12-02

“A Critique of Ng’s Third-Best Theory”

Richard G. Lipsey

February, 2012
A CRITIQUE OF NG's THIRD-BEST THEORY

February 2012

by

Richard G. Lipsey
Emeritus Professor of Economics
Simon Fraser University
640 Holmbury Place,
West Vancouver B.C.,
V7S 1P7
email: rlipsey@sfu.ca
ABSTRACT

The theory of second best established that the effect on community welfare of any one policy change varies with the specific context in which that change occurs. In paper that has been frequently quoted to justify specific policies, Ng argues that fulfilling first-best conditions piecemeal is an optimal policy under quite general conditions when neither full first nor second best optima are achievable. This paper first argues that Ng's conclusion does not follow from his own assumptions, which imply instead that the status quo should be maintained, whatever it might be. Next it gives one illustrative example showing how much damage can be caused by following Ng's advice. It then argues that when Ng's key assumption is replaced by one that is closer to the facts, there is no general a priori presumption for adopting any specific policy, including maintaining the status quo. The paper closes with some observations on the usefulness of welfare economics even when the full implications of second best theory are accepted.

Key Words: second best, third best, reaction function, piecemeal welfare policies.

JEL CLASSIFICATION: D60, D61
A CRITIQUE OF NG's THIRD-BEST THEORY

If piecemeal establishment in any one market of its first-best condition for a Partetian optimum was sufficient for an increase in community welfare, all that would be needed for a welfare-increasing policy change would be to know the present state of the market under consideration and the first-best condition for that market. The theory of second best established, however, that piecemeal satisfaction of any one first-best optimality condition is not sufficient to increase welfare. Instead, specific knowledge of the market in question, and its relations to other markets, is needed to establish that a policy change will necessarily increase the whole community's welfare. In short, if first-best conditions were sufficient when applied piecemeal, no context-specific information would be needed. In contrast, second best theory establishes that the effect on community welfare of any one policy change is context specific.

In a frequently quoted article, Ng (1977) argued that fulfilling first-best conditions is an optimal policy when policy makers are in a state of substantial ignorance and neither first- nor second-best global optima can be achieved. Hence, he argues, non-context-specific, welfare-increasing policies can be established quite generally.

Many authors have cited Ng in support of their advocacy of first-best rules in situations in which second-best optima cannot be achieved. Ebert (1985:264) argues that "first-best results can be reasonable approximations when prices are distorted". Brennan Buchanan (1980:213) states that "...if the complement-substitute relations are not known, uniformity of rates (the first-best condition) is to be preferred". A report by Australian Government Productivity Commission (2008:21) argues that when the first best is unattainable due to many unalterable "distortions" going for their first best and ignoring other distortions will be “...close to the mark". Maks (2005:217) argues that in situation of informational poverty (we do not know in which direction the second best lies) "...the expected value of the welfare is maximised by partial realization of the 'first-best' optimum conditions" while under informational scarcity (we know in which direction the second best lies but not how far away it is) "... it is not too far from optimal to equate the price to marginal cost as the average in the economy". Woo (2010:287-88) follows Ng in considering a case in which "...we do not know the direction and degree of departure of the second-best optimum from that resulting from the application of the first-best rule [in one particular market]". He then argues that "...the third-best policy in this situation is not to depart from the first-best rule." Ng, himself, uses his argument about the relevance of first-best conditions to third-best situations in a number of articles (Ng 1984, 1987a, 1987b, 2000) and repeats it with some extensions in his book, Ng (2004).

In Part I of this paper, I repeat the relevant parts of Ng's analysis. In Part II. I argue that, even if we accept all of Ng's assumptions, his conclusion that first-best policies are the preferred ones in third-best situations does not follow. Instead, the correct implication of his own assumptions is that the status quo should be maintained, whatever it might be. I then go on to show in one illustrative example how much damage can be caused by following Ng's advice to adopt first best policies piecemeal when the first best overall optimum cannot be established. In Part III, I criticise one of Ng's key assumptions and argue that when it is replaced by one that is closer to the facts, there is no general a priori presumption for adopting any specific policy, including maintaining the status quo or adopting the first-best rule piecemeal, as long as the first best is unattainable (as is always the case, as argued in Lipsey 2007). Finally in Part IV, I
conclude with some general observations concerning the ways in which piecemeal welfare theory can be correctly used in spite of the theory of second-best.

At the outset, some relevant terms need to be clarified. Lipsey (2007:352) writes: "Factors preventing attaining an efficient resource allocation are variously called ‘constraints’ or ‘distortions.’ Since neither of these terms cover everything that follows, I use the term ‘sources of divergence,’ sources for short. I define these as anything that if introduced on its own would prevent the achievement of a perfectly competitive, price-taking equilibrium that was Pareto efficient and otherwise attainable.”

Ng defines a second-best situation as one in which "...some second-best distortion is present but costs of information, etc. are negligible." In such a state, he argues, those markets in which there are no unalterable distortions can be manipulated to establish the second-best optimum. He then defines third-best situations as obtaining when both "...distortional and informational costs exist."1 In such a situation, he argues, the second-best optimum cannot be achieved and third-best rules are needed.

This usage contrasts with that of Lipsey and Lancaster. For them: "A ‘second-best situation’ referred to any situation in which the first best was unachievable. The ‘second-best optimum setting’ for any source referred to the setting of that source that maximises the value of the objective function, given settings on all the other existing sources." (Lipsey 2007:352). So when Lipsey and Lancaster speak of a second-best situation in which the second-best optimum is unattainable, Ng would speak of a third-best situation. Nothing of substance depends on which terminology is used, as long as one is clear about what is referred to by each term.

I. NG’S ANALYSIS.

Ng draws a curve, $F^1$, as shown in our Figure 1, that "...relates the value of the objective function to the direction and degree of divergence from the first-best rule of the variable under consideration, or relation curve for short” (Ng, 1977:1).2 For concreteness, I will let this variable be the price of good Z traded in market Z. Since the objective function is maximized when the first-best rule is fulfilled, we must assume that the rest of the economy also obeys its first-best rules so that there is an economy-wide first-best optimum allocation of resources. Ng goes on to state "...it is reasonable to expect that the relation curve is concave. As we diverge more and more from the first-best rule, the marginal damage increases (p.3).”3 For now, we accept this concavity assumption but will have more to say about it in Section III.3 [See figures at the end of the paper.]

Ng next introduces a source in some market other than Z. Under what he calls conditions of "informational poverty," policy makers do not know the magnitude and direction of the induced divergence of the second-best value of the price in market Z from its first-best value. So it is equally likely that the curve relating the setting of the objective function will have shifted to the right as to the left, as shown by the two curves $F^2$ and $F^3$ in the figure. Note that once the

1 This is not the place to debate Ng’s implied view that, given the existence of ‘sources’ that prevent the attainment of a first best optimum, the only thing that prevents the attainment of a second-best one is the absence of full and costlessly available information. Lipsey (2007) clearly believes otherwise.

2 Figure 1 is similar to Ng’s Figure 2 but with different labels.

3 This argument requires that the objective function provides a cardinal measure, not just an ordinal measure.
first-best, economy-wide solution is not achievable, each reaction curve reaches its maximum at
the second-best setting for the source in question. If the policy maker knew the relation curve, he
could impose a tax or subsidy on the price of Z to move its market to its second-best position. If
he knew only in which direction the maximum point on the new curve lay, say to the right of
point 0, he could make at least a small move in that direction by departing from the first-best
setting for market Z and be sure of making a gain. But if the policy maker is in informational
ignorance, the best policy is, according to Ng, to stay put at what was the first-best position for
market Z. This is because the concavity of the relation curve implies that the expected sign of the
change in the objective function for any move is negative. Say that he makes a small departure
from the first-best position for market Z by moving to position $f_1$. If the second-best relation
curve is in fact $F_3$, he gains the amount $j_1$ but if it is in fact $F_2$, he loses the amount $j_2$. By the
concavity assumption $j_1 < j_2$ so that the expected value of the change in the objective function is
negative, assuming that both relation curves, $F^2$ and $F_3$, are equally likely. From this, Ng
concludes that the third-best policy is to remain at the first-best position for market Z, and hence,
by repetition of the argument, in each market in which the setting of the source can be varied.

II. THE STATUS QUO RATHER THAN THE FIRST BEST CONDITION

But there is a catch. Ng has considered a situation in which a first best optimum exists
originally. One source, say one tax or monopoly, is then introduced into one market and the
question is asked: "Should the other first-best conditions be departed from in other markets?" But
this is not a situation in which any real-world policy maker finds him or herself. Instead, there
are many sources acting in many markets. So the initial situation facing any actual policy maker
must be one in which neither a first-best nor a second-best optimum obtains and the policy maker
asks: "Should I adopt a first-best rule in other market piecemeal?" Let us say, for example, that
market Z, which is shown in Figure 2, currently has a setting of its source of $f_3$, while its relation
curve is $F_4$. Hence, the second-best setting of its source is $f_2$, given all the settings of all the
other sources that are in operation. But the policy maker does not know in which direction the
second-best setting lies. If he guesses correctly and makes a small move to $f_3'$, he gains $j_3$. But if
he guesses incorrectly and makes a move of equal size to $f_3''$, he loses $j_3$. By the concavity
assumption, $j_3 < j_4$ so the expected value of a small move is negative.4

So Ng's argument does not establish a presumption for establishing the first-best
conditions but a presumption for staying where you are, whatever your current position, always
assuming informational poverty. It is an argument for maintaining the policy status quo, not for
imposing first-best rules in third-best worlds.

To take the matter one step further, let us investigate what would happen to the value of
the objective function in a simple illustrative case if the policy maker followed Ng's advice to
establish first best rules under conditions of informational poverty, or its implication, as stated by
Woo in the quotation cited above that if we know in which direction the second best lies but not
how far away it is "... it is not too far from optimal to equate the price to marginal cost as the
average in the economy”.

To illustrate we consider an additive community utility function:

---

4 The analysis in Figure 2 is simpler than Ng's shown in Figure 1. But it yields the same result as can be seen by
conducting the same operation on curve $F^2$ in Figure 1 by considering an equal increase and decrease in the setting
of the source around the setting of $f_1$. 

\[ U = aX^\alpha + bY^\beta + cZ^\gamma, \]

with production possibilities given by:
\[ R = X + gY + hZ. \]

Assuming that prices equal marginal costs, \( g \) and \( h \) are the prices of \( Y \) and \( Z \) in terms of \( X \). The demand equilibrium conditions that marginal utilities be proportional to prices are, with a little manipulation, given by:
\[ g\alpha aX^{\alpha-1} = b\beta Y^{\beta-1} \]
\[ h\alpha aX^{\alpha-1} = c\gamma Z^{\gamma-1} \]

where \( i \) and \( j \) are one plus the rate of tax or subsidy (a negative tax) on \( Y \) and \( Z \) respectively. The calculations shown in the appendix give \( U \) as a function of the consumption weights \( a, b \), and \( c \); the prices, \( g \) and \( h \); the taxes, \( i \) and \( j \); and the total endowment, \( R \).

For the calculation in Tables 1 and 2, we let \( R=100 \) and \( g = h= 1 \). Table 1 gives, for various values of the parameters \( a, b \) and \( c \) and \( i \), the tax on \( Y \), the value of \( j \) for which welfare is the same as when \( j \) is at its first best setting of unity (based on the assumption that \( \alpha = \beta = \gamma \)).

First consider the case in which the three commodities have equal weights in the utility function. If the tax on \( Y \) is 10 percent, the removal of any positive tax on \( Z \) of up to 9.6 percent will lower welfare, while if the tax on \( Y \) is 100 percent, the removal of any tax on \( Z \) of up to 61 percent will lower welfare, even though in both cases the removal establishes the first-best tax on \( Z \). In the lower part of the table, the good under consideration is of much less importance in the utility function but the critical tax on \( Z \), below which value its removal lowers welfare, is almost the same as in the equal-value case.\(^5\)

### Table 1

**Value of the Tax Below Which Welfare is Lowered by Adopting the First-Best Rule**

<table>
<thead>
<tr>
<th>Variable Specification</th>
<th>Value of ( i )</th>
<th>Value of ( j ) for which ( U (j) = U(1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a=b=c=1.0 )</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>1.096</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1.610</td>
</tr>
<tr>
<td>( a=b=1.4, c=0.2 )</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>1.096</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1.655</td>
</tr>
</tbody>
</table>

\(^5\) When \( i=1.1 \), the critical value of \( j \) is the same to three decimal places for both \( a=b=c=1 \) and \( a=b=1.4 \) and \( c=0.2 \). But when \( i=2.0 \), the critical values differ slightly for the two sets of values on the parameters, \( a, b \) and \( c \). Investigation suggests that this is not a rounding error but a real difference whose reasons we are unable to discern.
Evidently, following the advice that one should adopt first-best settings for sources that can be altered under conditions of both informational poverty and informational scarcity is not only misleading, it can be seriously harmful.⁶

III. THE CONCAVITY ASSUMPTION

As already noted, Ng assumes that the typical relation curve is concave, as shown in Figures 1 and 2. This implies that by increasing the setting of the source in question, the value of the objective function can be reduced to zero. Ng supports his concavity assumption by considering the market for a single commodity arguing that "...we have concavity if the algebraic slope of the demand curve is everywhere smaller than that of the marginal cost curve." This argument is valid if the objective function concerns the welfare to be derived by trading only in that one market. In this case, the demand curve can cut the price axis implying that the value of the local-market welfare function can be reduced to zero.

But welfare analysis that seeks to derive optimum conditions for the whole economy must use some objective function that covers all markets, such as the summation of all consumers’ and producers’ surpluses, or community indifference curves, or a Samuelsonian community welfare function. Also, to study most second-best issues, there must be at least three markets: one that is uncontrolled, one that has an unalterable source, and one in which the setting of the source can be manipulated by public policy.⁷ In all such cases, Ng's concave relation curve implies the impossible situation in which a large enough setting of a source in one single market, such as a high enough tax on peanuts or fuel oil, can drive community welfare to zero. Consideration of this impossible situation tells us that, even if the reaction curve is concave near its maximum point, it must go through at least one point of inflection and eventually become horizontal, once the source has reached its maximum welfare-reducing setting. If, for example, the source is a tax on one commodity, the horizontal position is reached once consumption of the commodity falls to zero, at which point the community's overall welfare will, of course, not be zero.

Indeed, there is no obvious reason why the curve should have only one point of inflection — especially when we consider the large number of different sources that are typically operating to cause myriad divergences from first-best optimum conditions. For example, Lipsey (2007) lists nine broad classes of sources that typically exist in static situations, each one of which can contain many different individual items; and there are several additional classes that can operate in dynamic situations. Thus until proven otherwise, we must assume that multiple inflection points may exist.

The results in this table suggest all sorts of interesting further cases for students to investigate. Examples are: (i) Find out what parameters (if any) other than i are important in causing the critical setting of j to change (ii) What are some of the effects of dropping the simplifying assumption that \( a=\beta=\gamma \), in which case the reduced form of the welfare equation shown by equation (17) in the mathematical appendix becomes much more complicated? (iii) What are some of the effects of dropping the simplifying assumption that \( a=b=c \) in equation (4) of the mathematical appendix ? (iv) Investigate other utility functions to see how the critical setting of j is determined and how its relation to the setting of the parameter i alters as all of the model's other parameters are altered, then explain how and why these results compare with those derived from the additive utility function.

For some cases, two markets are sufficient but for taxes, subsidies and other sources that alter market prices, three markets are needed. If there are only two markets and in one price exceeds marginal cost by \( x \) percent, a first best situation can be achieved if the policy makers intervene to set the price in the second market also to exceed marginal cost by \( x \) percent. But if there is a third market in which competition makes price equal to marginal cost, the policy maker cannot establish a first best optimum by any manipulation of the price in the second market only.
points are possible in many situations. All that we can say is that, assuming the curve to be concave near its maximum second-best optimum point, there must be at least one point of inflection (otherwise welfare could be reduced to zero by a large enough setting of a source in one single market) and that if there is more than one such point, there must be an odd number of them so that the partial derivative of the objective function with respect to the setting of the one source being manipulated will eventually approach zero.

To illustrate the need for an inflection point, we consider two cases. The first is noted by Lipsey in his brief, one-paragraph discussion of Ng’s paper (2007:358). Lipsey argues the need for a point of inflection in the reaction curve but appears to miss the important status quo point made in Section II above. He considers a case detailed in Lipsey (1970) in which the welfare function is \( U = x^\alpha y^\beta z^\gamma \) and the three goods \( X, Y, \) and \( Z \), are obtained at constant costs. In this case, even though the consumption of none of the goods can be reduced to zero by any finite tax, the relevant reaction curve has a point of inflection in the range of a positive tax.

For a second case, consider the additive utility function introduced in the previous section. The calculations are shown in the appendix and the relevant results are summarised in Table 2. (Again the marginal utilities are assumed to decline at the same rate for the three commodities, i.e., \( \alpha = \beta = \gamma \).) In all cases, the inflection point occurs at quite a high rate of tax on \( Z \) relative to the given tax on \( Y \). Thus all that this case illustrates is the existence of a point of inflection where the relation curve changes from being convex. Whether or not it is at a low enough setting of the source in question to be interesting in other cases is an open question. The cases considered here are, of course, highly restrictive examples in which only part of the economy is modelled and there are only two sources in operation, one that is parametric and one that is open to policy manipulation.

**TABLE 2**

**POINT OF INFLECTION IN THE REACTION CURVE AS THE TAX ON Z VARIES WITH THE TAX ON Y HELD CONSTANT**

<table>
<thead>
<tr>
<th>Variable Specification</th>
<th>Value of ( i )</th>
<th>Second-Best Value of ( t )</th>
<th>Value of ( t ) at Point of Inflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a=b=c=1.0 )</td>
<td>1.0</td>
<td>1.0</td>
<td>1.855</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>1.046</td>
<td>1.941</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1.261</td>
<td>2.349</td>
</tr>
<tr>
<td>( a=b=1.4, c=0.2 )</td>
<td>1.0</td>
<td>1.0</td>
<td>1.678</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>1.046</td>
<td>1.756</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1.261</td>
<td>2.117</td>
</tr>
</tbody>
</table>

Woo (2010: 290) argues that if Lipsey (2007) is right in arguing that the reaction curve may have a point of inflection (I argue that if it is convex around maximum point that occurs at the second-best optimum, it must have a point of inflection), this implies that the third-best policy of fulfilling first-best rules "...still applies for non-drastic deviation but not for drastic deviation." But the deviations where we can assume convexity of the reaction curve are not those around the first-best setting of the source in question. Instead, they are deviations around the
maximum value of the objective function, which occurs at the source's second-best setting. To know if the expected value of deviating marginally from either the first-best setting, assuming it to be established initially, or any other non-zero initial setting, one has to know the sign of the first and second derivatives of the objective function with respect to the setting of the source in question evaluated at its current value. To know the expected value of moving from the present non-zero setting of that source to the first-best setting, one has to know that value of the objective function evaluated at the value at both of those settings. If we had that kind of knowledge we might be just as likely to know the value of the second best setting itself.

Rather than upsetting Lipsey and Lancaster's conclusion that to increase overall community welfare by a piecemeal policy change in one sector of the economy one needs to have substantial context-specific empirical knowledge, Ng's analysis reinforces it, once the correct reaction curve is substituted for his invalid one — and Woo's observation does nothing to change this conclusion. The knowledge that we need to establish the sign, let alone the value, of the change in the objective function if we make a marginal change in the setting of the source in question, or alter it to the first-best setting, is context-specific and will necessarily vary from case to case.

IV. FURTHER OBSERVATIONS

"You cannot expect us to solve the whole economy before we decide whether or not to build a bridge or open up a harbour." Statements such as this are often made as criticisms of the relevance of second-best theory. The observation that such practical decisions are made by applied economists without solving for the welfare effects on the whole economy is often taken to support this view. In reply to such arguments, Lipsey (2007) reaffirms the message of Lipsey and Lancaster (1956) that the existence of many sources that prevent the attainment of a global first best implies the absence of any scientifically derived, non-context specific policy that ensures an increase in the welfare of the whole community. He then argues that many policy decisions that face applied economists concern much less general objective functions than a community welfare function. For examples: how to achieve a given amount of pollution for the minimum cost or where to build a bridge so as to minimise traffic congestion. These, and many other such examples, have the common aspect that the objective functions are much more focussed and context specific than that of maximizing the whole community's welfare.

One of the most enduring conflicts in economics is between those who believe that increasing the generality of theories is a valuable exercise and those who believe that to be valuable in most instances theories need to be tailored to specific contexts. Blaug (2009) argues cogently that the search for increasing generality is equivalent to a search for less and less applicability to particular issues. Hodgson (2002) points out that the issue of the applicability of highly general theories, and how much specificity is required for satisfactory theoretical explanations of actual phenomenon, was hotly debated by the 19th and early 20th century historical and the institutional economists in both Europe and the US. He goes on to argue (Hodgson, 2002: 22): “The domination of economics and sociology by general theorists, plus a minority of theoretical empiricists, has excluded this problem…. The methodological discussion of the general and the specific, of sameness and difference, is forgotten.” Most social scientists, he believes, assume without question that the more general a theory, the better it is. But this assumption ignores the probability that the more general a theory, the less empirical content it will have since, by ignoring the specific context in which many problems arise, they cannot be analysed in depth.
General equilibrium theories in the Arrow-Debreu tradition, and the first and second 'fundamental theorems' of welfare economics that are based on them, are very general, being devoid of any characteristics that would distinguish one market economy from another and one time period from another. Many who believe in the value of highly abstracted theories hold that these welfare theorems provide a useful defence of the free market system. In contrast, those who believe in the need for context specificity in useful theories argue that they are empty theorems that provide no useful guidance for any policy to be applied in any actual market-oriented economy. Ng's third-best theory is an example of the search for non-context specific guides to policy. In contrast, the general theory of second best is firmly on the side of those who argue in favour of more context specificity. Given the world of many sources in which we live, there are few, if any, general, scientifically derivable rules for developing piecemeal policies that apply to all market economies, in all circumstances, and at all times.

**MATH APPENDIX**

**Equilibrium Value of the Welfare Function**

Assume an additive utility function:

(1) \( U = aX^\alpha + bY^\beta + cZ^\gamma \quad 0 < \alpha = \beta = \gamma < 1 \)

The demand equilibrium conditions are that the marginal utilities be proportional to their prices. Cross multiplying this yields:

(2) \( giaX^{\alpha-1} = b\beta Y^{\beta-1} \)

(3) \( hja X^{\alpha-1} = c\gamma Z^{\gamma-1} \)

where \( g \) and \( h \) are the prices of \( Y \) and \( Z \) in terms of \( X \), \( i \) and \( j \) are one plus the rates of tax or subsidy on \( Y \) and \( Z \) respectively. The production possibilities are given by:

(4) \( R = X + gY + hZ \)

where \( R \) is the maximum production of \( X \) when all resources are dedicated to its production and opportunity costs are constant and equal to prices.

Using equations (2) and (3) to solve for each of the variables \( X \), \( Y \), and \( Z \) in terms of each of the other two gives equations: (5) - (10):

(5) \( X = (cZ^{\gamma-1}/hja)^{(1/\alpha-1)} \)

(6) \( Y = (gicZ^{\gamma-1}/hjb)^{(1/\beta-1)} \)

(7) \( X = (bY^{\beta-1}/gia)^{(1/\alpha-1)} \)

(8) \( Z = (hjbY^{\beta-1}/gic)^{(1/\gamma-1)} \)

(9) \( Y = (giaX^{\alpha-1}/b)^{(1/\beta-1)} \)

(10) \( Z = (hjaX^{\alpha-1}/c)^{(1/\gamma-1)} \)

Substituting equations (5) and (6) into (4):

---

8 See Blaug (2007) for a full discussion of these opposing views, which he argues reveal a fundamental schizophrenia among economists.
11 \[ R = \left( cZ^{\gamma-1}/hja \right)^{1/(\alpha-1)} + g\left( gicZ^{\gamma-1}/hjb \right)^{1/(\beta-1)} + hZ \]

Rearranging and solving for Z:

\[ Z = \frac{R}{\left( c/hja \right)^{1/(\alpha-1)} + g\left( gic/hjb \right)^{1/(\beta-1)} + h} \]

Substituting (7) and (8) into (4):

\[ R = \left( bY^{\beta-1}/gia \right)^{1/(\alpha-1)} + gY + h(hjbY^{\beta-1}/gic)^{1/(\gamma-1)} \]

Rearranging and solving for Y:

\[ Y = \frac{R}{\left( b/gia \right)^{1/(\alpha-1)} + g + h(hjb/gic)^{1/(\gamma-1)}} \]

Substituting (9) and (10) into (4):

\[ R = X + g\left( giaX^{\alpha-1}/b \right)^{1/(\beta-1)} + h(hjaX^{\alpha-1}/c)^{1/(\gamma-1)} \]

Rearranging and solving for X:

\[ X = \frac{R}{1 + g\left( gia/b \right)^{1/(\beta-1)} + h(hja/c)^{1/(\gamma-1)}} \]

Substituting (12), (14), and (16) into (1) gives the equilibrium value of the welfare function in terms of \( i \) and \( j \) and the parameters of the system:

\[ \text{Values in Table 1} \]

The values the two tables were computed by assuming that \( R = 100 \) and \( g = h = 1 \). The values in Table 1 were then obtained by setting the value of \( i \) and determining the value of \( U \) when \( j = 1 \) for each case. This value of \( U \) was then used to determine the value of \( j \) that yielded the same value of \( U \) on the other side of the maximum value. This was repeated for each value of \( i \) found in the table.

\[ \text{Values in Table 2} \]

The critical points in Table 2 were estimated using the first and second order ratios of discrete incremental increases in \( j \) of one ten-thousandth. The value of \( j \) that produced the maximum value of \( U \) was found when the first order ratio was equal to zero. The value of \( j \) that corresponds with the points of inflection in \( U \) was found when the second order ratio was equal to zero.

\[ \text{ACKNOWLEDGEMENTS} \]

I am indebted to Robin Boadway for critical advice and to my RA, Jessie Joice, for research, computational, and editorial assistance.
REFERENCES


Ng, Y.-K., 1987. 'Political Distortions' and the Relevance of Second and Third-Best Theories. Public Finance 42, 137-145.


FIGURE 1
THE OBJECTIVE FUNCTIONS RELATED TO THE SETTING OF ONE SOURCE
FOR THREE ALTERNATIVE SETTINGS OF OTHER SOURCES
FIGURE 2
ONE OBJECTIVE FUNCTION WHEN OTHER SOURCES
HAVE NON-ZERO SETTINGS