THREE ESSAYS ON THE PRICING OF CONVERTIBLE BONDS AND ON PUT-CALL PARITIES

by

Yuriy Zabolotnyuk
M.A. (Economics), Kyiv School of Economics (EERC), 2003
M.Sc. (Management), Kyiv University, 2003

THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

In the
Faculty of Business Administration

SIMON FRASER UNIVERSITY

Spring 2009

APPROVAL

Name: Yuriy Zabolotnyuk

Degree: Doctor of Philosophy

Title of Thesis: Three Essays on the Pricing of Convertible Bonds and on Put-call Parities

Examining Committee:

Chair:

David C. Thomas
Professor, Academic Chair, PhD Program, Faculty of Business Administration

Chris Veld
Senior Supervisor
Adjunct Professor, Faculty of Business Administration
Professor, University of Stirling

Rob Grauer
Supervisor
Endowed Professor, Faculty of Business Administration

Robert Jones
Supervisor
Professor, Department of Economics

Geoff Poitras
Supervisor
Professor, Faculty of Business Administration

Avi Bick
Internal Examiner
Associate Professor, Faculty of Business Administration

Sudipto Sarkar
External Examiner
Professor, McMaster University

Date Defended/Approved: February, 6 2009

Declaration of Partial Copyright Licence

The author, whose copyright is declared on the title page of this work, has granted to Simon Fraser University the right to lend this thesis, project or extended essay to users of the Simon Fraser University Library, and to make partial or single copies only for such users or in response to a request from the library of any other university, or other educational institution, on its own behalf or for one of its users.

The author has further granted permission to Simon Fraser University to keep or make a digital copy for use in its circulating collection (currently available to the public at the Branches & Collections' "Institutional Repository" link of the SFU Library website www.lib.sfu.ca), and, without changing the content, to translate the thesis/project or extended essays, if technically possible, to any medium or format for the purpose of preservation of the digital work.

The author has further agreed that permission for multiple copying of this work for scholarly purposes may be granted by either the author or the Dean of Graduate Studies.

It is understood that copying or publication of this work for financial gain shall not be allowed without the author's written permission.

Permission for public performance, or limited permission for private scholarly use, of any multimedia materials forming part of this work, may have been granted by the author. This information may be found on the separately catalogued multimedia material and in the signed Partial Copyright Licence.

While licensing SFU to permit the above uses, the author retains copyright in the thesis, project or extended essays, including the right to change the work for subsequent purposes, including editing and publishing the work in whole or in part, and licensing other parties, as the author may desire.

The original Partial Copyright Licence attesting to these terms, and signed by this author, may be found in the original bound copy of this work, retained in the Simon Fraser University Archive.

Simon Fraser University Library
Burnaby, BC, Canada

ABSTRACT

This thesis is a collection of three papers that have the valuation of derivative securities as a common theme. The first paper empirically compares three convertible bond valuation models. We use an innovative approach where all model parameters are estimated by the Marquardt (1963) algorithm using a subsample of convertible bond prices. The model parameters are then used for out-of-sample forecasts of convertible bond prices. The mean absolute deviation, which is calculated as the absolute difference between the model and the market price and expressed as a percentage of the market price, is 1.70% for the Ayache-Forsyth-Vetzal (2003) model, 1.74% for the Tsiveriotis-Fernandes (1998) model, and 2.12% for the Brennan-Schwartz (1980) model. For this and other measures of fit, the Ayache-Forsyth-Vetzal and the Tsiveriotis-Fernandes models outperform the Brennan-Schwartz model.

The second paper examines the market memory effect in convertible bond markets. More specifically, we look at the pricing of convertible bonds issued after the original issuer adversely redeemed previous issues without giving an opportunity for investors to benefit from bond value appreciation. We find evidence that the market underprices new convertible bond issues of firms that call their bonds early. We also find that the degree of market underpricing depends on whether the convertibles are more debt- or equity-like.

In the third paper, the European put-call parity condition is used to estimate the early exercise premium for American currency options traded on the Philadelphia Stock Exchange. Using a sample of 331 pairs of call and put options with the same exercise price and time to expiration, we find that the early exercise premium on average is 5.03% for put options and 4.60% for call options. The premia for both call and put options are strongly related to the interest rate differential and time to expiration. These results are important to consider when valuing American currency options using European option pricing models.

Keywords: convertible bonds, credit risk, currency options, put-call parity

Subject Terms: derivatives, fixed income

Присвячується моїм батькам

To my parents

ACKNOWLEDGEMENTS

I would like to express my gratitude to the faculty and my fellow students at the SFU Faculty of Business Administration and Department of Economics for helping in getting the right tools, and for inspiring and stimulating my research.

I cannot thank enough my senior supervisor, Dr. Chris Veld, for his expertise, support, and encouragement. His attitude toward research, his commitment and carefulness had and will have a significant impact on my academic endeavours.

I thank Dr. Robert Jones for bringing me into the world of advanced finance and for his continuous support; without the countless hours he spent with me this thesis would be impossible.

I am indebted to Dr. Rob Grauer for his invaluable advice and depth of knowledge provided in his classes. I want to thank Dr. Geoff Poitras for his encouragement and support in starting undertaking research at early stages. I thank Professor Peter Kennedy for being a good friend and such a phenomenal teacher.

Finally, heartfelt gratitude is owed to my parents and Oksana for their love, patience, and support.

TABLE OF CONTENTS

LIST OF TABLES

1: INTRODUCTION

This thesis is a collection of three papers with a common theme on the valuation of derivative securities.

In the first paper, we empirically compare three models for valuation of convertible bonds. These are the models of Ayache, Forsyth, and Vetzal (2003); the model of Brennan and Schwartz (1980); and the model of Tsiveriotis and Fernandes (1998). For empirical model comparison, we use a sample of 64 Canadian convertible bonds traded on the Toronto Stock Exchange. We judge the model's goodness-of-fit by looking at the difference between market prices and model-implied prices. Measures such as the mean absolute deviation (MAD), the mean deviation (MD), the root mean squared error (RMSE), and the percentage of pricing errors falling within a pre-specified range of market prices are used for comparing the models. Some models' parameters cannot be observed and have to be estimated. We divide the historical time series into two parts: we use the first "historical" subsample for calibration of the parameters; the second "forecast" subsample is used for predicting convertible bond prices by fixing parameters calibrated using the "historical" subsample.

Based on our estimation procedure, the MAD scores are 1.70%, 1.74%, and 2.12% for the Ayache-Forsyth-Vetzal, the Tsiveriotis-Fernandes, and the Brennan-Schwartz models respectively.

One of the drawbacks of using the Brennan-Schwartz model for predicting convertible bond prices is the need to estimate firm value. This estimation can be performed relatively simply for firms with simple capital structure consisting of stocks, single bond and single convertible bond. More sophisticated capital structures require simultaneous valuation of several securities, which complicates the estimation process. Considering this fact and the results presented above, we suggest using the Tsiveriotis-Fernandes and the Ayache-Forsyth-Vetzal models for valuation of convertible bonds of a wide range of issuers.

In the second paper, we study the optimal convertible bond call policy and its effects on convertible bond prices using a sample of Canadian and US bonds. Specifically, we examine how investors react to instances of optimal calls. An early call happens when the issuing firm calls convertible bond as soon as its value exceeds its call price. The early call policy is optimal from the point of view of the issuing firm but is disadvantageous for investors as they lose a chance to benefit from an increase in the convertible bond value triggered by an increase in the firm's stock price. We argue that investors, who observe firms calling their convertibles early, will put a downward pressure on the price of new convertible bonds issued by these firms.

To test whether investors punish early-call firms we calculate the differences between the theoretical (model) prices and the market prices. The models used in the study are the model of Ayache, Forsyth, and Vetzal (2003) and the model of Tsiveriotis and Fernandes (1998). Then, we perform regression analysis to find whether there is any systematic underpricing of the new convertible bonds issued by the early-call firms. Our sample consists of two Canadian and thirty-seven US convertible bonds issued by the firms that had multiple issues of convertible bonds and a history of calling their bonds. We find that new convertibles issued by the early-call firms are underpriced relative to the new convertibles issued by the firms that did not call their convertibles early. In addition, we argue that underpricing will also depend on the degree of convertible bond moneyness, which is the ratio of the underlying stock price (S) to the conversion price (K). Underpricing for new debt-like convertibles (with low S/K ratios) should be less severe as investors may not expect that the stock price will ever exceed the exercise price and thus will not look back at the firm's calling history. Chances of the stock price exceeding the conversion price for high S/K ratio (equity-like) convertible bonds are higher. Therefore, investors will pay more attention to the call history of the firms issuing equity-like convertibles. Our empirical findings support the hypothesis that underpricing is more severe for the new issues with high degree of moneyness.

In the third chapter, we estimate the early exercise premium (time value) of American currency options traded on the Philadelphia Stock Exchange using the put-call parity. The Black-Scholes model does not consider time value if used for the American currency options valuation. For accurate valuation of American currency options, the time value has to be estimated empirically. Importance of the correct estimation of currency options' time value is supported by the fact that American currency options are more likely to be exercised before maturity than other types of options (e.g. equity or index options). The prime candidates for early exercise are call options on high-interest rate currencies and put options on low-interest currencies. In our study, we use methodology of Zivney (1991). To the best of our knowledge, this is the first study to apply this methodology for estimating the early exercise premium of the currency options.

Using a sample of 331 pairs of call and put options, we find that the average early exercise premium (as a percentage of the option price) is 5.03% for put options and 4.60% for call options. Further regression analysis reveals that the early exercise premium is strongly related to the time to maturity and the interest rate differentials for both put and call options. These results are in line with the earlier results of Jorion and Stoughton (1989) who compute early exercise premia as differences between values of European currency options on the Chicago Board of Options Exchange and American currency options traded on the Philadelphia Stock Exchange. However, our results are statistically significant for both put and call options while they find significant results for call options only. Our findings are important to consider when valuing American currency options with European model.

2: EMPIRICAL COMPARISON OF CONVERTIBLE BOND VALUATION MODELS[1]

2.1 Introduction

Exchange-listed companies frequently attract capital by issuing convertible bonds. During the period from 1990 to 2003, there were globally more than seven-thousand issues of convertible bonds.[2] An important problem with convertible bonds is that they are difficult to value. This is caused by the fact that the exercise of the conversion right requires the bond to be redeemed in order to acquire the shares. For this reason, a conversion right is in fact a call option with a stochastic exercise price. In addition, most convertible bonds are callable in practice. This means that the issuing firm has the right to pay a specific amount, the call price, to redeem the bond before the maturity date. In some convertible bond contracts the *call notice period* is specified thereby requiring the firm to announce the calling date well before the redemption can be performed. Often, the call notice period is combined with a *soft call feature* where the bond can only be called if the underlying stock price stays above a certain pre-specified level for a pre-specified period. All these features complicate the valuation process for convertible bonds.

Despite the importance of convertible bond valuation for both academic and practical purposes, there is not much empirical literature on this topic. This

---

[1] This chapter is based on Zabolotnyuk, Jones, and Veld (2008).
[2] See Loncarski, ter Horst, and Veld (2006).

paper aims to fill this gap by empirically comparing three different convertible bond valuation models for a large sample of Canadian convertible bonds.

Convertible bonds are issued by corporate issuers and thus are subject to the possibility of default. There are two main approaches for valuing securities with default risk. The first approach, called *structural approach*, assumes that default is an endogenous event and bankruptcy happens when the value of firm's assets reaches some low threshold level. This approach was pioneered by Merton (1974) who assumes that the firm value follows a stochastic diffusion process and default happens as soon as the firm value falls below the face value of the debt. However, as pointed out by Longstaff and Schwartz (1995), a default usually happens well before the firm depletes all of its assets. Valuation of the multiple debt issues in Merton (1974) is subject to strict absolute priority where any senior debt has to be valued before any subordinated debt is considered. This creates additional computational difficulties for valuing defaultable debt of firms with multiple debt issues. Moreover, the credit spreads implied by the approach of Merton (1974) are much smaller than those observed in financial markets. In contrast, a default in the Longstaff and Schwartz (1995) model happens before the firm exhausts all of its assets and as soon as the firm value reaches some predefined level common for all issues of debt. The values of the credit spreads predicted by their model are comparable to the market observed spreads. The common default threshold for all securities allows valuation of multiple debt issues. Zhou (2001) developed a structural approach model where both diffusion and jumps are allowed in the asset value process. Addition of a jump process allows the possibility of instantaneous default caused by a sudden drop in firm value.

In the structural approach debt is viewed as an option on the value of firm assets, and an option embedded in the convertible bond can be viewed as a compound option on the value of firm assets. Therefore, the Black and Scholes (1973) methodology can be used for valuing convertible bonds. The models of Brennan and Schwartz (1977) and Ingersoll (1977) apply the structural approach for valuation of convertible bonds. In these models, the interest rates are assumed to be non-stochastic. Brennan and Schwartz (1980) correct this by incorporating stochastic interest rates. However, they conclude that for a reasonable range of interest rates, the errors from the non-stochastic interest rate model are small and for practical purposes, it is preferable to use the simpler model with non-stochastic rates.

Nyborg (1996) argues that one of the main problems inherent in the implementation of structural form models is that the convertible bond value is assumed to be a function of the firm value, a variable not directly observable.[3] To circumvent this problem, some authors model the price of convertible bonds as a function of the stock price, a variable directly observable in the market. The model of McConnell and Schwartz (1986) is such an example. In this model, they price Liquid Yield Option Notes (LYONs), which are zero coupon convertible

[3] Goldstein et al. (2001) developed a model where EBIT is used as a state variable instead of the firm value. However, considering that EBIT is reported non-frequently, its everyday observation or estimation may be problematic as well. To the best of our knowledge, there exist no *convertible bond* valuation models that use EBIT as a state variable.

bonds callable by the issuer and putable by the bondholder. However, one of the main drawbacks of their model is the absence of a bankruptcy feature.

The second approach used for the valuation of the defaultable corporate obligations is the reduced-form approach. In contrast to the structural approach, where default is an endogenous event tied to the firm's value and capital structure , in the reduced-form models default is an exogenous event. In the reduced-form approach the default risk of a firm and its value are not explicitly related; at any point in time the probability of default is defined by a Poisson arrival process and is described by a hazard function. The application of this approach for valuing defaultable non-convertible bonds can be found in the models of Jarrow and Turnbull (1995), Duffie and Singleton (1999), and Madan and Unal (2000).4 The attractiveness of this approach is that the convertible bond value can be modelled as a function of the stock price. The models of Tsiveriotis and Fernandes (1998), Takahashi, Kobayashi, and Nakagawa (2001), and Ayache, Forsyth, and Vetzal (2003) use the reduced-form approach for valuing convertible bonds. [5]

In contrast to the extensive theoretical literature on convertible bond pricing, there is very little empirical literature on this topic. Some researchers use

---

[4] See the paper of Andersen and Buffum (2004) for an excellent discussion of problems associated with the calibration and numerical implementation of reduced-form convertible bond valuation models.

[5] Another examples are the models of Cheung and Nelken (1994), Ho and Pfeffer (1996), and Hung and Wang (2002). The Cheung and Nelken (1994) and Ho and Pfeffer (1996) models are two-factor models, where the state variables are the stock price and the credit spread-adjusted interest rate. Cheung and Nelken (1994) assume no correlation between the interest rates and stock price changes; Ho and Pfeffer (1996) assume that the correlation is constant across periods. Hung and Wang (2002) add one more state variable, the default event, in addition to the stock price and interest rate.

market data to verify the degree of accuracy of their own models. Cheung and Nelken (1994) and Hung and Wang (2002) use market data on single convertible bonds to verify their models. Ho and Pfeffer (1996) use market data on seven convertible bonds to perform a sensitivity analysis of their two-factor multinomial model. King (1986) uses a sample of 103 US convertible bonds and finds that the average predicted prices of the Brennan-Schwartz model with non-stochastic interest rate are not significantly different from the mean market prices. Carayannopoulos (1996) uses the stochastic interest rate variant of the Brennan-Schwartz model. For a sample of 30 US convertible bonds, he finds a significant overpricing of deep-in-the-money convertible bonds. Takahashi, Kobayashi, and Nakagawa (2001) use data on four Japanese convertible bonds to compare their model to the models of Tsiveriotis-Fernandes (1998), Cheung and Nelken (1994), and Goldman-Sachs (1994). Based on the mean absolute deviation, which is calculated as the difference between the model and the market price expressed as a percentage of the market price, they conclude that their model produces the best predictions of convertible bond prices. Amman, Kind, and Wilde (2003) use 18 months of daily French market data and the Tsiveriotis-Fernandes model to find that, on average, market prices of French convertibles are 3% lower than the model-predicted prices.

Most companies that issue convertible bonds don't have straight bonds outstanding. For this reason, we cannot use straight bonds' parameters such as credit rating when calculating model prices for convertible bonds. Moreover, other model parameters, such as the underlying state variable volatility, the dividend yield, and the default rate are often not directly observable. Therefore, we use an innovative technique that allows for the calculation of model prices even when the values of the parameters are not observable. We divide our sample into two parts: a historical sample and a forecasting sample. Instead of using the values of the parameters inferred from the plain debt data or underlying stock market data, we use the information contained in the historical *convertible* bond prices to estimate the necessary parameters. This approach allows for forecasting the convertible bond prices using the convertible bond price series only. The data from the historical sample are used to calibrate model parameters. We then calculate model prices for the forecasting period and compare these to market prices.

The estimation procedure becomes very complicated if all features of convertible bond contracts are taken into account. Lau and Kwok (2004) show that if the soft-call feature is accommodated the dimension of the valuation procedure increases rapidly. They also show that the calling period essentially increases the optimal call price at which the issuers should call the bond. This effective call price can be viewed as the original call price multiplied by a call price adjustment, $(1+\pi)$, in which π is the excess calling cost, defined as the difference between the critical call price and the published call price. In our study we only account for call and call notice features and we do not price the soft-call feature. The call notice period is featured by including and calibrating the excess calling cost parameter. Therefore, the results of this study are subject to the fact that some convertible bond contract features are ignored.[6]

First, we estimate the Brennan-Schwartz model. However, since it is a structural form model, it requires a simultaneous estimation of all the other defaultable assets. This fact complicates estimation of the model. To eliminate this complication, we use a subsample of firms with a simple capital structure for the estimation of the Brennan-Schwartz model. A simple capital structure is defined as a capital structure that only consists of equity, risk-free straight debt, and convertible debt. The assumption of a simple capital structure substantially simplifies the estimation process. However, it also reduces the domain of applicability for this model.

In order to be able to value the convertible bonds of firms with a non-simple capital structure we need to rely on the reduced-form approach as it is not dependent on the capital structure of a firm. For this reason we use two other convertible bond valuation models. The first is the Tsiveriotis-Fernandes model, from now on to be referred to as the TF-model. The second is the model of Ayache, Forsyth, and Vetzal, from now to be referred to as the AFV-model.

We find that the mean absolute deviation of the model price from the market price, expressed as a percentage of the market price, is the smallest for the AFV-model (1.70%). This deviation is 1.74% for the TF-model, and 2.12% for the BS-model. The BS-model shows the smallest range of pricing errors. For the

---

[6] Notice that none of the bonds in our sample are putable, therefore this is not a problem for our estimation.

TF- and AFV-models, we find a negative relationship between moneyness and absolute values of the pricing errors while this relationship is positive for the BS-model. This means that the AFV- and TF-models misprice convertibles that have in-the-money conversion options less than convertibles with conversion options that are at-the-money or out-of-the-money. The negative relationship is also found between the moneyness and the actual value of the error terms for the BS-model meaning that the model tends to overprice the bonds that are deeper in-the-money. We find a positive relationship for the reduced-form models between absolute values of pricing errors and the volatility of the returns of the underlying stocks. The effect of volatility on the absolute deviations in the BS-model is statistically insignificant.

The remainder of this chapter is structured as follows. In Section 2.2, we present different convertible bond valuation models. Section 2.3 includes the data description. Section 2.4 is devoted to the estimation of the parameters. The results of the estimation are presented in Section 2.5. The chapter concludes with Section 2.6 where the summary and conclusions are presented.

2.2 Convertible bond valuation models

2.2.1 Model selection criteria

A comparison of valuation models is possible if all the model input variables are either directly observable or can be estimated. Structural models that use non-directly observable variables, such as firm value, are very difficult to estimate. Their estimation becomes easier if a simple capital structure of the firm is assumed. Reduced-form models, on the other hand, use directly observable market variables and are much easier to estimate. This reason explains their popularity among practitioners. The selection of models that are used for the comparison in our study is based on popularity with practitioners as well as their sound theoretical underpinnings. In this paper we compare the models of Brennan and Schwartz (BS) (1980), Tsiveriotis and Fernandes (TF) (1998), and Ayache, Forsyth, and Vetzal (AFV) (2003). Even though the Brennan and Schwartz model (1980) is the most well known theoretical model for valuation of convertible bonds, in their overview of convertible bond valuation models, Grimwood and Hodges (2002) argue that the approach of Tsiveriotis-Fernandes (1998) is the most popular among practitioners. The model of Ayache, Forsyth, and Vetzal (2003) is selected for its thorough treatment of the firm's stock price and bond payouts in the event of default.

2.2.2 Models used in the study

2.2.2.1 The BS-model

Brennan and Schwartz (1977, 1980) develop a structural type approach for valuing convertibles where the convertible bond value is modelled in terms of the firm value. The main assumptions of their approach are: (a) the firm value W

is the central state variable, the risk-adjusted return on which is the risk-free rate at each instant; (b) the dilution effect resulting from conversion must be handled consistently; (c) the effect of all cash flows on the evolution of the firm value must be accounted for; (d) assets must be sufficient to fund all assumed recoveries in default; and (e) the share price process is endogenously determined by all this.

The firm value W is assumed to be governed by the stochastic process $dW = (rW - D(W) - rB_s - cB_c)dt + \sigma W dW$ in which r is the instantaneous risk-free interest rate, B_s is the par value of senior straight bonds outstanding, B_c is the par value of convertible subordinated bonds outstanding, c is the annualized continuous coupon rate on convertible bonds, $D(W) = d\max\{0, W - B_s - B_c\}$ is the total continuous dividend payout on shares, d is the constant dividend yield on the book value of equity, and σ is the constant proportional volatility of the asset value. The stochastic process above is applied when the firm is not in default. Following Brennan and Schwartz (1980)[7] we further assume a constant default boundary prior to convertible debt maturity at the firm asset level $\underline{W} \equiv B_s + \rho B_c$, where ρ denotes the convertible bond early recovery rate as a fraction of par. This early default boundary implies W is just sufficient to fund full recovery on the

[7] The original Brennan-Schwartz (1977) paper does not permit early default, while the 1980 paper has an early default boundary.

senior straight debt, partial recovery on the convertibles, and zero recovery on equity at the time of default.[8]

The assumptions described earlier imply, due to standard arbitrage arguments, that the value of the entire convertible bond issue, *V*, has to follow the partial differential equation (PDE)

$$\frac{1}{2}V_{WW}\sigma^2W^2 + (rW - D(W) - rB_S - rB_C)V_W + cB_C + V_t - rV = 0$$

where the subscripts indicate partial differentiation.

Boundary conditions characterize the convertible bond value at maturity, at early default point $\underline{W}$, at rational early conversion level W^*, and at rational early call-level *W* if applicable. These conditions are as follows:

$$V(W,T) = \begin{cases} \max\{B_c, C(W)\} & \text{for } W \geq B_c + B_s \\ \max\{0, W\text{-}B_s\} & \text{for } W < B_c + B_s \end{cases} \quad \text{(maturity)}$$

$$V(W,t) \geq C(W) \text{ for all } W, t \quad \text{(voluntary conversion)}$$

$$V(\underline{W},t) = \rho B_c \quad \text{(early default)}$$

$$V(W,t) = \max\{P_c, C(W)\} \text{ for } V \geq (1+\pi)P_c \text{ for all } t \geq T_c \text{ (early call)}$$

---

[8] Note that we treat coupons as being continuously paid for purposes of the evolution of *W*. If the coupons are periodic, this is equivalent to saying that the accruing interest is continuously and irrevocably paid into a segregated escrow account which pays full accrued interest to bondholders in the event of default. In the actual computation, coupons are paid at discrete intervals and (linearly) accrued interest is assumed to be paid to the bondholder at the instant of conversion, call, or default.

In the above, T is the maturity date of the convertible bond, P_c is the early call price of the bond, T_c is the first call date of the bond, $C(W)$ is the conversion value of bond given W, and π is the excess value required for an early call.[9] The excess value π required for early call is introduced to accomodate the presence of a call notice period.[10] Note that upon notice of early call, bondholders exercise the conversion right if that gives a higher value. The BS-model is the only model that allows for the possibility of share dilution after the conversion of convertible bonds. The other models in our study ignore this feature by assuming no dilution after conversion. Solving the above PDE subject to the boundary conditions gives the theoretical value of the entire outstanding convertible bond issue.

There is little guidance on the empirical implementation of the original Brennan-Schwartz (1977, 1980) models. In our study we have filled in the missing elements of the BS-model as simple as possible by: (a) postulating fixed but unobservable senior claims B_s (bonds, bank loans, amounts due to government and suppliers, etc.); (b) specifying dividend flows in a way that they are non-negative, yet embody likely covenants in senior and subordinated debt; (c) selecting an early default barrier consistent with the assumed risk-free nature of senior debt and assumed partial recovery rate on subordinated debt; (d) assuming a floating coupon rate equal to r on senior debt so that its market value

---

[9] The bond conversion value $C(W)$ is defined as follows. Let K denote the exercise share price for the convertible bond, and N_0 the number of shares outstanding prior to exercise, and z as the fraction of firm assets, net of senior debt, owned by bondholders after conversion.

$q \equiv \frac{B_c / K}{N_0}$ $z \equiv \frac{q}{1+q}$. Then, $C(W) = (W - B_s)z$

[10] This might reflect either inattention on the part of management, but could also reflect issue costs associated with the replacement debt or credit market access concerns.

is constant over time; and (e) assuming agents expect that the risk-free rate $r(t)$ will follow a deterministic path implied by the term structure of Treasury rates at each time t. Note that the share price process implicit in all this cannot exhibit constant proportional volatility as typically assumed in competing models. Therefore, the stock price volatility will increase as assets fall closer to the default point. Similarly, the proportional dividend yield on the shares varies with the level of assets.

As mentioned before, determining the convertible bond value is a problematic task if the firm has a complex capital structure. The value of the convertible bonds has to be determined simultaneously with the values of all senior claims. In our study, we only estimate values for the BS-model for companies that have a simple capital structure. This is defined as a capital structure that only consists of equity, straight debt, and convertible debt. This means that we have to exclude companies that have preferred equity, warrants, and/or different types of subordinated debt in their capital structure. In this approach, the straight debt is assumed to be risk-free. The value of the firm is simply a sum of the values of the equity, convertible debt, and straight debt. This assumption eliminates the necessity of simultaneous valuation of convertibles and senior claims.

The list of observable constant parameters for the model is: c, T, T_c, P_c, K, and N_0. We further observe at each time t the then current risk-free forward rate structure $r(\tau)$, $\tau \geq t$, and the combined market value of shares plus convertibles (identical to firm assets net of senior claims) $W(t)–B_S$. The list of unobserved constant parameters, to be either specified or estimated, is: *d*, σ, ρ, π, and B_S. These parameter estimates are chosen via an extended Marquardt algorithm to minimize the sum-squared deviations of theoretical quotes from market quotes for the convertibles. This estimation procedure is, in effect, non-linear least squares, since the predicted quotes are non-linear functions of the parameters being estimated.

2.2.2.2 The TF-model

The TF-model, which is based on the methodology of Jarrow and Turnbull (1995), discriminates between two parts of the convertible bond: the bond-like or cash-only part (COCB) and the equity-like part. The COCB is entitled to all cash payments and no equity flows that an optimally behaving owner of a convertible bond would receive. Therefore, the value of the convertible bond, denoted as *V*, is the sum of the COCB value, denoted as Σ, and the equity value, $(V-\Sigma)$. The stock price is assumed to follow the continuous time process $dS = rSdt + \delta Sdw$, where *r* is the risk-free interest rate, δ is the standard deviation of stock returns, and *w* is a Wiener process. Since the bond-like part is subject to default, the authors propose to discount it at a risky rate. The equity-like part is default-free and is discounted at the risk-free rate. Convertible bond valuation then becomes a system of two coupled PDEs:

For V: $\frac{1}{2}V_{SS}\sigma^2 V^2 + rSV_S + V_t - r(V-\Sigma) - (r + r_C)\Sigma + f(t) = 0$

For Σ: $\frac{1}{2}\Sigma_{SS}\sigma^2 V^2 + rS\Sigma_S + \Sigma_t - (r + r_C)\Sigma + f(t) = 0$

S is the underlying stock price, r_c is the credit spread reflecting the pay-off default risk; *f(t)* specifies different external cash flows for cash and equity (e.g. coupons or dividends).

In order to find the value of the convertible bond, it is necessary to solve the system of PDEs. At each point in time, the convertible bond prices should satisfy boundary conditions. At the maturity date the following conditions should hold: $\mathrm{V(S,T) = max(aS, F + Coupon)}$, $\Sigma\mathrm{(S,T) = max\ (F, 0)}$ where *a* is the conversion ratio, and *F* is the face value of the bond. At the conversion points the constraints are: $\mathrm{V(S, t) \geq aS}$; $\Sigma = 0$ if $\mathrm{V(S, t) \leq aS}$. Callability constraints: $\mathrm{V \leq max\ (Call\ Price, aS)}$; $\Sigma = 0$ if $V \geq \mathrm{Call\ Price}$.

The prices of the convertible bond are first calculated for different stock prices at the maturity date. In the “equity-like” region of underlying stock prices, where the value of the bond if converted is higher than the face value plus accrued coupons, the convertible bond price is equal to the conversion value. In this range the price of the convertible bond is discounted one period back at the risk-free rate, *r*. In the stock price range, where the total of face value and accrued coupon is higher than the conversion value, the convertible bond prices are discounted at the risky rate, r+r_c. Working one period back, the convertible prices are calculated and the points are found where the issuer can call the bond. The iterations continue until the initial date is reached.

2.2.2.3 The AFV-model

The AFV-model is a modified reduced-form model that assumes a Poisson default process. The authors of this model argue that the TF-model does not properly treat stock prices at default, because it does not stipulate what happens to the stock price of a distressed firm in the case of bankruptcy.

The AFV-model boils down to solving the following equation $MV - p\max(aS(1-\eta), \rho X) = 0$, where MV is defined as $MV \equiv -\frac{1}{2}V_{SS}\sigma^2 V^2 - (r + p\eta - d)SV_s - V_t + (r+p)V$, subject to the boundary condition $\mathrm{V} \le \max\text{ (Call Price, aS)}$, where S is the stock price, *p* is the probability of default, η is the proportional fall in the underlying stock value after a default occurs, *r* is the interest rate, *a* is the conversion ratio, *d* is the underlying stock dividend yield.

In their original paper (Ayache et al., 2003) the authors argue that *X* can take many forms, be it the face value of the bond or the pre-default market value of the bond. In our study we use the version of the AFV-model where we assume *X* to be equal to the bond's face value. Thus, *ρ* is the proportion of the bond face value that is recovered immediately after a default, and *d* is the continuous dividend yield on the underlying stock.

The AFV-model assumes the probability of default to be a decreasing function of stock price: $p(S) = p_0^{'}(\frac{S}{S_0})^{\alpha}$ The symbols S_0, $p_0^{'}$, and α represent constants for a given firm; $p_0^{'}$ is the probability of default when the stock price is

S_0. We can also group S_0 and $p_0^{'}$ together, introduce $\gamma = \frac{p_0^{'}}{S_0^{\alpha}}$, and therefore rewrite the hazard function as $p(S) = \gamma S^{\alpha}$

2.3 Data description

In our study, we use the sample of 97 actively traded Canadian convertible and exchangeable bonds listed on the Toronto Stock Exchange as of November,1 2005.[11] These bonds have different issue dates going back from April 1997 to October 2005. The maturity dates range from March 2007 to October 2015. From the original sample of 97 bonds, we exclude all exchangeable bonds as well as bonds traded in currencies other than the Canadian dollar. None of the bonds in our sample is putable. After screening for the issues that have price series and prospectuses available, as well as information on underlying stocks and financial statements with dividend information, the sample reduces to 64 issues. Fifty-seven bonds out of 64 were issued by income trusts. This number is surprising since convertible bonds tend to be issued by young and growing firms while the income trust structure is more suited to stable, mature firms (Halpern, 2004). Forty-three convertible bonds of these 57 bonds were issued by income trusts operating in oil and gas and real estate industries.

[11] The pricing data for the convertible bonds in our study comes from the Toronto Stock Exchange. All historical price series reflect the actual prices and are not derived by extrapolation. This allows us to avoid the common problems associated with the pricing of privately traded bonds.

Seventeen firms from this sample have a simple capital structure consisting of equity, straight debt, and convertible debt only. These bonds are used for estimating the BS-model. Detailed information concerning the issues used in the study can be found in Appendix A.[12]

Even though there should theoretically be no difference in the pricing of convertible bonds issued by income trusts and ordinary corporations, we briefly outline the essence of income trusts, as there are no corresponding securities in US market. Income trusts raise funds by issuing units of securities to the public; they purchase most of the equity and debt of successful businesses with the acquired funds. Operating businesses act as subsidiaries of income trusts which in turn distribute 70% to 95% of their cash flows to unit holders as cash distributions (Department of Finance, Canada, 2005). Since most of the earnings are distributed to unit holders, little or no funds are left for Research and Development and/or capital expenditures. Therefore, the stable and mature types of businesses are deemed most suitable for an income trust structure. The preferential tax treatment of publicly traded investment vehicles makes income trusts a widespread business structure in Canada in the late 1990s and early 2000s. However, in October 2006 the Canadian Government introduced the changes in the tax legislation that effectively eliminated the preferential tax treatment of the income trusts.[13]

[12] The assumption of a simple capital structure reduces the sample to 17 firms. This subsample is only used only for evaluation of the Brennan-Schwartz model. All other models are evaluated using the complete sample of 64 bonds.

[13] For more detailed discussion on the changes in the income trust taxation rules see the Department of Finance news release at http://www.fin.gc.ca/n06/06-061-eng.asp

For the purpose of finding the convertible bond valuation model which predicts the prices that are the closest to market prices, the data is divided into two sub-samples: historical and forecasting. The pricing data that we use in both historical and forecasting samples is weekly data with prices being observed every Wednesday (to ensure a high trading activity on the market). The models' parameters are calibrated using the data from the historical sub-sample. Then, weekly model prices are calculated for each convertible bond for the forecasting period using the calibrated parameters. The best model is selected based on the distance between the actual forecasting period market prices and the model-predicted prices for the forecast period.

The historical sub-sample for each bond starts with its issue date and ends at the start of the forecast period. The first date of the forecast period for each bond varies with the bond's issue date. The historical sub-period is defined in a way that it is not shorter than one year. If the bond was issued before January 1, 2004, then the forecast period begins on January 1, 2005. If the bond was issued between January 1, 2004 and July 1, 2004, the starting date is set to be July 1, 2005. If the bond was issued after July 1, 2004, the starting date of the forecast period is set to be January 1, 2006. Thus, the bonds with the earlier issue dates have longer forecast sub-samples. This choice of starting date is stipulated by the need of a large enough historical sub-sample for the estimation of the model parameters.[14] The end of the forecasting period is fixed on April 28, 2006. We equally weigh the errors from observations in the historical sub-sample because the approach we use assumes that parameters stay constant over time, thus enabling us to use the calibrated values of parameters for out-of-sample convertible bond price predictions.

Globe Investor Gold database provides data on historical bond prices. We take detailed information on each issue including coupon rates, maturity dates, and conversion conditions from the prospectuses available at the SEDAR (*System of Electronic Document and Archive Retrieval*) and Bloomberg databases. We use the Dominion Bond Rating Service data on existing debt and issuer ratings. The information on underlying stocks' dividends comes from companies' websites and from the Toronto Stock Exchange. The information on the number of stocks and convertible bonds outstanding is taken from the Canadian Financial Markets Research Centre database. The descriptive statistics of the convertible bond characteristics are presented in Table 2.1. As can be seen from Table 2.1, the shortest time to maturity for the bonds in the sample is 1.25 years while the longest time to maturity is almost 10 years; the average time to maturity is around 5 years. The average degree of moneyness of the bonds for the sample period, S/K, ranges from 0.30 to 2.46. The average bond is slightly in-the-money with a ratio of the underlying stock price to the exercise price of 1.06. The least volatile underling stock has an annual standard deviation of 13%, the

[14] Loncarski, ter Horst, and Veld (2007) find that during the first six months after their issue convertible bonds are underpriced. This provides a possibility for convertible arbitrage. To avoid pricing biases we perform an alternative pricing procedure that uses reduced historical samples where the first six months of data are dropped. The results of the reduced sample estimation are similar to those using the original data samples.

most volatile – 60%. The average volatility of the underlying stocks is 25%. The average coupon rate for the convertible bonds in the sample is 7.2%.

Table 2.1 Convertible Bonds Sample Summary Statistics

This table presents the descriptive statistics of the sample of the 64 convertible bonds used in our study on the comparison of convertible bond pricing models. All the bonds in the sample are traded on the Toronto Stock Exchange. VOLAT refers to the annualized historical standard deviation of the returns on the underlying stock; TMAT refers to the remaining time to maturity (in years) of the bonds as of December 1, 2005. COUPON refers to the convertible bond coupon rates. S/K refers to the ratio of average stock price during the forecast period to the conversion price.

	VOLAT	COUPON	TMAT	S/K
MIN	0.13	0.050	1.25	0.30
MAX	0.60	0.100	9.92	2.46
MEAN	0.25	0.072	4.92	1.06
MEDIAN	0.23	0.068	4.72	1.02
STDEV	0.08	0.012	2.40	0.35

Since in the evaluation of the models risk-free interest rates are used, we use the forward interest rates, derived from the Bank of Canada zero coupon bond curves, as a proxy. The forward rates used are 3-month forward rates for horizons from 3 months to 30 years. Zero-coupon bonds data is taken from the Bank of Canada.

Andersen and Buffum (2004) develop a method for calibrating time-varying convertible bond valuation model parameters. They show in their study that the naïve assumption of constant non time-varying model parameters may cause significant convertible bond price estimation errors. Their estimation approach relies on the presence and abundance of data on the underlying firm's equity options and straight debt pricing. In contrast to the estimation techniques that require use of the firms' straight corporate bonds and/or equity options for estimating model parameters, we employ the method that uses information inherent in the *convertible* bond prices for calibrating the parameters of the models. Many of the firms in our sample issue convertible debt instead of straight bonds in order to save the costs of interest in the absence of a high credit rating. These firms offer investors convertible bonds with lower coupons. In exchange for these lower coupons, the conversion feature is added. The majority of these firms do not have other publicly traded corporate bonds in their capital structure. Therefore, using a method for convertible valuation that does not hinge on the presence of the firm's straight corporate debt promises to be valuable. Unlike the approach of Andersen and Buffum (2004), our approach relies solely on the presence of convertible bond pricing data. Assuming constancy of model parameters allows us to calibrate all parameters simultaneously without referring to the pricing data on other financial instruments of the firm (many of which may be non-existent). Our assumption of market efficiency makes the use of large historical series of pricing data justifiable since a wealth of information about model parameters is contained in these prices. The information contained in the prices of the convertible bonds may be helpful to calibrate parameters of the models in the absence of other types of bonds for the firm. Moreover, by using information contained in the historical convertible price series, it is possible to estimate all other convertible bond parameters such as the underlying state variable (stock price or firm value) volatility, dividend yield, and diffusion processes parameters.[15]

[15] Stock price or firm value volatility can be calculated using the time-dependent historical

Using historical convertible prices, we employ the Marquardt algorithm (Marquardt, 1963) to search for the model parameters that minimize the squared sum of residuals between model-predicted prices and market prices. Later, we use these parameters for forecasting the convertible prices for our forecasting sub-sample. Initial values and boundaries for parameters are provided based on the assumption of the corresponding models.

Given the convertible valuation model, the Marquardt algorithm finds the theoretical convertible bond prices given the initial values for model parameters. In the next step, the algorithm changes the model parameters until the values that return the minimum squared deviations of the model prices from the observed market prices are found.[16] The data needed for the estimation of the parameters and prediction of the out-of-sample theoretical convertible bond prices consist of the convertibles' market prices, conversion prices, issue-, settlement- and maturity dates, coupon rates, number of coupons per year, market prices of the underlying stock, call schedules and call prices, and numbers of outstanding convertible bonds and stocks for the BS-model.

We limit the parameters' values to ensure that the Marquardt algorithm does not assume unrealistic or non-plausible parameter values:

---

standard deviation of the stock prices or firm value respectively. However, all models in our study show larger pricing errors on average if the volatility is calculated instead of being calibrated. This may result from the fact that market may view historical volatility as a poor proxy of the real volatility. These results are not reported and are available from the author upon request.

[16] This technique allows estimation not just of the parameters of the hazard function, but also of the characteristics of the convertible bonds such as volatility and dividend yields that are implied by the convertible prices.

- The volatility σ has a lower floor of 0 and an upper floor of 1 (representing a range for standard deviation between 0% and 100% of stock returns or firm value returns);
- The bond recovery fraction ρ is assumed to be between 0 and 1;
- The dividend yield (*d*) is assumed to be between 0 and 30%;
- The excess calling cost π to be between 0 and 30% of the call price;
- The debt value for the BS-model to be between zero and hundred times the face value of convertible debt;
- The credit spread to have a lower floor of zero.

Unfortunately, it is not possible to guarantee that the estimated parameters represent the global minimum for the algorithm minimization score without complicating the estimation algorithm too much. To ensure that the calibrated parameters are robust to the starting values we repeat the calibration for each firm several times trying out different starting values until we find a parameter set that represents the lowest value of squared residuals between the algorithm predicted prices and the market prices. The descriptive statistics of the model parameters calibrated are included in Table 2.2.

Table 2.2 Calibrated Model Parameters

This table reports the descriptive statistics of the model parameters calibrated using the Marquardt algorithm. Parameter d is the implied dividend yield of underlying stock; σ is the implied volatility of underlying stock, π is the excess calling cost as the proportion of the original call price used to proxy the presence of the early call notice period; p shows the price implied proportion of the bond value recovered in case of default. The model specific parameters are: for the AFV-model α and γ are parameters of hazard function p(S), where $p(S) = p_0^{'}(\frac{S}{S_0})^{\alpha}$ $\gamma = \frac{p_0^{'}}{S_0^{\alpha}}$; for the TF-model r_c is an implied credit spread (in decimal form); for the BS-model debt denotes the calibrated value of the senior debt in the capital structure of a firm (per 100 dollars of the convertible bonds face value).

			Ayache-Forsyth-Vetzal			
	α	γ	σ	d	π	ρ
MEAN	-2.10	0.09	0.33	0.21	0.09	0.03
MEDIAN	-1.64	0.05	0.29	0.23	0.10	0.01
MIN	-16.29	0.00	0.00	0.01	0.00	0.00
MAX	2.03	0.48	0.94	0.30	0.30	0.34
			Tsiveriotis-Fernandes			
		r_c	σ	d	π	ρ
MEAN		0.08	0.33	0.21	0.06	0.19
MEDIAN		0.08	0.29	0.21	0.01	0.00
MIN		0.00	0.00	0.02	0.00	0.00
MAX		0.20	0.93	0.30	0.68	1.00
			Brennan-Schwartz			
		Debt	σ	d	π	ρ
MEAN		4049	0.43	0.26	0.10	0.10
MEDIAN		2653	0.33	0.29	0.10	0.00
MIN		0	0.07	0.03	0.00	0.00
MAX		10000	1.00	0.30	0.30	0.51

It can be seen that the parameters' values calibrated by the models are in the range consistent with the real market observations. The implied underlying return stock volatility (annualized standard deviation) is 33% for the AFV- and TF-models and the implied volatility of firm value is 43% for the BS-model. The calibrated dividend yield is on average equal to 21% for AFV- and TF-models and 26% for the BS-model. These numbers are reasonable considering that the majority of the bonds in the sample are issued by income trusts that historically have high cash distribution yields. The price implied recovery rates are on average 3% for the AFV-model, 19% for the TF-model, and 10% for the BS-model.

The excess cost values range from zero to 68% of the original call price. The average values for the excess calling cost are 6% to 10% depending on the model and fall in the range reported by Lau and Kwok (2004) for a 30-60 day call notice period.

Note that, together with convertible bond data, data on straight bonds can be used to calibrate the parameters. In this case, the "conversion price" of straight debt has to be specified as some unrealistically large number and "call dates" have to be set after the maturity date. This parameter calibration approach may yield superior results compared to the exploitation of only straight bond data, because the former approach uses a wider set of market information. However, since most of the firms in our sample do not have straight corporate debt, we only use convertible bonds' prices for calibration of parameters in our study.

2.4 Estimation

To be able to predict the theoretical convertible bond prices, parameters such as the underlying state variable volatility, dividend yield, and the credit spread are needed. Many of these parameters are not directly observable. Parameters like credit spread may be observed from the straight debt of the same firms. However, as we mentioned before, some firms issue only convertible bonds. For this reason, we use an approach where we use historical *convertible* bond prices for estimating *all* necessary parameters of the models.

For the Brennan and Schwartz (1980) model, the dividend payout is assumed to be a fixed proportion of the amount by which the firm value exceeds the principal owed on the debt (this is the sum of straight debt and convertible debt).[17] For the other models, the dividend yield is simply estimated as a constant proportion of the price of the underlying stocks. To account for the call notice period feature we introduce the call price adjustment parameter π; multiplying the original call price specified in the bond prospectus by $1+\pi$ gives us the *effective call price* as in Lau and Kwok (2004). This parameter is unknown and is calibrated for all models together with the dividend yield (d), stock/firm value volatility (σ), credit spread for the TF-model (r_c), default bond value recovery fraction (ρ), and hazard process parameters (α, γ) from the historical convertible bond prices. The dividend yield is assumed to be a constant proportion of the book value of the shares. The other unobservable variable

[17] Note that this is consistent with the numerical illustration of Brennan and Schwartz (1980). In addition, it has an advantage compared to their approach since their dividend specification can lead to negative dividends, while ours does not. Furthermore, it forces dividend payouts to stop while the firm value is still sufficient to repay senior debt completely, justifying our treatment of straight debt as risk-free.

necessary for the estimation of the BS-model is the value of the firm. We calculate the firm value as the sum of the values of its common shares (market price times the amount of shares outstanding), convertible bonds (market value of the bonds times the number of convertibles outstanding), and unobserved value of senior straight debt. The value of the senior straight debt can be comprised of many types of debt (such as bonds, bank loans, amounts due to government and suppliers), market values for which are often hard to observe. Obtaining data on market or face values for all these liabilities can be difficult. Thus, value of all senior claims is assumed equal to a constant amount over time and is calibrated by the Marquardt algorithm to provide the smallest possible deviation of the model price from the market price in the historical subsample.[18] In the original BS-model, the authors assume that convertible bondholders recover 2/3rd of the face value in case of bankruptcy. This implies that bankruptcy of a firm occurs as soon as the firm value net of senior debt drops to two-thirds of the value of the outstanding convertible bonds. In our study, we allow this recovery proportion to be calibrated for all models.

We use the Crank-Nicolson (1947) finite difference algorithm for solving the corresponding partial differential equations and the Marquardt iterative procedure for finding the values of parameters that produce the smallest deviations of model prices from the market prices. The Crank-Nicholson algorithm assumes a time dimension step of one month (0.0833 years). The

[18] We also estimate the version of the BS-model where we assume senior straight debt to be zero. In that case the model pricing errors tend to be larger than if we assume a constant positive value of the senior debt.

upper bound on the spatial grid is set at the level 15 times higher than the current value of the state variable (stock price in TF- and AFV-models or firm value in BS-model). The space between the lower and upper bounds in spatial grid has one hundred nodes.

To calculate the convertible bonds values with the TF-model the following data are needed: bond issue date, trading date, risk-free rate, price of the underlying stock at the settlement date, maturity date, coupon rate, conversion ratio, call schedule, and the credit spread that reflects the credit rating of the issuer.[19] The only input needed for calculating prices with the TF-model that cannot be directly observed from the market is the credit spread. We use the average value of the credit spread for bonds that have the same credit ranking. Many of the bonds in our sample are issued by small firms, and therefore do not have credit ratings assigned. For companies that do not have credit ratings assigned, we assume a BBB rating. The average credit spreads are taken from the Canadian Corporate Bond Spread Charts published by RBC Capital Markets.

Prices are also calculated for the AFV-model. This model allows for a different behaviour of stock prices in case the firm defaults on its corporate debt. The partial default version assumes that the price of the underlying stock is partially affected by the firm's default on its bonds. The total default model assumes that the stock price jumps to zero when default takes place. We also assume that in case of a default convertible bondholders recover the fraction ρ of

[19] Note that this model can also be estimated using the "cbprice" function of the Fixed Income Toolbox in Matlab.

the bond face value. In this study, we use the total default version of the AFV-model.[20]

It should be mentioned that we could not capture all the conditions of convertible bond contracts in our estimation. Some of the bonds in our sample have "soft call" provisions. This means that the bond can only be called if the underlying stock price trades above some barrier level for some pre-specified amount of time. In the typical case, the bond can be called only if the underlying trades above the barrier for 20 out of the previous 30 trading days, which results in a very high dimensional valuation problem. In addition, there are sometimes other special conditions. For example, with regard to the first bond mentioned in Appendix A, the issuer has the option to pay (either upon redemption or at maturity and subject to a prior notice period) either in terms of cash or in terms of additional shares, the number of which is determined by "95% of the weighted average trading price...for the 20 consecutive trading days ending on the fifth trading day preceding the date fixed for redemption on the maturity date (page 3 of the prospectus)". Such special conditions are not captured in our valuation and our results should be read against this background.

2.5 Results

Our comparison of convertible bond pricing models is based on the scores that show the ability of the models to generate prices that are close to market

[20] The results of the estimation of the partial default AFV-model with the assumed value of parameter $\rho=0$ (no change in the stock price at default) are very similar to the total default ($\rho=0$) AFV-model. These results are available on request from the authors.

prices. We will base our decision on several scores. The first and the most important score is the “Mean Absolute Deviation” (MAD). It is calculated as:

$$\text{MAD} = \text{average}\left[\text{abs}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]\right]$$

The MAD measures the pricing ability the best since it takes into account deviations from market prices from both sides. The second indicator is the “Mean Deviation” (MD). This is calculated as the average deviation of the model price from the market price as a percentage of the convertible bond market price:

$$\text{MD} = \text{average}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]$$

The MD gives an idea of the average model over- or underpricing of the convertible bonds. Another indicator of model fit is the “Root of Mean Squared Error” (RMSE), which is calculated as:

$$\text{RMSE} = \sqrt{\text{average}\left[\frac{\text{Market Price - Model Price}}{\text{Market Price}}\right]^2}$$

The “Mean Absolute Deviation” and “Mean Deviation” scores assign the same weight to all errors. There is no additional penalty for the instances when the model price is far from the market price. The “Root of Mean Squared Error” score gives larger weights to large deviations.

We also calculate the percentage of forecasted model prices that fall within specific intervals around the market price as one of the measures of model pricing precision. We use ten, five and one-percent intervals around market prices for this purpose.

Table 2.3 provides rankings of the models based on the indicators mentioned above.

Table 2.3 Mispricing Scores

This table reports the mispricing scores for all three models. The sample for the Ayache, Forsyth, and Vetzal (AFV) (2003) and Tsiveriotis and Fernandes (TF) (1998) models consists of 64 Canadian convertible bonds traded at the Toronto Stock Exchange. The subsample for the Brennan and Schwartz (BS) (1980) model consists of 17 firms, which have a capital structure that only consists of equity, straight debt, and convertible debt. Errors are calculated as the convertible bond market prices minus the corresponding model predicted prices.

The mean deviation (MD) expressed in dollars refers to the average pricing error (in dollars) for the entire sample. The MD in the percentage form refers to the average error as a percentage of the convertible market price and is calculated as: $\text{MD}=\text{average}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]$

The Mean absolute deviation (MAD) refers to the average absolute error as a percentage of the convertible market price and is calculated as: $\text{MAD}=\text{average}\left[abs\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]\right]$. The Root of Mean Squared Error (RMSE) is calculated as the square root of mean squared error: $\text{MRSE}=\sqrt{\text{average}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]^2}$. The last three rows report the percentage of all predictions that fall within the defined range, i.e. "within 10%" means that the pricing error was less than 10 per cent of the market value.

	TF	BS	AFV
MD, $	-0.33	-1.23	-0.43
t-statistics	(-5.68)**	(-9.93)**	(-7.01)**
MD, %	-0.32	-1.18	-0.43
t-statistics	(-6.32)**	(-11.23)**	(-8.15)**
MAD, %	1.74	2.12	1.70
RMSE	2.71	3.11	2.78

	TF	BS	AFV
Percentage of errors within 10% of market price	99.03	97.99	98.92
Percentage of errors within 5% of market price	95.29	90.34	95.55
Percentage of errors within 1% of market price	44.74	37.05	45.16

Based on the results in Table 2.3 it can be concluded that the AFV-model and the TF-model show the best predictive power. The AFV-model shows the lowest values of mean absolute deviations. The value of the MAD is 1.70%. The AFV-model reports a slight overpricing of convertibles: the value of the MD is -0.43%. Almost ninety-nine percent of model errors are lower than 10% of bond market prices, 45.16% of errors are smaller than 1% of bond market value.

The second best model is the TF-model. This model has an average MAD of 1.74%.[21] On average, the TF-model overprices convertibles by 0.32%. This result is in line with that of Ammann, Kind, and Wilde (2003) who find that on average the TF-model overprices French convertible bonds by 3%. Slightly more than ninety-nine percent of the model predictions fall within 10% of market prices; 44.74% of the predicted pricing errors are less than one percent of the market price as compared to 45.16 per cent for the AFV-model.

The BS-model shows the largest MAD-score: 2.12%. The BS-model, on average, overprices the convertibles relative to the market by 1.18%; 97.99% and 37.05% of pricing errors are less than 10% and 1% of the market price respectively. The average MD is significantly different from zero for all three models. Therefore, we can reject the null hypothesis that the models have a mean zero pricing error.

[21] Since the sample used for estimating the BS-model is different from the sample for the other two models, we run robustness checks by comparing the results of the BS-model to the results of the other two models using the same subsample of firms. Even though the magnitudes of mispricing scores for the other two models are marginally different, the newly calculated scores do not change the rankings of the models.

Based on the Root of Mean Squared Error (RMSE) the most accurate model is the TF- model with an RMSE equal to 2.71. The second best is the AFV-model with RMSE of 2.78. The BS-model has the largest RMSE score of 3.11. Since the RMSE-score weights large pricing errors more heavily, the ranking of the models by the means of RMSE differs from the ranking based on the MD.

To see whether there exist any differences in the way the models price convertibles of income trusts versus convertibles of ordinary corporations we separate our sample into two parts: the first part contains only income trusts while the second part only contains ordinary corporations. The results of these two subsamples are reported in Panel A of Table 2.4.

Table 2.4 Mispricing Scores for Subsamples of Income Trusts and Non-trusts and Debt-like and Equity-like Convertible Bond Issues

This table reports separate mispricing scores for income trusts and non-trusts and debt-like and equity-like convertible bond issues for all three models. The debt-or equity-likeness is calculated using the Black-Scholes delta: $\Delta = e^{-dT}\left[\frac{\ln(\frac{S}{X}) + (r - d + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\right]$ where S is the current price of the underlying stock, K is the conversion price, *d* is the continuously compounded dividend yield, r is the continuously compounded yield on a selected risk-free bond, σ is the annualized stock return volatility, T is the maturity of the bond, and N(.) is the cumulative standard normal probability distribution. The debt-like convertibles are those with Δ less than 0.5 and the equity-like convertibles are those with Δ greater or equal to 0.5.

The sample for the Ayache, Forsyth, and Vetzal (AFV) (2003) and Tsiveriotis and Fernandes (TF) (1998) models consists of 64 Canadian convertible bonds traded at the Toronto Stock Exchange. Fifty-seven of these bonds are issued by income trusts and remaining seven are issued by non-income trusts (ordinary corporations). Fifty-one out of 57 bonds issued by income trusts are debt-like and 6 are equity-like; 3 out of 7 bonds issued by ordinary corporations are debt-like, and 4 are equity-like. The subsample for the Brennan and Schwartz (BS) (1980) model consists of 17 firms (5 of which are ordinary corporations and 12 are income trusts) that have a capital structure that only consists of equity, straight debt, and convertible debt. Out of the 17 issues used in the BS-model 15 are debt-like and 2 are equity-like. Errors are calculated as convertible bond market prices minus the corresponding model predicted prices.

The mean deviation (MD) expressed in dollars refers to the average pricing error (in dollars) for the entire sample. The MD in the percentage form refers to the average error as a percentage of the convertible market price and is calculated as: $\text{MD=average}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]$

The Mean absolute deviation (MAD) refers to the average absolute error as a percentage of the convertible market price and is calculated as: $\text{MAD=average}\left[abs\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]\right]$

The Root of Mean Squared Error (RMSE) is calculated as the square root of mean squared error: $MRSE = \sqrt{\text{average}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]^2}$

The last three rows report the percentage of all predictions that fall within the defined range, i.e. "within 10%" means that the pricing error was less than 10 per cent of the market value. The last three columns of this table show the p-values of t-tests testing the equality of the sample means. Panel A reports the p-values for equal sample means tests for convertible bonds issued by income trusts versus ordinary corporations. Panel B reports the p-values for the equal sample mean tests for equity-like and debt-like convertible bonds.

The number of observations in brackets refers to the number of weekly observations for the AFV and TF-models. The number of observations for the BS-model differs, since for this model we use only the issues from the firms with simple capital structure. For the BS-model, we have 460 weekly observations of bonds issued by income trusts and 285 observations of the convertibles issued by ordinary corporations. Of these, 641 weekly observations are for the debt-like bonds and 104 weekly observations are for equity-like bond. Debt- and equity-likeness is defined on a weekly basis.

* and ** denote significance at the 5% and 1% levels, respectively

	TF	BS	AFV	TF	BS	AFV	TF	BS	AFV
			Panel A				P-values of tests of equal means for income trust and non-income trusts		
	Income trusts (2230 obs)			Non-income trusts (352 obs)					
MD, $	-0.43	-1.51	-0.24	0.27	-0.79	-1.60	0.003	0.005	0.000
t-statistics	(-8.29)**	(-10.81)**	(-4.36)**	(3.27)**	(-7.52)**	(-6.51)**			
MD, %	-0.41	-1.39	-0.25	0.19	-0.84	-1.50	0.004	0.009	0.000
t-statistics	(-8.97)**	(-11.20)**	(-5.56)**	(2.62)**	(-10.82)**	(-6.74)**			
MAD, %	1.57	2.50	1.57	2.76	1.66	2.50			

	TF	BS	AFV	TF	BS	AFV	TF	BS	AFV
RMSE	2.45	3.65	2.39	3.90	2.29	4.49			
Percentage of errors within 10% of market price	99.25	96.39	99.32	97.8	100	96.4			
Percentage of errors within 5% of market price	96.84	86.06	96.4	86.23	95.09	90.3			
Percentage of errors within 1% of market price	48.11	31.25	46.56	25.07	44.91	36.57			

Panel B

	Debt-like issues (2067 obs)			Equity-like issues (515 obs)			P-values of tests of equal means for debt-like and equity-like issues		
MD, $	-0.24	-1.45	-0.31	-0.66	0.12	-0.88	0.014	0.000	0.008
t-statistics	(-3.73)**	(-11.23)**	(-5.29)**	(-4.17)**	(0.34)	(-4.28)**			
MD, %	-0.23	-1.35	-0.33	-0.69	-0.17	-0.80	0.001	0.000	0.007
t-statistics	(-3.83)**	(-11.44)**	(-6.22)**	(-5.47)**	(-0.97)	(-4.29)**			
MAD, %	1.75	2.27	1.64	1.71	1.22	1.92			
RMSE	2.64	3.27	2.47	2.96	1.83	3.80			

	TF	BS	AFV	TF	BS	AFV	TF	BS	AFV
Percentage of errors within 10% of market price	99.18	97.66	99.23	98.46	100	97.67			
Percentage of errors within 5% of market price	95.97	89.24	96.23	92.71	97.12	92.82			
Percentage of errors within 1% of market price	42.35	33.7	44.12	53.74	57.69	49.32			

The average MD-score for the TF-model is -0.41% and 0.19% for trusts and non-trusts respectively. The AFV-model predicts errors of -0.25% and -1.50% and the BS-model comes to errors of -1.39% and -0.84% for trusts and non-trusts respectively. The sample means tests reject the null hypothesis of equal means at the 1% significance level. Therefore, we can conclude that the models price convertibles of income trusts and of ordinary corporations differently. Loncarski, ter Horst, and Veld (2008), who study motives for the issuance of convertible bonds in Canada, find that convertible bonds issued by income trusts are more debt-like and that convertible bonds issued by ordinary corporations are more equity-like. This difference may explain the fact that convertibles issued by income trusts are priced differently from convertibles issued by ordinary corporations. In order to test whether this is really the case, we follow the approach of Loncarski et al. (2008) and we divide the convertible bonds in our study into equity-like and debt-like convertibles. Their distinction is based on the delta measure that can be derived from the model of Black and Scholes (1973), corrected for continuous dividend payments. Convertible bonds with a delta higher than 0.5 are defined as equity-like and convertibles with a delta lower than 0.5 are defined as debt-like. Out of the 57 issues issued by income trusts in our sample, 51 issues were debt-like and 6 issues were equity-like. However, the percentage of equity-like issues among the convertibles issued by ordinary corporations was higher: 4 out of 7 convertibles issued by these corporations were equity-like. We calculate the mispricing of the debt-like and the equity-like convertibles and we test whether there is a difference in the mispricing. These results are included in Panel B of Table 2.4. In this panel, we can see that the MD for the debt-like convertibles is -0.23% for the TF-model and the score for the equity-like issues is -0.69% for the same model. The sample means tests reject the null hypothesis that the errors for the equity and debt-like issues come from the same distribution at the 5% significance level for this model. We find the same conclusion for the other two models. Based on these results it is likely that the difference in pricing between the trusts and the non-trust convertibles is driven by differences in equity- and debt-likeness between these two subsamples.

Table 2.5 provides the descriptive statistics for the mean deviations of the models' prices from the market prices both as the percentage of the market and the model prices.

Table 2.5 Descriptive Statistics of Models' Over/Underpricing Errors

This table provides the descriptive statistics for the deviations of the observed market prices from the model prices expressed as a percentage of market prices. The MD refers to the average error as a percentage of the convertible market price and is calculated as:

$$\text{MD}=\text{average}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]$$

The sample consists of 64 Canadian convertible bonds traded on the Toronto Stock Exchange. The start of the sample period depends on the issue date of the bond and starts on either January 1, 2005, July 1, 2005 or January 1, 2006. The sample period ends on April 28, 2006. The TF-model refers to the Tsiveriotis-Fernandes (1998) model. The AFV-model refers to the model of Ayache, Forsyth, and Vetzal (2003). The BS-model refers to Brennan-Schwartz (1980) model. The *t*-statistics are for the test of the average error being equal to zero.

	TF	AFV	BS
Mean	-0.32	-0.43	-1.18
Median	-0.17	-0.21	-0.74
Minimum	-23.55	-39.25	-13.93
Maximum	9.65	25.68	5.83
Range	33.20	64.93	19.77
t-statistics	-6.32	-8.15	-11.23

The BS-model shows the smallest range of errors.[22] The model' pricing ranges from underpricing the convertible securities by 5.83% of the market price to an overpricing by 13.93%; this creates a range of 19.77%. The pricing errors for the TF-model range from a negative 23.55% to a positive 9.65%. For the AFV- model, the errors span a range of 64.93%.

We also study whether there are convertible bond characteristics that affect the mispricing in a systematic way. In order to check for any such regularities we perform a regression analysis in which the pricing errors are regressed on characteristics of the convertible securities. These characteristics include the moneyness, measured as the ratio of the current market price to the conversion price (S/K), the annual historically observed volatility of the underlying stock (VOLAT), the convertible bond coupon rate (COUPON), and the remaining time to maturity of the convertible security (TMAT). We assume that the pricing errors are identically and independently distributed with normal distribution that has an expected value of zero and a finite variance. Table 2.6 shows the results of the regressions where the dependent variable is the MAD. The results in this table help to find the variables that explain the 'precision' of the models.

[22] However, the BS-model only uses a subsample of firms, while the other models use the full sample of 64 firms. If the same subsample of 17 firms is used, the BS-model still shows the smallest range of errors followed by the TF-model and the AFV-model. Detailed results are available from the authors on request.

Table 2.6 Factors Affecting Mean Absolute Deviations

This table shows regression results of the models' Mean Absolute Deviations (MADs) on the ratio of the stock market price to the conversion price (S/K), the time to maturity of the convertible security (TMAT), the historically observed volatility of the underlying stock (VOLAT), and the convertible bond coupon rate. MADs are defined as the absolute difference between market the price and the model price divided by the market price. The sample consists of 64 Canadian convertible bonds traded on the Toronto Stock Exchange. The start of the sample period depends on the issue date of the bond and starts on January 1, 2005, July 1, 2005 or January 1, 2006. The sample period ends on April 28, 2006. The TF-model refers to the Tsiveriotis-Fernandes model. The AFV-model refers to Ayache-Vetzal-Forsyth (2003) model. The BS-model refers to the Brennan-Schwartz (1980) model. The sample for the TF- and AFV-models consists of 64 firms traded on the Toronto Stock Exchange. For the estimation of the BS-model, the subsample of 17 firms with the capital structure consisting of equity, straight debt, and convertible debt is used. The *t*-values of the coefficient estimates are reported in parentheses.

Model	Intercept	S/K	VOLAT	COUPON	TMAT	R^2
TF	1.42	-0.94	4.49	6.43	-0.04	0.10
	(4.33)**	(-3.83)**	(3.33)**	(3.55)**	(-1.44)	
AFV	1.18	-0.63	5.50	-0.27	-0.02	0.05
	(2.58)**	(-3.22)**	(2.51)**	(-0.36)	(-0.51)	
BS	-6.82	0.47	-0.19	1.20	0.01	0.44
	(-5.16)**	(2.70)**	(-0.18)	(6.71)**	(0.15)	

* and ** denote significance at the 5% and 1% levels, respectively

As can be seen from Table 2.6, the Mean Absolute Deviation statistically depends on the degree of the convertible bond moneyness for all three models in our study. On average, convertible securities that are deep in-the-money have smaller MADs than convertible securities that are at-the-money or out-of-the-money in the TF- and AFV-models while the result is the reverse for BS-model.

The underlying stock volatility has a positive effect on the MADs for the TF- and AFV-models. This implies that all models, except the BS-model, misprice convertible bonds with highly volatile underlying stocks more heavily. This may happen because in the BS-model the firm volatility instead of the stock volatility is used for our calculations.

The AFV-model and the BS-model have smaller absolute deviations for the bonds with longer times to maturity. However, the coefficients are not statistically significant. This result contrasts the result of King (1986) who finds heavier mispricing for the convertibles close to maturity in the BS-model.

The convertible bond coupon rates have a statistically significant positive effect on the size of the absolute values of the pricing errors for all except the AFV-model. Both the TF- and BS-models predict larger absolute deviations for the bonds with higher coupon rates. This effect is much more profound in the TF-model.

Table 2.7 reports the regression results where the dependent variable is the MD. The regression of the actual values of the pricing errors helps to find the variables that explain the 'direction' of mispricing, i.e. whether the convertibles are under- or overpriced.

Table 2.7 Factors Affecting Mean Deviations

This table shows regression results of the Mean Deviations (MDs) on the ratio of the stock market price to the conversion price (S/K), the time to maturity of the convertible security (TMAT), the historically observed volatility of the underlying stock (VOLAT), and the convertible bond coupon rate (COUPON). MDs are defined as the market price minus the model price divided by the market price. The sample consists of 64 Canadian convertible bonds traded on the Toronto Stock Exchange. The start of the sample period depends on the issue date of the bond and starts on January 1, 2005, July 1, 2005 or January 1, 2006. The sample period ends on April 28, 2006. The TF-model refers to the Tsiveriotis-Fernandes model. The AFV-model refers to Ayache-Vetzal-Forsyth (2003) model. The BS-model refers to the Brennan-Schwartz (1980) model. The sample for the TF- and AFV-models consists of 64 firms traded on the Toronto Stock Exchange. For the estimation of the BS-model, the subsample of 17 firms with the capital structure consisting of equity, straight debt, and convertible debt is used. The *t*-values of the coefficient estimates are reported in parentheses.

Model	Intercept	S/K	VOLAT	COUPON	TMAT	R^2
TF	-1.46	-0.06	1.67	9.70	0.01	0.03
	(-2.73)**	(-0.18)	(0.86)	(4.13)**	(0.12)	
AFV	-0.39	0.37	-3.65	1.02	0.08	0.02
	(-0.72)	(1.48)	(-1.48)	(1.01)	(1.49)	
BS	9.35	-0.70	0.21	-1.44	0.07	0.42
	(5.70)**	(-2.85)**	(0.13)	(-7.17)**	(0.80)	

* and ** denote significance at the 5% and 1% levels, respectively.

From Table 2.7 it can be seen that the BS-model tends to overprice the bonds that are deeper in-the-money. This result is in line with the results of King (1986), Carayannopoulos (1996), and Ammann, Kind, and Wilde (2003) who report a positive relation between overpricing and moneyness.

Volatility and time to maturity do not have any effect on the direction of pricing for the models in our study according to the results in Table VII. This result is in contrast to Ammann, Kind, and Wilde (2003) who find a larger underpricing for bonds with longer terms to maturity.

The convertible bond coupon rate has a positive and statistically significant effect on the MD for the TF-model. However, the BS-model on average overprices the convertible bonds with *higher* coupon rates; this effect is statistically insignificant for the AFV-model.

2.6 Summary and conclusions

In this paper, we compare the price prediction ability of one structural and two reduced-form convertible bond pricing models using actual market data on convertible bonds traded on the Toronto Stock Exchange. As opposed to other studies, we estimate all model parameters from *convertible* bond price series. This approach allows for the calculation of theoretical convertible bond prices even when the issuing firms have no straight debt outstanding or when parameters such as the credit spread or the dividend yield are not observable from market data. The final sample consists of 64 convertible bonds and spans the period from January 1, 2005 to April 28, 2006 for the two reduced-form models. For the Brennan and Schwartz (BS) model, we use the subsample of 17 firms that have a simple capital structure that consists of equity, straight debt, and convertible debt only.

The results of our study show that the Ayache, Forsyth, and Vetzal (AFV) model and the Tsiveriotis and Fernandes (TF) models perform similarly while outperforming the Brennan and Schwartz (BS) model based on the magnitudes of the pricing errors. On average, the mean absolute deviation, which is calculated as the absolute difference between the market and the model price expressed as a percentage of the market price, is 1.70% for the AFV-model, 1.74% for the TF-model, and 2.12% for the Brennan-Schwartz model. This means that the model that requires more possibilities (non-discrete discount rate, possibility of total or partial default etc.), performs the best in terms of mean absolute deviations. It should be noted that, although the AFV-model returns the lowest MAD, the TF-model requires fewer parameters. An advantage of the AFV- and TF-models over the BS-model is that the former models can be used for pricing convertibles issued by a broader range of firms including bonds of companies with complex capital structures.

3: THE OPTIMAL CALL POLICY FOR CONVERTIBLE BONDS: IS THERE A MARKET MEMORY EFFECT?[23]

3.1 Introduction

Companies frequently issue convertible bonds. These bonds, at the option of the holder, can be converted into shares of common stock of the issuing company. Most convertible bonds are callable. This means that the issuing company has the right to redeem the bond before its maturity date at a pre-specified call price. In fact, according to survey evidence of Graham and Harvey (2001), the ability to call or force conversion of convertible debt if and when the company needs it is the third most important reason for companies to issue convertible bonds.[24] An optimal call policy of the issuing firm would be to maximize the value of the underlying shares by minimizing the value of the firm's convertible bonds. Brennan and Schwartz (1977) and Ingersoll (1977b) demonstrate that the firm's optimal call policy is to call the convertible bond as soon as the value of the uncalled bond is equal to the call price. In contrast to

[23] This chapter is based on Veld and Zabolotnyuk (2008).

[24] The most important reasons for US companies to issue convertible debt are that convertibles are an inexpensive way to issue "delayed" common stock (58.11%) and that the stock of the issuing company is currently undervalued (50.68%). The percentage of firms that find the ability to call or force conversion to be important is 47.59% of the US firms that have issued convertible debt. Bancel and Mittoo (2004) find a similar result (54.76%) for European firms. Brounen, De Jong, and Koedijk (2006) also ask this question for a sample of European firms and they find strong differences between the UK (80%) on one side and the Netherlands (16.67%), France (12.50%), and Germany (20%) on the other side.

this theoretically optimal call policy, Ingersoll finds that the median company waits until the conversion value is 43.9% higher than the call price for the convertible bonds and 38.5% higher for convertible preferred stocks.[25] Many empirical studies have investigated this phenomenon and they have suggested different explanations including information signalling, the difference in taxation between interest and dividends on the personal income tax level, differences in corporate taxation, and the firm's (rational) response to suboptimal actions of bondholders.

In this chapter, we test a new explanation that is based on the textbook of Weston and Copeland (1986). They write: "Another possible explanation has been termed "fair play" by some and "market memory" by others. The idea that management imposes self-restrictions on call policy out of some notion of fair play has not received much support. A more plausible view is that on successive issues of convertibles, the market will form an expectation of the company's call policies by its past behaviour. Anecdotal evidence can be found in the financial press that companies have to pay a somewhat higher interest rate on a new issue of convertibles because of a history of calling convertibles as soon as their conversion value has risen modestly above their call prices. The market will penalize erratic behaviour; what appears optimal is the adoption by the company of a consistent call policy". We test whether the policy to call at a conversion value substantially above call price or at a conversion value close to call price will

[25] Schultz (1993) argues that in contrast to convertible bonds, firms issuing callable warrants behave optimally and call warrants as soon as possible.

be "priced out" by the market. This is not a matter of "fairness" or self-imposed restrictions. It is a matter of providing investors with a basis for forming expectations. This explanation is in line with Agarwal, Fung, Loon, and Naik (2008) who cite Woodson (2002, page 28) who notes that convertible issuers in Japan usually do not call their bonds to avoid upsetting their investors.

In this chapter, we find that investors have memory of the past firm call policy and that they will punish the issuers that call their convertibles too early. We use a sample of thirty-six US and two Canadian firms that had previously redeemed convertibles and that had new convertibles issued afterwards. We test whether investors push down the prices of newly issued securities following an optimal call of already issued convertibles. We postulate that firms that had previously redeemed convertibles rationally (early and thus unfavourably for bondholders) will be punished by investors and that the prices of their new convertibles will be, ceteris paribus, lower than the prices of new convertibles of firms that redeemed earlier issues after allowing the bond value to substantially exceed the call price.

The degree of underpricing, in turn, should also depend on whether the newly issued convertibles are more debt- or equity-like. If the conversion price of the new issues is set too high, investors will view such convertibles as ordinary bonds as they will not expect the conversion value to exceed the call price. As investors do not expect to benefit from increased conversion value of the debt-like convertible bonds, their pricing will be less affected by the past call policy of the issuing firm. On the other hand, if the conversion price is set at a relatively low level where it is highly probable that the stock price will exceed the conversion price, investors will scrutinize the past firm call policy as they will expect to benefit from the increase in the conversion value. Therefore, we expect that underpricing will be larger for the newly issued convertible bonds with low conversion price and high ratio of the stock price to the conversion price.

Our empirical analysis supports the proposition that the new convertibles of the early-call firms will be underpriced by the market. We reject the null hypothesis that there is no difference in pricing of the newly issued convertible bonds of the firms that pursue rational call policy (call early) and of the firms that call convertibles late. Our results show that investors will underprice the new convertibles issued by the early-call firms. Moreover, we find that the degree of underpricing is positively related to the bond's moneyness ratio.

The market memory effect may also "fade" over time. That is, one may expect that the longer the period between the date the previous issue was called and the new issue date, the smaller the downpricing would be as investors may forget about firm call policy over time. However, our empirical analysis shows no statistical effect of the time since the last call on the pricing errors.[26]

The remainder of the chapter is structured as follows. Section 3.2 reviews the optimal call policy. The data and methodology are discussed in Section 3.3. The empirical results are presented in Section 3.4. We summarize and conclude with Section 3.5.

---

[26] The results of regressions including time since the call event as a dependent variable were not included in the paper to avoid cluttering. They are available from author upon request.

3.2 Convertible bonds call policy

Firm's convertible bonds call policy can be viewed as a negative signal by the investors and can be "priced out" by the market. In this section, we review different theories that explain firms' decision to call convertible bonds either early or late.

Dunn and Eades (1989) explain the late calling of convertibles as a rational response of the firm to suboptimal actions of the bond holders. They argue that many investors are too slow in converting their bonds when the conversion rights get in–the-money. Therefore, the late call policy of the firm may be optimal if it expects that enough investors deviate from the perfect-market conversion strategy. The study of Dunn and Eades concentrates on convertible preferred shares as this ignores the effects that the call may have on the tax shield of the firm. The authors document the existence of irrational investors who delay the conversion of their convertibles for long periods regardless of the possibility of realizing higher dividends after conversion.

Asquith and Mullins (1991) argue that the positive difference between the after-tax interest payments and the dividend payments is the principal reason why firms do not call convertibles in time. At the same time, as soon as the conversion value exceeds the call value (which in turn usually exceeds the face value) and the dividend rate exceeds the coupon rate, the optimal policy of the investors would be to convert the bonds into stocks voluntarily. Nevertheless, because of taxation, the investor decision to convert and the decision of the firm to call are not symmetric. There may exist some bonds that issuers do not call and that bondholders do not convert. The authors analyze a sample of 208 bonds that have conversion values that are higher than call prices. This sample is filtered using the following three rules. First, they eliminate all call-protected bonds. Secondly, they take out all the bonds with a conversion value less than 120% of call price.[27] Finally, they screen for convertibles with after-tax coupon rates that are lower than the dividend rates of the underlying stocks. Only one bond that violated the optimal call policy was left after applying all three criteria. Asquith (1995) calculates the median call delay for all US convertible bonds issued in 1980-82 to be less than four months. If a safety premium is required to make sure that the stock price will not fall below the conversion price, the median delay is less than a month.

Sarkar (2003) explains both early and late convertible bond calls by introducing the possibility of a convertible bond default and by taking into account corporate taxes. He argues that, because of the difference in taxation between bonds and stocks, it will be optimal for some firms to call bonds either early or late. The reason is that when the convertible coupon rate is low compared to the dividend yield that is forgone by not converting the bonds into stocks, it is beneficial for the firm not to call the convertibles and hence save the difference.

[27] When a call notice period is specified, there is a probability that the stock price may go down during this period and the firm will have to pay more than the conversion value. Thus, managers tend to wait until the conversion value exceeds the call price by some amount. Ingersoll (1977) and Mikkelson (1981) were unsuccessful in quantifying this value. Brigham (1966) reports in his survey that 120% is the most frequent percentage reported by managers and investment banks.

However, if the coupon is high as compared to the dividend yield on the underlying stock, it is optimal for the firm to force conversion by calling early. This way, the firm saves money by avoiding paying high coupon interest and paying lower dividends instead. Therefore, there exists a threshold value of coupon rate, which defines the optimal call policy for the firm. If the convertible coupon rate is lower than the lower threshold, the optimal policy for the firm is to call convertibles early; if the coupon rate is above the upper threshold, it is optimal to call late. Only when the coupon rate is between the threshold levels, the optimal call policy would be consistent with the policy where the convertible should be called as soon as the conversion value equals to the call price. The author argues that any variable that changes the optimal coupon rate thresholds will affect the optimal call policy. Therefore, the bonds with high (low) call premium, dividend income, tax rate and interest rate, and low (high) coupon and volatility will be associated with late (early) calls.

Mikkelson (1981) finds that, on average, common stock returns are significantly negative around the convertible call date. This result is inconsistent with the propositions of Brennan and Schwartz (1977) and Ingersoll (1977) who argue that a forced conversion is beneficial for common stock holders since it transfers the value of the conversion option from debt holders to stock holders. Since a delayed call is an adverse event for common stock holders, it is reasonable to expect the stock price to decrease until the call is announced at which point the stock price should increase to reflect favourable news. Contrary to this logic, Mikkelson (1981) finds opposite results. His finding contradicts the manager's optimality decision since it does not maximize the shareholders value.

Harris and Raviv (1985) develop a sequential signalling model that explains both delayed calls of convertibles and significantly negative returns for the underlying stocks around the call date. They show that there exists an equilibrium in which managers truthfully signal their private information by calling convertibles only if the information they signal is unfavourable. Thus, a decision to call is rationally perceived by investors as a signal of unfavourable private information.

Instances of calling convertible bonds that force bondholders to convert may convey information about the future of the firm. Campbell, Ederington, and Vankudre (1991) report that in asymmetric information markets, observation of the dividend and after-tax interest payment amounts during the forced conversion elicits managers' vision of the perspectives of the firm. When the dividend payments after the conversion are higher than the after-tax coupon interest but less than the before-tax coupon interest, the call reveals an unanticipated reduction in dividends or earnings. A dividend less than the after-tax coupon reveals the convergence of exceptional firm growth to the industry's average.

Company managers are the agents of the common stock holders. The optimal convertible bond call policy should be the policy that maximizes the value of common stocks and therefore minimizes the value of convertible bonds. Therefore, another strand of literature concentrates on the consequences of the call policy on the value of the firm's common stocks. If the decision to call the convertibles decreases the value of common stocks, it cannot be viewed as optimal since managers do not behave in a way consistent with maximization of common shareholders' value.

Ofer and Natarajan (1987) present evidence that confirms the information signalling hypothesis of convertible bond calls. The authors argue that if the call of a convertible conveys unfavourable private information, then there should be an unexpected decline in the firm's performance in the periods subsequent to the call date. Their study shows that there exists a significant decline in the performance, defined as a relative change in the profit variables, after the call date. This decline combined with the existence of significant negative stock returns associated with the call announcement and abnormal negative stock returns in the periods following the call support the hypothesis that convertible calls are motivated by unfavourable private information.[28]

Mazzeo and Moore (1992) offer an alternative hypothesis for explaining the stock price decrease at the call announcement dates. They claim that the price reaction is a result of early investors' sales of the stocks at lower prices. Thus, part of the call announcement effect on stock price is caused by liquidity costs. The authors find that stock prices rebound during the conversion period, which contradicts the hypothesis that a call conveys negative private information.

[28] Cowan, Nayar, and Singh (1990) question the large negative abnormal returns following the call reported by Ofer and Natarajan (1987). Cowan et al. (1990) reports that this result is a product of using the parameters of the model obtained from the pre-call period. Mikkelson (1981) finds that stock returns preceding the call of convertibles are positive and abnormally large. Because of this, models that use parameters obtained from the pre-call period will tend to overestimate the predicted returns for the post-call period. Cowan et al. (1990) argue that the results of Ofer and Natarajan are biased downwards and that this bias is acerbated by the assumption of non-serially correlated abnormal returns for each firm.

Datta and Iskandar-Datta (1996) try to use wealth-transfer effects to explain the stock reaction to the call. Since a call of convertible bond reduces the firm's leverage, they argue that wealth is transferred from stockholders to straight debt holders because of the debt becoming less risky. If the negative news conveyed by the call were the only explanation to the stock reaction to the call, there should also be negative wealth effect for the firm's bondholders. The authors find significant positive effect for straight bond holders at the call announcement date which suggests that the call event not just conveys unfavourable private information but also triggers wealth redistribution. At the same time the called convertible bond holders suffer wealth loss as a result of the conversion premium elimination. The authors also document an insignificant wealth effect for the holders of non-converted convertibles and explain it by the hybrid nature of the convertibles. Positive wealth transfer to the straight bondholders implies a wealth loss of common stock holders that may explain the negative reaction of stock prices to call announcement.

We argue that the call policy adopted by a firm will not just affect the value of its shares but also value of the subsequent issues of convertibles. Investors will form expectations of the future call policy by looking back at the previous record. If investors observe a line of behaviour of the firm in the past, they will expect the same policy to be applied in future. This will lead to situations when pricing of new issues by a firm will depend on past call policy of the firm. One of the examples of such behaviour would be downpricing of new convertible bonds issued by firm that previously called their bonds early.

3.3 Data and methodology

In this chapter, we study whether the market memory effect is applicable to the convertible bond issuers that limit gains of investors by calling the convertibles as soon as the bond value exceeds the call price. As mentioned earlier, Ingersoll (1977b) finds that that the median firm waits to call the convertible until the conversion value exceeds the call price by 43.9%. This suboptimal call policy allows investors to gain access to a source of excess returns and may be one of the reasons for investors to buy convertible bonds. We argue that investors are aware of this late convertible call policy and consider it when investing in convertible bonds. Therefore, we suggest that investors expect the issuing firms to call convertibles late. If the issuer calls convertibles in time, investors view this as a bad signal. Investors will punish subsequent issues of convertibles of such firms by offering lower prices. To see whether there exists this bad market memories effect we select a sample of firms that issued multiple convertible issues. The firms in our sample have previously called at least one of their convertibles and have one or more convertibles issued after the call date. We separate this sample in two parts. The first subsample consists of firms that called their convertibles non-optimally, letting the convertible bond value if non-called to exceed the call threshold (from now on, we will refer to these firms as the "late-call" firms).[29] The second subsample consists of firms that called their

[29] In order to check the robustness of our results we use thresholds of 110%, 120%, and 130% of the call price.

convertibles optimally (from now on, we will refer to these firms as the "early-call" firms) and thus eliminated the possibility for investors to experience a substantial increase in the convertible bond value.

We test the existence of market memory by studying whether investors push down prices of convertibles issued after the firm called some of outstanding issues in time. If the market memory effect exists, we will observe lower prices for the newly issued convertibles of the early-call firms than the prices of new convertibles issued by the late-call firms ceteris paribus. We attribute the call policy to the firm and not to the Chief Financial Officer (CFO). Thus, we assume the call policy of the firm to stay the same even after (possible) change of a CFO.[30]

To detect the subsequent down pricing of the early-call firms' convertibles we first identify the date of the early call for every firm in our sample. We then look at the pricing of the convertible bonds issued after the call date.[31] We calculate the theoretical prices for each convertible in both sub-samples and compare them to the market prices.[32] The market memory idea will be plausible if we detect any subsequent systematic differences in pricing of the early-call firm

---

[30] Without this assumption, one would have to control for the fact that the firm's call policy may change after the change of CFO, which, in turn, would have an effect on how investors will predict the future firms' call policy and price new issues.

[31] If a firm had more than one bond issued after the call event, we look only at convertible that was issued earlier. In the case of Symantec Corp, two different convertible bonds were issued at the same date following the call event. Both new issues of Symantec were included in the sample.

[32] Parameters are calibrated from the same that is later used for comparison. These parameters can be viewed as "market-implied". Considering that we use the same approach for both early-call and late-call convertibles, any systematic difference between two sub-samples would imply either over- or under-pricing. Another option would be to use the parameters calibrated from the data on called issued. However, since the parameters may change over time, we use the former approach.

convertibles relative to the pricing of the late-call firm convertibles. To calculate the theoretical values of the convertibles in our study we use the model of Tsiveriotis and Fernandes (1998) (from now on called TF-model) and the model of Ayache, Forsyth, and Vetzal (2003) (from now on called AFV-model).[33]

In order to estimate theoretical values of convertible bond prices using the TF-model we need to know the following characteristics of the convertible bond: coupon rate, coupon frequency, conversion ratio (or conversion price), maturity date, underlying stock volatility, underlying stock price, credit spread, and call schedule specifying call dates and prices. For estimation of theoretical prices using the AFV-model, we also need to estimate the parameters of the hazard function used for describing the probability of default. In order to determine whether the previous convertible issues of any given firm were called early (or late) we need to observe the values of the called bonds and prices of the underlying stock.

For the purpose of our analysis, we use data on firms issuing multiple convertible bonds in Canada and in the US. Prospectuses are collected from the System for Electronic Document Analysis and Retrieval Database (SEDAR) for Canadian issues and SEC Electronic Data Gathering, Analysis, and Retrieval System (EDGAR) for the US issues. These databases contain prospectuses of bonds issued on or after 1994. In Datastream we found 9,850 convertible bonds issued in the US and Canada since 1994. Four hundred convertibles were called

[33] In chapter 2 we study convertible pricing models of Tsiveriotis and Fernandes (1998), Ayache, Forsyth, and Vetzal (2003), and Brennan and Schwartz (1977) and show that TF-model and AFV-models exhibit the lowest mean absolute pricing errors.

before maturity. Our final sample consists of thirty-eight firms (36 in the US and 2 in Canada) that called their convertibles before maturity and issued new convertibles after the call date.[34]

We obtain convertible bond prices, underlying stock prices, and redemption dates from Datastream and the Canadian Financial Markets Research Centre (CFMRC) databases. The information on bond covenants as well as conversion prices, maturity dates, coupon payments, call schedule, and call prices are derived from the convertible bond prospectuses. Some model parameters, including implied volatility, hazard function parameters, and underlying stock dividend rate, are calibrated using the Marquardt (1963) algorithm.[35] This algorithm uses a set of parameters with preset initial values to calculate the implied convertible bond model prices. After calculating the model prices, the algorithm calculates the value of the loss function, which is defined as the sum of squared differences between the model and the market prices for each observation. The algorithm then changes the parameters until it finds the set of values that minimize the loss function. Given the nature of the algorithm, it is possible that the final set of parameters produces the local minimum of the

[34] Symantec Corporation issued two new convertibles at the same date after calling their existing issue. Therefore, we have 38 firms and 39 convertible bonds in our sample: 37 US bonds and 2 Canadian bonds.

[35] In regression analysis, we use realized volatility of underlying stock returns, which is estimated as the annualized historical volatility, calculated for every firm using the daily closing prices of the underlying stocks over the 2-year period preceding the redemption date of the convertible bond.

loss function.[36] To ensure that the algorithm does not reach a local minimum instead of a global minimum we repeat the calibration using a different set of parameters' initial values. We introduce and calibrate the call price adjustment parameter π to incorporate the call notice period and the Parisian option feature that is present in many convertible bond contracts; the call price specified in the prospectus multiplied by $1+\pi$ returns the *effective* call price.[37]

We calculate the model implied convertible bond prices for the first 200 trading days starting with the bond issue date. We then use the forecasted prices as the theoretical convertible bond prices and compare those to the market prices. The list of the convertible bond issues in our sample is reported in Table 3.1.

[36] To increase the chances of finding the global minimum it is possible to introduce the jump process where the values of parameters would be changed arbitrarily to random values with some probability. This method, however, may significantly increase the calculation time. Our estimation can be viewed as a specific case of a more general approach described above that uses a zero probability of such jump in parameter values.

[37] See Lau and Kwok (2004) for details of the Parisian option model implementation.

Table 3.1 Convertible Bonds Used in the Study[38]

This table reports the main characteristics of the convertible bonds used in our study. The sample consists of 37 US (all issues except for RSI and PVE) and 2 Canadian (RSI and PVE) convertible bonds that were issued by the companies with previous convertible bond call record. Symantec Corp. had issued two new convertible bonds at the same date; therefore, in total we have 36 US firms in our sample. The issuing dates for current bonds and the call dates for previous issues are reported in columns 2 and 9 respectively. CONVP denotes the conversion price. All prices are in local currency. Issue Size is in millions of dollars. VOLAT denotes the underlying stock volatility and is calculated as the annualized standard deviation of stock returns. The last column denotes whether the previously issued bond(s) of the same company was called early, that is before the convertible bond value exceeded the call price by 20%.

Issuer/Symbol	Issue Date	Issue Size	Maturity Date	Coupon Rate	CONVP	VOLAT	Industry	Call Date	Early Call
Acxiom Corp (ACXM)	Jul-02	175	Feb-09	3.75	18.25	0.24	IT	Apr-02	Yes
Alexion Pharmaceuticals (ALXN)	May-05	150	Feb-12	1.375	31.46	0.4	Pharmaceutical	Mar-05	Yes
Allergan Inc (AGN)	Jan-03	641.5	Nov-22	0	126.66	0.21	Pharmaceutical	Dec-03	Yes
Alpharma Inc (ALO)	Mar-07	300	Mar-27	2.125	32.5	0.53	Pharmaceutical	Jan-06	Yes
Advanced Micro Devices Inc (AMD)	Jul-07	2,200	May-15	6	28.08	0.49	IT	Feb-06	No
Amylin Pharmaceuticals (AMLN)	Aug-07	575	Jun-14	3	61.07	0.4	Biopharmaceutical	Aug-06	No
Anixter International (AXE)	May-07	300	Feb-03	1	63.48	0.3	Management Services	Jun-05	Yes
Avatar Holdings Inc (AVTR)	Nov-03	120	Apr-24	4.5	52.63	0.28	Real Estate	Nov-03	Yes
BioMarin Pharmaceutical (BMRN)	Apr-07	282.5	Apr-17	1.875	20.36	0.42	Pharmaceutical	Jan-07	No

[38] Nine out of thirty-nine convertible bond issues in the sample were called only after the bond value exceeded the call price by more than 10%, six issues were called after the bond value exceeded the call price by more than 20%, and three issues were called only after the bond value exceeded the call price by more than 30%. To avoid cluttering, we do not report these results in the table.

Issuer/Symbol	Issue Date	Issue Size	Maturity Date	Coupon Rate	CONVP	VOLAT	Industry	Call Date	Early Call
Charming Shoppes (CHRS)	Apr-07	275	May-14	1.125	15.38	0.36	Retail	Jun-07	Yes
Charter Communications (CHTR)	Jul-05	862.5	Nov-09	5.875	2.42	0.62	Communications	Dec-04	Yes
Commscope Inc (CTV)	Oct-04	250	Mar-24	1	21.75	0.43	Communications	Apr-04	No
Cypress Semiconductors (CY)	May-07	600	Sep-09	1	23.9	0.39	IT	Feb-07	Yes
Cymer Inc (CYMI)	Jun-02	250	Feb-09	3.5	50	0.77	IT	Mar-02	Yes
Doubleclick Inc (DCLK)	Jun-02	135	Jul-23	0	13.12	0.66	IT	Jul-03	No
Genesco Inc (GCO)	Sep-04	86.25	Jun-23	4.125	22.12	0.46	Retail	Jul-03	No
Genzyme Corp (GENZ)	Jul-01	575	May-21	3	70.3	0.65	Biotech/Health	Jun-01	No
Getty Images Inc (GYI)	Nov-03	265	Jun-23	0.5	61.08	0.53	Communications	Jul-03	Yes
Gilead Sciences Inc (GILD)	Nov-06	650	May-11	0.5	77.5	0.29	Biopharmaceutical	Nov-04	No
Health Management (HMA)	Nov-04	575	Aug-23	0	27.39	0.26	Healthcare	Aug-03	Yes
Hilton Hotels Corp (HLT)	Aug-03	575	Apr-23	3.375	22.5	0.24	Hospitality	May-03	Yes
Hutchinson Tech Inc (HTCH)	Jan-06	150	Jan-26	3.25	37.43	0.38	IT	Mar-03	Yes
IM Clone Systems Inc (IMCL)	Nov-04	600	May-24	1.375	94.69	0.68	Biopharmaceutical	Jun-04	Yes
Interpublic Group Inc (IPG)	Sep-07	400	Mar-23	4.25	12.42	0.26	Marketing Services	Dec-04	Yes
Lifepoint Hospitals Inc (LPNT)	Nov-05	225	Aug-25	3.25	61.22	0.29	Healthcare	Jun-05	Yes
LSI Logics Corp (LSI)	Nov-03	350	May-10	4	13.42	0.44	IT	Sep-03	Yes
Mentor Graphics (MENT)	Jun-06	200	Mar-26	6.25	17.97	0.4	IT	Mar-06	Yes

Issuer/Symbol	Issue Date	Issue Size	Maturity Date	Coupon Rate	CONVP	VOLAT	Industry	Call Date	Early Call
Micron Technology Inc (MU)	May-07	1,135	Jun-12	1.875	14.23	0.32	IT	Mar-06	Yes
PEP Boys (PBY)	Nov-02	150	Jun-07	4.25	22.4	0.67	Automotive	Nov-01	Yes
Provident Energy Trust (PVE)	Nov-05	100.8	Apr-11	6.5	14.75	0.21	Oil and Gas	May-05	Yes
Quantum Corp (DSS)	Feb-04	160	Aug-10	4.375	4.35	0.73	IT	Aug-03	Yes
Rogers Sugar Income Fund (RSI)	Mar-06	85	Jun-13	5.9	5.1	0.24	Food	Mar-06	Yes
Sandisk Corp (SNDK)	May-06	1,000	May-13	1	82.36	0.53	IT	Nov-04	No
Sepracor Inc (SEPR)	Jun-04	750	Dec-10	0	29.84	0.73	Pharmaceutical	Jan-04	Yes
St.Mary Land & Exploration (SM)	Jun-07	287.5	Apr-27	3.5	54.42	0.33	Oil and Gas	Mar-07	Yes
Symantec Corp (SYMC)	Dec-06	1,100	Jun-11	0.75	19.12	0.35	IT	Nov-04	Yes
Symantec Corp (SYMC)	Dec-06	1,000	Jun-11	1	19.12	0.35	IT	Nov-04	Yes
Waste Connections (WCN)	May-06	200	Apr-26	3.75	51	0.18	Utilities	Apr-04	Yes
XM Satellite Radio (XMSR)	May-05	400	Dec-09	1.75	50	0.49	Communications	Mar-04	Yes

From Table 3.1 we can see that the issuers represent different industries, many of them operating in pharmaceutical and technology sectors. The Canadian issuers (Rogers Sugar and Provident Energy) are structured as income trusts[39] while all US issuers are structured as corporations. The bonds in the sample have issue dates from 2001 to 2007 and maturity dates ranging from 2003 to 2027. Thirty out of the thirty-eight firms had previously called convertible bonds unfavourably for investors, while the remaining eight firms had called their convertibles favourably, that is when the value of convertible if non-called exceeded the call price by more than 20%. The issue sizes range from 85 to 2200 million dollars with the average issue size being just over 465 million dollars. The highest coupon rate paid on a bond in our sample is 6.5% and the lowest is zero. The underlying stock volatility spans a range from 0.18 to 0.73 (these numbers represent an annualized standard deviation of stock returns). The detailed descriptive statistics of convertibles used in the study is reported in Table 3.2.

[39] Income trusts purchase most of the equity and debt of successful businesses with the funds acquired by issuing units of securities to the public. Operating businesses act as subsidiaries of income trusts which in turn distribute high proportion of their cash flows to unit holders as cash distributions. The special tax treatment rules for income trusts makes them a widespread business structure in Canada.

Table 3.2 Summary Statistics

This table presents the summary statistics of the sample of the 37 US and 2 Canadian convertible bonds issued by companies that had convertible bonds issued in the past. Symantec Corp. had issued two new convertible bonds at the same date; therefore in total we have 36 US firms in our sample. Some of the previous issues were redeemed unfavourably for investors, without giving them an opportunity to benefit from appreciating underlying stocks and thus higher conversion values of the convertible bonds. The dummy variable D takes a value of 1 for bonds that were issued after the previous issue was called early and a value of 0 if the previous issue was redeemed late. VOLAT refers to the annualized historical standard deviation of underlying stock returns; TMAT refers to the time to maturity (in years) of the bond. COUPON refers to the convertible bond coupon rates. S/K refers to the ratio of the average stock price during the forecast period to the conversion price. TCALL refers to the time since the last call event. Pricing Error is defined as the market price minus the model (theoretical) price and is expressed either in dollar terms or as a percentage of bond prices and is provided for both AFV- and TF-models. Negative values of pricing errors occur when the market under prices the bond (relative to the theoretical model price).

	COUPON	VOLAT	Bond Price	Pricing Error, $		Pricing Error, %		TMAT	S/K	TCALL
				AFV	TF	AFV	TF			
MEAN	0.03	0.42	104.77	0.06	-0.02	0.00	0.00	11.28	0.77	1.44
MEDIAN	0.03	0.40	100.88	0.01	0.01	0.00	0.00	6.82	0.72	1.24
MAX	0.07	0.77	225.25	21.05	15.71	0.15	0.11	20.00	2.20	5.05
MIN	0.00	0.18	52.25	-8.06	-9.74	-0.14	-0.15	1.10	0.10	-0.94
STDEV	0.02	0.17	24.68	1.82	2.18	0.02	0.02	6.71	0.32	0.86

From Table 3.2 we can see that the average time to maturity of bonds in our sample is just over 11 years (the shortest time to maturity is just over one year and the longest is 20 years). The average bond in the sample is out-of-the-money with a ratio of average stock price to the conversion price of 0.77. The average market pricing errors show $0.02 underpricing by TF-model and $0.06 overpricing by AFV-model. The average pricing error as a percentage of the bond's price is zero. The difference between the market price and the theoretical convertible bond price ranges from $9.74 (15%) underpricing by the market to $15.71 (11%) overpricing if TF-model is used. Similarly, if AFV-model is used, the pricing ranges from $8.06 (14%) underpricing to $21.05 (11%) overpricing. Nine out of thirty-nine convertible bond issues in the sample were called only after the bond value exceeded the call price by more than 10%, six issues were called after the bond value exceeded the call price by more than 20%, and three issues were called only after the bond value exceeded the call price by more than 30%.

3.4 Results

We explain the difference between the market prices and the theoretical prices (*ΔP*) using the following regression model:

Model 1: $\Delta P = c + \alpha_1 SK + \alpha_2 TMAT + \alpha_3 COUPON + \alpha_4 VOLAT + \alpha_5 D$

where SK is the ratio of the current stock price to the conversion price, TMAT is the time remaining to maturity, COUPON is the coupon rate paid by the convertible bond, VOLAT is the annualized standard deviation of underlying stock returns, and D is a dummy variable that takes the value of 1 if the bond was issued after the previous convertible issue of the same firm was called early without giving the investors an opportunity to benefit from appreciating stock prices and 0 otherwise. We hypothesize that the subsequent convertible issues of firms that called bonds early will be underpriced as investors realize that if the firm called bonds early in the past they will more likely call the new issue early. Thus, we expect that the sign of the coefficient α_5 to be negative.

Our empirical analysis of the AFV- and TF-models' pricing errors in the previous paper suggests that we would also expect positive sign for coefficient α_3 in the regression using the TF-model errors. Since no other coefficients had statistically significant effect on the pricing errors, and we do not have any theoretical evidence in support of any specific coefficient values, we do not expect any particular sign for any other coefficients.

Our estimation sample consists of a pool of the first 200 available daily observations since the issue date for each of the 39 bonds in the sample. In total, we have 7380 daily observations.[40] Each of these observations contains an observed market price for a convertible, the model predicted price, the convertible coupon rate and conversion price, the underlying stock price and volatility, the time since the previous bond was redeemed, and the time until maturity of convertible. For each of the 200 observation dates we calculate the

[40] We have less than 200 daily observations for some bonds in our sample (AMD, Cypress Semiconductors, Genesco, Interpublic Group) because the cut off point for our sample is August, 15 2008.

pricing error as the difference between the market and the model prices. A negative value of the pricing error means that the market price was lower than the theoretical convertible price implying underpricing of the convertible. Positive values of the pricing errors evidence market overpricing. We use both the dollar differences between the model and the market prices and the percentage differences, calculated by dividing the dollar pricing errors by the market price of the convertible. We then estimate the regression coefficients by running a pooled OLS regression with heteroskedasticity and autocorrelation corrected errors.

To test whether the underpricing differs for the bonds that are more debt- or equity-like we use the following regression equation.

Model 2:

$$\Delta P = c + \alpha_1 SK + \alpha_2 TMAT + \alpha_3 COUPON + \alpha_4 VOLAT + \alpha_5 D + \alpha_6 D * SK$$

We hypothesize that bonds of early calling firms, which are more in-the-money (equity-like bonds), will suffer larger underpricing by the market than bonds with lower SK-ratios (debt-like bonds). Thus, we expect the sign of coefficient α_6 to be negative.

The results of the regression analysis are presented in Table 3.3.

Table 3.3 Underpricing of Convertible Bonds Issued by Firms Calling Early

Table 3.3 reports results of Model 1 panel regressions of convertible bond pricing errors on early call dummy variable and control variables. Pricing errors are calculated as market prices minus model predicted prices. The models used are the ones of Tsiveriotis and Fernandes (1998) and of Ayache, Forsyth, and Vetzal (2003). Pricing error is expressed either as a percentage of the convertible bond price or in dollar terms. The table reports coefficient values with t-statistic values underneath in brackets. TMAT is the time to maturity, COUPON is the convertible bond coupon rate, SK is the convertible bond moneyness ratio calculated as stock price divided by conversion price, VOLAT is the annualized historical volatility of the underlying stock. D is the dummy variable that takes value of 1 if the issuing firm has previously called its convertible bonds early and value of 0 otherwise. Early call happens when firm voluntarily calls its convertible bonds as soon as the convertible bond value exceeds 110% (alternatively 120% or 130%) of the call price. Coefficients in bold are significant at 5% level.

	TF Model						AFV Model					
	Error, %			Error, $			Error, %			Error, $		
	110	120	130	110	120	130	110	120	130	110	120	130
Intercept	**-0.883**	**-0.824**	**-0.286**	**-0.578**	**-0.454**	0.307	**-0.344**	-0.200	**0.723**	**-0.354**	-0.154	**1.067**
	(6.37)	(5.72)	(1.73)	(4.23)	(3.21)	(1.90)	(3.09)	(1.74)	(5.53)	(3.12)	(1.31)	(8.07)
TMAT	**0.014**	**0.012**	**0.015**	**0.014**	**0.011**	**0.015**	**0.012**	**0.009**	**0.013**	**0.016**	**0.012**	**0.018**
	(3.56)	(3.19)	(3.86)	(3.61)	(2.99)	(4.01)	(3.70)	(2.84)	(4.37)	(4.96)	(3.83)	(5.88)
COUPON	**3.485**	**4.159**	2.105	**4.906**	**6.018**	2.849	**5.250**	**6.507**	**2.786**	**6.495**	**8.202**	**3.209**
	(2.36)	(2.82)	(1.42)	(3.38)	(4.15)	(1.96)	(4.45)	(5.52)	(2.37)	(5.38)	(6.81)	(2.70)
SK	**0.665**	**0.681**	**0.759**	**0.501**	**0.522**	**0.645**	**0.326**	**0.350**	**0.494**	**0.406**	**0.437**	**0.630**
	(8.14)	(8.40)	(9.39)	(6.23)	(6.55)	(8.14)	(4.99)	(5.40)	(7.72)	(6.07)	(6.61)	(9.74)
VOLAT	**0.497**	**0.440**	**0.452**	**0.444**	**0.350**	**0.374**	**0.397**	**0.290**	**0.316**	**0.500**	**0.355**	**0.392**
	(3.18)	(2.81)	(2.90)	(2.89)	(2.27)	(2.45)	(3.17)	(2.32)	(2.57)	(3.90)	(2.78)	(3.14)
D	**-0.237**	**-0.281**	**-0.865**	**-0.377**	**-0.479**	**-1.306**	**-0.424**	**-0.543**	**-1.545**	**-0.573**	**-0.740**	**-2.066**
	(3.42)	(3.71)	(7.41)	(5.53)	(6.41)	(11.41)	(7.64)	(8.95)	(16.73)	(10.08)	(11.95)	(22.11)
R^2	0.013	0.013	0.019	0.013	0.014	0.026	0.016	0.019	0.044	0.026	0.031	0.074

The regression results in Table 3.3 show that the sign of the early-call dummy variable coefficient α_5 in Model 1 is negative and the coefficient is statistically significant. This result is robust with respect to the different models used for pricing convertibles as well as to the level of the early call threshold, and to the way the pricing error is defined. This finding confirms our market memory hypothesis that firms that call their convertibles early have their new convertible issues underpriced by the market. The values of coefficient α_5 tell us that on average new convertibles of the early calling firms are underpriced by the amount ranging from 0.24% to 1.55% ($0.37-$2.07).

Also, the degree of the early-call bonds underpricing increases with the level of threshold used for defining the “early call”. If we use a threshold of 110% (the firms that did not call their convertibles until the bond value exceeded call price by more than 10% are defined as “late calling”) then the average underpricing is 0.24% ($0.38) calculated using TF-model and 0.42% ($0.57) for the AFV-model. If the 120% threshold is used, magnitude of underpricing is 0.28% ($0.54) and 0.54% ($0.74) for the TF- and AFV-models respectively. The largest degree of underpricing is found using the 130% threshold: if we use the TF-model, on average, convertible bonds of early-call firms are underpriced by 0.87% ($1.31), and if we use the AFV-model, new convertibles are underpriced by 1.55% ($2.07) on average. One of the reasons for this finding may come from the fact that as we increase the late call threshold, more bonds from our sample will end up in the subsample of the early-call bonds.

In accordance with our hypothesis, the coupon rate has a positive effect on the pricing errors of the TF-model, which is evidenced by the positive and statistically significant value of coefficient α_3. The coefficient α_3 in the regression using the AFV-model pricing errors is also positive and statistically significant even though our previous empirical analysis found it to be insignificant. Surprisingly, the coefficients for TMAT, SK, and VOLAT are all significant and positive indicating that the larger the values for these bond characteristics, the larger the pricing errors we will observe on average.

The results of the regression analysis based on Model 2 are presented in Table 3.4. The regression results, namely, the negative and statistically significant values of the coefficient α_6 confirm that the degree of market underpricing depends on the bonds' ratio of the spot price to the conversion price. At the same time, the early-call dummy coefficients for some model specifications are positive and statistically significant and the statistical significance increases with the early-call threshold level. The *combined* effect of coefficients α_5 and α_6 shows that, on average, only bonds with relatively high moneyness ratios will suffer underpricing. Moreover, underpricing gets larger with the increase in moneyness ratio. For example, using the 130% threshold and the TF-model, we can see that only the bonds with the S/K ratio higher than 0.48 are underpriced relative to the bonds of late calling firms. For the AFV-model, the early-call firm bond underpricing will happen to the bonds with S/K ratio higher than 0.41. These results confirm our hypothesis that the degree of underpricing is different for equity- and debt-like convertibles of early calling firms.

The R^2s in regressions increase with the level of the early-call threshold. The lowest level of R^2 (0.013) is in the Model 1 regression that uses the 110% threshold, the highest value of R^2 (0.229) is in Model 2 using the 130% threshold. These results provide evidence that the variation in the early-call dummies values together with variation in coupon rates, volatility, time to maturity and degree of moneyness explain significant amount of variation of the pricing errors. This supports our proposition that firm's call policy does affect pricing of subsequent convertible bond issues.

Table 3.4 Underpricing and Degree of Moneyness of Convertible Bonds Issued by Early Call Firms

Table 3.4 reports results of Model 2 panel regressions of convertible bond pricing errors on early call dummy, interactive dummy, and a set of control variables. Pricing errors are calculated as market prices minus model predicted prices. The models used are the ones of Tsiveriotis and Fernandes (1998) and of Ayache, Forsyth, and Vetzal (2003). Pricing errors are expressed either as percentage of the convertible bond price or in dollar terms. TMAT is the time to maturity, COUPON is the convertible bond coupon rate, SK is the convertible bond moneyness ratio calculated as stock price divided by conversion price, VOLAT is the annualized historical volatility of the underlying stock. D is the dummy variable that takes value of 1 if the issuing firm has previously called its convertible bonds early and value of 0 otherwise. D*SK is the interaction dummy that takes value equal to SK if the underlying firm calls its bonds early and value of 0 otherwise. Early call happens when firm voluntarily calls its convertible bonds as soon as the convertible bond value exceeds 110%, 120%, or 130% of the call price. Coefficients in bold are significant at 5% level.

	TF Model						AFV Model					
	Error, %			Error, $			Error, %			Error, $		
	110	120	130	110	120	130	110	120	130	110	120	130
Intercept	**-1.239**	**-1.174**	**-3.416**	**-1.140**	**-1.016**	**-3.841**	**-0.838**	**-0.717**	**-3.107**	**-1.061**	**-0.893**	**-4.244**
	(7.08)	(6.64)	(13.87)	(6.63)	(5.86)	(16.18)	(5.99)	(5.08)	(16.39)	(7.43)	(6.21)	(23.16)
TMAT	**0.013**	**0.011**	0.006	**0.012**	**0.009**	0.003	**0.010**	**0.006**	0.002	**0.014**	**0.009**	0.003
	(3.30)	(2.75)	(1.56)	(3.19)	(2.29)	(0.91)	(3.25)	(2.05)	(0.83)	(4.34)	(2.73)	(1.07)
Coupon	**3.468**	**4.162**	**2.688**	**4.879**	**6.023**	**3.621**	**5.226**	**6.511**	**3.499**	**6.461**	**8.207**	**4.197**
	(2.35)	(2.82)	(1.84)	(3.37)	(4.16)	(2.58)	(4.44)	(5.53)	(3.12)	(5.38)	(6.85)	(3.87)
SK	**1.075**	**1.111**	**5.871**	**1.148**	**1.214**	**7.420**	**0.896**	**0.986**	**6.748**	**1.219**	**1.346**	**9.304**
	(7.28)	(7.40)	(18.76)	(7.91)	(8.23)	(24.60)	(7.59)	(8.23)	(28.02)	(10.12)	(11.04)	(39.96)
Volatility	**0.510**	**0.440**	**0.406**	**0.465**	**0.351**	**0.313**	**0.415**	**0.291**	**0.260**	**0.526**	**0.356**	**0.314**
	(3.26)	(2.82)	(2.66)	(3.03)	(2.29)	(2.13)	(3.32)	(2.33)	(2.21)	(4.13)	(2.81)	(2.76)
D	0.256	0.219	**2.650**	**0.402**	**0.325**	**3.353**	**0.261**	0.196	**2.756**	**0.406**	**0.315**	**3.899**
	(1.56)	(1.32)	(11.15)	(2.50)	(2.00)	(14.64)	(2.00)	(1.48)	(15.07)	(3.04)	(2.35)	(22.05)
D*SK	**-0.585**	**-0.605**	**-5.472**	**-0.924**	**-0.972**	**-7.252**	**-0.813**	**-0.894**	**-6.695**	**-1.161**	**-1.278**	**-9.285**
	(3.33)	(3.41)	(16.89)	(5.35)	(5.57)	(23.22)	(5.79)	(6.31)	(26.85)	(8.10)	(8.86)	(38.51)
R^2	0.015	0.015	0.055	0.017	0.018	0.092	0.020	0.024	0.129	0.034	0.041	0.229

3.5 Conclusions

In this chapter, we study the market memory effect in convertible bond markets. We argue that investors may treat certain convertible bond issues unfavourably because of the past call policy of the issuing firm. In particular, investors may put downward pressure on the price of new convertible bonds of the firms that have a history of calling their convertible bonds early. Early call happens when the issuer calls its bonds without giving the opportunity for bondholder to benefit from appreciation of the underlying stock prices and, as a result, increasing values of convertibles.

Brennan and Schwartz (1977) and Ingersoll (1977a) show that the firm's optimal strategy is to call the convertible bond as soon as the bond value, when not called, equals the call price. Evidence collected by Ingersoll (1977b) shows that many bonds are called late, that is when the bond value greatly exceeds the call price. As investors are aware of this late call behaviour, they may expect that any convertible bond issue is going to be called late. Thus, when they observe instances of rational (early) call behaviour, they may view those as non-friendly acts. The memory of the firm (unfavourable) past call policy will make investors to push down the new convertible bond issues prices. The degree of punishment will depend on how investors view the chances that convertible conversion value will exceed the call price. If the convertible's conversion price is specified to be much higher than the current stock price, the chances of the stock price exceeding the conversion price are much lower than if the conversion price is close to the current stock price. Thus, downpricing is expected to be more substantial for the new equity-like convertibles than for the new debt-like convertibles.

In our study, we use a sample of convertible bonds issued by firms that had previously called their convertibles. Thirty of the thirty-eight firms in our sample had called their previous issues rationally, without letting the conversion values to exceed call price by more than 20%. The results of the regression analysis using pricing errors from the models of Tsiveriotis and Fernandes (1998) and Ayache, Forsyth, and Vetzal (2003) show that the new issues of the early-call firms are underpriced relative to the new issues of firms that called late. The degree of market underpricing depends on the convertible bond ratio of underlying stock price to conversion price.

4: EUROPEAN PUT-CALL PARITY AND EARLY EXERCISE PREMIUM FOR CURRENCY OPTIONS[41]

4.1 Introduction

Since the collapse of the Bretton Woods system in 1973, companies have looked for ways to hedge their currency risks. This has led to the creation of a large market for currency options. Most exchange traded currency options are American style and can be exercised at any time up to the expiration date. This causes a problem because the benchmark formula for the valuation of currency options – the Garman-Kohlhagen (GK) currency option variant of the Black-Scholes model – does not take the premium for early exercise into account.[42] In order to apply the GK model to traded options, information on the size of the early exercise premium and the factors that determine this premium are important, if only for interpreting the behaviour of the implied volatilities estimated from traded options.

Jorion and Stoughton (1989) compute the early exercise premium as the difference between values of European currency options on the Chicago Board of Options Exchange (CBOE) and American currency options on the Philadelphia Stock Exchange. They find that the premiums for call options are significantly positively related to the ratio of the spot price and the exercise price, the foreign

[41] This chapter is based on Poitras, Veld, and Zabolotnyuk (2007, 2009).

[42] On the Garman-Kohlhagen model, see Garman and Kohlhagen (1983). Bollen and Rasiel (2003) provides a recent discussion of this and other currency option models.

interest rate, the volatility and the time to expiration. The premium is significantly negatively related to the domestic interest rate. For the same variables for put options they only find insignificant relationships.

Zivney (1991) provides a different methodology to derive the early exercise premium for index options using the put-call parity condition. He argues that the American option pricing models do not value the early exercise premium appropriately. Therefore, he suggests that this early exercise premium needs to be established empirically. De Roon and Veld (1996) refine Zivney's methodology for index options on an index in which dividends are reinvested. Another paper that builds on the original study by Zivney (1991) is the paper by Engström and Nordén (2000) who refine this methodology for equity options. To the best of our knowledge, no study has yet applied this methodology to currency options. This is remarkable, because currency options are more likely to be exercised early than both equity and index options. It is well known that rational investors only exercise equity options just before an ex-dividend date (see Merton, 1973). The early exercise decision for index options is less clear, because indexes consist of different stocks that pay dividends at different times. In general, it is not unrealistic to assume a continuous dividend yield for an index (see e.g. Hull, 2006). Under such assumptions, rational investors will exercise call options early if the dividend yield is higher than the risk-free interest rate. In practice, this will hardly ever be the case. Contrary to equity and index call options, currency options are a prime candidate for early exercise.[43] Call options

[43] See e.g. Bodurtha and Courtadon (1995).

on high-interest currencies and put options on low-interest currencies are the most likely to be exercised early. This is because a high-interest rate currency is expected to depreciate relative to the US Dollar and a low-interest rate currency is expected to appreciate relative to the US Dollar. The objective of this chapter is to study the early exercise premiums for currency options using the methodology originally suggested by Zivney (1991).

The empirical results examine 331 pairs of call and put currency options traded on the Philadelphia Stock Exchange between January 2, 1992 and September 24, 1997. Over the admissible range of exchange rate/exercise pairs considered, the average early exercise premium is 5.03% for put options and 4.60% for call options. The premiums for both call and put options are strongly related to the time to maturity and the interest rate differential. These results are largely in line with the earlier results of Jorion and Stoughton (1989). The major difference between our results and their results is that we find significant coefficients for both call and put options, while they only find significant results for call options. It is important to consider the results of our study when valuing American options with a closed-form solution of European model.[44]

The remainder of this chapter is organized as follows. Section 4.2 explains the decision to call the option early in details; Section 4.3 presents the methodology. Section 4.4 contains the data description, followed by Section 4.5, which covers the results. The chapter is concluded with Section 4.6.

[44] Another possibility to value an American option would be to use numerical methods, such as binomial trees or finite-difference schemes. However, our results allow getting a quick approximation of the European option price by finding the value of a similar American option using the Black-Scholes formula and adding the time early exercise premium to it.

4.2 The early exercise decision

Despite being essential to the valuation of American options, the trading fundamentals of the early exercise decision have not received much attention. An understanding of the optimal solutions for the relevant market participants is needed to explain the behaviour of the early exercise premium. Two general decision problems are encountered: maximizing the discounted expected value for the option holder; and maximizing arbitrage profit for a market maker. The solution of the arbitrage for the market maker determines the prices that are used in the option holder decision problem. In many cases, as the American option goes deep in the money the market maker will only be willing to quote the intrinsic value (time value equals zero) providing no incentive for the holder to sell the option in the market instead of exercising early.[45] When the discounted expected value of holding the option to maturity is less than that for exercising the option and holding the underlying security obtained until the expiration date, early exercise is triggered. For call options on stocks paying dividends at discrete intervals, this typically occurs for dividend payment dates close to the expiration date. For options on currencies, the difference between the foreign and domestic interest rates drives the early exercise decision. Two necessary conditions for the occurrence of an early exercise event are that there must be a traded American option to exercise; and that the option being exercised is in the money. In practical terms, this means that options for exercise were created at a prior time when the relation between spot price, exercise price and time to

[45] Intrinsic value is defined as the maximum of zero or the current spot price minus the exercise price for a call, and conversely for a put. The time value is the current option price minus the intrinsic value.

expiration required a non-trivial time value in the option price. Because option exercise involves a surrender of the time value that could be obtained if the option is sold instead of exercised, at the time of an exercise event, the prices quoted by the market maker are effectively equal to the intrinsic value, having little or no time value. To see this, consider the market maker's (bid) quote at time t for a deep in the money call option on a non-dividend paying stock (e.g., Poitras (2002), p.378-81). In order to hedge this purchase at price C(t), the market maker would short the stock at price s(t) and invest the balance in a riskless bond maturing on the expiration date. If the deep in the money option is quoted at intrinsic value (s(t) - K) – stock price minus exercise price – then after purchasing the call option and shorting the stock the market maker would be able to invest K in the riskless bond at interest rate r. Assuming perfect markets, this would generate a profit (π) on the expiration date, T, of:

$$\pi_m(T) = Ke^{r(T-t)} + \max[0, s(T) - K] - s(T)$$

Because this represents a profit in all future states of the world, it follows that the market maker would quote a higher price than the intrinsic value and the call option would be sold by the holder and not exercised. This is the basis for the well known result that the early exercise premium for an American call option on a non-dividend paying stock is zero because the probability of early exercise is zero.

For an option on a stock with discrete dividend payments, the short stock position will also be responsible for any dividends paid between the trade date and the expiration date. Taking D to be the future value of the dividend payment evaluated at T, the profit for the market maker is now:

$$\pi_m(T) = Ke^{r(T-t)} + \max[0, s(T) - K] - (s(T) + D)$$

For dividend payment dates close to the expiration date, this profit can be negative and there is no incentive to quote prices above intrinsic value. In this case, the option holder will receive $\max[0, s(T) - K]$ if the option is held to maturity. If the option is exercised early, an instant before the ex dividend date, the holder will borrow K and use the funds to purchase the stock. At expiration the profit will be:

$$\pi_h(T) = (s(T) + D) - Ke^{r(T-t)}$$

Assuming the option is far enough in the money and the time to expiration is short enough that the probability of s(T)<K is zero, then early exercise will be more profitable than holding the call option to maturity if the value of the dividend received exceeds the interest cost of the funds borrowed to exercise the option.

The early exercise decision for currency options differs substantively from equity options. In particular, let S(t) be the exchange rate measuring the value of 1 unit of foreign currency in terms of a reference (domestic) currency. As such, a call option giving the right to purchase 1 unit foreign currency in exchange for a stated amount of the reference currency is identical to a put option giving the right to sell the stated amount of reference currency in exchange for 1 unit of foreign currency. If the call option is quoted at intrinsic value at time t, then the market maker can borrow 1 unit of foreign currency at foreign interest rate rf, convert at S(t) to buy the call option for S(t)-K and invest the remaining funds at the domestic interest rate r in a riskless bond. The market maker's profit at T would be:

$$\pi_m(T) = Ke^{r(T-t)} + \max[0, S(T) - K] - (S(T)e^{r^*(T-t)})$$

Similarly, the market maker profit for a put option would involve borrowing K in the domestic market, buying the put option at intrinsic value K-S(t) and converting the balance of funds to foreign currency, investing at the foreign interest rate and converting the proceeds back to the reference currency at S(T). The market maker profit in this case is:

$$\pi_m(T) = (S(T)e^{r_f(T-t)}) + \max[0, K - S(T)] - Ke^{r(T-t)}$$

It follows that, as the call or put gets deep in the money, the incentive for the market maker to quote prices above intrinsic value depends on the difference between the domestic and foreign interest rates, r-rf.

Faced with quoted prices at intrinsic value, the deep in the money call option holder at time t will borrow K at interest rate r, exercise the option by paying K to receive S(t), converting to 1 unit of foreign currency, investing at rf and converting the proceeds back to the reference currency at S(T). At time T, the profit would be:

$$\pi_h(T) = S(T)e^{r_f(T-t)} - Ke^{r(T-t)}$$

Assuming the call option is deep enough in the money that the probability of S(T)<K is zero, early exercise will be triggered whenever this value exceeds

the profit from holding the option to maturity $\max[0, S(T) - K]$, i.e. when rf>r. Similarly, early exercise for the put option will be triggered when r>rf. Unlike equity options, where early exercise is triggered by discrete dividend payments close to the expiration date, currency options depend on the difference between rf and r, and how deep in the money the option is. Early exercise can occur for relatively long dated currency options, if the option is deep enough in the money that the probability of the option finishing out of the money is negligible. Because of the lack of incentives for the market maker to quote prices above intrinsic value for deep in the money options, early exercise has additional value for option holders seeking to sell the position prior to maturity.

4.3 Methodology

Deviations from the European put-call parity are used in order to measure the early exercise premium for currency options. The European put-call parity has the following form:[46]

$$c - p = Se^{-r_f(T-t)} - Ke^{-r(T-t)}$$

where *c* is the European call price; *p* is the European put price; *T-t* is the number of years to maturity of the option; r_f is the foreign risk-free interest rate; *r* is the domestic risk-free interest rate; *K* is the exercise rate (price) on the put and call options; and *S* is the spot foreign exchange rate at time t. Following Zivney (1991), the unobserved early exercise premium (*EEP*) can be estimated by

[46] For details, see Poitras (2002), page 477-479.

subtracting the observed theoretical European option price differentials from the observed American option price differentials, which leads to:

$$(C-P)-(c-p)=EEP_C-EEP_P=(C-P)-(Se^{-r_f(T-t)}-Ke^{-r(T-t)})$$

where the *EEP* are defined as: $C = c + EEP_C$ and $P = p + EEP_P$. Given that this equation produces an estimate for the difference of *EEP*'s, properties of the *EEP* specific to the currency options are used to enhance the estimate for the individual *EEP*'s.

The estimated early exercise premium for the currency options is then checked for consistency with the boundaries for the early exercise premium (EEP) according to the put-call parity.[47,48]

The options data are divided into two subgroups with respect to moneyness. Moneyness is defined as the ratio of the spot price to the exercise price (S/K). We identify in-the-money puts (S<K) and in-the-money calls (S>K). We do not consider the options that are near-the-money, since the early exercise premium for the options in this group can be attributed to both call and put options. The exact definitions are as follows:

Group 1: In-the-money put	S/K<0.99
Group 2: In-the-money call	S/K>1.01

[47] Following Merton (1973), Poitras (2002, page 477-479) shows that the lower bound for a European currency call option is the maximum of zero and the difference between the spot price discounted at the foreign risk-free interest rate and the exercise price discounted at the domestic risk-free interest rate. The lower bound for a European put option is the maximum of zero and the difference between the discounted exercise price and the discounted spot price. The lower bound for an American option is the same as the lower bound for a European option. In addition, American options can never be worth less than the immediate exercise value.

[48] In this context it should be noticed that another use of put-call parities is to derive implicit prices. See, for example, the study of Lung and Nishikawa (2005) on currency options.

A multiple regression model is used to test four hypotheses about the early exercise premium. The dependent variable in this model is the relative early exercise premium (REEP). The premium REEP is calculated as the absolute value of (a) – (b) divided by the option price in which (a) is the difference between the American call and put price and (b) is the difference between the call and put price as implied by the put-call parity.

$$REEP = \alpha + \beta_1(r\text{-}r_f) + \beta_2(T\text{-}t) + \beta_3(S/K) + \beta_4(\sigma_{call,t-1}) + \varepsilon$$

The first hypothesis is that the REEP depends on the domestic and foreign interest rates. For calls, the REEP should increase when the domestic interest rate is lower than the foreign rate. In that case the lower boundary becomes $Se^{-r_f(T-t)} - Ke^{-r(T-t)}$. This will be lower than S-K. In that case, there is an incentive for the call to be exercised early. This premium will rise if the difference becomes larger. The situation for the put is the reverse, because if the call is in-the-money, the put is out-of-the-money. Here if the domestic rate is higher than the foreign rate, the REEP will be larger for the put.

The second hypothesis is that the early exercise premium increases with time to maturity. This holds for both calls and puts.

The third hypothesis is that the REEP should increase for calls as the ratio of the spot price to the exercise price (S/K) increases. However, this relationship is not monotonic. On the one hand, when the spot price is higher than the exercise price, calls are in-the-money and thus, are more likely to be exercised. On the other hand, if the option gets very deep in the money, the theoretical trading value of the call reduces to $Se^{-r_f(T-t)} - Ke^{-r(T-t)}$, which is lower than the intrinsic value of S-K. In such a case early exercise will be optimal for the option holder and such options with be exercised and not be captured in the data set. The effect for puts is the opposite sign for calls. The early exercise premium is expected to decrease in absolute terms as the ratio of the spot price to exercise price increases because puts are moving in the direction of out-of-the-money.

The fourth hypothesis is that the REEP increases if the volatility increases. This hypothesis is not obvious. Jorion and Stoughton (1989) argue that a greater volatility raises the optimal exercise boundary for all maturities. However, it also increases the dispersion of future spot prices, which makes it more likely that this boundary is struck before the option's maturity. According to Jorion and Stoughton (1989) the net effect is that increases in volatility also increase the value of the early exercise premium. The volatility is estimated as the implied volatility of the call option with the same exercise price and time to maturity on the day before the estimation day ($\sigma_{call,t-1}$). Notice that we also use the implied volatility of call options for the put regressions. The reason for this is that the implied volatility of a call option gives a better estimate of the future volatility than the implied volatility of a put option. Finally, to take into account the effects of possible heteroskedasticity and autocorrelation, Newey-West standard errors for the regression estimates are reported.

In addition to the regression specification mentioned above we also tried two alternative specifications. Both specifications are related to the non-linearity of the hypothesized relationship between the early exercise premium and moneyness. In the first alternative specification we use S-K instead of S/K. This is in line with Zivney (1991) who also uses S-K. The second alternative specification uses log(S/K) instead of S/K. This is in line with Jorion and Stoughton (1989). The justification for the second alternative specification is that taking a logarithm tends to reduce the skewness and makes the variable more normally distributed.

4.4 Data description

Closing prices for currency options traded on the Philadelphia Stock Exchange (PHLX) are examined for the period from January 2, 1992 to September 24, 1997. Over this period, the currency options on the PHLX experienced active trading and high volumes. The data covers the six currencies that were the most actively traded, i.e. the Australian Dollar, the British Pound, the Canadian Dollar, the Deutsche (German) Mark, the Japanese Yen, and the Swiss Franc. Data on the exercise price, expiration date, spot exchange rate, and the closing prices of the options are derived from the PHLX database. The original database consists of 2,389 pairs of American call and put options that have the same trade date, underlying value, and exercise price. The data sorting process eliminates the options that are at-the-money (1,233), because in this case it is most difficult to attribute the EEP solely to either puts or calls. From the 1,156 options that are left, options with prices that are not consistent with the boundaries of the American put-call parity are eliminated (325). Following Dueker and Miller (2003), 459 observations are eliminated, because they have a negative EEP which is likely caused by non-synchronous reporting of the options and the spot. The remaining sample consisting of 372 observations is further filtered by removing 41 outlying observations where the REEP was greater than 15%. The final sample of 331 put-call pairs is composed of 186 with in-the-money puts and 145 with in-the-money calls. Three-month Eurodollar interest rates, obtained from the US Federal Reserve Board website, are used for the domestic interest rate.[49] These Eurodollar interest rates are applied to the covered interest rate parity condition to determine the foreign interest rates. For this purpose, the currency futures traded on the International Money Market Division of the Chicago Mercantile Exchange are used. These futures have the same expiration cycle as the options traded on the PHLX. The futures prices are from the Thomson Financial Datastream database.

4.5 Results

Table 4.1 summarizes the valuation of early exercise premium in the two groups.

[49] It is well known that Eurodollar interest rates are more appropriate then T-Bill rates in evaluation covered interest relationships, e.g., El-Mekkaoui and Flood (1998). Due to factors such as regulation and market structure the domestic T-Bill markets is less appropriate for capturing actual trade finance costs than the Eurodollar markets

Table 4.1 Market Valuation of Early Exercise Premium for Currency Put Options with S/K < 0.99 and Currency Call Options with S/K>1.01.

This table includes the relative early exercise premium (REEP) of put and call currency options traded on the Philadelphia Stock Exchange between January 2, 1992 and September 24, 1997. This premium is calculated as the absolute value of (a) – (b) divided by the option price in which (a) is the difference between the American call and put price and (b) is the difference between the call and put price as implied by the put-call parity. In total 331 pairs of options are used with an identical exercise price and time to maturity. Puts are taken into account if the ratio of spot price (S) to exercise price (K), S/K < 0.99. Calls are taken into account if S/K>1.01.

	No. of observations	Average US minus foreign interest rate, %	Average premium as % of option price	Median premium as % of option price	Standard deviation of premiums
			Puts		
Overall	186	1.78	5.03	3.74	4.02
Aus. Dollar	9	-1.79	5.27	4.04	4.85
British Pound	25	-1.94	2.78	1.74	3.42
Can. Dollar	19	-0.99	3.92	2.41	3.54
Deutsche Mark	36	1.94	4.59	3.90	3.20
Japanese Yen	65	4.11	6.37	4.89	4.53
Swiss Franc	32	2.43	5.01	4.53	3.33
			Calls		
Overall	145	-1.02	4.60	3.15	3.99
Aus. Dollar	19	-1.56	6.12	5.43	3.83
British Pound	39	-2.78	5.12	4.03	3.65
Can. Dollar	7	-2.49	7.57	6.23	5.62
Deutsche Mark	35	-1.35	4.24	2.66	3.99
Japanese Yen	19	1.38	4.04	1.90	4.43
Swiss Franc	26	1.10	2.78	2.06	2.93

The table shows average premiums as a percentage of the average call or put price. In Table 4.1, the average early exercise premium as a percentage of the put price is 5.03%. The Japanese yen puts show the largest average early exercise premium compared to the puts on the other currencies, as expected for a country with the largest positive interest rate differential. The average early exercise premium as a percentage of the call prices is 4.60%. As expected, the largest early exercise premium occurs for the countries with a negative interest rate differential. If we change the moneyness boundaries and consider calls with $S/K>1.005$ and puts with $S/K<0.995$ we will get the results presented in Table 4.2.

Table 4.2 Market Valuation of Early Exercise Premium for Currency Put Options with S/K < 0.995 and Currency Call Options with S/K>1.005.

This table includes the relative early exercise premium (REEP) of put and call currency options traded on the Philadelphia Stock Exchange between January 2, 1992 and September 24, 1997. This premium is calculated as the absolute value of (a) – (b) divided by the option price in which (a) is the difference between the American call and put price and (b) is the difference between the call and put price as implied by the put-call parity. In total 564 pairs of options are used with an identical exercise price and time to maturity. Puts are taken into account if the ratio of spot price (S) to exercise price (K), S/K < 0.995. Calls are taken into account if S/K>1.005.

	No. of observations	Average US minus foreign interest rate, %	Average premium as % of option price	Median premium as % of option price	Standard deviation of premiums
			Puts		
Overall	295	1.91	5.71	4.34	5.68
Aus. Dollar	17	-1.62	4.55	3.93	3.69
British Pound	35	-1.67	2.76	1.74	3.14
Can. Dollar	24	-0.41	4.99	3.04	5.28
Deutsche Mark	71	1.72	4.77	4.17	3.73
Japanese Yen	101	4.14	7.94	6.43	5.87
Swiss Franc	47	2.49	5.33	4.87	3.84
			Calls		
Overall	269	-1.37	6.88	4.12	7.21
Aus. Dollar	23	-1.54	6.20	5.25	4.08
British Pound	75	-2.43	6.23	4.11	6.35
Can. Dollar	19	-1.61	7.78	5.60	6.89
Deutsche Mark	79	-2.41	10.12	6.38	9.41
Japanese Yen	26	1.32	3.88	2.59	3.94
Swiss Franc	47	0.75	4.11	2.60	4.51

The overall results in Table 4.1 are somewhat different from the results of Table 4.2 where the reported premiums of 5.71% for puts and 6.88% for calls are significantly larger. Results in table 4.2 are form the sample where a narrower range (0.995 < S/K < 1.005) is used for defining at the money and the 41 outliers are included, the results for the (0.99 < S/K < 1.01) with the outliers included are 5.89% for puts and 7.76% for calls. The largest outlier premiums are for the Deutsche Mark in both groups, likely caused by the large fluctuations in German interest rates in 1992-1993. The large fluctuations also explain why the average difference between the US and the German interest rate is positive for put options and negative for call options. In general, the differences within each group between the early exercise premiums for the different currencies can be substantial. For example, in Group 1 (puts) the early exercise premium for the Japanese Yen is 6.37% while the premium for the British Pound is only 2.78%. In Group 2 (calls), the premium for the Canadian dollar is 7.57%, while it is only 2.78% for the Swiss Franc. These large differences are consistent with there being significant cross-sectional variation in foreign interest rates.

Table 4.3 includes descriptive statistics for the dependent and independent regression variables for the 331 put and call option pairs in the sample. Examination of the descriptive statistics reveals that the time to expiration of the options in the sample varies between five days and a little more than a year. The ratio of spot price to exercise price shows that most options are not very far in-the-money or out-of-the-money, since the ratio never exceeds 1.12

for call options and is never below 0.91 for put options. The mean values of 0.98 for puts and 1.03 for calls are close to the in the money boundaries of 0.99 and 1.01 used to define the sample. This is consistent with the theoretical result from Section 4.3 that very deep in-the-money options will be exercised and not sold because market makers will not quote prices above intrinsic value. The maximum EEP values of 14.62% for calls and 14.84% for puts can be compared to the maximum values for the sample with outliers included and more narrow in the money range of 22.00% for puts and 34.09% for calls.

Table 4.4 shows the results of the multiple linear regressions of the relative early exercise premium (REEP) on the different parameters for groups of puts and calls.

Table 4.3 Descriptive Statistics for the Sample of 331 Put and Call Options Traded on TSX Between January 2, 1992 and September 24, 1997

The table represents the descriptive statistics for the sample used. Time to maturity is expressed in years; S/K refers to the ratio of spot to exercise prices. REEP refers to the early exercise premium as percentage of option price. Price refers to the closing price of calls and puts. The early exercise premiums are calculated as the absolute value of (a) – (b) divided by the option closing price in which (a) is the difference between the American call and put option prices and (b) is the difference between the call and put price as implied by the European put-call parity. T-t is the remaining time to maturity of the option, in years; r is the domestic (US) risk-free interest rate, rf is the foreign risk-free rate, *historical volatility* is calculated as the 20-business day rolling standard deviation of the reference currency returns, *implied volatility* is derived from the respective call options.

	REEP, %	Time to maturity	S/K	S-K	log(S/K)	r-rf, %	Historical Volatility	Implied Volatility
			Calls (145 observations)					
MEAN	4.6	0.31	1.03	0.02	0.03	-0.01	0.006	0.124
MEDIAN	3.15	0.2	1.02	0.02	0.02	-0.01	0.006	0.123
MAX	14.62	1.01	1.12	0.15	0.11	0.05	0.015	0.218
MIN	0.1	0.02	1.01	0	0.01	-0.07	0.002	0.065
STDEV	3.98	0.27	0.02	0.02	0.02	0.03	0.003	0.031
			Puts (186 observations)					
MEAN	5.03	0.28	0.98	-0.01	-0.02	0.02	0.006	0.118
MEDIAN	3.77	0.18	0.98	-0.01	-0.02	0.02	0.005	0.112
MAX	14.84	0.95	0.99	0	-0.01	0.06	0.015	0.297
MIN	0	0.01	0.91	-0.13	-0.09	-0.06	0.002	0.040
STDEV	4.01	0.25	0.01	0.02	0.02	0.03	0.002	0.033

Table 4.4 Regression Results Using Implied Volatility

This table includes the results for the following regression equations that explain the relative early exercise premium (REEP) of put and call options traded on the Philadelphia Stock Exchange between January 2, 1992 and September 24, 1997: $REEP = \alpha + \beta_1(r - r_f) + \beta_2(T - t) + \beta_3(S / K) + \beta_4(\sigma_{call,t-1}) + \varepsilon$

$$REEP = \alpha + \beta_1(r - r_f) + \beta_2(T - t) + \beta_3(S - K) + \beta_4(\sigma_{call,t-1}) + \varepsilon$$

$$REEP = \alpha + \beta_1(r - r_f) + \beta_2(T - t) + \beta_3(\log(S / K)) + \beta_4(\sigma_{call,t-1}) + \varepsilon$$

REEP is calculated as the absolute value of (a) – (b) divided by the option price in which (a) is the difference between the American call and put price and (b) is the difference between the call and put price as implied by the put-call parity. In total 331 pairs of options are used with an identical exercise price and time to maturity. There are 145 observations for put options and 186 observations for call options. T-t is the remaining time to maturity of the option; r is the domestic (US) risk-free interest rate, rf is the foreign risk-free interest rate and $\sigma_{call,t-1}$ is the *implied volatility* of the call option with the same exercise price and time to maturity on the day before the estimation day. Puts are taken into account if the ratio of spot price (S) to exercise price (K), S/K < 0.995. Calls are taken into account if S/K>1.005 The equations are estimated using Newey-West standard errors..

	Constant	*r-r_f*	*T-t*	*S/K*	*S-K*	*Log (S/K)*	*$\sigma_{call,t-1}$*	*R^2*
					Puts			
Coefficients	0.49	0.77	0.05	-0.43			-0.43	0.3
t-statistics	2.43*	5.74*	3.49**	-2.13*			-4.20**	
Coefficients	0.06	0.79	0.05		-0.03		-0.36	0.28
t-statistics	6.34**	5.80**	3.46*		-0.2		3.76**	
Coefficients	0.07	0.77	0.05			-0.41	-0.43	0.3
t-statistics	6.67*	5.73*	3.49**			-2.14*	-4.20**	

	Constant	*r-r_f*	*T-t*	*S/K*	*S-K*	*Log (S/K)*	$\sigma_{call,t-1}$	R^2
				Calls				
Coefficients	0.23	-0.9	0.1	-0.24			0.38	0.43
t-statistics	1.11	-6.33**	4.92**	-1.16			2.80**	
Coefficients	-0.02	-0.92	0.1		-0.31		0.4	0.43
t-statistics	-1.16	-6.21**	5.01**		-1.49		2.98**	
Coefficients	-0.02	-0.9	0.1			-0.25	0.38	0.43
t-statistics	-1.1	-6.33**	4.92**			-1.1	2.79**	

*-significant at the 5%-level, **-significant at the 1%-level.

We first discuss the results for the regressions in which moneyness is defined as S/K. The results for the interest rate differential confirm the first hypothesis. The REEP is positively related to the difference between the domestic (US) and the foreign interest rate for puts and negatively for calls. This coefficient is significant at the 5%-level for puts and at the 1%-level for calls. The results for time to maturity for both calls and puts give the hypothesized sign as well. Moreover, in both cases the coefficient is significant at the 1%-level. The results for the moneyness give the hypothesized sign for put options. This coefficient is significantly different from zero at the 5%-level. The sign for call options is negative and insignificant, whereas we hypothesized a positive sign. This result is different from Zivney (1991) who finds that the coefficient of the moneyness is highly significant. This may be caused by the fact that he considers index options, and that we consider currency options. Another potential explanation for the unexpected result for call options is that the hypothesized relationship only holds if the options are not very far in-the-money. However, given the fact that the maximum value for S/K is only 1.121, this explanation is not likely to hold. The results for the fourth hypothesis give the expected significantly positive coefficient for call options. However, the effect for put options is significantly negative, which is contrary to our hypothesis.[50] In this context it is interesting to notice that the empirical study of Jorion and Stoughton

[50] A possible reason for this is that the data from American options are used in order to calculate implied volatilities. This may give volatility results that are not entirely reliable.

(1989) also finds a negative, albeit insignificant, relation between the early exercise premium for put options and volatility.[51]

In addition, we include the results for regressions with alternative specifications for the relationship between S and K. In the first of these specifications we replace S/K by S-K and in the second we replace S/K by log(S/K). The results for these alternative specifications are largely the same as for the original specification. The signs and the significance for the coefficients for time to maturity, interest rate differential and volatility are the same, only the level of significance changes between regressions (in all cases from 1% to 5% and vice versa). However, for put options the coefficient for the relationship between the spot price and the exercise price is no longer significant if we substitute (S/K) by (S-K). In the regression for log(S/K) the coefficient has again its hypothesized significant negative coefficient. All three different specifications of the regressions for call options show an insignificant negative coefficient for the relationship between S and K.

A final remark on the regression analysis concerns the R^2s of the regressions. These are high. They vary between 0.28 and 0.30 for the put options. The regressions for the call options all show an R^2 of 0.43, indicating that the variables in these regressions explain a large part of the variation in REEP.

As a robustness check, we include a series of regressions with historical volatility used as a measure of option's volatility. In these regressions the

[51] The t-statistics in their regression is -1.02.

volatility is calculated as the historical volatility of the underlying asset calculated as the 20-business day rolling standard deviation of the reference currency returns. The results of the regressions are presented in Table 4.5. As can be seen from the table, the REEP is positively (negatively) and significantly related to the interest rate differential for the put (call) options. The coefficients for the historical volatility are negative and significant for puts, and negative and non-significant for calls. The REEP for calls is positively and significantly related to the time to maturity; the same positive relationship is non-significant for put options. Finally, EEPs are not significantly related to the degree of option moneyness for both put and call options. Overall, the results are similar for different specifications of the relationship between S and K. The R^2s are equal to 0.18 and 0.29 in the puts and calls regressions respectively.

Table 4.5 Regression Results Using Historical Volatility

The table provides the results of the regressions that explain the variation of the relative early exercise premiums (REEP) of *call* options. The options were traded on the Philadelphia Stock Exchange (PLHX) during 01/1992-09/1997. REEP is calculated as the absolute value of (a) – (b) divided by the option closing price in which (a) is the difference between the American call and put option prices and (b) is the difference between the call and put price as implied by the European put-call parity. The sample contains 145 observations on call options and 186 observations on put options. T-t is the remaining time to maturity of the option, in years; r is the domestic (US) risk-free interest rate, r_f is the foreign risk-free rate and Volatility is the historical volatility of the underlying asset calculated as the 20-business day rolling standard deviation of the reference currency returns. To avoid problems with the attribution of the early exercise premium to either calls or puts we ignore the observations with at-the-money options. Call options are defined as in-the-money and included in the sample if the ratio of underlying spot price (S) to the exercise price (K), S/K>1.01. Put options are defined as in-the-money and included in the sample if the ratio of underlying spot price (S) to the exercise price (K), S/K < 0.99. The standard errors are estimated using the heteroskedasticity and autocorrelation consistent covariance matrix of Newey and West (1987). * denotes significance at the 5%-level; ** denotes significance at the 1%-level.

	Intercept	$r-r_f$	*T-t*	*S/K*	*log(S/K)*	*S-K*	*Volatility*	R^2
					Puts			
Coefficient	-0.01	0.52	0.02	0.06			-2.48	0.18
t-Statistics	-0.04	4.40**	1.91	0.34			-2.71**	
Coefficient	0.05	0.52	0.02		0.06		-2.49	0.18
t-Statistics	6.81**	4.40**	1.91		0.34		-2.71**	
Coefficient	0.05	0.49	0.02			0.09	-2.47	0.18
t-Statistics	7.23**	3.97**	1.92			0.77	-2.83**	

	Intercept	$r\text{-}r_f$	*T-t*	*S/K*	*log(S/K)*	*S-K*	*Volatility*	R^2
					Calls			
Coefficient	0.19	-0.53	0.05	-0.15			-1.17	0.29
t-Statistics	1.50	-4.83**	3.43**	-1.23			-1.11	
Coefficient	0.04	-0.53	0.05		-0.16		-1.17	0.29
t-Statistics	4.65**	-4.83**	3.43**		-1.24		-1.11	
Coefficient	0.03	-0.53	0.05			-0.03	-1.19	0.29
t-Statistics	4.48**	-4.49**	3.34**			-0.36	-1.10	

4.6 Conclusions

The original Black-Scholes model does not consider the early exercise premium when valuing the American options. Therefore, the size of these premiums has to be estimated empirically. Importance of the correct estimation of currency options' time value is emphasized by the fact that American currency options are more likely to be exercised before maturity than other types of options, such as equity or index options. The prime candidates for the early exercise are call options on high-interest rate currencies and put options on low-interest currencies.

Zivney (1991) provides a methodology for derivation of the early exercise premium for index options using the put-call parity condition. We use methodology of Zivney (1991) to estimate the premiums for American currency options and find that they are slightly higher for put options than for call options. More specifically, using a sample of 331 pairs of call and put currency options traded on the Philadelphia Stock Exchange between January 2, 1992 and September 24, 1997 we find that the average early exercise premium is 5.03% for put options and 4.60% for call options. Using the regression analysis, we find that the early exercise premiums are affected by the time to maturity and the interest rate differential. However, the exact relationship between the REEP and the interest rate differential, time to maturity, moneyness, and volatility depends on the volatility measure used in the regression.

The R^2's of the regressions are high indicating that a large part of the variation in the early exercise premiums is explained by the variation in the interest rate differential, time to maturity, moneyness ratio, and volatility.

APPENDIX

Characteristics of the Convertible Bonds used in the Study

This table reports the main characteristics of the convertible securities used in our study. The sample consists of 64 Canadian convertible bonds traded on the Toronto Stock Exchange. CONVR shows the number of stocks obtained in case of conversion for each 100 dollars of bond face value. The underlying stock volatility is expressed as the annualized standard deviation. In the call schedule the first number refers to the call price per 100 dollars of face value, the second number refers to the starting date of calling at this price. Calling continues until the next call date (if any) or until the maturity date if not specified otherwise. Conversion ratio is per hundred dollars of face value. VOLAT refers to the underlying stock volatility.

Credit spreads are in basis points and are derived from the corporate credit rating using the 2005 Royal Bank of Canada relative value curves for Canadian corporate bonds. The "Income trust" column reports whether the issuing entity was an income trust. The "Call notice period" provides the minimum number of days between the calling announcement and actual call date. "Industry" specifies the area of specialization of the issuing firm.

Soft call lists conditions to be met before the bond can be called. For example, "20 days cumulative (consecutive) above 125% of conversion" means that the stock has to be traded above 125% of its conversion price for at least 20 (consecutive) days in any given 30-day period before the issuer can call the bond; "N" refers to the absence of the soft call condition for a given bond.

Asterisks (*) denote the firms with a simple capital structure consisting of equity, straight debt, and convertible debt. These firms are used in the estimation of the Brennan-Schwartz model.

Issuer (Symbol)	Issue Date	Maturity Date	Coupon, %	CONVR	VOLAT	Credit Spread	Call schedule	Income Trust	Call notice period	Industry	Soft call
Advantage Energy AVN.DB.A	3-Jul	8-Aug	9	5.9	0.21	45	105 - 08/01/06, 102.5 -08/01/07	Y	30-60	Oil and gas	N
Advantage Energy AVN.DB.B	3-Dec	9-Feb	8.25	6.1	0.21	65	105 - 02/01/07, 102.5 -02/01/08	Y	30-60	Oil and gas	N

Issuer (Symbol)	Issue Date	Maturity Date	Coupon, %	CONVR	VOLAT	Credit Spread	Call schedule	Income Trust	Call notice period	Industry	Soft call
Advantage Energy AVN.DB.C	3-Jul	9-Oct	7.5	4.9	0.21	65	105 - 10/01/07, 102.5 -10/01/08	Y	30-60	Oil and gas	N
Advantage Energy AVN.DB.D	5-Jan	11-Dec	7.75	4.8	0.21	75	105 - 12/01/07, 102.5 -12/01/08	Y	30-60	Oil and gas	N
Agricore United AU.DB	2-Nov	7-Nov	9	13.3	0.30	65	100 - 12/01/05	N	30-60	Agriculture	N
Alamos Gold AGI.DB	5-Jan	10-Feb	5.5	18.9	0.48	65	100 - 02/15/08	N	30-60	Metals and mining	20 days cumulative above 125%
Alexis Nihon AN.DB*	4-Aug	14-Jun	6.2	7.3	0.14	128	100 - 06/30/08	Y	30-60	Real estate	20 days cumulative above 125%
Algonquin Power APF.DB*	4-Jul	11-Jul	6.65	9.4	0.17	69	100 - 07/31/07	Y	30-60	Utilities	20 days cumulative above 125%
Baytex Energy BTE.DB	5-Jun	10-Dec	6.4	6.8	0.26	115	105 - 12/31/08, 102.5 -12/31/09	Y	30-60	Oil and gas	N
Bonavista Energy BNP.DB	4-Jan	9-Jun	7.5	4.4	0.26	83	105 - 02/01/07, 102.5 -02/01/08	Y	30-60	Oil and gas	N
Bonavista Energy BNP.DB.A	4-Dec	10-Jul	6.75	3.5	0.26	105	105 - 12/31/07, 102.5 -12/31/08	Y	30-60	Oil and gas	N
Boyd Group BYD.DB*	3-Sep	8-Sep	8	11.6	0.60	45	105 - 09/30/04, 102.5 -09/30/05	Y	N/A		

Issuer (Symbol)	Issue Date	Maturity Date	Coupon, %	CONVR	VOLAT	Credit Spread	Call schedule	Income Trust	Call notice period	Industry	Soft call
Calloway REIT CWT.DB	4-Apr	14-Jun	6	5.9	0.22	88	100 - 06/30/08	Y	30-60	Real estate	Previous day price above 125%
Cameco Corp CCO.DB*	3-Sep	13-Oct	5	4.6	0.40	82	100 - 10/01/08	N	30-60	Metals and mining	N
Can Hotel Inc. HOT.DB	2-Feb	7-Sep	8.5	10.4	0.15	65	100 - 03/01/05	Y	30-60	Real estate	20 days consecutive above 115%
Can Hotel Inc. HOT.DB.A	4-Nov	14-Nov	6	8.5	0.15	128	100 - 11/30/08	Y	30-60	Real estate	20 days consecutive above 125%
Chemtrade CHE.DB*	2-Dec	7-Dec	10	6.9	0.23	45	105 - 12/31/05, 102.5 -12/31/06	Y	30-60	Chemicals	Previous day price above 125%
Cineplex Galaxy CGX.DB*	5-Jul	12-Dec	6	5.3	0.27	75	100 - 12/31/08	Y	30-60	Media	20 days consecutive above 125%
Clean Power CLE.DB	4-Jun	10-Dec	6.75	9.8	0.34	69	100 - 06/30/07	Y	30-60	Utilities	Previous day price above 125%
Clublink LNK.DB*	Apr-98	8-May	6	5	0.18	65	100 - 03/15/03	N	30-60	Leisure	N
Cominar CUF.DB	4-Sep	14-May	6.3	5.8	0.15	88	100 - 06/30/08	Y	30-60	Real estate	20 days consecutive above 125%
Creststreet Power CRS.DB	5-Jan	10-Mar	7	10	0.19	65	100 - 03/15/08	Y	30-60	Utilities	Current price above 125%

Issuer (Symbol)	Issue Date	Maturity Date	Coupon, %	CONVR	VOLAT	Credit Spread	Call schedule	Income Trust	Call notice period	Industry	Soft call
Daylight Energy DAY.DB*	4-Oct	9-Dec	8.5	10.5	0.21	65	105 - 12/01/07, 102.5 -12/01/08	Y	N/A	Oil and gas	N
Dundee REIT D.DB	4-May	14-Jun	6.5	4	0.17	117	100 - 06/30/08	Y	30-60	Real estate	20 days consecutive above 125%
Dundee REIT D.DB.A	5-Apr	15-Mar	5.7	3.3	0.17	117	100 - 03/31/09	Y	30-60	Real estate	20 days consecutive above 125%
Esprit Energy EEE.DB*	5-Jul	10-Dec	6.5	7.2	0.23	65	105 - 12/31/08, 102.5 -12/31/09	Y	30-60	Oil and gas	N
Fort Chicago Energy FCE.DB.A	3-Jan	8-Jun	7.5	11.1	0.22	45	100 - 01/31/06	Y	30-60	Oil and gas	20 days consecutive above 125%
Fort Chicago Energy FCE.DB.B	3-Oct	10-Dec	6.75	9.4	0.22	55	100 - 12/31/06	Y	30-60	Oil and gas	20 days consecutive above 125%
Gerdau AmeriSteel Corp. GNA.DB*	Apr-97	7-Apr	6.5	3.8	0.44	65	100 - 04/30/02	N	30-60	Metals and mining	N
Harvest Energy HTE.DB	4-Jan	9-May	9	7.1	0.31	90	105 - 05/31/07, 102.5 -05/31/08	Y	30-60	Oil and gas	N
Harvest Energy HTE.DB.B	5-Jul	10-Dec	6.5	3.2	0.31	90	105 - 12/31/08, 102.5 -12/31/09	Y	40-60	Oil and gas	N
InnVest INN.DB.A	4-Mar	11-Apr	6.25	8	0.19	105	100 - 04/15/08	Y	30-60	Real estate	20 days consecutive above 125%

Issuer (Symbol)	Issue Date	Maturity Date	Coupon, %	CONVR	VOLAT	Credit Spread	Call schedule	Income Trust	Call notice period	Industry	Soft call
Inter Pipeline IPL.DB	2-Nov	7-Dec	10	16.7	0.22	45	100 - 12/31/05	Y	30-60	Oil and gas	20 days consecutive above 125%
IPC US REIT IUR.DB.U	4-Nov	14-Nov	6	10.5	0.21	90	100 - 11/30/08	Y	30-60	Real estate	20 days consecutive above 125%
IPC US REIT IUR.DB.V	5-Sep	10-Sep	5.75	9.1	0.21	65	100 - 09/30/08	Y	30-60	Real estate	20 days consecutive above 125%
Keyera KEY.DB*	4-Jun	11-Jun	6.75	8.3	0.24	65	100 - 06/30/07	Y	30-60	Oil and gas	20 days consecutive above 125%
Legacy Hotels LGY.DB*	2-Feb	7-Apr	7.75	11.4	0.20	65	100 - 04/01/04	Y	30	Hotels	20 days consecutive above 115%
Magellan Aerospace MAL.DB*	2-Dec	8-Jan	8.5	22.2	0.44	45	100 - 01/31/06	N	40-60	Aerospace	20 days consecutive above 125%
MDC Partners MDZ.DB	5-Jun	10-Jun	8	7.1	0.34	83	100 - 06/30/08	N	30-60	Marketing services	20 out of 30 consecutive days above 125%
Morguard Real Estate MRT.DB.A	2-Jul	7-Nov	8.25	10	0.13	45	100 - 11/01/05	Y	30-60	Real estate	20 days consecutive above 125%
NAV Energy NVG.DB	4-May	9-Jun	8.75	9.1	0.27	65	105 - 06/30/07, 102.5 -06/30/08	Y	30-60	Oil and gas	N
Northland Power NPI.DB*	4-Aug	11-Jun	6.5	8	0.25	69	100 - 06/30/07	Y	30-60	Utilities	20 days consecutive above 125%

Issuer (Symbol)	Issue Date	Maturity Date	Coupon, %	CONVR	VOLAT	Credit Spread	Call schedule	Income Trust	Call notice period	Industry	Soft call
Paramount Energy PMT.DB	4-Aug	9-Sep	8	7	0.22	65	105 - 09/30/07, 102.5 -09/30/08	Y	40-60	Oil and gas	N
Paramount Energy PMT.DB.A	5-Apr	10-Jun	6.25	5.2	0.22	65	105 - 06/30/08, 102.5 -06/30/09	Y	30-60	Oil and gas	N
Pembina PIF.DB.A	1-Dec	7-Jun	7.5	9.5	0.24	45	100 - 06/30/05	Y	30-60	Oil and gas	20 days consecutive above 125%
Pembina PIF.DB.B	3-Jun	10-Dec	7.35	8	0.24	55	100 - 06/30/06	Y	30-60	Oil and gas	20 days consecutive above 125%
Primaris REIT PMZ.DB*	4-Jun	14-Jun	6.75	8.2	0.20	117	100 - 06/30/08	Y	40-60	Real estate	20 days consecutive above 125%
Primewest Energy PWI.DB.A	4-Aug	9-Sep	7.5	3.8	0.24	83	105 - 09/30/07, 102.5 -09/30/08	Y	30-60	Oil and gas	N
Primewest Energy PWI.DB.B	4-Aug	11-Dec	7.75	3.8	0.24	105	105 - 12/31/07, 102.5 -12/31/08	Y	30-60	Oil and gas	N
Progress Energy PGX.DB	5-Jan	10-May	6.75	6.7	0.26	65	105 - 12/31/07, 102.5 -12/31/08	Y	30-60	Oil and gas	N
Provident Energy PVE.DB.A	3-Sep	8-Dec	8.75	9.1	0.21	65	100 - 01/01/07	Y	30-60	Oil and gas	N
Provident Energy PVE.DB.B	4-Jul	9-Jul	8	8.3	0.21	83	100 - 07/31/07	Y	30-60	Oil and gas	N

Issuer (Symbol)	Issue Date	Maturity Date	Coupon, %	CONVR	VOLAT	Credit Spread	Call schedule	Income Trust	Call notice period	Industry	Soft call
Provident Energy PVE.DB.C	5-Feb	12-Aug	6.5	7.3	0.21	105	100 - 08/31/08	Y	30-60	Oil and gas	N
Retirement Res REIT RRR.DB.B	3-Jul	11-Jan	8.25	8.1	0.25	95	100 - 07/31/07	Y	30-60	Real estate	20 days consecutive above 125%
Retirement Res REIT RRR.DB.C	5-Apr	15-Mar	5.5	8.8	0.25	95	100 - 03/31/09	Y	30-60	Real estate	20 days consecutive above 125%
Retrocom Mid-Market RMM.DB*	5-Jul	12-Jul	7.5	12.1	0.29	75	100 - 08/31/09	Y	30-60	Real estate	20 days consecutive above 125%
Rogers Sugar RSI.DB.A	5-Mar	12-Jun	6	18.9	0.24	75	100 - 06/29/08	Y	30-60	Food	20 days consecutive above 125%
Royal Host Real Estate RYL.DB	2-Feb	7-Mar	9.25	14.3	0.19	65	100 - 03/01/05	Y	30	Real estate	20 days consecutive above 125%
Summit Real Estate SMU.DB	4-Feb	14-Mar	6.25	4.7	0.21	88	100 - 03/31/08	Y	30-60	Real estate	20 days consecutive above 125%
Superior Propane SPF.DB	1-Jan	7-Jul	8	6.3	0.27	45	100 - 02/01/04	Y	30-60	Diverse	20 days consecutive above 125%
Superior Propane SPF.DB.A	2-Dec	8-Nov	8	5	0.27	45	100 - 11/01/05	Y	30-60	Diverse	20 days consecutive above 125%
Superior Propane SPF.DB.B	5-Jun	12-Dec	5.75	2.8	0.27	69	100 - 07/01/08	Y	30-60	Diverse	20 days consecutive above 125%

Issuer (Symbol)	Issue Date	Maturit y Date	Coupon , %	CONVR	VOLAT	Credit Spread	Call schedule	Incom e Trust	Call notice period	Industry	Soft call
Superior Propane SPF.DB.C	5-Oct	15-Oct	5.85	3.2	0.27	88	100 - 10/31/08	Y	30-60	Diverse	20 days consecutive above 125%
Taylor NGL TAY.DB*	5-Mar	10-Sep	5.85	9.7	0.27	75	100 - 09/10/08	Y	30-60	Oil and gas	20 days consecutive above 125%

BIBLIOGRAPHY

Agarwal, V., W.H. Fung, Y.C. Loon, and N.Y. Naik, 2008, "Risk and return in convertible arbitrage: Evidence from the convertible bond market.", *Working paper*, Georgia State University, CFR, London Business School, and SUNY-Binghamton.

Ammann, M., A. Kind, and C. Wilde, 2003, "Are convertible bonds underpriced? An analysis of the French market", *Journal of Banking and Finance 27*, 635-653.

Andersen, L. and D. Buffum, 2004, "Calibration and implementation of convertible bond models", *The Journal of Computational Finance 7*, 1-34.

Asquith, P., 1995, "Convertible bonds are not called late", *The Journal of Finance 50*, 1275-1289.

Asquith, P. and D.W. Mullins, 1991, "Convertible debt: corporate call policy and voluntary conversion", *The Journal of Finance 46*, 1273-1289.

Ayache, E., P.A. Forsyth, and K.R. Vetzal, 2003, "Valuation of convertible bonds with credit risk", *The Journal of Derivatives 11*, 9-30.

Bancel, F. and U.R. Mittoo, 2004, "Cross-country determinants of capital structure choice: A survey of European firms", *Financial Management 33*, 103-132.

Black, F. and M. Scholes, 1973, "The pricing of options and corporate liabilities", *Journal of Political Economy 81*, 637-654.

Bodurtha, J.N. and G.R. Courtadon, 1995, "Probabilities and values of early exercise: spot and futures foreign currency options", *The Journal of Derivatives 3*, 57-75.

Bollen, N. and E. Rasiel, 2003, "The performance of alternative valuation models in the OTC currency options market", *Journal of International Money and Finance 22*, 33-64.

Brennan, M.J. and E.S. Schwartz, 1977, "Convertible bonds: valuation and optimal strategies for call and conversion", *The Journal of Finance 32*, 1699-1715.

Brennan, M.J. and E.S. Schwartz, 1980, "Analyzing convertible bonds", *Journal of Financial and Quantitative Analysis 15*, 907-929.

Brigham, E.F., 1966, "An Analysis of Convertible Debentures: Theory and Some Empirical Evidence", *The Journal of Finance 21*, 35-54.

Brounen, D., A. De Jong, and K. Koedijk, 2006, "Capital structure policies in Europe: Survey evidence", *Journal of Banking and Finance 30*, 1409-1442.

Campbell, C.J., L.H. Ederington, and P. Vankudre, 1991, "Tax shields, sample-selection bias, and the information content of conversion-forcing bond calls", *The Journal of Finance 46*, 1291-1324.

Carayannopoulos, P., 1996, "Valuing convertible bonds under the assumption of stochastic interest rates: an empirical investigation", *Quarterly Journal of Business and Economics 35*, 17–31.

Cheung, W. and I. Nelken, 1994, "Costing the converts", *RISK 7*, 47-49.

Cowan, A.R., N. Nayar, and A.K. Singh, 1990, "Stock returns before and after calls of convertible bonds", *The Journal of Financial and Quantitative Analysis 25*, 549-554.

Crank, J. and P. Nicolson, 1947, "A practical method for numerical evaluation of solutions of partial differential equations of the heat conduction type", *Proceedings of the Cambridge Philosophical Society 43*, 50–64.

Datta, S. and M. Iskandar-Datta, 1996, "New evidence on the valuation effects of convertible bond calls", *Journal of Financial and Quantitative Analysis 31*, 295-307.

Department of Finance, Government of Canada, 2005, "Tax and other Issues Related to Publicly Listed Flow-Through Entities (Income Trusts and Limited Partnerships)", *Consultation Paper*, September.

De Roon, F. and C. Veld, 1996, "Put-call parities and the value of early exercise for put options on a performance index", *The Journal of Futures Markets 16*, 71-80.

Dueker, M.J. and T.W. Miller, 2003, "Directly Measuring Early Exercise Premiums Using American and European S&P 500 Index Options", Journal of Futures Markets 23, 287-313.

Duffie, D. and K.J. Singleton, 1999, "Modelling term structure of defaultable bonds", *Review of Financial Studies 12*, 687-720.

Dunn, K.B. and K.M. Eades, 1989, "Voluntary conversion of convertible securities and the optimal call strategy", *Journal of Financial Economics 23*, 273-301.

El-Mekkaoui, M. and M.D. Flood, 1998, "Put-call parity revisited: intradaily tests in the foreign currency options market", *Journal of International Financial Markets, Institutions and Money 8*, 357-376.

Engström, M. and L. Nordén, 2000, “The early exercise premium in American put option prices”, *Journal of Multinational Financial Management 10,* 461-479.

Garman, M. and S. Kohlhagen, 1983, “Foreign Currency Option Values”, *Journal of International Money and Finance* 2, 231-237.

Goldman-Sachs, 1994, “Valuing convertible bonds as derivatives”, *Goldman-Sachs Quantitative Strategies Research Note*, November.

Goldstein, R., N. Ju, and H. Leland, 2001, “An EBIT-based model of dynamic capital structure”, *Journal of Business 74,* 483-512.

Graham, J.R. and C.R. Harvey, 2001, "The theory and practice of corporate finance: evidence from the field”, *Journal of Financial Economics 60*, 187-243.

Grimwood, R. and S. Hodges, 2002, “The valuation of convertible bonds: A study of alternative pricing models”, *Working paper*, Warwick University.

Halpern, P., 2004, “Is the trust in trusts misplaced? A study of business income trusts and their role in Canadian capital markets”, *Working paper*, Rotman School of Management, University of Toronto.

Harris, M. and A. Raviv, 1985, "A sequential model of convertible debt call policy”, *The Journal of Finance 40,* 1263-1282.

Ho, T. and D. Pfeffer, 1996, “Convertible bonds: model, value attribution and analytics”, *Financial Analysts Journal 52*, 35-44.

Hull, J, 2006, “Options, futures, and other derivatives”, 6th edition, Prentice-Hall, Englewood Cliffs, New Jersey.

Hung, M.W. and J.Y. Wang, 2002, “Pricing convertible bonds subject to default risk”, *The Journal of Derivatives 10*, 75-87.

Ingersoll, J., 1977a, “A contingent claims valuation of convertible securities”, *Journal of Financial Economics 4*, 289-322.

Ingersoll, J., 1977b, "An examination of corporate call policies on convertible securities”, *The Journal of Finance 32,* 463-478.

Jarrow, R. and S. Turnbull, 1995, “Pricing derivatives on financial securities subject to credit risk”, *The Journal of Finance 50*, 53-85.

Jorion, P. and N.M. Stoughton, 1989, “An empirical investigation of the early exercise premium of foreign currency options”, *The Journal of Futures Markets 9*, 365-375.

King, R., 1986, “Convertible bond valuation: An empirical test”, *The Journal of Financial Research 9*, 53–69.

Laprise, S., Fu, M., S. Marcus, A. Lim, and H. Zhang, 2006, "Pricing American-Style Derivatives with European Call Options", *Management Science 52*, 95-110.

Lau, K. W. and Y. K. Kwok, 2004, "Anatomy of option features in convertible bonds", *Journal of Futures Markets 24*, 513–532.

Loncarski, I., J. ter Horst, and C. Veld, 2006, "Why do companies issue convertible bonds? A review of theory and empirical evidence", In: L.Renneboog (editor), Advances in Corporate Finance and Asset Pricing, Elsevier, Amsterdam, 311-339.

Loncarski, I., J. ter Horst, and C. Veld, 2007, "The rise and demise of the convertible arbitrage strategy", *Working Paper*, Tilburg University.

Loncarski, I., J. ter Horst, and C. Veld, 2008, "Why do companies issue convertible bonds: an empirical analysis for the Canadian market", Canadian Journal of Administrative Sciences (forthcoming).

Longstaff, F. and E.S. Schwartz, 1995, "A simple approach to valuing risky fixed and floating rate debt", *The Journal of Finance 50*, 789-819.

Lung, P.P. and T. Nishikawa, 2005, "The implied exchange rates derived from option premiums: A test of the currency option boundary", *The Review of Futures Markets 14*, 167-198.

Madan, D. and H. Unal, 2000, "A two-factor hazard rate model for pricing risky debt and the term structure of credit spreads", *Journal of Financial and Quantitative Analysis 35*, 43-65.

Marquardt, D., 1963, "An algorithm for least-squares estimation of nonlinear parameters", *SIAM Journal of Applied Mathematics 11*, 431-441.

Mazzeo, M.A. and W.T. Moore, 1992, "Liquidity costs and stock price response to convertible security calls", *The Journal of Business 65*, 352-369.

McConnell, J. and E.S. Schwartz, 1986, "LYON Taming", *The Journal of Finance 50*, 53-85.

Merton, R.C., 1973, "Theory of rational option pricing", *Bell Journal of Economics and Management Science 4*, 141-173.

Merton, R.C., 1974, "On the pricing of corporate debt: the risk structure of interest rates", *The Journal of Finance 29*, 449-470.

Mikkelson, W.H., 1981, "Convertible calls and security returns", *Journal of Financial Economics 9*, 237-264.

Newey, W.K. and K.D. West, 1987, "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix", *Econometrica* 55, 703-708.

Nyborg, K., 1996, "The use and pricing of convertible bonds", *Applied Mathematical Finance 3*, 167-190.

Ofer, A.R. and A. Natarajan, 1987, "Convertible call policies: an empirical analysis of an information-signalling hypothesis", *Journal of Financial Economics 19*, 91-108.

Poitras, G., 2002, "Risk Management, speculation, and derivative securities", Academic Press, New York.

Poitras, G., C. Veld, and Y. Zabolotnyuk, 2007, "Put-Call Parity and the Early Exercise Premium for Currency Options", *Review of Futures Markets 16*, 159-169.

Poitras, G., C. Veld, and Y. Zabolotnyuk, 2009, "European Put-Call Parity and the Early Exercise Premium for American Currency Options", *Multinational Finance Journal* (forthcoming).

Sarkar S., 2003, "Early and late calls of convertible bonds: theory and evidence", *Journal of Banking and Finance 27*, 1349-1374.

Schultz, P., 1993, "Calls of warrants: timing and market reaction", *The Journal of Finance 48*, 681-696.

Takahashi, A., T. Kobayashi, and N. Nakagawa, 2001, "Pricing convertible bonds with default risk", *The Journal of Fixed Income 11*, 20-29.

Tsiveriotis, K. and C. Fernandes, 1998, "Valuing convertible bonds with credit risk", *The Journal of Fixed Income 8*, 95-102.

Veld, C., and Y. Zabolotnyuk, 2008, "The Optimal Call Policy for Convertible Bonds: Is there a Market Memory Effect?", *Working paper,* Carleton University and University of Stirling.

Weston, J.F. and T.E. Copeland, 1986, "Managerial Finance", 8th edition, The Dryden Press.

Woodson, H., 2002, "Global convertible investing: the Gabelli way", Wiley, New York.

Zabolotnyuk, Y., R. Jones, and C. Veld, 2008, "An empirical comparison of convertible bond valuation models", *Working paper,* Carleton University, Simon Fraser University, and University of Stirling.

Zhou, C., 2001, "The term structure of credit spreads with jump risk", *Journal of Banking and Finance 25*, 2015-2040.

Zivney, T.L., 1991, "The value of early exercise in option prices: an empirical investigation", *Journal of Financial and Quantitative Analysis 26*, 129-138.