq. 7.5).

zone, the near-surface equivalent temperature is over 6 K cooler than in the reference run in models using flow-law coefficient $A$ (Fig. 7.2d). The equivalent temperature differences are again slightly larger using the enhanced flow coefficient $A_e$. Because of the small amount of temperate ice, the enhancement factor in Eq. (6.47) plays a limited role.

Relative to strain heating and meltwater entrapment, basal heat sources (not shown) play only a small role. Sufficient basal heating causes the bed to reach the melting point. Temperate conditions do not extend to the glacier interior because of the negligible thermal diffusivity $\kappa_t$.

Allowing the glacier geometry to evolve does not substantially change the results in this

Table 7.3: Results of heat source removal (Experiment 1) with fixed glacier geometry and flow-law coefficient $A$ (Eq. 6.46).

<table>
<thead>
<tr>
<th>Test</th>
<th>mean $\Delta K'$ (K)</th>
<th>Temperate ice fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control (REFT)</td>
<td>—</td>
<td>0.69</td>
</tr>
<tr>
<td>No strain heating ($Q_{str} = 0$)</td>
<td>$-0.55$</td>
<td>0.57</td>
</tr>
<tr>
<td>No entrapment ($Q_m = 0$)</td>
<td>$-3.42$</td>
<td>0</td>
</tr>
<tr>
<td>No basal heating ($Q_b = 0$)</td>
<td>$-0.07$</td>
<td>0.68</td>
</tr>
</tbody>
</table>
Figure 7.3: Effect of flow-law coefficient parametrization on glacier thermal structure. Glacier geometry evolves in these simulations. The basal temperate layer thickness (a) is normalized to ice thickness. The cold layer thickness in (b) is equivalent to the cold-temperate transition depth.

In the absence of strain heating, the cold-temperate transition in the REFT model becomes deeper compared to the control near the glacier terminus. In the absence of meltwater entrapment, the glacier grows thicker and longer due to the higher viscosity of cold ice. A thin temperate zone forms at depth in the lower half of the glacier. This temperate layer is thicker than in the fixed geometry model because of the higher stresses in the larger steady-state glacier (Fig. 7.3a).

**Flow-law coefficient coupling**

The effect of water content on ice rheology as represented by $A_e$ has a large impact on glaciers with a high temperate ice fraction, and a plausibly small impact on those that are mostly cold. Unlike other sources of strain enhancement such as lattice preferred orientation and ice impurity content, water content is directly connected to thermal structure, and by extension, to climate conditions.
When glacier geometry evolves freely, both REFT and REFC models with enhanced flow-law coefficient $A_e$ exhibit a steady-state that is thinner than that using $A$ because of the higher fluidity in the large regions of temperate ice. Using the enhanced flow-law coefficient $A_e$ with the REFC model yields a glacier thickness 3% (8 m) smaller than with $A$ (not shown). The thickness of the REFC basal temperate layer, on the other hand, decreases by 40% (14 m), such that simulations with $A$ predict a thicker temperate layer (Fig. 7.3a). This thinning with the enhanced coupling of $A_e$ is consistent with the findings of Aschwanden et al. (2012). The cold layer thickness in the REFT model, which depends on both advection rates and strain heating, is slightly greater with $A_e$ than with $A$ (Fig. 7.3b).

### 7.2.2 Experiment 2: Parameter sensitivity

**Independent variables**

Changes in boundary heat sources/sinks and internal heat generation require the overall thermal regime to shift in response. Thermal structure within the control models also varies slightly with the choice of $A$ or $A_e$. The sensitivities of the steady-state control models to changing environmental parameters is similar regardless of whether $A$ or $A_e$ is chosen, so in the following section I focus on results based on $A$ alone (Figs. 7.4, 7.5).

Shifts in air temperature exert a strong control on thermal structure (Fig. 7.4a). The transition from a fully cold to a mostly temperate glacier occurs over an air temperature range of approximately 3 K. An intermediate thermal structure develops between the two end-member models with a temperate core derived from strain heating. With further warming, meltwater entrapment in the lower accumulation zone produces temperate ice through the full thickness of the glacier in the central region of the flowband (Fig. 7.6c). Cold ice advected from high elevations persists as a cold region up-glacier from the temperate zone.

The expansion of the temperate ice region with increasing air temperature occurs by two mechanisms. First, in a warmer climate less heat is lost during the winter and the cold layer that forms in the ablation zone does not penetrate as deeply into the ice. Secondly, larger amounts of heat derived from meltwater entrapment are added to the glacier in the accumulation zone and advected downstream. Within the model, this second effect is partially muted when the firn aquifer becomes saturated, however the shortening of the cold season associated with higher $\Delta T$ causes incrementally less heat to be lost to the atmosphere over the entire elevation range.
Figure 7.4: The results of varying parameters in Table 7.2, given in terms of mean equivalent temperature difference (solid line, dots) and the fraction of temperate ice (dashed line, crosses). (a) Air temperature offset. (b) Aquifer thickness. (c) Degree-day factor. (d) Equilibrium line altitude, recast as accumulation area ratio assuming a rectangular glacier. (e) Maximum englacial water content. (f) Advection multiplier. The values used for the reference model are indicated by the vertical grey bars.

Figure 7.5: The effect of varying pairs of variables on temperate ice fraction (contoured). Equilibrium line altitude has been recast as accumulation area ratio. Air temperature offset (ΔT) and aquifer thickness (h_{aq}) are co-varied in (a), air temperature offset (ΔT) and AAR in (b), and aquifer thickness (h_{aq}) and AAR in (c). Dashed lines denote hypothetical trajectories through the parameter space based on a linear mass balance sensitivity and a constant lapse rate. See text for details.
In Fig. 7.4a, there is a sharp transition from a largely temperate to a nearly cold glacier for temperatures cooler than that used to produce the reference model (REFT). In REFT, upstream heating from meltwater entrapment is the source of much of the temperate ice in the glacier interior, so eliminating this heat source produces a transition to a largely cold glacier. The transition is not as rapid if defined in terms of the metric mean $\Delta K'$, indicating that englacial heat storage is not strongly affected by the transition from less diffusive temperate ice to more diffusive cold ice. Although the Péclet number typically drops as ice cools to sub-melting temperatures, the effect on total heat storage within the glacier is small.

The thickness of the near-surface aquifer $h_{aq}$ (Fig. 7.4b) and the value of the degree day factor $f_{dd}$ (Fig. 7.4c) are important for similar reasons as surface temperature. In the case of a thin surface aquifer, less water from the previous melt season is entrapped, and refreezing and heat loss to the atmosphere occur more efficiently during the cold season. A thicker aquifer captures more water and preserves more energy at depth because of the insulating properties of the overlying snow and firn. The thermal structure of REFT is relatively insensitive to near-surface aquifer water content thresholds $\omega_{aq} \geq 5\%$ (not shown) and declines substantially below that.

There is a tendency for temperate ice generation in the accumulation zone to be greater at intermediate elevations. In this region, temperatures are high enough in the summer to produce significant quantities of melt, and burial rates are high enough to advect large amounts of heat into the glacier before it is lost to cooling in the winter. Nearer the equilibrium line, submergence rates are lower and the ratio of vertical advection to diffusion is smaller. Our assumption of a constant near-surface aquifer thickness causes the upper glacier transition from complete refreezing to producing temperate ice to be different than it would be in the case of a tapered aquifer. Therefore, different aquifer geometries could conceivably alter the zone in which heat from meltwater entrapment is pumped into the glacier.

Reducing the extent of the accumulation zone by raising the equilibrium line (Fig. 7.4d) has an important effect on temperate ice generation because it diminishes the region over which heat can be added. Increasing the maximum permitted water content $\omega_{eng}$ results in higher fractions of temperate ice (Fig. 7.4e). The ablation zone cold layer thins due to the higher quantity of water that must refreeze at the cold-temperate transition surface. This heat transfer requires a steeper thermal gradient, forcing the cold-temperate transition toward the glacier surface. The increase in temperate ice fraction begins to level off with a water content
threshold $\omega_{\text{eng}}$ of 2–3%. The mean equivalent temperature difference relative to the REFT model increases steadily with $\omega_{\text{eng}}$ because more heat is stored within the glacier as liquid water.

Finally, I explore the effect of altering the rate of heat advection by a constant coefficient $C_u$ across the glacier. In assigning $C_u \neq 1$, the velocity field used for energy advection is no longer consistent with the glacier geometry. Nevertheless, this experiment is useful for illustrating the effect of advection on thermal structure. In the case of transient fluctuations in glacier velocity (as in a surge), advection will cause the thermal regime to move toward the results developed in these simulations. For low advection rates, the ablation zone cold layer penetrates deeper into the glacier, restricting the extent of temperate ice derived from the accumulation zone. With high advection rates, the resulting thermal structure is similar to that with high allowable water content (Fig. 7.4e). In both cases, the rate of water transport to the cold-temperate transition increases, causing the transition to occur nearer the glacier surface.

Model sensitivity to changes in variables depends on the choice of reference glacier (e.g. REFT versus REFC). In terms of temperate fraction, the high sensitivity of meltwater-dominated polythermal glaciers (such as REFT) to small perturbations develops from the large amounts of heat potentially captured ($Q_m$) or lost through the glacier surface.

**Coupled variables**

In the absence of other changes, higher air temperatures cause polythermal glaciers to become more temperate. Reductions in firn thickness and accumulation area extent have the opposite effect (Fig. 7.5). In the case of a thick near surface aquifer (Fig. 7.5a) and a large accumula-
tion area (Fig. 7.5b), surface air temperature acts as a nearly independent control on thermal structure, primarily through the effect of meltwater entrapment. The situation reverses when aquifer thickness becomes less than \( \sim 2 \) m or when the accumulation area ratio is less than about one-third.

In Fig. 7.5, plausible trajectories across the parameter space have been mapped as dashed lines. In Fig. 7.5a, I make the assumptions that near-surface aquifer thickness is equivalent to the annual net balance (in firn-equivalent) and that changes in net balance can be represented by a scalar mass balance sensitivity (Eq. 7.3). Published mass balance sensitivities over a 1 K range vary widely, so I choose \( \partial b_n / \partial T = 0.5 \) m K\(^{-1}\) a\(^{-1}\) (w.e.). For a warming environment, the quasi-steady-state trajectories through the \( h_{aq} - \Delta T \) and AAR–\( \Delta T \) parameter spaces are non-monotonic (Fig. 7.5a,b), in contrast to those through the AAR–\( h_{aq} \) parameter space. If a glacier is in a region of the \( \Delta T \) parameter space where it is relatively insensitive to changes in air temperature (i.e. \( \Delta T < -2.0 \) for REFT) only the correlated changes in accumulation area and near-surface aquifer thickness are important. In this case, increasing air temperature will produce a reduction in the fraction of temperate ice within the glacier (similar to Fig. 7.5c as \( h_{aq} \) and AAR are reduced). Alternatively, in a scenario where the accumulation area ratio is roughly static but the firn thinning and temperature rise occur (as in Fig. 7.5a), a glacier may become more temperate before cooling again as meltwater entrapment is further inhibited.

The sensitivity tests provide a preliminary estimate of how thermal structure may respond to changing climates. A 1 K increase in temperature for the REFT model corresponds to a roughly 8% increase in the amount of temperate ice (Fig. 7.4a). At the same time, if rising temperatures increase summer ablation, the near-surface aquifer thickness (Fig. 7.4b) should decrease and the equilibrium altitude should rise (Fig. 7.4d). Assuming that net balance sensitivity to temperature \( \partial b_n / \partial T = 0.5 \) m K\(^{-1}\) a\(^{-1}\), the combined effects of aquifer thinning and accumulation zone reduction sum to a much larger decrease in temperate ice fraction. This calculation may be altered if a higher winter balance accompanies rising temperatures, so the potential exists for polythermal glaciers to become either colder or warmer in a warming environment (c.f. Rippin et al., 2011).

**Modelled polythermal structures**

For the glacier geometries modelled here, the strongest controls on thermal structure are those that modify the capacity for meltwater entrapment in the accumulation zone. As discussed
above, this includes both surface temperature (Fig. 7.4a) and near-surface aquifer thickness (Fig. 7.4b). In the sensitivity tests and simulations shown in Fig. 7.6, glaciers with low temperate fractions have a polythermal structure similar to REFC, with the highest heat content at depth (Fig. 7.6a,f). In simulations with the greatest meltwater entrapment and heat preservation, steady-states more closely resemble REFT (Fig. 7.6e,j). The intermediate thermal states vary, but are often characterized by cold ice in the upper glacier and thicker cold layers in the ablation zone (Fig. 7.6c,h).

In cold climates or environments where meltwater is not efficiently captured in the accumulation area, strain heating represents the primary source of englacial heat. This source is largely restricted to the deeper part of the ice column. If surface ablation rates are high enough, the layer of ice warmed by strain-heating will eventually be near enough the surface to lose much of this heat to the atmosphere. Such a layered structure (Figs. 7.6a–b, f–g) is similar to thermal structures observed in Svalbard (Björnsson et al., 1996), the Canadian Arctic (Blatter and Kappenberger, 1988), and in ice streams draining large continental ice sheets (Truffer and Echelmeyer, 2003). Alternatively, when environmental conditions permit meltwater entrapment at the surface, latent heat quickly becomes a dominant heat source. This situation (Fig. 7.6d–e, i–j) is similar to that observed in largely temperate glaciers such as Storglaciären (Pettersson et al., 2004).

7.2.3 Experiment 3: Prognostic modelling

The final experiment examines the transient responses of the REFT and REFC models to changing climate. Equilibrium line altitude $z_{ELA}$ and near-surface aquifer thickness $h_{aq}$ co-vary with a prescribed air temperature evolution in a manner consistent with Eq. (7.3). With rising air temperature, the model net balance decreases linearly. Hypothetical increases in winter balance are prescribed to offset ablation predicted by Eq. (7.3) such that a high winter balance diminishes the effect of rising air temperature on $z_{ELA}$ and $h_{aq}$.

For the REFT model, a wide range of thermal responses is possible in a warming climate (Fig. 7.7). Many trajectories show decreasing temperate fraction over time, with this effect being most pronounced when winter balance does little to offset increased summer melt. When large increases in winter balance are prescribed, the fraction of temperate ice (as well as the glacier volume) remains relatively steady. More than 80% of the ablation increase must be offset by increased accumulation in order to maintain or increase the temperate ice fraction for REFT.
Figure 7.7: Temporal evolution of the fraction of temperate ice with changing environmental conditions for time-dependent models. Each line in (a) represents a scenario with a different net accumulation response to a single function for temperature (b). Near-surface aquifer thickness $h_{aq}$ decreases and equilibrium line altitude $z_{ELA}$ rises when net balance is lowered (all scenarios). Models in which winter accumulation offsets 20%, 50%, and 80% of the increased summer melt are shown by the solid, dashed, and dotted bold lines, respectively. Small glaciers become prone to large fluctuations in temperate fraction when their sizes are not large relative to the fixed horizontal discretization, so lines are terminated if the glacier length falls under 3 km.

In this scenario, an 80% accumulation offset for 2.5 K warming is equivalent to an increase in winter balance of 1 m [w.e.].

The thermal evolution of the colder REFC model is different, with many of the high accumulation offset scenarios resulting in substantially increased temperate ice fraction. Rising air temperatures increase the meltwater availability in the accumulation area. The increased meltwater is captured in scenarios where the near-surface aquifer thickness and extent remain large. For smaller winter accumulation sensitivities (balance offsets less than $\sim$50%), the glacier remains dominated by cold ice. In many of the scenarios where winter balance does not increase substantially, both REFT and REFC become small and thin.

Based on an assumed accumulation increase of 10% and a temperature rise of 1 K, de Woul and Hock (2005) find that increased winter balance offsets increased ablation by approximately
20%. The range of accumulation offsets for glaciers north of 60°N that they report stretches from 54.4% to less than 5%. Our results suggest that offsets $>85\%$ for the REFT model and $>60\%$ for the REFC model are required to maintain or increase the temperate ice fractions in the prescribed warming scenario (2.5 K warming). This leads me to conclude that many polythermal alpine glaciers will experience a net cooling as climate warms.
Chapter 8

Simulating South and North Glaciers

The objective of this chapter is to interpret the distribution of temperate (radar-scattering) ice in South Glacier and North Glacier using the thermomechanically-coupled flowband model described in Chapters 6 and 7. Solutions for the steady state thermal structure are presented and compared to the radar data described in Chapter 3. The degree to which the model is capable of reproducing observed thermal structure is discussed in terms of the adequacy of the model physics and limitations of a two-dimensional representation of glacier geometry and dynamics. The importance of model parameter choices to both local and glacier-wide thermal structure is presented in the form of sensitivity tests. Finally, the flowband glacier geometry and thermal structure are evolved in time to compare hypothetical future thermal regimes in South Glacier and North Glacier.

8.1 Model inputs and configuration

8.1.1 Glacier geometry

Profiles of glacier width and bed and surface elevation are inputs required by the coupled thermal model. Because the model is two-dimensional, bed and surface geometries are extracted along flowline profiles. The approximate flowband used in the model of South Glacier is based on the surface flowline used by De Paoli and Flowers (2009), and was chosen based on a steepest descent algorithm applied to the glacier surface DEM. Differences from the original flowline include the addition of a single point on the upper glacier to allow the model domain to terminate closer to the real glacier headwall.
The North Glacier flowband is based on a hand-picked line that roughly follows the valley thalweg. This choice causes the flowband to pass between the lobes of lateral scattering described in Chapter 5. The valley thalweg is not necessarily the path of a flowline, but should be a close approximation. Because of the asymmetrical valley geometry, part of the surface flowline passes glacier-left of the glacier centerline.

Flowline surface elevations are derived from the elevation models described in Chapter 3. Using the horizontal coordinates of the flowlines introduced above, I sample elevation every 20 m with a nearest-neighbour scheme. These data are convolved with a boxcar function of the same width as the flowband model horizontal grid resolution in order to prepare the data for down-sampling onto the model grid. Likewise, I sample bed elevation data from the bed elevation models created in Chapter 5. Glacier width is calculated based on hand-digitized outlines of glacier extent. The half-width is determined by calculating the length of line segments stretching from the flowline to the outline boundary with a direction orthogonal to the flowline. The final half-width profile is one-half of the length of the flowline-orthogonal line segment that terminates at the glacier boundaries. A cubic spline function is used for down-sampling the geometrical profiles onto the model domain.

The cross-sectional shape of South Glacier is wide and shallow along most of its length, with width-to-thickness ratios frequently greater than ten. The shape of South Glacier approaches that of a rectangular channel. By contrast, North Glacier cross-sections are typically narrower, deeper, and closer to semi-elliptical in shape. Although glacier width is accounted for in the
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\kappa_c)</td>
<td>Cold ice diffusivity</td>
<td>(9.92 \times 10^{-4})</td>
<td>kg m(^{-1}) s(^{-1})</td>
</tr>
<tr>
<td>(\kappa_t)</td>
<td>Temperate ice diffusivity</td>
<td>(1.0 \times 10^{-5})</td>
<td>kg m(^{-1}) s(^{-1})</td>
</tr>
<tr>
<td>(c_p)</td>
<td>Specific heat of ice</td>
<td>2097</td>
<td>J kg(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>(L_f)</td>
<td>Latent heat of fusion</td>
<td>(3.335 \times 10^5)</td>
<td>J kg(^{-1})</td>
</tr>
<tr>
<td>(\partial T_m/\partial P)</td>
<td>Pressure-melting slope</td>
<td>(9.8 \times 10^{-8})</td>
<td>K Pa(^{-1})</td>
</tr>
<tr>
<td>(Q_c)</td>
<td>Creep activation energy</td>
<td>(1.15 \times 10^5)</td>
<td>J mol(^{-1})</td>
</tr>
<tr>
<td>(R)</td>
<td>Ideal gas constant</td>
<td>8.314</td>
<td>J mol(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>(g)</td>
<td>Gravitational acceleration</td>
<td>(-9.81)</td>
<td>m s(^{-2})</td>
</tr>
<tr>
<td>(\rho_i)</td>
<td>Ice density</td>
<td>910</td>
<td>kg m(^{-3})</td>
</tr>
<tr>
<td>(\rho_w)</td>
<td>Water density</td>
<td>1000</td>
<td>kg m(^{-3})</td>
</tr>
</tbody>
</table>

Table 8.1: Physical constants used for model runs.

Lateral drag parameterization in (6.23), the narrowing of a non-rectangular glacier cross-section with depth is not. Therefore, a uniform shape factor \((f_s)\) is applied to North Glacier for the velocity calculation. The shape factor is estimated from results provided by Nye (1965) for both rectangular and semi-elliptical channels. The shape factor adjusts the flowband velocity by a coefficient \(f^n_s\) (where \(n\) is Glen’s flow-law exponent) to account for lateral drag due to the shape of the glacierized channel.

### 8.1.2 Surface air temperature

Surface air temperature data are required for two reasons. The surface air temperature is used in calculating the upper Dirichlet boundary condition on ice enthalpy, as well as for determining the amount of surface melt required to apply the meltwater heat source (6.25). Air temperature measurements are available for both South Glacier and North Glacier from automatic weather stations (AWS, Figure 8.2) and Onset™HOBO Data Logger installations (H00–H04, Figure 8.2). The AWS temperatures are recorded with a tripod-mounted and shielded HMP45C212 TRH probe, and have a nominal accuracy of \(\pm 0.28^\circ\text{C}\). The nominal sensor height is 2 m. The HOBO units contain H08-032-08 sensors shielded by an RS1 radiation shield, and are installed on poles at a nominal height of 2 m above the snow surface. The nominal installation heights are prone to changing throughout the year due to snowfall, and for the HOBO sensors, ice ablation.

The data from South Glacier span a longer period than those from North Glacier (Figure 8.2). Atmospheric lapse rates that are constant in time and space are not capable of accurately capturing differences in temperature at different elevations throughout the year. Summer tem-
Figure 8.2: The period of record for individual temperature sensors on South Glacier and North Glacier. Instruments are installed from the glacier termini (H00) to the accumulation zone (H04).

Temperatures tend to exhibit a strong negative gradient with elevation, but this weakens and may invert during the winter. To represent surface temperature in the model, I have constructed a simple empirical model of surface temperature \( G \) as a function of time \( t \), elevation \( z \), and fitting parameters \( p \):

\[
G(p, t, z) = p_0 + p_1 \sin (2\pi (t - p_2)) + p_3 \sin (\pi (t - p_4)) \\
+ z^2 (p_5 \sin (2\pi (t - p_6)) + p_7) + z (p_8 \sin (2\pi (t - p_9)) + p_{10}).
\] (8.1)

The model describes the annual cycle in temperature (first line) superimposed on a time-dependent quadratic function of elevation (second line). Depending on \( p \), the quadratic component may describe a variety of lapse rate functions while transitioning smoothly between seasons. Smooth transitions reduce the significance of model timestep choices. This approach ignores daily variations in temperature (addressed separately). Differential shading is assumed to be of secondary importance along the relevant glacier flowlines, but could play a greater role over a three-dimensional domain. The model (8.1) is intended to provide a better parameterization of surface temperatures than selecting a standard time-invariant lapse rate. I determine the fitting parameters \( p \) by optimizing the cost function

\[
C(p) = ||G(p, t, z) - T(t, z)|| + M(p),
\] (8.2)

where \( ||G(p, t, z) - T(t, z)|| \) is an \( L^2 \) norm of the modelled temperature \( G(p, t, z) \) against the observations \( T(t, z) \) and \( M(p) \) is a regularization functional that reduces December-to-January
Figure 8.3: Observed and modelled mean monthly surface temperatures interpolated over elevation for South Glacier. The results from North Glacier (not shown) are similar.

The misfit

\[ M(p) = \sqrt{\sum_z (G(p, \text{Dec}, z) - G(p, \text{Jan}, z))^2}. \]  

(8.3)

A minimization of (8.2) that neglects the misfit term \( M(p) \) tends to exhibit non-physical jumps in temperature from December 31st to January 1st. A simplex optimization scheme (Nelder and Mead, 1965) searches for suitable parameters \( p \) that minimize (8.2).

Fitting the surface temperature model (8.1) to the available data in the form of monthly mean temperatures yields a function for surface air temperature that captures many of the broad characteristics of the observations (Figures 8.3, 8.4, 8.5). The maximum mean summer temperatures of approximately 3.0°C and the minimum mean winter temperatures of roughly −16°C for South Glacier are reproduced by the model. The data for North Glacier are sparser than for South Glacier, but the modelled temperature field is similar. A feature of the raw data on both glaciers is the lack of congruency between the mid-glacier AWS temperatures at roughly 2300 m and the small HOBO temperature sensors at other elevations. On South Glacier, the air temperatures measured at the AWS are higher on average than the reported temperatures at the nearest HOBO by 0.3 K in the winter and 0.6 K in the summer. On North Glacier, these differences are 1.5 K and 0.4 K, respectively. This discrepancy in measured temperatures may be related to the differences in the radiation shields. Slightly better residuals may be possible by discarding the AWS data from the model optimization, but not knowing whether the AWS or HOBO data are more accurate, I have retained all of the data. The integrated annual negative
Figure 8.4: Modelled temperature timeseries at the five HOBO temperature sensors (H00–H04) and the mid-glacier AWS from South Glacier. Observed monthly mean air temperatures averaged over the period 2006–2012 shown as grey dots with $1\sigma$ error bars.

degree days over one year at the South Glacier AWS averages to $-2479$ K-days, compared to the modelled value of $-2524$ K-days. At the nearest South Glacier HOBO (40 m lower in elevation), the measured and modelled values are $-2569$ K-days and $-2488$ K-days, respectively. The model lies between the two sets of observations, and reproduces the negative temperatures well in both cases. On North Glacier, performance is slightly worse. At the North Glacier AWS, the integrated value of negative degree days is $-2401$ K-days, while the modelled value is $-2585$ K-days. The poorer performance of the model on North Glacier reduces confidence in the surface boundary condition applied to the subsequent glacier models.

The second application of surface air temperature data is melt calculation. A classical temperature index (degree day) method (6.25) provides meltwater quantities. Daily temperature variations are important when using a temperature index model to calculate melt. If the average daily temperature is below the freezing point, melt may still occur over parts of the day (Figure 8.6). To account for this without introducing short timesteps and daily oscillations into the temperature model, I have made a semi-analytic correction to the temperatures used to calculate melting. I start with the assumption that daily variations are approximately sinusoidal, such that for any time $(t)$ within a day $(d)$, the temperature is

$$T(t) = \alpha \sin t + G(p, t, z),$$

(8.4)
Figure 8.5: Modelled and observed mean monthly air temperatures plotted against sensor elevation. The lapse rate of the modelled temperatures changes seasonally to approximate the observations. The mid-glacier AWS data point is circled in each plot. Error bars are $1\sigma$ of the monthly observations.
for daily amplitude $\alpha$ and daily average temperature $G(p, t, z)$. The quantity relevant for melting is the mean positive (Celsius) temperature $T_+$ over the course of the day:

$$\overline{T_+} = \frac{1}{2\pi} \int_{a}^{b} \alpha \sin t + G(p, t, z)dt$$

$$= \frac{1}{2\pi} \left( \alpha(\cos a - \cos b) + G(p, t, z)(b - a) \right)$$

$$a = \arcsin \left( -\frac{G(p, t, z)}{\alpha} \right)$$

$$b = \pi - a.$$  

When meltwater entrapment is calculated in (6.25), the surface air temperature that I use is the corrected temperature (8.5).

### 8.1.3 Snow depth

Snow depth is important insofar as it affects the near surface heat conductivity (e.g. Equation 6.9). Temperature sensors protruding from the ice on the cable at South Glacier borehole SG-MIDMET15M-2011 demonstrate this effect. In September and October of 2011, the temperatures measured at a point near the ice-snow interface are higher and experience dampened oscillations relative to temperatures at a point higher up (Figure 8.7). A simple function is used to describe the seasonal snowpack in the ablation area. A layer of snow with thickness $h_{sd}$ is represented as part of the modelled surface for a portion of the year bounded by $t_1$ and $t_2$. The
Figure 8.7: Temperatures from two sensors protruding from the SG-MIDMET15M-2011 borehole (Chapter 3). Both sensors are above the ice surface, but can become buried in snow. Sensor 1 is higher than Sensor 2, and the latter becomes more deeply buried in late September. Sensor 2 exhibits very little variability in temperature because of the insulating effect of the snow, and has a higher mean daily temperature than Sensor 1.

Daily snow depth is

\[
\text{snow depth}(d) = \begin{cases} 
    h_{sd} & \text{for } d < t_1 \lor d > t_2 \\
    0 & \text{for } d \geq t_1 \land d \leq t_2 
\end{cases}, 
\]

where \( d \) is days since January 1st. The density of the snow layer is \( \rho_f \). The modelled snow layer thickness is constant throughout the ablation zone and does not vary or densify throughout the winter. Within the accumulation zone, a snow layer is implicitly included as part of (6.27), and does not change in time.

### 8.1.4 Sliding parameters

The sliding law proposed by Schoof (2005) and implemented in the flow dynamics model depends on two parameters that describe the maximum slope of small-scale bed obstacles \( (m_{\text{max}}) \) and their dominant wavelength \( (\lambda_{\text{max}}) \). Flowers et al. (2011) experiment with different values of \( m_{\text{max}} \) and \( \lambda_{\text{max}} \) on South Glacier, and demonstrate that higher values of the former result in lower sliding speeds and higher values of the latter produce higher sliding speeds. They choose \( m_{\text{max}} = 0.25 \) and \( \lambda_{\text{max}} = 6.0 \) for steady state simulations and \( m_{\text{max}} = 0.5 \) and \( \lambda_{\text{max}} = 13.5 \) for prognostic simulations. The sliding rates are a function of both these parameters and the pre-
scribed basal water pressures. I choose $m_{\text{max}} = 0.5$ and $\lambda_{\text{max}} = 6.0$ for all experiments because I find that the observed surface velocities can then be reasonably reproduced, and that the convergence of the numerical model improves with higher values of $m_{\text{max}}$. The choice of sliding parameters is non-unique (c.f. Flowers et al., 2011), and depends on the choice of the flow-law coefficient $A$ (here, enthalpy-dependent).

### 8.1.5 Thermal model parameters

Thermal model parameters are estimated from field data or taken from the literature (Table 8.2). The geothermal flux ($Q_{\text{geo}}$) is selected based on heat flows typical of northwestern North America (Blackwell and Richards, 2004). Local heat fluxes may vary greatly, particularly in mountainous regions. The degree day factor ($f_{\text{dd}}$) is chosen to be close to the mean calibrated value of $5.4 \text{ mm K}^{-1} \text{ d}^{-1}$ for a temperature-index model listed by MacDougall et al. (2011). This value applies to snow-covered surfaces on both glaciers and is within the range of values tabulated by Hock (2003) for a variety of glaciers. Degree-day factors are typically lower for snow than for ice because of the difference in albedo; I use values tuned for snow because within this model, the melt parameterization is restricted to the accumulation zone. Calibrated values for individual years on South Glacier and North Glacier range from $2.6–8.2 \text{ mm K}^{-1} \text{ d}^{-1}$, so annual and basin-scale variability make it difficult to choose a single value. This variability and the simplicity of this parameterization compared to more advanced models (e.g. MacDougall et al., 2011) lead me to avoid choosing different values for each glacier. I do test the effect of changing the degree-day factor. The run-off ratio ($r$) falls in the middle of a range of values reported by Braithwaite et al. (1994) and references therein. Other measurements and estimations of the run-off ratio tend to be similar (e.g. Pfeffer et al., 1991; Huybrechts et al., 1991).

Englacial water content ($\omega_{\text{eng}}$) is not frequently observed above 1%, so this is taken to be the drainage threshold, following Greve (1997a) and Aschwanden et al. (2012). Englacial drainage appears to be more complex than a simple threshold (e.g. Lliboutry, 1976), but remains poorly understood. The aquifer water content threshold ($\omega_{\text{aq}}$) is based on firn porosity and saturation measurements by Fountain (1989). According to Jansson et al. (2003), roughly 40% of the pore volume of firn may be occupied by pore water with the remainder inaccessible due to pore close-off.

Firn aquifers form above the firn-ice transition of the accumulation zone (e.g. Schneider, 1999). The aquifer thickness ($h_{\text{aq}}$) in Table 8.2 is comparable to a range of values from disparate
Table 8.2: Parameters for South and North Glacier simulations. The column labelled “Source” indicates the original source of the values: D – estimated from data; L – estimated from literature.

<table>
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<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
<th>Source</th>
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<tr>
<td>$Q_{geo}$</td>
<td>Geothermal flux</td>
<td>55</td>
<td>mW m$^{-2}$</td>
<td>L$^a$</td>
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<tr>
<td>$f_{dd}$</td>
<td>Degree-day factor</td>
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<td>mm K$^{-1}$ d$^{-1}$</td>
<td>L$^b$</td>
</tr>
<tr>
<td>$r$</td>
<td>Runoff fraction</td>
<td>0.4</td>
<td>–</td>
<td>L$^c$</td>
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<tr>
<td>$\omega_{eng}$</td>
<td>Max ice water content</td>
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<td>%</td>
<td>L$^d$</td>
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<tr>
<td>$\omega_{aq}$</td>
<td>Max aquifer water content</td>
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<td>%</td>
<td>L$^e$</td>
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<tr>
<td>$h_{aq}$</td>
<td>Aquifer thickness</td>
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<td>m</td>
<td>D, L$^f$</td>
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<td>$\partial b_n/\partial z$</td>
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<td>m a$^{-1}$ m$^{-1}$ [ice eq.]</td>
<td>D</td>
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<td>$\dot{b}_{MAX}$</td>
<td>Maximum balance</td>
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<td>m a$^{-1}$ [ice eq.]</td>
<td>L$^g$</td>
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<tr>
<td>$z_{ELA}$</td>
<td>ELA</td>
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<td>m</td>
<td>D$^h$</td>
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<tr>
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<td>m</td>
<td>D$^k$</td>
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<td>–</td>
<td>D$^l$</td>
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<tr>
<td>$\rho_l$</td>
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<td>$n_{zd}$</td>
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<td>–</td>
<td>–</td>
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<td>Domain length, North Glacier</td>
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<td>D$^o$</td>
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<td>$f_s$</td>
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<td>–</td>
<td>D$^p$</td>
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<tr>
<td></td>
<td>Shape factor, North Glacier</td>
<td>0.93</td>
<td>–</td>
<td>D$^q$</td>
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$^a$Blackwell and Richards (2004)
$^b$Hock (2003); MacDougall et al. (2011)
$^c$Braithwaite et al. (1994)
$^d$Pettersson et al. (2004); Aschwanden et al. (2012)
$^e$Fountain (1989)
$^i$Compare with values reported by Fountain (1989), Braithwaite et al. (1994), and Jansson et al. (2003)
$^g$Rough estimate based on pole measurements and modelled mass balances (MacDougall, 2010).
$^h$Based on modelled mass balances (MacDougall, 2010) and maps of shallow radar scattering (Ch. 5).
$^i$Chosen based on snow pit depths.
$^j$Based on observed period of snow cover
settings reported by *Fountain* (1989), *Braithwaite et al.* (1994), and *Jansson et al.* (2003). The true aquifer thicknesses of South Glacier and North Glacier are not known. Little information is available regarding how aquifer thickness changes over multi-year time spans.

Glacier net balance is spatially and temporally variable (e.g. *MacDougall*, 2010), and difficult to summarize in a simple parameterization. Changes in net balance for the field site due to climate variability over periods beyond 2006–2012 may be relevant to the present thermal structure, but are not known. Here, net balance parameters are only used for prognostic experiments, and are roughly based on ablation stake data and modelling results (c.f. *MacDougall*, 2010). Physical parameters related to ice creep are taken from Chapter 3 of *Cuffey and Paterson* (2010).

### 8.1.6 Spatial and temporal discretization

Selecting spatial and temporal discretizations requires both numerical and practical considerations. Stable model results can frequently be obtained from the FOA model using a spatial domain discretized into $50 \times 30$ horizontal and vertical nodes with timesteps of 0.5 a (Table 8.2). To minimize errors introduced by interpolation, the thermal model operates on the same horizontal grid as the dynamics model. However, the vertical discretization of the thermal model domain must be much finer than is required by the dynamics model to properly predict temperatures in the near surface (c.f. *Reijmer and Hock*, 2008, for a similar problem). The thermal model domain is discretized into 49 vertical cells, with the uppermost cells on the order of 0.1 m thick. I use linear interpolation to transfer model variables between the flow dynamics and thermal model. In order to resolve seasonal variability in surface temperature and melt rates, the thermal model is asynchronously-coupled to the flow dynamics model, using a timestep of 0.1 a (Chapter 6). When near-surface advection rates are large, the timestep of the thermal model is permitted to decrease adaptively in order to preserve numerical stability.

### 8.2 Simulated steady-state thermal structure and sensitivities

Thermal steady state models of both glaciers with their present geometries have been constructed by using the methods described above. The ice surface is not permitted to evolve in any of the following simulations, so the only feedback is between the velocity and thermally-dependent viscosity fields. I test both water-content insensitive ($A$) and enhanced ($A_e$) viscosity
coupling relations (6.46, 6.47). Where it is helpful to do so, I report results in terms of equivalent ice temperature. This metric is defined in Celsius as

\[ T_{eq} = \frac{H_m - H}{c_p}, \]  

(8.8)

where \( H_m \) is the enthalpy at 0°C with zero water content (c.f. Eq. 7.5). This is the equivalent to reporting enthalpy adjusted by a multiplicative factor that makes the units more clear. Positive equivalent temperatures imply non-zero water content.

8.2.1 South Glacier

Modelled thermal structure with and without tuned \( P_w \)

I use a zero water pressure profile (\( P_w(x) = 0 \)) to generate a control model of South Glacier. This model approximately reproduces the observed thermal structure, including the temperate accumulation zone and cold ablation zone (Figure 8.8a). Water content increases with depth in the mid-glacier, but is at the water content threshold (\( \omega_{eng} \)) throughout much of the upper accumulation zone. Immediately above the equilibrium line altitude, the ice is nearly temperate (a result of the meltwater latent heat parameterization that is limited to the accumulation zone). The mean equivalent ice temperature (8.8) is slightly below -1°C.

Flowers et al. (2011) prescribe positive basal water pressures in order to match the observed flow velocities in South Glacier, assuming a constant flow coefficient of \( A = 2.4 \times 10^{-24} \text{ Pa}^{-3} \text{ s}^{-1} \). A similar exercise performed with the thermomechanically-coupled model uses the values in Figure 8.9. Sliding rates are higher in the model tuned in this way (Figure 8.9a), which increases the rate of advective heat transfer from the accumulation zone and elongates the region of temperate ice in the ablation zone (Figure 8.8b). The convexity of the CTS becomes more pronounced. Although this technique is ad hoc, the resulting surface velocity profile matches the observed surface velocities more closely (Figure 8.9a). For the purposes of comparing modelled thermal structure to observed thermal structure, I use the model with the tuned water pressure profile. In sensitivity experiments I use the control model with zero water pressure as a null hypothesis to better isolate variables in the absence of the additional uncertainty regarding water pressure. This is done with the caveat that the control model under-predicts expected basal velocities in the modern glacier.

The control model (\( P_w = 0 \)) underestimates the longitudinal extent of the basal temperate
Figure 8.8: Diagnostic models of South Glacier thermal structure. The control model (top) assumes zero basal water pressure ($P_w(x) = 0$), while the tuned model uses the water pressure profile shown in Figure 8.9. The water-insensitive flow-law coefficient $A$ (6.46) is used. Solid contours show temperature, while dashed contours show water content by volume. The location of the temperature acquisition cable (TAC) is off the central flowline by 280 m.
Figure 8.9: Modelled (lines) and observed (bars) annual surface velocities for South Glacier (2006–2011, a) and North Glacier (2007–2009, b). Observations on South Glacier are derived from real-time kinematic GPS measurements of velocity stakes positioned along the glacier centreline. Observations on North Glacier are from the same but incorporate some hand-held GPS data, and are of lower accuracy. The prescribed water pressure ($P_w$) profiles are given in terms of a fraction of overburden pressure.

layer inferred from the radar data (Figure 8.10). Radar evidence of a nearly contiguous basal temperate layer extends to 2.7 km from the headwall, and patchy temperate ice continues as far as nearly 4 km from the headwall. The control glacier is cold at the bed beyond 2.6 km. The situation improves in the model with a water pressure profile tuned to reproduce observed surface velocities, in which a basal temperate ice zone extends to 2.8 km (Figure 8.8b).

The provenance of the most distal temperate ice (visible in Figure 8.10) is unknown, but it exists at locations on and near the flowline that are below $-4^\circ$C in the tuned model (Figure 8.8b). Discrepancies in the extent of temperate conditions between the model and observation may be rooted in the lack of any detailed treatment of basal hydrological processes or meltwater inputs in the model. Crevasses, moulins, and ice-marginal streams represent gateways for large amounts of energy to the basal environment that are not accounted for in the thermal model. Two glacial tributaries (discussed below) are unaccounted for in flowband model. Other factors include uncertainty regarding geothermal fluxes and the possibility that former dynamical and thermal states have left imprints on the modern glacier. The latter possibility is briefly addressed in Section 8.3.
Figure 8.10: Comparison of modelled and observed temperate ice. Modelled CTS shown by the bold line and the observed upper layer of radar scattering on the South Glacier model flowband shown by diamonds. Observed scattering across a swath surrounding the central flowband (see inset) is indicated by shades of grey. Darker shades indicate that scattering is observed across a greater width of the swath.

The instrumented borehole SG-UPPER85M-2011 in the upper ablation zone of South Glacier is 280 m off the model flowband. At the nearest modelled point, the modelled cold-temperate surface (CTS) is about 40 m below the ice surface. This is shallower than the observed CTS at the borehole, which is closer to 50 m deep. Heat at the nearest modelled point is mostly derived from meltwater entrapment and advected down-glacier from the accumulation area. Near-surface heat is quickly removed in the ablation zone by cold winter temperatures. In an advecting column of ice, the depth of penetration by the cold layer increases with time, unless it is offset by internal heating. The depth of the CTS in the upper ablation zone should be sensitive to the equilibrium line altitude, advection rates, and winter temperatures.

Modelled temperatures nearest the SG-UPPER85M-2011 borehole fall below $-4^\circ$C, while the borehole instruments report minimum temperatures below 16 m depth between $-1.5^\circ$C and $-2^\circ$C. Lateral heterogeneity is observed in South Glacier. Both shallow radar scattering and deep transparency are observed near the borehole. Although the borehole and modelled flowband represent different parts of the glacier, the difference in temperatures raises the question of whether the model underestimates cold layer temperatures. One possibility is that the surface boundary condition is not accurately represented. No ablation area surface heat sources are included in the thermal model, however a small amount of refreezing may occur due to water trapped in cryoconite surfaces and discontinuous snow patches. Alternatively, a cryo-hydrologic
system (c.f. Phillips et al., 2010) might moderate englacial temperatures that deviate far from the pressure melting point. Should ablation zone refreezing play a role, the temperature gradient within the cold layer would be shallower, and less heat would be removed by conduction from within the glacier. Experiments (not shown) in which the thickness of the snow layer \( h_{\text{sd}} \) changes affect englacial temperatures, but have only a small effect on the depth to the CTS.

**Flow-law coefficient**

Using the enhanced flow-law coefficient \( A_e \) (6.47), it is not possible to reproduce the observed surface velocities in the flowband model of South Glacier with the reference value of the viscosity prefactor \( A_0 \) (Table 8.2) from (6.46). The reduced viscosity of the temperate ice that forms in the control run combined with the steep surface gradients in the first 1.5 km from the headwall of South Glacier allows the ice to flow at rates in excess of 50 m a\(^{-1}\) at the surface, both with basal water pressures held at zero \( P_w = 0 \) and with basal sliding prohibited. One feature of the physical glacier geometry that is not represented in the model is a 90°+ right-hand bend in the central flowline direction between 0.6 km and 1.7 km from the headwall, where the glacier flow direction changes from eastward to southward. The increased deformation required for viscous flow through this bend may cause lower velocities than would be predicted by a simple flowband model, however it has not been investigated whether this is sufficient to explain the discrepancy in surface flow velocities.

The steady state thermal structure of the above simulation with \( P_w = 0 \) exhibits less temp-
perate ice than the control model with the water-insensitive (A) flow-law coefficient. This is an artefact of holding the ice surface fixed in the diagnostic model: the higher flow rates cause ice to submerge faster in the accumulation zone and as a corollary, less meltwater is entrapped (Figure 8.11). Despite the more limited englacial extent of temperate ice, the basal ice remains at the pressure-melting point up to 2.5 km from the valley headwall, slightly further than in the control model.

8.2.2 North Glacier

Modelled thermal structure with and without tuned $P_w$

The temperate accumulation zone and the cold-ice region below the ablation area are reproduced in the control model representing North Glacier. Flow velocities with $P_w = 0$ are similar to the observed values in the ablation zone based on stake measurements (Figure 8.9b). Modelled surface velocities are lower than those observed in the region 1–2.5 km from the headwall. At roughly 2.7 km from the headwall, a gap in the valley wall permits a steep distributary glacier to flow south into the neighbouring valley. As with South Glacier, prescribing elevated water pressure in this area improves the match between the modelled and observed surface velocities. In general, the zero-water pressure velocity profile of the control model is a better fit to the observations on North Glacier than on South Glacier. The control model for North Glacier is about 1°C colder than the equivalent model for South Glacier, with a mean equivalent temperature just below −2°C. The CTS boundary is concave-upwards, with a long tail of temperate ice at the base (Figure 8.12). Patchy basal temperate ice inferred from the radar data extends as far as 3 km from the valley headwall (Figure 8.13). Within the tuned model, this is roughly consistent with the furthest extent of the 0°C isotherm (Figure 8.12). The point along the flowline at which temperate ice becomes extinct in the model is up-glacier from the downstream limit of the radar scattering lobes (4.4 km from the headwall) noted in Chapter 5. The absence of temperate ice at this point in the model implies that the lobes result from some process that is not represented in the flowband model. Despite this difference, the distribution of cold and temperate ice is reproduced within the control model very closely (Figure 8.13).

The borehole NG-MID75M-2011 is not near the modelled flowband, so it is not expected that the measured temperatures would be reproduced by the model. The temperatures of the modelled cold layer are up to 2°C lower than the minimum temperatures at the borehole.
(≈ −3°C). In addition to the off-flowband location of the borehole, other possible reasons for this discordance partly mirror those from South Glacier. Incomplete treatment of the surface boundary conditions (discussed above) and the lack of a cryo-hydrological system to moderate englacial temperatures may contribute. In addition, the borehole was intentionally drilled in a location in which radar scattering was observed far from the bed, and is therefore a sample biased in favour of warmer ice. Finally, shear along the yz-plane may lead to additional strain heating in the vicinity of the borehole (discussed in Section 8.2.6).

**Flow-law coefficient**

Using the enhanced flow-law coefficient $A_e$ and reverting to the zero water pressure profile, the modelled glacier surface velocities are again larger than the observed velocities. In this case, the difference is roughly a factor of three in the accumulation zone, which reflects the amount of enhancement added through (6.47). Due to the small role played by advection in North Glacier as a whole, the modelled thermal structure (not shown) is very similar to that with the water-insensitive flow-law coefficient $A$. The most visible difference is a thickening of the temperate ice layer at the mid-glacier base, however the longitudinal ($x$) temperate extent remains close to the control model.

**8.2.3 Role of advection**

Some of the differences between South Glacier and North Glacier, and between the control and tuned models of South Glacier, are related to differences in the respective roles played by diffusion and advection. The heterogeneous partitioning of heat flow across the two components of the advection-diffusion equation (6.19) can be represented in terms of the Péclet number. The Péclet number (Pe) describes the relative importances of advection and diffusion at a given length scale ($L$): 

$$ Pe = L \frac{u}{\kappa}, $$  

with flow velocity $u$ and thermal diffusivity $\kappa$. A high Péclet number indicates that advection dominates, while a low Péclet number is characteristic of a system controlled by diffusion. The length scale is important because it specifies the domain of interest; at sufficiently small length scales diffusion always dominates, while at sufficiently large length scales, advection is more effective (*Cuffey and Paterson*, 2010, Ch. 11). I take the local ice thickness as the
Figure 8.12: Diagnostic models of North Glacier thermal structure. The control model (top) assumes zero basal water pressure ($P_w(x) = 0$), while the tuned model uses the water pressure profile shown in Figure 8.9. The water-insensitive flow-law coefficient $A$ (6.46) is used. Solid contours show temperature, while dashed contours show water content by percent volume. The location of the temperature acquisition cable (TAC) is off the central flowline by 265 m.

Figure 8.13: Comparison of modelled and observed temperate ice. Modelled CTS shown by the bold line and the upper layer of radar scattering on the North Glacier model flowband shown by diamonds. Observed scattering across a swath surrounding the central flowband (see inset) is indicated by shades of grey. Darker shades indicate that scattering is observed across a greater width of the swath.
Figure 8.14: Péclet numbers calculated for control and tuned models of South Glacier (a,b) and North Glacier (c,d), reported on a log_{10} scale. Positive values (shaded) indicate advection-dominance, while negative values (white) indicate diffusion-dominance.
length scale relevant to heat loss through the ice surface. With this choice, the Péclet number for the steady state South Glacier and North Glacier models depends largely on whether the ice is temperate or cold (Figure 8.14). The low effective heat diffusivity in temperate ice (and to a lesser extent, higher accumulation zone flow velocities) causes the accumulation zone to be advection-dominated. The slow flow and higher cold ice diffusivity of the ablation zone lead to a diffusion-controlled regime. In the ablation zone, advection of upstream heat content is locally less important than the diffusive heat loss to the surface. The temperatures of the ablation zone are more likely to be altered by additional source terms, obstacles to diffusive heat loss, or changes in boundary conditions than by changes in the advection of heat from the accumulation zone. In a thicker or faster-flowing glacier, the balance would be shifted toward an advection-controlled regime.

8.2.4 Influence of sliding

De Paoli and Flowers (2009) suggest that basal sliding represents a large fraction of the observed surface velocity over the central part of the South Glacier flowband. In the model of South Glacier with prescribed zero basal water pressure, basal sliding accommodates a quarter of the surface displacement in the upper glacier (< 1.2 km from the headwall), but much less further down-glacier. When water pressures are tuned to reproduce the observed surface velocities, sliding accommodates most of the surface movement between 1.7–2.6 km from the headwall. Beyond 3 km, surface velocities are very low, and the fraction accommodated by sliding is less meaningful. In the model of North Glacier, sliding in the control model is less important than in the model of South Glacier, with sliding comprising 10–30% of modelled surface velocities.

The effect of sliding on thermal structure should be twofold. First, sliding increases advection rates at and near the glacier base. This means that more heat from up-glacier should be transported at the base of the ice column. Secondly, sliding reduces the role of strain heating (6.22) by diminishing the amount of ice deformation that occurs within the glacier. In the following model runs, I disable basal sliding for South Glacier and North Glacier and compare the predicted thermal structure with the control models introduced above.

Compared to the control model of South Glacier with \( P_w = 0 \), enforcing a no-slip basal boundary in the thermal model causes only modest changes in the thermal structure that develops (Figure 8.15a). Upper glacier surface velocities fall by one-third, and the already slowly
Figure 8.15: Influence of sliding on thermal structure within South Glacier. No sliding (a, 42% temperate, $\Delta K' = -0.2$ K), sliding and $P_w(x) = 0$ (b, 45% temperate, $\Delta K' = 0$ K), and sliding with $P_w(x)$ tuned to approximate observed surface velocities (c, 59% temperate, $\Delta K' = 0.6$ K). $\Delta K'$ is defined as in (7.5).
In the model of North Glacier, enforcing no-sliding causes only slight changes in the predicted thermal structure relative to the control model (Figure 8.15a,b). The 0 K and −2 K isotherms intersect the bed only slightly closer to the headwall in the no-slip model. The temperate ice extent away from the bed is nearly identical to the control model. The cold ice in the no-sliding model tends to be slightly cooler than when sliding is permitted. In both models,
Table 8.3: Ranges of selected model parameters used for sensitivity tests.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Range</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>ωaq</td>
<td>5.0–15.0 %</td>
<td>(c.f. Fountain, 1989, 30–100% firn saturation)</td>
</tr>
<tr>
<td>ωeng</td>
<td>0.0–2.0 %</td>
<td>(c.f. Pettersson et al., 2004, and references therein)</td>
</tr>
<tr>
<td>fdd</td>
<td>4.0–8.0 mm K$^{-1}$ d$^{-1}$</td>
<td>(c.f. Hock, 2003, and references therein)</td>
</tr>
<tr>
<td>r</td>
<td>0.3–0.5</td>
<td>(c.f. Braithwaite et al., 1994, and references therein)</td>
</tr>
<tr>
<td>h$_{aq}$</td>
<td>2.0–5.0 m$^{-1}$</td>
<td>(c.f. Fountain, 1989; Braithwaite et al., 1994)</td>
</tr>
<tr>
<td>Q$_{geo}$</td>
<td>0–100 W m$^{-2}$</td>
<td>(c.f. Blackwell and Richards, 2004)</td>
</tr>
</tbody>
</table>

the bulk of the temperate ice is at the drainage threshold of 1% water content, which prevents any significant differences from developing. Using the North Glacier model with $P_w(x)$ tuned to better reproduce observed surface velocity increases the rate of advection and causes the basal temperate layer of the mid-glacier to thicken (Figure 8.16c).

8.2.5 Parameter sensitivity

Varying model parameters such as drainage threshold ($ω_{eng}$), runoff ratio ($r$), or degree day factor ($f_{dd}$) causes changes in the predicted thermal structure. I have selected a range of parameter values that bracket the reference values in Table 8.2 and run the model with fixed surface geometry for each. The parameters varied are those from Table 8.2 that are the most poorly constrained (e.g. $ω_{eng}$, $h_{aq}$, $Q_{geo}$), the most likely to be non-stationary (e.g. $r$, $h_{aq}$), and the most likely to be limited by the simplistic nature of the heat source parameterizations (e.g. $f_{dd}$, $r$, $h_{aq}$). The parameter ranges are chosen to extend over what I estimate to be realistic limits for each variable based on reported values for settings relevant to South Glacier and North Glacier (Table 8.3). Because these choices are imprecise and contain a degree of subjectivity, it is more useful to compare how sensitivities differ between South Glacier and North Glacier rather than between disparate variables applied to a single glacier. South Glacier and North Glacier exhibit different sensitivities to model parameters (Figure 8.17).

Both South Glacier and North Glacier exhibit large differences in temperate fraction when the englacial drainage threshold ($ω_{eng}$) is varied (Figure 8.17). Temperate ice fractions are relatively insensitive to changes in the geothermal flux ($Q_{geo}$), although the mean equivalent temperature of North Glacier (and to a lesser extent, South Glacier) is strongly affected by reducing or increasing the geothermal flux (Figure 8.17).

The local effects of changing parameters vary. Intuitively, changing the geothermal flux
Figure 8.17: Results of sensitivity tests for South Glacier (a,c) and North Glacier (b,d) in terms of mean equivalent temperature (8.8) (a,b) and temperate ice fraction (c,d). The values for the control models are indicated by the vertical dashed lines.
Figure 8.18: Flowband representations of enthalpy differences reported as equivalent degree (8.8) differences for each sensitivity test pair. The differences shown are the enthalpy increases from the lower to the higher energy model within each pair. (a,b) $[\omega_{aq} = 15\%] - [\omega_{aq} = 5\%]$; (c,d) $[\omega_{eng} = 2\%] - [\omega_{eng} = 0\%]$; (e,f) $[f_{dd} = 8 \text{ mm K}^{-1} \text{ d}^{-1}] - [f_{dd} = 4 \text{ mm K}^{-1} \text{ d}^{-1}]$; (g,h) $[r = 0.3] - [r = 0.5]$; (i,j) $[h_{aq} = 5 \text{ m}] - [h_{aq} = 3 \text{ m}]$; (k,l) $[Q_{geo} = 100 \text{ W m}^{-2}] - [Q_{geo} = 0 \text{ W m}^{-2}]$. 

South Glacier

North Glacier
(Q_{geo}, Figures 8.17k,l) affects the temperature of the cold basal ice the strongest, while changing the englacial drainage threshold (\omega_{eng}, Figures 8.17c,d) mostly affects the temperate ice beneath the accumulation zone. A clear difference can be seen between the parameters that effect accumulation zone processes (e.g. f_{dd}, r, and h_{aq}, Figures 8.17e,f,g,h,i,j) and the geothermal flux, which only has a major effect in regions of cold ice because of the low heat diffusivity within temperate ice.

Notable differences in parameter sensitivity between the two glaciers are visible for degree-day factor (f_{dd}) and near-surface aquifer thickness (h_{aq}). The temperate ice fraction of South Glacier is strongly affected by both of the above, while the temperate ice fraction in North Glacier is less sensitive to these parameters (Figure 8.17c,d). Changes to accumulation zone parameters that increase the latent heat delivery have less effect on North Glacier, because the underlying temperate ice in the control model is frequently saturated and has little capacity to entrap more water. A pattern of higher sensitivity to accumulation zone parameters persists in the alternative metric of mean equivalent temperature (Figure 8.17a,b). The low modelled sensitivity of North Glacier is related to its flow velocities and the condition of the diagnostic model that net balance compensate perfectly for the surface velocities calculated through continuity. As the accumulation zone velocity gradients of South Glacier are higher, implicit accumulation rates must be greater to compensate. The resulting high submergence rates reduce the enthalpy per volume incorporated into the ice from meltwater entrapment. Variations in aquifer parameters such as thickness and the degree-day factor alter the quantity of heat entrained in the ice. On North Glacier, in contrast, submergence rates are low and the aquifer becomes saturated with meltwater. Small changes have no effect on the glacier-wide thermal regime. If thresholds such as \omega_{aq} exist in nature, glaciers with higher submergence rates in the accumulation zone (such as the modelled South Glacier) will be more sensitive to change.

8.2.6 Lateral heterogeneity and limitations of flowband modelling

The models applied to South Glacier and North Glacier above reduce the problem domain to a single two-dimensional transect. They assume that glacier dynamics can be described by a flowband. Real glaciers exhibit lateral variability in bed topography and ice thickness. Slightly different choices of the central flowline result in differences in the model inputs. Because South Glacier has a box-like cross-section, different flowlines maintain roughly the same ice thickness, but have slightly different bed profiles. In order to test the relevance of the flowline choice to
modelled thermal structure, I have run the above model with the water-insensitive flow-law coefficient $A$ on two additional South Glacier flowlines that are 100 m toward glacier left and glacier right of the central flowline (Figure 8.19a). The starting and ending locations of the alternative flowlines are forced to be identical to those of the central flowline. The offset of 100 m is small enough that I assume that the glacier width and valley wall stress parameterization identical to that used for the central flowline (Figure 8.1) are valid, while large enough that bed topography and ice thickness along the flowlines display appreciable differences (Figure 8.19b). I do not repeat this experiment for North Glacier, because its lower cross-sectional aspect and U-shaped bed cause substantial differences in ice thickness on alternative flowlines and invalidate the assumptions above.

The South Glacier basal water pressure is prescribed to be zero, as in the control model. The results show that with the alternative flowlines, the modelled thermal structure remains very similar to that derived from the central flowline (Figure 8.20). Climate controls the modelled distribution of temperate and cold ice at the glacier surface such that the boundary is very near the equilibrium line altitude. For the central and left glacier flowlines, this boundary occurs within 2 km of the headwall (Figure 8.20a,b). On the right flowline, the shallow temperate ice extent stretches slightly further (Figure 8.20c). The ice surface along this flowline is higher 2 km from the headwall. As a result, the meltwater parameterization, which is prescribed in terms of an ELA, causes the temperate ice extent near the surface to stretch further.

The longitudinal extent of the temperate ice zone at depth depends on the water content that amasses in the accumulation zone, the surface temperatures (which control the amount of heat lost to the surface by diffusion), and the rate of heat advection. These are similar between runs,
Figure 8.20: Thermal structure modelled along the alternative flowlines in Figure 8.19. The glacier-left (a) and glacier-right (c) flowlines are offset from the central flowline (b) by 100 m.
which causes the modelled temperate ice extent to be relatively unchanged across the three
flowlines. Longitudinal variations in temperate ice extent at the ice base are roughly 100–200 m.

There are other ramifications of representing glaciers using flowband models. Variations
in stresses caused by flow in curved valleys are not accounted for, nor are ice fluxes from
tributaries (South Glacier) or distributaries (North Glacier). The flowband assumption captures
the primary flow of ice within a glacier valley, but neglects the dynamics of secondary flows that
may be orthogonal to the transect represented by the flowband. Schoof and Clarke (2008) show
that in some physically-relevant scenarios, secondary flow may be important enough to explain
géomorphological features such as fluted topography. Transect-orthogonal velocity components
would cause ice affected by off-transect dynamics to be introduced into the assumed flowband.

The flowband model is furthermore incapable of representing laterally heterogeneous thermal
structures. In South Glacier, radar scattering is observed further down-glacier on the eastern
side of the valley than on the western side. The termination of radar scattering on the eastern
side of the valley corresponds to the point where the eastern tributary valley merges with
the main valley. Differences in upstream source areas have been suggested as a source
of lateral variability in CTS water content in Storglaciären by Pettersson et al. (2007). In South
Glacier, the main body of temperate ice (west side of valley) also coincides with a zone of high
upstream area (Appendix B). Upstream area is function of hydraulic potential, and high up-
stream area predicts higher fluxes within the basal hydrological system. In the lower glacier
(> 3 km from the headwall), the zone of high upstream area is focused on the eastern side of
the valley because of the bed geometries and ice thickness (Figure B.6).

Lateral variability in thermal structure may also result from heterogeneity in the strain heat-
ing source term (6.22). Local temperature and water content records the integrated heating
along the upstream flowline (e.g. Aschwanden and Blatter, 2005), and so lateral differences in
stress and strain regimes that are not represented in a flowband model may lead to observed
secondary structure. The strain heating distribution caused by isolated basal sliding in the
context of North Glacier is considered in greater detail in Appendix F.

8.3 Time-dependent simulations of thermal structure

Glaciers in southwestern Yukon have undergone notable mass loss in the past fifty years (e.g.
Barrand and Sharp, 2010; Berthier et al., 2010), and continue to lose mass presently (Luthcke
et al., 2008). While the modelling experiments above have relied on a steady state assumption, here I apply a net balance and allow the ice surface to vary with time in order to simulate future evolution.

The initial model geometries are a geometrical and thermal steady-states for each glacier with the measured bed profiles and a longitudinal extent similar to that observed in the real glaciers. In order to achieve steady state, a mass balance profile different from that measured must be used. The equation governing the mass balance profile is the same piece-wise linear function of elevation as was used in Chapter 7, which is:

\[ \dot{b}(z) = \begin{cases} 
  z \left( \frac{\partial b_n}{\partial z} \right) - z_{ELA} \left( \frac{\partial b_n}{\partial z} \right) & \text{if } z < z_{\text{max}} \\
  b_{\text{max}} & \text{if } z \geq z_{\text{max}} 
\end{cases} \tag{8.10} \]

with \( z_{\text{max}} = \frac{\dot{b}_{\text{max}}}{(\partial b_n/\partial z)} + z_{\text{ELA}} \). The maximum net balance for both glaciers (\( b_{\text{MAX}} \)) is estimated to be 1.0 m a\(^{-1}\), loosely based on MacDougall (2010, Fig. 2.4). An alternative approach to represent the net balance is to approximate the observed net balance with a polynomial (e.g. Flowers et al., 2011, using 2007 balance year for South Glacier). Whether using (8.10) or a polynomial approximation, when steady state is reached, the net balance must by definition be equal to zero. While Flowers et al. (2011) shift the entire balance profile by a scalar, (8.10) depends on ice surface elevation, so I adjust the equilibrium line altitude \( z_{\text{ELA}} \) instead. For the initial mass balance, this approach is similar to that of Flowers et al. (2011), however it makes is clear how the balance should evolve in time according to (8.10).

Once a steady state glacier geometry and thermal stricture have been reached, I shift the net balance profiles of the modelled glaciers by adding a constant balance perturbation \( \Delta \dot{b}_n \) to (8.10). Reasonable choices for the balance perturbation are taken from the literature. Based on interpolated elevation changes, Berthier et al. (2010) provide an estimate of the balance rate for the St. Elias and Wrangell mountains of \(-0.47 \pm 0.09\) m a\(^{-1}\) for the period 1962–2006. Using area-volume scaling, Barrand and Sharp (2010) estimate the ice loss in Yukon glaciers to have been \(-0.78 \pm 0.34\) m a\(^{-1}\) over 1957–2008. These losses have occurred alongside winter and summer temperature increases over the period 1950–2002 of \(2.0 \pm 0.8^\circ\text{C} \) and \(1.0 \pm 0.4^\circ\text{C} \) respectively (Arendt et al., 2009). Observed mean balance perturbations incorporate both the external climate forcing (e.g. increased ablation due to rising temperatures) and the internal balance feedback (i.e. lower net balance as the ice surface declines due to a positive \( \frac{db_n}{dz} \)).
Figure 8.21: Flowband area (fine black line) and temperate ice area (dashed line) during prognostic experiments on South Glacier (a,b) and North Glacier (c,d) forced by perturbing net balance ($\dot{b}_n$). The area averaged temperature (thick black line) is in the lower axes. The notches in the South Glacier curves occur when a downstream ice mass becomes disconnected from the main glacier as the ice retreats. Note the differences in $x$ axes between the runs with $\Delta \dot{b}_n = -0.47\,\text{m a}^{-1}$ and those with $\Delta \dot{b}_n = -0.78\,\text{m a}^{-1}$.
Predictions of future temperature changes in northwestern North America for the next century follow or exceed this trend (IPCC, 2007).

I conservatively assume a balance perturbation equal to the average over the past half-century. The balance perturbation changes in the future as a result of the feedback in (8.10) between net balance ($\dot{b}_z$) and elevation ($z$). No additional climate forcing, due for example to increasing temperatures, is assumed. The coupled ice dynamics and enthalpy models are stepped forward in time using the same parameters as the control models (Table 8.2). The degree-day factor ($f_{dd}$), near-surface aquifer thickness ($h_{aq}$) and temperature ($G$) are held constant. If the near-surface aquifer were to thin meltwater entrapment would decline (e.g. Chapter 7), so the results of these experiments may understate effects of atmospheric warming.

Using both estimates of the balance perturbation above, the glaciers experience significant retreat over the first two centuries. With the balance perturbation $\Delta \dot{b}_n = -0.47 \text{ m a}^{-1}$, the total ice volume and temperate ice volume of South Glacier both fall rapidly (Figure 8.21a). The South Glacier temperate ice volume nears a steady value after the first century. The ice volume has not reached a new steady state after 500 a have passed, at which point the South Glacier flowband is roughly one-fifth of its original area (Figure 8.21a). The average glacier temperature along the flowband changes very little for the first two decades, and then falls by $2^\circ \text{C}$. Average temperature is relatively stable after 100 a, remaining near $-2^\circ \text{C}$ for the rest of the simulation. Following the initial decline, the average temperature rises slightly after 200 a because subsequent wastage occurs in the cold ice region of the glacier. With the more drastic balance perturbation of $-0.78 \text{ m a}^{-1}$, the temperate ice nearly disappears after 70 a have passed (Figure 8.21b). The mean temperature follows a trend similar to the previous simulation, with a slightly lower minimum temperature of $\approx -3^\circ \text{C}$ reflecting the loss of most of the accumulation zone. South Glacier all but disappears shortly after 150 a have elapsed (Figure 8.22).

The initial response of North Glacier to the milder balance perturbation ($\Delta \dot{b}_n = -0.47 \text{ m a}^{-1}$) is similar to South Glacier but more delayed. The slower response to the balance shift is a result of both the slower flow rates and greater length of North Glacier. The temperate ice volume reaches a new steady value after $\approx 200$ a (Figure 8.21c). The response of the mean temperature accelerates after roughly 50 a, after which it falls to $\approx -7^\circ \text{C}$. In contrast to the South Glacier simulations, the average temperature does not eventually rise because strain heating plays a larger role than in South Glacier, so the loss of cold ice (that would tend to increase the mean temperature) is offset by diminishing strain heating. With the larger balance
Figure 8.22: Modelled glacier geometry and thermal structure at 30 a intervals during the first 150 a of time-dependent simulations. Temperate ice is shown in black, and cold ice in light grey for simulations of South Glacier (cols. 1,3) and North Glacier (cols. 2,4) and mass balance perturbations of $\Delta \dot{b}_n = -0.47 \text{ m a}^{-1} \text{ (cols. 1,2)}$ and $\Delta \dot{b}_n = -0.78 \text{ m a}^{-1} \text{ (cols. 3,4)}$. 
perturbation ($\Delta \dot{b}_n = -0.78 \text{ m a}^{-1}$), the temperate ice region disappears after 80 a and the total ice volume drops below 1/10 of its original value after 200 a (Figure 8.21c,d).

The modelled colder temperatures in North Glacier are partly due to the lower elevation upper valley as compared to South Glacier, as well as generally lower flow velocities. The lower elevation of the upper valley causes the accumulation zone to be more restricted as the glacier thins, which limits the part of the model boundary over which meltwater is entrapped. The lower velocities reduce the rate at which heat is distributed throughout the glacier (Figure 8.14), and cause the ablation zone to grow colder.

South Glacier and North Glacier are unlikely to be in a steady-state at present. An additional set of experiments (not shown) uses an initial state in which the thermal structure has been spun-up with the modern ELA (but steady state geometry) to better approximate the observed thermal structure. The results of time-dependent experiments using these initial states and the same climate forcings are very similar to the results above after 50 model years have elapsed. In the first 50 a, the temperate ice area is smaller, and the initial decline of temperate ice area (dashed lines, Figure 8.21) is slower.

### 8.4 Summary

The thermal model outlined in Chapter 6 has been applied to simulate steady-state thermal structures of South Glacier and North Glacier. The distribution of cold and temperate ice predicted in diagnostic simulations is comparable to that inferred based on radar backscatter, and the major features (temperate accumulation zone, cold ice above a temperate layer in the ablation zone) are reproduced. Temperate ice in the ablation zone of South Glacier is observed to extend further than shown by the model. Minimum temperatures in the modelled ablation zones are lower than temperatures measured in off-flowband boreholes. If the borehole temperatures are representative of temperatures on the flowband, then this mismatch may indicate that the treatment of the ice surface with a Dirichlet boundary condition tracking parameterized air temperature is not complete. The model results confirm that accumulation zone water entrapment is a dominant source of englacial heat, and is alone capable of describing much of the observed thermal structure.

Sensitivity tests applied to both modelled glaciers show that the response of South Glacier to changes in model parameters sometimes differs from the response of North Glacier. While
accumulation zone parameters exercise an important role in controlling the modelled thermal structure of South Glacier, simulated North Glacier is relatively insensitive to degree-day factor ($f_{dd}$) and run-off ratio ($r$). Under the model assumptions, low advection rates in the diagnostic model cause the North Glacier accumulation zone to be closer to saturation.

In time-dependent simulations, the net balance profiles of steady-state South Glacier and North Glacier flowband models are shifted by a balance perturbation ($\Delta \dot{b}_n$) that raises the equilibrium line altitude and restricts the accumulation zone. Because an important heat source is then deactivated across part of the surface boundary, the modelled temperate ice volumes of South Glacier and North Glacier fall. Reductions in the accumulation zone near-surface aquifer thickness are not considered, but are known from the sensitivity results above to also reduce the temperate ice extent. In the case of South Glacier, the average ice temperature may eventually begin to increase after an initial decline as the cold lower glacier ablates away. In North Glacier, no such rise in temperatures occurred in either model run, due to the lower elevation of the upper valley and slower modelled velocities that limit advective heat transfer. The balance perturbations observed in the Wrangell-Saint Elias Mountains and Yukon, Canada do not permit net increases in modelled temperate ice volume. The temperate fraction may increase despite the net loss of temperate ice if the rate of cold ice wastage is great enough. Increases in in air temperature may influence ablation zone temperatures because of the high relative importance of diffusive heat transfer there, but do not increase the temperate fraction if meltwater entrapment becomes inefficient due to near-surface aquifer thinning.
Chapter 9

Synthesis and Conclusions

Glacier thermal structure reflects climate and glacier dynamics, both of which influence the rate at which glaciers incorporate and transport heat. Climate exerts direct control on the ice-surface boundary, while glacier dynamics are important for strain heating and for advecting internal heat. Dynamics are additionally influenced by thermal structure through the temperature sensitivity of ice rheology and water content-dependent enhancement, the latter of which increases the fluidity of temperate ice (Duval, 1977; Gusmeroli et al., 2010; Cuffey and Paterson, 2010). Glaciers in Alaska and Western Canada have been shown to make a measurable contribution to sea level rise (Berthier et al., 2010), which underscores the importance of better understanding their state and dynamics. In light of the relevance of small glaciers, thermal regime is of interest both for what it reveals about existing conditions and for what it means for the future.

9.1 Summary

The strong contrast in dielectric permittivity between ice and water causes radar scattering in water-bearing temperate ice. It is possible to exploit this to map the boundaries between cold and temperate ice layers (Chapter 2). Using radar survey techniques, I have developed maps of the thermal structure within two glaciers in the Saint Elias Mountains of Yukon, Canada. These maps build upon the bed mapping of South Glacier (De Paoli, 2009) by expanding the mapping to englacial features, incorporating additional data (Chapter 3), and extending the interpretation to North Glacier. New processing methods were introduced in order to map the bed in regions where englacial scattering reduces the strength of bed reflections. Additional filters designed in
Chapter 4 permit the accurate demarcation of englacial features. For the purpose of mapping thermal structure, I use a conceptual model that assumes temperate ice is overlain by cold ice, as observed elsewhere (e.g. Blatter, 1987; Holmlund and Eriksson, 1989; Björnsson et al., 1996; Gusmeroli et al., 2012). Thermal structure maps estimating temperate layer thickness are based on the timing difference between the onset of the radar scattering layer and the basal reflection (Chapter 4).

I have used multiple forms of validation for the resulting maps (Chapter 5). Common-midpoint surveys performed on South Glacier are not accurate enough to distinguish between cold and temperate ice strata, but they do provide field confirmation of radar ice velocities which are required for accurate ice thickness calculations. I test radar-derived ice thicknesses using cross-over analyses of independent radar surveys, ensuring that surveys performed on different dates with different radar configurations yield similar results. From this analysis, I found that valley-perpendicular surveys tend to give more reproducible results than valley parallel surveys because off-nadir reflections lead to radar clutter in the latter. Comparison of radar-derived depths with borehole lengths on South Glacier reveals a close match, lending confidence in the radar data and its interpretation. Finally, temperature measurements from boreholes on South Glacier and North Glacier compare well with the radar-based interpretations of glacier thermal structure.

In the final maps of thermal structure, both noteworthy similarities and differences exist between South Glacier and North Glacier (Chapter 5). Both glaciers appear to be largely temperate within the accumulation zone, suggesting that meltwater entrapment in snow and firn plays an important role in determining thermal structure. Within the ablation zone, both glaciers consist largely of radar-transparent (i.e. cold) ice, with laterally and longitudinally heterogeneous layers of temperate basal ice at higher elevations. While the basal temperate ice in the ablation zone of South Glacier consists of a single, discontinuous body, North Glacier exhibits a bilobate lateral symmetry with a cold trough running roughly down the valley thalweg. The fraction of South Glacier’s centerline length that contains temperate ice is furthermore greater than that of North Glacier (roughly 3/4 and 1/2, respectively).

In order to investigate the internal and boundary processes that control the thermal structure of similar glaciers, I developed a flowband thermal model (Chapter 6) based on an enthalpy method (e.g. Aschwanden et al., 2012). The model solves the energy balance equation while incorporating heat sources in the form of basal heat fluxes, strain heating, and latent heat cap-
ture associated with entrapped meltwater. I have applied the model to synthetic glaciers using zeroth-order shallow ice approximation flow dynamics in order to investigate the sensitivity of thermal structure to physical and environmental variables (Chapter 7). In glaciers for which meltwater entrapment is a major control, thermal structure is sensitive to changes in accumulation zone firn properties. In hypothetical warming scenarios without the invocation of high accumulation rates, simulated glaciers tend to become cooler with less temperate ice.

Finally, by coupling the thermal model to a first-order shallow ice model of glacier flow (see Pimentel et al., 2010), I performed thermomechanically-coupled diagnostic and prognostic experiments using South Glacier and North Glacier flowband geometries (Chapter 8). Applying parameters estimated to be representative of the present day, simulated versions of both glaciers reproduce many characteristics of the observed thermal structure. The observed extent of temperate ice in South Glacier is not fully reproduced, which may be due to rapid advection and cold ice wastage as part of the glacier’s recent surge and retreat history. Prognostic models based on a simplified mass balance parameterization and estimated rates of ice wastage (Barrand and Sharp, 2010; Berthier et al., 2010) show rapid reductions in temperate ice volume when evolved from an initial steady state. While the average temperature of South Glacier stays within a few degrees of the melting point, the average temperature of North Glacier falls by 5–10 K as time passes, depending on the forcing applied. The difference between the two glaciers is related to differences in bed geometry and flow dynamics.

9.2 Discussion

9.2.1 Radar frequencies

Radar surveys of glaciers often use multiple antennas, with bed reflections mapped using low frequencies and englacial scattering mapped with higher frequencies. Despite testing radar antennas with three different center frequencies (10–50 MHz), I observed little variation in the resulting englacial reflections. This similarity across nominal frequencies is likely due to the broadband nature of dipole antennas. The frequency range used here is much smaller than that in the antenna comparisons of Ødegård et al. (1997) and Pettersson (2005), in which antennas spanning ranges of roughly 1 GHz are used. Within the range tested, the choice of dipole antennas is a compromise between size and transmitted power, rather than transmitted power and imaging capacity. I propose that for future surveys of similar glaciers using the same
radar system as available here, antennas with center frequencies in the range of 10–20 MHz best satisfy these criteria.

9.2.2 Polythermal structure

Polythermal structures have been observed and discussed in a range of environments beyond North and South Glaciers, from mostly temperate cases with thin cold layers (e.g. Aschwanden and Blatter, 2005, 2009), nearly cold high-Arctic glaciers with basal temperate ice (e.g. Blatter, 1987; Rabus and Echelmeyer, 2002; Wohlleben et al., 2009), and ice caps (e.g. Breuer et al., 2006). As in this thesis, comparable thermal structure has been observed across glaciers that share similar climate regimes (e.g. Bamber, 1987; Björnsson et al., 1996). To a large degree, this can be attributed to the control exercised by climate on accumulation zone processes. The amount of heat incorporated into glaciers through meltwater entrapment is an important first-order influence on the resulting thermal regime. Because of the basic consistency enforced by climate, the primary thermal structure of glaciers expected to have similar dynamics could be confirmed with a small number of radar transects that cover the full glacier length. Care should be taken to not generalize these results to very different climate or dynamic settings. For example, maritime climates with air temperatures constant close to the melting point will cause the amount of heat lost to the atmosphere to in the ablation zone to be much less and perhaps negligible. Thick and fast flowing glaciers will generate much larger amounts of strain heating, such that this heat source may be of importance comparable to that of meltwater entrapment.

Properties unique to individual glaciers serve as secondary controls on thermal structure. Ice advected from tributaries and off-trunk cirques may incorporate ice with a different thermal state into the downstream glacier (Björnsson et al., 1996). Surging periodically changes the partitioning of advective and diffusive heat transfer, causing transient thermal structures. Conversely, Clarke et al. (1986) and Fowler et al. (2001) argue that thermal regime is a contributing factor for some glacier surges. In light of this, the high concentration of surging glaciers in some parts of the world (e.g. Meier and Post, 1969) and the aforementioned climate dependence of thermal structure would seem to be complementary.

The documented softening of ice as water content rises (Duval, 1977) has important repercussions for modelling ice flow of glaciers in to the future. Diagnostic modelling in which the flow-law prefactor $A_0$ is tuned to observations may in many cases implicitly account for water-softening. This is because deformation rates tend to be highest at depth where the effective
stress is large, and in common thermal regimes water contents are also higher here. On the other hand, Breuer et al. (2006) report that the impact of including water content in the rheology of a polythermal ice cap yields a nonlinear response in flow velocities, implying that in this case a simple scaling accommodated through tuning $A_0$ would not work. The future cooling of small glaciers (explored in Chapter 7) could lead to an apparent decline in ice viscosity in addition to that expected from temperature change alone. For this reason, prognostic modelling of temperate and polythermal glacier dynamics should consider the effect of a moisture-dependent flow-law coefficient in the form of $A_e$ (6.47).

9.2.3 Thermal regime of South Glacier and North Glacier

The temperate ice within these two polythermal glaciers depends on meltwater entrapment. Similar results are likely for other small glaciers in cold climates that undergo accumulation zone melting in the summer. Because of the latent heat source, climate and conditions in the shallow accumulation zone exert a powerful influence on thermal structure. In the experiments of Chapters 7 and 8, both accumulation zone parameters and accumulation area ratio are perturbed, resulting in reduced temperate ice volumes. As climate warms, it is likely that the heat pumped into the accumulation areas of small glaciers in this particular region will decline.

The ablation zones of both glaciers investigated here are subject to lower flow velocities than the accumulation zones, which makes the fraction of advected heat smaller. The diffusivity of heat in cold ice is also much higher than in temperate ice, owing to the nearly isothermal nature of the latter. These two factors provide a mechanism by which diffusive heat exchange with the atmosphere becomes important to temperatures within the ablation zones of South Glacier and North Glacier. As the accumulation area ratio declines in the future, the thermal regime may shift from the present one, in which meltwater entrapment plays an important role, into one in which more of the thermal structure can be explained in terms of basal heating processes (e.g. Clarke et al., 1984) and air temperatures (e.g. Pettersson et al., 2007).

Lateral heterogeneity in thermal structure may also be time-dependent. If ice influxes from the tributaries of South Glacier are partially responsible for the observed distribution of cold and temperate ice, then the thermal structure will respond as contributions from the tributaries fall. The glacier-left (eastern) tributary of South Glacier has likely already ceased to be an important source of ice into the main trunk. As North Glacier thins, the reduced driving stresses will cause a reduction in the rate of strain heating which contributes to the bilobate structure (Appendix F).
Changes in meltwater delivery to the bed and the basal conditions affecting sliding rates may also be important.

### 9.2.4 Thermal disequilibrium

Time-dependent models (Chapter 8) show that timescales of temperature adjustment in these glaciers are on the order of centuries for balance perturbations similar to those observed over the past 50 a. Following a rapid change in climate or geometry, mean glacier temperature takes a long time to approach equilibrium. The period of mean ice temperature change is on the same scale as that of geometric adjustment, but mean temperatures in simulations of both South Glacier and North Glacier exhibit a multiple decade lag following step perturbations. The non-stationary nature of climate makes it likely that thermal disequilibrium exists even in small glaciers. This is similar to the conclusions of Rippin et al. (2011), who report observations consistent with thermal disequilibrium in the small Scandinavian glacier Kårsaglacären.

The surging history (and corresponding recent retreat) of South Glacier may be reflected by the observed temperate ice at points much further along the glacier flowline than reproduced by the steady-state model. Historical events may also be reflected in the high observed temperatures near the SG-UPPER85M-2011 borehole, which are roughly $6^\circ$C higher at 16 m depth than the mean annual air temperature and $2–3^\circ$ higher than the modelled temperatures on the flowband 280 m away. While investigating thermal disequilibrium in John Evans Glacier in the Canadian Arctic, Wohlleben et al. (2009) report that although such a disequilibrium exists, its effect is dwarfed by that of modern heat transport related to the hydrological system, which is alone capable of explaining much of the hypothesized disequilibrium.

### 9.2.5 Limitations of the present model

Several implications of the model's flowband reduction were discussed in Chapter 8. Additional limitations relate to the incomplete mapping of the real world to the model, independent of dimensionality. Heat transport through hydrological systems, as suggested by Wohlleben et al. (2009), may be an important pathway. A minimalistic treatment of englacial water transport exists in the form of the moisture flux term (6.18) presented in Chapter 6, but this is neither grounded in theory nor well-constrained by observation. It is arguable that a Darcy diffusion law rather than the Fickian diffusion embodied by (6.18) may be appropriate (Fowler, 1984).
Macro-scale structures such as crevasses are not considered, and likely affect hydrology, as well as increase diffusive heat loss by increasing glacier surface area.

Physics governing englacial hydrology are poorly understood. Englacial water drainage must occur in some form, and the drainage function based on a constant threshold value ($\omega_{\text{eng}}$, Chapter 6) often fails to reproduce variable water contents similar to those observed (e.g. Lliboutry, 1976; Murray et al., 2000; Gusmeroli et al., 2010). The sensitivity tests of Chapters 7 and 8 show that differences in the threshold $\omega_{\text{eng}}$ change the extent of ablation zone temperate ice formed by advecting meltwater. A drainage function that is spatially- or temporally-dependent could therefore affect the temperate ice extent, with a range bracketed by the results with constant thresholds representative of a characteristic minimum and maximum threshold.

Further limitations are predicated on the simplifications made to the heat source physics in the thermal model (Chapter 6). Ablation zone refreezing, either in the cryoconite surface, in crevasses, or in remnant firn as the accumulation zone retreats are not considered. The affect of refreezing or entrapment in the remnant firn layer would have a dampening effect on sensitivity of thermal structure to accumulation zone retreat.

In the accumulation zone, the calculation of melt by a classical temperature index model has been shown to provide less accuracy than more sophisticated techniques. In particular, on South Glacier much of the heat available for melting is radiative rather than sensible (MacDougall, 2010). Uncertainty introduced into the melt calculation prevents the model from being useful for calculating absolute thresholds for temperate ice production in the accumulation zone because the latent heat delivered by meltwater may not be accurately quantified. A similar limitation pertains to the simplistic near-surface aquifer model, in which water is distributed within a fixed number of upper surface layers. Aquifer density is prescribed rather than calculated, and even if the present state were well-constrained by field observations, there is no obvious way in which the aquifer parameters should be prescribed for the future.

No aquifer diffusivity or percolation rates are considered (e.g. Reijmer and Hock, 2008), so some fraction of the meltwater reaches the base of the aquifer instantaneously. Instantaneous movement of meltwater may lead to an overestimation of latent heat penetration into the firn layer. The end-result of such an overestimation is that heat preservation can occur with a smaller aquifer thickness in the present model than would occur otherwise. On the other hand, Darcy-type diffusion of meltwater into an unconfined firn aquifer will eventually result in an unsaturated upper layer in the aquifer akin to the vadose zone of soil hydrology (Jansson et al., 1976).
2003). The unsaturated layer could have an insulating effect, and reduce the required aquifer thickness for heat preservation. The ultimate balance between these two opposing influences on the model predictions is not clear. In the present model, variables such as aquifer thickness, despite their physical meaning, are effectively tuning parameters. Useful results could therefore be obtained by jettisoning the physical metaphors in the current aquifer model and instead testing prescribed enthalpies for the surface boundary condition.

9.3 Conclusions

Based on the work described above, I make the following conclusions:

1. Both glaciers exhibit polythermal structure that is strongly influenced by meltwater entrapment. The polythermal structure observed through radar scattering has been confirmed at two borehole locations. The presence of temperate ice in the accumulation zones of both glaciers requires a surface heat source, the most plausible being the latent heat associated with meltwater entrapment. The primary thermal structure of either glacier is identifiable with a single longitudinal radar line, and a similar approach could be useful on other glaciers. Secondary features such as lateral heterogeneity would require additional radar lines to resolve, but do not fundamentally change the interpretation above.

2. The primary features of observed temperate ice geometries are largely reproduced in steady-state diagnostic models. In particular, the North Glacier thermal structure is closely reproduced by the model. The higher flow velocities of South Glacier result in accumulation zone heat being advected further down-glacier before being dissipated into the atmosphere by diffusion. In contrast, temperate ice is more restricted in North Glacier because of low ice flow velocities. Thermal disequilibrium may be responsible for low elevation temperate ice observed in South Glacier that is not reproduced in modelled steady-state scenarios. The surge history of South Glacier should result in periodically changing advection rates that result in thermal disequilibrium.

3. As a corollary to the previous point, thermal structure can be predicted with a flowband model, but accurate predictions will rely on information regarding flow velocities, melting rates, and accumulation zone parameters. Additional data in the form of air temperatures and historical glacier dynamics are useful and of secondary importance. In terms of
thermal structure, South Glacier and North Glacier are likely to be representative of many similarly-sized glaciers in the northern Saint Elias Range, Yukon, as well as glaciers in comparable climate settings. In a more maritime setting, the cooling of the ablation zone will occur to a smaller extent. In a colder setting, accumulation zone melting may be insufficient to generate temperate ice in the upper glacier. Caution should be exercised in extending these results where the findings relevant to South Glacier and North Glacier are violated, such as to much larger or much smaller ice masses. In the former, strain heating may play a more dominant role resulting in more heat generation downstream of the equilibrium line, while in the latter, diffusion may prevail over advection (e.g. Gilbert et al., 2012).

4. In simulations representing both synthetic and real glaciers, temperate ice volume is found to be sensitive to parameters that affect the efficiency of meltwater entrapment. Thinning near surface aquifers and rising run-off ratios reduce meltwater entrapment. In the polythermal South Glacier and North Glacier, perturbations to net balance that reduce the accumulation area cause reductions in temperate ice volume and changes to mean flowband temperature. After an initial decline, the mean temperature of the South Glacier flowband rises because of cold ice ablation. No subsequent temperature rise occurred in the North Glacier simulation.

9.4 Future Work

An application for accumulation zone data develops directly from the meltwater-dominated nature of the observed thermal regimes in South Glacier and North Glacier. Data such as density, temperature, and permeability profiles could yield insights into how the accumulation zone changes over seasonal and decadal timescales. Some of these data exists in the form of the snow pits that are dug annually by the Simon Fraser University Glaciology Group (unpublished data), however it would be useful for the purposes of aquifer modelling to have profiles that penetrate through more than the latest balance year (unlike the present data), or data collected during the summer and fall, when aquifer saturation should be highest. These data could be used to quantify the latent heat capture, storage, and fluxes in the firn aquifer in greater detail than is presently possible.

Building on the above, an important step in terms of predictive modelling is the implementa-
tion of a better model of the accumulation zone firn layer, which was identified as a weakness in Section 9.2.5. Such a model is important first because it would improve on the present simplistic handling of meltwater and secondly because it would have greater predictive power when coupled to a mass balance model for prognostic simulations. By coupling the thermal model with a more realistic firn model, expected aquifer changes with rising temperatures and changing surface balances could be directly addressed. One set of possible approaches is discussed by Reijmer and Hock (2008) and references therein.

The limitations imposed by the flowband reduction of the model described in Chapter 6 are relatively small, but could be directly addressed by expanding the model into three dimensions. Because the ice flow model that is more appropriate for valley glaciers (Pimentel et al., 2010) is limited to two dimensions in its present implementation, it would need to be generalized as well. Given the existence of advanced open-source ice dynamics models such as Elmer (Gagliardini and Zwinger, 2008), it may be easier to use one of these as a basis for three-dimensional modelling.

Regional-scale modelling of thermal structure may be useful to test hypotheses regarding the respective roles of climate and individual glacier features. A model useful for this purpose could perhaps be based on a computationally-efficient flow dynamics simplification such as the Second-Order Shallow Ice Approximation (SOSIA) described by Baral et al. (2001) and Egholm et al. (2011). Such an approach would permit a better understanding of the variability of thermal structure within mountain ranges, and would be applicable to efforts to extend the modelling presented here to unmeasured glaciers.


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Appendix A

Apparent Dip vs. True Dip

The subsurface geometries expected from visually examining raw data from techniques such as ground-penetrating radar methods are not necessarily representative the true geometry (e.g. Bauder et al., 2003). One important reason for this is that the reflections recorded in each radar trace represent an unfocused summation of all existing reflectors. Migration methods can be used to recalculate the reflector geometry that caused the observed reflections (Daniels, 2004). Migration is commonly used in many engineering and geophysical fields (Daniels, 2004; Claerbout, 2010).

One characteristic that migration attempts to correct is reflector dip angle (Scales, 1995). As a radar system moves from position $S_1$ to $S_2$ on the glacier surface (Figure A.1a), it sounds the bed at $R_1$ and $R_2$. Prior to migration, reflections are interpreted as having come from $Q_1$ and $Q_2$, respectively. As a result, the change in thickness from $S_1$ to $S_2$ observed from the surface, $\Delta z'$, is expected to be the vertical change $\Delta z$, while in reality, it is a change in the length of a line orthogonal to the reflecting surface. If the bed dip angle is $\theta$, then

$$\frac{\Delta z'}{\Delta x} = \sin \theta$$  \hspace{1cm} (A.1)

$$\frac{\Delta z'}{\Delta x} = \tan \theta$$  \hspace{1cm} (A.2)

Focusing on the upper triangle (shown in Figure A.1b), the apparent bed dip $\phi$ is the arctangent of the apparent vertical and horizontal offsets $\Delta z'$ and $\Delta x$, but because $\Delta z'$ is not vertical,
the true dip $\theta$ is the arcsin. The apparent dip $\phi$ is related to the true dip by

$$\tan \phi = \sin \theta.$$  \hspace{1cm} (A.4)

This is sometimes known as the migrator’s equation (e.g. Stolt, 1978; Scales, 1995; Upadhyay, 2004).
Appendix B

Repeated Radar Lines

The strength of the reflections observed in radar surveys has been used to elucidate bed properties (e.g. Bentley et al., 1998; Gades et al., 2000; Jacobel et al., 2009; Pattyn et al., 2009). Repeated radar soundings can also be used to track temporal changes in bed reflectance, which are often interpreted in the context of diurnally- or seasonally-changing hydrological and thermal conditions (e.g. Jania et al., 2005; Irvine-Fynn et al., 2006). Three radar lines on South Glacier (Figure B.1) were surveyed repeatedly in 2011. This appendix describes the methodology used to analyse these lines and presents the results. As of yet, no firm conclusions have been drawn from these data.

B.1 Methodology

In order to compare radar lines from different dates, it is necessary to first project them onto the nominal repeat lines shown in Figure B.1.

The nominal coordinates for the repeat line endpoints are given in Table B.1. I transform the UTM coordinates for each radar trace to a new equirectangular projection in coordinates \((p, q)\), such that on the line \(q = 0\) and \(p\) is the distance from the westernmost point on the nominal line.

The nominal line is represented by a vector \(\overrightarrow{AC}\). Then, the unit vector \(\hat{AC}\) is

\[
\hat{AC} = \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|},
\]

where vertical bars indicate the vector norm. Then, letting \(\overrightarrow{AD}\) be the UTM vector from the starting point of the nominal line to the radar trace location and \(\overrightarrow{CD}\) be the vector from the radar

Figure B.1: Repeat radar lines indicated by black bars. The nearby poles and “MidMet” meteorological station are also indicated.
trace to the nearest point on the nominal line,
\[ p = \mathbf{\hat{A}C} \cdot \mathbf{\hat{A}B} \]
\[ q = |\mathbf{\hat{C}D}|. \]

The nominal lines are gridded at an interval \( \Delta p \), and for each position \( P_i \) on the line, all radar traces that with \( P_i - \frac{1}{2} < p < P_i + \frac{1}{2} \) and \( |q| < q_{\text{max}} \) are averaged into a new composite trace, where \( q_{\text{max}} \) is a threshold value chosen to be 40 meters under the assumption that bed properties change acceptably little in the along-glacier direction over this distance. The result of this operation is that new radar lines are created, with the values from the field data incorporated into the nearest grid node on each nominal line. To within GPS accuracy, corresponding traces from separate surveys are now directly comparable.

I use bed reflection power (BRP) to compare radar signatures over each line over time. I have dewowed and migrated the data, and picked the direct coupling and bed reflections for all traces in the same way as described in Chapter 4. BRP is defined as by Gades et al. (2000) and Pattyn et al. (2009) as
\[
\text{BRP} \propto \frac{1}{2(n_2 - n_1 + 1)} \sum_{i=n_1}^{n_2} V_i^2
\]
for window boundaries \( n_1 \) and \( n_2 \) and signal voltage \( V \). The bed and direct wave summation windows begin at the first break of their respective wavelets, and end at a prescribed constant time offset afterward. For the bed reflection, the summation window has been chosen to be 140 ns long. This window length is shorter than that used by Gades et al. (2000) and Pattyn et al. (2009) because they used lower frequency radar antennas. The direct wave is wider because of the superposition of the air and ground waves, so the direct coupling window is chosen to be 200 ns. As long as englacial reflections are faint (true over most of the repeat lines) and the glacier is sufficiently thick that the direct wave summation window does not include the bed reflection (greater than 16 m, also true), then these windows are adequate.

To diagnose bed properties in the absence of other data, attenuation correction must be considered for its effect on BRP. Attenuation correction is less important for observing temporal changes that occur from one survey date to the next. Attenuation is a function of temperature and ice impurities (MacGregor et al., 2007; Matsuoka, 2011), neither of which is likely to change over a time span of months. Still important for this application is ensuring that the energy directed into the glacier is constant or normalized. Two normalization schemes are tested: one in which the BRP is normalized to average direct coupling power (DCP), and one in which the BRP is normalized to average BRP within the line. The former assumes that antenna-ice cou-

<table>
<thead>
<tr>
<th>Line</th>
<th>Start</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper</td>
<td>(601800, 6743680)</td>
<td>(602410, 6743780)</td>
</tr>
<tr>
<td>Middle</td>
<td>(601920, 6743410)</td>
<td>(602510, 6743540)</td>
</tr>
<tr>
<td>Lower</td>
<td>(602080, 6742840)</td>
<td>(602320, 6742890)</td>
</tr>
</tbody>
</table>

Table B.1: Coordinates of endpoint locations for the three repeat radar lines surveyed in 2011. Coordinates are eastings and northings from the UTM Grid 7 North projection, with the WGS84 spheroid.
pling is constant, and so DCP is a reliable indicator of not only the energy propagated through the air, but also that directed into the ice. This may not be true, because the glacier surface transitioned from dry snow to wet snow and finally to bare ice over the course of the observation period. Also, the glacier surface may be spatially heterogeneous, particularly in the early summer. The second normalization scheme assumes that mean bed reflectivity is constant over all survey dates. This is unlikely to be true, and as a consequence temporal changes in mean BRP are lost to the normalization. However, temporal variability in spatial patterns is preserved, such that a bright anomaly in one survey will remain bright after normalization. This normalization scheme permits comparison of relative amplitude changes in BRP across surveys performed on different dates.

BRP is generally higher in radar measurements made over thinner ice because, because there is less attenuation within the ice. I use the method of Gades et al. (2000) to correct for ice thickness variations. I fit an exponential curve to the empirical relationship between ice thickness $h$ and bed reflectivity,

$$BRP_{\text{calc}} = ae^{-bh}$$

where $a$ and $b$ are fitting parameters. I then normalize the observed BRP to the expected BRP calculated from this curve

$$BRP' = \frac{BRP}{BRP_{\text{calc}}}.$$  

(B.3)

B.2 Results

![BRP-depth relationship](image)

Figure B.2: BRP-depth relationship, with observed data (grey diamonds) and the calculated fitting curve (black line). This relationship is poorer than that found by (Gades et al., 2000) in Antarctica, but qualitatively similar to that found by (Pattyn et al., 2009) for non-centreline data.

The three sets of radargrams derived as described above are shown in Figure B.3. The similarity of the projected radargrams acquired on difference dates attests that the dominant observable shapes represent persistent radar reflectors, and that the GPS-based georeferencing methods applied to the data work.

Some noticeable differences do exist, particularly in the apparent amplitude of both the bed
reflection and the direct coupling waves. In particular, the direct coupling waves from July 25th appear to be weaker than those from the two previous surveys, despite the fact that the radar and antenna configuration were kept as similar as possible. Computing the DCP for each radar line confirms this (Figure B.4). The direct coupling power typical for each survey day changes, although it remains nearly constant from one location to the next on the same day. Moreover, the DCP from July 10th appears to exhibit a large variability, although it is centred around a value that does not appear to change (Figure B.4b). Identical equipment and acquisition settings were used for each survey date, so I would speculate that antenna coupling with the ice is changing. One hypothesis suggests that differences in surface roughness and the presence of water on the glacier surface could lead to variable attenuation of the direct wave on different survey dates.

The non-normalized BRP does not exhibit such a clear offset from one survey date to the next. It is unlikely that the reflectivity of the glacier bed changes in inverse relation to the direct coupling power between the radar antennas. It is more likely that the DCP does not closely indicate how much energy is actually transferred into the ice. This last conclusion suggests that normalizing the BRP to direct coupling power is not fruitful, because DCP is likely unrelated to BRP. In the case of the July 10th survey, DCP is so variable that a lack of correlation between DCP and BRP would introduce noise within the survey. For these reasons, the second normalization scheme is described from this point on.

BRP is relatively small in deeper (60 m+) regions of the surveys, but varies from low to high values in shallower parts (Figure B.2). The depth–BRP relationship is $BRP_{\text{calc}} = 1.91e^{-0.013h}$. The residuals between the observed BRP and that predicted from this curve are large, indicating that differences in ice thickness do not explain all of the variability in BRP. The normalized and depth-corrected BRP is shown in Figure B.5.

In surveys of the upper line, there are few clear structures, aside from regions of higher bed reflectivity at the transect peripheries relative to the zones of lower reflectivity near glacier centre. Even after depth correction, these regions of high reflectivity correspond to the shallower parts of the profile. The temporal persistence of these brightly reflecting regions through time improves confidence in using BRP as a metric of temporal change.

The middle profile is notable for the existence of numerous spikes in reflectivity despite the relatively constant ice thickness (Figure B.5b). One set of spikes is exists from 200–280 m into the profile. On May 6th, there were no isolated regions of highly elevated BRP, although there is a wide region of slightly elevated BRP from 220–280 m with a small peak at 280 m. By July 10th, a pair of large spikes had developed: one near the preexisting maximum at 280 m, and one about 40 meters toward the west (240 m). Both of these had subsided by July 25th, but a third and larger spike appeared slightly further west, about 25 meters west of the western peak from July 10th (215 m).

Viewed in the radargrams (Figure B.3, middle row), this region is marked by the existence of an englacial point reflector in the lower section of the glacier. Though difficult to see in Figure B.3, this point reflector exhibits a polarity that matches the air wave, and is reversed from the bed reflector. This is most clearly expressed in the July 25th survey. As discussed in Chapter 2, this reversal is consistent with reflection from an interface of a material with lower dielectric permittivity than the transmitting material. Compared to ice, air and some geological materials may have similar or slightly lower dielectric permittivity.

A second ephemeral bright spot is visible at 420 m in the middle profile surveyed on July 10th (Figure B.5b). This is a region where bed reflectivity is typically slightly elevated, perhaps reflecting the shallower ice. The spike in reflectivity is not observed in either the preceding or the subsequent survey.
The lower repeat radar profile is in a part of the glacier observed to be stagnant based on pole velocities below $10 \text{ m a}^{-1}$. Because of technical problems, this line was not surveyed on July 10th. In general, the BRP observed in both May and late July appear to match each other reasonably well (Figure B.5c).
Figure B.3: Repeat line radargrams. Rows represent locations while columns represent dates. Data have been dewowed and migrated, but are otherwise unfiltered. Vertical white bars indicate areas where data are not available. The depth scale is approximate. The figures are plotted looking up glacier.
Figure B.4: Direct coupling power. The figure is plotted looking up glacier, so that the left side is glacier right (i.e. west).
Figure B.5: Bed reflection power, normalized to mean power over each radar line (B.3). The figure is plotted looking up glacier, so that the left side is glacier right (i.e. west).
B.3 Interpretation

Because the changes in DCP are much larger than any observed changes in BRP, I consider it unlikely that the power supplied or emitted by the transmitting antenna varied significantly. I have also found that the 10 MHz antennas used are relatively insensitive to the precise mounting configuration with the radar sled, so small differences from survey to survey do not appear adequate to explain the variation in DCP. Two other possibilities are that changes in DCP may result from differences in direct wave propagation along the ice-air interface, or from differences in interaction between the superimposed air wave and ground wave that comprise the direct coupling.

Changes in surface character from May to late July were large, with the glacier surface transitioning from mostly dry snow on May 6th to wet and patchy snow on July 10th, and to bare, wet, and cryoconite-covered ice by July 25th. It is not obvious how this change explains why DCP should increase from May 6th to July 10th, and then decrease substantially for July 25th.

Examination of the direct coupling waves from the three different survey dates reveals that the shape of the wavelet changes considerably from date to date. On May 6th, the wavelet appears to consist of two half-wavelength lobes. The first lobe is double peaked, with the peaks separated by about 12 nanoseconds. On July 10th, this spacing is 24 nanoseconds. On July 25th, the two-lobed model breaks down, and the direct coupling waves instead look like an asymmetrical sawtooth wave. These effects may be the result of both small changes in antenna offset, or changes in the relative strength of the air and ground waves. This latter effect could reasonably be expected to depend on changing antenna-to-ground coupling with changing surface properties.

In order for the comparisons made above to be valid, the georeferencing error for each radar line should be small relative to the horizontal sampling rate. GPS accuracy estimates range from 3 m for the Magellan GPS with WAAS-enabled, to under 10 m for the on-board GPS with WAAS disabled. Where the radar was stationary, random variability in reported GPS location is typically below 3 m, however it is on occasion much worse. The close correspondence of subglacial features seen in repeat profile radargrams as well as correlation between static BRP features appear to validate expectations about GPS accuracy. Temporal shifts by BRP spikes over distances shorter than 20 m cannot be proven using the available GPS coordinates.

B.4 Hydrological implications

The low-permittivity englacial point reflectors and spikes in BRP observed in the neighbourhood of 200-260 meters along the middle profile may support a hypothesis involving periodic water flow through this area of the glacier. The signals returned by the point reflectors are consistent with an englacial void, which would provide a conduit for water. The spikes in BRP may occur when a drainage pathway is occupied by water, because the high dielectric permittivity of water would increase the reflectance. If this is the case, then the exact location of channels is not fixed but varies over time. Impermanence of subglacial and englacial features over the course of an ablation season has also been observed using ground-penetrating radar by Irvine-Fynn et al. (2006). Significant variability of glacier radar signatures on short time scales has been observed by Jania et al. (2005). They also observed an oscillation in BRP correlated with the diurnal cycle. In light of these previous results, it is plausible that hydrological reorganization could cause many of the observed changes in BRP.
Upstream areas can be used to predict likely locations for channels. Given a digital elevation model (DEM), an upstream area calculation sums the area that may contribute runoff to any given grid cell. In a subglacial environment, hydraulic potential gradients drive the movement of water, and can be approximated by

$$\Psi = z + fh,$$

for bed elevation $z$, ice thickness $h$, and floatation fraction $f$. The glacier ice thickness maps and bed elevation maps generated in Chapter 5 have been used to calculate hydraulic potential. Before calculating upstream areas, parts of the hydraulic potential map without any outflow direction (sinks) are filled following the method of Wang and Liu (2006). Then, flow-routing is performed using the $D_{\infty}$ method of Tarboton (1997). The repeat radar transects are shown in Figure B.7 with the resulting upstream area for multiple floatation fractions.

There is no strong correspondence between the predicted channel locations derived from the upstream area calculation and areas of high BRP in Figure B.5. There does appear to be some correlation between July spikes in BRP and upstream area at high floatation fractions in the middle profile. On the other hand, the section of the upper profile with high upstream area around 240 m from the western profile boundary does not match any peaks in the BRP.

The available data are too limited and too afflicted by sampling problems to make strong conclusions about the distribution and movement of subglacial channels. At the same time, this technique has potential to yield useful information with additional data.
Figure B.6: Upstream area calculated with flotation fractions $f = 0.5$ and $f = 1.0$ for South Glacier (a,b) and North Glacier (c,d).
Figure B.7: Upstream area along the repeat radar profiles. Upstream area has been calculated for 10 different flotation fractions in the range 0.1–1.0. The figure is plotted looking up glacier, so that the left side is glacier right (i.e. west).
Appendix C

Additional Wavelet Transforms

Additional wavelet transforms have been computed following Torrence and Compo (1998) for selected traces collected with 35 MHz and 50 MHz antennas. Traces are chosen from North Glacier radar lines similar to that shown in Figure 4.1.

Figure C.1: Wavelet transforms of traces collected during (a,b) 35 MHz and (c,d) 50 MHz radar surveys on North Glacier in August 2011. The brightness scale is a relative log scale, with warmer tones indicating higher power. The data from the 50 MHz survey attenuates more rapidly, so a radar trace from shallower ice was chosen.
Appendix D

Additional TAC Comparisons

In addition to the six radar-TAC comparisons shown in Chapter 5, additional nearby radar lines at 10 MHz on South Glacier and 35 MHz and 50 MHz on North Glacier are available.

Figure D.1: Additional 35 MHz radar line from South Glacier collected in 2011, passing 112 meters northwest of the SG-UPPER85-2011 borehole.
Figure D.2: Additional 35 MHz radar lines from North Glacier collected in 2011, passing over and around the NG-MID75-2011 borehole. Transverse lines are shown in (a,b) and a longitudinal line is shown in (c).
Figure D.3: Additional 50 MHz radar lines from North Glacier collected in 2011, passing over and around the NG-MID75-2011 borehole. Transverse lines are shown in (a, b) and a longitudinal line is shown in (c).
Appendix E

Additional Radargrams

The collection of unprocessed (left) and processed (right) radargrams on the following pages illustrates the englacial scatter processing used for 10 MHz radar surveys. These data were collected in May 2011 on both South Glacier and North Glacier.
South Glacier (May 2011)
Line 27, 10 MHz

South Glacier (May 2011)
Line 28, 10 MHz

South Glacier (May 2011)
Line 29, 10 MHz

South Glacier (May 2011)
Line 31, 10 MHz

South Glacier (May 2011)
Line 32, 10 MHz

South Glacier (May 2011)
Line 33, 10 MHz
Appendix F

Lobate Symmetry in North Glacier

Localized strain heating provides another mechanism to generate lateral variability in thermal structure. I suggest that focused zones of strain heating caused by basal sliding is sufficient to explain the laterally-symmetrical lobate pattern of radar scattering observed in North Glacier. This derivation follows that of Nye (1965). The Stokes equations in a channel inclined at angle $\alpha$ (such that the $x$ coordinate is perpendicular to the valley plunge, and $y$ is horizontal) are

$$
\frac{\partial \tau_{xx}}{\partial x} - \frac{\partial P}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \rho g \sin \alpha \\
\frac{\partial \tau_{yy}}{\partial y} - \frac{\partial P}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} = 0 \\
\frac{\partial \tau_{zz}}{\partial z} - \frac{\partial P}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = \rho g \cos \alpha ,
$$

for pressure $P$, deviatoric stress $\tau_{ij}$, density $\rho$ and gravitational acceleration $g$. Assuming uniformity in $x$ (which is the case if the valley is long compared to its width and depth), the gradients of $x$ become negligible. Nye (1965) gives the reduced equation as

$$
\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \rho g \sin \alpha .
$$

Incorporating a form of Glen’s flow law with effective stress $\tau_E$ and a definition of viscosity ($\nu$)

$$
\nabla u = A \tau_E^{-1} \tau_{ij} \\
\tau_{ij} = \eta \nabla u ,
$$

allows (F.2) to be rewritten as

$$
\frac{\partial}{\partial y} \left( \eta \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( \eta \frac{\partial u}{\partial z} \right) = \rho g \sin \alpha
$$

for velocity $u$.

Equation (F.5) is a nonlinear Poisson equation. Amundson et al. (2006) use this simplified version of the nonlinear Stokes equation to investigate spatial and temporal variability of the flow field in Black Rapids Glacier. I use a similar approach to study the spatial distribution of strain heating in North Glacier. I use the open-source software package FEniCS (Logg et al., 2012) to calculate a solution to (F.5) using finite elements. Picard iteration is used to
handle the viscosity-velocity nonlinearity. The problem domain is an unstructured triangular mesh representing a flowband-orthogonal slice from a mid-glacier section of North Glacier, approximately near temperature sensor H03. To simplify the problem, I use a constant flow-law coefficient \( A = 2.4 \times 10^{-24} \text{ Pa}^{-3} \text{ s}^{-1} \). Velocities in the area are roughly 9 m a\(^{-1}\), based on pole displacements over the period July 2007–May 2009. As a control experiment, I prescribe a no-slip \((u_b = 0)\) condition as a Dirichlet boundary along the glacier bed, and a zero shear stress \((\nabla u = 0)\) condition as a Neumann boundary condition at the ice-air interface. The surface slope \((\alpha)\) is chosen to be 5\(^\circ\), which is approximately the average surface slope of the middle stretch of North Glacier.

The solution gives the highest flow velocities at the surface, near the center of the glacier (Figure F.1a), which is qualitatively similar to analytical solutions with idealized geometries (Nye, 1965). The maximum surface velocity with the chosen value of \( A \) is only 5 m a\(^{-1}\), which is slower than observed over the period 2007–2009. The highest rates of strain heating are concentrated at depth and broadly distributed across the glacier valley (Figure F.1b).

Sliding likely occurs at the base of the glacier valley, so in a second simulation, I prescribe a sliding velocity as a Dirichlet boundary condition. The choice of a sliding velocity function has been considered by Amundson et al. (2006). Following them, I choose a polynomial that causes surface flow velocity to roughly match observations of 9–10 m a\(^{-1}\). For simplicity, I limit the polynomial to second order.

When sliding is prescribed, the pattern of ice velocities changes because there is reduced basal traction restraining the ice over a section of the basal boundary (Figure F.2a). Zones of high heat generation appear near the glacier base on either side of the sliding region (Figure F.2b). This occurs because there is a zone of high strain rates, primarily in the \(yx\)-plane, where the basal boundary condition changes. The locations of the high strain heating lobes are similar to the locations of the laterally-symmetric scattering zones observed in the radar data, maintaining the possibility that they form for similar reasons. The maximum heating rates are sufficient to account for roughly 0.7 g kg\(^{-1}\) a\(^{-1}\) of melt. If the sliding pattern is only maintained seasonally the melt may be less, however it is still a large amount of water when integrated over the number of years a parcel of ice spends travelling through the shear zone.

The results of this simple experiment do not exclude other possible processes that might lead to the observed structure. One alternative idea is that variations hydraulic potential could concentrate water in the radar-scattering zones. Were this to happen, the effect of high water contents on viscosity could cause a minor feedback, by which increased strain further contributes to high local water contents. Other possibilities not investigated include distributions of surface features that concentrate meltwater influx on either side of the central flowline, or misinterpretation of the radar scatter.
Figure F.1: Spatial variation in modelled down-glacier flow velocity (a) and strain heating rates (b) in a cross-section for North Glacier, with a no-slip basal boundary condition.

Figure F.2: Spatial variation in modelled down-glacier flow velocity (a) and strain heating rates (b) in a cross-section for North Glacier, with a prescribed basal sliding velocity.

Figure F.3: Observed radar scattering (light grey) in the North Glacier cross-section.