KU–BAND IMAGE REJECTION SLIDING–IF
TRANSMITTER IN A 0.13–µm CMOS PROCESS

by

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ABSTRACT

The effects of gain/phase mismatch, and phase noise in a multiband double-image-rejection transmitter are investigated. Although a direct-I/Q modulator architecture is simple, the I/Q gain and phase mismatches directly affect the image rejection ratio (IRR) over the operating frequencies. However, the double-image-rejection transmitter (DIRT) has low sensitivity to a small I/Q phase mismatch while the IRR is predominantly dependent on the IF gain mismatch. Furthermore, there is a region of insensitivity to both gain and phase mismatches in the DIRT. For characterizing the effects of the I/Q mismatch and phase noise in the DIRT, the IRR, EVM, and SER are theoretically analyzed and simulated at the system level. The proposed DIRT with sliding-IF is implemented on a 0.13-µm CMOS process to prove the insensitivity to the I/Q mismatch effects over the 11 GHz to 15 GHz multiband frequency ranges. For supporting the multiband functionality, frequency dividers-by-4/8/16 are utilized to generate 0.675 GHz, 1.35 GHz, and 2.7 GHz quadrature IF LO signals using a 10.8 GHz RF LO signal. The measurement results show that the in-band image rejection and LO leakage suppression are greater than 48.8 dBc and 43.5 dBc, respectively, over the wideband frequency range. The output-referred 1-dB compression point is obtained as high as -4 dBm with a 1.5 V power supply. A multiband DIRT operating above 10 GHz has not been previously reported.

Keywords: CMOS multiband transmitter, Ku-band applications, image rejection transmitter, Sliding-IF transmitter
To my beloved wife, daughter, and parents!
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<tr>
<td>BB</td>
<td>Base Band</td>
</tr>
<tr>
<td>BPF</td>
<td>Band Pass Filter</td>
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<tr>
<td>CMOS</td>
<td>Complementary Metal Oxide Semiconductor</td>
</tr>
<tr>
<td>DCT</td>
<td>Direct Conversion Transmitter</td>
</tr>
<tr>
<td>DIRT</td>
<td>Double Image Rejection Transmitter</td>
</tr>
<tr>
<td>DR</td>
<td>Dynamic Range</td>
</tr>
<tr>
<td>EVM</td>
<td>Error Vector Magnitude</td>
</tr>
<tr>
<td>IF</td>
<td>Intermediate Frequency</td>
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<tr>
<td>IRR</td>
<td>Image Rejection Ratio</td>
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<tr>
<td>IP3</td>
<td>Third-order intercept point</td>
</tr>
<tr>
<td>I/Q</td>
<td>In-phase/quadrature</td>
</tr>
<tr>
<td>LO</td>
<td>Local Oscillator</td>
</tr>
<tr>
<td>LTCC</td>
<td>Low Temperature Co-fired Ceramic</td>
</tr>
<tr>
<td>MMIC</td>
<td>Monolithic Microwave Integrated Circuit</td>
</tr>
<tr>
<td>MC</td>
<td>Monte Carlo</td>
</tr>
<tr>
<td>PA</td>
<td>Power Amplifier</td>
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<tr>
<td>PCB</td>
<td>Printed Circuit Board</td>
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<tr>
<td>PLL</td>
<td>Phase Locked Loop</td>
</tr>
<tr>
<td>Acronym</td>
<td>Abbreviation</td>
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<tr>
<td>PN</td>
<td>Phase Noise</td>
</tr>
<tr>
<td>PPF</td>
<td>Polyphase Filter</td>
</tr>
<tr>
<td>Q</td>
<td>Quality Factor</td>
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<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
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<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>SCL</td>
<td>Source Coupled Logic</td>
</tr>
<tr>
<td>SER</td>
<td>Symbol Error Rate</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
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<tr>
<td>VCO</td>
<td>Voltage Controlled Oscillator</td>
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CHAPTER 1: INTRODUCTION

A multiband transmitter is an area of interest in wireless communication systems. Various transmitter architectures have been widely investigated for implementing a multiband transmitter. Superheterodyne and direct-conversion architectures are well-known structures in wireless communication systems. However, the superheterodyne architecture requires two separate LO generators, an IF filter and an image rejection filter, which make it difficult to implement a single chip at high frequencies. The direct-conversion architecture suffers from local oscillator (LO) leakage, severe in-phase/quadrature (I/Q) mismatch effects, and frequency pulling problems. Therefore, a dual-conversion transmitter with sliding-IF using an LO generator is more beneficial in consideration of both chip integration and multiband system performance in comparison with the traditional architectures. Moreover, the multiband transmitter is required to be insensitive to I/Q mismatch effects due to variations over the operating frequencies. The mismatch effects directly affect the image rejection ratio (IRR), resulting in channel interference and a degradation of receiver sensitivity over the wideband frequency range. The phenomenon is easily observed in the microwave/millimetre-wave frequency range because of the high frequency of operation. Therefore, better insensitivity to I/Q mismatches is required to implement a multiband transmitter operating over the microwave/millimetre-wave range. This thesis focuses on the analysis and implementation of a multiband microwave/millimeter-wave transmitter which is
In insensitive to I/Q mismatch effects. Error vector magnitude (EVM), image rejection ratio (IRR), and symbol error rate (SER) are investigated to evaluate the effects of the I/Q mismatches and phase noise on system performance. The complicated IRR equations are approximated to show the relationship between EVM and IRR, while giving the insight into I/Q mismatch effects on the transmitter architecture. Furthermore, the feasibility of the multiband transmitter over the microwave frequency range is proven by implementing the transmitter on a 0.13-μm CMOS process.

1.1 Background

The need of CMOS microwave/millimeter-wave transceivers for ISM bands at 17 GHz, 24 GHz, and 60 GHz has grown dramatically over the last decade due to: the expansion of high data-rate communication systems, reduction of the system cost, increased integration, and congestion of WLAN frequency (2.4/5 GHz) bands [1]–[6]. Short-range communication is more attractive to the development of low power CMOS transceivers. Although the requirements result in the development of CMOS transceivers at microwave/millimeter ISM bands, Ku/Ka band transceivers have been mainly implemented by up/down converter monolithic microwave-integrated-circuit (MMIC) chips, a power amplifier (PA) MMIC, and an embedded low-temperature co-fired ceramic (LTCC) bandpass filter suppressing the image signal [7]–[10]. Fig. 1.1 shows the typical architecture of the Ku/Ka band transmitter with a direct I/Q modulator. Although the transmitter architecture with a direct I/Q modulator is simple, the I/Q gain and phase mismatches can directly affect the IRR variations over the operating frequencies. However, in the double-image-rejection transceivers based on the Weaver architecture,
the IRR is insensitive to a phase mismatch. In the Weaver receiver architecture [11]–[14], making the RF phase mismatch equal to the IF phase mismatch is sufficient to reject the image signal. In the double-image-rejection transmitter (DIRT) [15]–[16], phase mismatches of the IF and RF mixer pairs can cancel each other for small phase mismatches, leaving the IF and RF gain mismatches as the most significant mismatch term. Therefore, the DIRT architecture can have an advantage on the I/Q gain and phase mismatch effects for wideband and multiband transmitters.

In the reported Ku-band transmitter [8], the measured results of the entire transmitter showed an output power of 26 dBm, LO rejection of 20 dBc, and an image rejection of more than 40 dBc at 14 GHz band. The PA MMIC is essential to obtain an output power of 26 dBm, but for lower output power requirements an integrated CMOS DIRT can be considered. The transmitter must have the ability to reject the LO leakage and image signal to compensate for the removal of the RF filter. In addition to the suppression ability, it is very important to select a suitable multiband CMOS transmitter architecture in order to exploit the wideband frequency range for Ku-band applications.
1.2 Transmitter architectures

The typical configurations of microwave/millimeter-wave dual-conversion transmitter architectures are shown in Fig. 1.2. System architecture of a dual-conversion transmitter with a sliding-IF in Fig. 1.2(a) has several advantages over the traditional superheterodyne and direct-conversion transmitter configurations [17]–[20]. However, the dual-conversion transmitter architecture has its own drawbacks when used for a multiband transmitter. The rejection ratio on the image signal of the IF-to-RF mixer is dependent on the attenuation characteristic of the RF front-end, as indicated by the dashed lines shown in Fig. 1.3(a). This could include the characteristics of antenna and inductive loads of the output amplifier. As the IF frequency goes up, the IF-to-RF image signal can be further suppressed, as indicated by the direction of the arrow in Fig. 1.3. Hence, the IF should be set close to 1/2 or 1/3 of the output frequency to significantly suppress the image signal of an IF-to-RF mixer. However, the higher IF frequency makes the microwave/millimeter-wave transmitter more sensitive to I/Q mismatch effects. In addition, IF harmonic frequency can overlap the output frequency due to the relation between the output \( f_{RF} \) and intermediate \( f_{IF} \) frequencies. The relations among the output \( f_{RF} \), intermediate \( f_{IF} \), and RF LO \( f_{OSC} \) frequencies are expressed by (1.1) and (1.2), where N means frequency divider ratio for IF frequency.

\[
f_{IF} = \frac{f_{RF}}{N + 1} \quad (1.1)
\]

\[
f_{OSC} = \frac{N}{N + 1} \cdot f_{RF} \quad (1.2)
\]
Figure 1.2: (a) Dual-conversion architecture with sliding-IF. (b) Half-RF architecture. (c) Double-image-rejection transmitter architecture with sliding-IF.
Figure 1.3: Spectrum of a transmitter with complex signals. (a) Dual-conversion architecture with sliding-IF (b) Dual-conversion sliding-IF architecture with double-image-reject mixers. (dashed lines: the attenuation characteristics of the RF front-end, arrows: the direction of the intermediate ($f_{IF}$) and image ($f_{IM}$) signals).

For these reasons, the frequency plan should be carefully determined while taking into consideration the attenuation level of the RF front-end. In the reported paper [3], a half-RF dual-conversion transmitter was proposed for 60 GHz applications. In such a case, the quadrature LO generator was not required due to the RF polyphase filter (PPF). Although the reported architecture is attractive for an on-chip voltage-controlled oscillator (VCO) in Fig. 1.2(b), the architecture is limited to a single-band operation and the image signal at ($3f_{OSC} - f_{IF}$) should be eliminated, as shown in Fig. 1.3(a). In addition,
for the multiband functionality, the millimeter-wave transmitter requires a wide VCO tuning range and the output frequency of the on-chip VCO must be set at half the output frequency of the transmitter architecture. Fig. 1.2(c) shows a block diagram of a DIRT with a sliding-IF, where the image signal suppression level is ideally independent of the RF front-end attenuation, as shown in Fig 1.3(b). The IF can be far away from the RF output frequency regardless of the RF front-end attenuation characteristic. Hence, the I/Q-matching requirements can be alleviated and the harmonics of the IF signal can be neglected. For these reasons, the DIRT architecture with sliding-IF can be more attractive for the multiband microwave/millimeter-wave transmitter.

In the reported literature, an CMOS DIRT architecture was proposed over 900 MHz and 1800 MHz frequency bands [21] and the DIRT with a sliding-IF of 1 GHz was demonstrated for the 5 GHz IEEE 802.11a WLAN band [22]. The DIRT architecture was also used with harmonic-rejection mixers [23] and a 24 GHz single-band phased-array transmitter [24]. However, a multi-band DIRT operating over 10 GHz has not been previously reported. In addition to the inherent IRR characteristics, the sliding-IF architecture can be applied to the multiband transmitter because the IF is not limited to a fixed IF. The total die area can be reduced by using a single LO for the quadrature IF and RF LO generators. The output frequency of the multiband transmitter can be determined by the sliding IF and RF LO frequencies. If the frequency tuning range of the RF LO generator covers 10–11.5 GHz, the sliding IF frequency is below 3 GHz through a combination of frequency divider blocks, as shown in Fig. 1.4.
Therefore, 11–15 GHz multiband output frequencies can be achieved by sliding IF frequencies below 3 GHz. The sliding IF frequency can be expressed by

\[
f_{IF} = \frac{1}{4k + 1} \cdot f_{OUT} \bigg|_{k = \text{int}}.
\]  

(1.3)

The quadrature IF LO signals at 1/16\(f\text{OSC}\), 1/8\(f\text{OSC}\), and 1/4\(f\text{OSC}\) frequencies are generated by the frequency divider blocks. The RF output frequencies of 17/16\(f\text{OSC}\), 9/8\(f\text{OSC}\), and 5/4\(f\text{OSC}\) can be obtained by summing the RF LO frequency of \(f\text{OSC}\) and the sliding-IF frequencies of 1/16\(f\text{OSC}\), 1/8\(f\text{OSC}\), and 1/4\(f\text{OSC}\).
1.3 Motivation and contributions

1) The in-band image signal in a microwave/millimeter-wave transmitter can not be eliminated by the attenuation characteristic of the RF front ends. Additional mismatch compensation circuits make the transmitter more complex. Hence, symmetric/asymmetric gain- and phase-mismatch effects should be analyzed and simulated to compare the in-band IRR characteristics between the DCT and DIRT architectures.

2) The DIRT has been widely used for higher-order QAM systems due to the robust I/Q mismatch effect and the higher suppression of the in-band signal. Although the DIRT has been reported for receivers [11]–[13] and transmitters [14]–[16] in many wireless applications, the analytical expressions of EVM and SER have not been previously reported with gain/phase mismatches and LO phase noise. Moreover, the reported expressions for the IRR are too complicated to give insights into I/Q mismatch effects. Therefore, a theoretical analysis of the DIRT architecture is presented using the EVM, IRR and a union bound on SER. In addition, the relationship between IRR and EVM is required to quantify the system performance.

3) The DCT has a lower power consumption than the DIRT, because it uses a fewer number of components than the DIRT, which is composed of three mixer pairs and IF/RF local oscillator (LO) generator blocks. In spite of a larger power consumption, the DIRT has been used for higher-order QAM systems. Therefore, a comparison between the DCT and DIRT should be undertaken via an analysis of the I/Q gain and phase mismatches and RF phase noise effects to show the advantage and disadvantages of the
architectures. In addition, the analytic results of the DIRT should show the improvement of the signal-to-noise ratio (SNR) performance on higher-order QAM systems.

4) A multi-band CMOS DIRT operating over 10 GHz has not been reported. Therefore, the implementation of the transmitter can be a reference for multiband transmitters over 10 GHz.

The contributions of this work include:

- Two distinct gain conditions are proposed for the analysis of the I/Q mismatch effects of the double-image-rejection transmitter.
- In a proposed gain condition-II, the double-image-rejection transmitter is insensitive to both gain and phase mismatches.
- The complex envelope-based matrix model is proposed for the double-image-rejection transmitter.
- The analysis on the insensitivity of the double-image-rejection transmitter is carried out by EVM, IRR, and SER calculations.
- Image rejection ratio is approximated to show the insight into I/Q mismatch effects of the double-image-rejection transmitter architecture.
- The relationship between EVM and IRR is proposed for the double image rejection transmitter.
- The phase noise effects on EVM and SER are investigated and compared with DCT to show the advantage of the DIRT for higher-order QAM systems. The proposed equations for EVM and SER can give the insight into the transmitter architectures.
A multiband double-image-rejection transmitter operating over 10 GHz is implemented on a 0.13-\(\mu\)m CMOS process. A multiband DIRT operating above 10 GHz frequency has not been previously reported.

The measured in-band image rejection ratio has the best performance in comparison with previously reported CMOS transmitters operating around 10 GHz.

The feasibility of the multiband transmitter for microwave/millimetre wave applications is proven.

1.4 Organization of thesis

In this chapter, transmitter architectures have been investigated for the multiband operation. Subsequent chapters focus on the analysis of the DIRT architecture. Furthermore, the proposed multiband transmitter is implemented on a 0.13-\(\mu\)m CMOS process. The measured image rejection ratio shows the feasibility of the multiband transmitter operating above 10GHz. The rest of the chapters are organized as follows:

Chapter 2 focuses on the analysis of the IRR, EVM and SER characteristics of the DIRT in two distinct conditions. The analytical approximation for the IRR is derived to show the relationship between IRR and EVM. Moreover, the phase noise effect is examined as a function of phase mismatch using the matrix channel model. The simulation results show the insensitivity to the I/Q mismatch in the two distinct conditions. The principle conclusion is that the DIRT has lower sensitivity to the phase mismatch in any conditions. Furthermore, the transmitter
is insensitive to both gain and phase mismatches in the proposed gain condition-II.

- In chapter 3, the proposed DIRT with sliding-IF is designed on a 0.13-µm CMOS process. The DIRT is composed of IF mixer pairs, RF mixer pair, preamplifier, frequency divider block, and RF quadrature blocks. The schematic block diagrams for the building blocks are shown in the circuit level. For supporting the multiband functionality, a frequency divider scheme is proposed for quadrature IF LO signals using a single RF LO generator. The post-layout simulation results show that the transmitter is properly operating over the Ku band frequency range with the parasitic capacitances and resistances.

- In chapter 4, the insensitivity to the gain/phase mismatch effects is demonstrated for multiband operation by showing the in-band image rejection ratio. In addition, the measurement setup and results are shown for the fabricated test chips. A test PCB board was fabricated for mounting the bare die and biasing the DC power supply voltage. An external RF LO signal is directly fed on the chip pads to avoid high frequency attenuation loss characteristic. The measurement results show the image signals were suppressed lower than 48.8 dBc over 12.5 to 14.5 GHz frequency range and the LO leakage suppression ratios were lower than 43.5 dBc.

- Chapter 5 presents the conclusion and discusses future work.
CHAPTER 2: IMAGE REJECTION TRANSMITTER

In this chapter, the effects of I/Q gain and phase mismatches are investigated by analyzing the rejection ratio of the image signals and LO leakages as well as showing the I/Q mismatch relations between two IF mixer pairs and an RF mixer pair. Fig. 2.1 shows that an in-band image (Image1), two out-of-band images (Image2 and Image3), and two LO leakage signals are emitted by the transmitter. The in-band image (Image1) can be defined as the cross-talk between I/Q data streams. The two out-of-band images (Image2 and Image3) can be defined as spurious signals from the IF-to-RF mixer pair. The frequencies of the RF output, and in-band image, and two out-of-band image signals are defined as

\[
\begin{align*}
\omega_{RF} &= \omega_{LO2} + \omega_{LO1} + \omega_{BB} \\
\omega_{RFim} &= \omega_{LO2} + \omega_{LO1} - \omega_{BB} \\
\omega_{IMim} &= \omega_{LO2} - \omega_{LO1} + \omega_{BB} \\
\omega_{IM} &= \omega_{LO2} - \omega_{LO1} - \omega_{BB}.
\end{align*}
\] (2.1)

The frequencies of the LO leakage signals are defined by

\[
\begin{align*}
\omega_{LO(2)} &= \omega_{LO2} \\
\omega_{LO(1,2)} &= \omega_{LO2} + \omega_{LO1}.
\end{align*}
\] (2.2)
The out-of-band image signals and the LO-leakage signals are attenuated through the bandlimited characteristic of the RF front-end. However, the in-band image signal includes the I/Q mismatch effects regardless of the attenuation characteristic of the RF front-end. Hence, the I/Q mismatch effects can be examined by analyzing the in-band image rejection ratio ($\text{IRR}_{\text{RFim}}$) in the DIRT architecture. Moreover, the $\text{IRR}_{\text{RFim}}$ degrades the sensitivity of the receiver or creates channel interference. Therefore, the $\text{IRR}_{\text{RFim}}$ should be analyzed carefully with I/Q gain and phase mismatches of the IF and RF mixer pairs.

### 2.1 Image signals as a function of gain mismatches

The double-image-rejection transmitter (DIRT) is composed of two IF mixer pairs (A and B) and an RF mixer pair (C) as shown in Fig. 2.2. If the input quadrature signals I and Q are injected into the IF mixer pairs, the output signal can be expressed as a function of the input magnitude mismatch and gain/phase mismatches for the IF and RF
mixture pairs (A, B, and C). The input magnitude mismatch ratio (k) can be generated through the DAC and active-filter blocks. The IF mixer gain mismatches ($\alpha_m1/\alpha_m2$ and $\alpha_m3/\alpha_m4$), RF gain mismatch ($\alpha_5/\alpha_6$), IF LO phase mismatches ($\theta_{1-2}$), RF LO phase mismatches ($\theta_{3-4}$), and DC-offsets ($O_{i/q}$, $O_{2i2q2}$, $O_{LOI/Q}$, and $O_{LOI2/Q2}$) are independent random variables. The IF phase mismatch ($\theta_{IF1-4}$) and RF phase mismatch ($\theta_{RFI/Q}$) result from each mixer and connection lines. Furthermore, the entire IF gain mismatches including the input magnitude mismatch ($k\alpha_m1/\alpha_m2$ and $k\alpha_m3/\alpha_m4$) can be replaced with the IF gain mismatches at the output of IF mixers ($\alpha_1/\alpha_2$ and $\alpha_3/\alpha_4$) for the simplicity of calculation. The phase variation effects ($\theta_{IF1-4}$ and $\theta_{RFI/Q}$) resulting from each mixer will be investigated by Monte Carlo simulations in the next section. To derive the output power spectrum, the input quadrature I/Q signals, the quadrature LO signals, and the output signal ($s_{RFOUT}(t)$) are expressed by (2.3)–(2.13).

\[ i_{BB}(t) = k \cdot \cos(\omega_{BB}t) + O_i \] (2.3)

\[ q_{BB}(t) = -\sin(\omega_{BB}t) + O_q \] (2.4)

\[ i_{LO1}(t) = A_{LOQ} \cos(\omega_{LO1}t + \theta_1) + O_{LOI} \] (2.5)

\[ q_{LO}(t) = -A_{LOQ} \sin(\omega_{LO1}t + \theta_2) + O_{LOQ} \] (2.6)

\[ s_{IFI}(t) = \alpha_1 \cdot i_{BB}(t) \cdot i_{LO1}(t) - \alpha_2 \cdot q_{BB}(t) \cdot q_{LO1}(t) \] (2.7)

\[ s_{IFQ}(t) = \alpha_3 \cdot i_{BB}(t) \cdot q_{LO1}(t) + \alpha_4 \cdot q_{BB}(t) \cdot i_{LO1}(t) \] (2.8)

\[ s_{IOUT}(t) = s_{IFI}(t) + O_{2i2} \] (2.9)
\[ s_{QOUT} (t) = s_{IFQ} (t) + O 2_{q2} \]  
\[ i_{LO2} (t) = A2_{LO2} \cos(\omega_{LO2} t + \theta_3) + O_{LO2} \]  
\[ q_{LO2} (t) = -A2_{LO2} \sin(\omega_{LO2} t + \theta_4) + O_{LO2} \]  
\[ s_{RFOUT} (t) = \alpha_5 \cdot s_{IOUT} (t) \cdot i_{LO2} (t) - \alpha_6 \cdot s_{QOUT} (t) \cdot q_{LO2} (t) \]

If the RF output signal \( s_{RFOUT}(t) \) is derived without any assumptions, the analysis will not yield useful insight into the performance effects. Therefore, the analysis will be carried out by the following steps: 1) gain mismatch effects will be investigated with small IF and RF phase mismatches, which can cancel each other at the output of the DIRT [15]-[16], 2) LO phase mismatch effects will be analyzed together with the gain mismatches, and 3) the asymmetric phase mismatch effects of the mixers will be analyzed by Monte Carlo simulations. To analyze the gain mismatch effects of the DIRT, the I/Q gain mismatch effect of one mixer pair is investigated using the signal constellation, where the input I and Q signals are either −1 or +1 as QPSK data stream [31]. Fig. 2.3(a) shows the block diagram of one I/Q mixer pair with gain mismatch by \( \alpha_1 = (1+\Delta x/2) \) and \( \alpha_2 = (1-\Delta x/2) \), where \( \Delta x \) is gain mismatch in the I and Q paths. By multiplying the input signals with the quadrature LO signals, the resultant signal constellation can be obtained with gain mismatch variations as shown in Fig. 2.3(b). If the IF and RF gain mismatch variations are randomly considered in the DIRT, it is difficult to offer the insight into the I/Q gain mismatch effect. Therefore, the IF and RF gain mismatches are assumed as \( \Delta x_1, \Delta x_2, \) and \( \Delta x_3 \) in the same manner as with one I/Q
modulator in Fig. 2.3(a). Under this assumption, the IRR$_{RFin}$ can be derived as (2.14). The derivation of (2.14) is shown in Appendix A.

\[
 IRR = \frac{\text{Mag}_{RFin}}{\text{Mag}_{RF}} = \frac{1}{64} \left(2(\Delta x_1 + \Delta x_2) + (\Delta x_1 - \Delta x_2)\Delta x_3\right)^2 \tag{2.14}
\]

where $\Delta x_1$, $\Delta x_2$, and $\Delta x_3$ are IF gain mismatch of IF mixer pair A, IF gain mismatch of IF mixer pair B, and RF gain mismatch, respectively. Fig 2.4(a) shows the graph of the IRR$_{RFin}$ as a function of IF gain mismatch of the mixer pair B without any RF gain mismatch. When IF gain mismatch of the mixer pair A is changed from -10 % to 10 % or -5 % to 5 %, the IRR$_{RFin}$ is not sensitive to IF gain mismatch at $\Delta x_1 = -\Delta x_2$, but
highly sensitive to the IF gain mismatch at $\Delta x_1 = \Delta x_2$. The effect of RF gain mismatch is shown in Fig. 2.4(b). When the IF gain mismatch on the mixer pair A is 10%, the $\text{IRR}_{\text{RFim}}$ varies as a function of IF gain mismatch of mixer pair B and RF gain mismatch. The DIRT has the worst IRR curve (bold line) as an upper bound when the RF gain mismatch is 10% as shown in Fig. 2.4(b). This simulation result demonstrates that 1) higher input I/Q magnitude mismatch results in lower image suppression by $\Delta x_1 = \Delta x_2$, for small IF mixer gain mismatches, 2) if the magnitudes of input I/Q signals are identical, the IRR variation depends on the variations of IF mixer gain mismatches from $\Delta x_1 = \Delta x_2$ to $\Delta x_1 = -\Delta x_2$, 3) the image signal can be significantly suppressed by adjusting the IF path gains to the gain condition, $\Delta x_1 = -\Delta x_2$, even though there are RF gain mismatch variations. Therefore, the phase mismatch effects need to be investigated under the specified gain conditions: 1) gain condition-I ($\alpha_1 = \alpha_3$ and $\alpha_2 = \alpha_4$) as the region sensitive to gain mismatch but insensitive to phase mismatch and 2) gain condition-II

---

**Figure 2.3:** a) Block diagram of one direct I/Q mixer pair. b) Effect of I/Q gain mismatch on QPSK signal constellation.
Figure 2.4: a) IRR variations as function of IF gain mismatch of mixer pair B in the variation of IF gain mismatch of mixer pair A by -10%, -5%, 5% and 10% when no RF mixer gain mismatch. b) IRR variations as function of IF gain mismatch of mixer pair B in the variation of RF gain mismatch by -10%, -5%, 5% and 10% when IF mixer gain mismatch of mixer pair A is 10%.*.*the bold line indicates the lowest IRR curve in the variations of the RF mismatch.
($\alpha_1=\alpha_4$ and $\alpha_2=\alpha_3$) as the region insensitive to both gain and phase mismatches. The IF gain mismatch ($\alpha_1/\alpha_2$) and RF gain mismatch ($\alpha_5/\alpha_6$) are given by $1+\varepsilon_1$ and $1+\varepsilon_2$, respectively. The IF and RF phase mismatches are defined as $\theta_1-\theta_2=\theta_A$ and $\theta_3-\theta_4=\theta_B$, respectively.

### 2.2 Image rejection ratio as a function of I/Q gain & phase mismatches

#### 2.2.1 In gain condition-I ($\alpha_1=\alpha_3$ and $\alpha_2=\alpha_4$):

Under gain condition-I, the image rejection ratios $\text{IRR}_{RFim}$, $\text{IRR}_{IMim}$, and $\text{IRR}_{IM}$ are expressed by

\[
\text{IRR}_{RFim} = \frac{N_0 - N_1 - N_2 + N_3 + N_4}{D_0 + D_1 + D_2 + D_3 + D_4} \tag{2.15}
\]

\[
\text{IRR}_{IMim} = \frac{N_0 + N_1 + N_2 - N_3 - N_4}{D_0 + D_1 + D_2 + D_3 + D_4} \tag{2.16}
\]

\[
\text{IRR}_{IM} = \frac{N_0 + N_1 - N_2 - N_3 - N_4}{D_0 + D_1 + D_2 + D_3 + D_4} \tag{2.17}
\]

where,

\[
D_0 = (4E_1 + 2E_2 + 2E_3 + E_4) \\
D_1 = (4E_1 + 2E_2) \cos(\theta_A) \\
D_2 = 4E_1 \cos(\theta_B) \\
D_3 = 4(1+\varepsilon_2) \cos(\theta_A) \cos(\theta_B) \\
D_4 = (4\varepsilon_1(1+\varepsilon_2) + 2E_3) \cos(\theta_A + \theta_B) \tag{2.18}
\]
Figure 2.5: Image rejection 1) as function of the IF phase mismatch, for IF and RF gain mismatches 2% and RF phase mismatch 3° in DIRT*. 2) as function of the phase mismatch for gain mismatch 2% in a direct I/Q modulator. *Whichever phase mismatch is chosen in the DIRT, the RF or IF phase mismatch effect on the DIRT is the same in (2.19) and (2.28).

\[
\begin{align*}
N_0 &= (4E_1 + 2E_2 + 2E_3 + E_4) \\
N_1 &= (4E_1 + 2E_2) \cos(\theta_A) \\
N_2 &= 4E_1 \cos(\theta_B) \\
N_3 &= 4(1 + \varepsilon_2) \cos(\theta_A) \cos(\theta_B) \\
N_4 &= (4\varepsilon_1 (1 + \varepsilon_2) + 2E_3) \cos(\theta_A + \theta_B)
\end{align*}
\]

and

\[
\begin{align*}
E_1 &= 1 + \varepsilon_1 + \varepsilon_2 + \varepsilon_1 \varepsilon_2 \\
E_2 &= \varepsilon_2^2 (1 + \varepsilon_1) \\
E_3 &= \varepsilon_1^2 (1 + \varepsilon_2) \\
E_4 &= (\varepsilon_1 \varepsilon_2)^2
\end{align*}
\]
The IRR\textsubscript{RFim}, IRR\textsubscript{IMim}, and IRR\textsubscript{IM} are derived in Appendix B. Fig. 2.5 shows the graph of the IRR\textsubscript{RFim} as a function of IF phase mismatch ($\theta_A$). The RF phase mismatch ($\theta_B$) is set equal to 3°. The IF and RF gain mismatches ($\varepsilon_1$ and $\varepsilon_2$) are 2 %. The analytical results show that the in-band image signal can be suppressed more than 40dBc even though the IF phase mismatch ($\theta_A$) varies from 0° to 50°. However, this implies that less than 2 % IF gain mismatch ($\varepsilon_1$) is required to suppress more than 40dBc regardless of the phase mismatch ($\theta_A$). Therefore, the IRR\textsubscript{RFim} is limited by the IF gain mismatch. Although it is possible to numerically graph the IRR\textsubscript{RFim}, (2.15) is too complicated to offer any useful insight into the transmitter behavior.

\begin{equation}
IRR_{RFim} \approx \frac{4\varepsilon_1^2 + (\theta_A \theta_B)^2}{16}
\end{equation}

\begin{equation}
\approx \frac{\varepsilon_1^2}{4}, \text{by } (\varepsilon_1^2 >> (\theta)^4) |\theta_A = \theta_B.
\end{equation}

Equation (2.19) is approximated from (2.15) under the two assumptions reported in [32]. The first assumption is that the gain mismatches ($\varepsilon_1$ and $\varepsilon_2$) are much smaller than unity with perfect quadrature phase matches of the quadrature IF and RF LO signal generators. Under the first assumption, equation (2.15) can be approximated by the IF gain mismatch ($\varepsilon_1$) as the first term in (2.19). The second assumption assumes that the phase mismatches ($\theta_A$ and $\theta_B$) exist with perfect gain matches. Under the second assumption, the IRR\textsubscript{RFim} is expressed via the multiplication of the phase mismatches ($\theta_A$ and $\theta_B$) in the second term of (2.19). The IF phase mismatch effect is equal to RF phase mismatch effect in (2.19).
By summing the derived gain and phase mismatches, an approximation for the total $\text{IRR}_{RF_{im}}$ can be obtained in (2.19). Under this approximation, the $\text{IRR}_{RF_{im}}$ is dominated by the IF gain mismatch and the phase mismatch can be negligible as in (2.20).

$$\text{IRR}_{IM_{im}} \approx \frac{(\varepsilon_1 \varepsilon_2)^2 + 4\theta_A^2}{16}$$  (2.21)

$$\approx \frac{\theta_A^2}{4}, \text{by } (\theta^2 \gg \varepsilon^4) |\varepsilon_1 = \varepsilon_2$$  (2.22)

$$\text{IRR}_{IM} \approx \frac{\varepsilon_2^2 + \theta_B^2}{4}$$  (2.23)

The $\text{IRR}_{IM_{im}}$ of (2.21) is similarly derived from (2.19). The image2 signal is predominately suppressed by the IF phase mismatch, as shown in (2.22). The image3 signal is suppressed by the RF gain and phase mismatches in (2.23).

2.2.2  In gain condition-II ($a_1 = a_4$ and $a_2 = a_3$):

Under gain condition-II, the image rejection ratios are expressed by

$$\text{IRR}_{RF_{im}} = \frac{N_0 - N_1 - N_5 + N_6}{[D_0 + D_1 + N_5 + N_6]}$$  (2.24)

$$\text{IRR}_{IM_{im}} = \frac{N_0 - N_1 + N_5 - N_6}{[D_0 + D_1 + N_5 + N_6]}$$  (2.25)
\[ IRR_{IM} = \frac{[N_0 + N_1 - N_5 - N_6]}{[D_0 + D_1 + N_5 + N_6]} \]  

\[ N_5 = (4E_1 + 2E_3) \cos(\theta_B) \]  
\[ N_6 = 4E_1 \cos(\theta_A) \cos(\theta_B). \]  

(2.26)  

The IRRs are derived using a similar procedure as in Appendix B. Fig. 2.5 shows the graph of the \( IRR_{RFim} \) as a function of IF phase mismatch under the gain condition-II. The RF phase mismatch \( (\theta_B) \) is 3°. The IF and RF gain mismatches \( (\varepsilon_1 \text{ and } \varepsilon_2) \) are 2%. The in-band image signal can be suppressed by 50 dBC, 45 dBC, and 40 dBC even though the phase mismatches of the quadrature IF LO generator are 10°, 20°, and 40°, respectively. In comparison with the \( IRR_{RFim} \) under the gain condition-I, the in-band image signal is suppressed by an additional 18dB for 5° IF phase mismatch under the gain condition-II. Furthermore, the DIRT has lower sensitivity to the gain mismatch for variations of phase mismatch from -10° to 10°. To gain further insight, the approximations of (2.24) to (2.26) are derived assuming small gain and phase mismatches as follows:

\[ IRR_{RFim} \approx \frac{(\varepsilon_1 \varepsilon_2)^2 + (\theta_A \theta_B)^2}{16} \]  

(2.28)  

\[ IRR_{IMim} \approx \frac{\varepsilon_1^2 + \theta_A^2}{4} \]  

(2.29)  

\[ IRR_{IM} \approx \frac{\varepsilon_2^2 + \theta_B^2}{4} \]  

(2.30)
The $\text{IRR}_{\text{RFim}}$ is expressed by the multiplications of the phase mismatches ($\theta_A^2$ and $\theta_B^2$) and the gain mismatches ($\varepsilon_1^2$ and $\varepsilon_2^2$) under the two assumptions used in the gain condition-I. In comparison to (2.19), the gain mismatch is reduced from $4\varepsilon^2$ to $\varepsilon^4$. However, the IF and RF phase mismatch effects of (2.28) are equal to those of (2.19). These results also demonstrate that the in-band image can be suppressed by more than 58dBc for $-5^\circ$ to $5^\circ$ phase variations even though the image rejection transmitter has 2% gain and $3^\circ$ RF phase mismatches. Similarly as seen in (2.28), the $\text{IRR}_{\text{IMim}}$ is obtained from summing the IF gain and phase mismatches. The image signal is dependent on the RF gain and phase mismatches in (2.30). If we assume that $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $\theta_1 - \theta_2 = \theta_3 - \theta_4 = \theta$, (2.31)–(2.32) are a further simplification.

$$\text{IRR}_{\text{RFim}} \approx \frac{(\varepsilon)^4 + (\theta)^4}{16} \tag{2.31}$$

$$\text{IRR}_{\text{IMim}} = \text{IRR}_{\text{IM}} \approx \frac{\varepsilon^2 + \theta^2}{4} \tag{2.32}$$

where the $\text{IRR}_{\text{IMim}}$ and $\text{IRR}_{\text{IM}}$ are the same as the IRR for a single I/Q modulator described in [30]. Furthermore, the $\text{IRR}_{\text{RFim}}$ can be expressed by the square of each mismatch terms of a single I/Q modulator ($\varepsilon^2/4$ and $\theta^2/4$). Therefore, the in-band IRR can be suppressed greater than the IRR of a direct I/Q modulator as shown in Fig. 2.5. The lower and upper bound of the IRR can be decided by gain condition-I and II in the case of small gain/phase mismatches. Furthermore, the insensitivity to the gain and phase mismatch effects can make the DIRT more suitable to the multiband operations.
2.3 EVM as a function of I/Q gain and phase mismatches

The output signal of a quadrature modulator is a real bandpass signal with a carrier frequency ($\omega_c$), such as $\text{Re}[\nu(t)\exp(j\omega_c t)]=I\cos(\omega_c t)−Q\sin(\omega_c t)$, where $\nu(t)$ denotes a complex envelope, and the I and Q include the RF imperfection in the transmitter, as shown in Fig. 2.6. The complex envelope for the DCT can be represented by a 2 X 2 matrix, containing real and imaginary parts in the first and second rows [25]. The matrix model is shown in a transmission system, as seen in Fig. 2.7. Using the EVM definition, the error vector ($ev$) can be expressed by the system imperfections ($m_{ev}$) and the noise ($m$) at the output of demodulator in the following equations [26].
\[ ev = m_{ev} + m \]  

(2.33)

where

\[ m_{ev} = \sqrt{E_s} (H - I)s \]  

(2.34)

\[ m = D \cdot n \]  

(2.35)

\[ H = DRM \]  

(2.36)

where \( H \) is the system matrix for a modulator (M), channel (R), and demodulator (D). The \( E_s, s, \) and \( n \) indicate average symbol energy, transmitted symbol, and thermal noise, respectively. To evaluate only the transmitter mismatch effects, the demodulator (D) matrices can be expressed with no imperfections (D=I).
2.3.1 In gain condition-I ($\alpha_1 = \alpha_3$ and $\alpha_2 = \alpha_4$):

Under gain condition-I, the complex envelope for the DIRT is expressed by

$$M = \frac{1}{2} \left[ (\alpha_A \alpha_B + \alpha_A \beta_B) \cos \left( \left( \phi_A + \phi_B \right) / 2 \right) - (\beta_A \alpha_B - \beta_A \beta_B) \sin \left( \left( \phi_A - \phi_B \right) / 2 \right) \right]$$  \hspace{1cm} (2.37)

where the IF gains are given by $\alpha_1 = \alpha_3 = \alpha_A$ and $\alpha_2 = \alpha_4 = \beta_A$, and the RF gains are given by $\alpha_5 = \alpha_B$ and $\alpha_6 = \beta_B$. The IF and RF phase mismatches, $\theta_A$ and $\theta_B$, are defined as $\phi_A$ and $\phi_B$, respectively. The derivation of (2.37) is shown in Appendix C. The EVM, normalized to the average energy, can be expressed by [26]

$$EVM_{\text{rms, avg}}^2 = \frac{N_A}{E_s} + \left[ 1 + \frac{\text{Tr}(H^TH)}{2} - \text{Tr}(H) \right]$$

$$= \frac{1}{\text{SNR}} \left[ 1 + \frac{\text{Tr}(H^TH)}{2} - \text{Tr}(H) \right]$$ \hspace{1cm} (2.38)

To calculate the EVM of (2.38), the traces, $\text{Tr}(H^TH)$ and $\text{Tr}(H)$, can be obtained by (2.39) and (2.40).

$$\text{Tr}(H^TH) = \frac{1}{4} \left[ \alpha_A^2 \alpha_B^2 + \alpha_A^2 \beta_B^2 + \beta_A^2 \alpha_B^2 + \beta_A^2 \beta_B^2 + 2 \alpha_A \alpha_B \beta_B \cos(\phi_A + \phi_B) + 2 \beta_A \alpha_B \beta_B \cos(\phi_A - \phi_B) \right]$$ \hspace{1cm} (2.39)
\[ Tr( H ) = Tr( H^T ) = \frac{1}{2} \cdot (\alpha_B + \beta_B)(\alpha_A \cos((\phi_A + \phi_B)/2) + \beta_A \cos((\phi_A - \phi_B)/2)) \] (2.40)

The linear transforms caused by the transmitter imperfection (M) should preserve the energy of the input vector signal and noise. In [25] and [26], the power normalization of the matrix M was achieved by \( \alpha^2 + \beta^2 = 2 \), which can preserve the energy of the information signals. The preserved energy can be obtained when the covariance of \( H \cdot s \) or the trace of \( H^T H \) is unchanged. In the same manner, the trace, \( Tr(H^T H) = 2 \), can leave the power of the signal unchanged. Hence, the gains of the mixer pairs can be related by the following equations.

\[ \beta_A^2 \beta_B^2 = \frac{8}{CUS} \] (2.41)

\[ \alpha_A^2 \alpha_B^2 = \gamma_1^2 \cdot \gamma_2^2 \cdot \frac{8}{CUS} \] (2.42)

\[ \alpha_A^2 \beta_B^2 = \gamma_1^2 \cdot \frac{8}{CUS} \] (2.43)

\[ \beta_A^2 \alpha_B^2 = \gamma_2^2 \cdot \frac{8}{CUS} \] (2.44)

where

\[ CUS = (\gamma_1^2 \gamma_2^2 + \gamma_1^2 \gamma_2^2 + \gamma_1^2 \gamma_2^2 + \gamma_1^2 \gamma_2^2 + \gamma_1^2 \gamma_2^2 + \gamma_1^2 \gamma_2^2 + \gamma_1^2 \gamma_2^2 + \gamma_1^2 \gamma_2^2) \] (2.45)

and

\[ \frac{\alpha_A}{\beta_A} = \gamma_1, \quad \frac{\alpha_B}{\beta_B} = \gamma_2 \] (2.46)
Fig. 2.8: The comparison of EVM variations: 1) as function of IF gain mismatch when 2dB and 10° RF gain and phase mismatches are given* in the variations of IF phase mismatch by 0°, 5°, and 10°, in the DIRT and 2) as function of gain mismatch while varying phase mismatch by 0°, 5°, and 10° in a direct IQ modulator. * The 2dB/10° RF gain/phase mismatches of the DIRT are given as the worst gain/phase mismatches of the direct IQ modulator.

Fig. 2.8 shows the comparison of the EVM variations on a direct I/Q modulator and DIRT when the SNR is 45 dB. The EVM variations of a direct I/Q modulator are achieved by phase mismatches of 0°, 5°, and 10°. The EVM variations of the DIRT under two distinct conditions are obtained under the variation of IF phase mismatches by 0°, 5°, and 10°. The RF gain and phase mismatches are 2 dB and 10°, respectively, which correspond to the worst gain/phase mismatches in the direct I/Q modulator. In the simulation results, the EVM variations of the direct modulator depend on both gain and phase mismatches, but those of the DIRT are not sensitive to phase mismatch but limited to the IF gain mismatch under the gain condition-I. The IF gain condition-II of $\alpha_1=\alpha_4$ and
$\alpha_2 = \alpha_3$ is proposed to alleviate the effect of the IF gain mismatch. The analysis of the proposed condition-II is in the following section.

### 2.3.2 In gain condition-II ($\alpha_1 = \alpha_4$ and $\alpha_2 = \alpha_3$):

Under gain condition-II, the complex envelope for the DIRT is expressed by

$$M = \frac{1}{2} \left[ \begin{array}{cc} (\alpha_A \alpha_B + \beta_A \beta_B) \cos \left( (\phi_A + \phi_B)/2 \right) & (\beta_A \alpha_B - \alpha_A \beta_B) \sin \left( (\phi_A - \phi_B)/2 \right) \\ (\alpha_A \alpha_B - \beta_A \beta_B) \sin \left( (\phi_A + \phi_B)/2 \right) & (\beta_A \alpha_B + \alpha_A \beta_B) \cos \left( (\phi_A - \phi_B)/2 \right) \end{array} \right]. \quad (2.47)$$

The derivation of (2.47) is obtained using a similar procedure as in Appendix C.

To calculate the EVM via (2.38), the traces, $\text{Tr}(H^TH)$ and $\text{Tr}(H)$, can be expressed by

$$\text{Tr}(H^TH) = \frac{1}{4} \left( \alpha_A^2 \alpha_B^2 + \alpha_A^2 \beta_B^2 + \beta_A^2 \alpha_B^2 + \beta_A^2 \beta_B^2 + 2\alpha_A \beta_A \alpha_B \beta_B \cos (\phi_A + \phi_B) + 2\alpha_A \beta_A \alpha_B \beta_B \cos (\phi_A - \phi_B) \right) \quad (2.48)$$

$$\text{Tr}(H) = \text{Tr}(H^T) = \frac{1}{2} \left( \alpha_A \alpha_B + \beta_A \beta_B \right) \cdot \cos \left( (\phi_A + \phi_B)/2 \right) + \left( \beta_A \alpha_B + \alpha_A \beta_B \right) \cdot \cos \left( (\phi_A - \phi_B)/2 \right) \quad (2.49)$$

The signal power can be held constant using the relationships in (2.41)–(2.44), where the CUS is given by

$$\text{CUS} = \left( \gamma_1^2 \gamma_2^2 + \gamma_1^2 + \gamma_2^2 + 1 + 2 \cdot \gamma_1 \gamma_2 \cdot \cos \left( \phi_A + \phi_B \right) \right) \quad (2.50)$$
Figure 2.9: EVM variations as function of IRR, when gain condition-I: \( a_1 = a_3 \) and \( a_2 = a_4 \).

The EVM variations for gain condition-II were also plotted in Fig. 2.8. Under this condition, the EVM variations are insensitive to both gain and phase mismatches.

### 2.4 EVM variation by image rejection ratio

As previously mentioned in Section 2.2 and 2.3, the IRR is dominated by the IF gain mismatch. Therefore, the relations between EVM and IRR can be expressed by

\[
EVM = \sqrt{\frac{1}{SNR} + 2 \left( \frac{I - IRR}{I + IRR} \right) \frac{\gamma_i + I}{\sqrt{\gamma_i}}},
\]  

(2.51)
where $\gamma_1$ is replaced with $1 + 2\sqrt{\text{IRR}}$ using (2.20). The derived EVM is expressed by

$$EVM = \sqrt{\frac{1}{\text{SNR}}} + 2 - \frac{1 - \text{IRR}}{\sqrt{1 + \text{IRR}}} \cdot \frac{2(1 + \sqrt{\text{IRR}})}{\sqrt{1 + 2\sqrt{\text{IRR}}}}. \quad (2.52)$$

Fig. 2.9 shows the relationship between EVM and IRR in the variations of SNR from 10 dB to 45 dB.
Figure 2.10: Image rejection of the DIRT when (a) $\alpha_1/\alpha_2 = \alpha_3/\alpha_4 = 1$ dB, and $\alpha_5/\alpha_6 = 1$ dB, and (b) $\alpha_1/\alpha_2 = \alpha_4/\alpha_3 = 1$ dB, $\alpha_5/\alpha_6 = 1$ dB in the phase $\theta_A = \theta_B = 5^\circ$.

2.5 System simulation results

System level simulations are performed for various IRRs of the DIRT. The input frequency, the quadrature IF LO frequency, and the quadrature RF LO frequencies are 0.1 GHz, $f_{LO}/4$ GHz, and $f_{LO}$ GHz, respectively. The resultant output frequency is $(5/4)f_{LO} + 0.1$ GHz and the corresponding image signal is at $(5/4)f_{LO} - 0.1$ GHz. Two distinct mismatch configurations are simulated in Fig. 2.10. In the first configuration, the IF phase mismatch ($\theta_A$) and RF phase mismatch ($\theta_B$) are $5^\circ$ for 1.0 dB IF gain condition-I ($\alpha_1/\alpha_2 = \alpha_3/\alpha_4$) and 1.0 dB RF gain mismatch ($\alpha_5/\alpha_6$). In the second configuration, the IF gain mismatch is replaced with 1 dB IF gain condition-II ($\alpha_1/\alpha_2 = \alpha_4/\alpha_3$). The simulation results show that the image signal under gain condition-II is suppressed 20 dBc more than under gain condition-I. The sensitivity of gain and phase mismatches is investigated in Fig. 2.10 when the RF gain and phase mismatches are 1 dB and $5^\circ$, respectively. The simulation results show that the image signals are suppressed by up to 25 dBc and 46 dBc.
Figure 2.11: Image rejection by (a) IF gain mismatch variations when $\alpha_5/\alpha_6=1\text{dB}$, and $\theta_A=\theta_B=5^\circ$ assuming condition-I and -II; (b) phase mismatch variations when $\alpha_5/\alpha_6=1\text{dB}$ under 1dB gain condition-I and 1dB gain condition-II.
over −1 dB to 1 dB gain mismatch variations under gain condition-I and gain condition-II, respectively, in Fig. 2.11(a). Fig. 2.11(b) shows the insensitivity of the IF phase mismatches. The dependence of the phase mismatch can be reduced under both gain condition-I and −II, and the gain mismatch dependency can be alleviated under gain condition-II. However, statistical phase mismatch asymmetries between the mixers cannot be neglected at the operating IF frequency. Therefore, the statistically asymmetric effect of mixer phase mismatch is analyzed by Monte Carlo simulation. The IF phase mismatch terms (θ_{IF1,2,3,4}) are assumed as uniform random variables in the range of -1° to 1° to show the mismatch effects. In the symmetric IF phase mismatch condition, the LO phase mismatch depends on θ_A = θ_1 − θ_2 and the IF phase mismatch terms are expressed by θ_{IF1} = θ_{IF4} and θ_{IF2} = θ_{IF3}. Therefore, in the gain condition-II with symmetric IF phase mismatch, both gain and phase mismatch effects are reduced at the RF output port as shown in Fig. 2.12. However, the asymmetric IF phase mismatch terms can have different mismatch effects such as θ_{IF1} = θ_{IF3} and θ_{IF2} = θ_{IF4}. In such a case, the IF phase mismatches cannot cancel each other at the RF output port and the phase mismatches directly affect the IRR. Fig. 2.12 shows that gain condition-I has the lower sensitivity and gain condition-II has higher sensitivity to the asymmetric IF phase mismatches. In gain condition-II of Fig. 2.12(c), the gain mismatch effects can be eliminated at the RF output port by gain condition-II, but the IF phase mismatch dominates the IRR due to the asymmetric phase mismatches such as θ_{IF1} = θ_{IF3} and θ_{IF2} = θ_{IF4}. In gain condition-I of Fig. 2.12(c), both the gain and asymmetric phase mismatches affect the IRR, and the gain mismatch still dominates the IRR more than the asymmetric IF phase mismatch. However the sensitivity of condition-II is not significant, because the IRR is suppressed.
Figure 2.12: Monte Carlo simulation results by IF/RF phase mismatch variations: (a) 1° phase mismatch when 1 dB gain mismatch under condition-I and condition-II. (b) 2° phase mismatch when 0.5 dB gain mismatch (c) 2° phase mismatch when 0.2 dB gain mismatch under condition-I and condition-II.
enough to neglect the mismatch effect as well as the worst IRR of the DIRT depends on gain condition-I. Moreover, as the input magnitude mismatch is decreased, the condition-I level depends on the IF mixer pair mismatch as the upper bound of the IRR. Therefore, IF gain mismatches must be reduced to obtain greater IRR by (2.19) and (2.28). In addition, the electrical length of I/Q connection lines for mixers and IF LO generator should be equivalent, and the layout of the mixer pairs should be a duplicate of each other and arranged symmetrically to reduce the asymmetric IF phase mismatches.

Figure 2.13: SER variations as function of the Es/No, when IF gain mismatch of 3 dB, RF gain mismatch of 1dB, IF phase mismatch of 10° and RF phase mismatch of 10°.
2.6 I/Q mismatch effects on symbol error rate

In this section, the I/Q mismatch effects on SER are examined. Although EVM is independent of the individual transmitted symbols, the symbol error rate depends on the transmitted symbols and the correlation properties of the noise vector. In addition, the SER is more sensitive to I/Q mismatches of the transmitter than to those of the receiver. The union bound on the SER can be defined as the sum of the individual symbol error probabilities between the transmitted symbols. The derivation of the SER for equal energy symbols was reported in [26]. However, in the high QAM system with unequal energy symbols, the difference of the correlation metrics \(x_{ji}\) is given by \(CM_{ji} - CM_{ij} = r_j^T(s_j - s_i) - E_{ji}/2\), where \(r, s\) and \(E_{ji}\) are the received signal, a transmitted symbol and energy difference between \(s_j\) and \(s_i\) symbols, respectively [33]. Hence, the mean \(mx_{ji}\) and variance \(\sigma^2x_{ji}\) of the \(x_{ji}\) can be expressed by \(\sqrt{Es}s_j^T H^T(s_j - s_i) - E_{ji}/2\) and \((No/2)(s_j - s_i)^TDD^T(s_j - s_i)\), respectively. In this way, the union bound on the SER \(Q(mx_{ji}/\sigma x_{ji})\) for unequal energy symbols can be expressed by

\[
P_s \leq \frac{1}{N} \sum_{j,i} \sum_{i \neq j} \left( Q \left( \sqrt{\frac{2Es}s_j^T H^T(s_j - s_i) - E_{ji}/2} \right) \right).
\]

(2.53)

where \(j\) and \(i\) are the indices of the \(N\) transmitted symbols. The SERs of the DIRT under gain condition-I and gain condition-II are calculated using quadrature phase shift keying (QPSK) by (2.54) and (2.55), respectively.
where $\phi_S=(\phi_A+\phi_B)/2$ and $\phi_D=(\phi_A-\phi_B)/2$. Fig. 2.13 shows the SER variations as a function of $E_s/N_o$ under the following conditions: IF-gain mismatch of 3dB, RF gain mismatch of 1dB, IF phase mismatch of 10°, and RF phase mismatch of 10°. The ideal QPSK SER and the SER for a direct I/Q modulator are plotted to compare with the SER of the DIRT. The SER of the DIRT under the gain condition-I has an improvement of around 1dB at $10^{-6}$ SER compared with the direct I/Q modulator. The SER under the gain condition-II is close to the ideal QPSK SER even though the gain and phase mismatches are included in the matrix. These SER results match with the simulation results of EVM and IRR.
2.7 Summary of I/Q mismatch effects

The DIRT was analyzed with gain and phase mismatch effects. To characterize gain and phase mismatch effects in the architecture, EVM, IRR, and a union bound on SER were examined. The derived EVM equations were obtained using the matrix model for a transmission system. The EVM variations of the DIRT were dependent on the IF gain mismatch for gain condition-I. However, under gain condition-II, the EVM variation of the DIRT was insensitive to both gain and phase mismatches. The IRRs were also derived from the two proposed gain conditions. The IRRs are approximated to give insight into the gain and phase mismatch effects. The system simulation results showed that more than 40 dBc image suppression can be obtained for 1-dB IF gain mismatch and phase mismatch variations of -8° to 8° in the gain condition-II. Furthermore, the simulation results of EVM and the union bound on SER also prove the validity of the proposed concept. The relationship between EVM and IRR was also given from the derived equations.
2.8 Phase noise effect

In the quadrature modulator, both gain and phase mismatches affect the IRR in DCT, but, in DIRT architecture, gain mismatch effect mainly dominates the IRR in the case of a small phase mismatch. In other words, the gain mismatch effect is common to the DCT and DIRT, but the cancellation of phase mismatches at RF output node is a distinct characteristic of the DIRT. Therefore, in this section, a comparison between the DCT and DIRT is undertaken via an analysis of the I/Q phase mismatch and RF phase noise effects. For comparison of the transmitter architectures, phase synchronization error in the channel (R) and the RF impairments of the quadrature demodulator (D) are removed in the derivation of the EVM and SER equations. The analytic results of the DIRT will show that its insensitivity to phase mismatch improves the signal-to-noise ratio (SNR) performance on higher-order QAM systems.

2.8.1 In direct conversion transmitter

The phase mismatch and phase noise effects are investigated in the modulator matrix (M), while the demodulator (D) matrix should be expressed with no imperfections (D = I). Therefore, the system matrix (H), which incorporates a phase mismatch of modulator (M) with no gain mismatch, can be expressed with the channel (R) by

\[ H = \begin{bmatrix} \cos(\phi_{ch}) & -\sin(\phi_{ch}) \\ \sin(\phi_{ch}) & \cos(\phi_{ch}) \end{bmatrix} \begin{bmatrix} \cos(\phi_m / 2) & \sin(\phi_m / 2) \\ \sin(\phi_m / 2) & \cos(\phi_m / 2) \end{bmatrix} \]

\[ = \begin{bmatrix} \cos(\phi_{ch} + \phi_m / 2) & -\sin(\phi_{ch} - \phi_m / 2) \\ \sin(\phi_{ch} + \phi_m / 2) & \cos(\phi_{ch} - \phi_m / 2) \end{bmatrix} \]

\[ (2.56) \]
where $\phi_m$ and $\phi_{ch}$ indicate a deterministic phase mismatch of the modulator and a phase difference between the transmitter and receiver LO, respectively. The channel phase difference results in a phase synchronization error between transmitter and receiver. In a real transmitter, the phase difference term is affected by the random phase variation of LO phase noise. Therefore, the phase difference can be expressed by the sum of a deterministic variable and a Gaussian random variable as follows [30]:

$$\phi_{ch} = \phi_{chd} + \phi_{chr}, \quad \phi_r \sim N(0, \phi_{rms}^2) \quad (2.57)$$

where $\phi_{chd}$ and $\phi_{chr}$ indicate a deterministic phase difference and Gaussian random variable with rms LO phase error, $\phi_{rms}$, respectively. By assuming $\phi_{rms} << 1$, the system matrix is derived as (2.58).

$$H = \begin{bmatrix}
\cos(\phi_{ch} + \phi_m / 2) - (\phi_{chr}) \sin(\phi_{ch} + \phi_m / 2) & -\sin(\phi_{ch} - \phi_m / 2) - (\phi_{chr}) \cos(\phi_{ch} - \phi_m / 2) \\
\sin(\phi_{ch} + \phi_m / 2) + (\phi_{chr}) \cos(\phi_{ch} + \phi_m / 2) & \cos(\phi_{ch} - \phi_m / 2) - (\phi_{chr}) \sin(\phi_{ch} - \phi_m / 2)
\end{bmatrix}
\cos(\phi_{ch} + \phi_m / 2) - (\phi_{chr}) \sin(\phi_{ch} - \phi_m / 2) \\
\sin(\phi_{ch} + \phi_m / 2) & \cos(\phi_{ch} + \phi_m / 2)
\end{bmatrix}
+ (\phi_{chr}) \begin{bmatrix}
-\sin(\phi_{ch} + \phi_m / 2) & -\cos(\phi_{ch} - \phi_m / 2) \\
\cos(\phi_{ch} + \phi_m / 2) & -\sin(\phi_{ch} - \phi_m / 2)
\end{bmatrix} \quad (2.58)$$

If there is no phase synchronization error between a transmitter and a receiver, the matrix can be expressed by a random phase noise term, $\phi_{chr}$, and a deterministic phase mismatch term, $\phi_m$, in
\[ H = \begin{bmatrix} \cos(\frac{\phi_m}{2}) & \sin(\frac{\phi_m}{2}) \\ \sin(\frac{\phi_m}{2}) & \cos(\frac{\phi_m}{2}) \end{bmatrix} + \phi_{ch} \begin{bmatrix} -\sin(\frac{\phi_m}{2}) & -\cos(\frac{\phi_m}{2}) \\ \cos(\frac{\phi_m}{2}) & \sin(\frac{\phi_m}{2}) \end{bmatrix} \]
\[ = H_d + \phi_{ch} H_r \quad (2.59) \]

The resultant EVM for the DCT can be expressed by [30]

\[ EVM_{rms,avg} = \frac{N_o}{E_s} + \left[ 1 + \frac{\text{Tr}(H_d^T H_d)}{2} - \text{Tr}(H_d) \right] + \phi_{rms}^2 \quad (2.60) \]

Therefore, the EVM is derived, with the Tr(H_d)=2\cos(\phi_m/2), Tr(H_d^T H_d)=2, and Tr(H_r^T H_r)=2, as

\[ EVM_{rms,avg} = \frac{1}{SNR} + 2 - 2\cos(\phi_m/2) + \phi_{rms}^2 \quad (2.61) \]

Assuming that \( \phi_m \) is small, and using Taylor series expansion, \( \cos(\phi)=1-\phi^2/2! \text{[rad]} \), yields

\[ EVM_{rms,avg} = \frac{1}{SNR} + \phi_d^2 + \phi_{rms}^2 \quad (2.62) \]
2.8.2 In double image rejection transmitter

The DIRT is composed of two IF mixer pairs and one RF mixer pair, as shown in Fig. 2.14. To investigate the LO phase noise and IF/RF phase mismatches, the IF and RF path gains are assumed to be $\alpha_1$ and $\alpha_2$, respectively. The IF and RF phase mismatches are defined as $\phi_A$ and $\phi_B$, respectively. The system matrix ($H$) of the DIRT is expressed by (2.63).

$$H = \alpha_1 \alpha_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\phi_{ch}) & -\sin(\phi_{ch}) \\ \sin(\phi_{ch}) & \cos(\phi_{ch}) \end{bmatrix} \begin{bmatrix} \cos((\phi_A + \phi_B)/2) & 0 \\ 0 & \cos((\phi_A - \phi_B)/2) \end{bmatrix}$$

$$= \alpha_1 \alpha_2 \cdot \begin{bmatrix} \cos(\phi_{ch}) \cdot \cos\left(\frac{\phi_{ad} + \phi_{bd}}{2}\right) & -\sin(\phi_{ch}) \cdot \cos\left(\frac{\phi_{ad} - \phi_{bd}}{2}\right) \\ \sin(\phi_{ch}) \cdot \cos\left(\frac{\phi_{ad} + \phi_{bd}}{2}\right) & \cos(\phi_{ch}) \cdot \cos\left(\frac{\phi_{ad} - \phi_{bd}}{2}\right) \end{bmatrix}$$

The derivation of the complex envelope matrix is shown in Appendix A. The power normalization of the matrix is achieved by $\text{Tr}(H^\dagger H)=2$, which can preserve the energy of the information signals. Under the consideration of LO phase noise, the phase difference is decomposed into a deterministic variable and a Gaussian random variable.

$$\phi_{ch} = \phi_{chd} + \phi_{chr}, \quad \phi_{chr} \sim N(0, \sigma_{chr}^2)$$

Under the same assumption of $\phi_{IFrms(RFrms)} \ll 1$, the matrix can be simplified as
Figure 2.14: Block diagram of a double-image-rejection transmitter.

\[
H = \alpha_1\alpha_2 \begin{bmatrix} 
\cos (\phi_S) & -\phi_{ch} \cos (\phi_B) \\
\phi_{ch} \cos (\phi_s) & \cos (\phi_B)
\end{bmatrix}
\]

\[
= \alpha_1\alpha_2 \begin{bmatrix} 
\cos (\phi_s) & 0 \\
0 & \cos (\phi_B)
\end{bmatrix} + \phi_{ch}\alpha_1\alpha_2 \begin{bmatrix} 
0 & \cos (\phi_B)
\cos (\phi_s) & 0
\end{bmatrix}
\]

\[
= H_d + \phi_{ch}H_{IF}
\]

where \( \phi_S = (\phi_{Ad} + \phi_{Bd})/2 \) and \( \phi_S = (\phi_{Ad} - \phi_{Bd})/2 \). In the DIRT architecture, the traces, \( \text{Tr}(H_d^T H_d) \) and \( \text{Tr}(H_r^T H_r) \), are given by

\[
\text{Tr} (H_d^T H_d) = (\alpha_1\alpha_2)^2 \left( 1 + \cos (\phi_{Ad}) \cos (\phi_{Bd}) \right)
\]

(2.66)
\[ \text{Tr} \left( H_r^T H_r \right) = (\alpha_1 \alpha_2)^2 \left( 1 + \cos(\phi_{Ad}) \cos(\phi_{Bd}) \right) \]  
(2.67)

In the same manner, the trace, \( \text{Tr}(H_r^T H_r) = 2 \), derives the relationship \((\alpha_1 \alpha_2)^2 = 2/(1+\cos(\phi_{Ad})\cos(\phi_{Bd}))\). To calculate the EVM of (2.60), the trace, \( \text{Tr}(H_d) \), is given by

\[ \text{Tr}(H_d) = 2\alpha_1 \alpha_2 \cos(\phi_{Ad}/2) \cdot \cos(\phi_{Bd}/2) \]  
(2.68)

\[ \text{EVM} \quad \text{rms, avg} = \frac{1}{\text{SNR}} + 2 - 2\sqrt{2} \frac{\cos(\phi_{Ad}/2) \cdot \cos(\phi_{Bd}/2)}{\sqrt{1 + \cos(\phi_{Ad}) \cos(\phi_{Bd})}} + \phi_{rms}^2 \]  
(2.69)

Assuming that \( \phi \) is small, and using the Taylor series expansion \( \cos(\phi) = 1 - \phi^2/2 |\phi|_{\text{rad}} \), yields

\[ \text{EVM} \quad \text{rms, avg} = \frac{1}{\text{SNR}} + 2 - 2\sqrt{1 - \frac{\phi_{Ad}^2 \phi_{Bd}^2}{16}} + \phi_{rms}^2 \]  
(2.70)

In comparison with (2.62) of the DCT, the effect of the phase mismatch term in the DIRT is much smaller and can be neglected in (2.70). However, the LO phase error terms must be carefully considered, because the architectures of DCT and DIRT use single up-conversion and double up-conversion transmitters, respectively. In the section III-B, the comparison of LO phase error difference between DCT and DIRT is discussed.

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2.8.3 Calculation of phase noise

Fig. 2.14 showed the generic architecture of the DIRT with sliding-IF, which was comprised of an LO generator for RFLO signal and a frequency divider for the IFLO signal. The LO phase error of the DIRT and DCT can be calculated by using a cascaded phase noise concept [34], [35] and the calculation is compared with harmonic balance (HB) simulation. The IF I/Q output signals are given as \( s_{IFI} = \cos(\omega_{IF}t + \phi_n(t)/N) \) and \( s_{IFQ} = -\sin(\omega_{IF}t + \phi_n(t)/N) \), which result from the multiplication of baseband signal and the IFLO signal. \( \phi_n(t) \) and \( N \) are defined as a random phase noise and frequency divider ratio, respectively. Furthermore, RFI and RFQ output signals are derived as \( s_{RFI} = 0.5 \cdot \cos(\omega_{RF}t + \phi_n(t)/N + \phi_n(t)) \) and \( s_{RFQ} = 0.5 \cdot \cos(\omega_{RF}t + \phi_n(t)/N + \phi_n(t)) \). The RF output signal is expressed as \( s_{RFOUT} = \cos(\omega_{RF}t + \phi_n(t)/N + \phi_n(t)) \) by summing RFI and RFQ output signals, where the RF output frequency, \( \omega_{RF} \), is given by the sum of IF and RF LO frequencies. Therefore, the output phase noise \( L_{RF}(f_m) \) can be obtained by

\[
L_{RF} (f_m)_{\text{dB}} = 20 \log \left\{ 10^{\frac{L_{IFI}(f_m)_{\text{dB}}}{20}} + 10^{\frac{L_{RFLO}(f_m)_{\text{dB}}}{20}} \right\}
\]

(2.71)

where \( L_{IFI}(f_m) \), \( L_{RFLO}(f_m) \), and \( f_m \) are defined as the phase noise of the IF output, the phase noise of the RFLO signal, and the offset frequency from the carrier frequency, respectively.
Figure 2.15: RF output phase noise by LO phase noise in (a) DCT and (b) DIRT.
The IF phase noise can be given by

\[ L_{\text{IF}}(f_m)_{\text{dB}} = L_{\text{RFLO}}(f_m)_{\text{dB}} - 20 \cdot \log_{10}(N) \]  \hspace{1cm} (2.72)

where \( L_{\text{RFLO}} \) and \( N \) are RFLO phase noise and frequency divider ratio, respectively. Therefore, the resultant output LO phase noise error can be calculated by

\[ \Phi_{\text{rms}}^2 = 2 \cdot \int_{f_1}^{f_2} 10^{L_{\text{IF}}/10} \, df = \left( \frac{N + 1}{N} \right)^2 \cdot \Phi_{\text{RFLO rms}}^2 \]  \hspace{1cm} (2.73)

Fig. 2.15 shows the HB simulation result by RFLO phase noise term. In the DCT, the output LO phase noise is the same with the RFLO phase noise, but the output LO phase noise of DIRT is increased by the \(((N+1)/N)^2\) term. By using the calculated LO phase error terms, the resultant EVM of DIRT is expressed with a phase noise of the RFLO in the form of

\[ \text{EVM}_{\text{rms avg}}^2 \approx \frac{1}{\text{SNR}} + 2 - 2 \left[ \frac{1}{16} \Phi_{\text{IF}}^2 + \frac{(N + 1)^2 \Phi_{\text{RFLO rms}}^2}{N} \right] \]  \hspace{1cm} (2.74)
Figure 2.16: (a) Output phase error in the variation of in-band RFLO phase noise; (b) EVM variation in the variation of RFLO phase error when no phase mismatches; (c) EVM variation in the variation of RFLO phase error when 3° phase mismatch.
Fig. 2.16 shows the relationship between EVM and rms LO phase-noise errors when the SNR is infinite. The bandwidth for the calculation of phase error is 12.2 KHz. The graphs of (2.62) and (2.74) are compared with system simulation results. If the RFLO phase errors are the same in DCT and DIRT, the total LO phase error increases with in-band phase noise, as shown in Fig. 2.16(a), where the LO phase error of DCT is smaller than that of DIRT. If there are no phase mismatch terms of the modulator, the EVM increases with the RFLO phase error, as shown in Fig. 2.16(b), where the resultant EVM of the DCT is smaller than the EVM of DIRT. However, when the phase mismatch of the modulator is considered as 3° in variation of RFLO phase error, the EVM of the DIRT is smaller than the EVM of DCT within 1.8° RFLO phase error, as shown in Fig. 2.16(c). After the 1.8° RFLO phase error, the EVM of DIRT becomes greater than the DCT.
Figure 2.18: Signal constellation in (a) DCT and (b) DIRT for QPSK (left) and 16-QAM (right). x: transmitted symbols, o: received symbols.

However, the RFLO frequency of the DIRT is lower than the RFLO frequency of DCT, because the RF output frequency of the DIRT is expressed with RFLO frequency and the divider ratio (N) by

$$f_{out} = \frac{N+1}{N} \cdot f_{RFLO}$$

(2.75)

where $N = 2, 4, 8, 16$. 
Therefore, the RF LO frequency and phase error can be scaled down by the ratio of $N/(N+1)$ to have the same RF output frequency. If the RFLO phase error is scaled down, the RF output phase error can be the same as the output phase error of the DCT in (2.74). Fig. 2.17 shows the EVM of DCT and DIRT for variations of RF output phase noise error, for the case of the scaled-down phase error of the DIRT. The EVM of the DIRT depends not on the phase mismatch but on the LO phase error, but the EVM of DCT depends on both phase mismatch and LO phase error terms. The eliminated phase mismatch phenomenon can be confirmed in the signal constellation of QPSK and 16-QAM systems, as shown in Fig. 2.18, where the phase mismatch effect of DIRT cancel each other at the RF output port.
Figure 2.19: Output phase noise: (a) in the variation of input phase noise when mixer noise figure of 5dB is given; and (b) in the variation of mixer noise figure when input phase noise of -150 dBc is given.
2.8.4 Effects of mixer noise figure in DCT and DIRT

The noise figures of the mixers in the DCT and DIRT also affect the output phase noise term. The assumptions used to compare the noise figure effects of mixers in the DCT and DIRT are: 1) identical input phase noise is applied to the transmitters 2) the mixer gain is unity 3) perfect quadrature phase match. The power of the RF output nodes in the DCT and DIRT is unity with different input power levels of the input mixers. In the cascaded phase noise calculation, the output phase noise of the DCT is given by

$$\frac{N_o}{C_o} \bigg|_{DCT} = 10 \log \left( 10^{ \frac{(N_o/C_o)_m}{10}} + 10^{ \frac{F_{\text{mix}} + 10 \log (10) + kT - C_i}{10}} \right)$$

(2.76)

In the same manner, the output phase noise of the DIRT is given by

$$\frac{N_o}{C_o} \bigg|_{DIRT} = 10 \log \left( 10^{ \frac{(N_o/C_o)_m}{10}} + 10^{ \frac{F_{\text{mix}} + 10 \log (10) + kT - C_i}{10}} \right)$$

$$+ 10^{ \frac{10}{10} \left( \frac{F_{\text{mix}} + 10 \log (10) + kT - C_i}{10} \right) - 10^{ \frac{10}{10} \left( \frac{10 \log (10) + 10}{10} \right)} }$$

(2.77)

Fig. 2.19 shows the calculation of the output phase noise and HB system simulation results. If the noise figure of the mixer is assumed to be 5 dB, the output phase noise increase as a function of the input phase noise is shown in Fig. 2.19(a). Although the mixer noise figure on the DCT and DIRT has an effect, the contribution to the output phase noise is small above the input phase noise of -150 dBe. The output phase noise
increase with noise figure when the input phase noise is -150 dBc is shown in Fig. 2.19(b). The phase noise difference between the DCT and DIRT is within 0.55 dB at a noise figure of 10 dB. The calculations of (2.76) and (2.77) match with the simulation result, as shown in Fig. 2.19.

### 2.9 Phase noise effect on symbol error rate

In a similar manner as mentioned in section 2.6 using the system matrix of (2.59), the SER of the DCT for a QPSK modulation is derived as

\[
P_s \leq \frac{1}{2}, \quad \left\{ \begin{array}{l}
Q \left( \frac{E_s}{N_o} (1 + \phi_{nw}) \cdot \left[ \cos(\phi_n / 2) + \sin(\phi_n / 2) \right] \right) \\
Q \left( \frac{E_s}{N_o} (1 - \phi_{nw}) \cdot \left[ \cos(\phi_n / 2) + \sin(\phi_n / 2) \right] \right) \\
Q \left( \frac{E_s}{N_o} (1 - \phi_{nw}) \cdot \left[ \cos(\phi_n / 2) - \sin(\phi_n / 2) \right] \right) \\
Q \left( \frac{E_s}{N_o} (1 - \phi_{nw}) \cdot \left[ \cos(\phi_n / 2) - \sin(\phi_n / 2) \right] \right)
\end{array} \right.
\] \tag{2.78}

The SER for DIRT using QPSK modulation with system matrix (2.65) is derived as

\[
P_s \leq \frac{1}{2}, \quad \left\{ \begin{array}{l}
Q \left( \frac{E_s}{N_o} \left( \alpha_i \alpha_z \right) \cdot \left[ \cos(\phi_D) + \phi_{chr} \cos(\phi_S) \right] \right) \\
Q \left( \frac{E_s}{N_o} \left( \alpha_i \alpha_z \right) \cdot \left[ \cos(\phi_D) - \phi_{chr} \cos(\phi_S) \right] \right) \\
Q \left( \frac{E_s}{N_o} \left( \alpha_i \alpha_z \right) \cdot \left[ \cos(\phi_S) + \phi_{chr} \cos(\phi_D) \right] \right) \\
Q \left( \frac{E_s}{N_o} \left( \alpha_i \alpha_z \right) \cdot \left[ \cos(\phi_S) - \phi_{chr} \cos(\phi_D) \right] \right)
\end{array} \right.
\] \tag{2.79}
where \( \phi_S = (\phi_{Ad} + \phi_{Bd})/2 \) and \( \phi_S = (\phi_{Ad} - \phi_{Bd})/2 \).

For the closed-form expression, the SER can be approximated by using the Taylor series in (2.78) and (2.79). Furthermore, under the assumption of \( \phi_m \ll 1 \) and \( \phi_{\text{IFrms(RFrms)}} \ll 1 \), taking the first order term in the derivation simplifies the approximation. Therefore, the approximated SER for DCT using QPSK is derived as

\[
P_S \leq \frac{1}{2} \left\{ Q\left( \frac{E_s}{\sqrt{N_\epsilon}} \right) + Q\left( \frac{E_s}{\sqrt{N_\epsilon}} \right) \right\}
\]

(2.80)

In the same manner, the SER for DIRT is approximated as

\[
P_S \leq \left\{ Q\left( \frac{E_s}{\sqrt{N_\epsilon}} \right) + Q\left( \frac{E_s}{\sqrt{N_\epsilon}} \right) \right\}
\]

(2.81)

If there is no phase mismatch and LO phase error, the SERs in DCT and DIRT are the same as

\[
P_S \leq 2 \cdot Q\left( \frac{E_s}{\sqrt{N_\epsilon}} \right)
\]

(2.82)

In a similar manner, the SER for DCT using 16-QAM and 64-QAM is approximated as
The SER for DIRT using 16-QAM and 64-QAM is approximated as

\[
P_s \leq \sum_{i=0}^{\sqrt{M} - 1} \frac{2 \sqrt{M} - 1}{M} \left\{ Q\left( \frac{3}{M - 1} E_s \left(1 + (2k + 1) \cdot \left(\frac{\phi_m}{2} + \phi_{\text{chr}}\right)\right) \right) + Q\left( \frac{3}{M - 1} E_s \left(1 + (2k + 1) \cdot \left(\frac{\phi_m}{2} - \phi_{\text{chr}}\right)\right) \right) + Q\left( \frac{3}{M - 1} E_s \left(1 - (2k + 1) \cdot \left(\frac{\phi_m}{2} + \phi_{\text{chr}}\right)\right) \right) + Q\left( \frac{3}{M - 1} E_s \left(1 - (2k + 1) \cdot \left(\frac{\phi_m}{2} - \phi_{\text{chr}}\right)\right) \right) \right\}
\] (2.83)

If there is no phase mismatch and phase error, the SER in DCT and DIRT using 16-QAM is the same as

\[
P_s \leq \sum_{i=0}^{\sqrt{M} - 1} \frac{4 \sqrt{M} - 1}{M} \left\{ Q\left( \frac{3}{M - 1} E_s \left(1 + (2k + 1) \cdot \phi_{\text{chr}}\right) \right) + Q\left( \frac{3}{M - 1} E_s \left(1 - (2k + 1) \cdot \phi_{\text{chr}}\right) \right) \right\}
\] (2.84)

In (2.83) and (2.84), the Q-function is a function of the deterministic phase mismatch and random phase difference. Therefore, the SER, due to the zero mean Gaussian noise with rms LO phase error, can be calculated by

\[
P_s \leq 3 \cdot Q\left( \frac{1}{5} \frac{E_s}{N_o} \right)
\] (2.85)
\[ P_{\text{SymErr}} = \int_{-\infty}^{+\infty} P_{s} \cdot p_{\phi_{ch}}(\phi_{ch}) \, d\phi_{ch} \] 

(2.86)

\[ = \int_{-\infty}^{+\infty} P_{s} \cdot \frac{1}{\sigma \sqrt{2\pi}} \cdot \exp\left(\frac{-\phi_{ch}^2}{2\sigma^2}\right) \, d\phi_{ch} \] 

(2.87)

Fig. 2.20 shows the SER comparison between the complete equation (2.53) and the approximated equations (2.83) and (2.84) for 16 and 64-ary QAM systems. In Fig. 2.20(a), a 5° RF phase mismatch and 0.5/2.5° LO phase errors are assumed in the DCT for 16-QAM, as well as 5° IF/RF phase mismatch and 0.5/2.5° LO phase error are also assumed in the DIRT for 16-QAM. For 64-QAM in Fig. 2.20(b), 5° RF phase mismatch and 0.5/1.5° LO phase errors are assumed in the DCT, as well as 5° IF/RF phase mismatch and 0.5/1.5° LO phase error are also assumed in the DIRT. The graph of (2.83) and (2.84) match well with the graph of complete equation (2.53). The effects of RFLO phase errors between DCT and DIRT are investigated. Fig. 2.21 shows the SER as a function of the variation in RFLO phase error. In the case of same RFLO phase error with no phase mismatch, the SER of DIRT is greater than the SER of DCT in QPSK, 16 QAM and 64 QAM, because the total output phase error of DIRT is greater than that of DCT in (2.83), as shown in Fig. 2.20(a). However, if the RFLO phase noise is scaled down by \( N/(N+1) \), the SER of DCT and DIRT are the same as each other, because the total output phase noise is the same, as shown in Fig. 2.20 (b). In consideration of phase mismatch terms in the DCT and DIRT, if there are 5° phase mismatches, the SER of DCT is degraded more than that of the DIRT, even though the RFLO phase errors are the same in the DCT and DIRT, as shown in Fig. 2.21(a). In addition, for a scaled-down RFLO phase
noise error, the SER of the DIRT is superior to that of the DCT, as shown in Fig. 2.21(b). The analytical SER results match with the EVM results. As mentioned in section III, for transmitters with $5^\circ$ phase mismatch, the SER of the DCT is sensitive to both phase noise and phase mismatch, but the SER of the DIRT is degraded by only LO phase error, as shown in Fig. 2.21 and Fig. 2.22. The LO phase noise term depends on the voltage controlled oscillator (VCO), the inherent semiconductor process characteristics, and the phase-locked loop (PLL) parameters. However, the I/Q phase mismatch effects can be minimized by the DIRT’s own architecture. Therefore, the insensitivity to phase mismatch effects in the DIRT architecture can provide the DIRT with more advantages over the DCT architecture in higher-order QAM systems.
Figure 2.20: Comparison between complete equation (27) and the proposed approximations (33) and (34) for (a) 16-QAM and (b) 64-QAM when 5° phase mismatch and 5° IF/RF phase mismatches in DCT and DIRT, respectively.
Figure 2.21: SER comparison when no phase mismatch: a) when RFLO phase errors are 4°, 2°, and 1.2° for QPSK, 16-QAM, and 64-QAM, respectively. b) when RFLO phase noise in DIRT is scaled down by 3.2°, 1.6°, and 0.96° for QPSK, 16-QAM, and 64-QAM, respectively.
Figure 2.22: SER comparison in cases of phase mismatch of 5° in DCT and IF/RF phase mismatch of 5° in DIRT: a) when RFLO phase errors are 4°, 2°, and 1.2° for QPSK, 16-QAM, and 64-QAM, respectively. b) when RFLO phase noise in DIRT is scaled down by 3.2°, 1.6°, and 0.96° for QPSK, 16-QAM, and 64-QAM, respectively.
2.10 Summary of phase noise effect

The LO phase noise error and phase mismatch effects of the DCT and DIRT architectures were investigated by using a complex envelope-based matrix model. The EVM and SER of the DIRT are degraded more than that of the DCT with the same RFLO phase noise error, when there is no phase mismatch. However, if there are phase mismatches in the transmitters, the EVM and SER of DIRT are superior to those of DCT in case of small phase mismatches. Furthermore, the RF output phase noise for the DIRT should be calculated carefully in consideration of the operating frequency and divider ratios, because the RF output frequency is higher than the RFLO frequency. For the case where RF output phase noise error in both DCT and DIRT is the same, the EVM and SER of the DIRT is sensitive to only phase noise error, whereas both phase mismatch and phase error of the DCT directly affect EVM and SER. Therefore, the DIRT architecture can be attractive for improving SNR degradation of higher order QAM systems.

2.11 LO leakage with I/Q gain and phase mismatches

Active double-balanced mixers were used for the DIRT to suppress the LO leakage. The LO leakage can result from DC offsets of the transconductance stages in the mixers and substrate isolation. Hence, the mismatch effects on the LO leakages are considered. The analysis is carried out assuming higher image suppression with small IF gain mismatch. The LO leakage rejection ratio at frequency $\omega_{LO2} + \omega_{LO1}$ can be expressed by

$$LORR_{(1,2)} \approx 2O_i^2 \cdot \frac{2[1 + \cos(\theta_A)\cos(\theta_B)]}{[\cos(\theta_A) + 1][\cos(\theta_B) + 1]}$$  \hspace{1cm} (2.88)
The derivations of the leakage rejection ratio LORR\(_{(1,2)}\) are obtained using a similar procedure as in Appendix B. The DC offsets (\(O_i\) and \(O_q\)) of the IF mixer pairs (A and B) are assumed to be equal (\(O_i = O_q\)) and the DC offsets (\(O_{2i_2}\) and \(O_{2q_2}\)) of the RF mixer pair (C) are assumed to be equal (\(O_{2i_2} = O_{2q_2}\)). Under the assumption that the phase mismatches (\(\theta_A\) and \(\theta_B\)) are small and can be neglected, the LORR\(_{(1,2)}\) is proportional to the square of the DC offset (\(O_i\)) of the IF mixer pairs. The amplitude of the LO leakage \(L_{O(2)}\) at frequency \(\omega_{LO2}\) is expressed by

\[
L_{O(2)} \approx (A_{2,LO})^2 [(1 + \varepsilon_2)^2 O_{2i_2}^2 \\
+ (OF_q)^2 + 2(1 + \varepsilon_2)O_{2i_2} OF_q \sin(\theta_B)]
\]

(2.89)

\[
OF_q = O_i O_{LOQ} + O_q O_{LOI} + O_{2q_2}
\]

(2.90)

The \(OF_q\) consists of the input DC offset of the IF mixer pair B and the input DC offset on the lower mixer of the RF mixer pair C. The derivation of the leakage signal \(L_{O(2)}\) is shown using a similar procedure as in Appendix B. The \(O_{LOI}\) and \(O_{LOQ}\) are the DC offsets of the quadrature IF LO signals, and \(O_{2q_2}\) is the DC offset of the lower mixer in the RF mixer pair. If the DC offsets (\(O_i\) and \(O_q\)) of the IF mixer pair B are eliminated in (2.90), the LO leakage does not depend on the DC offsets (\(O_i, O_q, O_{LOI}\) and \(O_{LOQ}\)) of the IF mixer pairs.
Therefore, to reduce such DC-offset effects, a DC-blocking capacitor could be used between the IF and RF mixers. If the DC offsets (O_i and O_q) of the IF mixer pairs (A and B) are eliminated, the OF_q can be expressed by the DC offset (O_{2q2}) of the RF mixer pair C in (2.90). Finally, the rejection of the LO leakage is given by

$$LORR_{(2)} = \frac{2(O_2)^2}{(A_{LOQ})^2} \cdot \left[ \frac{4[1 + \sin(\theta_B)]}{\{\cos(\theta_A) + 1\}\{\cos(\theta_B) + 1\}} \right]$$ (2.91)

The input DC offsets (O_{2i2} and O_{2q2}) of the RF mixers are assumed as O_{2i2} = O_{2q2} = O2. The derivation of the leakage rejection ratio LORR_{(2)} uses a similar procedure as in Appendix B. If the phase mismatches (\theta_A and \theta_B) are assumed to be negligible, the LO leakage signal depends on the DC offset (O2) of the RF mixer pair C.
2.12 LO self-mixed signals with I/Q gain and phase mismatches

DC-offset generated by LO-self mixing of a receiver is most critical for receiver performance. However, LO-to-IF feedthrough of the receiver is not critical, because the LO signal can be suppressed by an LPF following the IF-mixer. For the transmitter, the LO-to-Baseband leakage signal can also be suppressed by the gm-stage having an LPF characteristic. If the mixers are assumed to be ideal and there are reflections between the input of the IF-stage mixer and the output of the LPF following the DAC, the output power spectrum can be as shown in Fig. 2.23. The equation pairs (2.92) and (2.93) express the amplitudes of the self-mixed signals at the outputs of the IF mixer pairs (A and B) in case of small IF gain mismatch. These equations are used to investigate the spurious signals for the case where small IF gain mismatch effects exist.

\[
S_{ IFI } @ 0Hz : \varepsilon_1 + \frac{1}{2} \varepsilon_1^2
\]
\[
S_{ IFI } @ 2\omega_{1stLO} :
\cos^2(\theta_2)(1 + 2\varepsilon_1 + \varepsilon_1^2) + (\varepsilon_1^2 + \varepsilon_1^3 + \frac{1}{4}\varepsilon_1^4)
\]  
(2.92)

\[
S_{ IFQ } @ 0Hz : 1 + \varepsilon_1 + \frac{1}{2} \varepsilon_1^2
\]
\[
S_{ IFQ } @ 2\omega_{1stLO} :
1 - \cos^2(\theta_2)(1 + 2\varepsilon_1 + \varepsilon_1^2) + (2\varepsilon_1^2 + 2\varepsilon_1 + \varepsilon_1^3 + \frac{1}{4}\varepsilon_1^4).
\]  
(2.93)

In (2.92) and (2.93), the self-mixed signals on SFI and SFQ nodes are eliminated at 0 and 2\(\omega_{1stLO}\) (Hz), respectively, for small IF-gain mismatch, \(\varepsilon_1 \approx 0\). In
addition, the self-mixed signals through the IF mixer pairs (A and B) can be emitted through the IF-to-RF feedthrough and by multiplying the quadrature RF LO signals as shown in Fig. 2. Although the leakage signals at \( f_{\text{osc}} \) can be generated by multiplying the DC-offsets of the IF mixers and the quadrature RF LO signal, the leakage signals can be eliminated via ac-coupling capacitors between IF- and RF mixers.
CHAPTER 3: CIRCUIT IMPLEMENTATION

In this chapter, the insensitivity to the gain and phase mismatch effects is demonstrated for multiband operation by showing the in-band image rejection ratio. The proposed DIRT with sliding-IF is implemented on a 0.13-μm CMOS process.

3.1 System block diagram

Fig. 3.1 shows the frequency plan of the DIRT with sliding-IF, where the IF frequency should be correspondingly set from 0.75 GHz to 3.0 GHz to cover the output frequency over 11 GHz to 15 GHz. Fig. 3.2 shows the block diagram of the DIRT with sliding-IF, which is comprised of two IF I/Q mixer pairs, one RF mixer pair, and a two-stage RF pre-amplifier. An external signal generator is used as a common LO source instead of an on-chip voltage-controlled oscillator (VCO) for this chip version. To achieve the multiband functionality, the frequency dividers-by-4/8/16 are used for attaining quadrature IF LO signal. The IF I/Q mixer pairs operate over the wideband frequency range of 0.75 GHz to 3.0 GHz. A two-stage RC polyphase filter is used to generate the quadrature RF LO signals with an external signal source covering 10 GHz to 11.5 GHz. Although the polyphase filter has inherent loss, the required phase accuracy can be achieved and the path loss can be compensated by buffer amplifiers.
3.2 Block specifications

The typical characteristics of an I/Q modulator include carrier feed-through, sideband suppression, amplitude/phase mismatches and inter-modulation performance. This section describes the typical system performance.
Fig. 3.3 shows a cascaded three-stage system. The first stage, second stage, and third stage can be defined as a BB-to-IF up stage, IF-to-RF up stage, and RF preamplifier, respectively. For completely matched input/output impedances, the total available gain can be expressed by

$$G_{total} = G_1 \cdot G_2 \cdot G_3$$

(3.1)

where $G_n$ is defined as the power gain in each stage. However, in a real IC transmitter, the input and output impedances of each stage are hard to be matched. Therefore, the total cascaded gain is different from the total available power gain as follows:

$$G_{total} \neq G_1 \cdot G_2 \cdot G_3.$$  

(3.2)

In this case, voltage gain is more convenient. The loaded voltage gain and power gain are related by
\[ G_{V,n} = \frac{V_{\text{out},n}^2}{R_{\text{in},n+1}} \left/ \frac{V_{\text{in},n}^2}{R_{\text{in},n}} \right. = A_{V,n}^2 \frac{R_{\text{in},n}}{R_{\text{in},n+1}}. \]  

Therefore, the power gain can be expressed with loaded voltage gain and resistance by

\[ G_{V,1} \cdot G_{V,2} \cdot G_{V,3} = A_{V,1}^2 \cdot A_{V,2}^2 \cdot A_{V,3}^2 \cdot \frac{R_{\text{in},1}}{R_L}. \]  

(3.4)

For the linearity and available output power of the cascaded system, the cascaded voltage 1-dB compression point is given in

\[ \frac{1}{A_{\text{1dB}}} = \frac{1}{A_{\text{1dB},3}^2} + \frac{1}{A_{\text{1dB},2}^2 \cdot A_{V,3}^2} + \frac{1}{A_{\text{1dB},1}^2 \cdot A_{V,3}^2 \cdot A_{V,2}^2}. \]  

(3.5)

where \( A_v \) and \( A_{\text{1dB}} \) are voltage gain and 1-dB compression voltage at each output load, respectively. Therefore, 1-dB power compression point can be derived as

\[ \frac{1}{\text{OP1dB}_{\text{total}}} = \frac{1}{\text{OP1dB}_3} + \frac{1}{\text{OP1dB}_2 \cdot G_{V,3}} + \frac{1}{\text{OP1dB}_1 \cdot G_{V,3} \cdot G_{V,2}} \]  

(3.6)

where 1-dB compression point of each stage is
\[ OP_{dB_n} = \frac{A_{idB,n}}{R_{m,n+1}}. \quad (3.7) \]

The nonlinearity of the cascaded system can be evaluated by input third-order intermodulation as

\[ \frac{1}{A_{iP3}^3} \approx \frac{1}{A_{iP3,1}^3} + \frac{A_1^2}{A_{iP3,2}^3} + \frac{A_2^2}{A_{iP3,3}^3}. \quad (3.8) \]

where \( A_{iP3} \) is defined as the input IP3 of each stage. If each stage gain is greater than unity, the nonlinearity of the latter stages becomes more critical. The input third-order intercept point is related with the output third intercept point by

\[ A_{oIP3} = A_v \cdot A_{iP3}. \quad (3.9) \]

Therefore, the output third-order intercept point can be given by

\[ \frac{1}{A_{oIP3}^3} \approx \frac{1}{A_{oIP3,1}^3} \cdot A_1^2 + \frac{1}{A_{oIP3,2}^3} \cdot A_2^2 + \frac{1}{A_{oIP3,3}^3}. \quad (3.10) \]

The input 1-dB compression point is approximately related with third intercept point by
Table 1.1 Transmitter link budget

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<tr>
<th>Parameters</th>
<th>Input</th>
<th>1&lt;sup&gt;st&lt;/sup&gt; stage</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; stage</th>
<th>3&lt;sup&gt;rd&lt;/sup&gt; stage</th>
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<td>-24</td>
<td>-16</td>
<td>-16</td>
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<tr>
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<td></td>
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<td>&gt;21 dBv</td>
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</tbody>
</table>

\[
A_{\text{IIP3}} = 9.6dB + A_{1-dB,\text{in}} \quad (3.11)
\]

\[
A_{\text{OIP3}} = 10.6dB + A_{1-dB} \quad (3.12)
\]

Therefore, the third-order intercept point can be approximately evaluated from the 1-dB compression point. Table 1 shows the transmitter link budget.
3.3 Circuit Design

3.3.1 Current source with power down function

Current source is used for each building block. Fig. 3.4 shows a schematic diagram of the current source and its bias circuit. The cascode transistors M1 and M2 increase the output impedance of the current source, and the transistors M3, M4, M6 and M8 are used for DC-bias circuit of a wide swing cascode current mirror, where the transistor M3 can be replaced with cascode transistors connected with the gates of M1 and M2 for making more symmetric current mirror. The transistors M7 and M9 are utilized for power down function, which toggles power-on and -off of the implemented block.

![Figure 3.4: Schematic diagram of current source block.](image)

3.3.2 Image-reject mixer and amplifier

Fig. 3.5 shows a simplified schematic diagram of the entire double-image-rejection mixer pairs. Standard double-balanced Gilbert-cell mixer pairs were used for the
input-to-intermediate frequency translation. The mixer consists of a differential transconductance stage, a differential switching stage, and loads for a differential port. In the differential pairs, the LO signal and IF signal feed-through can be ideally eliminated, if there are mismatch effects. Therefore, the isolation characteristic is superior to the single-balance mixer type. The amplitude of LO voltage swing directly affects the conversion gain and the noise of the mixer. When the LO switching pair is simultaneously on, the input signal is canceled as a common-mode signal and the conversion gain is decreased. The reduced conversion gain increases the noise contributed by LO switching pairs. Therefore, the switching of the LO signal should be abrupt by decreasing drain current, or increasing the transistor width of the switching stage [31]. If the drain current is decreased, the source input impedance of LO switching pair is increased by the relationship of \( Z_s = 1/gm \), where \( Z_s \) denotes the input impedance seen at the source, while reducing the transconductance of the LO switching pairs in terms of

\[
g_m = \sqrt{2\mu C_{ox} \frac{W}{L} I_D} .
\]  

(3.13)

In consideration of (3.13), if the width of the LO switching MOS is increased at a fixed drain current, the input impedance seen at the source is decreased, and the switching transition can be abrupt. However, in such a case, the input signal can be shunted through the increased gate-source capacitance of the switching transistors at drain nodes of the transconductance stage.
Moreover, the capacitance can result in signal distortion by increasing the load capacitance of the transconductance stage. Therefore, the transistor size of the switching stage should be smaller than the transistor size of the transconductance stage, for optimized LO switching. In addition to the effects of the transistor size, the mixer can be linearized by increasing the gate-source overdrive voltage of the input transistors. However, for a given bias current, a higher overdrive voltage of the transconductance stage leads to a lower transconductance in terms of

\[ g_m = \frac{2I_D}{V_{GS} - V_{TH}}, \]  

(3.14)
and thus, the decreased $g_m$ results in an increase of noise figure and the reduction of the conversion gain. Therefore, the trade-off among the linearity, noise, and the gain of the mixer can be optimized by the drain current, dc-biasing and the width of the transistors. Moreover, for the optimized gain characteristic, the load effects of the mixer are important parameters. The up-conversion gain of the mixer is ideally given with the LO square wave in terms of

$$\text{square}(\omega_{\text{LO}}, t) \sin(\omega_{\text{a}}, t) = \left\{ \frac{4}{\pi} \sin(\omega_{\text{LO}} t) + \cdots \right\} \cdot \sin(\omega_{\text{bb}} t) = \frac{2}{\pi} \cos((\omega_{\text{bb}} + \omega_{\text{LO}}) t), \quad (3.15)$$

and the maximum switching loss is $20 \log(2/\pi) = -3.92$ dB. Therefore, the available mixer conversion gain can be calculated with the constant, $2/\pi$, by

$$G = \frac{2}{\pi} G_m \cdot Z_{\text{out}} \quad (3.16)$$

where $G_m$ and $Z_{\text{out}}$ are the transconductance and the output load impedance, respectively. For the LO switching stage, the input bias voltage of switching transistors is

$$V_{GS} - V_{TH} < V_{DS}, \quad (3.17)$$
and, practically, it depends on the drain current of the mixer. The load impedance $Z_{\text{out}}$ can be calculated by the inductor quality factor $Q$ and input/output capacitances between two cascaded stages. The RLC tank impedance is given by

$$Z(\omega) = \frac{j\omega L}{1 + j\omega \frac{L}{R_L} - \omega^2 LC}. \quad (3.18)$$

The Q value is expressed by

$$Q_{\text{tan}k} = R_L \sqrt{\frac{C}{L}} = \frac{R_L}{\omega_o L} = \omega_o R_L C. \quad (3.19)$$

Therefore, the normalized tank impedance is given by

$$\frac{Z(\omega)}{R_L} = \frac{1}{1 + jQ_{\text{tan}k}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}. \quad (3.20)$$

At the resonant frequency, $\omega = \omega_o$, the $Z_{\text{out}}$ is given by

$$Z(\omega_o) = Q_{\text{tan}k} \sqrt{\frac{L}{C}} = R_L, \quad (3.21)$$
where $R_L$ is the total load impedance in terms of

$$R_L = R_p \parallel R_{\text{load}}, \quad (3.22)$$

where $R_p$ and $R_{\text{load}}$ denote the loss of the inductor and output load resistance. The loss of inductor, $R_p$, can be derived from the quality factor of the inductor and the internal resistance, $R_S$, by

$$R_p = Q_{\text{ind}}^2 R_S. \quad (3.23)$$

If the output load, $R_{\text{load}}$, is unloaded, the total load $R_L$ is equal to the inductor resistance $R_p$. Therefore, the mixer gain is given by

$$G = \frac{2}{\pi} G_m \cdot Z_{\text{out}} = \frac{2}{\pi} G_m \cdot Q_{\text{ind}}^2 \sqrt{L/C} = \frac{2}{\pi} \frac{G_m}{C} \cdot \frac{L}{R_S}. \quad (3.24)$$

The mixer gain is proportional to the inductor $Q_{\text{ind}}$. Furthermore, the wideband response of the gain can be obtained by lowering the $Q_{\text{ind}}$. The load capacitance is composed of the input and output capacitances between two cascaded stages by
\[ C_{tank} = C_{out,n} + C_{in,n+1}. \] (3.25)

Through optimization of the circuits, the transistor length of the transconductance stages (M1-M4) was chosen as 250 nm for improving the effects of transistor matching. The layouts of two IF I/Q mixer pairs were identically and symmetrically drawn to reduce the mismatch effects between the mixer pairs. The electrical length of the I/Q connection lines for mixers and IF LO generator should be identical to reduce asymmetric phase mismatch effects. The IF I/Q modulator achieves about 3 dB voltage gain over the 2 GHz IF band. The transistor widths of the transconductance stage (M1-M4) and LO switching stage (M5-M8) are 180 µm and 30 µm, respectively. The RF I/Q modulator covers the 11 GHz to 15 GHz output frequency range. The transistor widths of the transconductance stage (M9-M10) and LO switching stage (M11-M12) are 60 µm and 50 µm, respectively. The RF I/Q modulator achieves 1dB voltage gain and -3 dB bandwidth over 11.5 to 14.5 GHz at the output node.

In order to drive a power amplifier, a two-stage preamplifier with transistors of 50 µm width are used for the RF output stage, which has 5-8dB voltage gain over 11.5 to 14.5 GHz. To minimize the RF signal distortion, the nonlinearity of the amplifier should be considered with the RF output power and current as follows:
The even-order signals of the differential amplifier cancel each other at the differential output port. However, the third harmonic distortion can be derived from the drain currents. The output current of the differential amplifier is given by

\[ I_{D1} - I_{D2} = \frac{1}{2} \mu_m C_{ox} \frac{W}{L} V_{dd} \sqrt{4(V_{GS} - V_{TH})^2 - V_{id}^2}. \]  

(3.27)

If \( V_{id} = V_{GS} + V_m \cos(\omega t) \), the drain current is given by

\[ I_{D1} - I_{D2} = g_m [V_m - \frac{3V_m^3}{32(V_{GS} - V_{TH})^2}] \cos(\omega t) - g_m \frac{V_m^3}{32(V_{GS} - V_{TH})^2}] \cos(3\omega t). \]  

(3.28)
Therefore, the third harmonic distortion is given in

\[ HD_3 = \frac{1}{32} \left( \frac{V_m^2}{(V_{gs} - V_{th})^2} \right) = \frac{1}{32} \frac{V_m^2}{V_p^2}. \] (3.29)

Using the relationship between voltage and current, the third harmonic distortion is given by the current in

\[ HD_3 = \frac{1}{32} \left( \frac{I_{RF}}{I_D} \right)^2. \] (3.30)

Total harmonic distortion (THD) can be quantified by summing the power of all harmonics and normalizing the result to the power of the fundamental as follows.

\[ THD = HD_2^2 + HD_3^2 + \cdots \] (3.31)
Figure 3.7: Voltage Gain of the IF and RF-stage mixer and amplifier.

In the cascaded system, the cascaded harmonic distortion can be calculated by summing the harmonic distortions of each stage. Therefore the total nth harmonic distortion is given by

\[(HD_n)_{1+2} = (HD_n)_1 + (HD_n)_2\]  

(3.32)

where \(n\) is defined as n-th harmonic.

By optimizing the parameters of the amplifiers, the inductances of 385 pH and 1nH are used for the 1\textsuperscript{st} stage and 2\textsuperscript{nd} stage amplifiers as shown in Fig. 3.6. The simulated voltage gain graph for the IF and RF stage is shown in Fig. 3.7. The RF stage includes the RF I/Q modulator and preamplifier.
3.3.3 IF Quadrature LO Generator

A cascaded divide-by-4/8/16 static frequency divider generates the quadrature IF LO signals by dividing an external signal source, as shown in Fig. 3.8(a). The frequency divider is composed of two dividers-by-4 (A and B) and a divider-by-2 for multiband operation. The frequency dividers-by-4 consist of two master-slave flipflops (MS-FF) with negative feedback.

Figure 3.8: (a) Block diagram of frequency dividers-by-4/8/16 and (b) Schematic diagram of D-latch and buffer.
The MS-FF is composed of two D-latches with current mode logic (CML) and the topology is often used for high speed operation, as shown in Fig. 3.8(b). The static frequency divider operation is explained by the edge-triggered master/slave D flip-flop (DFF), as shown in Fig. 3.9. The first DFF is commonly called the master DFF and the second one is normally referred to as the slave DFF. The master and slave are activated by the complementary. In the first cycle, each input positive duty clock (diagonal lines in Fig. 3.9) is fed into the MS-DFF. In the next cycle, the inverted output is fed back to the input, which makes the output toggled. For every two input clock cycles, the same procedure is repeated. Therefore, the output frequency is half of the input frequency. Based on the above-mentioned operation, the frequency divider can be characterized in the frequency domain by its sensitivity curve shown in Fig. 3.10 [36]-[38]. The minimum input voltage, $V_{\text{min}}$, is defined as the minimum voltage amplitude of the input signal, where the divider functions properly. The $f_{\text{so}}$ is defined as self-oscillation frequency, where any injected input signal is not required at the input port.
Therefore, in the D-latch shown in Fig. 3.8(b), self oscillation condition should be required for the oscillation frequency. The input impedance, $R_{\text{in}}$, seen at the drain of latch transistor $M_L$ can be viewed as the negative impedance $-2/g_{m,ML}$ by the positive feedback in Fig. 3.8(b). Thus, if the input impedance is less than or equal to the equivalent load resistance, the D-latch circuit can be considered as a free running oscillator. Therefore, the self oscillation condition is given by

$$g_m \cdot R_L > 1.$$  \hfill (3.33)

However, this oscillation condition is not mandatory. Although a clock divider does not have this condition, the divider can still function at the cost of a larger full-swing input voltage.
Figure 3.11: Input sensitivity of frequency divider-by-4.

Figure 3.12: Input sensitivity of frequency divider-by-4(B) at the FD2 node in the variations of the latch transistor widths.
Figure 3.13: Post-layout simulation results of the dividers-by 4/8/16; divider-by-4 (Top), divider-by-8 (Middle), and divider-by-16 (Bottom).

clock amplitude. In addition to the oscillation condition, the size of latch transistors ($M_L$) of a D-flipflop should be increased to lower the self-oscillation frequency [37]. In Fig. 3.8(a), the following frequency divider-by-4(B) and divider-by-2 require larger transistors than those of the preceding frequency divider-by-4(A), because they are used for lower operating frequency. In addition, the output voltage swing level of the preceding frequency divider-by-4(A) affects the sensitivity of the following frequency dividers. Fig. 3.11 shows the input voltage sensitivity of the frequency divider-by-4, where the width of latch transistors ($M_L$), driver transistors ($M_D$), and clock transistors ($M_C$) are 60µm, 60µm, and 40µm, respectively, with 78 ohm loads. The self-oscillation frequency is between 10 GHz and 11.5 GHz, which corresponds to RFLO operating frequency at $FD_1$-point. The output frequency 2.5GHz to 3GHz can be obtained by dividing the input frequency range. If the output voltage swing magnitude at $FD_2$-point
can be kept higher than the input voltage sensitivity 250 mV over 2.5 GHz to 3 GHz, the
minimum latch transistor size of the following dividers can be the same latch transistor
size of the preceding frequency divider-by-4(A). Therefore the latch transistor sizes of the
following frequency dividers can be optimized by considering the output voltage swing
of the output buffer amplifier. Fig 3.12 shows the relationship between input sensitivity
and the ratio of latch transistor widths of the divider-by-4(A) and (B). Fig. 3.13 shows the
post layout simulation results for the proposed divider blocks.
3.3.4 Quadrature RFLO generator

Fig. 3.14 shows the schematic diagram of a quadrature RFLO generator, which is composed of polyphase filter (PPF) and buffer amplifiers. The buffer amplifiers are used to compensate for the insertion loss at the input and output ports of the PPF. An on-chip inductor of 1.2 nH was utilized for the output buffer loads. The PPF was implemented as a two-stage RC filter to cover the wide bandwidth. In order to implement a four-phase RC polyphase filter, the input terminals must be properly connected. The RC PPF shown in Fig. 3.15 provides a constant magnitude response at all operating frequencies whereas 90° phase difference is obtained at the pole frequencies. Moreover, the filter is preferred when the sensitivity to the mismatch effects is considered.
Figure 3.15: RC PPF for quadrature signal generation.

\[ \omega_1 = 1/R_1 C_1 \quad \omega_2 = 1/R_2 C_2 \]

Figure 3.16: Computed results of (a) phase error and (b) attenuation of Type-II for two-and three-stage RC PPF.
The transfer function of the two-stage polyphase filter is given by [41]

\[
\frac{V_{\text{I,Q}}(s)}{v_{\text{in}}} = \frac{1}{R_1} \cdot \frac{(1 \pm sR_1C_1) \pm (1 \mp sR_1C_1) \cdot sR_2C_2}{\alpha \cdot (1 + sR_2C_2)^{-1} / R_2 (1 + (sR_2C_2)^{-2})}
\]  

(3.34)

where

\[
\alpha = \frac{1}{R_1} (1 + sR_1C_1) + \frac{1}{R_2} (1 + sR_2C_2).
\]  

(3.35)

The frequencies, \( f_{\text{max}} \) and \( f_{\text{min}} \) are defined as the edge frequencies of the bandwidth. A phase error below 2° can be achieved at the 1.5 bandwidth (=R_2C_2/R_1C_1). Fig. 3.16 shows the phase error and attenuation characteristic of the PPF. For the frequency variations due to process tolerance, ± 10 % tolerance of R and C result in a maximum 3° phase mismatch over 10 GHz to 11.5 GHz as shown in Fig. 3.17. Fig. 3.18 shows the post-layout simulation results of the polyphase filter for the RF mixer pair, where R_1, R_2, C_1, and C_2 corresponds to 86 ohms, 57 ohms, 188 fF, and 188 fF, respectively. The phase difference between I/Q signals was kept within 2° over 9 to 14 GHz frequency range. The I/Q RF LO generator is comprised of differential input and output ports. Therefore the 90°, 180°, 270° phase differences are shown in Fig. 3.18.
Figure 3.17: Phase difference of the two-stage RC PPF with R and C variations.

Figure 3.18: Post-layout simulation results of the two-stage RC PPF.
3.4 Transmitter simulation results

Fig. 3.19 shows the post layout simulation results of the entire transmitter. In the post layout simulation, the parasitic capacitors and resistors are extracted. The input ($f_{bb}$) and RFLO ($f_{RFLO}$) frequencies are 0.1 GHz and 11.2 GHz, respectively. The output frequency 14.1 GHz is obtained by $f_{bb} + 5 \cdot f_{RFLO} / 4$. The input LO power is given as 3dBm.
The IRR and LO leakage signals were suppressed greater than 50 dBC and 38 dBC, respectively. Fig. 3.20 shows the output power variations over temperature range from -50° to 90°. The output power variations were normalized to output power of 25° in circuit level simulation. Fig. 3.21 shows the chip layout of the DIRT with sliding-IF.

![Graph showing output power variation over temperature range from -50° to 90°.](image)

*Figure 3.20: Output power variation over temperature range from -50° to 90°.*
Figure 3.21: Chip layout of the designed DIRT with sliding-IF.
CHAPTER 4: FABRICATION AND MEASUREMENT

4.1 Fabrication and test setup

Fig. 4.1 shows the chip photograph of the DIRT with sliding-IF. The total chip area including bonding pads and dummy metal areas is \(2 \times 2 \text{ mm}^2\). The output pads were arranged so that they are compatible with GSGSG (G:GND, S:signal) probes. The power supply voltage is 1.5 V. An I/Q input signal generator, an RF LO signal generator, and an output spectrum analyzer were used for measurement setup. A 1-dB attenuator was used to make input magnitude mismatch of the input I/Q signals. Fig. 4.2 shows the test board layout. The board size is 6 mm by 6 mm. The DC supply voltage is applied through the PCB board lines.

Figure 4.1: Chip photograph of the fabricated DIRT with sliding-IF.
To change the IF frequency divider ratios, the pads 26, 27, and 35 were toggled between 0V and 1.5V. Bypass capacitors were used on the DC power lines to reduce the effect of the power supply noise and stabilize the DC power. Fig. 4.3 shows the photographs of the measurement setup.
Figure 4.3: The photograph of test measurement setup.
Figure 4.4: Measured narrowband output spectra. (a) using a 1-dB attenuator at an input port (b) not using any attenuators at the I/Q input ports.
4.2 Measurement Results

Fig. 4.4 shows the measurement results of the narrowband output spectra by using a 1-dB attenuator at an input port. Inserting a 1-dB attenuator into an input port makes the IF gain condition dominated by the input magnitude mismatch. The IF gain mismatch corresponds to the gain condition-I and 23.16 dBc in-band IRR was achieved. When the input I/Q signals were injected with the same magnitudes, the image suppression was obtained as 47.3 dBc, which results from IF gain mismatch of the IF mixer pairs. Four test sample dies were measured for the investigation of the in-band IRR tolerances resulting from the different samples.

![Figure 4.5: Measured image rejection ratios of four fabricated transmitter samples.](image-url)
Figure 4.6: In-band image rejection using an attenuator 1) as function of the IF phase mismatch, for RF gain mismatches 0.2 dB and 0.5 dB, and RF phase mismatch 3° in DIRT.

Figure 4.7: Measured in-band IRR and LO leakages of the transmitter over 11 GHz to 14.5 GHz.
The in-band IRRs were measured lower than 45 dBc at 13.5 GHz as shown in Fig. 4.5. The measurement results show that the IF gain mismatch is much smaller than 1 dB and the asymmetric IF LO phase mismatch was kept small. Fig. 4.6 shows the IRR variations as a function of IF gain mismatch by (2.15) and (2.19). The analytical result matches with the measurement result, where the 1 dB IF gain mismatch result in 24 dBc in-band IRR. The external RF LO signal was swept from 10 GHz to 11.4 GHz to examine the insensitivity to the mismatch effects due to the frequency variation from 11 GHz to 14.5 GHz. Fig. 4.7 shows the measured in-band IRRs in using dividers-by-4/8/16 with 10 GHz, 10.8 GHz, and 11.4 GHz RF LO signals. The image signals were suppressed lower than 48.5 dBc over 12.5 to 14.5 GHz frequency range and the LO leakage suppression ratios were lower than 43.5 dBc by using divider-by-4. In addition, the worst in-band IRR was 46.5 dBc over 11 GHz to 14.5 GHz by using divider-by-16. The offset frequencies from output frequency were 170 MHz and 85 MHz for image and LO leakage signals, respectively. The measured IRR was plotted with the reported IRRs of the identical DIRT architecture in Fig. 4.8 to show the IRR characteristic of the DIRT architecture over 0.9 GHz to 15 GHz. With the small IF gain mismatch, the out-of-band IRR_{\text{IM}} at \omega_{\text{LO2}}-\omega_{\text{LO1}}+\omega_{\text{BB}} depends on IF phase mismatch in (2.21) and (2.29), and the out-of-band IRR_{\text{IM}} at \omega_{\text{LO2}}-\omega_{\text{LO1}}-\omega_{\text{BB}} mainly depends on RF LO gain and phase mismatch in (2.23) and (2.30). However, in the measurement results, the out-of-band IRRs include attenuation of the front-end circuit. Therefore, the attenuation level should be compensated, after measuring the attenuation, by turning off the power of an IF IQ mixer pair.
Fig. 4.8: Measured image rejection ratios of the DIRT architecture over 0.9 GHz to 15 GHz. * suppression of spectral images.

Fig. 4.9 shows the out-of-band $\text{IRR}_{IM}$ and $\text{IRR}_{IMim}$ over 11GHz to 14.5 GHz. The suppression $\text{IRR}_{IM}$ over 28 dBC to 32 dBC means that there are 3° to 4.5° RF phase mismatch and 0.45 dB to 0.65 dB RF gain mismatch as shown in Fig. 4.10. In addition, the suppression $\text{IRR}_{IMim}$ over 28 dBC to 32 dBC means that there are 3° to 4.5° IF phase mismatch. Although there is 3° to 4.5° IF phase mismatch given by the $\text{IRR}_{IMim}$, the IF phase mismatch doesn’t affect the in-band IRR in the DIRT architecture. Therefore, the IF phase mismatch insensitivity can be proven in the DIRT architecture. Fig. 4.11 shows the in-band IRR over the 13.585 GHz, 12.24 GHz, and 11.56 GHz RF output frequencies, which were obtained by using dividers-by-4, -8, and -16 with a common 10.8 GHz RF LO signal, respectively.
Figure 4.9: Measured out-of-band IRR and LO leakages of the transmitter over 11 GHz to 14.5 GHz.

Figure 4.10: Out-of-band image rejection as function of the gain and phase mismatches.
Figure 4.11: Measured narrowband output spectra of the DIRT using a frequency divider with RF LO signal of 10.8 GHz: (a) a divider-by-4; (b) a divider-by-8; and (c) a divider-by-16.
The measured results show that the IRR is insensitive to the mismatch effects due to frequency variations over 11 GHz to 14 GHz. However, the LO leakage is higher than the in-band image suppression. The LO leakage results from input dc-offset and lack of adequate substrate isolation. The dc-offset can result in constellation origin offset while deteriorating the EVM error. If the LO leakage signal increases, it can cause in-band spurious emissions. The measured image signal, LO leakage, and spurious signal suppressions of the DIRT with a divider-by-4 are compared with the reported CMOS transmitters working around 10 GHz, [42]–[45], in Table I. The measured in-band image signals were suppressed greater than other reported architectures operating around 10 GHz even though the multiband DIRT is working above 10 GHz frequency range. The measurement results imply the better insensitivity to the mismatches due to the frequency variations. Fig. 4.12 shows the wideband output spectra of the transmitter using dividers-by-4/8/16 and an external 10.8 GHz RF LO source. In Fig. 4.12(a), the transmitter using a divider-by-4 suppressed spurious signals lower than 25 dBc. In addition, 12.24 GHz and 11.56 GHz output frequencies were measured in the multiband sliding-IF transmitter using divider-by-8/16 as shown in Fig. 4.12(b) and Fig. 4.12(c), respectively. The spurious signals were suppressed lower than 22 dBc and 18 dBc in the transmitter using divider-by-8 and divider-by-16, respectively. Fig. 4.13 shows the measured gain curve and 1-dB compression point. The output referred 1-dB compression point is obtained as high as -4 dBm with a 1.5 V power supply and 3dBm LO power at 13.5 GHz.
Table 4.1 Comparison of the CMOS transmitters operating around 10 GHz

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<td>17.45</td>
<td>12.5 ~ 14.5</td>
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<td>Dual Conversion</td>
<td>Direct I/Q</td>
<td>Dual Conversion</td>
<td>DIRT</td>
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<td>-33.8 dBC</td>
<td>-35 dBC</td>
<td>-48.8 dBC</td>
</tr>
<tr>
<td><strong>LO leakage</strong></td>
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<td>-35.2 dBC</td>
<td>&lt; -43dBm (with VCO)</td>
<td>-43.5 dBC</td>
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<td><strong>Spurious Tones</strong></td>
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<td>-28 ~ -45.5</td>
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<td>-18 dBC</td>
<td>-25 dBC</td>
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<td>1.8 V</td>
<td>1.2 V</td>
<td>1.5 V</td>
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<td><strong>Power</strong></td>
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<td><strong>139 mW</strong></td>
<td>#72 mW</td>
<td>*114 mW</td>
<td>*133 mW</td>
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* 2-stage upconversion mixers + 2-stage preamplifier,

**2-stage upconversion mixers + output buffer + synthesizer

# PA + IQ mixer + IQ divider
Figure 4.12: Measured wideband output spectra of the DIRT using a frequency divider with RF LO signal of 10.8 GHz: (a) a divider-by-4; (b) a divider-by-8; and (c) a divider-by-16.
Figure 4.13: Measured gain curve (a) and Output power (b).
CHAPTER 5: CONCLUSION

In this thesis, the effects of gain/phase mismatch and LO phase noise were analyzed by IRR, EVM, and SER for the multiband CMOS transmitter. Moreover, the feasibility of the DIRT was demonstrated for Ku-band applications by using a 0.13 um CMOS process. The in-band image signals can be suppressed in the multiband frequency range because the proposed DIRT architecture has an inherent suppression characteristic and is insensitive to the I/Q mismatch effects due to frequency variations. The approximated equations for the image rejection ratios provide insight into the I/Q mismatch effects in the multiband DIRT. Furthermore, the relationship between IRR and EVM is demonstrated through the derived equations. The system simulation results also showed that higher in-band image signal suppression was obtained with IF gain and phase mismatch variations. In the measurement results, the in-band IRRs from 48.8 dBc to 50 dBc were obtained over 12.5 GHz to 14.5 GHz. The insensitivity to the frequency variations makes the DIRT architecture more suitable for multiband operation. The multiband CMOS DIRT operating over Ku-band has not been previously reported.
APPENDICES

Appendix A

Derivation of Image Rejection Ratio in the DIRT

Neglecting the DC-offset terms of the double-image-rejection transmitter in Fig. 2.2, the input quadrature signals \((i_{BB}, q_{BB})\), IF LO signals \((i_{LO1}, q_{LO1})\), and RF LO signals \((i_{LO2}, q_{LO2})\) can be expressed by

\[
i_{BB}(t) = k \cdot \cos(\omega_{BB} t) \tag{A1}
\]

\[
q_{BB}(t) = -\sin(\omega_{BB} t) \tag{A2}
\]

\[
i_{LO1}(t) = A_{LOQ} \cos(\omega_{LO1} t + \phi_A / 2) \tag{A3}
\]

\[
q_{LO1}(t) = -A_{LOQ} \sin(\omega_{LO1} t - \phi_A / 2) \tag{A4}
\]

\[
i_{LO2}(t) = A_{LOQ2} \cos(\omega_{LO2} t + \phi_B / 2) \tag{A5}
\]

\[
q_{LO2}(t) = -A_{LOQ2} \sin(\omega_{LO2} t - \phi_B / 2) \tag{A6}
\]

where \(k\) is the magnitude mismatch of input signals, \(\phi_A\) and \(\phi_B\) are the IF and RF phase mismatches, and \(A_{LOQ}\) and \(A_{LOQ2}\) are IF and RF LO amplitudes.
The IF output signals can be expressed with the IF mixer gains ($\alpha_{m1}$, $\alpha_{m2}$, $\alpha_{m3}$, and $\alpha_{m4}$) by

\[ s_{IF} (t) = k \cdot \alpha_{m1} \cdot i_{BB} (t) \cdot i_{LO1} (t) - \alpha_{m2} \cdot q_{BB} (t) \cdot q_{LO1} (t) \]  \hspace{1cm} (A7)

\[ s_{IFQ} (t) = k \cdot \alpha_{m3} \cdot i_{BB} (t) \cdot q_{LO1} (t) + \alpha_{m4} \cdot q_{BB} (t) \cdot i_{LO1} (t) . \]  \hspace{1cm} (A8)

Therefore, the RF output signal can be expressed with the RF mixer gains ($\alpha_5$ and $\alpha_6$) by

\[ s_{RFOUT} (t) = \alpha_5 \cdot A^2_{LOQ2} \cdot s_{IF} (t) \cdot \cos(\omega_{LO2}t + \phi_B /2) \]
\[ + \alpha_6 \cdot A^2_{LOQ2} \cdot s_{IFQ} (t) \cdot \sin(\omega_{LO2}t - \phi_B /2) . \]  \hspace{1cm} (A9)

To investigate the gain mismatch effects with small phase mismatches, the ratio of the magnitudes at RF and image frequencies, $\omega_{RF} = \omega_{LO2} + \omega_{LO1} + \omega_{BB}$ and $\omega_{RFim} = \omega_{LO2} + \omega_{LO1} - \omega_{BB}$, is given by

\[ IRR = \frac{\text{Mag}_{RFim}}{\text{Mag}_{RF}} = \frac{(-\alpha_{m4}\alpha_6 - \alpha_{m2}\alpha_5 + k \cdot \alpha_{m3}\alpha_6 + k \cdot \alpha_{m1}\alpha_5)^2}{(\alpha_{m4}\alpha_6 + \alpha_{m2}\alpha_5 + k \cdot \alpha_{m3}\alpha_6 + k \cdot \alpha_{m1}\alpha_5)^2} \]  \hspace{1cm} (A10)

where the IF gain terms ($k\alpha_{m1}$, $\alpha_{m2}$, $k\alpha_{m3}$ and $\alpha_{m4}$) at each IF mixer outputs can be replaced with the entire IF gains ($\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$) for the simplicity of calculation.
Therefore, the image rejection ratio (IRR) is given by

\[
IRR = \frac{\text{Mag}_{\text{RF}_\text{im}}}{\text{Mag}_{\text{RF}}} = \frac{((\alpha_3 - \alpha_4) + (\alpha_1 - \alpha_2) \alpha_5/\alpha_6)^2}{((\alpha_3 + \alpha_4) + (\alpha_1 + \alpha_2) \alpha_5/\alpha_6)^2}
\]  \hspace{1cm} (A11)

where IF and RF gains (\(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5,\) and \(\alpha_6\)) are expressed with common IF and RF gains (\(G_{\text{IFM}}\) and \(G_{\text{RFM}}\)) by \(G_{\text{IFM}}(1+\Delta x_1/2), G_{\text{IFM}}(1-\Delta x_1/2), G_{\text{IFM}}(1+\Delta x_2/2), G_{\text{IFM}}(1-\Delta x_2/2), G_{\text{RFM}}(1+\Delta x_3/2)\) and \(G_{\text{RFM}}(1-\Delta x_3/2)\), respectively. Therefore the image rejection ratio (IRR) is given by

\[
IRR = \frac{\text{Mag}_{\text{RF}_\text{im}}}{\text{Mag}_{\text{RF}}} = \frac{1}{64} \left(2(\Delta x_1 + \Delta x_2) + (\Delta x_1 - \Delta x_2) \Delta x_3\right)^2
\]  \hspace{1cm} (A12)
Appendix B

Derivation of Image Rejection Ratios in the condition-I

Assuming $\alpha_1=\alpha_3$ and $\alpha_2=\alpha_4$, the gain mismatches $\alpha_1/\alpha_2=\alpha_3/\alpha_4$ and $\alpha_5/\alpha_6$ can be expressed by

$$\Delta A_1 = \alpha_1/\alpha_2 = \alpha_3/\alpha_4, \Delta A_2 = \alpha_5/\alpha_6 \quad (B1)$$

When only the RF frequency at $\omega_{RF}=\omega_{LO2}+\omega_{LO1}+\omega_{BB}$ is considered, the output power is given by

$$S_{OUT}(t) \text{at } \omega_{RF} = \frac{1}{4} A_{LOQ}^2 A_{LOQ2}^2 \cos(\omega_{RF} t) \ast [\Delta A_1 \Delta A_2 \cos(\theta_1 + \theta_3)$$
$$+ \Delta A_2 \cos(\theta_2 + \theta_4) + \Delta A_1 \cos(\theta_2 + \theta_4) + \cos(\theta_1 + \theta_4)]$$
$$+ \frac{1}{4} A_{LOQ} A_{LOQ2}^2 \sin(\omega_{RF} t) \ast [-\Delta A_1 \Delta A_2 \sin(\theta_1 + \theta_3)$$
$$- \Delta A_2 \sin(\theta_1 + \theta_2) - \Delta A_1 \sin(\theta_2 + \theta_4) - \sin(\theta_1 + \theta_4)] \quad (B2)$$

Substituting $\Delta A_1$ and $\Delta A_2$ with $(1+\varepsilon_1)$ and $(1+\varepsilon_2)$, the amplitude of the RF output power is

$$Mag_{RF} = \frac{1}{16} (A_{LOQ} A_{LOQ2})^2$$
$$[4(1+\varepsilon_1+\varepsilon_2+\varepsilon_1\varepsilon_2)+2\varepsilon_2^2(1+\varepsilon_1)+2\varepsilon_1^2(1+\varepsilon_2)$$
$$+(\varepsilon_1\varepsilon_2)^2+4(1+\varepsilon_1+\varepsilon_2+\varepsilon_1\varepsilon_2)+2\varepsilon_2^2(1+\varepsilon_1)) \cos(\theta_A) \quad (B3)$$
\[
+ 4(1 + \varepsilon_1 + \varepsilon_2 + \varepsilon_1 \varepsilon_2) \cos(\theta_B) \\
+ 4(1 + \varepsilon_2) \cos(\theta_A) \cos(\theta_B) \\
+ (4 \varepsilon_1 + 4 \varepsilon_1 \varepsilon_2 + 2 \varepsilon_1^2 + 2 \varepsilon_1 \varepsilon_2^2) \cos(\theta_A + \theta_B) \]
\]

where \( \theta_A = \theta_1 - \theta_2 \) and \( \theta_B = \theta_3 - \theta_4 \).

Substituting \( \varepsilon_1 \) and \( \varepsilon_2 \) with \( E_k |_{k=\text{integer}} \), \( D_k |_{k=\text{integer}} \) and \( N_k |_{k=\text{integer}} \), the magnitude of RF output power is simplified as

\[
Mag_{RF} = \frac{1}{16} (A_{LOQ} A_{2LOQ_1})^2 [(4E_1 + 2E_2 + 2E_3 + E_4) \\
+ (4E_1 + 2E_2) \cos(\theta_A) + 4E_1 \cos(\theta_B) + 4(1 + \varepsilon_1) \cos(\theta_A) \cos(\theta_B) \\
+ (4\varepsilon_1 (1 + \varepsilon_2) + 2E_3) \cos(\theta_A + \theta_B)] \\
= \frac{1}{16} (A_{LOQ} A_{2LOQ_1})^2 \cdot [D_0 + D_1 + D_2 + D_3 + D_4]
\]

where,

\[
D_0 = (4E_1 + 2E_2 + 2E_3 + E_4) \\
D_1 = (4E_1 + 2E_2) \cos(\theta_A) \\
D_2 = 4E_1 \cos(\theta_B) \\
D_3 = 4(1 + \varepsilon_1) \cos(\theta_A) \cos(\theta_B) \\
D_4 = (4\varepsilon_1 (1 + \varepsilon_2) + 2E_3) \cos(\theta_A + \theta_B)
\]

and

\[
E_1 = 1 + \varepsilon_1 + \varepsilon_2 + \varepsilon_1 \varepsilon_2 \\
E_2 = \varepsilon_2^2 (1 + \varepsilon_1) \\
E_3 = \varepsilon_1^2 (1 + \varepsilon_2) \\
E_4 = (\varepsilon_1 \varepsilon_2)^2.
\]
By the same procedure, the magnitudes of the in-band image signal at 
\(\omega_{RF_{im}}=\omega_{LO2}+\omega_{LO1}-\omega_{BB}\), the out-of band image signal2 at \(\omega_{IM_{im}}=\omega_{LO2}-\omega_{LO1}+\omega_{BB}\), and the out-of band image signal3 at \(\omega_{IM}=\omega_{LO2}-\omega_{LO1}-\omega_{BB}\) are derived as

\[
Mag_{RF_{im}} = \frac{1}{16} (A_{LOQ} A_{2\,LOQ}_{2})^{2} \cdot [N_{0} - N_{1} - N_{2} + N_{3} + N_{4} ] \tag{B5}
\]

\[
Mag_{IM_{im}} = \frac{1}{16} (A_{LOQ} A_{2\,LOQ}_{2})^{2} \cdot [N_{0} - N_{1} + N_{2} - N_{3} - N_{4} ] \tag{B6}
\]

\[
Mag_{IM} = \frac{1}{16} (A_{LOQ} A_{2\,LOQ}_{2})^{2} \cdot [N_{0} + N_{1} - N_{2} - N_{3} - N_{4} ] \tag{B7}
\]

where

\[
N_{0} = (4E_{1} + 2E_{2} + 2E_{3} + E_{4})
\]

\[
N_{1} = (4E_{1} + 2E_{2}) \cos(\theta_{A})
\]

\[
N_{2} = 4E_{1} \cos(\theta_{B})
\]

\[
N_{3} = 4(1 + \varepsilon_{2}) \cos(\theta_{A}) \cos(\theta_{B})
\]

\[
N_{4} = (4E_{1} (1 + \varepsilon_{2}) + 2E_{3}) \cos(\theta_{A} + \theta_{B})
\]

\[
N_{5} = (4E_{1} + 2E_{3}) \cos(\theta_{B})
\]

\[
N_{6} = 4E_{1} \cos(\theta_{A}) \cos(\theta_{B})
\]

Therefore, the image rejection ratios can be expressed by
\[ IRR_{RFim} = \frac{Mag_{RFim}}{Mag_{RF}} = \frac{[N_0 - N_1 - N_2 + N_3 + N_4]}{[D_0 + D_1 + D_2 + D_3 + D_4]} \] \hspace{1cm} (B8)

\[ IRR_{IMim} = \frac{Mag_{IMim}}{Mag_{RF}} = \frac{[N_0 - N_1 + N_2 - N_3 - N_4]}{[D_0 + D_1 + D_2 + D_3 + D_4]} \] \hspace{1cm} (B9)

\[ IRR_{IM} = \frac{Mag_{IM}}{Mag_{RF}} = \frac{[N_0 + N_1 - N_2 - N_3 - N_4]}{[D_0 + D_1 + D_2 + D_3 + D_4]} \] \hspace{1cm} (B10)
Appendix C

Derivation of Matrix model for DIRT under gain condition-I

The IF output complex signals can be expressed by

\[
S_{IFI}(t) = \begin{bmatrix}
\alpha_A \cos(\phi_A / 2) & \beta_A \sin(\phi_A / 2) \\
\alpha_A \sin(\phi_A / 2) & \beta_A \cos(\phi_A / 2)
\end{bmatrix} \begin{bmatrix}
I_{in} \\
Q_{in}
\end{bmatrix}
\]  
(C1)

\[
S_{IFQ}(t) = \begin{bmatrix}
-\alpha_A \sin(\phi_A / 2) & \beta_A \cos(\phi_A / 2) \\
-\alpha_A \cos(\phi_A / 2) & \beta_A \sin(\phi_A / 2)
\end{bmatrix} \begin{bmatrix}
I_{in} \\
Q_{in}
\end{bmatrix}
\]  
(C2)

The output complex signal can be expressed with the matrix \(M_{RF}\) of RF mixer pair by

\[
S_{RFOUT} = M_{RF} \begin{bmatrix}
\text{Re}(S_{IFI}) + j \text{Im}(S_{IFI}) \\
\text{Re}(S_{IFQ}) + j \text{Im}(S_{IFQ})
\end{bmatrix}
\]  
(C3)

where

\[
M_{RF} = \frac{1}{2} \begin{bmatrix}
\alpha_b \cos(\phi_b / 2) & \beta_b \sin(\phi_b / 2) \\
\alpha_b \sin(\phi_b / 2) & \beta_b \cos(\phi_b / 2)
\end{bmatrix}
\]  
(C4)

Therefore, the output signal is given by
\[ S_{\text{OUT}} = \frac{1}{2} \cdot \left( \alpha_A \alpha_B + \alpha_A \beta_B \right) \cos \left( \left( \phi_A + \phi_B \right)/2 \right) I_{\text{in}} + \left( \beta_A \alpha_B - \beta_A \beta_B \right) \sin \left( \left( \phi_A - \phi_B \right)/2 \right) Q_{\text{in}} \]

\[ + \frac{1}{2} j \cdot \left( \alpha_A \alpha_B - \alpha_A \beta_B \right) \sin \left( \left( \phi_A + \phi_B \right)/2 \right) I_{\text{in}} + \left( \beta_A \alpha_B + \beta_A \beta_B \right) \cos \left( \left( \phi_A - \phi_B \right)/2 \right) Q_{\text{in}} \]  

(C5)

The output complex signal of (B5) can be separated in real and imaginary components by

\[
\begin{bmatrix}
I_{\text{out}} \\
Q_{\text{out}}
\end{bmatrix}
= M
\begin{bmatrix}
I_{\text{in}} \\
Q_{\text{in}}
\end{bmatrix}
\]

(C6)

where

\[
M = \frac{1}{2} \begin{bmatrix}
(\alpha_A \alpha_B + \alpha_A \beta_B) \cos \left( \left( \phi_A + \phi_B \right)/2 \right) & (\beta_A \alpha_B - \beta_A \beta_B) \sin \left( \left( \phi_A - \phi_B \right)/2 \right) \\
(\alpha_A \alpha_B - \alpha_A \beta_B) \sin \left( \left( \phi_A + \phi_B \right)/2 \right) & (\beta_A \alpha_B + \beta_A \beta_B) \cos \left( \left( \phi_A - \phi_B \right)/2 \right)
\end{bmatrix}
\]

(C7)
REFERENCE LIST


