

THE FORMATION AND DECAY OF
THE COMPOUND NUCLEUS Ge^{68}

BY

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ABSTRACT

Experimental excitation functions have been measured by radiochemical means for a number of reactions proceeding through the compound nucleus Ge^{68} . The target-projectile pairs were: $\text{He}^4 + \text{Zn}^{64}$, $\text{C}^{12} + \text{Fe}^{56}$, and $\text{O}^{16} + \text{Cr}^{52}$ leading to the following reactions: (α, γ) , (x, p) , (x, n) , (x, pn) , and $(x, 2n)$. Recoil ranges were measured for products of the alpha-induced reactions in order to determine those reactions which proceeded by compound nucleus formation and decay. It has been found that the "independence hypothesis" is verified for reactions induced by different target-projectile pairs but proceeding through compound nuclei of nearly equal angular momentum.

Excitation functions have been calculated with the SFU IBM System 360/40 computer via the statistical theory of nuclear reactions according to a formalism containing the explicit dependence of nuclear emission probabilities on angular momentum. Probabilities for γ -ray emission were calculated according to the single-particle model coupled with enhancement functions chosen to reflect experimentally observed collective effects.

Agreement between experiment and calculations was found to be good for the $\text{Zn}^{64} + \text{He}^4$ excitation functions. The "high energy tails" of the excitation functions were found to be accounted for by effects of γ -ray competition with particle emission when the latter was suppressed by angular

momentum effects. Agreement in the $\text{Fe}^{56} + \text{C}^{12}$ case was found to be very encouraging, considering the complexity of the target-projectile system and attendant theoretical difficulties.

RESUME

Les fonctions d'excitation expérimentales ont été mesurées par les moyens radiochimiques pour plusieurs réactions qui ont lieu comme résultat de la formation du noyau composé Ge^{68} . Les paires de projectile-cibles étaient: $\text{He}^4 + \text{Zn}^{64}$, $\text{C}^{12} + \text{Fe}^{56}$, et $\text{O}^{16} + \text{Cr}^{52}$ qui produisent les réactions suivantes: (α, γ) , (x, p) , (x, n) , (x, pn) , et $(x, 2n)$. On a mesuré les distances d'atomes reculants pour les produits des réactions alpha-induites pour déterminer ces réactions qui ont résulté de la formation et de la décomposition de noyaux composés. On a vérifié "l'hypothèse d'indépendance" pour les réactions, induites par de différentes paires de projectile-cibles, mais qui résultent de noyaux composés d'à peu près la même quantité de mouvement angulaire.

On s'est servi de l'ordinateur SFU IBM Système 360/40 pour calculer les fonctions d'excitation par moyen de la théorie statistique de réactions nucléaires et selon le formalisme, qui contient la dépendance explicite, des probabilités d'émission nucléaire, du quantité de mouvement angulaire. On a calculé les probabilités d'émission de rayons γ selon le modèle à une seule particule en conjonction avec les fonctions d'augmentation, qui ont été choisies pour démontrer les effets collectifs observés par de différents expérimentateurs.

L'accord entre l'expérience actuelle et les calculs pour les fonctions d'excitation $\text{Zn}^{64} + \text{He}^4$, était bon. On pourrait expliquer la persistance de grandes valeurs pour la section efficace aux hautes énergies par les effets de la concurrence entre les rayons γ et l'évaporation de particules quand celle-la fut supprimée par les effets de quantité de mouvement angulaire. L'accord au cas de $\text{Fe}^{56} + \text{C}^{12}$ était très encourageant, si l'on tient compte de la complexité du système de projectile-cibles et des difficultés théoriques.

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I. INTRODUCTION

A. Nuclear Reaction Mechanisms

1. Compound Nucleus Reactions

The term nuclear reaction is applied to a variety of processes involving the collisions of nucleons or groups of nucleons.

One of the simplest types of nuclear reaction is the interaction of a neutron with a target nucleus. Early workers^{1.}, measuring the variation of neutron reaction cross-sections over small bombarding energy ranges, found large fluctuations (termed "resonances"), the widths (≈ 0.1 eV) of which are small compared to the spacing ($1 - 10^3$ eV) between them. Bethe^{2.} attempted to explain neutron reactions in terms of a potential-well model, but was unable to account for the narrowness of the resonances. The postulate that the resonances corresponded to many-particle excited states of the product nucleus rather than virtual single-particle states of a neutron in a nuclear potential well led to the conclusion by Bohr^{3.} and by Breit and Wigner^{4.} that the incoming neutron rapidly shared its energy with the nucleons of the target nucleus to form a "compound nucleus".

In classical terms, Bohr visualised the reaction as a two-step process in which the bombarding energy is first distributed among all of the nucleons to form a metastable state which may then decay in a number of ways.

The excitation energy, if reconcentrated on a nucleon or group of nucleons near the surface of the nucleus as a result of random nucleon-nucleon collisions may supply enough energy to eject the particle or particles in question, or the compound nucleus could de-excite by the emission of gamma rays. In the case of higher mass nuclides, de-excitation could also occur by division of the compound nucleus into two fragments of nearly equal mass, a process called fission.

Bohr concluded that the compound nucleus lifetime must be long compared to nuclear relaxation times³., but the validity of the model has been found to extend to short-lived reaction systems⁵.. The two steps, compound nucleus formation and decay, were assumed to be independent; that is, the decay of a compound nucleus would depend only on the constants of motion of that particular compound nucleus, not on how it was formed. This is the so-called independence hypothesis; one consequence of it is that the probability for a given reaction may be factored into two parts, one corresponding to the formation probability (the cross-section for capture of the incoming particle) and a second to the decay probability as follows:

$$\sigma(a,b) = \sigma_{\text{cap}}(a) \left[\frac{P_b}{\sum_i P_i} \right]$$

The quantity in square brackets (the branching ratio) is the quotient of the probability for de-excitation via the reaction channel "b" (ie., to form a particular residual

nucleus and particle with a given kinetic energy) and the probability sum over all possible exit channels.

The principle of detailed balance must hold for compound nucleus reactions, that is, the reaction system must be symmetric with respect to the exchange of entrance and exit channels. Thus,

$$\frac{\sigma(a,b)}{\lambda_a^2} = \frac{\sigma(b,a)}{\lambda_b^2}$$

Where λ_a and λ_b are the reduced De Broglie wave lengths of the system in the entrance and exit channels^{6,7}.

2. Direct Interactions

A necessary condition in compound nucleus reactions is that energy equilibration be achieved after the incoming particle enters the target nucleus. However, a second group of reactions proceeds via a quite different mechanism in which much less than complete projectile momentum is transferred to the compound nucleus. These processes are called "direct interactions". In one form, for instance, the incoming particle may enter the target nucleus, excite a small number of nucleons to bound excited or unbound states and retain enough energy to leave the nucleus.

Another well characterised form of direct interaction is the Oppenheimer-Phillips reaction⁸. This reaction involves the interaction of a deuteron with a nucleus resulting in the absorption by the nucleus of a

neutron to populate single particle neutron states accompanied by the emitting of a high energy proton (with the proton energy perhaps exceeding the initial deuteron energy).

Compound nucleus reactions and direct interactions may be thought of as two opposite extreme mechanisms for low energy nuclear reactions. A particular reaction may proceed by an intermediate mechanism with more characteristics of one extreme than the other.

3. Experimental Distinction of Reaction Mechanisms

The predominant mechanism may often be deduced from experimental results. In the case of a compound nucleus reaction, the cross-section for formation of a specific product nucleus (say by means of an (α, n) reaction) should increase rapidly with bombarding energy until the threshold for a competing reaction (e.g., the emission of two nucleons instead of one) is reached; at this point the cross-section for the (α, n) reaction will begin to fall as the competing reaction cross-section rapidly rises. This follows from the strong dependence of the excitation energy of the compound nucleus on bombarding energy.

On the other hand, the incoming particle in a direct interaction deposits an excitation energy which is weakly related to the bombarding energy, thus rapid changes in the cross-section with the bombarding energy due to the competition described above would not be expected.

The statistical model predicts that the emission of particles from the compound nucleus is a stochastic process, and, therefore, that a compound nucleus (with zero angular momentum) emits particles isotropically. However, if a compound nucleus has a non-zero angular momentum, particles will be emitted preferentially in the equatorial plane. Classically, the compound nucleus may be thought of as a flywheel, emitting particles tangentially in its plane of rotation. The axis of rotation is perpendicular to the beam direction, hence preferential emission will be found at 0° and 180° to the beam direction in the center-of-mass system, with the overall distribution symmetric about 90° .

Most direct interactions, however, will show preferential emission at small angles to the incident beam direction as a result of the fact that a substantial fraction of the incident particle momentum is transferred directly to the emitted particles.

The emitted particle energy spectrum in the case of compound nucleus reactions is expected to be Maxwellian (except for coulomb barrier effects), but the energy spectrum for direct interactions will be more intense at higher energies, and often peaks may be resolved, corresponding to the low-energy single particle states which have been excited in the target nucleus.

The residual nuclei of a nuclear reaction will have more recoil energy in the compound nucleus case, since the linear momentum of the bombarding particle is transferred completely to the compound system.

If a projectile of energy E_a and mass A_a strikes a target of mass A_t to form a compound nucleus which emits a particle to form a residual nucleus with mass A_r and recoil energy E_r , then,

$$E_r = \frac{E_a A_a A_r}{(A_a + A_t)^2} = CE_a \quad (1-a)$$

where C is a constant. Furthermore, from experiment it is found that the recoil range in matter $R_r = C'E_r$ so that $R_r = C''E_a$, or a plot of the range of recoiling nuclei vs. the bombarding energy should be a straight line.

This result would not be expected in direct interactions, on the other hand, since the amount of energy transferred to recoiling nuclei is weakly related to the bombarding energy and is always less than E_r calculated from equation (1-a). Having found proportionality between range and energy over a specific bombarding energy region, one may resort to theoretical range-energy relationships to calculate a recoil energy corresponding to an experimental range and then compare the energy with that calculated for complete momentum transfer. (see appendix I). Confidence in the theoretical calculation is gained by comparing theory to experiment at the reaction threshold where full momentum transfer must be achieved for the reaction to proceed.

B. The Statistical Model

1. Weisskopf Evaporation Formula

Bethe^{10.}, Weisskopf^{11.}, and Weisskopf and Ewing^{12.} developed early statistical models to deal with compound nucleus reactions. Weisskopf^{11.}, comparing the emission of nucleons by an excited nucleus to the evaporation of molecules by a liquid drop (an analogy first suggested by Frenkel^{13.}), performed the thermodynamic derivation which follows. If one considers a nucleus B with energy E_B and a neutron with energy between ϵ and $\epsilon + d\epsilon$ and velocity $v = \left(\frac{2\epsilon}{M} \right)^{\frac{1}{2}}$ enclosed in a volume Ω , the probability P_C per unit time that the neutron will be captured to form nucleus A with energy between E_A and $E_A + d\epsilon$ is given by

$$P_C = \sigma(E_A, \epsilon) \frac{v}{\Omega} \quad (1-b)$$

where $\sigma(E_A, \epsilon)$ is the mean cross-section for the collision of the neutron with nucleus B ($E_B = E_A - E_0 - \epsilon$) producing compound nucleus A (E_A), and E_0 is the binding energy of the neutron to nucleus B. The probability for the reverse process is obtained by dividing P_C by the number ($\omega_A(E_A)d\epsilon$) of states into which the neutron can be captured and multiplying by the number ($\omega_B(E_A - E_0 - \epsilon)$) of states into which A(E_A) can decay. In the volume Ω , there are

$$\frac{\Omega}{2\pi^2} \frac{gm}{h^3} (2m\epsilon)^{\frac{1}{2}} d\epsilon$$

translational states which may be occupied by the neutron (g is the spin degeneracy of the neutron), so that by detailed balance

$$P_n(\epsilon) d\epsilon = P_c \frac{\omega_B(E_B)}{\omega_A(E_A)} \frac{\Omega}{2} \frac{gm}{\pi^2 \hbar^3} (2 m\epsilon)^{\frac{1}{2}} d\epsilon$$

or

$$P_n(\epsilon) d\epsilon = \sigma(E_A, \epsilon) \frac{gm}{\pi^2 \hbar^3} \frac{\omega_B(E_B)}{\omega_A(E_A)} d\epsilon \quad (2-b)$$

The entropy of a nucleus with energy between E and $E + dE$ is defined by

$$S = \log \omega(E)$$

Then,

$$P(\epsilon) d\epsilon = \sigma(E_A, \epsilon) \frac{gm}{\pi^2 \hbar^3} e^{S_B(E_A - E_0 - \epsilon) - S_A(E_A)} d\epsilon$$

If $E_A \gg E_0$ and $E_A \gg \epsilon$ and ω_A and ω_B are assumed to be identical functions ($\omega_A(E) = \omega_B(E)$), a Taylor expansion on S_B yields

$$S_B(E_A - E_0 - \epsilon) = S_A(E_A) - (E_0 + \epsilon) \left(\frac{dS_A}{dE} \right)_{E_A}$$

Also $\frac{dS_A}{dE} = \frac{1}{T_A(E)}$ where $T_A(E)$ is the temperature (in units of kT) at which the most probable energy of the body A is equal to E . Thus,

$$P(\epsilon) d\epsilon = \sigma(E_A, \epsilon) \frac{gm}{\pi^2 \hbar^3} e^{-E_0/T_A(E)} \epsilon e^{-\epsilon/T_A(E_A)} \quad (3-b)$$

Which is the usual form of Weisskopf's evaporation formula.

2. The Fermi Gas Model

Little is known about the densities of levels in highly excited nuclei; the variation of level density with energy has been derived from appropriate models. A widely used formulation may be derived from the Fermi gas model, in which the nucleus is described as a collection of degenerate fermions enclosed in a sphere of radius R . Under these restrictions, one may use the principles of statistical thermodynamics to derive a level density for the system.¹⁴ The number of states occupied by nucleons with momentum less than P_F (the momentum corresponding to the "Fermi energy") is given by

$$N = \frac{2}{(2\pi\hbar)^3} \frac{4}{3} \pi \Omega P_F^3 \quad \left(\Omega = \frac{4}{3} \pi R^3 \right)$$

At complete degeneracy (ie., the ground state),

$$P_F^{\text{proton}} = (3\pi^2)^{\frac{1}{3}} \hbar \left(\frac{Z}{\Omega} \right)^{\frac{1}{3}} \quad (4-b)$$

$$P_F^{\text{neutron}} = (3\pi^2)^{\frac{1}{3}} \hbar \left(\frac{A-Z}{\Omega} \right)^{\frac{1}{3}}$$

Where Z and A are the proton and mass numbers respectively. The total kinetic energy of the protons considered as a degenerate gas at absolute zero is given by

$$\begin{aligned} E_0^{\text{protons}} &= \frac{2\Omega}{(2\pi\hbar)^3} \int_0^{P_F} \frac{p^2}{2m} 4\pi p^2 dp \\ &= \frac{\pi \frac{4}{3} \frac{5}{3^3}}{10} \left(\frac{Z}{\Omega} \right)^{\frac{2}{3}} Z \frac{\hbar^2}{m} \\ &= \frac{3}{5} \frac{P_F^2}{2m} Z = \frac{3}{5} E_F Z \end{aligned}$$

(m is the mass of the proton)

If the Fermi gas is heated to a temperature T (ie., the nucleus is excited), the excitation energy may be related to the temperature of the gas using formulas valid for a perfect gas of fermions of high degeneracy.¹⁵ One obtains¹⁴. for protons

$$(E_T - E_0)^{\text{protons}} = \frac{\pi^2}{4} Z \frac{k T^2}{E_F}$$

where k is Boltzman's constant. Substituting the value of E_F calculated from the P_F of equation (4-b),

$$(E_T - E_0)^{\text{protons}} = \frac{\pi^2}{72} \frac{\Omega m}{\hbar^2} \left(\frac{Z}{\Omega} \right)^{\frac{1}{3}} (kT)^2$$

Similarly, for neutrons

$$(E_T - E_0)^{\text{neutrons}} = \frac{\pi^2}{72} \frac{\Omega m}{\hbar^2} \left(\frac{A-Z}{\Omega} \right)^{\frac{1}{3}} (kT)^2$$

Summing the neutron and proton energies

$$\begin{aligned} (E_T - E_0)^{\text{total}} &= 0.08 A^{\frac{2}{3}} [Z^{\frac{1}{3}} + (A-Z)^{\frac{1}{3}}] (kT)^2 \\ &= a (kT)^2 \end{aligned}$$

Where a is called the Fermi gas constant. The density of states at a given energy is related to the entropy of the gas by

$$S = k \log \frac{\rho(E)}{\rho(0)}$$

or,

$$\rho(E) = \rho(0) e^{S/k} \quad (5-b)$$

Thermodynamically,

$$\begin{aligned}
 S(T) &= \int_C^T \frac{dE}{T} \\
 &= 2 ak^2 \int_0^T T \frac{dT}{T} \\
 &= 2 ak^2 T \\
 &= 2 (ak^2 E)^{\frac{1}{2}}
 \end{aligned} \tag{6-b}$$

Equating quantities in equations (5-b) and (6-b)

$$k \log \frac{\rho(E)}{\rho(0)} = 2 (ak^2 E)^{\frac{1}{2}}$$

or

$$\begin{aligned}
 \frac{\rho(E)}{\rho(0)} &= e^{2(aE)^{\frac{1}{2}}} \\
 \rho(E) &= Ce^{2\sqrt{aE}}
 \end{aligned} \tag{7-b}$$

Many calculations have been performed using a level density of the above form (equation (7-b)) where C and a were assumed to be adjustable parameters which were varied to give a best fit to experimental data.^{16,17}

The Fermi gas level density expression must, however, be corrected for nucleon angular momentum pairing. Weisskopf and Ewing¹². suggested adjustment of the pre-exponential constant as follows:

$$C_{\text{odd-odd}} = 4C_{\text{even-even}} \text{ and } C_{\text{even-odd}} = C_{\text{odd-even}} = 2C_{\text{even-even}}$$

Hurwitz and Bethe¹⁸. concluded that it is more realistic to consider odd-even effects on level densities as arising from displacements of the ground-state energies caused by nucleon pairing.

Thus, the energy appearing in the level density expression must be measured from a virtual ground state, displaced upward from the true ground state by an amount equal to the pairing energy. For even-even nuclei, the proton and neutron pairing energies cause an additive displacement, thus,

$$\delta_{oe} \approx \delta_{eo} \quad \text{and} \quad \delta_{ee} \approx 2 \delta_{oe}$$

Cameron¹⁹. has calculated pairing energies from a comparison of results of his semi-empirical mass equation with measured atomic masses. Newton²⁰. has taken shell effects into account by calculating a characteristic value of a for each proton and neutron number.

Dostrovsky, Fraenkel, and Friedlander²¹. have corrected for shell effects by including an additional energy shift which is positive just before and negative after closed shells.

3. Capture and Inverse Cross-Sections

A neutron, striking a target with impact parameter b, if captured, will form a system with an angular momentum vector normal to the direction of the relative momentum (p) of the neutron and target and of magnitude

$$L = pb = \frac{\hbar b}{\lambda} \quad (8-b)$$

Where λ is the reduced De Broglie wave length of the system in the entrance channel. The angular momentum is quantized in units of \hbar , so that

$$L = i\hbar \quad i = 0, 1, 2, 3, \dots, i_{\max} \quad (9-b)$$

The maximum value of l is limited by the maximum value of the impact parameter which is the sum of radii of the neutron and target ($b_{\max} = R$), and from combination of equations (8-b) and (9-b),

$$b_{\max} = l_{\max} \lambda$$

or

$$l_{\max} = R / \lambda$$

Classically, one may think of the probability for formation of an intermediate system with a specific angular momentum as corresponding to the probability that the neutron will hit the nucleus with a velocity and an impact parameter corresponding to that particular value, thus⁹.

$$\begin{aligned} \sigma_l &= \pi \lambda^2 [(l+1)^2 - l^2] \\ &= \pi \lambda^2 (2l+1) \end{aligned} \tag{10-b}$$

The total cross-section is the sum of these partial cross-sections over l ⁹.

$$\begin{aligned} \sigma_r &= \sum_{l=0}^{l_{\max}} \sigma_l \\ &= \pi \lambda^2 \sum_{l=0}^{l_{\max}} (2l+1) \\ &= \pi \lambda^2 (l_{\max} + 1)^2 \\ &= \pi (\lambda + R)^2 \end{aligned} \tag{11-b}$$

Equation (11-b) represents an upper limit for σ_r since, in reality, not all collisions will result in capture. Barrier penetration (or transmission) coefficient must be introduced into the sum as follows:

$$\sigma_r = \pi \lambda^2 \sum_{l=0}^{l_{\max}} (2l+1) T_l \quad (12-b)$$

where

$$T_l \leq 1$$

In the case of a charged particle interacting with a target nucleus, the value of the relative momentum at the point of contact will be

$$p = (2\mu)^{\frac{1}{2}}(\epsilon-B)^{\frac{1}{2}} = (2\mu\epsilon)^{\frac{1}{2}}(1 - \frac{B}{\epsilon})^{\frac{1}{2}}$$

Where ϵ is the energy of charged particle, μ is the reduced mass, and the coulomb barrier, B , is given by

$$B = \frac{Z_X Z_T e^2}{R}$$

In this case, the upper limit for the cross-section may be shown⁹. to be

$$\sigma_r = \pi R^2 (1 - B/\epsilon) \quad (13-b)$$

Equation (12-b) is valid for charged particle cross-sections, provided that the coulombic as well as the centrifugal barrier has been included in the calculation of the transmission coefficients.

Equations such as (13-b) have often been used for the approximate calculation of inverse reaction cross-sections and capture cross-sections.³⁵

However, the development of the optical model (sec. I-C(1.)) has made possible more detailed calculation of transmission coefficients and several optical model computer programs are available for this purpose.^{23-25.}

C. Modern Compound Nucleus Theories

1. The Optical Model

When considering the interactions of incoming particles with target nuclei, an exact treatment of the many body problem is prohibitively complicated; many simplifying assumptions are necessary for all but two-body cases. Most treatments of particle-nucleus interactions neglect all structure of the nucleus and adopt a model which will describe the nuclear properties of interest.

Bethe². described the nucleus as a real potential well, characterised only by its radius and depth. However, several features of this model were found to be inadequate; the magnitude of predicted neutron capture cross-sections, the wide spacing of resonances, and the slow change of cross-sections with energy were not in agreement with experimental results.

Compound nucleus theories, alone, were also found to be inadequate, since no account is taken of direct interaction contributions.

The addition of an imaginary term to the potential of the well in Bethe's model was found to extend its range and accuracy. Early work by Serber²⁶. and by Fernbach, Serber, and Taylor²⁷. resulted in the proposal that the elastic scattering of neutrons be compared with the scattering of a wave by a refracting and absorbing sphere of refractive

index

$$\eta = (1 - \frac{V_{\text{nuc}}}{E})^{\frac{1}{2}}$$

Where the wave-length of the light corresponds to the energy of the neutrons ($E = h\nu$). The nuclear potential is of the form

$$V_{\text{nuc}} = V + iW$$

Where V is the average single particle real potential deflecting particles from straight-line motion, and W is the "absorption potential" representing the particles lost in the formation of compound nuclei²⁸. The nuclear potential is a function of the radial distance, r , between the centers of the interacting bodies, and spin-orbit interactions must be considered (except where both particles are of zero spin), leading to the following form:

$$V_{\text{nuc}}(r) = Vf(r) + iWg(r) + (\frac{\hbar}{\mu c})^2 (V_{\text{so}} + iW_{\text{so}}) (\frac{df}{rdr}) \sigma \cdot \mathbf{r}$$

Where V and W are the depths of the real and imaginary potentials, V_{so} and W_{so} are the real and imaginary spin-orbit potentials, σ is the Pauli spin operator, \mathbf{r} is the orbital angular momentum in units of \hbar , and $f(r)$ and $g(r)$ are form factors.²⁹

According to the Woods-Saxon³⁰ configuration

$$f(r) = [1 + \exp(\frac{r-R}{a})]^{-1} \quad (\text{c-1a})$$

Where R is the sum of the nuclear radii of the interacting particles ($R = r_0(A_T^{\frac{1}{3}} + A_X^{\frac{1}{3}})$) and a is a parameter which measures the real radial diffuseness.

Two different assumptions have been made about the form factor $g(r)$.^{29.}

(1.) $g(r) = f(r)$ This assumption corresponds to volume absorption.

(2.) Surface absorption.

In the latter case (a purely phenomenological approach) the imaginary form factor is usually assumed to be Gaussian.

$$g(r) = \exp\left[\frac{-(r-R)^2}{b^2}\right] \quad (c-1b)$$

Where b corresponds to the imaginary radial diffuseness, but is usually treated as an adjustable parameter. The real form factor is usually taken from the Woods-Saxon configuration in both cases.^{29.}

The total potential is

$$V_T(r) = V_c(r) + \frac{l(l+1)\hbar^2}{2\mu r^2} + V_{nuc}(r) \quad (c-2)$$

Where the first two terms correspond to coulombic and centrifugal potentials. The transmission coefficients for equation (12-b) may be calculated from optical model phase shifts (δ_l) .^{5.}

$$T_l(\epsilon) \equiv 1 - |e^{2i\delta_l \epsilon}|$$

The phase shifts being obtained by solution of the Schrodinger equation:

$$\left[-\left(\frac{\hbar^2}{2M}\right)\nabla^2 + V(r)\right]\psi = E\psi$$

using the $V(r)$ of equation (c-1).

Since, if a spin-orbit interaction is used, the calculated transmission coefficients will be angular momentum dependent, the form of the summation to obtain the cross-section will be dictated by the way in which the angular momenta are coupled. If intermediate coupling, (see appendix II) is used, the angular momentum distribution of the compound nuclear states is given by⁶.

$$\sigma_c(I, \epsilon, J_c) = \frac{\pi\lambda^2(2J_c+1)}{(2s+1)(2I+1)} \sum_{S=|I-s|}^{I+s} \sum_{\iota=|J_c-S|}^{|J_c+S|} T_{\iota}(\epsilon) \quad (c-4)$$

For cases where $I = s = 0$, (c-4) reduces to

$$\sigma_c(\epsilon, J_c) = \pi\lambda^2 \sum_{\iota} (2\iota+1) T_{\iota}(\epsilon) \quad (c-5)$$

which is identical in form to equation (12-b) except that σ_c and T_{ι} now refer to a specific angular momentum.

The total reaction cross-section of equation (12-b) is given by

$$\sigma_r = \sum_{J=0}^{\infty} \sigma_c(\epsilon, J_c)$$

2. Angular Momentum Considerations

Since the impact parameter is not unique in nuclear reactions, compound nuclei will be formed with a distribution in angular momentum which is a function of the nature and energy of the reacting system. Equation (2-b) which was derived without reference to angular momentum, may be rewritten to include angular momentum effects as follows:

$$P(E_F, J_F; E_C, J_C) d\epsilon = \frac{(2s_x+1)}{\pi^2 \hbar^3} \cdot \epsilon \cdot \mu \cdot \sigma(\epsilon, J_C, J_F) \frac{\omega(E_F, J_F)}{\omega(E_C, J_C)} d\epsilon \quad (c-6)$$

Where, for clarity, the subscripts A and B have been changed to C and F, signifying "compound" and "final" respectively. The spin degeneracy, g , has been replaced by its equivalent, $2s_x + 1$, where s_x is the spin of the particle x in the entrance channel.

The total probability Γ_{xJ} for emission of particle x from a compound nucleus with excitation energy E_C and angular momentum J_C is the probability given in equation (c-6), summed over all possible J_F and integrated over all possible energies of emission.

$$\Gamma_{xJ}(E_C, J_C) = \frac{(2s_x+1)\mu}{\pi^2 \hbar^3} \int_{\epsilon} \sum_{J_F} \sigma_x(\epsilon, J_C, J_F) \frac{\omega(E_F, J_F)}{\omega(E_C, J_C)} d\epsilon \quad (c-7)$$

The cross-section for a particular reaction, e.g.

$X + b \rightarrow Y + x$ is given by

$$\sigma(b, x) = \sum_{J_C} \sigma_{cap}(\epsilon_b, J_C, I) \frac{\Gamma_{xJ}(E_F, J_F)}{\sum_i \Gamma_{iJ}(E_F, J_F)} \quad (c-8)$$

Where I is the spin of the target nucleus and the sum over I includes all possible particles which may be emitted by the compound nucleus. It can be seen that factoring of the formation and decay probabilities is no longer possible, so that the independence hypotheses as previously stated is no longer valid. The independence hypothesis is valid for a particular J_C however, since

$$\sigma(b, x) = \sum_{J_C} \sigma_{J_C}(b, x)$$

where

$$\sigma_{J_C}(b, x) = \sigma_{\text{cap}}(\epsilon_b, J_C, I) \frac{\Gamma_{xJ}(E_F, J_F)}{\sum_I \Gamma_{iJ}(E_F, J_F)}$$

The capture and inverse cross-sections in equations (c-8) and (c-6) may be calculated as in section C-1.

The level density will also be affected by the inclusion of angular momentum considerations. From the general theoretical grounds of statistical mechanics, the dependence of the level density on angular momentum is expected to be²⁹.

$$(2J + 1) \exp \left[-\frac{(\text{rotational energy})}{(\text{temperature})} \right]$$

If M is the total angular momentum projection on a Z -axis and the number of particles, N , is large, the statistical central limit theorem predicts that the distribution of

M will be Gaussian

$$\omega(M) \propto \exp \left[-\frac{M^2}{2\sigma^2} \right]$$

or

$$\omega(E, M) = \left[2\pi\sigma^2 \right]^{\frac{1}{2}} \omega(E) \exp \left[-\frac{M^2}{2\sigma^2} \right] \quad (c-9)$$

Where $\sigma^2 = N \langle m^2 \rangle$ is the mean square deviation of M, m being the projection of the angular momentum of a single particle.

The projections of the total angular momentum are

$M = J, J-1, \dots, -J$, and

$$\omega(E, M) = \sum_{J=|M|}^{J_{\max}} \omega(E, J)$$

so that

$$\omega(E, J) = \omega(E, M=J) - \omega(E, M=J+1) \quad (c-10)$$

Since $\omega(E, M=J)$ and $\omega(E, M=J+1)$ may be expanded in a Taylor series about $M = J + \frac{1}{2}$,^{44, 45.}

$$\omega(E, M=J) = \omega(E, J+\frac{1}{2}) + (M-J-\frac{1}{2}) \left. \frac{\partial \omega}{\partial M} \right|_{M=J+\frac{1}{2}} + \dots$$

$$= \omega(E, J+\frac{1}{2}) - \frac{1}{2} \left. \frac{\partial \omega}{\partial M} \right|_{M=J+\frac{1}{2}} + \dots$$

$$\omega(E, M=J+1) = \omega(E, J+\frac{1}{2}) + \frac{1}{2} \left. \frac{\partial \omega}{\partial M} \right|_{M=J+\frac{1}{2}} + \dots$$

it follows, (neglecting terms $\leq \frac{1}{96} \left. \frac{\partial^3 \omega}{\partial M^3} \right|_{M=J+\frac{1}{2}}$)

from (c-10) and (c-9) that

$$\begin{aligned} \omega(E, J) &\approx \left. \frac{\partial}{\partial M} \right|_{M=J+\frac{1}{2}} \left[\omega(E, M) \right]_{M=J+\frac{1}{2}} \\ &= \omega(E) \frac{(2J+1)}{2(2\pi)^{\frac{1}{2}}} \left(\frac{\hbar}{\sigma^2} \right)^{\frac{3}{2}} \exp \left[-\frac{\hbar^2 (J+\frac{1}{2})^2}{2\sigma^2} \right] \end{aligned}$$

(c-11)

The derivation of equation (c-9) depends on use of the central limit theorem which requires that the number of particles involved be large, but Lang³³. has added further theoretical justification of the Gaussian form by the following derivation. For a Fermi gas with a density of single particle states g and the following equation of state

$$E = at^2 - \frac{3}{2}t + \frac{M^2}{2\langle m^2 \rangle_g} \quad (c-12)$$

the density of states is given by

$$\omega(E, M) = \omega\left(E - \frac{M^2}{2\langle m^2 \rangle_g}, 0\right)$$

that is, the rotational and thermal energies are thought of as separate. Since ³³.

$$\log[\omega(E, M)] \approx \log[\omega(E, 0)] - \frac{M^2}{2\langle m^2 \rangle_g} \frac{\partial}{\partial E} \log[\omega(E, 0)]$$

and

$$\frac{1}{\mathfrak{J}} \equiv \frac{\partial}{\partial E} \log [\omega(E, 0)]$$

we obtain

$$\omega(E, M) = \omega(E, 0) \exp \left[\frac{-M^2}{2c\mathfrak{J}} \right]$$

The quantity c is the moment of inertia in units of \hbar ($c = \frac{I}{\hbar}$) and is related to the spin cut off parameter by

$$\sigma^2 = ct$$

Where t is the thermodynamic temperature related to the "nuclear temperature" \mathfrak{J} by

$$\mathfrak{J} \approx t + \frac{3}{4} a^{-1}$$

For a Fermi gas with the equation of state given by (c-12)⁴⁷,

$$\omega(E) = \omega(E,0) = \frac{1}{12} \pi^{\frac{1}{2}} a^{-\frac{1}{4}} \left(E + \frac{3}{2}t\right)^{-\frac{5}{4}} \exp(2\sqrt{aE})$$

and, substituting into (c-11),

$$\omega(E,J) = \frac{\sqrt{2}}{48} a^{\frac{1}{2}} \left(\frac{\hbar^2}{2I}\right)^{\frac{3}{2}} \left(E + \frac{3}{2}t\right)^{-2} (2J+1) \exp\left[2(aE)^{\frac{1}{2}} - \frac{n^2 J(J+1)}{2It}\right] \quad (c-13)$$

Bethe¹⁰. and others³⁵. have arrived at similar forms for the level density. This form is unrealistic, however, in that it predicts the same proportion of high and low angular momentum states at all energies, when, in reality, higher J states appear only at higher energies.⁵ Lang³⁶. has attempted to correct for the superfluity of high J states at low energies by introducing a higher order term in M as follows:

$$\omega(E,J) = \frac{\omega(E,0)}{2\pi\left(\frac{I}{\hbar^2} - \frac{3M^2}{5gt}\right)} \exp\left[\frac{-M^2\hbar}{2It} - \frac{M^4\hbar^2}{(2It)^2 \cdot 5gt}\right]$$

Sarantites and Pate³¹. have derived a level density of the following form:

$$\omega(E,J) = \frac{\sqrt{2}}{48} \left(\frac{\hbar^2}{2I}\right)^{\frac{3}{2}} a^{\frac{1}{2}} (E+t-E_r)^{-2} (2J+1) \exp\left\{2[a(E-E_r)]^{\frac{1}{2}}\right\} \quad (c-14)$$

where

$$E_r = \frac{\hbar^2 J(J+1)}{2I}$$

This expression results in a much sharper decrease in the number of levels just before the highest value of J allowed (defined by $E = \frac{\hbar^2 J_{\max}(J_{\max}+1)}{2I}$), but does not appreciably relieve the persistence of high J levels at lower energies.

It can be seen that the value chosen for I greatly influences the angular momentum dependence of the level density. More will be said about the choice of I in the following section.

3. The Superconductor Analogy

The Fermi gas model (sec. B-2) is among the simplest of macroscopic models of the nucleus, since the nucleons are assumed not to interact. It is unrealistic, however to assume that the motions of the nucleons are completely independent within a finite nucleus.

Theories developed to describe the motion of electrons in metals have shown that even small interactions between fermions of a system may result in changes in the macroscopic properties of the system. Bardeen, Cooper, and Schrieffer³⁷ have shown that correlations between pairs of electrons arise from interactions with lattice vibrations in a crystal. For energies near the Fermi surface, electrons with equal and opposite angular momenta may pair to form

quasi-bound states, thus lowering the total energy of the system and causing the appearance of a gap in the originally continuous energy spectrum.^{38.}

The observed energy gap in the spectra of even-even nuclei led to the suggestion by Bohr, et. al.^{39.} that nucleon correlations exist which are similar to the fermion correlations of the electrons of a superconductor. The theories of superconductivity have been applied extensively to the nuclear case^{40,41,42.} where lattice interactions are replaced by forces between the nucleons which lead to angular momentum pairing. The strength of the nucleon interactions are proportional to the correlation function θ . Lang^{42.} has shown that the energy gap is equal to 2θ in even-even nuclei which is approximately equal to 2Δ , where Δ is the energy difference between the ground state masses of even and odd nuclei corrected for surface, coulomb, and symmetry energies. A more detailed set of calculations by Vonach, et. al.^{43.} led to the value

$$\theta = 1.3\Delta$$

For the ground state, the theories of Bardeen, Cooper, and Schrieffer^{37.} give a condensation energy^{43.}

$$C_0 = \frac{1}{4}g\theta_0^2 = 0.47 \text{ atc}^2$$

where

$$a = \frac{1}{6}\pi^2 g$$

$$t_c = \frac{4\theta}{7}$$

It is assumed that $2\theta_0$ is the energy gap for $t = 0$.

"g" is the single particle level density of protons and neutrons for a nucleus without residual interactions.

The quantity C_0 decreases with energy and reaches zero at U_c , the energy corresponding to the critical temperature, t_c , at which all pairs are broken. Above t_c , the nucleus may be described as a Fermi gas with a ground state energy shifted upward by an amount equal to the condensation energy. The critical energy is given by⁴³.

$$U_c = at_c + C_0 = 1.47 at_c^2$$

Below U_c the concept of a level density begins to lose meaning, although theoretical forms have been derived.^{55,56}

In the nuclear case, superconductor theories have been extended to describe changes in the rotational moment of inertia, which is expected to decrease with energy below t_c . Belyaev⁴⁰ and others^{39,44} have derived approximate forms for the rotational moment of inertia as a function of energy below the critical energy. At higher energies, the moment of inertia is expected to be equal to the moment of inertia of a rigid body of the nuclear dimensions.

4. Isobaric Spin

Since the proton and the neutron are often thought of as different quantum states of the same particle (the nucleon), it is convenient to introduce a quantity which specifies charge state.⁴⁵ This variable, the isobaric spin or i-spin, takes on the values $t = +\frac{1}{2}$ and $t = -\frac{1}{2}$ for protons and neutrons respectively. An isobaric spin operator, \mathfrak{J} , may be introduced which has properties analagous to the Pauli spin operator, σ ; in fact, the quantum mechanical behavior of t parallels that of s , the ordinary spin (however, spin-space and i-spin-space are completely independent).

The total i-spin of a nucleus is⁴⁶.

$$T = \sum_{i=1}^A t_i$$

which has Z component

$$T_Z = \frac{1}{2}(Z - N)$$

For light nuclei ($A \lesssim 50$), T is a good quantum number, but as Z^2/A increases or as the neutron excess increases, T becomes less valid, in the former case because of coulombic effects on commutation properties, and in the latter because different single particle shells are being filled by neutrons and protons.²⁹

Isobaric spin conservation is important in the reactions between elementary particles and light nuclei. In the scattering of deuterons by helium, an intermediate system with $T = 0$ is expected. The first three excited states of Li^6 are the following:^{7.}

$$J = 3+ \quad T = 0 \quad 2.19 \text{ MeV}$$

$$J = 0+ \quad T = 1 \quad 3.57 \text{ MeV}$$

$$J = 2+ \quad T = 0 \quad 4.52 \text{ MeV}$$

Resonances are found corresponding to the $T = 0$ states, but the $T = 1$ state is not formed as a compound nucleus.^{7.}

The exact mass and energy regions of applicability of isobaric spin conservation are at present undefined, since most experiments to date only test relative isobaric spin relationships between neighboring isobars (charge exchange reactions), and little information is available for more complicated systems.^{47.}

Studies of alpha particles and protons emitted from the compound nucleus Cu^{63} produced by alpha ($T = 0$) bombardment of Co^{59} ($T = -5/2$) and proton ($T = \frac{1}{2}$) bombardment of Ni^{62} ($T = -2$) have yielded results, the anomalies in which have been discussed in terms of isobaric spin arguments^{48,49.}; however, these arguments are still in the formative stage, and the validity of isobaric spin conservation at such high energies is doubtful.

D. Present Work.

The present work is a radiochemical study of reactions proceeding through the compound nucleus Ge^{68} excited to energies of 10-50 MeV.

Previous studies^(50-52.) have dealt with production of the compound nucleus Ge^{68} by bombardment of Zn^{64} with He^4 ions, but have not included product range measurement to determine predominant reaction mechanisms. Different formalisms of the statistical model have been employed^{50,21,53,31.} with varied success in describing the magnitude of the observed cross-sections. Calculations by Porile^{50.}, which did not include angular momentum dependence, required variation of the level density parameter between 0.8 and 2.8 MeV^{-1} to obtain agreement with experiment. From the shapes and high energy tails of some of the excitation functions Porile concluded that a sizable direct interaction contribution was probably present or that the statistical model was not completely applicable to this case.

Dostrovsky, Friedlander, and Fraenkel^{21.} required anomalously low ($a = A/40$) level density parameters to obtain agreement between their non angular momentum dependent statistical model calculations and the data of Porile^{50.}.

Esterlund and Pate^{53.} employed a primitive angular momentum dependent formalism with a variable moment of inertia. In order to obtain agreement with the data of Porile^{50.}, a moment of inertia of 1.2 times the rigid body

value was required; this value is in obvious conflict with physical possibility.

Saratites and Pate³¹. used Monte Carlo techniques and an angular momentum dependent formalism in their calculations. For calculations involving the emission of one particle by the compound nucleus, excellent agreement was obtained with the data of Porile⁵⁰. using a realistic value of the level density parameter and the rigid body moment of inertia. For the two particle case, agreement was less satisfactory.

In light of the anomalies observed in previous studies of this reaction system, a reinvestigation of the reactions involved was undertaken. In order to detect any major direct interaction contributions, recoil ranges were measured. Also, the same compound nucleus was formed in three different ways; namely, He⁴ bombardment of Zn⁶⁴, C¹² bombardment of Fe⁵⁶, and O¹⁶ bombardment of Cr⁵². Cross-sections were measured as function of energy for the following reactions:

Product	Target	Zn ⁶⁴	Fe ⁵⁶	Cr ⁵²
Ge ⁶⁷		(α ,n)	(C ¹² ,n)*	(O ¹⁶ ,n)*
Ga ⁶⁷		(α ,p)	(C ¹² ,p)	(O ¹⁶ ,p)*
Ge ⁶⁶		(α ,2n)	(C ¹² ,2n)	(O ¹⁶ ,2n)*
Ga ⁶⁶		(α ,pn)	(C ¹² ,pn)	(O ¹⁶ ,pn)*
Ge ⁶⁸		(α , γ)	————	————

(* only upper limits on the cross-sections were measurable)

At the energies of the measurements, the average angular momentum of the compound nucleus is expected to differ in the three cases when considered at equal excitation energies, providing a test for the angular momentum dependence of the excitation functions. Also, direct interaction contributions are expected to differ substantially for three such different reaction systems.

The isotopic spin is, however, invariant and plays no role for the three target projectile systems studied, since projectiles with zero isotopic spin are impinging on targets with isotopic spin -2 in all three cases considered.

A Fortran IV coding of statistical model calculations was undertaken for an IBM system 360/40 computer. Large memory capacities have made possible direct calculations and summation of reaction probabilities, and this method was chosen, rather than Monte Carlo techniques which have been used earlier^{50,21,53,31}.

II. Experimental Procedures

A. Targets

Thin metal foil targets were prepared from the materials described in Table I. Materials enriched in an isotope of interest were used in those cases where activities would be produced via reactions with other constituents of the natural isotopic mixture and would interfere with the radioassay of the nuclide of interest.

1. Zinc

Zinc targets were prepared by vacuum deposition of metallic zinc onto thin aluminum foils. The aluminum surface had to be pretreated; aluminum metal which has been exposed to air always has an oxide coating and zinc will not readily condense onto non-metallic surfaces⁵⁴. This difficulty may be overcome by using high vapor intensities, but deposits thus formed are granular and uneven. In the case of zinc enriched in Zn^{64} , this would have been especially troublesome, since only small amounts of the enriched isotope were available.

It has been found⁵⁴, however, that a thin layer ($\sim 1 \text{ \AA}$) of silver will provide nucleating sites for zinc on a non-metallic surface. Therefore, the aluminum foils were pre-treated by evaporating a thin deposit of silver, the thickness of which was determined by means of a quartz crystal and frequency monitor, the frequency of a crystal being proportional to the mass deposited upon it.

TABLE 1

Abundance (atomic percent)

<u>Element</u>	<u>Isotope</u>	<u>Natural</u> ^{56.}	<u>Enriched</u> ^{57.}
Cr	50	4.31	—
	→ 52	83.76	—
	53	9.55	—
	54	2.38	—
Fe	54	5.82	0.1 ± 0.02
	→ 56	91.66	99.7 ± 0.1
	57	2.19	0.1 ± 0.05
	58	0.33	0.1 ± 0.02
Zn	→ 64	48.89	99.85 ± 0.04
	66	27.81	0.14 ± 0.04
	67	4.11	<0.01
	68	18.57	0.01
	70	0.62	<0.01

In all cases, the overall average thickness of the silver deposit was less than the atomic radius, and the amount deposited was known to $\pm 10\%$ ⁵⁵. The zinc deposits evaporated onto a foil thus pretreated were examined by means of a microscope fitted with a calibrated eyepiece, and the grain size was found to be much less than the thickness of the deposit in all cases.

High purity, commercially available⁵⁸, natural zinc foils of .0005 inch thickness were used at higher energies when the beam energy loss in such foils was less than 1 MeV.

2. Enriched Iron

Iron foils, enriched in Fe^{56} , were prepared by electroplating iron onto aluminium foil from basic tartrate solutions. The methods of Blann et. al.⁵⁹ were modified by lowering the pH to 9.5 to avoid reaction of the basic solution with the aluminium substrate. A 5 cm. diameter tantalum disc was used as the cathode at a distance of 0.5 cm. from the aluminium foil anode. Foils were cut from the center of the plated area to avoid variations in the deposit thickness at the edges of the foils.

3. Natural Iron and Chromium

Natural iron and chromium foils were prepared by vacuum evaporation onto aluminium foil using standard techniques⁵⁴.

4. Uniformity

The uniformity of all foils was checked by accurately weighing segments of the foil which had been cut with a small die of known area (7.55 mm^2). None of the foils used for bombardment had a thickness which varied by more than 3%.

B. Bombardments

1. He^4

Bombardments were performed at the Brookhaven National Laboratory 60-inch Cyclotron, the University of Washington (Seattle) 60-inch Cyclotron, and the Washington University 45-inch Sector-focused Cyclotron.

Incident alpha-particle beam energies were measured via the observed ranges in stacks of aluminium plus blue cellophane foils. An amount of aluminium was used corresponding to most of the range, with blue cellophane used downstream for the final few percent. A brief ($\sim 1 \mu\text{A-sec}$) passage of charged particles through blue cellophane causes ionization-induced bleaching, and the thickness beyond which bleaching no longer occurs corresponds to the range of the incident beam. The energy was then calculated from the previously measured⁶⁰. range-energy relationship for aluminium and blue cellophane.

The incident beam energies were found to be:

U. of Washington 60-inch Cyclotron	41.6 ± 0.5 MeV
Brookhaven N.L. 60-inch Cyclotron	41.0 ± 0.3 MeV
Washington U. 45-inch Cyclotron	25.7 ± 0.3 MeV

The target configuration used on the U. of Washington 60-inch cyclotron consisted of a water-cooled aluminium plate onto which were clamped the foil stacks to be irradiated. The beam itself was not well collimated; therefore collimation at the target was effected by means of a thick aluminium mask clamped around the edges of the foil stack. In most cases, a special target chamber was used to avoid overheating of the target foils. This so-called "bell jar" assembly, (see figure 1.) allowed a flow of helium to pass across the face of the target stack, but lowered the incident beam energy to 37.4 ± 0.5 MeV.

A similar target block was used on the Washington U. 45-inch cyclotron, but no special cooling was required, because of smaller beam intensity and more efficient water-cooling of the block.

In both of the above cases, the beam intensity was measured by means of the production of Zn^{65} in copper foils. The excitation function for the combined $\text{Cu}^{63} (\alpha, 2n) \text{Ga}^{65}$ and $\text{Cu}^{63} (\alpha, pn) \text{Zn}^{65}$ reactions was determined in a separate experiment (see appendix III) and found to be in agreement with the results of Houck and Miller⁶¹.

The projectile energy at various positions through the foil stack was calculated using the range-energy curves due to Atkinson and Willis⁶². The range of alpha particles in zinc was determined relative to aluminium by a series of blue cellophane experiments in which different portions of the beam were intercepted by zinc or aluminium degraders before entering the cellophane stack. From the relative positions of the stopped beams in the stack, the range in zinc was calculated from the range difference and beam energy.

In the Washington University bombardments, during which the $\text{Zn}^{64} (\alpha, \gamma) \text{Ge}^{68}$ excitation function was determined, only thin foils ($< 2 \text{ mg/cm}^2$) were used. The more accurate incident energy determined from the average of an extensive series of range measurements by Reeder⁶³ was used in calculations ($25.9 \pm 0.1 \text{ MeV}$).

A detailed description of the Brookhaven target assembly and Faraday cup used to monitor the beam intensities is given elsewhere⁶⁴.

2. Heavy Ion (O^{16} and C^{12})

Bombardments were performed at the Yale University Heavy Ion Linear Accelerator. The most probable velocity of the fully accelerated beam particles is a constant (independent of the charge of the ions) as is the energy per projectile mass unit (10 MeV). To insure uniform energy, the C^{12} and O^{16} beams (consisting of charge +5 ions) were

passed through an analyzing magnet before being brought into focus in the experimental area.

A typical one inch diameter target stack consisted of one target foil ($\sim 250 \mu\text{g}/\text{cm}^2$) preceded by an appropriate thickness of beam-degrading aluminium foils and followed by several thin ($\sim 200 \mu\text{g}/\text{cm}^2$) aluminium catchers. This stack was clamped onto a brass cylinder which slipped into a $1\frac{1}{2}$ inch x 6 inch Faraday cup (see figure 2). The charge from the Faraday cup was integrated by a calibrated Cary electrometer. The efficiency of the Faraday cup in retaining secondary electrons had been shown to be 100% under the influence of a strong magnetic field⁶⁵..

The energy of the beam particles at various positions through the target stack was determined from the range-energy curves of Northcliffe^{66,67}..

C. Chemistry and Radiation Assay Techniques

1. Chemical Separations

Standard radiochemical procedures^{68,69}. were adapted⁵⁰. for use on all targets in this work. Each target was dissolved in a 6N HCl solution containing known amounts of Ge(IV) and Ga(III) carrier plus Zn, Ni, Co, and Mn holdback carriers. To insure that the germanium and gallium activities were in the appropriate oxidation states, a few drops of 40% H_2O_2 solutions were added.

Germanium activities were separated by distillation as GeCl_4 from the above solution. The gaseous GeCl_4 was

dissolved in water and the germanium was precipitated as GeS_2 with H_2S , filtered and mounted for counting. Chemical yields were usually greater than 90%.

Following distillation of the germanium activity, the remaining solution was extracted with isopropyl ether to remove the gallium activities into the ether phase as GaCl_4^- . The ether phase was washed several times with 6N HCl before the gallium activity was back-extracted with water. The water solution was then buffered to pH 7.0 with $\text{NH}_3\text{-NH}_4\text{Cl}$ before addition of a few drops of saturated 8-hydroxyquinoline solution to precipitate the gallium activity as $\text{Ga}(\text{C}_9\text{H}_6\text{NO})_3$; this was heated to expel water and mounted for counting. Chemical yields varied between 45 and 70%.

2. Radiation Assay

Ge^{66} (2.4 hours) - A 7.6 cm x 7.6 cm $\text{NaI}(\text{Tl})$ detector was employed and the intensity of the 381.4-keV full energy peak was measured. Efficiency data due to Heath⁷⁰ were employed. The relative gamma-ray intensity was taken as $.334 \pm .045$ photons/disintegration⁷¹. In some cases this nuclide was allowed to decay to its gallium daughter which was then assayed.

Ge^{67} (19 minutes) - Allowed to decay to Ga^{67} which was assayed.

Ge^{68} (280 days) - The 511-keV annihilation radiation from the decay of 68-minute Ga^{68} in secular

equilibrium was assayed via a 7.6 cm x 7.6 cm NaI(Tl) detector. The annihilation radiation detection efficiency was calibrated with a Na^{22} standard source of known disintegration rate which was calibrated by conventional⁹. 511-511 keV gamma-ray coincidence measurements. Positron emission was taken to occur in 87.9%⁷² of the Ga^{68} disintegrations. (see appendix IV)

Ga^{66} (9.5 hour) - Annihilation radiation was assayed via 7.6 cm x 7.6 cm NaI(Tl) detectors. Positron emission was taken to occur in 56.54% of the disintegrations⁷³. The decay of the nuclide was followed in some cases by means of beta proportional counters described elsewhere⁶⁵.

Ga^{67} (78 hour) - The 91.22-93.26 keV and 184.5-206 keV gamma-ray peaks were assayed via a 7.6 cm x 7.6 cm NaI(Tl) detector. The relative gamma-ray intensities were taken as 0.473 photons/disintegration and 0.274 photons/disintegration respectively⁷⁴. Representative gamma-ray spectra were inspected for impurity activities via Ge(Li) detectors described elsewhere⁷¹.

A series of measurements of the above specified kind spaced in time was taken for each sample and the resulting decay curves were subjected to least squares analysis via CLSQ⁷⁵, the Brookhaven computer program for radioactive decay analysis.

III Treatment of Data

A. Recoil Range Calculations

1. Influence of Particle Emission on Recoil Ranges

In a compound nucleus reaction, the energy E_R imparted to the recoiling excited compound nucleus will be determined by the conservation of linear momentum in the target-projectile system.

$$E_R = \frac{E_b A_b A_R}{(A_b + A_T)}$$

The subsequent emission of particles by the compound nucleus (symmetric about 90° to the beam direction in center of mass coordinates) will result in a distribution of recoil energies about an average value close to, but not equal to, that of the recoiling compound nucleus. Blann^{76,77} has calculated that the emission of a proton by a mass 59 compound nucleus will result in no more than a 10-15% decrease from the recoil energy of the compound nucleus.

In this work recoil ranges were corrected for effects of particle emission using the approximations of Blann^{76,77}, namely:

(1) The evaporation energy spectrum is replaced with a unique average energy.

(2) The evaporated particle angular distribution is replaced with 50% of the particles emitted forward and 50% emitted backward along the beam direction.

Detailed calculations by Blann^{76,77} have shown that these approximations produce in the recoil ranges of the product when α -particles are emitted from Ni^{59} a less than 1% error and a less than 7% error respectively.

In the present calculations, average ranges were corrected by a maximum of 16% for emission of two nucleons. It was assumed that the average neutron and proton emission energies were 3.2 and 6.5 MeV respectively. In calculations of Blann⁷⁷ a factor of two change in the assumed average particle energies was found not to effect the calculated recoil velocities by more than 1.5%.

2. Effects of Target Thickness

Projected ranges were calculated using the formalism described in Appendix I-1 (namely equation I-8) which takes into account the variation of the reaction cross-section within the target due to degradation of the beam energy. This correction never exceeded 35% in the cases considered.

3. Scattering Corrections

The experimentally measured projected ranges were converted to total path lengths using the data of Lindhard, Scharff, and Schiøtt⁷⁸ (see Appendix I-2). This correction was normally approximately one third of the projected range. Effects due to scattering of recoils at the target-catcher foil interface were assumed to be small and were ignored.

B. Excitation Function Calculations

1. Cross Section Calculations

During the bombardment, the number of product nuclei, N , present at time t is⁹:

$$N = R/\lambda (1 - e^{-\lambda t})$$

where R is the rate of formation and λ is the decay constant of the product nuclei. The rate of formation of nuclei in a reaction with cross section σ , induced by I particles per unit time impinging upon the target (of superficial atomic density N_T) is:

$$R = I\sigma N_T$$

and if I is constant throughout the bombardment,

$$\sigma = \frac{N}{N_T I (1 - e^{-\lambda t})}$$

The quantity σ (which is often, as at present, that of interest) may be extracted if all the quantities on the right hand side of the equation are known or measured.

A complication may arise when one measures the number of nuclei of a particular species to determine the cross section for a particular reaction. For example, the product of the $\text{Zn}^{64}(\alpha, n)$ reaction, Ge^{67} , decays by positron emission with a 19-minute half life to an isotope, 77-hour Ga^{67} which is also the product of the $\text{Zn}^{64}(\alpha, p)$ reaction. In this

case, the amount of Ga^{67} produced both during the bombardment and before chemical separation of the gallium fraction must be calculated and subtracted from the measured total to obtain the amount of Ga^{67} formed directly in the reaction of interest. This calculation, which is described in detail elsewhere⁹, was performed also for the $\text{Zn}^{64}(\alpha, \text{pn})\text{Ga}^{66}$ reaction and the corresponding heavy ion cases.

If the beam current varies during the bombardment (as it did in the HILAC bombardments), R will not be constant. Under these conditions, the bombardment may be divided into smaller time intervals, Δt_i , during which the rate of formation, R_i , is approximately constant. Under these conditions⁹:

$$N = \frac{1}{\lambda} \sum_{i=1}^n R_i (1 - e^{-\lambda \Delta t_i}) e^{-(t - t_i)}$$

where t_i is the time at the end of the i^{th} interval.

2. Excitation Energy

In order to compare the excitation functions for various target projectile systems forming the compound nucleus Ge^{68} , it is convenient to convert the bombarding energies into corresponding excitation energies of the compound nucleus.

$$E^* = E_{\text{cm}} + E_{\text{BE}}$$

$$= E_b \frac{A_T}{A_{cn}} + (A_b + A_T - A_{cn}) c^2$$

where E^* is the excitation energy of the compound nucleus, E_{cm} is the center of mass energy of the target projectile system, and E_{BE} is the binding energy of the projectile to the target. c is the velocity of light.

The projectile, target, and compound nucleus masses (A_b , A_T , and A_{cn} respectively) were obtained from the mass table of Mattauch, et al.⁷⁹.

3. Coulomb Barriers and Reaction Thresholds

Coulomb barrier energies, V , were calculated via the approximation of spherical nuclei in contact⁸⁰, i.e.

$$V = \frac{Z_b Z_T e^2}{R_b + R_T}$$

where e is the charge of the electron and other symbols have their usual meaning with b and T designating the bombarding particle and target nucleus respectively.

Reaction threshold energies were calculated as differences between the compound nucleus and product nucleus binding energies as tabulated by Mattanch, et al.⁸¹.

Because of the high coulomb barrier for the C^{12} and O^{16} reaction systems, all measurements were taken on these systems at bombarding energies far above the thresholds of the reactions studied (see table II).

TABLE II

System	V(MeV)	E_{BE} (MeV)
$Cr^{52} + O^{16}$	29.5	+6.423
$Fe^{56} + C^{12}$	24.6	+5.968
$Zn^{64} + He^4$	10.3	+2.998

Reaction	Reaction Threshold (MeV)*
$Ge^{68} \rightarrow Ge^{67} + n$	12.180
$Ge^{68} \rightarrow Ge^{67} + 2n$	21.980
$Ge^{68} \rightarrow Ga^{67} + p$	7.000
$Ge^{68} \rightarrow Ga^{67} + p + n$	18.230

Compound Nucleus	Particle	Binding Energy
Ge^{68}	n	12.180
	p	7.000
	α	2.998
Ge^{67}	n	9.800
	p	6.050
	α	3.483
Ge^{66}	n	12.550
	p	5.730
	α	3.321
Ga^{67}	n	11.230
	p	5.273
	α	5.775

* All energies refer to excitation energies in Ge^{68} .

TABLE II (cont.)

Compound Nucleus	Particle	Binding energy
Ga^{66}	n	9.102
	p	5.078
	α	5.041

IV Experimental Results

A. Radiative Capture

The experimentally measured $\text{Zn}^{64} (\alpha, \gamma) \text{Ge}^{68}$ excitation function is shown in figure 3. The previously measured data of Porile⁵⁰ are shown on the same figure for comparison. The heavy lines were drawn to fit the data of the present work.

Above 17 MeV no (α, γ) cross-sections were measurable because of the interfering Ge^{68} activity produced via the $(\alpha, 2n)$ reaction on Zn^{66} .

The four lowest energy points give a fairly good definition of the $\text{Zn}^{64} (\alpha, \gamma) \text{Ge}^{68}$ excitation function. Differences between these results and those of Porile⁵⁰ may be explained in terms of different energy resolution in the two sets of experiments. The measurements of Porile⁵⁰ were achieved by degrading a full energy beam, of 41.0 ± 0.5 MeV to energy values in the region of interest. The present data was measured at energies resulting from degradation of a 25.9 ± 0.1 MeV⁶³ full energy beam. The maximum error in the energy values plotted for the present work is not expected to exceed 0.5 MeV, while the uncertainty in the energy values of Porile⁵⁰ is expected to be many times this value. It can be seen from figure 5 that Porile's cross-section value at the highest energy he employed may have included the effects of interfering activity from the $\text{Zn}^{66} (\alpha, 2n) \text{Ge}^{68}$ reaction, while his lowest energy cross-section may have extended into the energy region below the

coulomb barrier (as may the lowest energy cross-section in the present work).

The error bars on the cross-section magnitudes in this work are mainly a measure of the uncertainty in detector efficiencies during the assay of the various samples. Chemical yields were greater than 90% in most cases. The absolute uncertainty for any of the points shown in figure 3 is not expected to exceed 15% and the relative uncertainty of one value with respect to others is 5%.

Unfortunately, recoil ranges for the $\text{Zn}^{64}(\alpha, \gamma)\text{Ge}^{68}$ reaction were unmeasurable in these experiments. Long irradiation times were necessary for the production of useful activity of 280 day Ge^{68} , and only a small fraction of the Ge^{68} recoils were found in the catcher foils as a result of low recoil energies.

Aside from known collective effects⁸²⁻⁸⁸., the majority of radiative capture events are expected to proceed via a compound nucleus mechanism and complete momentum transfer. Also, the absence of either a proton or neutron excess in the bombarding particle would further hinder any possibility of direct capture⁸⁹..

The $\text{Cr}^{52}(\text{O}^{16}, \gamma)\text{Ge}^{68}$ and $\text{Fe}^{56}(\text{C}^{12}, \gamma)\text{Ge}^{68}$ excitation functions were unmeasurable in the present work.

B. Reactions Involving the Emission of One Nucleon by the Compound Nucleus

1. $\text{Zn}^{64} (\alpha, n) \text{Ge}^{67}$

The $\text{Zn}^{64} (\alpha, n) \text{Ge}^{67}$ excitation function from the present study is shown in figure 4, together with the data of Porile⁵⁰. The heavy line was drawn to fit the data of the present work.

Although the positions in energy of the excitation function peaks are in reasonable agreement for the two sets of measurements, the low and high energy ends differ considerably. Above a bombarding energy of 30.8 MeV, interference from the $\text{Zn}^{66} (\alpha, 3n) \text{Ge}^{67}$ reaction is possible and may be present in the two highest energy points measured in this work.

The error bars on the cross-section magnitudes in figure 4 are due mainly to a combination of uncertainties from radioactivity assay statistics and detector efficiencies as well as uncertainties in chemical yields. The uncertainty in the magnitude of any of the cross-sections does not exceed 20% and the relative uncertainty of the points is approximately 7%.

Ranges for the product of this reaction were experimentally unmeasurable directly in the present work, because of the low activities in the catcher foils. However, the long duration of bombardment for the production of Ga^{67} by the (α, p) reaction resulted in a substantial

amount of Ga^{67} formed indirectly (through the (α, n) reaction followed by β^+ decay), so that the ranges measured by determining the fraction of Ga^{67} in the catcher foils are actually combined ranges for the products of the (α, n) and (α, p) reaction. These ranges are presented and discussed in the following section.

2. $\text{Zn}^{64} (\alpha, p) \text{Ga}^{67}$

The $\text{Zn}^{64} (\alpha, p) \text{Ga}^{67}$ excitation function for the present work is shown in figure 5 together with the data of Porile⁵⁰. It can be seen that agreement between the two sets of data is within experimental error throughout the entire energy range of the measurements.

Sources of error in the present work include uncertainties in the chemical yields, beam currents, target thickness, incident and degraded particle energy, and radioactivity assay statistics and efficiencies. The latter uncertainties account for most of the magnitude of the error bars on the cross-sections of figure 5. These error bars represent less than 20 % uncertainties in all cases. The relative uncertainties are less than 10%.

The combined $\text{Zn}^{64} (\alpha, n) \text{Ge}^{67}$ and $\text{Zn}^{64} (\alpha, p) \text{Ga}^{67}$ ranges are also shown in figure 5. Within experimental error, the ranges are directly proportional to bombarding energy. The ranges at the highest energy has the largest uncertainty and may contain an appreciable direct interaction component.

In figure 6, the experimental ranges are shown plotted together with the range-energy curve calculated by means of the formalism of Lindhard, Scharff, and Schiøtt⁷⁸.. (see Appendix I). Agreement between the data and theory is within experimental error.

On the basis of the analysis of the range data for the mixed (α ,n) and (α ,p) products, it may be concluded that both of these reactions proceed primarily via the compound nucleus mechanism.

3. $\text{Fe}^{56} (\text{C}^{12}, \text{p}) \text{Ga}^{67}$ and $\text{Fe}^{56} (\text{C}^{12}, \text{n}) \text{Ge}^{67}$

Only the sum of the cross-sections for these two reactions was measurable in the present work. The cross-sections obtained are shown in figure 7 plotted as a function of the Ge^{68} excitation energy together with the sum of the experimentally measured $\text{Zn}^{64} (\alpha, \text{p}) \text{Ga}^{67}$ and $\text{Zn}^{64} (\alpha, \text{n}) \text{Ge}^{67}$ cross-sections.

Errors in the experimentally measured cross-sections arise from the same sources discussed in the previous section, but in the present work, uncertainties in the beam energy and small non-constant beam currents provide the major sources of error. The uncertainties in the cross-section magnitudes are about 60%, and the energy uncertainty is about ± 1.5 MeV.

The cross-sections for the C^{12} induced reactions are seen to be in agreement within experimental error with an extrapolation of the α -induced excitation function.

This agreement is not unexpected, since the compound nucleus angular momenta resulting from the two reaction systems are calculated to be very nearly equal in this energy region. In figure 8, the average Ge^{68} angular momentum as calculated with ABACUS-2*^{23,90} is plotted vs. excitation energy for the three reaction systems: $\text{Zn}^{64} + \text{He}^4$, $\text{Fe}^{56} + \text{C}^{12}$, and $\text{Cr}^{52} + \text{O}^{16}$. At the two bombarding energies studied for the C^{12} case, the average compound nucleus angular momentum is expected to differ by less than 2 \hbar from the corresponding alpha cases. It is not surprising, therefore, that behavior similar to that first observed by Ghoshal⁹¹ is seen for these two reaction systems.

No ranges were measured, but it may be assumed that C^{12} -induced reactions resulting in products close to the mass of the compound nucleus proceed predominantly by a compound nucleus mechanism, since a direct mechanism whereby the incident C^{12} nucleus imparts the majority of its momentum to one or two nucleons of the target nucleus, is quite improbable⁹².

4. $\text{Cr}^{52} (\text{O}^{16}, \text{n}) \text{Ge}^{67}$ and $\text{Cr}^{52} (\text{O}^{16}, \text{p}) \text{Ga}^{67}$

Upper limits determined for the magnitude of the sum of the $\text{Cr}^{52} (\text{O}^{16}, \text{n}) \text{Ge}^{67}$ and $\text{Cr}^{52} (\text{O}^{16}, \text{p}) \text{Ga}^{67}$ cross-sections are shown plotted as a function of Ge^{68} excitation energy in figure 7. The lower energy cross-section was measured at an energy below the coulomb barrier and the low value indicated by the experimental limit is

expected. The higher energy cross-section is seen to be below the value one would expect by extrapolating the α and C^{12} excitation functions to this energy. This apparent violation of the independence hypothesis is expected, however, since the average angular momentum of the compound nucleus formed by O^{16} bombardment of Cr^{52} is lower at this energy than either the α or C^{12} induced average angular momentum (see figure 8). Excitation functions for systems with higher angular momenta are expected⁹ to be shifted to higher energies, and this is seen to be consistent with the present experimental results.

C. Reactions Involving the Emission of Two Nucleons
by the Compound Nucleus.

1. $\text{Zn}^{64}(\alpha, 2n)\text{Ge}^{66}$

The cross-sections measured for the $\text{Zn}^{64}(\alpha, 2n)\text{Ge}^{66}$ reaction are shown in figure 9, plotted together with the data of Porile⁵⁰. The magnitudes of the latter cross-sections were adjusted by a factor corresponding to the replacement of the rather approximate Ga^{66} positron branching ratio, $\sim 66\%$ ⁹³, used by Porile in his Ga^{66} assay, with the more accurate value 56.54% ⁷³, now available. The solid line in figure 11, was drawn to fit the present data.

The main discrepancy between the two sets of data exists at energies between the threshold and the peak of the excitation function. The present data are expected to be correct to within $\pm 20\%$ in cross-section magnitude and within ± 0.8 MeV in energy. The relative accuracy of the cross-sections is expected to be better than $\pm 10\%$.

The recoil ranges measured for the product of this reaction are also shown in figure 9, plotted on a linear ordinate scale vs. bombarding energy.

Figure 10, shows the experimental and calculated recoil ranges plotted against the calculated average recoil energy for the compound nucleus. The solid line was calculated using the formalism of Lindhard, Scharff, and Schiøtt⁷⁸. The experimentally measured projected ranges agree well within experimental error with the calculated projected ranges.

This analysis provides strong evidence that the $\text{Zn}^{64}(\alpha, 2n)\text{Ge}^{66}$ reaction proceeds predominantly via the compound nucleus mechanism.

2. $\text{Zn}^{64}(\alpha, \text{pn})\text{Ga}^{66}$

The cross-sections measured for the $\text{Zn}^{64}(\alpha, \text{pn})\text{Ga}^{66}$ reaction are shown in figure 11 plotted together with the data of Porile⁶³, which were again adjusted to correct for the calibration error as described in the previous section. The solid line was drawn to fit the present data.

The discrepancy between the two sets of data appears to be a displacement along the energy axis. The present data consist of a combination of data measured via bombardments at two different cyclotrons (Brookhaven, and University of Washington). The Brookhaven experiments also resulted in points on the $\text{Zn}^{64}(\alpha, \text{p})\text{Ga}^{67}$ excitation function which agreed with the data of Porile⁵⁰. (see section IV-B-4). The present data are expected to be accurate to within $\pm 20\%$ in cross-section magnitude and within $\pm 0.8\text{MeV}$ in energy. The relative accuracy of the cross-sections is expected to be better than $\pm 10\%$.

The recoil ranges measured for the product of this reaction are also shown in figure 11. plotted vs. bombarding energy on a linear ordinate scale. The two highest energy ranges appear to break from the linearity established by the lower energy points.

Figure 12. shows the recoil ranges plotted against the calculated average recoil energy of the compound nucleus. The solid line was calculated using the formalism of Lindhard, Schaff, and Schiøtt.⁷⁸ The higher energy disparity between the calculated ranges and experimental ranges is quite pronounced in this graph.

On the basis of this analysis, it may be deduced that the $\text{Zn}^{64}(\alpha, \text{pn}) \text{Ga}^{66}$ reaction proceeds predominantly via the compound nucleus mechanism at lower energies ($< 32 \text{ MeV}$). At higher energies, the lower ranges would indicate the onset of a noncomplete momentum transfer process (although this assumption is based essentially on the validity of one range measurement.)

3. $\text{Fe}^{56}(\text{C}^{12}, 2\text{n}) \text{Ge}^{66}$ and $\text{Fe}^{56}(\text{C}^{12}, \text{pn}) \text{Ga}^{66}$

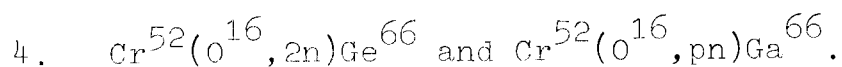
The cross-sections measured for the $\text{Fe}^{56}(\text{C}^{12}, 2\text{n}) \text{Ge}^{66}$ and $\text{Fe}^{56}(\text{C}^{12}, \text{pn}) \text{Ga}^{66}$ reactions are shown plotted together as a function of C^{12} bombarding energy in figure 13., the dotted and solid lines were drawn to fit the cross-sections for the respective reactions.

The $\text{Fe}^{56}(\text{C}^{12}, 2\text{n}) \text{Ge}^{66}$ and $\text{Zn}^{64}(\alpha, 2\text{n}) \text{Ge}^{66}$ cross-sections are plotted as a function of Ge^{68} excitation energy in figure 14., with the dotted and solid lines drawn to fit the two respective excitation functions. The lowest energy $(\text{C}^{12}, 2\text{n})$ cross-section is seen to be in agreement with the $(\alpha, 2\text{n})$ excitation function, but, unfortunately, the higher energy portion of the $(\alpha, 2\text{n})$ excitation function is not sufficiently well-defined to enable a meaningful comparison with the higher energy $(\text{C}^{12}, 2\text{n})$ cross-sections.

The $\text{Fe}^{56}(\text{C}^{12}, \text{pn}) \text{Ga}^{66}$ and $\text{Zn}^{64}(\alpha, \text{pn}) \text{Ga}^{66}$ cross-sections are shown plotted vs. Ge^{68} excitation energy in figure 15. Although the high energy portion of the (α, pn) excitation function is more well-defined experimentally in this case, some uncertainty arises

from the apparent presence of a non-compound nucleus reaction component as discussed in section IV-C-2. The triangles of figure 15. represent an upper limit to the compound nucleus contribution to the cross-section. These upper limits were determined using the approximation that all catcher activity resulted from compound nuclear processes; target foil activities (and the corresponding cross-sections) were calculated with equation I-5 (Appendix I) using the range values calculated by means of the formalism of Lindhard, Scharff, and Schiøtt⁷⁸. Again, the lowest energy heavy ion cross-section is seen to be in agreement with the (α ,pn) excitation function as defined by either the measured or corrected cross-sections, but little can be inferred about agreement with the higher energy heavy ion cross-sections.

A comparison of the alpha- and C^{12} -induced results in the form of cross-section ratios is deferred to section V-C-2 to follow.



Only upper limits were measurable for the $Cr^{52}(O^{16},2n)Ge^{66}$ and $Cr^{52}(O^{16},pn)Ga^{66}$ reactions. These limits are plotted in figures 14. and 15., respectively. The decreased magnitude of these cross-sections when compared to the corresponding C^{12} - and α -induced excitation functions may be explained by arguments analagous to those used in section IV-B-4.

V. Statistical Model Calculations

A. Formalism

The formalism used for the calculation of reaction cross-sections is based on the statistical model theory discussed in sections I-B and I-C.

1. Particle Emission

The probability for transitions between nuclear states was calculated via equation (c-6), namely;

$$P(E_F, J_F; E_C, J_C) d\epsilon = \frac{(2s_x+1)}{\pi^2 \hbar^3} \epsilon \mu \sigma(\epsilon, J_C, J_F) \frac{\omega(E_F, J_F)}{\omega(E_C, J_C)} d\epsilon$$

Where

$$\omega(E, J) = \frac{\sqrt{2}}{48} a^{\frac{1}{2}} \left(\frac{\hbar}{I}\right)^{\frac{3}{2}} (E + \frac{3}{2}t)^{-2} (2J+1) \exp \left[2(aE)^{\frac{1}{2}} - \frac{\hbar^2 J(J+1)}{2It} \right]$$

and

$$\sigma(\epsilon, J_C, J_F) = \frac{\pi \hbar^2 (2J_C+1)}{(2s_x+1)(2J_F+1)} \sum_{S=|J_F-s_x|}^{J_F+s_x} \sum_{t=|J_C-S|}^{J_C+S} T_t(\epsilon)$$

For non-interacting free nucleons confined within a nucleus of radius $R = r_0 A^{\frac{1}{3}}$, the Fermi gas model predicts the level density parameter to be the following:^{35.}

$$a = 2 \left(\frac{\pi}{3}\right)^{\frac{4}{3}} \frac{m r_0^2}{\hbar^2} A$$

Experimental evidence, as interpreted by Lang^{94.} indicates the value $a = \frac{A}{8.0} \text{ MeV}^{-1}$ (corresponding to a radius parameter $r_0 = 1.15 \text{ f}$) which was used in this work.

The energy used in the level density expression was corrected for nucleon pairing with the pairing energies of Cameron^{19.} Since Ge^{68} lies far from shell closure, no

shell correction was used.

For each excitation energy, E^* , a maximum value of the angular momentum, J_{\max} , is expected⁹⁵. to exist above which the level density is zero. If the excitation energy is partitioned into thermal and rotational energies and the rotational energy is taken to be^{96.,33}.

$$E_{\text{rot}} = \frac{J(J+1)\hbar^2}{2I}$$

it follows that

$$E^* = \frac{J_{\max}(J_{\max}+1)\hbar^2}{2I}$$

For a rigid spherical body, the moment of inertia, I_r , is given by²⁹.

$$I_r = \frac{2}{5} AR^2$$

For a nucleus with a constant pairing energy between particles in doubly occupied pair states (see section I-C-3) the moment of inertia is expected to be considerably less than the rigid body value for energies less than a neutron binding energy.^{40.,42.,44}. For energies greater than a neutron binding energy, the rigid body moment of inertia is expected to be a good approximation to the nuclear moment of inertia.^{40.,42}. To reflect these expectations, the moment of inertia was taken to vary between some value at zero energy (I_0), and the rigid body value (I_r) at higher energies as follows:

$$I = I_r(1 - be^{-0.693E/c})$$

where

$$I_r(1-b) = I_0$$

It was found that the magnitude of the assumed value for I_0 (which determines b) could be changed from $.04 I_r$ to $0.4 I_r$ with an effect of only a few per cent on the calculated cross-sections. The value $I_0 = .04 I_r$ or $b = 0.96$ was used throughout the calculations. The calculation results were found, however, to be quite sensitive to the value chosen for c , which controls the rate of approach to rigidity with energy; this was taken as 3.0 MeV, resulting in a moment of inertia equal to 93% of the rigid body value at the neutron binding energy for Ge^{68} (see figure 16.)

Below a certain energy (which was also taken as 3 MeV, but was unrelated to the above choice), the level density becomes unrealistically large when compared with experiment^{97.,98.} and was replaced with an extrapolation to zero energy, namely:

$$\omega'(E,J) = \left(\frac{E}{3.0}\right)^2 \omega(E=3.0,J) + \left(\frac{3.0-E}{3.0}\right) \omega_0$$

Here ω_0 is the "level density" at zero energy which was chosen from the trends of experimentally measured level densities.^{97.,98.}

The optical model parameters (see equations c-1, c-1-a, and c-1-b) used in calculating transmission coefficients are summarized in Table III. Parameters for p-, n-, and α -penetration were chosen for optimum agreement with the tabulations of Hodgson^{99.} The parameters for the C^{12} and O^{16} cases are those suggested by Vogt.^{100.}

TABLE III*

Reaction	<u>V</u>	<u>W</u>	<u>a, b</u>	<u>r₀</u>	<u>V_{so}</u>	<u>W_{so}</u>
$\text{Zn}^{64} + \text{He}^4 \rightarrow \text{Ge}^{68}$	-50 MeV	-11 MeV	0.59f	1.10f	0	0
$\text{Fe}^{56} + \text{C}^{12} \rightarrow \text{Ge}^{68}$	-100 MeV	-10 MeV	0.6f	1.10f	0	0
$\text{Cr}^{52} + \text{O}^{16} \rightarrow \text{Ge}^{68}$	-100 MeV	-10 MeV	0.6f	1.10f	0	0
$\text{Ge}^{68} \rightarrow \text{Ge}^{67} + n$	-52.5 MeV + 0.6E	-2.5 - 0.3E	0.6f	1.10f	-9.5 MeV	0
$\text{Ge}^{68} \rightarrow \text{Ga}^{67} + p$	-55.0 MeV + 0.7E	-0.50 - 0.45E	0.65f	1.10f	-9.5 MeV	0

* See equation c-1, c-1(a), and c-1(b) for the meaning of the symbols.

All transmission coefficients were calculated with ABACUS - 2*, the optical model program of Auerbach²³. as revised by Donnelly⁹⁰. for use on the University of British Columbia IBM 7040 computer. It has been found that the coulomb wave functions used in ABACUS - 2²³. may not give proper convergence in energy regions near coulomb barriers or at low excitation energies. The revisions in ABACUS - 2* include changes in the coulomb wave functions, however, and proper convergence is expected¹⁰¹. with these changes.

2. Gamma Ray Emission

The probabilities for gamma ray emission were calculated using the formalism derived from the single particle model^{101.,29}. as follows:

$$P_{\gamma} (E_F, J_F; E_C, J_C) = C_{\iota} (\epsilon_{\gamma}) \frac{\omega(E_F, J_F)}{\omega(E_C, J_C)} \epsilon_{\gamma}^{2\iota+1} \quad (A-1)$$

where ι corresponds to the multipolarity of the emitted gamma ray of energy

$$\epsilon_{\gamma} = E_C - E_F$$

The factors $C_{\iota} (\epsilon_{\gamma})$ (which have been assumed to be constants by previous authors^{31.,96.,103-105}. were altered from the single particle estimates to take into account experimentally observed collective effects not predicted by the single particle model.

Dipole gamma rays show in all nuclei a broad absorption resonance of 4 - 10 MeV full width at half maximum peaked at about 20 MeV.²⁹ This so-called giant dipole resonance was taken to be of the Breit-Wigner⁴ form as follows:

$$C_i(\epsilon_\gamma) = C_i' f(\epsilon_\gamma) \quad (i=1)$$

with

$$f(\epsilon_\gamma) = \frac{E_d^2 + \frac{1}{4}\Gamma^2}{(\epsilon_\gamma - E_d)^2 + \frac{1}{4}\Gamma^2}$$

where E_d is the energy at the peak of the resonance and Γ is the full energy width at half maximum. The factor C_i' ($i=1$) is taken to be a constant the magnitude of which is determined by empirical fitting of the calculation results to the measured (α, γ) excitation function. The hydrodynamic model predicts that the energy of the resonance peak will vary as $A^{-\frac{1}{3}}$ ⁸², and the following form⁸² is consistent with experimental results,^{106, 107}.

$$E_d = 82A^{-\frac{1}{3}}$$

giving for Ge⁶⁸ the value 20 MeV. The resonance width was chosen, consistent with experimental evidence, to be 5 MeV. It has been found⁸³ that calculated cross-sections are "relatively insensitive" to variations of 1 MeV in the resonance peak energy and of 2 MeV in the resonance width.

Other gamma-ray resonances are known to exist,^{84, 85} but not enough experimental characterization is available

to warrant their inclusion in the present calculations.

Many experimental electric quadrupole (E2-) transition intensities are found to be strongly enhanced over single particle estimates.²⁹. According to the single-particle model alone, one would expect the dipole gamma-ray emission rate to be 10^2 to 10^3 times faster than the quadrupole gamma-ray emission rate,^{102.,103.} but compilations of experimentally known reduced gamma-ray emission rates¹⁰⁸. show many E2 emission rates are 10 - 100 times faster than single-particle estimates. This effect has been explained⁸⁶. in terms of rapid de-excitation through intra-rotational band cascades. An attempt was made in some of the present calculations to reflect this behavior by introducing an enhancement factor into the quadrupole term for gamma emission as follows:²⁹.

$$C_i(\epsilon_\gamma) = C_i' \frac{J_c(J_c+1)(J_c-2)}{2(2J_c-1)} \left[\frac{\Delta E(J_c \rightarrow J_c-2)}{\Delta E(J_c \rightarrow J_c-1)} \right]^5$$

where C_i' is again taken to be a constant. The quantities ΔE correspond to level spacings at the excitation energy E^* . The above expression was not allowed to exceed

$$C_i(\epsilon_\gamma) = C_i' Z^2$$

the theoretical upper limit.⁸⁸.

For quadrupole transitions not involving a change of two units of angular momentum, no enhancement factor was used, ie.

$$C_i(\epsilon_\gamma) = C_i'$$

Since the present calculations did not include parity considerations, no distinction could be drawn between electric quadrupole and magnetic quadrupole transitions. The latter are not expected to be enhanced.

Although the above form for the enhancement factor has some theoretical grounds²⁹, it was used only as a rough attempt to reflect experimentally observed effects, and is in no way to be considered a quantitative treatment of the problem.

The factors $C_i(\epsilon_\gamma)$ for $i = 3$ and $i = 4$ were taken to be constants. Higher multipoles than that corresponding to $i = 4$ were not considered. The constants C_i were chosen to reflect the single-particle estimates for each multipolarity and, since the single particle model predicts smaller contributions from higher multipoles^{29,84,102,103}, these constants were rather arbitrarily taken to decrease by factors of ten for each unit increase in i . The magnitude of the four constants was set by fitting of the calculated value of the (α, γ) cross-section to the experimentally measured value.

B. SFUSMAP - Program Logic

The calculation was performed via direct calculation and summation of the various transition probabilities, according to a procedure similar to that of Blann.¹⁰⁹

The initial emitting system was taken to be a compound nucleus with a unique excitation energy ($E^* = E_{cm} + E_{BE}$) and a distribution in angular momentum, $\sigma(Z,A,E^*,J)$, characteristic of the reaction through which it was formed.

The possible modes of de-excitation considered were gamma, alpha, proton, and neutron emission. Arrays dimensioned in residual energy and angular momentum were considered for the product nuclei formed by these respective modes of de-excitation, namely $\sigma(Z,A,E,J)$, populated by gamma emission by the compound nucleus (Z,A) ; $\sigma(Z-2,A-4,E',J')$, populated by alpha emission; $\sigma(Z-1,A-1,E'',J'')$, populated by proton emission; and $\sigma(Z,A-1,E''',J''')$, populated by neutron emission. To conserve computer space these arrays were divided into bins $2\hbar$ units wide in angular momentum and 2 MeV wide in energy. Reducing the energy width to 1 MeV was found to effect the outcome of sample calculations by less than 5%.

The initial angular momentum population of the compound nucleus ($\sigma(Z,A,E^*,J)$) was calculated via ABACUS - 2*⁹⁰. (see preceding section) and read in as input data to the program. The relative probabilities for population of bins in the arrays for the several daughter

nuclei were calculated for each angular momentum value of the compound nucleus, multiplied in turn by the capture cross-section leading to each particular compound nucleus angular momentum value, and summed to give the total population cross-section of the daughter arrays by the decay of the compound nucleus. Thus, for neutron emission,

$$\sigma(Z, A-1, E', J') = \sum_J \sigma(Z, A, E^*, J) \frac{P_n(E^*, J; E', J')}{\sum_{i=\gamma, \alpha, p, n} \sum_{E'} \sum_{J'} P_i(E, J; E', J')}$$

The populations $\sigma(Z, A-2, E, J)$, $\sigma(Z-1, A-1, E, J)$, and $\sigma(Z-1, A-2, E, J)$ are then considered in order.

The formation cross section for a particular nucleus is taken as the sum of the bins in the array populated by gamma emission in that nucleus. This approximation will overestimate the formation cross-section slightly, since some states populated by gamma emission may still emit particles; however, the contribution of these states to the total cross-section will be small (<0.1%) as a result of the fact that gamma emission is normally several orders of magnitude less probable than particle emission when the latter is energetically allowed. Thus, the cross-section for formation of the product nucleus resulting from emission of two neutrons by the compound nucleus is,

$$\sigma(x, 2n) = \sum_{E'} \sum_{J'} \left[\sum_E \sum_J \sigma(Z, A-2, E, J) \frac{P_\gamma(E, J; E', J')}{\sum_{i=\gamma, \alpha, p, n} \sum_{E'} \sum_{J'} P_i(E, J; E', J')} \right]$$

The de-excitation process is only considered up to and including emission of three particles. This limitation is a result of lack of computer space only, and the program may easily be expanded to include the consideration of further particle evaporation.

The output of each set of calculations contains the population distribution for each nucleus considered both before and after emission of one gamma-ray, as well as the final formation cross-sections. A more detailed description of the computer program is given in Appendix V.

C. Calculation Results

1. General Features of Calculated Results

Sample results for residual nuclei resulting from particle emission are shown in figures 17, 18, and 19. The calculated quantities $\sigma(Z,A,E,J)$ are shown plotted vs. E and J in the form of contour diagrams, with successive contours representing factors of ten in cross-section magnitude. The emitting system in all three cases is Ge^{68} excited to 40 MeV and with $20\hbar$ units of angular momentum. The vertical dotted line represents the angular momentum of the emitting compound nucleus, and the horizontal dotted line represents the excitation energy of the emitting nucleus minus the binding energy of the emitted particle. The region of no states corresponds to J values greater than J_{max} as defined in section V-A

In figures 17, and 18, representing the results of neutron and proton emission respectively, the average change in angular momentum is approximately $2.5\hbar$ in the direction of lower angular momentum. This decrease in angular momentum is a result of the greater availability of levels at J values lower than $20\hbar$ as compared to the availability at higher J values.

Figure 19, representing the results of alpha particle emission, shows an average decrease in angular momentum of $6\hbar$. Furthermore, the average change in angular momentum is seen to increase sharply with increasing alpha energy. This effect is a consequence of the fact that

higher energy alpha particles are able to carry off (or carry in) many more units of angular momentum (cf. figure 8). This same effect is also seen for neutrons and protons, but to a much lesser extent because of the smaller masses of these particles. In the region bordering the rotational cutoff ($J \approx J_{\max}$), these trends are quite important.

The star in figure 20 represents a state in a compound nucleus and the verticle components of the arrows correspond in length to the binding energy of an emitted particle. For purposes of illustration, states in the daughter nucleus are represented on the same diagram. The horizontal and curved dotted lines represent a displacement of one binding energy from zero energy and the rotational cutoff, respectively. It is seen that for the compound nucleus in a state represented by the star, particle emissions with a zero or positive change in the nuclear angular momentum are inhibited (dotted arrow), but that emissions resulting in negative changes in the nuclear angular momentum are allowed (full arrow). Unless the change in angular momentum is very large indeed, the energy of the emitted particle will, of necessity, be low. For low energy particles, transmission coefficients for higher ℓ values are very small, so that the corresponding emission probability will be small. Under these conditions, gamma ray emission is expected^{109.,104.} to compete favorably with particle emission.

Figures 21 and 22 illustrate a particular case where population of states near the rotational cutoff is an important consideration. In the upper right hand corner of the diagram, the cross-section (in millibarns) for population of a given compound nucleus angular momentum value during bombardment is shown plotted against angular momentum (in units of \hbar). The cross-section distribution is seen to be peaked at higher J values for the $\text{Fe}^{56} + \text{C}^{12}$ case (figure 21) than for the corresponding $\text{Zn}^{64} + \text{He}^4$ case (figure 22). The contour diagrams (which are identical to those described previously, except that now the contours represent the sum of contributions from each angular momentum value along the accompanying σ_J distribution in the emitting compound nucleus) reflect this angular momentum difference even after emission of two nucleons. It can be seen, particularly in the Ge^{66} and Ga^{66} product nuclei, that the population of states along the rotational cutoff is much more dense in the $\text{Fe}^{56} + \text{C}^{12}$ case. Since, therefore, these two product nuclei have a much higher probability for gamma ray emission in the $\text{Fe}^{56} + \text{C}^{12}$ case, one would expect the $\text{Fe}^{56}(\text{C}^{12}, \text{pn})$ and $\text{Fe}^{56}(\text{C}^{12}, 2\text{n})$ reaction cross-sections to be larger at this energy than the corresponding $\text{Zn}^{64}(\alpha, \text{pn})$ and $\text{Zn}^{64}(\alpha, 2\text{n})$ cross-sections which will be depleted by particle emission. For the same reason, reaction thresholds for emission of three nucleons should be shifted to higher energies for the higher angular momentum case. Corresponding shifts to higher energies are expected for excitation functions corresponding to reactions involving evaporation of one particle by the compound nucleus.

An even more pronounced case of highly populated states in the vicinity of the rotational cutoff is seen in figure 23 for the system $\text{Cr}^{52} + \text{O}^{16}$ at 64MeV excitation.

The effects of gamma ray competition with particle emission are well illustrated by the calculated excitation functions shown in figure 24.

The differences in slope between the two excitation functions below 40MeV are due mainly to differences in the total reaction cross-section. Above 40MeV, however, where both total reaction cross-sections change slowly with energy, the slope of the $\text{Fe}^{56}(\text{C}^{12}, \text{pn})\text{Ga}^{66}$ excitation function increases in relation to the $\text{Zn}^{64}(\alpha, \text{pn})\text{Ga}^{66}$ excitation function; this is due to effects such as that just described following the rapidly increasing average angular momentum of the $\text{C}^{12} + \text{Fe}^{56}$ system as compared to the $\text{He}^4 + \text{Zn}^{64}$ system (see figure 8).

2. Comparisons of Calculations with Experiment.

The calculated $\text{Zn}^{64}(\alpha, \gamma)\text{Ge}^{68}$ excitation function is shown in figure 25 together with the present experimental data. The magnitude of the calculated excitation function was set by adjustment of the gamma strength constants (C_1') in formula V-A-1. The apparent discrepancy in energy is probably due to inaccuracy in the values used for particle binding energies⁸¹, which are subject to large uncertainties.^{20, 81, 110} The magnitudes of the gamma strength constants obtained for Ge^{68} from this calculation were assumed to be identical for neighboring nuclei.

The calculated $\text{Zn}^{64}(\alpha, n)\text{Ge}^{67}$ excitation function is shown in figure 26 plotted together with the experimental cross-sections. Agreement is seen to be quite good above 20 MeV, whereas agreement is poor for lower energy values. This disparity is probably due to inaccuracy in the neutron binding energy used⁸¹. and concomittant inaccuracy of the reaction threshold.

The $\text{Zn}^{64}(\alpha, p)\text{Ga}^{67}$ excitation function is shown in figure 27. Although the cross-section magnitudes are generally lower than those measured, the excitation function shape is seen to reflect the shape of the experimental excitation function quite well.

The $\text{Zn}^{64}(\alpha, 2n)\text{Ge}^{66}$ excitation function is shown in figure 28. Again there appears to be an energy shift between the experimental and calculated excitation functions which may probably be attributed to inaccurate binding energies. Otherwise, both the shape and the magnitude of the calculated excitation function reflect those of the experimental excitation function quite well.

The calculated $\text{Zn}^{64}(\alpha, pn)\text{Ga}^{66}$ excitation function is shown in figure 29. The calculated excitation function shape is seen to closely reproduce that of the experimentally measured one, but, as in the (α, p) case, the calculated cross-section magnitudes are low. This suggests possible inaccuracies in the calculated proton transmission

coefficients, although the disparity may be due to other causes.

The calculated excitation function for the sum of the $\text{Fe}^{56}(\text{C}^{12}, \text{n})\text{Ge}^{67}$ and $\text{Fe}^{56}(\text{C}^{12}, \text{p})\text{Ga}^{67}$ excitation functions are shown in figure 30 plotted together with the experimental sum cross-sections. There is a more serious lack of agreement in this case.

The calculated $\text{Fe}^{56}(\text{C}^{12}, 2\text{n})\text{Ge}^{66}$ excitation function is shown in figure 31. Agreement is seen to be good between the calculated excitation function and the two higher energy cross-section values. The calculation is seen not to be in agreement with the lowest energy cross-section value. This lack of agreement is very likely due to inaccuracy of the coulomb barrier as calculated by ABACUS - 2*. The rigid sphere in contact approximation (see section III-B) predicts a much lower coulomb barrier energy, and this prediction appears to be in better agreement with experiment.

The calculated $\text{Fe}^{56}(\text{C}^{12}, \text{pn})\text{Ga}^{66}$ excitation function is shown in figure 32. Again, agreement with experiment is good for the two highest energy points, but poor for the lowest energy point. Again, the inaccurate coulomb barrier probably accounts for this discrepancy.

Some ambiguity arises when comparing cross-section magnitudes because of the different probabilities of forming the compound nucleus for different target projectile systems. This ambiguity is removed when one considers reaction cross-section ratios, however.

Figure 33 shows the calculated and experimental ratios of the $\text{Zn}^{64}(\alpha, \text{pn})\text{Ga}^{66}$ cross-section to the $\text{Zn}^{64}(\alpha, 2\text{n})\text{Ge}^{66}$ cross-section as well as the $(\text{C}^{12}, \text{pn})/(\text{C}^{12}, 2\text{n})$ ratios plotted vs. excitation energy. The discrepancy between the calculated $(\alpha, \text{pn})/(\alpha, 2\text{n})$ ratio and experiment is probably to be attributed to binding energy errors. At higher energies the calculated and experimental ratios are seen to merge within experimental error. The calculated and experimental $(\text{C}^{12}, \text{pn})/(\text{C}^{12}, 2\text{n})$ ratios are seen to agree well within experimental error.

It is interesting to note that the experimental ratios form a continuous curve for the two systems. The coincidence of the two sets of data in the region from 36 - 40 MeV where the angular momenta of the two systems is very closely matched (cf. figure 8) provides strong evidence that the predominant mechanism for both of these reactions is compound nucleus formation and decay and is a convincing verification of the independence hypothesis. Since any direct interaction component would not be expected to be of the same magnitude for two such differing target-projectile systems, it must be concluded that the highest energy range measurement for the $\text{Zn}^{64}(\alpha, \text{pn})\text{Ga}^{66}$ reaction (see figure 11.) is probably in error.

VI. Conclusions

It may be stated, on the basis of this work, that, contrary to previous observations^{34,63}, the compound nucleus model provides a satisfactory account of the reactions of alpha particles with Zn^{64} . This conclusion is based on experimental recoil range evidence as well as the agreement of calculated excitation functions with experiment. The "high energy tails" of the measured excitation functions may be explained in terms of angular momentum effects and gamma ray competition with particle emission, rather than by assuming large direct interaction contributions to the reaction mechanism. Furthermore, all calculation parameters employed were based on available independent experimental evidence, rather than adjusted to achieve best agreement between calculation and experiment as has been done by other workers^{34,44,63,66}.

The main source of difficulty in performing calculations of excitation functions was found, in the present work, to be lack of reliable optical model parameters as well as uncertainty in particle binding energies.

The outcome of the $\text{Fe}^{56} + \text{C}^{12}$ calculations seems encouraging in light of the fact that one would expect the approximations inherent in the optical model to be an oversimplification for such a complex target-projectile system. The success of the statistical model on this reaction system would suggest further such experiments for investigation of angular momentum effects (see Appendix VI).

The treatment of gamma ray emission used in this work is in obvious need of refinement. The single particle estimates are inadequate, and the nature of collective enhancement of gamma ray emission probabilities is still a largely unexplored field. The rather empirical approach used in this work was seen to be adequate only to a first approximation.

In conclusion, it may be stated that, provided one has a reasonably accurate knowledge of the constants of motion of a compound nucleus formed by a given target-projectile system, the decay of this compound nucleus may be accurately described by the statistical model.

TABLE IV

Zn⁶⁴ and He⁴ Cross-sections.

<u>E_α (MeV)</u>	<u>σ(α,γ)</u>	<u>σ(α,p)</u>	<u>σ(α,n)</u>	<u>σ(α,pn)</u>	<u>σ(α,2n)</u>
9.1		21.8			
9.4	.259				
11.3	.771				
11.9		115			
12.9			2.0		
13.5	.720				
14.3		269			
14.4			11.2		
15.6	.218				
15.9			67.5		
16.4		305			
18.0		404		0.64	
19.6		452			
19.7			233		
21.6				84.1	
21.8			206		
22.4		339			
24.8		225		551	
25.0					0.97
26.1			82.3		
27.2					50.9
28.3		84.8		938	
29.5					126
29.7			45.1		
31.0				969	
31.5			36.1		
32.3					129
34.5			20.0		

TABLE IV (cont'd)

<u>E_{α} (MeV)</u>	<u>$\sigma(\alpha, \gamma)$</u>	<u>$\sigma(\alpha, p)$</u>	<u>$\sigma(\alpha, n)$</u>	<u>$\sigma(\alpha, pn)$</u>	<u>$\sigma(\alpha, 2n)$</u>
34.7					109
34.9				539	
36.9		11.7		415	
37.3					82.2

(All cross-sections are expressed in millibarns)

TABLE V

Zn⁶⁴ and He⁴ Recoil Ranges.

<u>E_α (MeV)</u>	<u>R_O [(α,n)+(α,p)]</u>	<u>R_O [(α,pn)]</u>	<u>R_O [(α,2n)]</u>
8.6	87		
11.5	172		
12.9	193		
15.6	274		
17.0		145	
19.0	301		
19.7		228	
23.1	374		
24.1		256	
25.0			342
27.2			353
27.6		333	
29.5			376
30.2		361	
32.3			319
34.5		333	
34.7			435
36.3	477	272	
37.3			495

(All ranges are expressed in μg/cm².)

TABLE VI

Fe⁵⁶ and C¹² Cross-sections

<u>E_{C¹²}(MeV)</u>	<u>E*(MeV)</u>	<u>σ(C¹²,n)+σ(C¹²,p)</u>	<u>σ(C¹²,pn)</u>	<u>σ(C¹²,2n)</u>
36.9	36.4	2.57	370	98.0
39.1	38.2	1.49	90.0	24.2
43.8	42.1	- - -	17.2	5.3
75.4	68.1	- - -	<0.5	<0.9

(all cross-sections are expressed in millibarns)

TABLE VII

Cr⁵² and O¹⁶ Cross-sections

<u>E_{O16}(MeV)</u>	<u>E*(MeV)</u>	<u>$\sigma(O^{16},n)+\sigma(O^{16},p)$</u>	<u>$\sigma(O^{16},pn)$</u>	<u>$\sigma(O^{16},2n)$</u>
39.2	36.4	<1.8	<0.23	<.061
47.5	42.7	<2.7	<0.34	<.090
131.0	106	<2.6	<0.33	<.088

<u>E_{O16}(MeV)</u>	<u>E*(MeV)</u>	<u>$\sigma(O^{16},\alpha 2n)$</u>	<u>$\sigma(O^{16},\alpha pn)$</u>	<u>$\sigma(O^{16},3pn)$</u>
39.2	36.4	<3.1	<0.70	<0.28
47.5	42.7	<2.1	<0.48	<0.19
131.0	106	<3.0	<0.69	<0.27

(all cross-sections are expressed in millibarns)

HILAC TARGET HOLDER

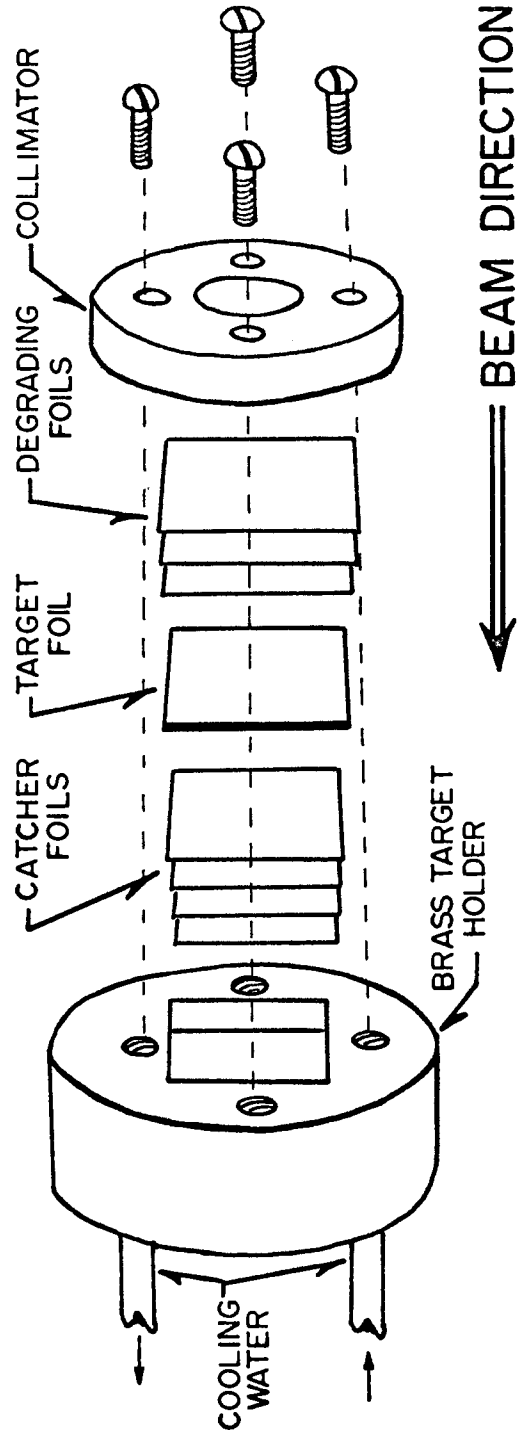


FIGURE 1

UNIVERSITY OF WASHINGTON "BELL JAR" TARGET ASSEMBLY

(Schematic)

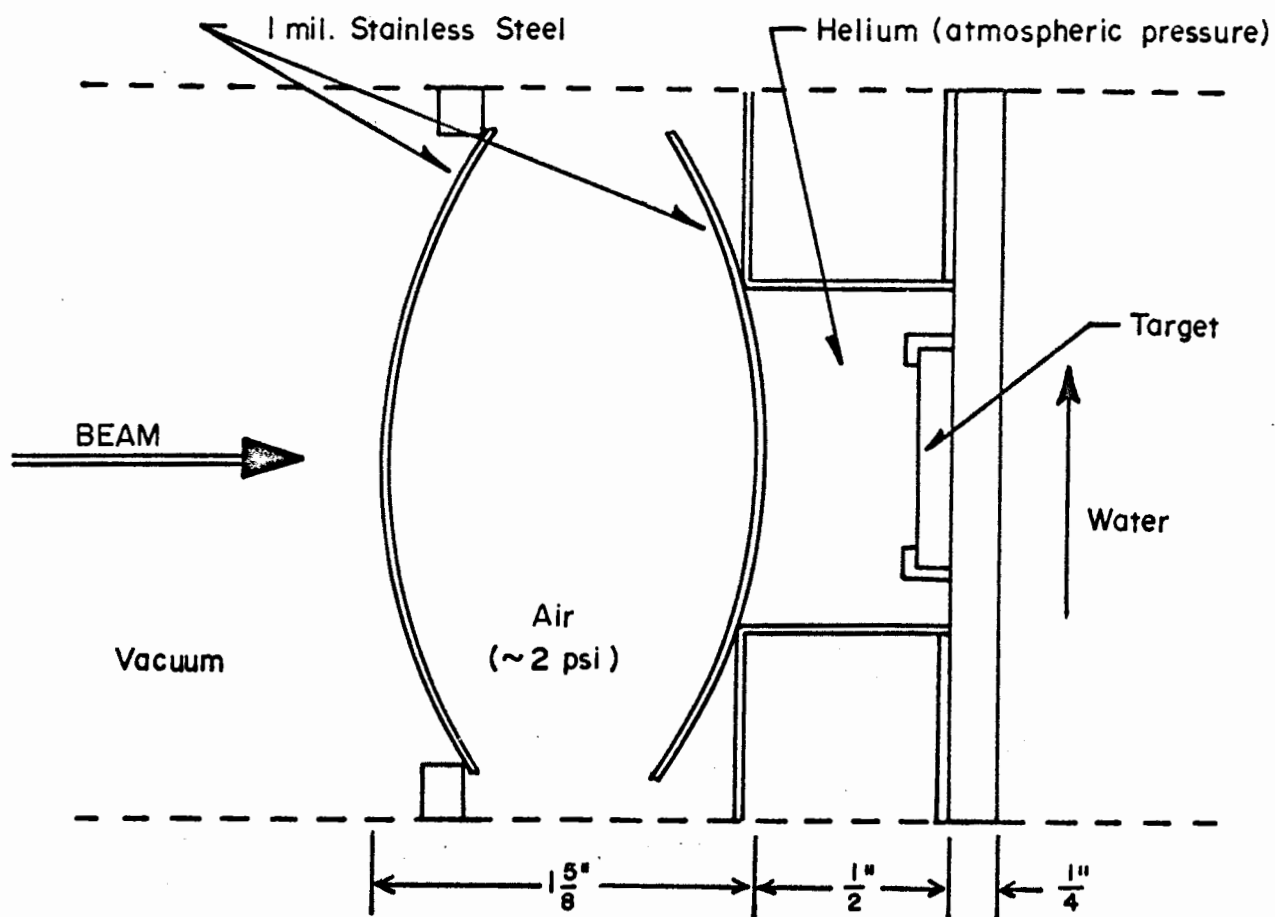


FIGURE 2

FIGURE 3

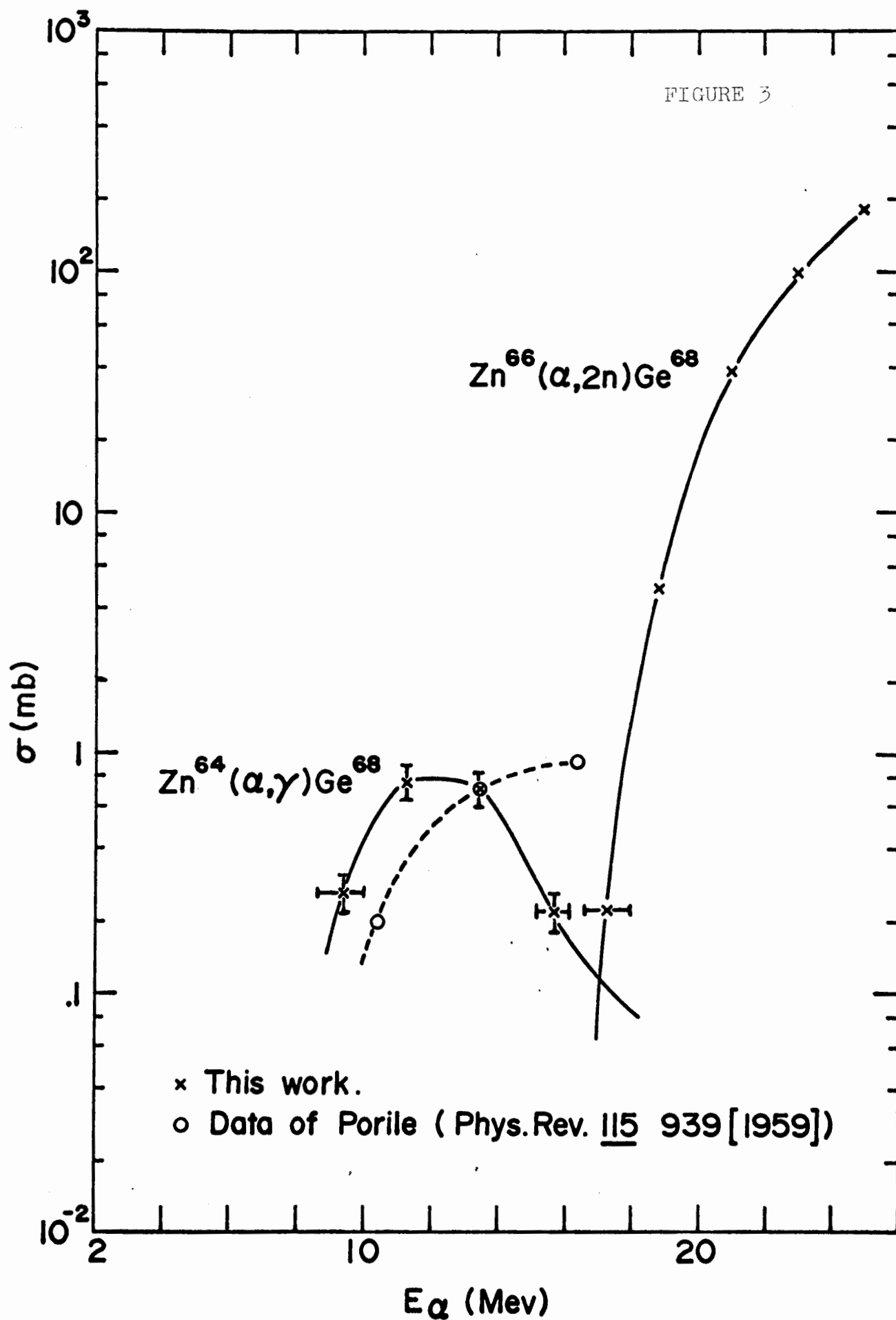


FIGURE 4

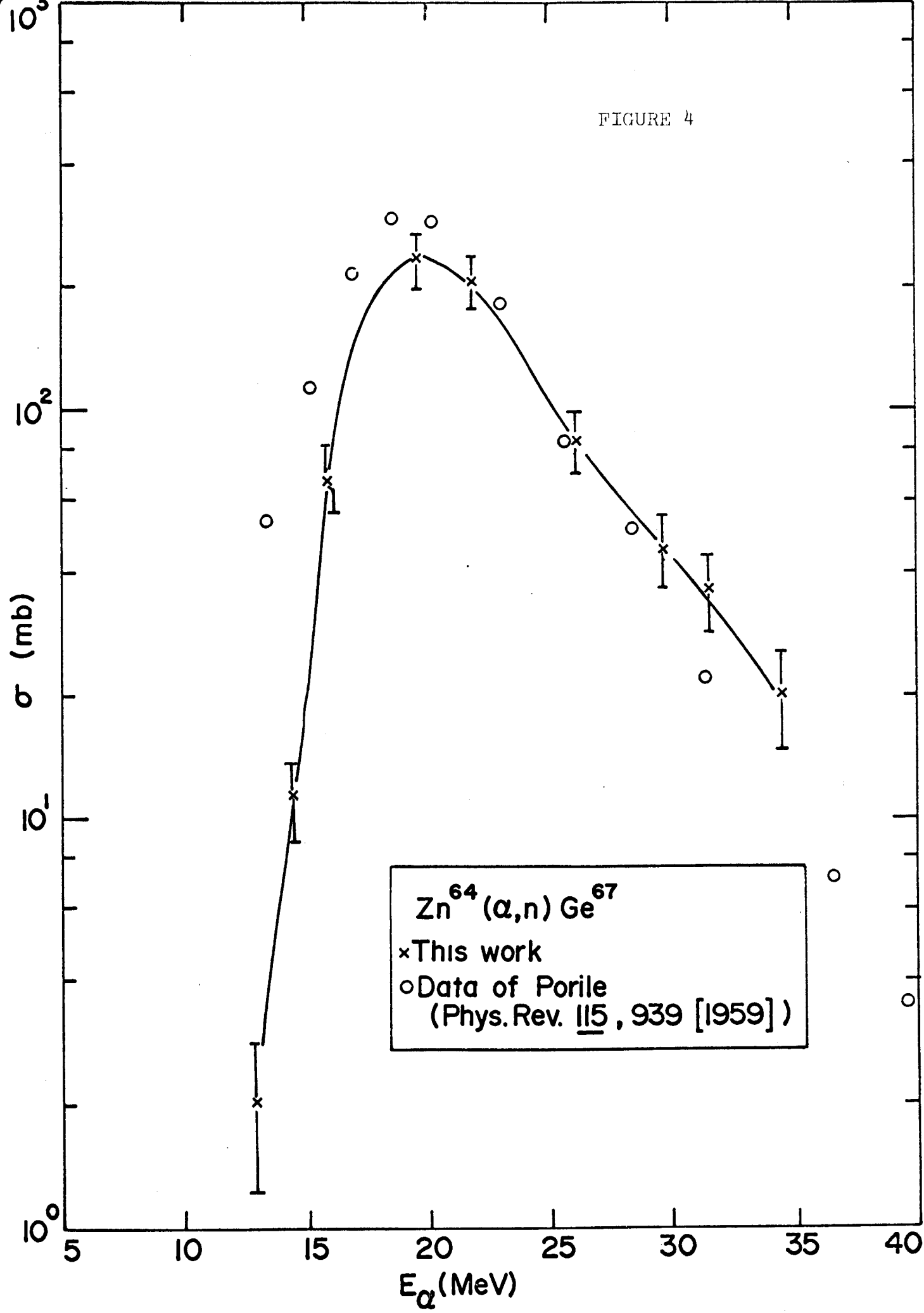


FIGURE 5

$\text{Zn}^{64}(\alpha, p)\text{Ga}^{67}$

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 • Data of Porile
 (Phys. Rev. 115, 939
 [1959])

○ R_0

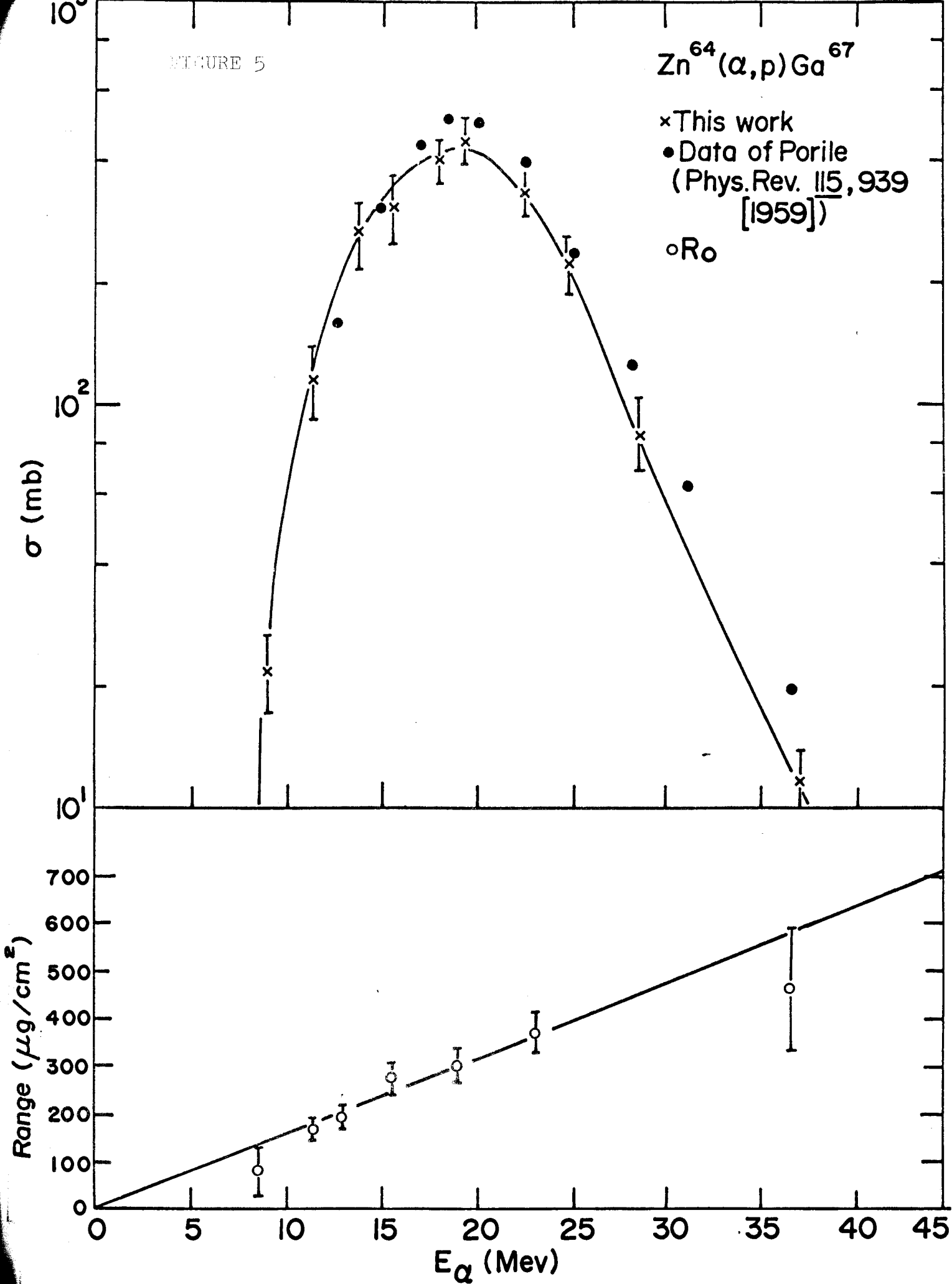


FIGURE 6

Range of Recoiling Mass - 67 Ions in Zinc

x $\text{Zn}^{64}(\alpha, p) \text{Ga}^{67} + \text{Zn}^{64}(\alpha, n) \text{Ge}^{67}$

— LSS

Range ($\mu\text{g}/\text{cm}^2$)

E_R (Mev)

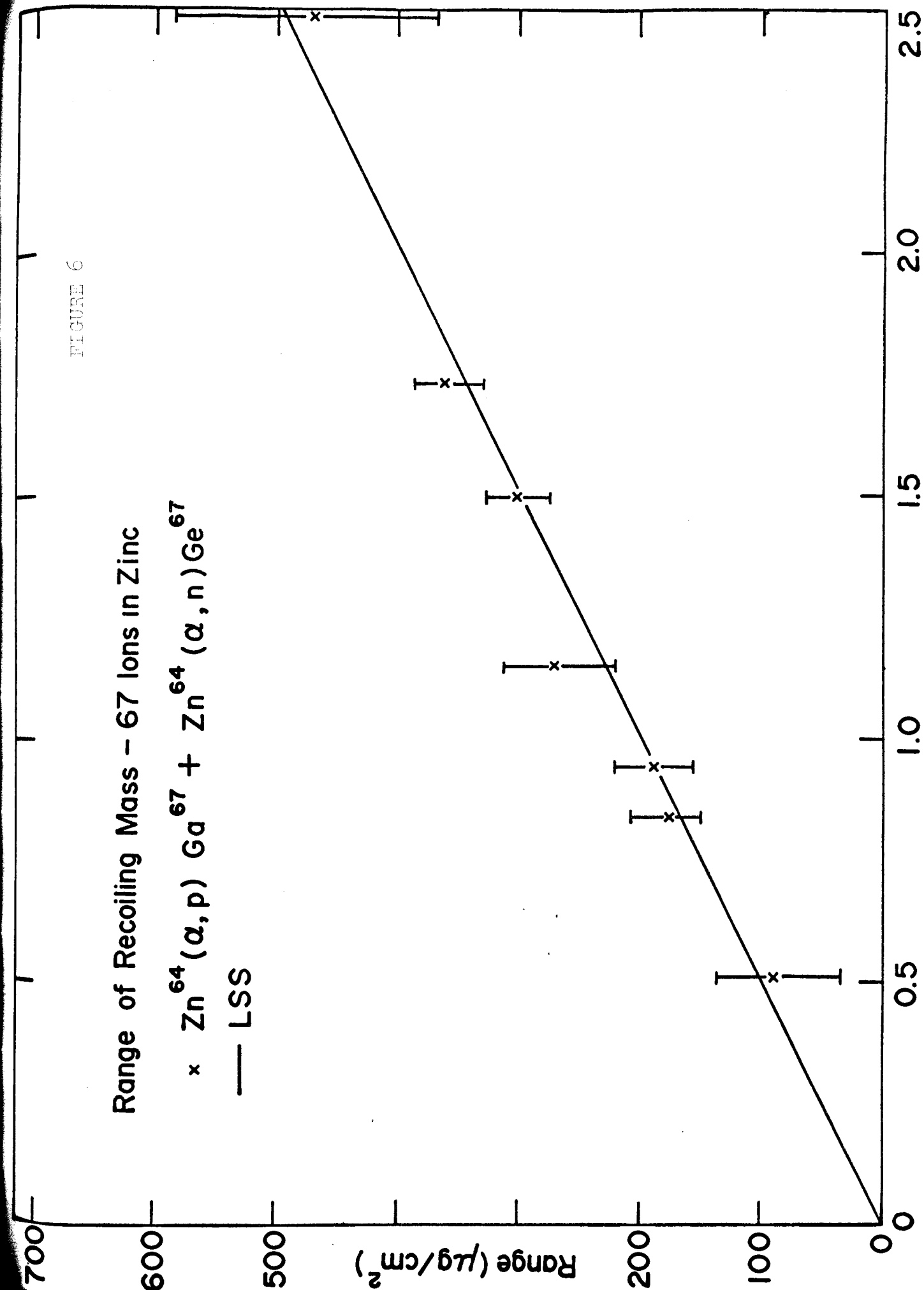


FIGURE 7

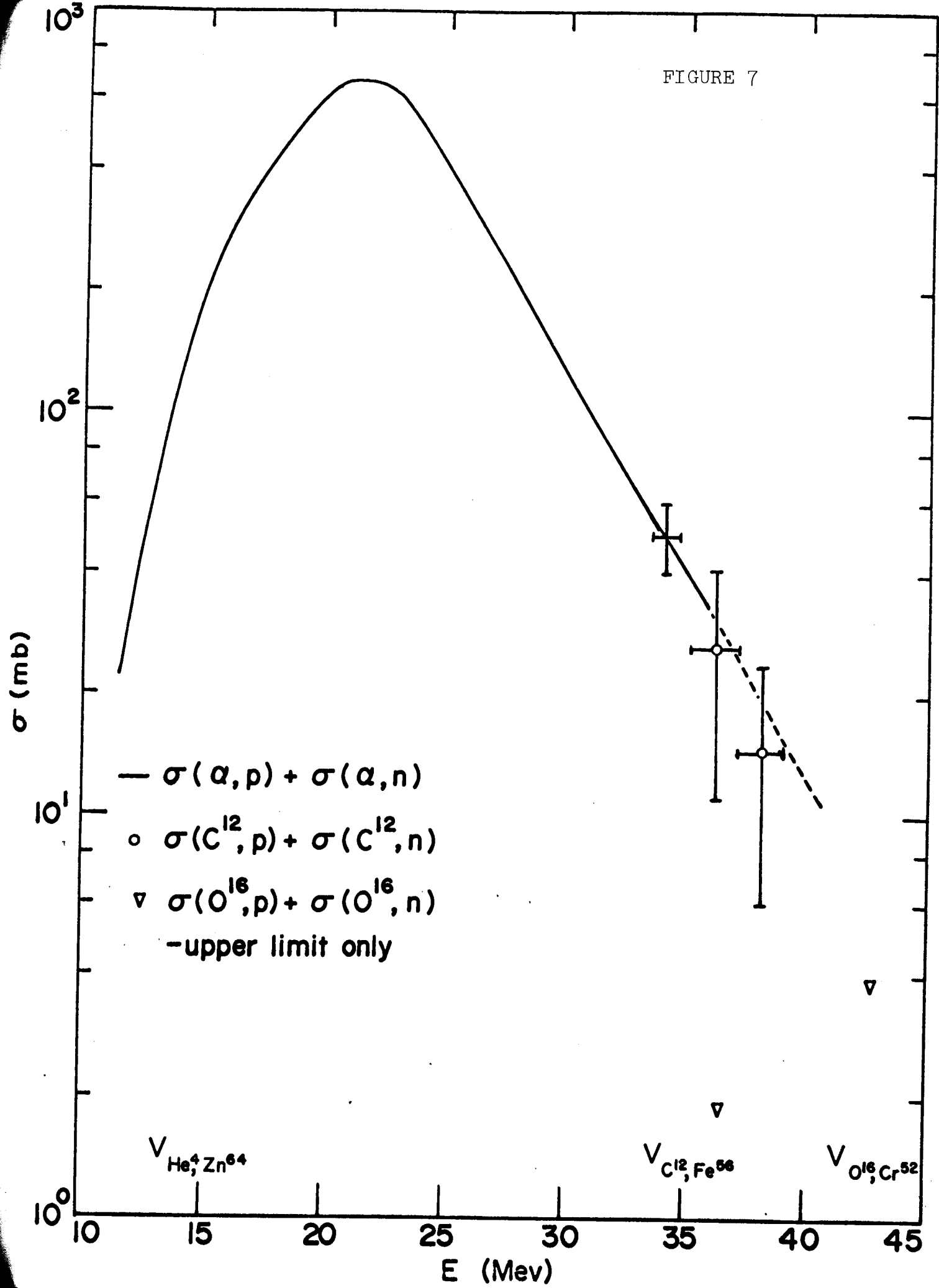
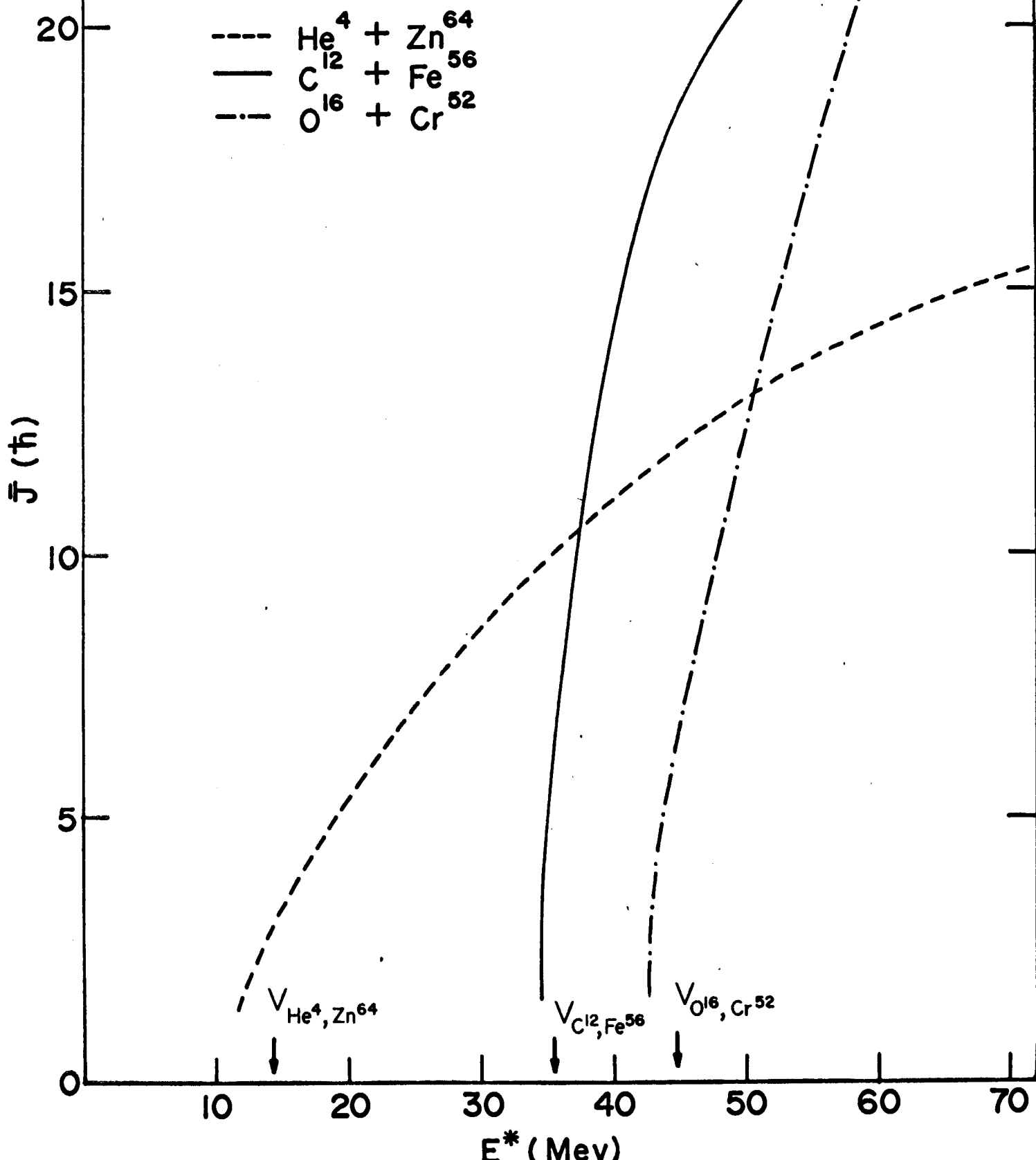


FIGURE 8

Average Angular Momentum
vs. Excitation Energy for
the Compound Nucleus Ge^{68}



$\text{Zn}^{64}(\alpha, 2n)\text{Ge}^{66}$

FIGURE 9

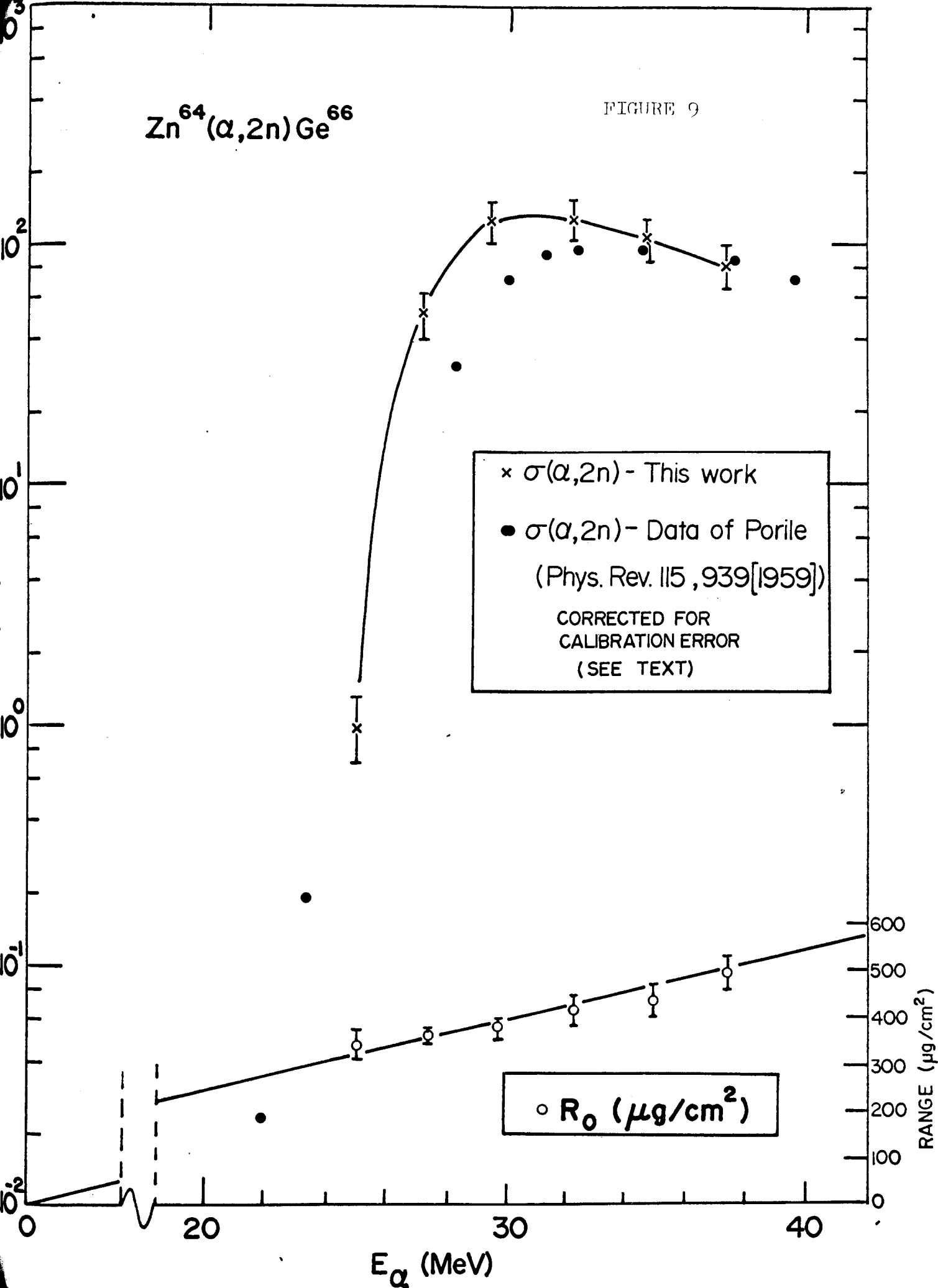


FIGURE 10

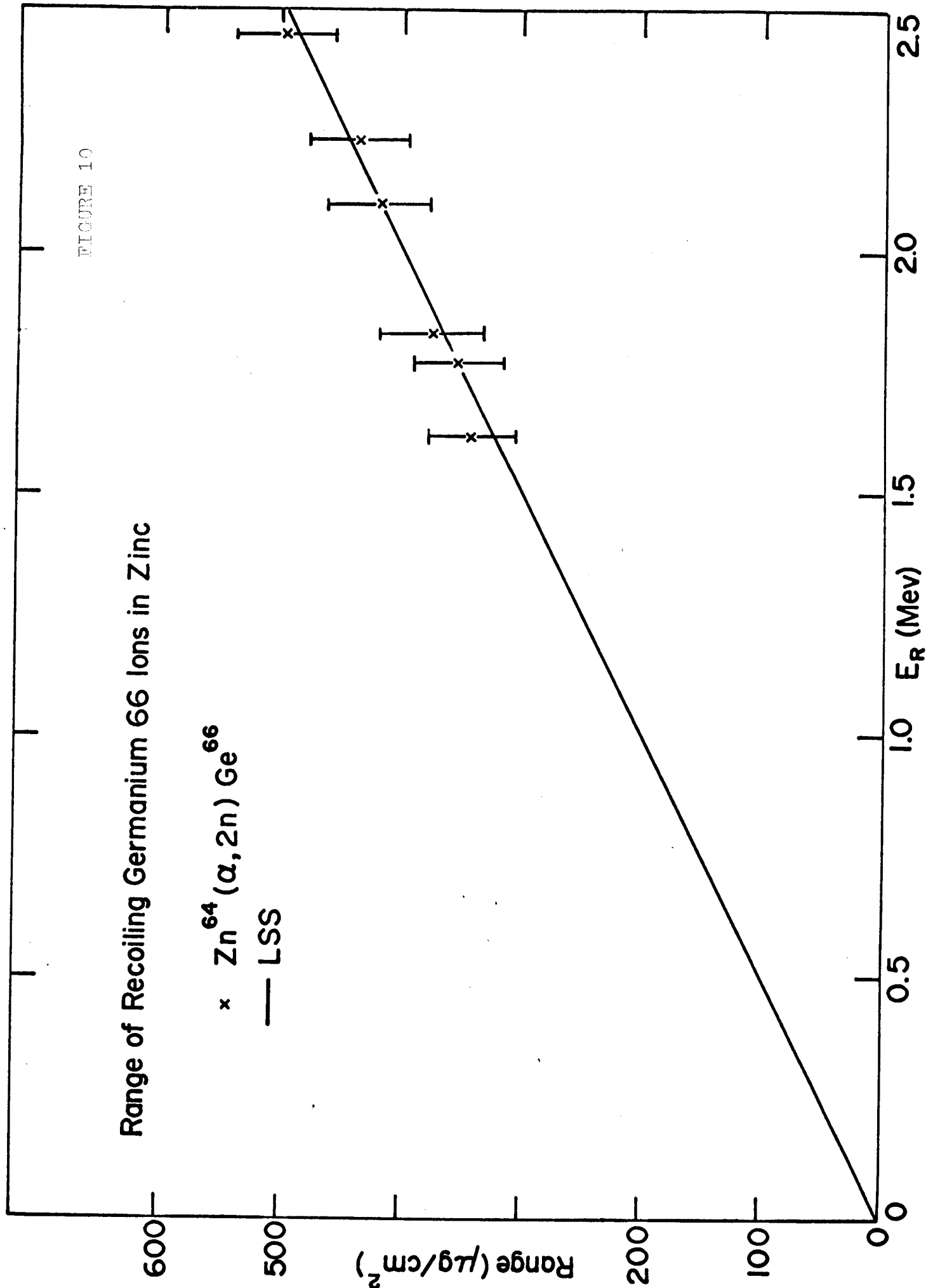
Range of Recoiling Germanium 66 Ions in Zinc

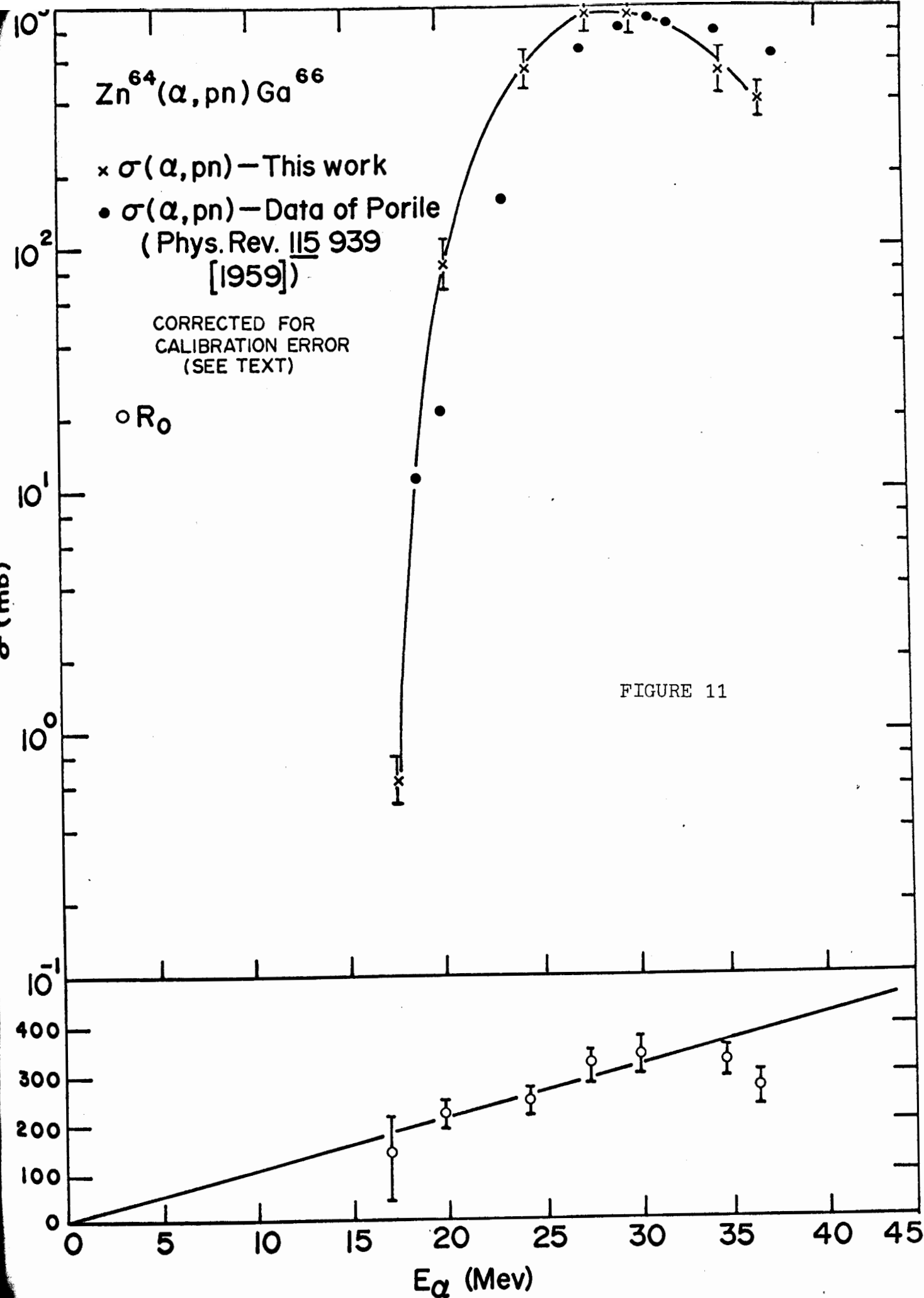
x $\text{Zn}^{64}(\alpha, 2n)\text{Ge}^{66}$

— LSS

Range ($\mu\text{g}/\text{cm}^2$)

E_R (Mev)





Ranges of Recoil Gallium Ions
in Matter

x $\text{Zn}^{64}(\alpha, \text{pn})\text{Ga}^{66}$
— LSS

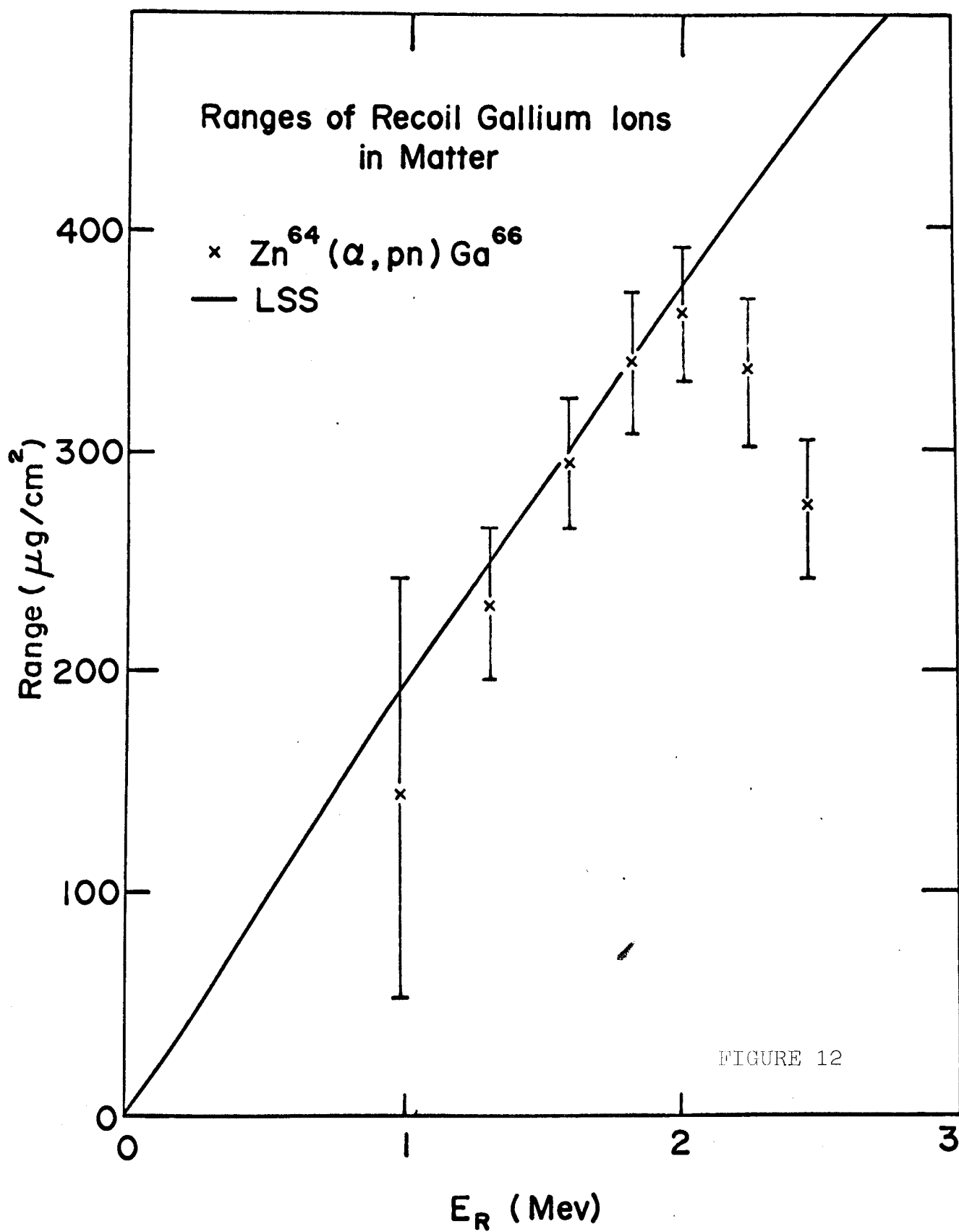


FIGURE 12

FIGURE 13

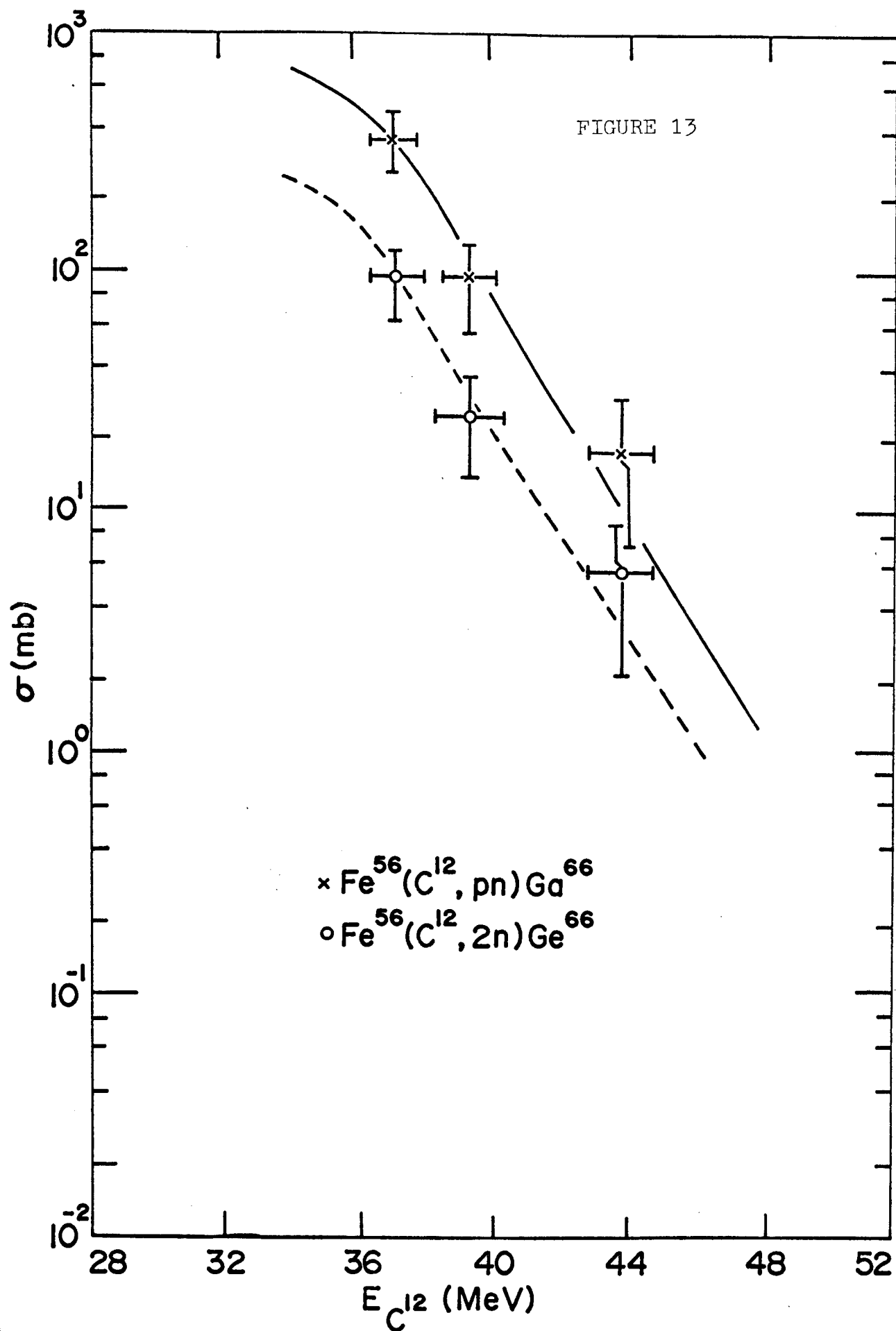


FIGURE 14

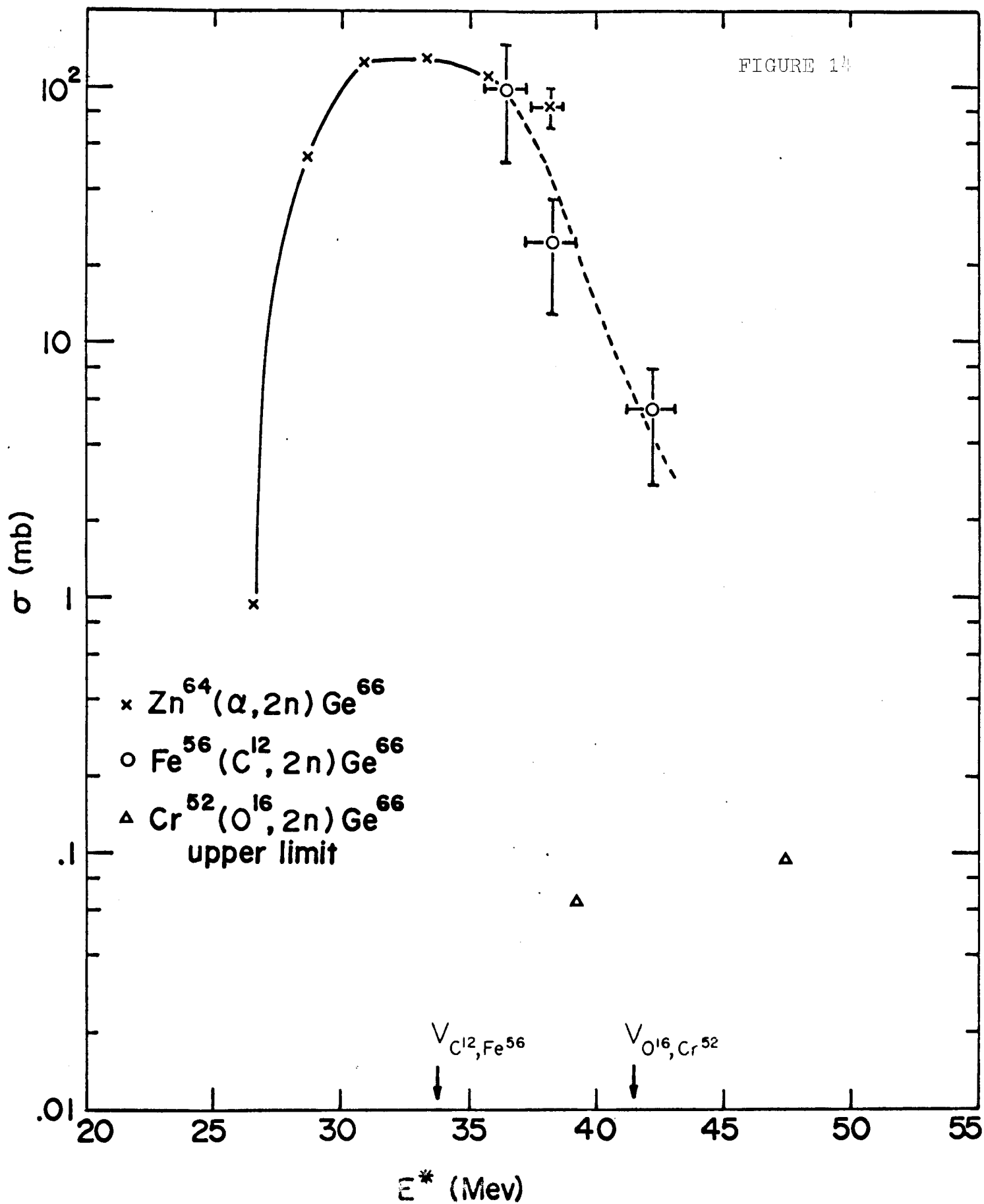
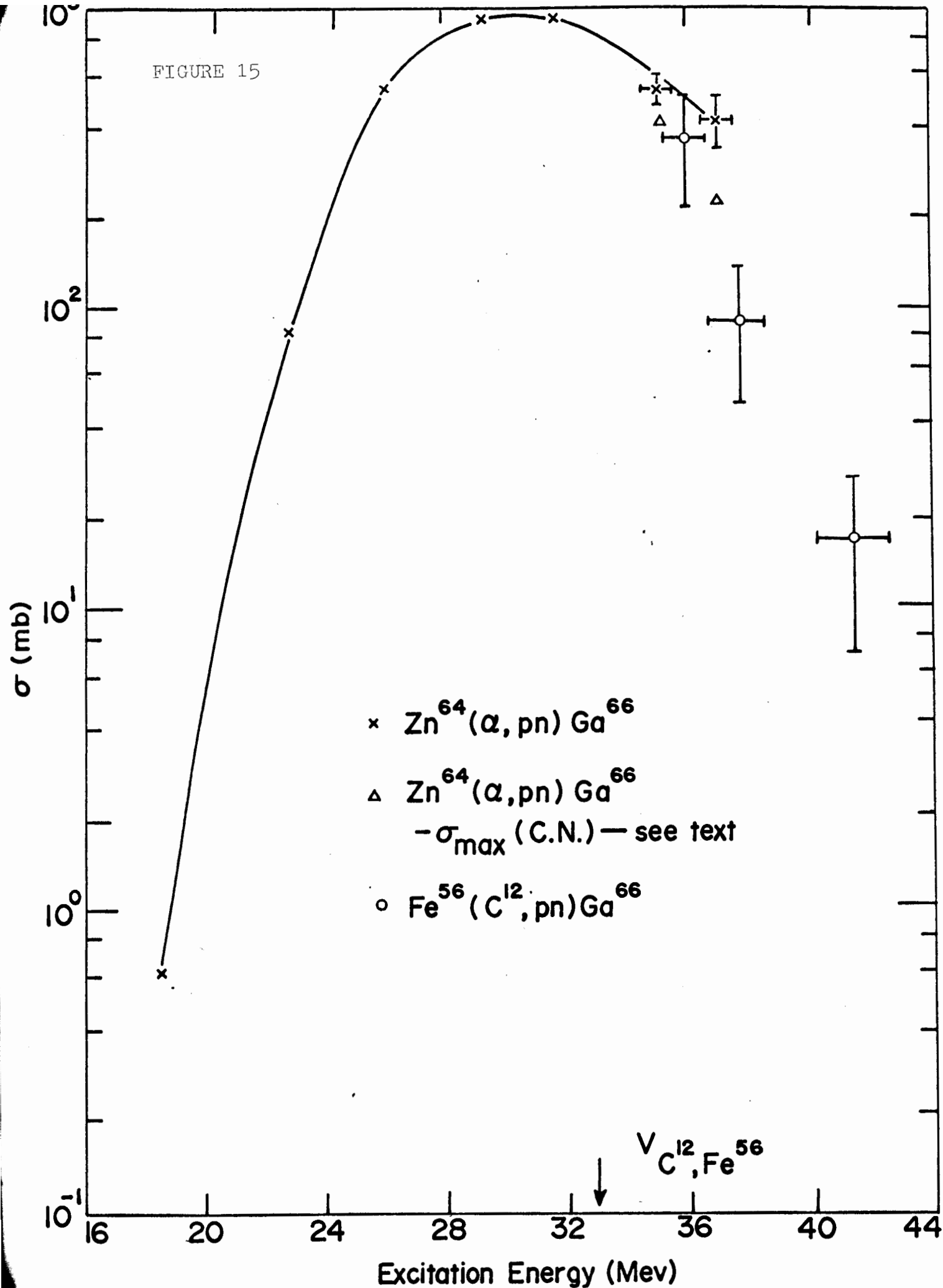


FIGURE 15



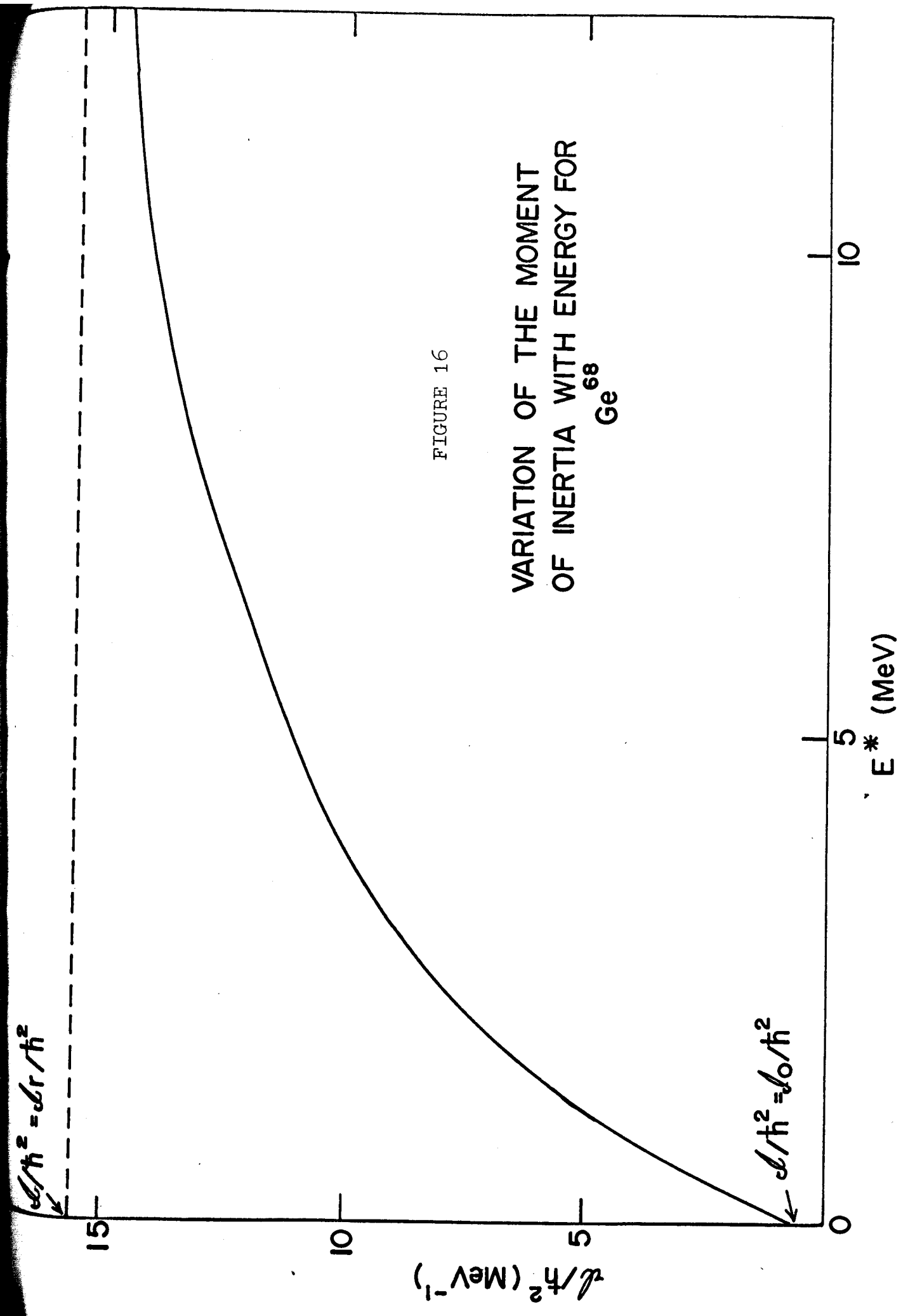


FIGURE 17

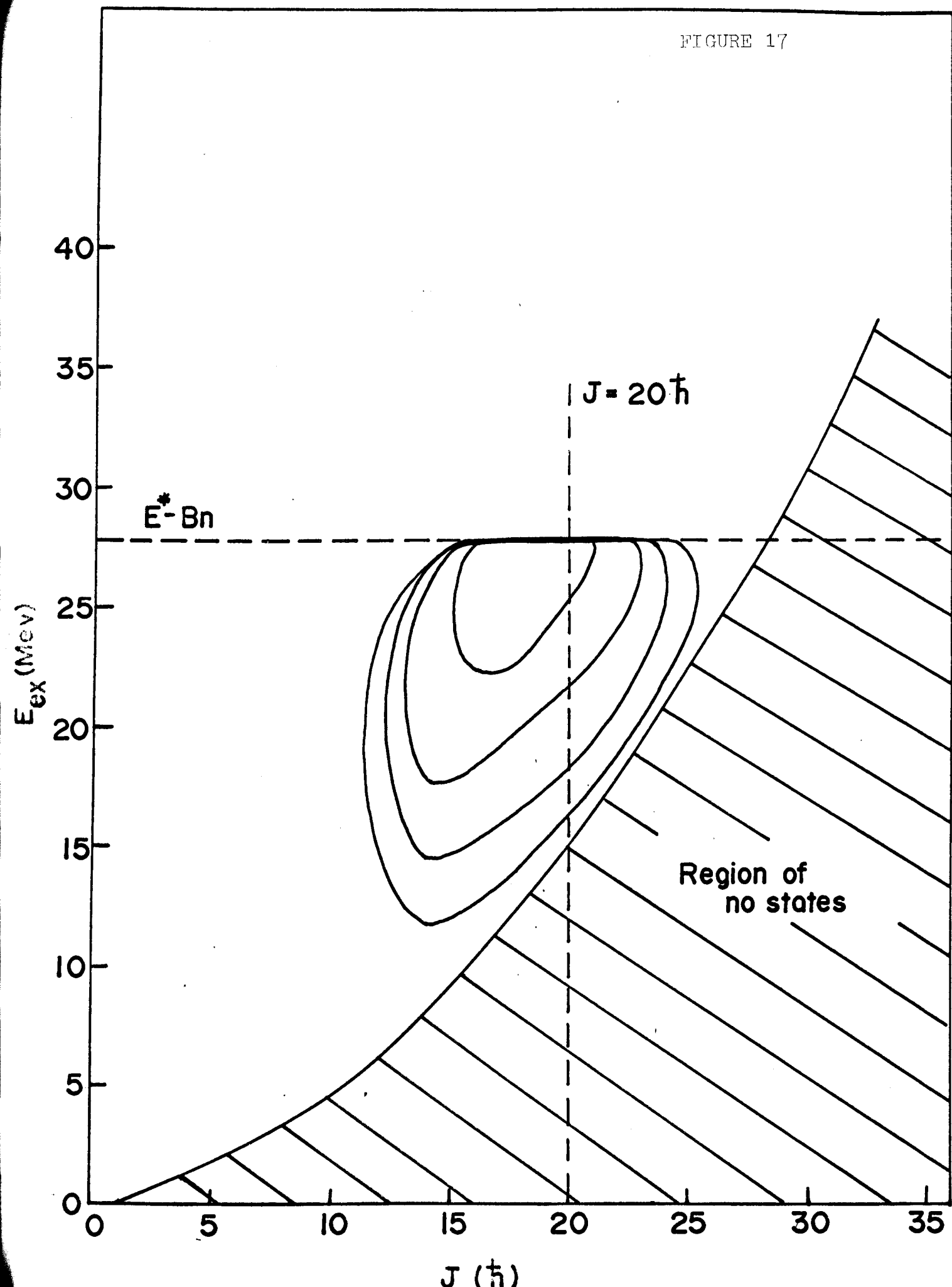


FIGURE 13

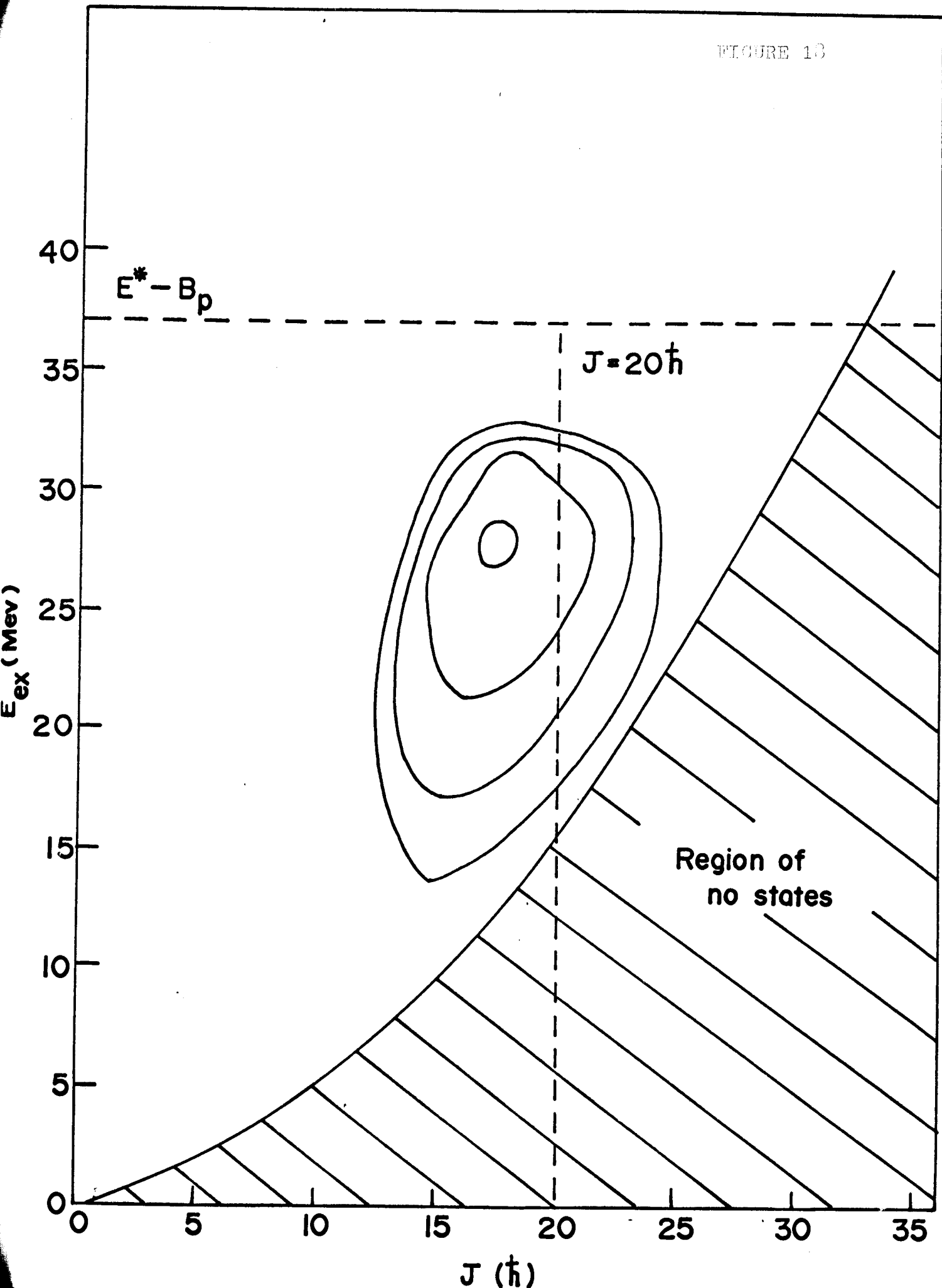


FIGURE 19

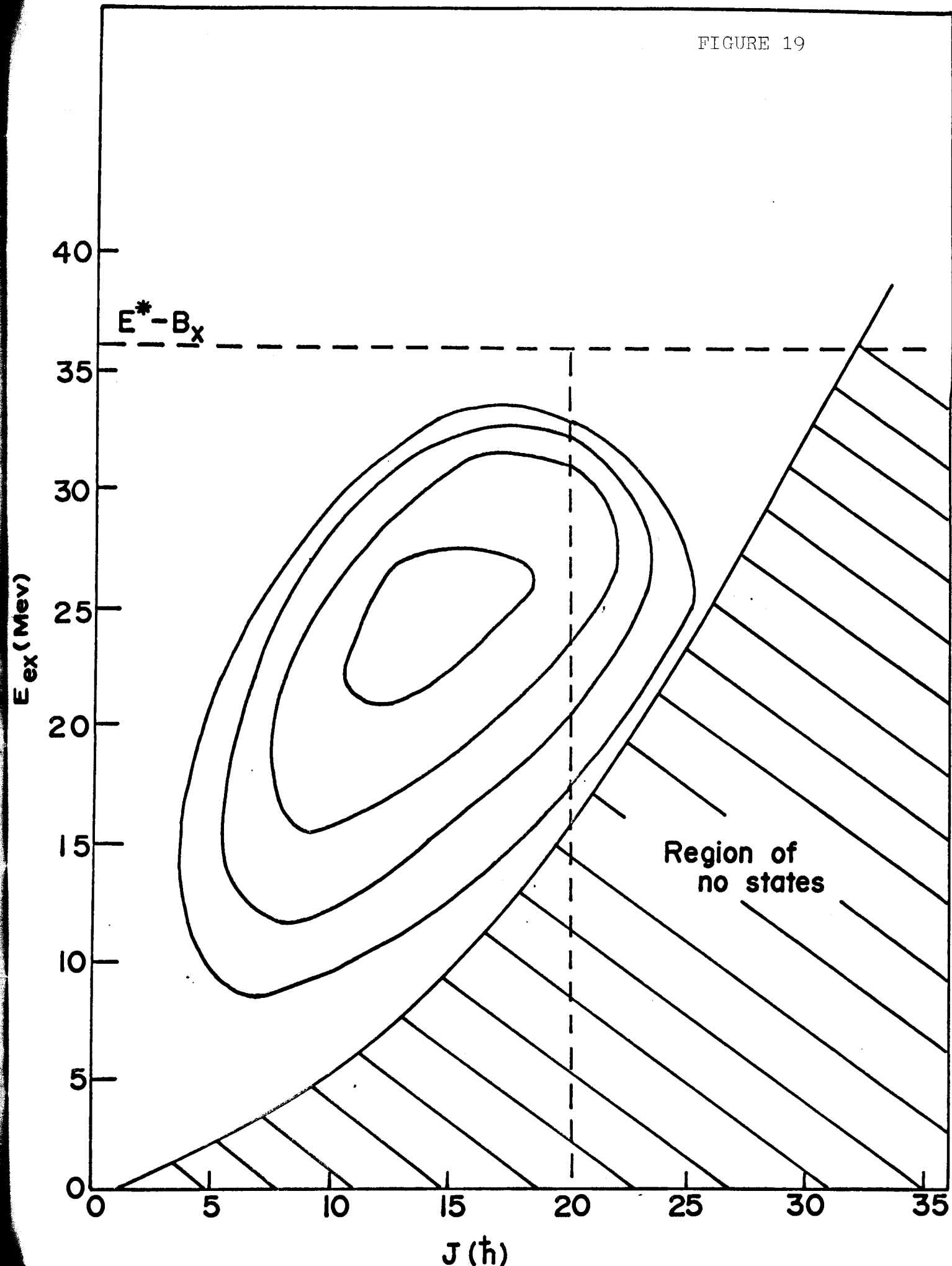


FIGURE 20

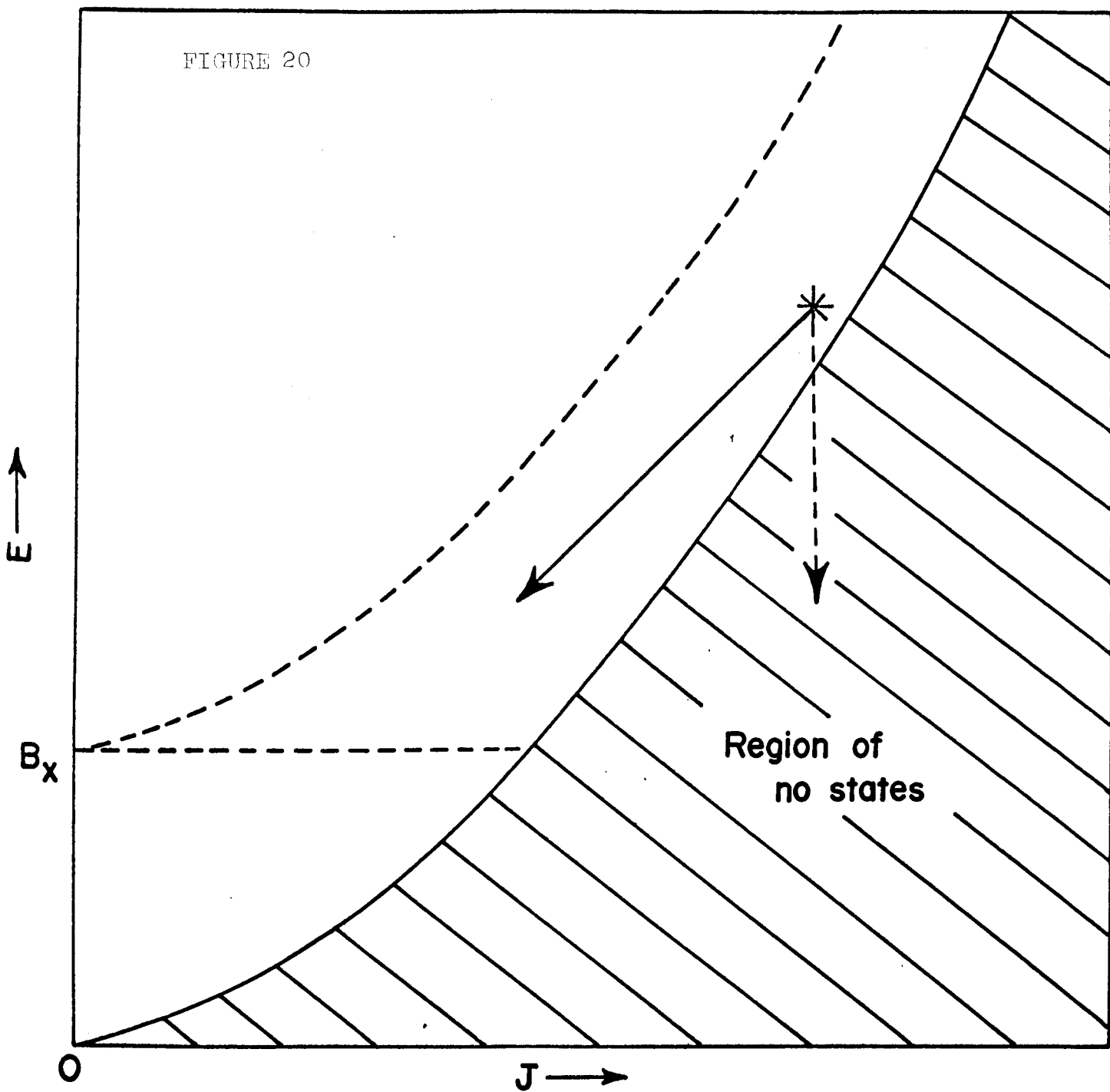


FIGURE 21

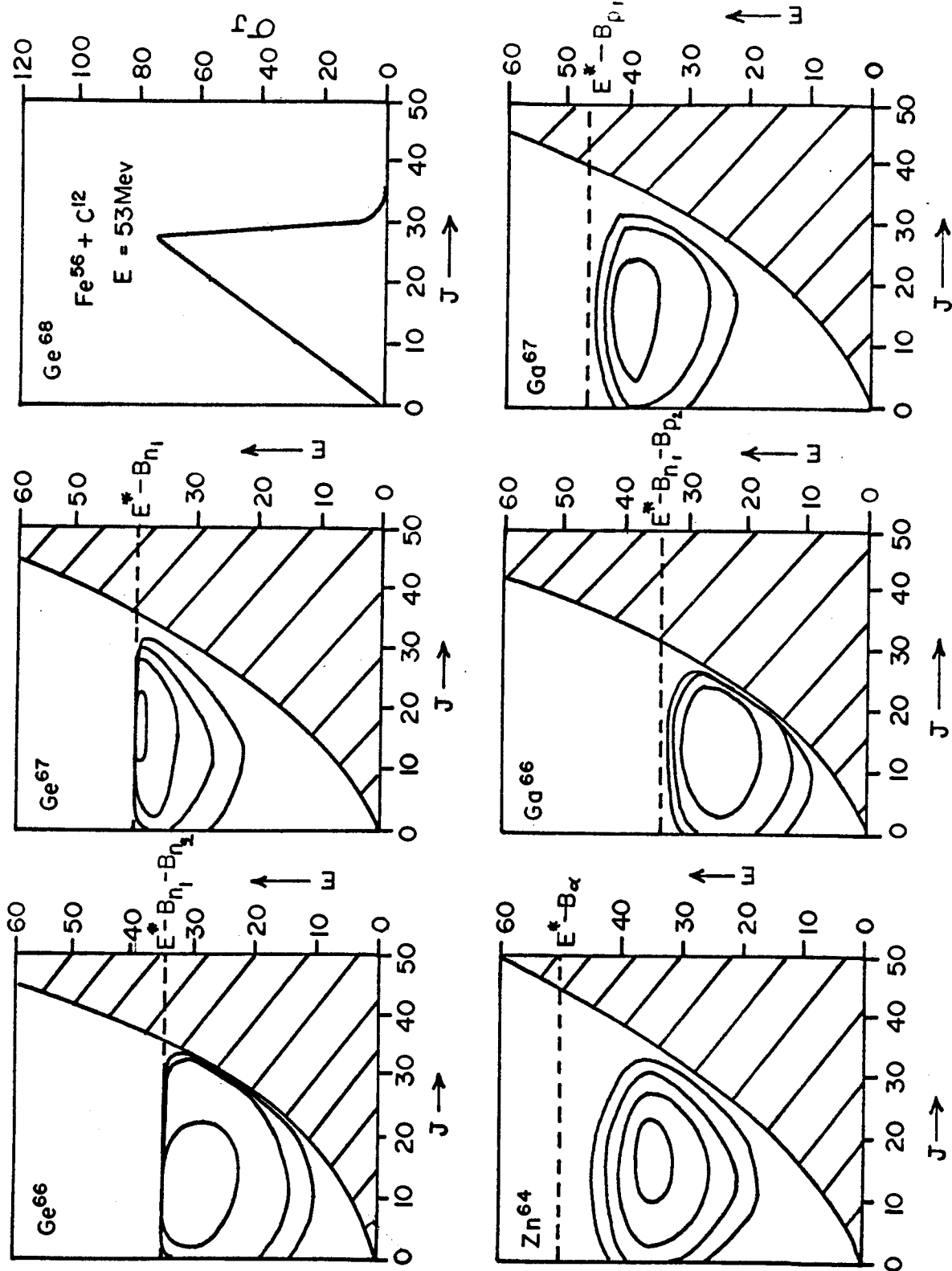


FIGURE 22

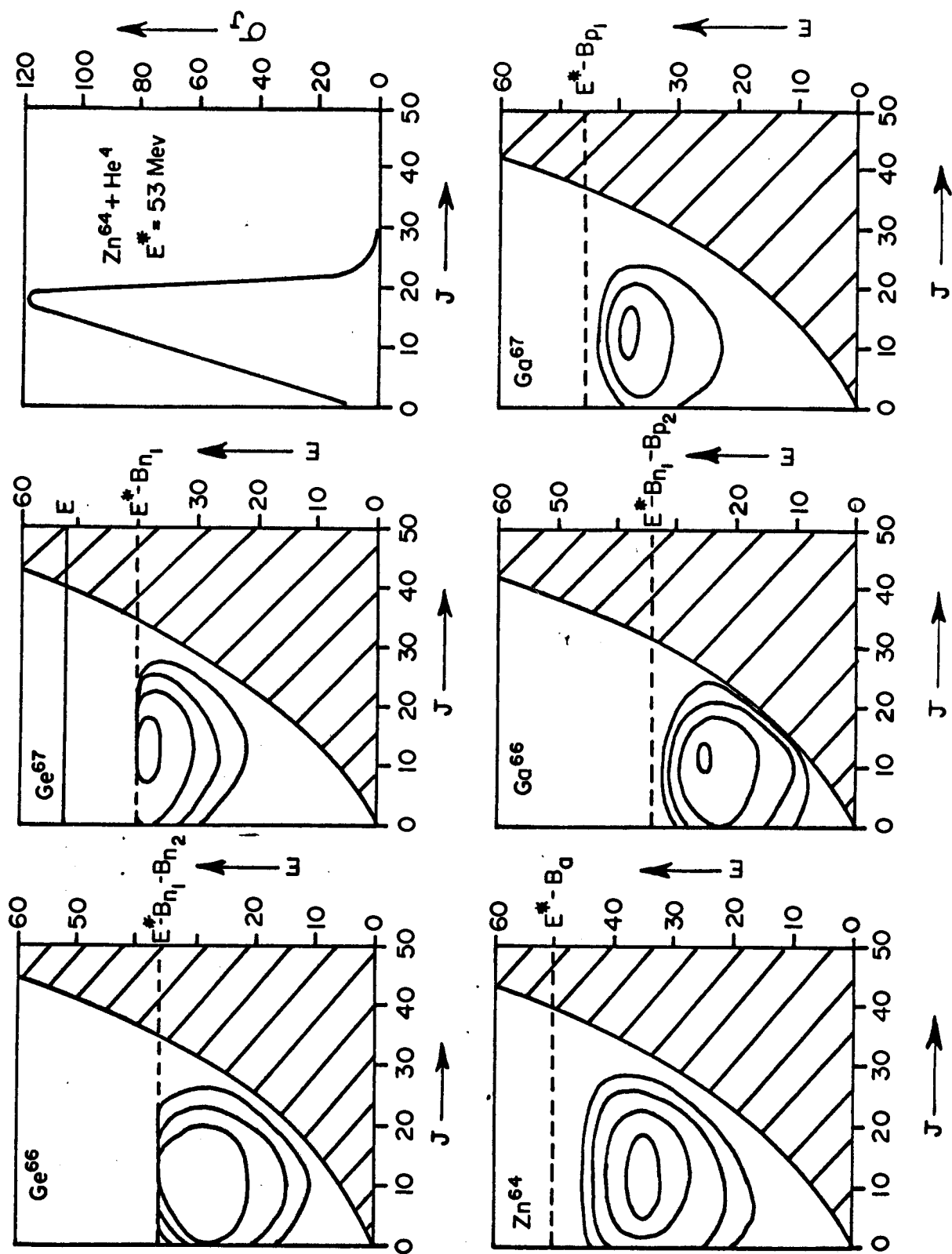


FIGURE 23

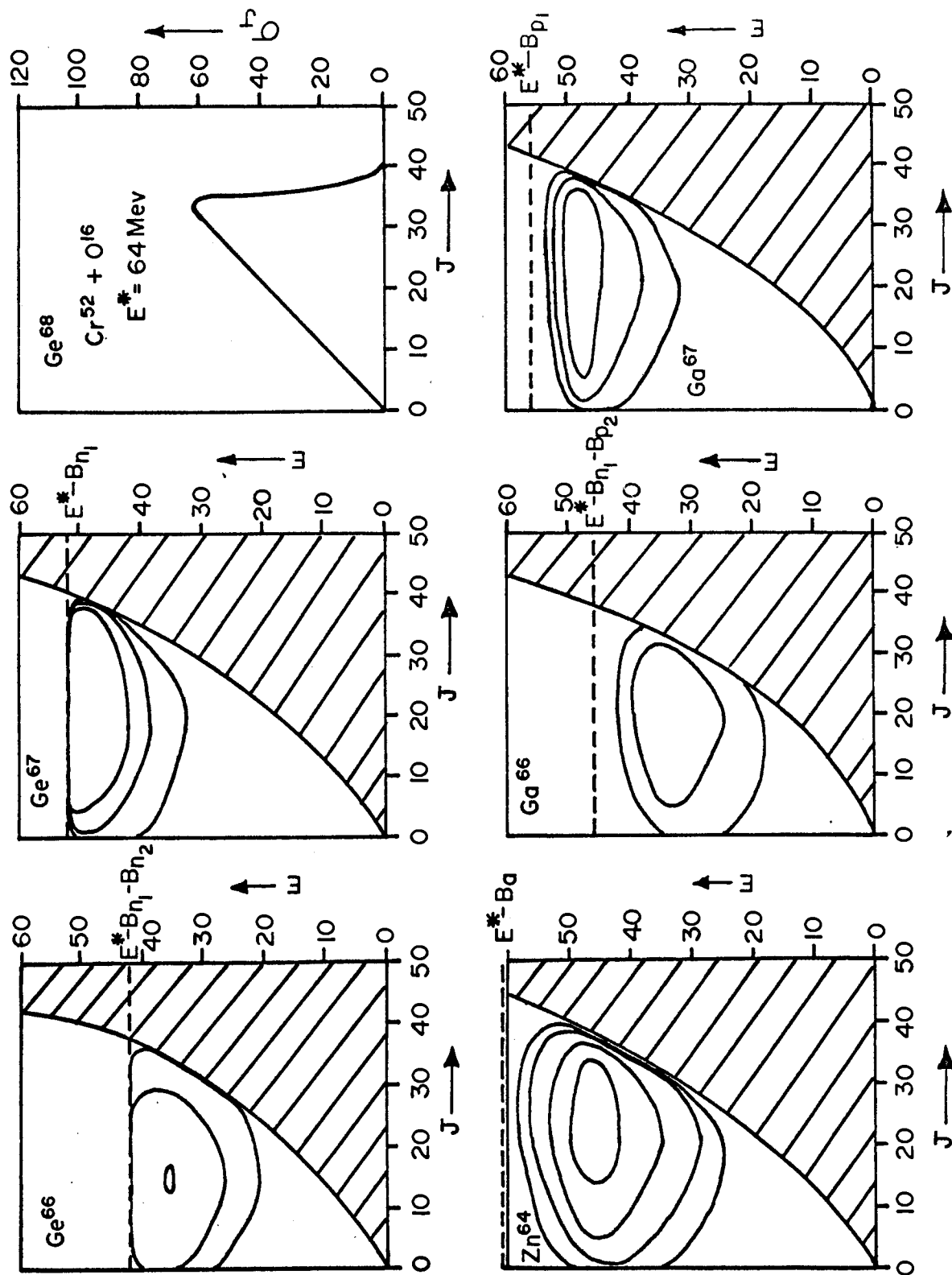


FIGURE 24

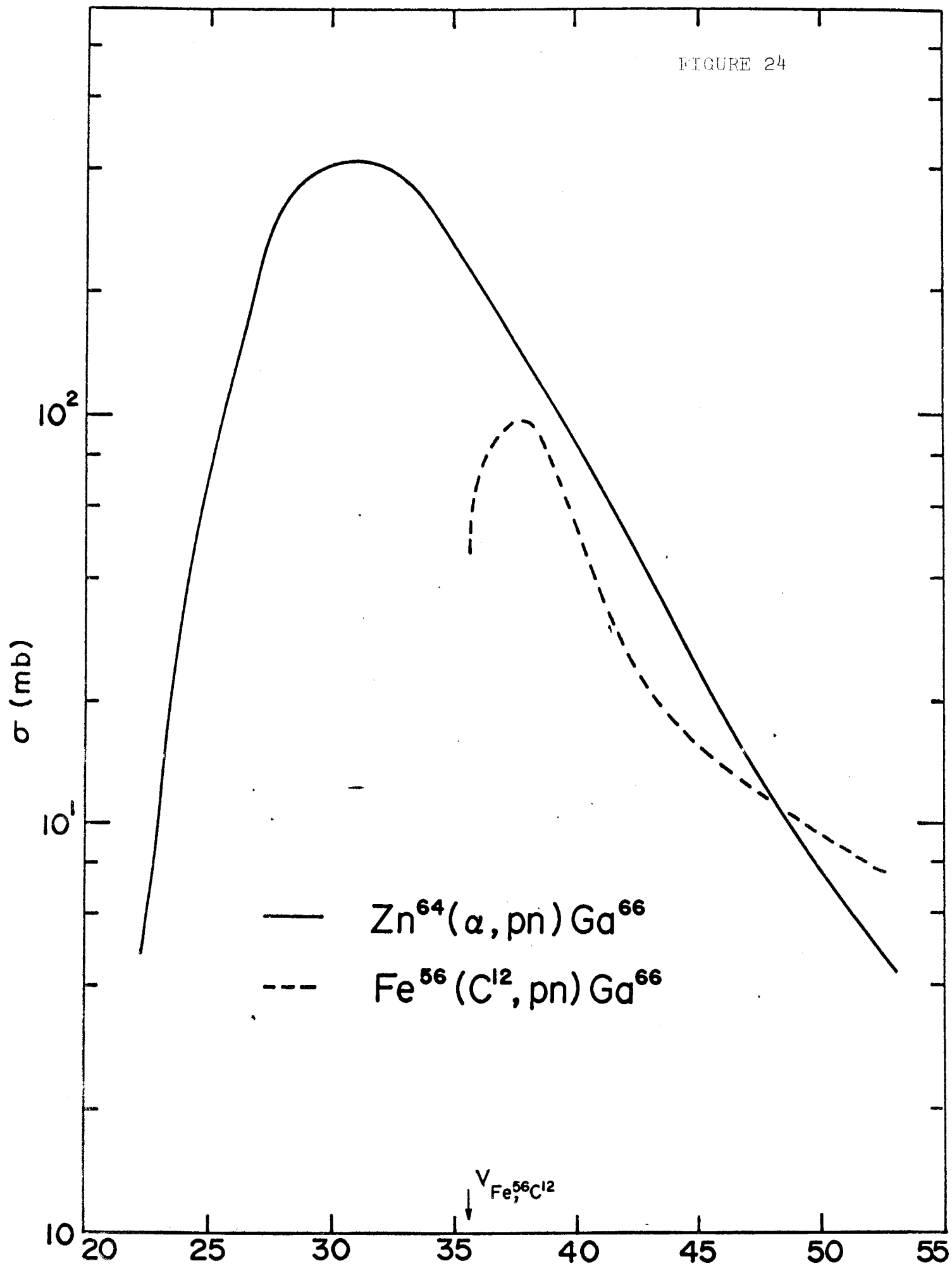


FIGURE 25

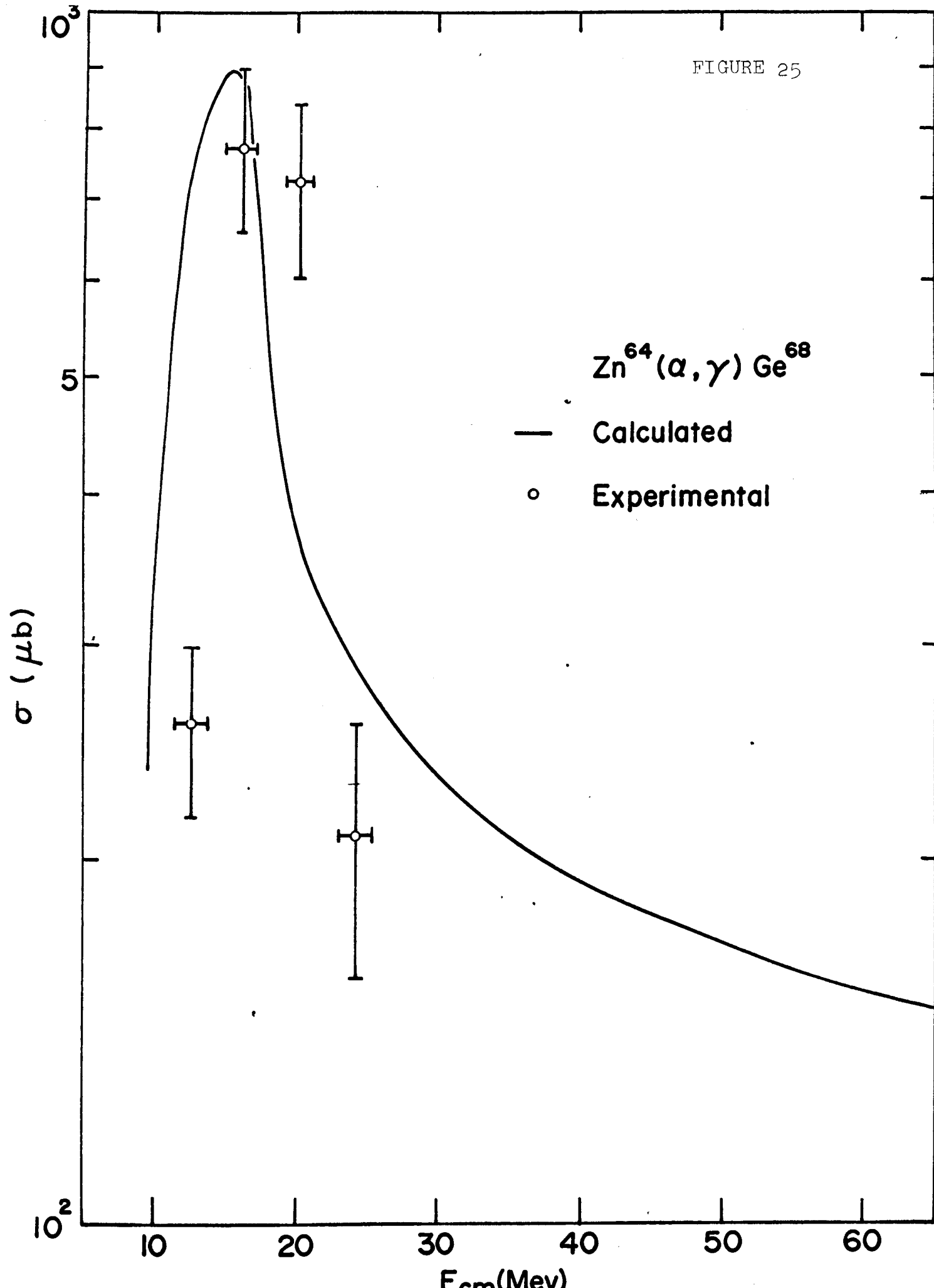


FIGURE 26

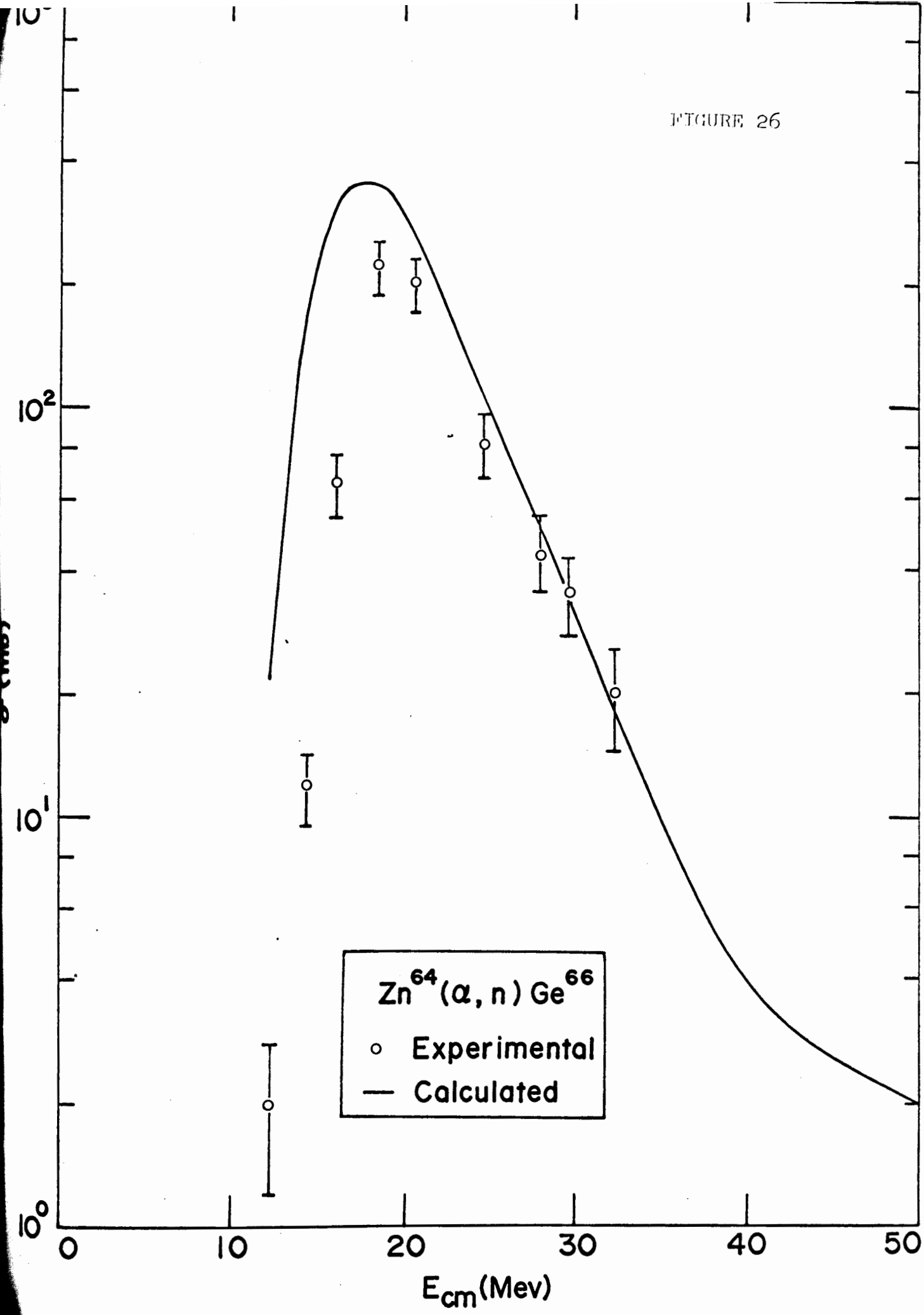
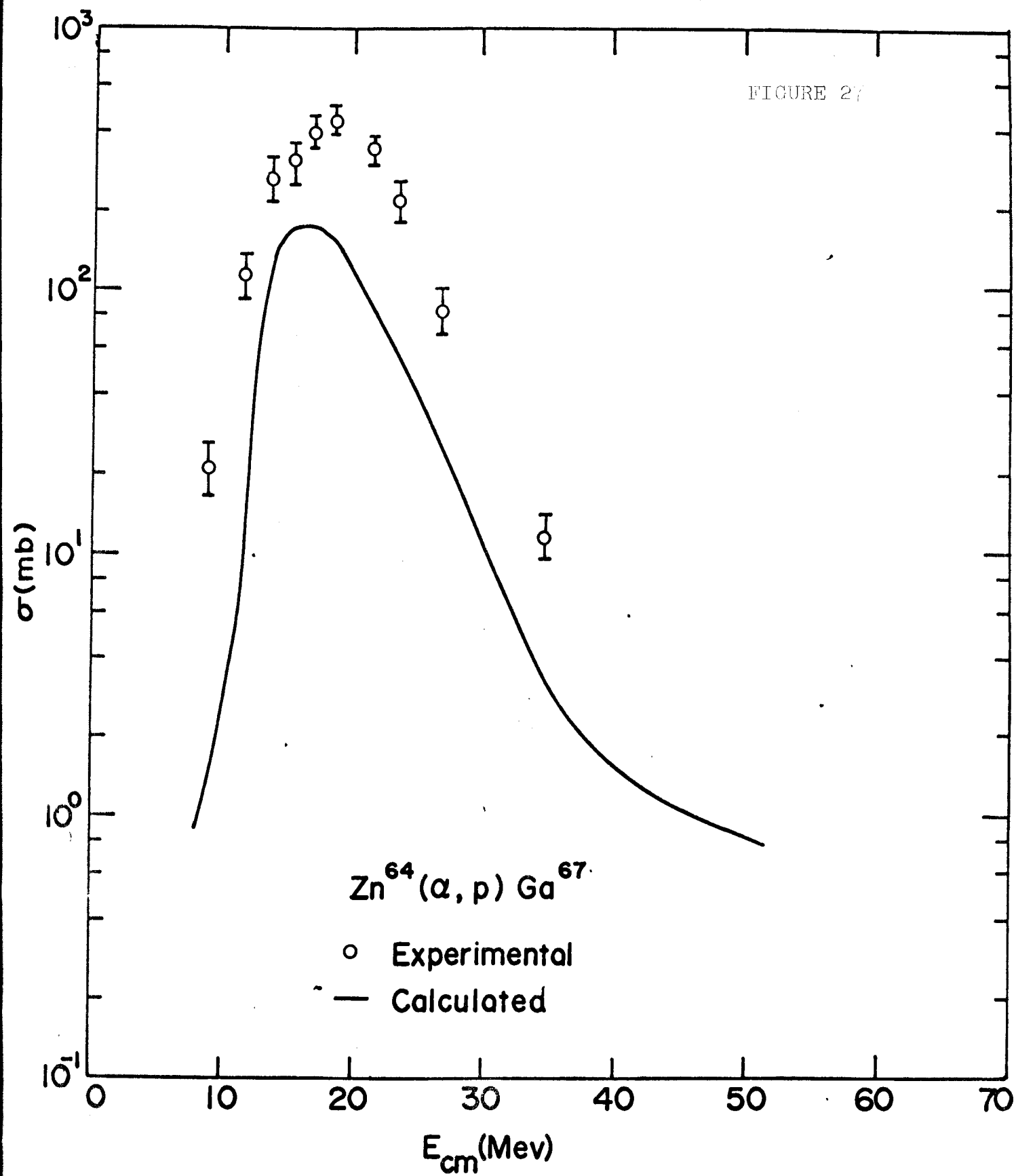


FIGURE 27



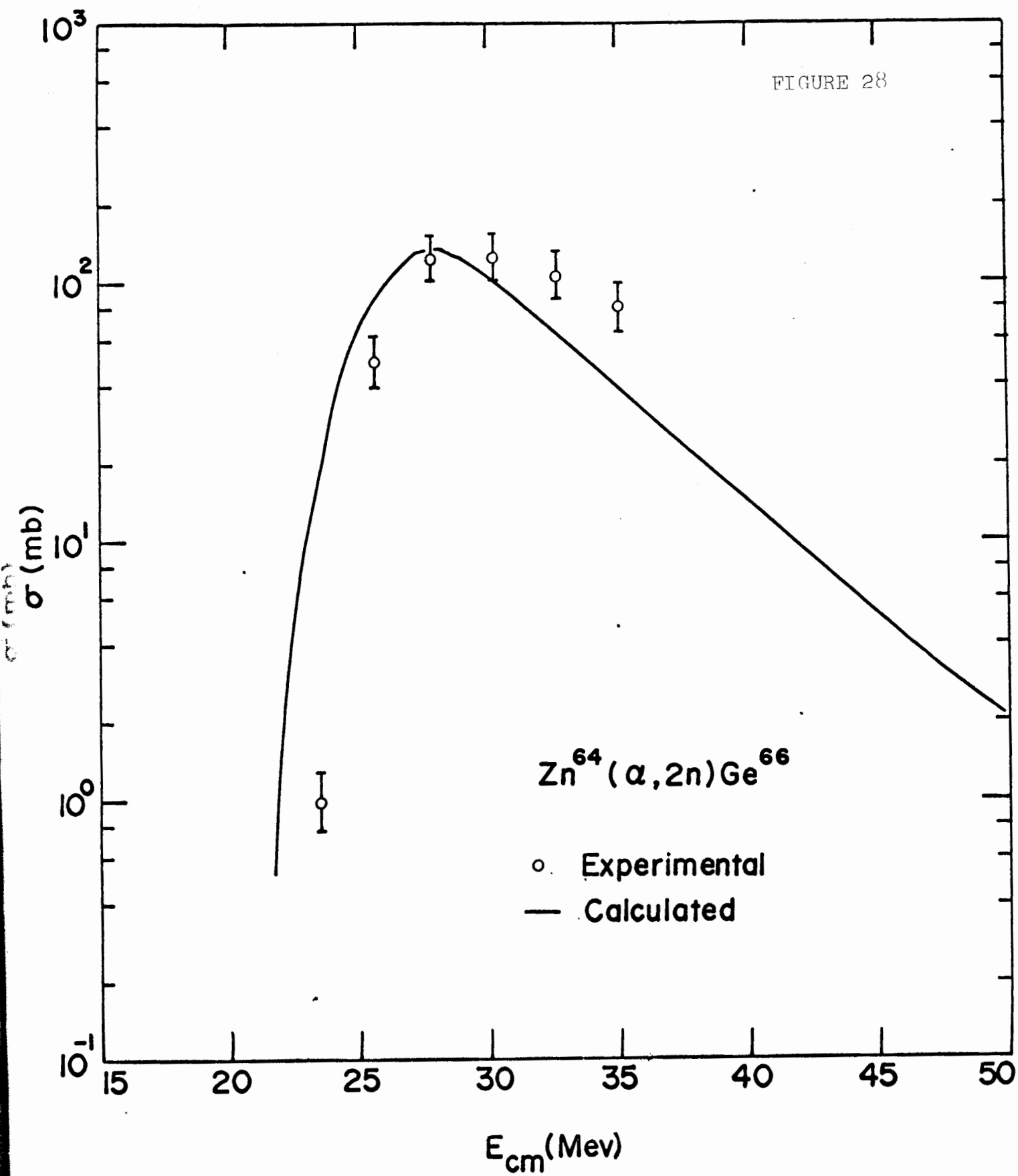


FIGURE 29

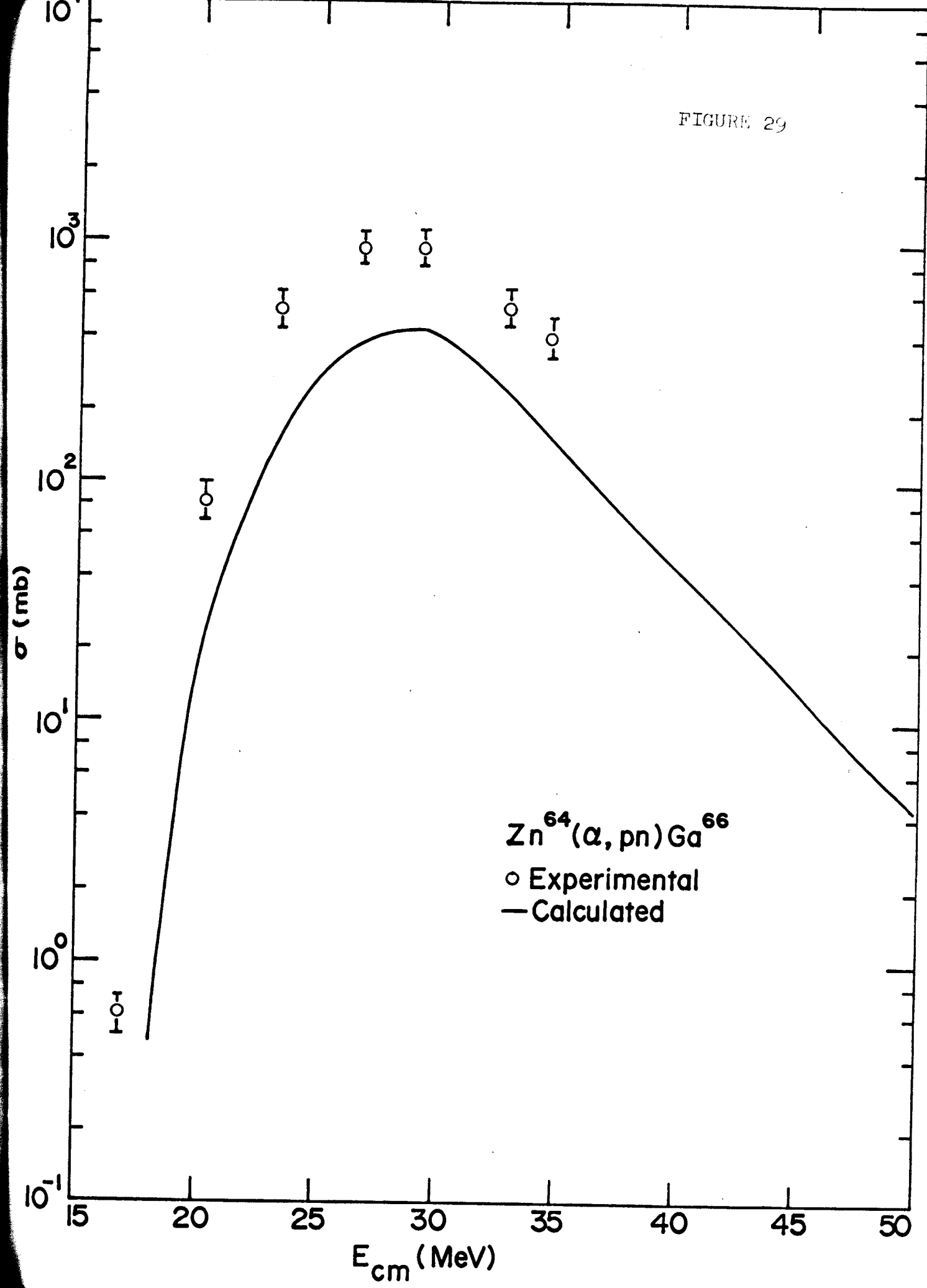


FIGURE 30

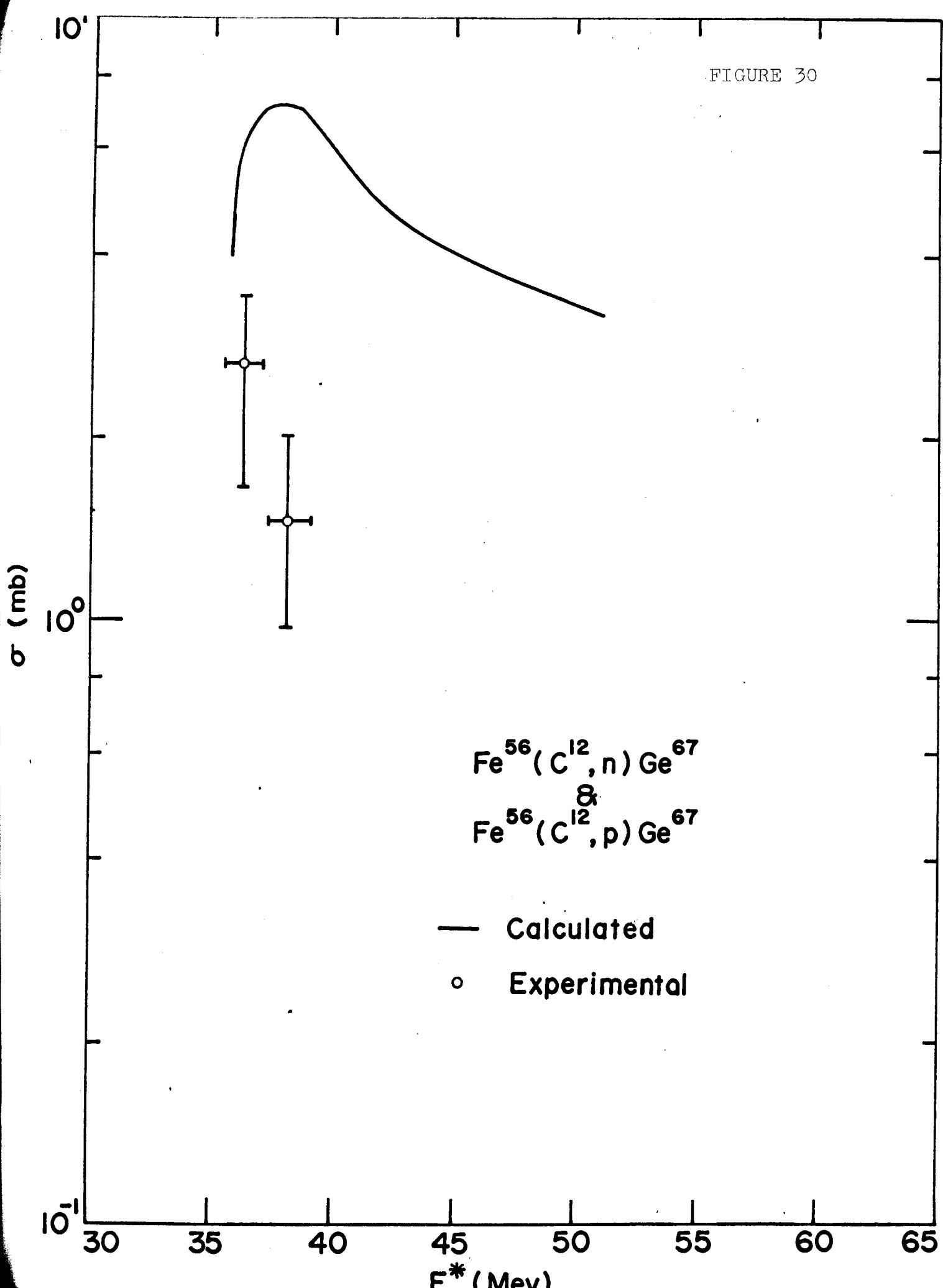
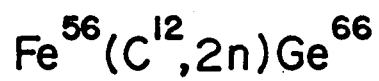


FIGURE 31



○ Experimental
— Calculated

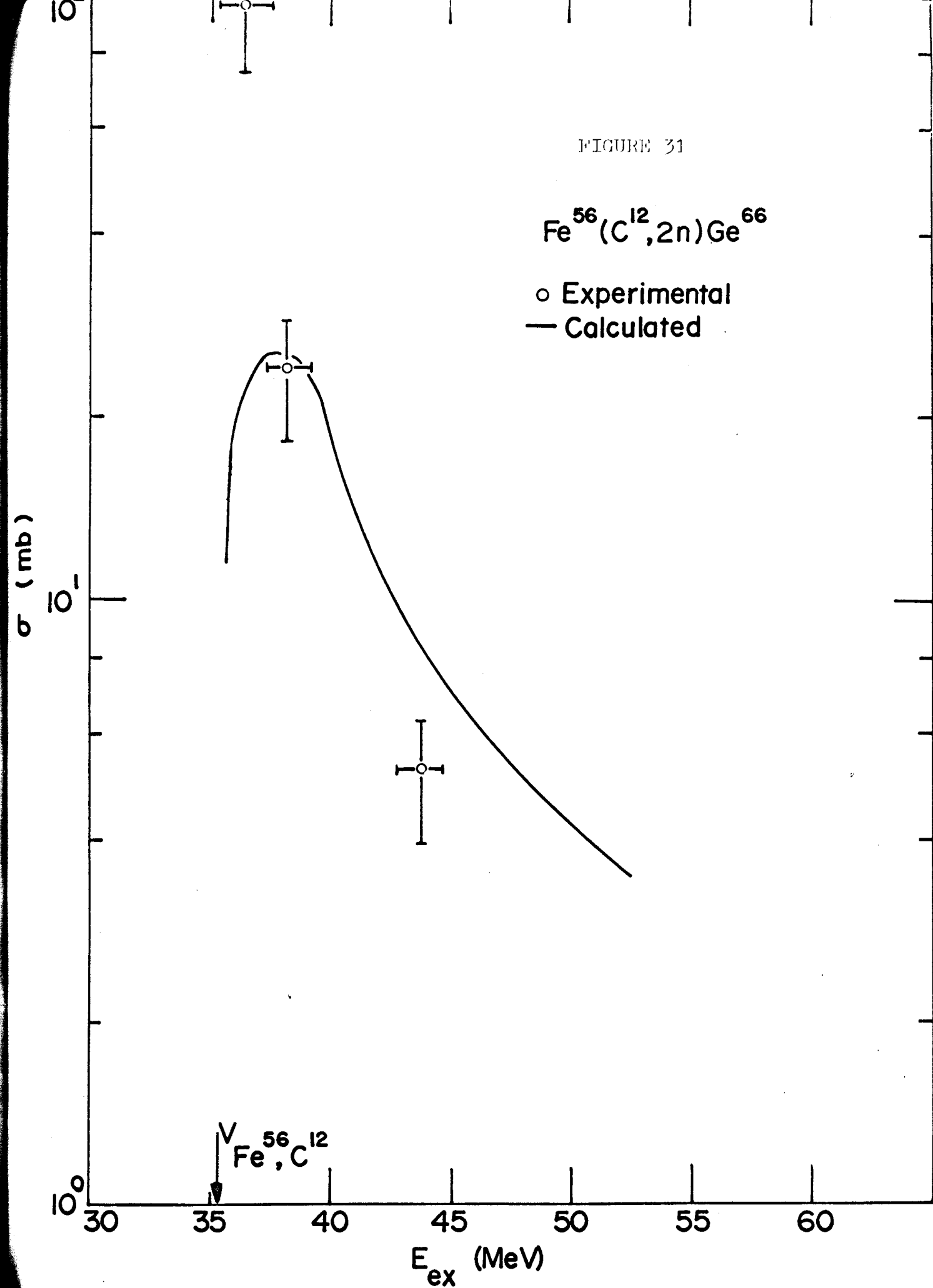
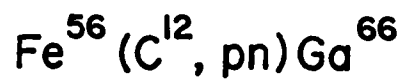
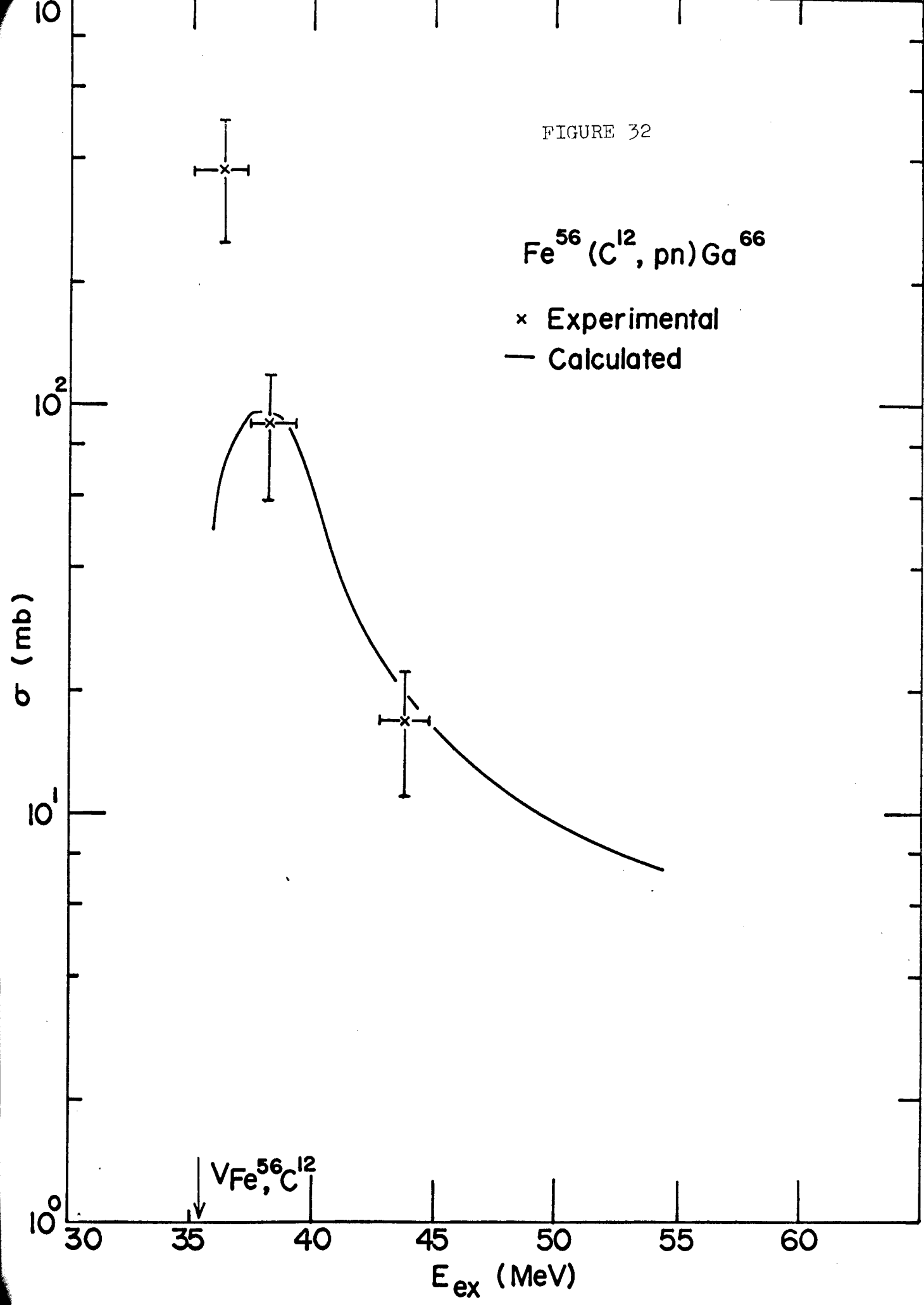
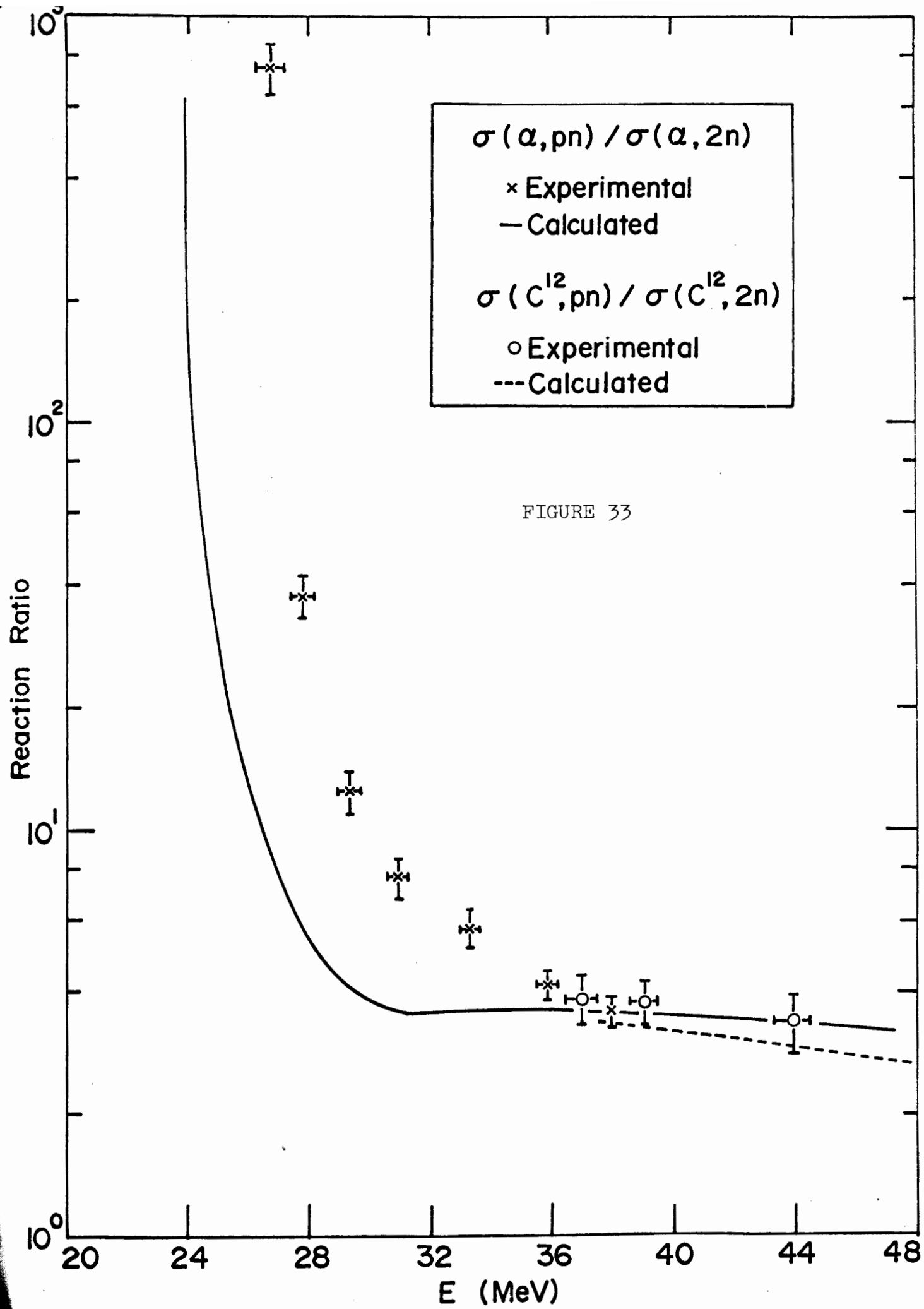


FIGURE 32



× Experimental
— Calculated





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Appendix I

Recoil Range Theories.

1. Recoil Range measurements.

Recoil ranges may be studied in a variety of ways,¹¹¹ the simplest of which is to measure projected ranges along the beam direction. For a given reaction at a specified energy, the observed ranges of nuclei recoiling from the target into some stopping medium will center about an average value R_0 . Deviations from this average value, called range straggling, arise from a number of sources:¹¹¹⁻¹¹⁴

(1.) Straggling from effects of the nuclear reaction, ie., resultant velocities of individual recoiling atoms will differ because of non-unique allignment of the velocity vectors of the emitted particles. (ρ_n)

(2.) Straggling inherent in the stopping process. (ρ_s)

(3.) Straggling caused by finite target thickness, ie., the individual recoiling nuclei originate from different depths in the target and escape with different degrees of the full range. (ρ_w)

(4.) Straggling caused by inhomogeneities in the stopping medium. (ρ_f)

The overall range straggling parameter is the resultant of parameters corresponding to the above combined in quadrature

$$\rho^2 = \rho_n^2 + \rho_s^2 + \rho_w^2 + \rho_f^2$$

and the range straggling is given by $R_0\rho$. If the distribution

in range values is Gaussian, then¹¹².

$$P(R)dR = \frac{1}{R_0 \rho \sqrt{2\pi}} \exp \left[- \left(\frac{R-R_0}{\sqrt{2R_0 \rho}} \right)^2 \right] dR \quad (I-1)$$

Assuming uniform production of recoils across a layer of target atoms of thickness W, the fraction of recoils which remain in the target is^{111,112}.

$$F_W = \frac{1}{(2\pi)^{\frac{1}{2}} R_0 W} \int_0^W \int_0^W \exp \left\{ - \left(\frac{r-s-R_0}{\sqrt{2R_0 \rho}} \right)^2 \right\} dr ds \quad (I-2)$$

where s is the distance from the edge of the layer to the point at which the recoiling atom originates and r-s is the distance that it travels. The integration of the above equation leads to the following^{111,112}.

$$F_W = \frac{\sqrt{2R_0 \rho}}{W} \left[\mathfrak{I} \left(\frac{R_0-W}{\sqrt{2R_0 \rho}} \right) - 2\mathfrak{I} \left(\frac{R_0}{\sqrt{2R_0 \rho}} \right) + \mathfrak{I} \left(\frac{R_0+W}{\sqrt{2R_0 \rho}} \right) \right] \quad (I-3)$$

where

$$\mathfrak{I}(y) = \frac{1}{2\sqrt{\pi}} \exp(-y^2) - \frac{1}{2} [1 - I(y)] y$$

and

$$I(y) = \frac{2}{\sqrt{\pi}} \int_0^y \exp(-u^2) du$$

F_W may be approximated by the first term for $W \gg R_0$

$$F_W \cong \frac{\sqrt{2R_0 \rho}}{W} \mathfrak{I} \left(\frac{R_0-W}{\sqrt{2R_0 \rho}} \right) \quad (I-4)$$

and if the thickness is many times the average range, (D-4) reduces to

$$R_0 = W(1-F_W) \quad (I-5)$$

If one bombards a thick target backed by a thick catcher foil, F_W is calculable from the activities observed in the target and catcher foils,

$$1 - F_W = \frac{A_C}{A_T + A_C} \quad (I-6)$$

and the calculation of R_0 follows.

In practice, recoiling atoms will not be produced uniformly across the target thickness. If one assumes a linear variation of cross-section over the energy range corresponding to the target thickness, ie.,

$$\sigma_s = \frac{(\sigma_W - \sigma_O)s}{W} + \sigma_O \quad (I-7)$$

where σ_O is the cross-section at the target-catcher interface ($s=0$) and σ_W is the cross-section at the opposite surface ($s=W$), and all recoiling ions are assumed to be formed within their range, R_0 , from the surface, then⁷⁶.

$$1 - F_W = \frac{A_C}{A_T + A_C} = \frac{\int_0^{R_0} \sigma_s ds}{\int_0^W \sigma_s ds} = \frac{R}{W^2} \cdot \frac{2\sigma_O W + (\sigma_W - \sigma_O)R_0}{\sigma_O + \sigma_W}$$

For small differences between σ_O and σ_W ($|\sigma_W - \sigma_O| < \sigma_O$), and $R \ll W$.⁷⁶

$$R_0 = (1 - F_W) W \frac{(\sigma_O + \sigma_W)/2}{\sigma_O} \quad (I-8)$$

that is, the range must be corrected by a factor corresponding to the ratio of the average cross-section in the foil to the cross-section at the target-catcher interface.

Such thick target, thick catcher, or integral, range experiments provide a simple means for determining ranges if the variation of cross-section is small across available target thicknesses and the distribution in ranges is known to be Gaussian. If these conditions are not met, however, a differential method may be appropriate.

A convenient differential method for measuring average projected ranges in the beam direction is to bombard thin targets ($W \ll R_0$) backed by several thin catchers. If F_t , the fraction of activity that passes through thickness t , is plotted on a probability scale vs. t , the t value corresponding to $F_t = \frac{1}{2}$ defines R_0 . The t value for F_t equals 0.0787 specifies $R_0(1 + \sqrt{2\rho})$ ¹¹².

Also, any deviation from Gaussian distribution about the average projected range may be observed directly in this type of experiment. However, one must be studying a product which is produced in high yield and the average range must be relatively large ($>200\text{mg}/\text{cm}^2$) in order for this type of differential measurement to be experimentally feasible.

2. The Stopping Process.

In order for interpretation of the results of recoil range measurements to be possible, the nature of the interactions of recoiling atoms with matter must be known. According to the formulation due to Bohr,¹¹⁵ the stopping process is dependent on the velocity (v) of the moving atom.

If v is greater than the orbital velocities of the electrons of the stopping atoms, stopping is mainly by interaction with these electrons. If v is less than the orbital electron velocity, stopping is by interaction with the atoms as a whole (e.g. with the crystal lattice of the stopping medium.) In the latter case, Bohr derived the following formula¹¹⁵. for R_0 expressed in mg/cm^2 and E in MeV.

$$R_0 = 0.600 \frac{A_S(A_S+A_R)}{A_R} \frac{(Z_R^{\frac{2}{3}}+Z_S^{\frac{2}{3}})^{\frac{1}{2}}}{Z_S Z_R} E$$

which holds when the recoiling particle masses are much larger than the stopping atomic masses ($A_R \gg A_S$).

In general, the stopping power may be represented (neglecting channeling effects) as the following¹¹⁶.

$$\frac{dE}{dX} = k[E^{\frac{1}{2}} + c_1]$$

The first term corresponds to electronic stopping and the second to elastic (atomic) stopping. Lindhard, Scharff, and Schiøtt⁷⁸. (LSS) have derived a general stopping theory where atomic stopping was described by a Thomas-Fermi potential and the proportionality constant, k , for the electronic stopping term was given as⁷⁸.

$$k = \xi_e \frac{0.0793 Z_R^{\frac{1}{2}} Z_S^{\frac{1}{2}} (A_R + A_S)^{\frac{3}{2}}}{(Z_R^{\frac{2}{3}} + Z_S^{\frac{2}{3}})^{\frac{3}{4}} A_R^{\frac{3}{2}} A_S^{\frac{1}{2}}} \quad (\text{I-9})$$

$$\xi_e = Z_R^{\frac{1}{6}}$$

The LSS theory makes use of dimensionless range and energy parameters, ρ and ϵ , given by

$$\begin{aligned}\rho &= N A_S 4\pi a^2 A_R (A_R + A_S)^{-2} \\ \epsilon &= E a A_R [Z_R Z_S e^2 (A_R + A_S)]^{-1} \\ a &= a_0 \{0.8853 (Z_R^{\frac{2}{3}} + Z_S^{\frac{2}{3}})^{-\frac{1}{2}}\}\end{aligned}$$

Where N is the number of stopping atoms per unit volume, e is the electronic charge, and a_0 is the first Bohr radius in the hydrogen atom. The LSS calculations produce a set of ρ - ϵ curves, each characterised by a value of k . The straggling, or mean square deviation from the mean, is given by plotting the quantity

$$\frac{(A_R + A_S)^2}{4 A_R A_S} \frac{\overline{\Delta R}^2}{\overline{R}^2}$$

vs. ϵ for different values of k . The range along the total path length of the recoiling atom is related to the projected range by the following correction factor given by LSS.

$$R = R_0 \frac{1}{4} [-1 - 3\mu + (5 + \mu) \cdot \frac{1 + \mu}{2\mu^{\frac{1}{2}}} \arccos(\frac{1 - \mu}{1 + \mu})]$$

where $\mu = A_S/A_R$. LSS have given curves of $\frac{1}{\mu} \frac{R - R_0}{R_0}$ and $\frac{R}{R_0} - 1$ vs. ϵ for several values of k .

At low energies

$$\frac{R}{R_0} \cong 1 + \frac{\mu}{3} \quad (I-11)$$

although several inadequacies exist in the theories of LSS¹¹¹, they have, in general, provided agreement within

experimental error with the type of integral and
differential ranges measured by the previously described
experiments. 76,77,117-121.

Appendix II

Coupling Schemes.

For the reaction $X+b \rightarrow Y+x$, the target and projectile will have a mutual orbital angular momentum, \vec{l} , as will the products, \vec{l}' . X and Y will have spins \vec{I} and \vec{I}' and orbital angular momenta \vec{L} and \vec{L}' , respectively, and b and x will have spins \vec{s} and \vec{s}' , respectively. Angular momentum must be conserved; however, the method of addition of these vectors depends on the type of interaction assumed or "coupling" scheme used²⁹.

(a) j-j Coupling.

$$\vec{J} = \vec{I} + \vec{L} \qquad \vec{J}' = \vec{I}' + \vec{L}'$$

$$\vec{j} = \vec{l} + \vec{s} \qquad \vec{j}' = \vec{l}' + \vec{s}'$$

$$\vec{J} + \vec{j} = \vec{J}_c = \vec{J}' + \vec{j}'$$

(b) L-S (Russel Saunders) Coupling.

$$\vec{S} = \vec{I} + \vec{s} \qquad \vec{S}' = \vec{I}' + \vec{s}'$$

$$\vec{L}_1 = \vec{L} + \vec{l} \qquad \vec{L}_2 = \vec{L}' + \vec{l}'$$

$$\vec{S} + \vec{L}_1 = \vec{J}_c = \vec{S}' + \vec{L}_2$$

(c) Intermediate Coupling.

$$\vec{J} = \vec{I} + \vec{L} \qquad \vec{J}' = \vec{I}' + \vec{L}'$$

$$\vec{S} = \vec{J} + \vec{s} \qquad \vec{S}' = \vec{J}' + \vec{s}'$$

$$\vec{S} + \vec{l} = \vec{J}_c = \vec{S}' + \vec{l}'$$

The quantity S is called the channel spin²⁹.

If the intermediate coupling scheme is used, the distribution in angular momentum of intermediate states is given by⁶.

$$\sigma_c(I, \epsilon, J_c) = \frac{\pi \lambda^2 (2J_c + 1)}{(2s + 1)(2I + 1)} \sum_{s=|I-s|}^{(I+s)} \sum_{t=|J_c-s|}^{(J_c+s)} T_t(\epsilon)$$

Appendix III.

The $\text{Cu}^{63}(\alpha, \text{pn})\text{Zn}^{65}$ and $\text{Cu}^{63}(\alpha, 2\text{n})\text{Ga}^{65}$ Reactions.

The excitation function for the production of Zn^{65} in natural copper foils via the $\text{Cu}^{63}(\alpha, 2\text{n})\text{Ga}^{65}$ and $\text{Cu}^{63}(\alpha, \text{pn})\text{Zn}^{65}$ reactions was measured by the bombardment of 3.64 mg/cm^2 natural copper foils⁵⁸. backed by thick aluminium catchers and interspersed between aluminium degrading foils. The beam intensity was monitored via the production of Ga^{67} in zinc foils by the $\text{Zn}^{64}(\alpha, \text{n})\text{Ge}^{67}$ and $\text{Zn}^{64}(\alpha, \text{p})\text{Ga}^{67}$ reactions.

All 15 minute Ga^{65} activity was allowed to decay to 245 day Zn^{65} . A $7.6 \text{ cm} \times 7.6 \text{ cm}$ NaI(Tl) detector was employed to monitor the intensity of the 511-kev and 1115-kev peaks. Efficiency data due to Heath⁷⁰. were employed. Positron emission was taken to occur in 51.7% ⁷⁴. of the decays and the intensity of the 1115-kev peak was taken as 0.49 photons/disintegration⁷⁴.. The disintegration rates determined separately from the two peaks were found to agree within a few percent in all cases.

The measured excitation function is shown in figure 34.

Appendix IV.

Characterization of 280 day Ge^{68} .

The activity of 280 day Ge^{68} was determined by radioactivity assay of its 67 minute daughter after secular equilibrium had been established. Ge^{68} decays by pure electron capture, and therefore does not emit any easily detectable radiation. The radiations emitted during the decay of Ga^{68} are easily detectable, however.

The identity of Ga^{68} was established by a series of measurements immediately after separation of GeCl_4 from the dissolved target (several months after the irradiation to eliminate shorter lived Ge activities), followed by analysis of the radioactive growth curve. Figure 35 is such a curve. The crosses denote measured activity plotted versus time from the end of the separation to the time of the measurement of the activity. The open circles represent the difference between measured activity and the equilibrium activity. The half life was measured from the decay curve defined by the open circles in order to establish the identity of this isotope, and was found in all cases to agree well with the value measured in an independent study of the nuclide Ga^{68} .

Appendix V.

SFUSMAP

The Fortran VI statistical model program, SFUSMAP, consists of a main program, four subprograms, and a function subroutine.

a.) MAIN.

The main program reads the input data and performs the summations and multiplications of the various probabilities for populations of the E-J grids corresponding to the various nuclear states. Most of the written output from the program is executed by MAIN.

b.) Subroutine WRITE.

This subprogram was written as a result of core space restrictions on the main program and is used to write out the E-J matrix ONE.

c.) Subroutine BRANCH.

Probabilities for the population of final states are calculated for a specific compound nucleus energy and angular momentum by the subprogram. Most of the statistical model mathematics are performed by this subroutine which calls the remaining two subroutines and the function subroutine.

d.) Subroutine FURG.

Transmission coefficients (which are read in as data) are interpolated and summed to give inverse reaction cross-sections by this subroutine.

e.) Subroutine BIND.

This subroutine returns a value of the binding energy of a particular particle for a given compound nucleus.

f.) Function ALEVEL.

This function calculates the level density corresponding to a particular E and J.

A listing of SFUSMAP immediately follows this discussion. The program has been revised substantially since this listing to remove programming steps which were included to avoid inadequacies in early versions of the SFU system 360/40 compiler programs.

A typical input data deck consists of:

(1) 36 cards containing binding energies (punched in columns 1-6) for the nine nuclei surrounding (and including) the compound nucleus at lower neutron and proton numbers.

(2) 108 cards, each containing 8 transmission coefficients for alpha penetrations (punched in each group of 10 columns). Each group of four cards represents one energy value with each value separated by 2 MeV. This data designates the matrix TIA (J,JXE) where J and JXE signify angular momentum and energy respectively.

(3) 135 cards, each containing eight proton transmission coefficients as above. Each group of five cards represents one energy value. This data populates the matrix TPL(J,L,JXE) where L signifies orbital angular momentum.

(4) 135 cards containing neutron transmission coefficients, populating the matrix TLN (J,L,JXE).

(5) 1 card containing the four single-particle *gamma* strength constants (C_i) punched in the first four groups of 10 columns.

(6) An eight card packet which contains:

- (a) The center of mass energy of the target-projectile system (punched in columns 1-8).
- (b) The binding energy of the projectile and the target (columns 2-8).
- (c) 5 cards containing the capture cross-sections for population of each J state from J=0 to J=39 $\frac{1}{2}$ for the energy specified by the first card in the packet (a). The data is punched in each group of 10 channels.
- (d) The angular momentum value above which the capture cross-section is zero.

The number of data packets which may be used is not limited by the program. The last card in the deck should contain a zero punched in place of the energy.

C SFUSMAP, THE COMPUTER BOGGLER.

0001

DIMENSION GAMMA(4),CL(4)

0002

DIMENSION E(4,9)

0003

DIMENSION TLA(32,27), TLP(2,20,27), TLN(2,20,27)

0004

REAL ONE

0005

DIMENSION ONE(30,20), TWO(30,20), THREE(30,20), FIVE(30,20)

0006

DIMENSION FOUR(30,20), SIG(9)

0007

DIMENSION SFUBAR(30,20,4)

0008

REAL NINE

0009

DIMENSION NINE(30,20)

0010

COMMON B,AN,ZN,IX,EF

0011

COMMON U

0012

COMMON CL

0013

COMMON TLA,TLP,TLN

0014

COMMON SFUBAR

0015

READ(5,1)((B(IX,ICN),IX=1,4),ICN=1,9)

0016

1 FORMAT(1F6.3)

0017

READ(5,2)((TLA(J,JXE),J=1,32),JXE=1,27)

0018

READ(5,2)((TLP(J,L,JXE),J=1,2),L=1,20),JXE=1,27)

0019

READ(5,2)((TLN(J,L,JXE),J=1,2),L=1,20),JXE=1,27)

0020

READ(5,6)(CL(L),L=1,4)

0021

6 FORMAT(4E10.3)

0022

2 FORMAT(8F10.8)

0023

3 FORMAT(8E10.3)

0024

4 FORMAT(2F8.3,13,E20.8)

0025

5 FORMAT(8F15.8)

0026

9 FORMAT(2F12.8,5X,2F12.8,5X,2F12.8,5X,2F12.8)

0027

7 FORMAT(1F10.5)

0028

2008 FORMAT(E20.8)

0029

200 FORMAT(4E20.8)

0030

CONTROL PROGRAM STARTS HERE

0031

FDC=1.3E-07

0032

DIMENSION SIGCAP(40)

0033

READ(5,7) ECM

0034

READ(5,7) Q

0035

READ(5,3)(SIGCAP(JCN),JCN=1,40)

0035	READ(5,7) BB
0036	XY=BB
0037	7515 CONTINUE
0038	WRITE(6,7) ECM
0039	WRITE(6,7) Q
0040	WRITE(6,7) BB
0041	AA=ECM+Q
0042	DO 3000 I=1,30
0043	DO 3000 J=1,20
0044	JEF=I
0045	JJF=J
0046	ONE(JEF,JJF)=0.0
0047	TWO(JEF,JJF)=0.0
0048	THREE(JEF,JJF)=0.0
0049	FIVE(JEF,JJF)=0.0
0050	FOUR(JEF,JJF)=0.0
0051	NINE(JEF,JJF)=0.0
0052	3000 CONTINUE
0053	ICN=1
0054	SIG(ICN)=0.0
0055	4004 CONTINUE
0056	DO 10 K=1,30
0057	DO 10 J=1,20
0058	DO 10 I=1,4
0059	JEF=K
0060	JJF=J
0061	IX=I
0062	SFURAR(JEF,JJF,IX)=0.0
0063	10 CONTINUE
0064	JCN=BB+1.
0065	SFURAB=0.0
0066	DO 20 IA=1,4
0067	IX=IA
0068	AN=68.
0069	ZN=32.
0070	CALL BRANCH(GAMMA,AA,BB)
0071	SFURAB=SFURAB+GAMMA(IX)
0072	20 CONTINUE
0073	IF((SFURAB).LE.0.0) GO TO 12002
0074	DO 2002 ID=1,30

0075	DO 2002 IE=1,20	
0076	DO 2002 IF=1,4	
0077	JEF=ID	
0078	JJF=IE	
0079	IX=IF	
0080	SFURAR(JEF,JJF,IX)=SFURAR(JEF,JJF,IX)/SFURAR	
0081	2002 CONTINUE	
0082	12002 CONTINUE	
0083	IX=1	
0084	DO 2003 IG=1,30	
0085	DC 2003 IH=1,20	
0086	JEF=IG	
0087	JJF=IH	
0088	ONE(JEF,JJF)=ONE(JEF,JJF)+SFURAR(JEF,JJF,IX)*SIGCAP(JCN)	
0089	ICN=1	
0090	SIG(ICN)=SIG(ICN)+SFURAR(JEF,JJF,IX)*SIGCAP(JCN)	
0091	2003 CONTINUE	
0092	IX=4	
0093	DO 2004 IC=1,30	
0094	DO 2004 IP=1,20	
0095	JEF=IC	
0096	JJF=IP	
0097	TWO(JEF,JJF)=TWO(JEF,JJF)+SFURAR(JEF,JJF,IX)*SIGCAP(JCN)	
0098	2004 CONTINUE	
0099	IX=3	
0100	DO 2005 IQ=1,30	
0101	DO 2005 IR=1,20	
0102	JEF=IQ	
0103	JJF=IR	
0104	FOUR(JEF,JJF)=FOUR(JEF,JJF)+SFURAR(JEF,JJF,IX)*SIGCAP(JCN)	
0105	2005 CONTINUE	
0106	IX=2	
0107	DC 2106 IS=1,30	
0108	DO 2106 IT=1,20	
0109	JEF=IS	
0110	JJF=IT	
0111	NINE(JEF,JJF)=NINE(JEF,JJF)+SFURAR(JEF,JJF,IX)*SIGCAP(JCN)	
0112	2106 CONTINUE	
0113	PR=PR-1.	
0114	IF((BB).GE.0.0) GO TO 4004	

0115	ICN=1	
0116	WRITE(6,4) ECM,Q,ICN,SIG(ICN)	
0117	WRITE(6,101)	
0118	CALL WRITE(ONE)	
0119	WRITE(6,102)	
0120	WRITE(6,204)	
0121	DC 2104 I=1,30	
0122	JEF=I	
0123	WRITE(6,203)(JEF,(TWO(JEF,JJF),JJF=1,10))	
0124	2104 CONTINUE	
0125	WRITE(6,100)	
0126	WRITE(6,205)	
0127	DC 2105 I=1,30	
0128	JEF=I	
0129	WRITE(6,203)(JEF,(TWO(JEF,JJF),JJF=11,20))	
0130	2105 CONTINUE	
0131	WRITE(6,103)	
0132	WRITE(6,204)	
0133	DC 2111 I=1,30	
0134	JEF=I	
0135	WRITE(6,203)(JEF,(FOUR(JEF,JJF),JJF=1,10))	
0136	2111 CONTINUE	
0137	WRITE(6,100)	
0138	WRITE(6,205)	
0139	DC 2112 I=1,30	
0140	JEF=I	
0141	WRITE(6,203)(JEF,(FOUR(JEF,JJF),JJF=11,20))	
0142	2112 CONTINUE	
0143	WRITE(6,100)	
0144	DC 2119 I=1,30	
0145	JEF=I	
0146	WRITE(6,203)(JEF,(NINE(JEF,JJF),JJF=1,10))	
0147	2119 CONTINUE	
0148	WRITE(6,100)	
0149	DC 2113 I=1,30	
0150	JEF=I	
0151	WRITE(6,203)(JEF,(NINE(JEF,JJF),JJF=11,20))	
0152	2113 CONTINUE	
0153	ICN=2	
0154	DC 9800 I=1,30	

0155	DO 9800 J=1,20	
0156	JEF=I	
0157	JJF=J	
0158	ONE(JEF,JJF)=C.0	
0159	9800 CONTINUE	
0160	ENSUM=C.0	
0161	SIG(ICN)=0.0	
0162	JA=1	
0163	8500 CONTINUE	
0164	JR=1	
0165	2006 CONTINUE	
0166	DO 3001 I=1,30	
0167	DO 3001 J=1,20	
0168	DO 3001 K=1,4	
0169	JEF=I	
0170	JJF=J	
0171	IX=K	
0172	SFURAR(JEF,JJF,IX)=0.0	
0173	3001 CONTINUE	142
0174	SFURAR=0.0	
0175	DO 4000 JC=1,4	
0176	IX=JC	
0177	AN=67.	
0178	ZN=32.	
0179	EEF=2*JA-1.75	
0180	FJ=40.0-2*JB	
0181	JJEF=EEF/2+1	
0182	JJJF=FJ/2.0+1.5	
0183	FURAR=IWC(JJEF,JJJF)	
0184	IF((FURAR).LT.FDG) GO TO 12019	
0185	IF((JA-3).LT.C) GC TC 700	
0186	CALL BRANCH(CAMMA,EEF,FJ)	
0187	SFURAB=SFURAR+GAMMA(IX)	
0188	4000 CONTINUE	
0189	IF((SFURAB).LT.FDG) GC TC 700	
0190	GO TO 701	

0191	700 IX=1	
0192	SFUBAR(JJEF,JJF,IX)=1.0	
0193	GO TO 2109	
0194	701 CONTINUE	
0195	ENSUM=ENSUM+SFUBAR	
0196	DO 2010 I=1,30	
0197	DO 2010 J=1,20	
0198	DO 2010 K=1,4	
0199	JEF=I	
0200	IX=K	
0201	JF=J	
0202	SFUBAR(JEF,JF,IX)=SFUBAR(JEF,JF,IX)/SFUBAR	
0203	2010 CONTINUE	
0204	2109 CONTINUE	
0205	DO 2007 JD=1,30	
0206	DO 2007 JE=1,20	
0207	JEF=JD	
0208	JF=JE	
0209	IX=I	
0210	SIG(ICN)=SIG(ICN)+SFUBAR(JEF,JF,IX)*FUBAR	
0211	ONE(JEF,JF)=CNE(JEF,JF)+SFUBAR(JEF,JF,IX)*FUBAR	
0212	IX=4	
0213	THREE(JEF,JF)=THREE(JEF,JF)+SFUBAR(JEF,JF,IX)*FUBAR	
0214	IX=3	
0215	FIVE(JEF,JF)=FIVE(JEF,JF)+SFUBAR(JEF,JF,IX)*FUBAR	
0216	2007 CONTINUE	
0217	12019 CCNTINUE	
0218	J8=JB+1	
0219	IF((JB-20).LE.0) GO TO 2006	
0220	8520 CONTINUE	
0221	JA=JA+1	
0222	IF((JA-30).LE.0) GO TO 8500	
0223	8510 CONTINUE	
0224	ENDRM=ENSUM	
0225	WRITE(6,4) ECM, Q, ICN, SIG(ICN)	
0226	WRITE(6,10+)	
0227	WRITE(6,204)	
0228	DO 2107 I=1,30	
0229	JEF=I	
0230	WRITE(6,203)(JEF,(THREE(JF,JF),JJF=1,10))	

0231	2107	CONTINUE
0232		WRITE(6,100)
0233		WRITE(6,205)
0234		DO 2108 I=1,30
0235		JEF=I
0236		WRITE(6,203)(JEF,(IFREE(JEF,JJF),JJF=11,20))
0237	2108	CONTINUE
0238		WRITE(6,105)
0239		WRITE(6,204)
0240		DO 2115 I=1,30
0241		JEF=I
0242		WRITE(6,203)(JEF,(FIVE(JEF,JJF),JJF=1,10))
0243	2115	CONTINUE
0244		WRITE(6,100)
0245		WRITE(6,205)
0246		DO 2116 I=1,30
0247		JEF=I
0248		WRITE(6,203)(JEF,(FIVE(JEF,JJF),JJF=11,20))
0249	2116	CONTINUE
0250		WRITE(6,105)
0251		CALL WRITE(ONE)
0252		ICN=3
0253		ENSUM=C.0
0254		SIG(ICN)=0.0
0255		DO 9801 I=1,30
0256		DO 9801 J=1,20
0257		JEF=I
0258		JJF=J
0259		ONE(JEF,JJF)=0.0
0260	9801	CONTINUE
0261		IA=1
0262	8502	CONTINUE
0263		IR=1
0264	5006	CONTINUE
0265		DO 5001 I=1,30
0266		DO 5001 J=1,20
0267		DO 5001 K=1,4
0268		JEF=I
0269		JJF=J
0270		IX=K

0271	SFUBAR(JEF,JJF,IX)=0.0
0272	5001 CONTINUE
0273	SFURAB=C.0
0274	DO 5000 IC=1,4
0275	IX=IC
0276	AN=66.
0277	ZN=32.
0278	EEF=2*IA-1.75
0279	FJ=40.0-2*IB
0280	JJEF=EEF/2+1
0281	JJF=FJ/2.0+1.5
0282	FUBAR=THREE(JJEF,JJF)
0283	IF((FUBAR).LT.FDG) GO TO 15011
0284	IF((IA-3).LT.0) GO TO 702
0285	CALL BRANCH(GAMMA,EEF,FJ)
0286	SFURAB=SFURAB+GAMMA(IX)
0287	5000 CONTINUE
0288	IF((SFURAB).LT.FDG) GO TO 702
0289	GO TO 703
0290	702 IX=1
0291	SFUBAR(JJEF,JJF,IX)=1.0
0292	GO TO 15010
0293	703 CONTINUE
0294	ENSUM=ENSUM+SFURAB
0295	DO 5010 I=1,30
0296	DO 5010 J=1,20
0297	DO 5010 K=1,4
0298	JEF=I
0299	JJF=J
0300	IX=K
0301	SFUBAR(JEF,JJF,IX)=SFUBAR(JEF,JJF,IX)/SFURAB
0302	5010 CONTINUE
0303	15010 CONTINUE
0304	DO 5007 ID=1,30
0305	DO 5007 IE=1,20
0306	JEF=ID

0307	JJF=IE	
0308	IX=1	
0309	ICN=3	
0310	SIG(ICN)=SIG(ICN)+SFUBAR(JEF,JJF,IX)*FUPAR	
0311	ONE(JJF,JJF)=CNE(JEF,JJF)+SFUBAR(JEF,JJF,IX)*FUBAR	
0312	5007 CONTINUE	
0313	15011 CONTINUE	
0314	I8=I8+1	
0315	IF((I8-30).LE.0) GO TO 5006	
0316	8522 CONTINUE	
0317	IA=IA+1	
0318	IF((IA-30).LE.0) GO TO 8502	
0319	8512 CONTINUE	
0320	ENORM=ENSUM	
0321	WRITE(6,4) ECM, Q, ICN, SIG(ICN)	
0322	WRITE(6,110)	
0323	CALL WRITE(ONE)	
0324	ICN=4	
0325	ENSUM=C.C	
0326	SIG(ICN)=0.0	
0327	DO 9802 I=1,30	
0328	DO 9802 J=1,20	
0329	JEF=I	
0330	JJF=J	
0331	ONE(JEF,JJF)=0.0	
0332	9302 CONTINUE	
0333	JA=1	
0334	8501 CONTINUE	
0335	J8=1	
0336	6005 CONTINUE	
0337	DO 6001 I=1,30	
0338	DO 6001 J=1,20	
0339	DO 6001 K=1,4	
0340	JEF=I	
0341	JJF=J	
0342	IX=K	
0343	SFUPAR(JEF,JJF,IX)=0.0	
0344	6001 CONTINUE	
0345	SFURAB=0.0	
0346	DO 6000 JC=1,4	

0347	IX=JC	
0348	AN=67.	
0349	ZN=21.	
0350	EEF=2*JA-1.75	
0351	FJ=40.0-2*JB	
0352	JJEF=EEF/2+1	
0353	JJF=FJ/2.0+1.5	
0354	FUBAR=FOUR(JJEF,JJF)	
0355	IF((FUBAR).LT.FCG) GO TO 16011	
0356	IF((JA-3).LT.O) GO TO 704	
0357	CALL BRANCH(GAMMA,EEF,FJ)	
0358	SFURAB=SFURAB+GAMMA(IX)	
0359	6000 CONTINUE	
0360	IF((SFURAB).LT.FDG) GC TC 704	
0361	GO TO 705	
0362	704 IX=1	
0363	SFUBAR(JJEF,JJF,IX)=1.C	
0364	GO TO 16010	
0365	705 CONTINUE	
0366	ENSUM=ENSUM+SFUBAR	
0367	DO 6010 I=1,30	
0368	DO 6010 J=1,20	
0369	DO 6010 K=1,4	
0370	JEF=I	
0371	JJF=J	
0372	IX=K	
0373	SFUBAR(JEF,JF,IX)=SFUBAR(JEF,JF,IX)/SFUBAR	
0374	6010 CONTINUE	
0375	16010 CONTINUE	
0376	DO 6007 ID=1,30	
0377	DO 6007 IE=1,20	
0378	JEF=ID	
0379	JJF=IE	
0380	IX=4	
0381	FIVE(JEF,JJF)=FIVE(JEF,JJF)+SFUBAR(JEF,JF,IX)*FUBAR	
0382	IX=1	
0383	SIG(ICN)=SIG(ICN)+SFUBAR(JEF,JF,IX)*FUBAR	
0384	ONE(JEF,JJF)=ONE(JEF,JJF)+SFUBAR(JEF,JF,IX)*FUBAR	
0385	6007 CONTINUE	
0386	16011 CONTINUE	

0387	JB=JB+1	
0388	IF((JB-20).LE.0) GO TO 6006	
0389	8521 CONTINUE	
0390	JA=JA+1	
0391	IF((JA-30).LE.0) GO TO 8501	
0392	8511 CONTINUE	
0393	ENORM=ENSUM	
0394	WRITE(6,4) ECM, Q, ICN, SIG(ICN)	
0395	WRITE(6,106)	
0396	CALL WRITE(ONE)	
0397	WRITE(6,107)	
0398	WRITE(6,204)	
0399	DO 2306 I=1,30	
0400	JEF=I	
0401	WRITE(6,203) (JEF, (FIVE(JEF, JJF), JJF=1,10))	
0402	2306 CONTINUE	
0403	WRITE(6,100)	
0404	WRITE(6,205)	
0405	DO 2307 I=1,30	
0406	JEF=I	
0407	WRITE(6,203) (JEF, (FIVE(JEF, JJF), JJF=11,20))	
0408	2307 CONTINUE	
0409	ICN=5	
0410	ENSUM=0.0	
0411	SIG(ICN)=0.0	
0412	DC 9803 I=1,30	
0413	DC 9803 J=1,20	
0414	JEF=I	
0415	JJF=J	
0416	ONE(JEF, JJF)=0.0	
0417	9803 CONTINUE	
0418	IA=1	
0419	9502 CONTINUE	
0420	IB=1	
0421	9006 CONTINUE	
0422	DO 9001 I=1,30	

0423	DO 9001 J=1,20	
0424	DO 9001 K=1,4	
0425	JEF=I	
0426	JJF=J	
0427	IX=K	
0428	SFURAR(JEF,JJF,IX)=0.0	
0429	9001 CONTINUE	
0430	SFURAB=0.0	
0431	DO 9000 IC=1,4	
0432	IX=IC	
0433	AN=66.	
0434	ZN=31.	
0435	EEF=2*IA-1.75	
0436	FJ=40.0-2*IR	
0437	JJEF=EEF/2+1	
0438	JJF=FJ/2.0+1.5	
0439	FUBAR=FIVE(JJEF,JJF)	
0440	IF((FUBAR).LT.FDG) GO TO 19011	
0441	IF((IA-3).LT.0) GO TO 706	
0442	CALL BRANCH(GAMMA,EEF,FJ)	
0443	SFURAB=SFURAB+GAMMA(IX)	
0444	9000 CONTINUE	
0445	IF((SFURAB).LT.FDG) GO TO 706	
0446	GO TO 707	
0447	706 IX=1	
0448	SFUBAR(JJEF,JJF,IX)=1.0	
0449	GO TO 19010	
0450	707 CONTINUE	
0451	ENSUM=ENSUM+SFURAB	
0452	DO 9010 I=1,30	
0453	DO 9010 J=1,20	
0454	DO 9010 K=1,4	
0455	JEF=I	
0456	JJF=J	
0457	IX=K	
0458	SFURAR(JEF,JJF,IX)=SFUBAR(JEF,JJF,IX)/SFURAB	
0459	9010 CONTINUE	
0460	19010 CONTINUE	
0461	DO 9007 ID=1,30	
0462	DO 9007 IE=1,20	

0463	JEF=ID	
0464	JJF=IE	
0465	IX=1	
0466	ICN=5	
0467	ONE(JEF,JJF)=CNE(JEF,JJF)+SFUBAR(JEF,JJF,IX)*FUBAR	
0468	SIG(ICN)=SIG(ICN)+SFUBAR(JEF,JJF,IX)*FUBAR	
0469	9007 CONTINUE	
0470	15011 CONTINUE	
0471	IB=IB+1	
0472	IF((IB-20).LE.0) GO TO 9006	
0473	9522 CONTINUE	
0474	IA=IA+1	
0475	IF((IA-30).LE.0) GO TO 9502	
0476	9512 CONTINUE	
0477	WRITE(6,108)	
0478	CALL WRITE(ONE)	
0479	ENORM=ENSUM	
0480	WRITE(6,4) ECM, Q, ICN, SIG(ICN)	
0481	ICN=1	
0482	WRITE(6,4) ECM,Q,ICN,SIG(ICN)	
0483	ICN=4	
0484	WRITE(6,4) ECM, Q, ICN, SIG(ICN)	
0485	ICN=3	
0486	WRITE(6,4) ECM, Q, ICN, SIG(ICN)	
0487	ICN=2	
0488	WRITE(6,4) ECM, Q, ICN, SIG(ICN)	
0489	ICN=5	
0490	WRITE(6,4) ECM, Q, ICN, SIG(ICN)	
0491	READ(5,7) ECM	
0492	IF((ECM).LT.1.0) GO TO 11111	
0493	READ(5,7) C	
0494	READ(5,3)(SIGCAP(JCN),JCN=1,40)	
0495	READ(5,7) RB	
0496	XY=RB	
0497	GO TO 7515	
0498	11111 CONTINUE	
0499	100 FORMAT(1H1///)	
0500	101 FORMAT(1H1,6X,58HPOPULATION OF STATES IN CERTAINIA 67 AFTER BATHA 15MISSION//)	
0501	102 FORMAT(1H1,6X,35HPOPULATION OF STATES IN CERTAINIA 67 AFTER BATHA 15MISSION//)	

0502	ION OF A NEUTRON BY THE COMPOUND NUCLEUS//)	
	103 FORMAT(1H1,6X,87HPOPULATION OF STATES IN GALLIUM 67 AFTER EMISSION	
0503	IN OF A PRGION BY THE COMPOUND NUCLEUS.//)	
	104 FORMAT(1H1,6X,88HPOPULATION OF STATES IN GERMANIUM 66 AFTER NEUTRON	
0504	IN EMISSION FROM STATES IN GERMANIUM 67//)	
	105 FORMAT(1H1,6X,85HPCPOPULATION OF STATES IN GALLIUM 66 AFTER PRGION F	
0505	MISSION FROM STATES IN GERMANIUM 67//)	
	106 FORMAT(1H1,6X,56HPCPOPULATION OF STATES IN GALLIUM 67 AFTER GAMMA E	
0506	MISSION//)	
	107 FORMAT(1H1,6X,95HPCPOPULATION OF STATES IN GALLIUM 66 AFTER NEUTRON	
0507	MISSION FROM STATES IN GALLIUM 67 ADDITIVE<//)	
	108 FORMAT(1H1,6X,56HPCPOPULATION OF STATES IN GALLIUM 66 AFTER GAMMA E	
0508	MISSION//)	
	109 FORMAT(1H1,6X,58HPCPOPULATION OF STATES IN GERMANIUM 67 AFTER GAMMA	
0509	MISSION//)	
	110 FORMAT(1H1,6X,58HPCPOPULATION OF STATES IN GERMANIUM 66 AFTER GAMMA	
0510	MISSION//)	
	203 FORMAT(//13,5X,1CE12,3)	
0511	8505 FORMAT(2F10,5)	
0512	204 FORMAT(3HJEF,10X,3HJ#C,5X,3HJ#1,9X,3HJ#2,5X,3HJ#3,9X,3HJ#4,9X,3HJ	151
	1#5,9X,3HJ#6,9X,3HJ#7,9X,3HJ#8,9X,3HJ#9//)	
0513	205 FORMAT(3HJEF,9X,4HJ#10,3X,4HJ#11,9X,4HJ#12,8X,4HJ#13,8X,4HJ#14,8X	
	1,4HJ#15,8X,4HJ#16,8X,4HJ#17,8X,4HJ#18,9X,4HJ#19//)	
0514	206 FORMAT(3HJEF,9X,4HJ#20,8X,4HJ#21,9X,4HJ#22,8X,4HJ#23,3X,4HJ#24,8X	
	1,4HJ#25,8X,4HJ#26,8X,4HJ#27,9X,4HJ#28,8X,4HJ#29//)	
0515	207 FORMAT(3HJEF,5X,4HJ#30,8X,4HJ#31,8X,4HJ#32,8X,4HJ#33,8X,4HJ#34,8X	
	1,4HJ#35,8X,4HJ#36,8X,4HJ#37,3X,4HJ#38,8X,4HJ#39//)	
0516	STOP	
0517	DEBUG UNIT(6), INIT(ENORM)	
0518	AT 9001	
0519	END	

TOTAL MEMORY REQUIREMENTS 007700 BYTES

FURTRAN IV G LEVEL 0, MOD 0

WRITE

DATE = 68017

30/13/41

0001 SUBROUTINE WRITE(ONE)
0002 DIMENSION ONE(30,20)
0003 DO 1 I=1,30
0004 JEF=1

0005 WRITE(6,203)(JEF,(ONE(JEF,JJF),JJF=1,10))
0006 1 CONTINUE
0007 WRITE(6,100)
0008 DO 2 I=1,30
0009 JEF=1

0010 WRITE(6,203)(JEF,(ONE(JEF,JJF),JJF=11,20))
0011 2 CONTINUE
0012 100 FORMAT(1H1///)
0013 203 FORMAT(//13,5X,10E12.3)
0014 RETURN
0015 END

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TOTAL MEMORY REQUIREMENTS 000264 BYTES

0001 SUBROUTINE BRANCH(GAMMA,AA,BB)
 0002 DIMENSION GAMMA(4),CL(4)
 0003 DIMENSION B(4,9)
 0004 DIMENSION TLA(32,27), TLP(2,20,27), TLM(2,20,27)
 0005 DIMENSION SFUBAR(30,20,4)
 0006 REAL JF
 0007 REAL JR
 0008 COMMON B,AN,ZN,IX,EF
 0009 COMMON U
 0010 COMMON CL

0011 COMMON TLA,TLP,TLM
 0012 IF((AA).LE.0.0) GO TO 33
 0013 AB=.96
 0014 CC=3.0
 0015 RI=15.58
 0016 C=(1.-AB*EXP(-0.693*CC*AA))*RI

0017 AJR=-0.5+0.5*SQR(1.C+8.0*AA*C)
 0018 IF((PB-AJR).GT.0.0) GO TO 33
 0019 COMMON SFUBAR
 0020 GAMMA(IX)=0.0
 0021 SFUSUM=0.0

1113 CONTINUE

0022
 0023 DIMENSION PN(6), PZ(6)
 0024 PZ(1)=-1.36
 0025 PZ(2)=0.0
 0026 PZ(3)=-1.09
 0027 PZ(4)=0.0
 0028 PZ(5)=-1.47

0029 PZ(6)=0.0
 0030 PN(1)=-1.46
 0031 PN(2)=0.0
 0032 PN(3)=-1.44
 0033 PN(4)=0.0
 0034 PN(5)=-1.46

0035 PN(6)=0.0
 0036 ACN=68.

0037	ZCN=32.
0038	IZ=7CN-ZN+1.
0039	IN=ACN-AN-7CN+ZN+1.
0040	IF(AA-3.0) 4000,4000,4001
0041	4000 E=3.0+PN(IN)+PZ(IZ)
0042	W=ALEVEL(E,BP)*AA*2.0/9.0+(3.0-AA)*10.0
0043	GO TO 4002
0044	4001 E=AA+PN(IN)+PZ(IZ)
0045	W=ALEVEL(E,BB)
0046	4002 CONTINUE
0047	GO TO 17
0048	33 GAMMA(IX)=0.0
0049	GO TO 20
0050	17 CONTINUE
0051	GO TO (4,5,6,7),IX
0052	4 BX=0.0
0053	DZ=C.0
0054	DA=0.0
0055	GO TO 9
0056	5 CALL BIND(BX)
0057	DZ=2.0
0058	DA=4.0
0059	GO TO 8
0060	6 CALL BIND(BX)
0061	DZ=1.0
0062	DA=1.0
0063	GO TO 8
0064	7 CALL BIND(BX)
0065	DZ=0.0
0066	DA=1.0
0067	8 CONTINUE
0068	U=AA-BX
0069	IF(U).GT.0.0) GO TO 19
0070	GO TO 33
0071	19 CONTINUE
0072	AN=AN-CA
0073	ZN=ZN-DZ
0074	FACTA=AN*DA/(AN+DA)
0075	IZ=ZCN-ZN+1.
0076	IN=ACN-AN-ZCN+ZN+1.

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0077 DO 111 I=1,30
0078 DO 2 K=1,20
0079 EF=2*I-1.75
0080 XE=AA-EF-BX
0081 IF((XE).LT.0.0) GO TO 3
0082 JEF=EF/2+1
0083 C=(1.-AB*EXP(-.693*CC*EF))*RI
0084 JR=-0.5+0.5*SQRT(1.0+8.0*EF*C)
0085 JF=40-2*K
0086 JJF=JF/2.0+1.5
0087 IF((JF-JR).GT.0.0) GO TO 2
0088 IF((JF).LT.0.0) GO TO 2
0089 CALL FURG(IX,XE,BB,JF,TLA,TLP,TLN,SIGMA,AN,ZN)
0090 IF(EF-3.0) 3000,3000,3001
0091 3000 E=3.0+PN(IN)+PZ(IZ)
0092 YAJ=SIGMA*(ALEVEL(E,JF)*EF**2.0/5.0+10.0*(3.0-EF))
0093 GO TO 3002
0094 3001 E=EF+PN(IN)+PZ(IZ)
0095 YAJ=SIGMA*ALEVEL(E,JF)
0096 3002 CONTINUE
0097 SFUBAR(JEF,JJF,IX)=FACTA*XE*YAJ/W
0098 IF((IX).EQ.2) GO TO 3012
0099 EF=2*I-0.75
0100 XE=AA-EF-BX
0101 IF((XE).LT.0.0) GO TO 3012
0102 IF((XE).GT.10.0) GO TO 3012
0103 CALL FURG(IX,XE,PB,JF,TLA,TLP,TLN,SIGMA,AN,ZN)
0104 IF(EF-3.0) 3010,3010,3011
0105 3010 CONTINUE
0106 E=3.0+PN(IN)+PZ(IZ)
0107 YAJ=SIGMA*(ALEVEL(E,JF)*EF**2.0/9.0+10.0*(3.0-EF))
0108 GO TO 3013
0109 3011 CONTINUE
0110 F=EF+PN(IN)+PZ(IZ)
0111 YAJ=SIGMA*ALEVEL(E,JF)
0112 3013 CONTINUE
0113 SFUBAR(JEF,JJF,IX)=(SFUBAR(JEF,JJF,IX)+FACTA*XE*YAJ)/2.0
0114 3012 CONTINUE
0115 SFUSUM=SFUSUM+SFUBAR(JEF,JJF,IX)
0116 2 CONTINUE

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0117 111 CONTINUE
0118 3 GAMMA(IX)=SFUSUM
0119 GO TO 20
0120 9 CONTINUE
0121 U=AA-BX
0122 IZ=ZCN-ZN+1.
0123 IN=ACN-AN-ZCN+ZN+1.
0124 DO 13 LL=1,4
0125 L=LL
0126 DO 11 I=1,30
0127 EF=2*I-1.75
0128 XE=AA-EF
0129 IF((XE+2.0).LT.0.0) GO TO 13
0130 IF((XE).GT.0.0) GO TO 7000
0131 XE=0.20
0132 EF=AA-XE
0133 7000 CONTINUE
0134 JEF=EF/2+1
0135 C=(1.-AB*EXP(-.693*CC*EF))*21
0136 JR=-0.5+C*.5*SQR(1.0+8.0*EF*C)
0137 DJ=L
0138 JD=BB+DJ+0.5
0139 LF=BB-DJ+0.5
0140 DO 12 K=LF,JD
0141 JF=K
0142 JJF=JF/2+1
0143 IF((JF).LT.0.0) GO TO 12
0144 IF((JF-JR).GT.0.0) GC TC 12
0145 IF(EF-3.0) 2000,2000,2001
0146 2000 E=3.0+PN(IN)+PZ(IZ)
0147 YAJ=ALEVEL(E,JF)*EF*2.C/9.0+(3.0-EF)*10.0
0148 GC TC 2002
0149 2001 E=EF+PN(IN)+PZ(IZ)
0150 YAJ=ALEVEL(E,JF)
0151 2002 CONTINUE
0152 IF((L).EQ.1) GO TO 5000
0153 IF((BR).LT.4.0) GO TO 202
0154 IF((AA).LT.3.0) GC TC 202
0155 IF((L).EQ.2) GO TO 200
0156 202 CONTINUE

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GO TO 5001
5000 CONTINUE
GIDIP=41C.26/((20.1-AA)**2+6.25)
SFUBAR(JEF,JJF,IX)=SFUBAR(JJF,JJF,IX)+GIDIP*CL(L)*XE**((2.*DJ+1.))
1*YAJ/W
GO TO 5002
5001 CONTINUE
SFUBAR(JEF,JJF,IX)=SFUBAR(JEF,JJF,IX)+CL(L)*XE**((2.*DJ+1.))*YAJ/W
5002 CONTINUE
SFUSUM=SFUSUM+SFUBAR(JEF,JJF,IX)
GO TO 204
200 CONTINUE
DELJ=ARS(JF-RR)
IF((DELJ).EQ.2.0) GO TO 201
GO TO 202
201 CONTINUE
CONJ=2.*((2.*BB+1.)/((BB+1.))*((BB-1.))*2-1.))
WD=1.0/ALEVEL(F,RR)
WE=1.0/YAJ
ETWC=CONJ*(WD/(WD+WE))**5
SFUBAR(JEF,JJF,IX)=SFUBAR(JEF,JJF,IX)+CL(L)*XE**((2.*DJ+1.))*YAJ/W
1ETWC
ETMAX=ZN**2
IF((ETWC-ETMAX).GT.0.0) GO TO 250
GO TO 251
250 ETWC=ETMAX
251 CONTINUE
GO TO 5002
204 CONTINUE
12 CONTINUE
11 CONTINUE
13 CONTINUE
GAMMA(IX)=SFUSUM
20 RETURN
END
```

TOTAL MEMORY REQUIREMENTS 001396 BYTES

35/13/71

DATE = 63017

FORTRAN IV G LEVEL 0, MOD C ALEVEL

FUNCTION ALEVEL(E,J)

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0020

0021

REAL M

REAL J

CC=3.0

AB=.96

RI=15.58

CCC=CC*5.0

IF(E-CCC)4,5,5

5 C=RI

GO TO 6

4 C=(1.-AB*EXP(-.693*CC*E))*EI

6 CONTINUE

2 A=8.5

FACT=5.282

T=(1.5+SQRT(2.25+4.*A*E))/(2.*A)

10 CONTINUE

M=J+.5

EI=(E+1.5*T)

ALEVEL=FACT*2.*M*EXP(2.*SQRT(A*E)-M*M/(2.*C*E))/(EI*(1

3 RETURN

END

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TOTAL MEMORY REQUIREMENTS 0002F2 BYTES

FORTRAN IV G LEVEL 0, MOD 0			FURG	DATE = 68017	30/13/71
0001	SUBROUTINE FURG(IX,XE,BB,JF,TLA,ILP,TLN,SIGMA,AN,ZN)				
0002	DIMENSION TLA(32,27), ILP(2,20,27), TLN(2,20,27)				
0003	REAL JF				
0004	JIX=IX-1				
0005	JYE=XE				
0006	JXEA=JXC/2+1				
0007	JXEB=JXEA+1				
0008	XEA=2*(JXEA-1)				
0009	INTER=0.5*(XF-XEA)				
0010	GO TO (2,1,1), JIX				
0011	1 S=0.0				
0012	DA=1.0				
0013	GO TO 3				
0014	2 S=0.5				
0015	IF((XE).LE.5.0) GO TO 7				
0016	DA=4.0				
0017	3 JS=ABS(JF-S)				
0018	JJS=JF+S				
0019	FACTA=.6566*(2.*BB+1.)*(AN+DA)/((2.*JF+1.)*(2.*S+1.)*AN*DA*XE)				
0020	20 CONTINUE				
0021	SIGSUM=C.0				
0022	DO 4 JJ=JS,JJS				
0023	BJ=JJ				
0024	LL=ABS(2B-PJ)				
0025	LLL=BB+BJ				
0026	TALLY=C.0				
0027	DO 5 L=LL,LLL				
0028	J=JJ-JS+1				
0029	GO TO (11,12,13),JIX				
0030	11 IF((L-32).GT.C) GO TO 6				
0031	TLX=TLA(L,JXFA)-(TLA(L,JXFA)-TLA(L,JXEB))*INTER				
0032	GO TO 14				
0033	12 IF((L-20).GT.0) GO TO 5				
0034	TLX=ILP(J,L,JXEA)-(ILP(J,L,JXEA)-ILP(J,L,JXEB))*INTER				
0035	GO TO 14				
0036	13 IF((L-20).GT.0) GO TO 6				

0037		TLX=TLN(J,L,JXEA)-(TLN(J,L,JXEA)-TLN(J,L,JXEP))*INTER
0038	14	TALLY = TALLY+TLX
0039	5	CONTINUE
0040	6	SIGSUM=SIGSUM+TALLY
0041	4	CONTINUE
0042		SIGMA=SIGSUM*FACTA
0043		GO TO 8
0044	7	SIGMA=0.0
0045	8	CONTINUE
0046		RETURN
0047		END

FORTRAN IV G LEVEL 0, MCD 0

FURG

DATE = 63017

30/13/41

TOTAL MEMORY REQUIREMENTS 00076E BYTES

FORTRAN IV G LEVEL 0, MCD 0 BIND DATE = 68017 20/13/71

0001	SUBROUTINE BIND(BX)
0002	DIMENSION B(4,9)
0003	COMMON B,AN,ZN,IX,EF
0004	ACN=68.0
0005	ZCN=32.0
0006	ICN=ACN-AN+2.0*(ZCN-ZN)+1.0
0007	BX=B(IX,ICN)
0008	RETURN
0009	END

TOTAL MEMORY REQUIREMENTS 000195 PYTES

Appendix VI. The $\text{Ar}^{40} + \text{Si}^{28}$ Target-Projectile System.

Extension of the study of the compound nucleus Ge^{68} to reactions induced by the target-projectile pair Si^{28} and Ar^{40} would provide a test of the effects of extremely high angular momentum on the decay of this compound nucleus.

The pertinent data for this target-projectile system are contained in table VIII. The large negative binding energy makes possible the extension of measurements to much lower Ge^{68} excitation energies than were possible for either the C^{12} or O^{16} systems. However, the high projectile energies necessary to form the compound nucleus might present a problem, since recent work¹²² has shown that the cross-section for complete fusion processes above 100 MeV decreases with increasing energy of the bombarding particle. Attempts were made¹²² to explain this behavior by considering a sharp cut-off in the spin distribution of the compound nucleus. Collisions which would lead to higher compound nucleus spins were assumed to result instead in noncomplete fusion processes.

Obvious experimental difficulties would be expected for this system (e.g. low beam currents and beam energy uncertainty), however theoretical problems might be even more formidable. For instance, the optical model description of nuclear interactions will likely prove to be inadequate for a complex system such as this. A model which considers interactions with two potential wells might be more appropriate

Also, for higher angular momentum states, fission may become a predominant mode of de-excitation for the compound nucleus¹²³.

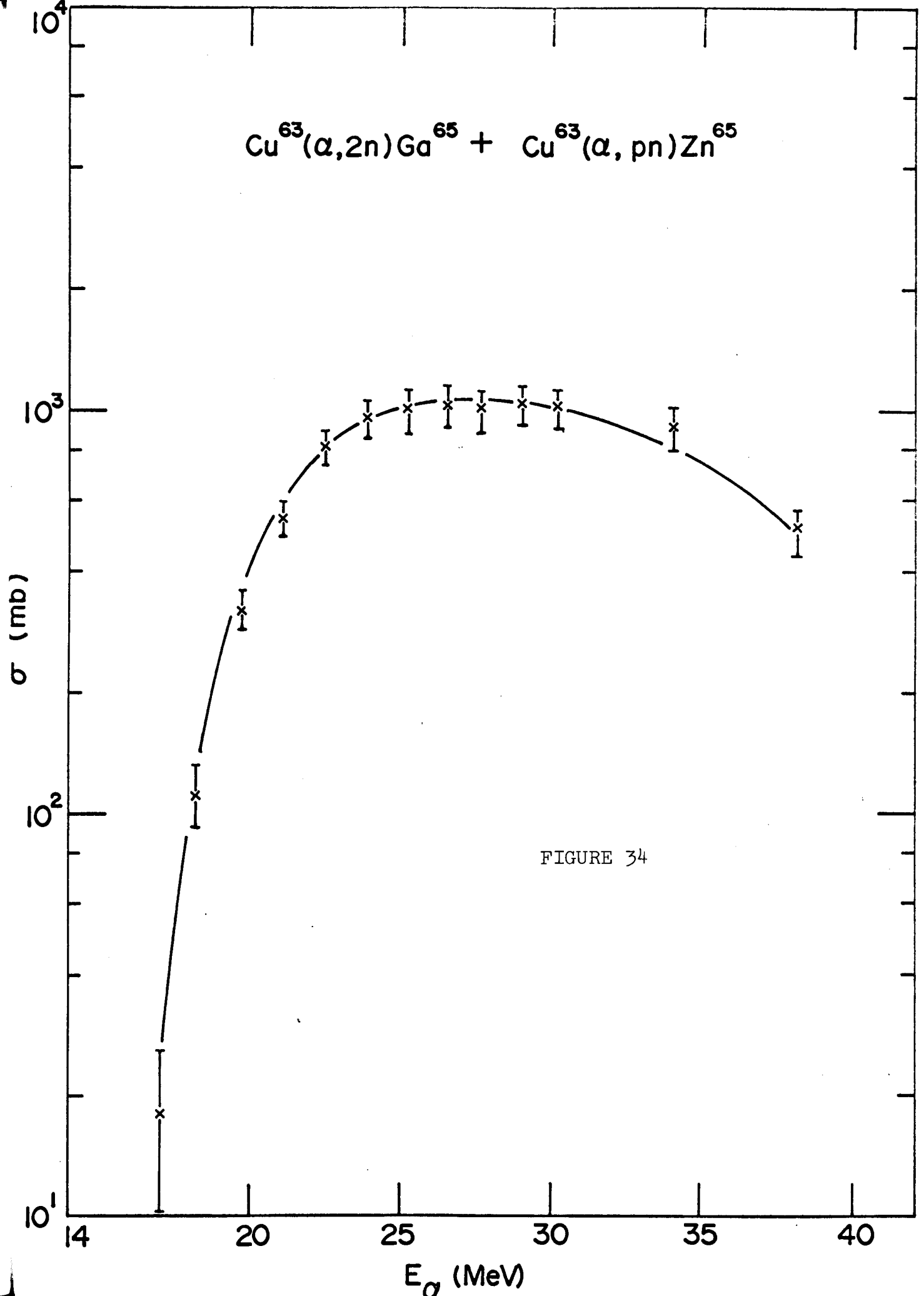
The reactions studied would be those leading to products one or two mass units removed from the compound nucleus, in order to avoid reactions which would proceed predominantly via direct interaction mechanisms.

TABLE VIII

<u>Ar⁴⁰ + Si²⁸</u>			
<u>V(MeV)</u>	<u>E_{BE}(MeV)^{79.}</u>	<u>E_{Ar40}(MeV)[‡]</u>	<u>E*(MeV)[‡]</u>
37.7	-10.042	91.5	27.7

[‡] Listed values are minimum energies required to overcome the coulomb barrier.

Element	Isotope	Natural Abundance (atomic %) ^{56.}
Si	→ 28	92.21
	29	4.70
	30	3.09
Ar	36	0.337
	38	0.063
	→ 40	99.60



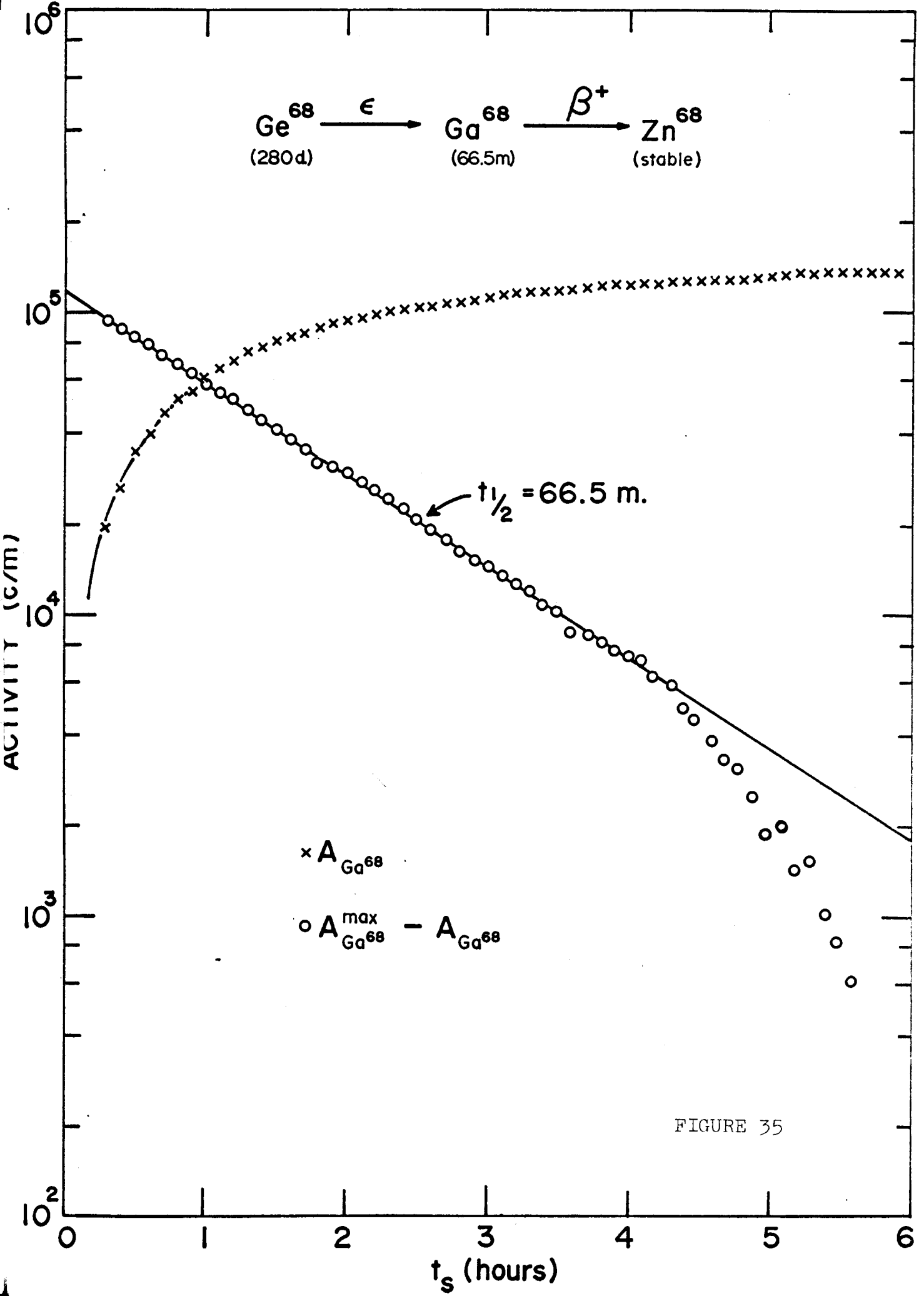


FIGURE 35