NEIGHBORHOOD MODELS:

AN ALTERNATIVE FOR THE MODELING OF SPATIAL STRUCTURES

by

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Neighborhood Models: An Alternative for the Modeling of Spatial Structures

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Abstract

In the last three decades there has been a widespread use of quantitative models in geography. The majority of models have been applied for descriptive, predictive and hypothesis-testing purposes. Quantitative geography is at a stage where the benefits and limitations of most of these models have been tested. Consequently, there has been a tendency to seek models considered to be more "appropriate" for geographical analysis. This work examines some of the most commonly used mathematical tools in human geography and presents a new family of models as an alternative for the mathematical representation of spatial structures.

The proposed models are based on the notion of "geographical neighborhood" and are named neighborhood models. In the formalization process mathematical concepts such as space and subspace are used to model the notion of "neighborhood"; two quasi-mathematical structures (geo-spaces and geo-subspaces) are defined as an aid in this modeling.

As a first step in the construction of neighborhood models a set of measures of local variation (heterogeneity indices) are introduced. The role played by these indices from a

mathematically formal point of view is fundamental, since through them it is possible to combine and benefit from two mathematical areas of knowledge: topology and fuzzy set theory.

In the second part of the thesis the indices are applied to two geographical problems: 1) the design of classification algorithms with regionalization purposes; 2) an aid in the selection of sites for the allocation of resources in an educational planning environment. In the first case the index is used to define the degree of membership of an element to the interior (or border) of a region through the topological concept of neighborhood and that of fuzzy set. In the second application several indices are used to describe spatial and temporal characteristics of the demand for schools in an area in central Mexico.

Finally, some future areas of research are proposed. Although these ideas concerning neighborhood models have not been completely explored and developed, it is clear that the approach provides a fruitful avenue for research.

To Rodolfo, Fito and Pablo,

Chata and Chato,

Maru, Ara, and Paco.

To my friends.

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Chapter 1.

INTRODUCTION

Since the beginning of the Quantitative Revolution in Geography a considerable amount of mathematical models have been used to represent different aspects of the geographical landscape. There are, however, certain facets of spatial structures that have received less attention from modelers. Such is the case of the geographical notion of neighborhood. The main aim of this thesis is to build mathematical models designed to allow the formal representation of the geographical concept of neignborhhod.

In the first part of this chapter, an overview of the role played by mathematical models in the discipline is given, while in the second part, intuitive notions of the neighborhood concept are discussed.

1.1 Mathematical Models

Models have played a fundamental role in the development of various areas of knowledge such as physics, engineering, architecture, computing, economics and geography. Commonly a model is said to be a representation of objects, events and processes of the real world (Johnson, 1963, p.218). Depending on their purpose, models are classified into different groups. For example, Forrester (1973, p.49) distiguishes between physical and abstract, dynamic and static, linear and nonlinear and stable and unstable models. A model is said to be mathematical when mathematical language is used in its representation.

Mathematical models have been used extensively in the natural sciences. For example, in physics practically all knowledge is expressed in mathematical language, and theoretical advancement has been accomplished to a large degree through mathematical modeling. On the other hand, in the social sciences the use of mathematical models has not been as extensive or successful. In geography, models such as maps have always been intimately related to geographic knowledge. However, mathematical models did not have a notable presence in the development of geographical theory until the so-called "Quantitative Revolution" which took place in the 1950's. As a consequence, a new branch of geography, quantitative geography, was established.

1.2 Quantitative Geography

As noted by Gregory (1983, p.80) mathematics has been used in geography for a long time, particularly in the construction and use of maps. According to his view, trigonometry, Euclidean geometry and space transformations are areas in which geographers are traditionally trained.

The quantitative era in geography is characterized not simply by the presence of mathematical techniques but by the intensification and expansion of their use and the introduction of mathematical modeling and abstract theory. Although quantitative geography originated in the United States in the 1950's, its development in other countries shows distinctive characteristics. According to recent literature there are two major schools of quantitative geography: the Anglo-American and the Continental European schools. They can be identified by the facts and circumstances of their development. For example, Bennett (1981, p.1) characterizes the German and French tradition as being more concerned with deep methodological questioning, while in the English-speaking countries more importance has been given to the development of analytical techniques, and a more pragmatic approach has been pursued.

In the 1980's, three decades after the initiation of the quantitative revolution, European researchers are engaged in the historical analysis of the quantitative approach and the contrastive analysis of the state of the art in the two principal schools. Several academic meetings have been organized to discuss these topics and their relevance to the future of quantitative and theoretical geography (Haining, 1984, Bennett, 1981 and Beaumont, 1983).

As a result of these meetings, a consensus seems to have emerged regarding the initial development stages of quantitative geography: In early research, too much importance was given to the techniques and not enough to their role in the development of geographic theory. For example, the main purpose in applying mathematical techniques was often to test untried tools. In this respect Bennett and Wrigley's more theoretical perspective (1981, p.6) regarding "core" and "frame" disciplines are particularly interesting.

According to these authors, core disciplines are "those areas of thinking which are providing new systems, concepts, and developing new explanatory paradigms." In contrast, frame disciplines "are those which derive in methodology, object of study, and terminology, from other external subjects." Adopting these terms, quantitative geography sought to be a core discipline in its initial stage while in most recent years it has become more of a frame discipline. Consequently, researchers in various branches of geography are making more extensive use of quantitative techniques without necessarily considering themselves "quantitative geographers."

It should be mentioned that quantitative geography has evolved not only in its approach to the use of mathematical models but also in the kind of techniques used. For example, statistical inference in the 1960's "was seized as a panacea for geographical methodology" (Bennett and Wrigley, 1981, p.8), while in the 1970's the appropriatness of this methodology was questioned. As a result, statistical techniques for specific geographical purposes were designed. Such is the case of Cliff and Ord's autocorrelation (1973) and Clark's geostatistics (1979). Bennett and Wrigley (1981, p.9) have anticipated that statistical inference will continue to be used in the future but not to the extent that it was in the past.

In general, early quantitative methods adapted mathematical models designed in other areas of knowledge to represent the "geographical landscape." This is the case of such widely used models as the gravity model, factor analytic methods, regression models, cluster analysis as well as a wide range of statistical techniques. However, as a result of critical analysis by both geographers and scientists from other disciplines, in more recent years more appropriate models and discipline-specific techniques have been developed.

1.2.1 Galton's Problem

The criticisms made by Sir Francis Galton at the end of the last century of the use of correlation analysis in anthropological studies is the point of departure of the present work.

In 1889 at a meeting of the Royal Anthropological Institute, E. B. Tylor recognized that anthropology needed a scientific methodology for the analysis of ethnographic data and proposed a cross-cultural survey methodology. He applied this method to data he had collected on different tribes and societies. The importance of Tylor's work resides in the fact that he was able to correlate distinct traits which were present in the various human groups under study. During the discussion period of the meeting, a comment made by Sir Francis Galton had a significant impact on the future development of both anthropology and geography:

> It was extremely desirable for the sake of those who may wish to study the evidence for Dr. Tylor's conclusions, that full information should be given as to the degree in which the customs of the tribes and races which are compared together are independent. It might be, that some of the tribes had derived them from a common source, so that they were duplicate copies of the same original. Certainly, in such an investigation as this, each of the observations ought, in the

language of the statisticians, to be carefully "weighted." It would give a useful idea of the distribution of the several customs and of their relative prevalence in the world, if a map were so marked by shadings and colour as to present a picture of their geographical ranges (Tylor, 1889).

As a consequence of this remark spatial dependence became an issue in both anthropological and geographical studies and spatial analysts became aware of the need to incorporate the "spatial dimension" in their mathematical models. One of the concepts that has been used in modeling spatial dependence is that of "geographical neighborhood."

1.2.2 Neighborhoods

A retrospective analysis of mathematical models and techniques makes it clear that certain aspects of the "geographical landscape" have been much more widely modeled than others. For example, the modeling of properties such as distance and shape were part of the "new geometric tradition" of the early 1960's (Haining, 1983, p.86) There are, however, other aspects of the geographical landscape which have received little attention from modelers. This is the case of the geographical notion of "neighborhood."

The concept of neighborhood is treated in geographical studies at different levels and with different meanings. For example in urban studies neighborhoods are considered as sub-areas with homogeneous characteristics such as level of income, age of population, etc. and are used as the basic elements for analysis (Hall, 1983). In other instances geometric characteristics of the geographical landscape are studied through the relationship among neighbors. Such is the case of the cardinal neighbor method (Elliott, 1983) where the geometric patterns of cities and population centers are identified and analysed, based on the neighborhood relationship among them.

Neighborhoods also play an important role in other areas of knowledge. For example in mathematics, there is a whole discipline which is based on the formal notion of neighborhood, namely topology. In it, concepts that are usually defined in an euclidean space are generalized through the concept of neighborhood and the "topological characteristics" of mathematical objects are studied (Firby and Gardiner, 1982). Furthermore these mathematical concepts have been applied in other disciplines such as chemistry, where the topological characteristics of three dimensional networks are used in the analysis of the chemical characteristics of compounds (Springer, 1973) and in the classification and characterization of gold compounds (Hall, Gilmour and Mingos, 1984), among other applications.

Although the notion of neighborhood is conceived in different manners among distinct disciplines, at an intuitive level the concept is based on the idea that the "space" that surrounds an entity or a specific portion of "space" is of special significance. At elementary levels of analysis in both mathematics and geography, the concept of neighborhood is related with the intuitive notion of "surrounding space". However, in both disciplines different definitions of neighborhood are used according to the problem at hand. For example in some quantitative techniques such as autocorrelation, geostatistics and topological data structures, specific meanings are given to the concept of neighborhood. There is however, no general treatment of this concept in the geographical literature. To exemplify some of the methods and techniques, specific definitions of neighborhood are used throughout the presentation, but emphasis is done on the general concept of geographical neighborhood.

Briefly, the principal objective of this thesis is the design of a family of formal tools that allow the modeling of the concept of "geographical neighborhood."

In Chapter 2 some classical models as well as those that have incorporated the spatial dimension through the concept of neighborhood are reviewed. In Chapter 3 the notion of neighborhood model is introduced, and the mathematical concepts that are used in the modeling process are defined.

In Chapters 4 and 5 two applications of neighborhood models are presented. The first is a theoretical application in regionalization. The second is an application to a specific problem in educational planning. Finally, the appendix contains a mathematical discussion of some of the concepts used in the thesis.

Chapter 2.

MATHEMATICAL MODELS IN HUMAN GEOGRAPHY

From the period of the "Quantitative Revolution" in geography to the present, a considerable number of mathematical models and methods have been used for geographical analysis purposes. Many of them are adaptations of formal tools used in other sciences such as physics, biology, botany, economics and psychology. The development of quantitative geography is at a stage where it has become necessary to analyse the benefits and limitations of the techniques used in the past and to propose new ones.

Mathematical models that have been used in the past have represented distinct aspects of spatial structures using various formal tools. Since the principal objetive of this work is to present neighborhood models as an alternative for the modeling of spatial structures, it is convenient to study some of the mathematical structures that have been incorporated into existing models.

In this chapter some of the mathematical models that can be considered as classical in human geography are reviewed as well as those that have incorporated the notion of neighborhood in the representation of the geographic landscape.

2.1 Classical Models

Three of the models that have been extensively used in human geography are: factor models, the gravity model and networks. According to the purpose of application of each one of them, the geographical landscape has been modeled in different ways. Whether the models actually include the entities and the relationships among them that are most important for geographical studies remains an open question. The reason for presenting these three models is to establish which elements of the geographical landscape have been included and to discuss to what extent the spatial structure has been modeled.

2.1.1 Factor Models

Factor analysis was originally developed to aid scientists in testing hypotheses concerned with the organization of mental ability. At the beginning of the century, psychologists were interested in measuring "general intelligence" by defining and quantifying its components. The point of departure is an nxm matrix which includes for "n" persons the scores of "m" tests associated to each one of them. By means of factor analysis a small number (r<<m) of hypothetical factors are determined. These factors allowed psychologists to isolate what they viewed as fundamental personality components or "factors of the mind" (Rees, 1971, p.220). According to Lawley (1971, p.1) the method was initially restricted to psychometrics, and for some time it remained the black sheep of statistical theory. Although it is considered to be a "complicated" technique, its use is increasingly widespread today due to the existence of easy-to-use computer packages which facilitate its application.

This mathematical model has been extensively described both for researchers with a mathematical and statistical background (Lawley, 1971 and Harman, 1976) and for those who lack such training (Taylor, 1977 and Berry, 1971). Since this study is principally concerned with the specific application of the model, only those mathematical aspects which are relevant to a geographic context will be discussed.

Factorial Ecology

Factorial ecology is the branch of quantitative geography which is concerned with the use of factor models in geographical studies. Although principal component analysis was developed by Pearson (1901) and Hotelling (1933), factor analysis began with the work of Spearman (1904, 1926). Research in factorial ecology does not appear until the mid-fifties (Bell, 1955). However, the technique has become well-established since then, and it is presently taught as part of the regular curricula in the geography programs offered by most universities.

According to Taylor (1977, p.255) factorial ecology developed in a period in which two opposing groups of researchers were studying the structure of urban centers. On the one hand, there were the human ecologists who were interested in the ecology of urban areas and had proposed several spatial models (concentric ring model, sector model and the multiple nuclei model). On the other hand, there were the social area analysts who hypothesized that the urban social structure could be characterized through three indices or dimensions: economic status, family status and ethnic status. In fact, the initial application of ecological factor analysis was undertaken by Bell (1955). Its purpose was to test the hypothesis that urban populations could be adequately described by the three-status criteria.

The results obtained by Bell were encouraging enough to cause widespread acceptance and use of the technique among urban social geographers. Studies of various cities in the U.S. (Salins, 1971, p.235) allowed researchers to undertake comparative (congruence) analysis: among the different metropolitan areas and over several points in time. In other major urban centers in the world, factor models were applied similarly (Haynes, 1971, p.324, Janson, 1971, Johnston, 1971, p.315).

The extensive use of the factor analysis technique for hypothesis testing and as a descriptive tool has also made researchers increasingly aware of its limitations. In some urban ecology studies the use of the technique has been successful but that, as with any other formal tool, the appropriate use of factor models depends on the understanding that the researcher has of both the problem and the technique.

Factorial Ecology as a Geographical Model

Why and how does factorial ecology qualify as a geographical model? The fact that the model is applied through the use of areal units has been considered enough to automatically classify it as a spatial analysis technique. There have been some attempts to introduce the location variable as one of the variates in the factor model in order to transform it into a "more" explicitly geographic model. However, the results obtained from these procedures have come under strong criticism. For example, when latitude and longitude are included as variables in the factor model, two main disadvantages have been found: the circularity of method and the lack of invariance to the selection of axes (Taylor, 1977, p.275). The first objection refers to the fact that in the procedure location variables (latitude and longitude) are used to calculate scores which are latter located on a map, producing as an effect the location of locations. In the second case, criticism is based on the fact that the selection of orthogonal axes is arbitrary so that the factors obtained in each case are not necessarily equal. In other words, the factorial model is sensitive to the change of axes of the location variables.

In order to understand how factorial ecology models the geographical landscape it is important to review some basic concepts behind the modeling process.

When the decision to use a mathematical model is made, the common procedure is to develop or select the most appropriate from the existing models. The most relevant objects of the phenomenon under study, together with the most important known relationships among them, are usually expected to be included in the model.

One of the advantages of using a mathematical model is that once the real environment has been identified with a mathematical structure, all mathematical knowledge is at the service of the researcher.

Expected and unexpected relationships among the objects can emerge as the result of applying the mathematical techniques. This may allow the researcher to find optimum solutions to specific problems. In fact, the possibilities of using mathematical models as aids in research and in the solution of specific problems is limited only by the researcher's ability to apply existing tools or to develop new ones.

The history of the application of the model under discussion reveals that researchers took no special interest in the spatial relationships among objects (in this case areal units). In fact, there has been no real conceptual difference in the use of the technique by geographers and psychologists. Once the areal units and the various census variables are defined and identified with a mathematical structure (in this case a matrix), all other crucial relationships beyond the scope of the model tend to be ignored.

For example, factors that are extremely important for certain geographical analyses such as distance, nearness, relative position, and contiguity are in no way subject to analysis by the factorial model. The model is insensitive to all these factors. As applied in Bell's study, the model is unable to consider the relevance of the size and spatial distribution of the census tracts. Whether the census tracts of the city of Los Angeles were arranged in a regular shape such as a square or in a chain mode or as they actually are, and whether similar tracts were close together or far apart, could not influence the conclusions Bell arrived at based on the factorial model. In short, the problem is the incapacity of factorial ecology to model any spatial structure.

The question that remains is not so much whether introducing the relative position or absolute location makes factorial ecology a "more" geographic model, but rather how important it is that the models used by urban social geographers take account of those spatial relationships which have been ignored until now. This question is still subject to debate.

2.1.2 Gravity Models

The law of universal gravitation announced by Newton in 1666 is one of the cornerstones of modern science. The law can be stated as follows:

> Every particle in the universe attracts every other particle with a force that varies directly as the product of the masses of the two particles and inversely as the square of their distance apart. The direction of the force is along the straight line joining the two particles (Fowles, 1970, p.139).

The importance of movement in social phenomena inspired some social geographers to use models similar to those developed by physicists. A family of models analogous to the law of universal gravitation have been developed in geography to study such phenomena as migration between population centers and retail trade areas: the movements of persons (journey to work, journey to shop, etc.) and the movement of goods.

The first models, such as the one used by Ravenstein in 1885 for migration studies in England and Wales, were strongly inspired by the Newtonian model. However, more recently geographers have developed a whole new family of interaction models that differ substantially from the physical model (Wilson, 1971). As Wilson shows (1971, p.1) "gravity model" has become something of a misnomer. For example, a conceptual distinction worth making is that while in physical terms the gravitational force is exerted in equal magnitude on both particles, and motion comes as a effect of that force, in geographical applications the equivalent of force is identified with movement (flows).

Gravity models have been mainly designed for use as predictive and descriptive tools. In the past, planning decisions have been based on these models' predictions of traffic flows in several metropolitan areas in the United States, population migration between cities, and sales in a shopping center (Taylor, 1977, p.287).

The mathematical expression of the model varies according to predictive or descriptive objectives. Below are two of the most common equations.

Migration Model

The migration Tij between an origin i and a destination j is directly proportional to the product of the sizes of the areas Oi and Dj and inversely proportional to the distance dij between them, raised to some power n.

$$T_{ij} = k\ O_i\ D_j\ (d_{ij})^{-n} \qquad (2.1)$$

Interaction Models

The interaction between zone i and j is directly proportional to the product of the "mass terms" Wi and Wj associated with the zones and inversely proportional to a measure of distance or cost of travel, raised to some power n.

$$T_{ij} = k\ W_i\ W_j\ (C_{ij})^{-n} \qquad (2.2)$$

In these types of models it is common to find additional constraints. These restrictions are often related to knowledge of the total interaction, emerging or arriving flows of a zone. The full interaction model can be written as:

$$Tij = Ai\ Bj\ Oi\ Dj\ f(Cij) \qquad (2.3)$$

$$Ai = \left[\sum_j Bj\ Dj\ f(Cij)\right]^{-1} \qquad (2.4)$$

$$Bj = \left[\sum_i Ai\ Oi\ f(Cij)\right]^{-1} \qquad (2.5)$$

The difference between equation (2.3) and previous mathematical expressions of the model is that the constant k is substituted by the product of Ai and Bj. Equations (2.4) and (2.5) are derived from the constraints imposed on the model.

$$\sum_j Tij = Oi \qquad (2.6)$$

$$\sum_i Tij = Dj \qquad (2.7)$$

According to Shepard (1969, p.8) there are two established methodologies to estimate the parameters of the gravity model: the regression and entropy maximization approaches. In the first case the researcher deals with the model:

$$Tij = k\ Oi\ Dj\ (dij)^{-n}\ \varepsilon ij \qquad (2.8)$$

where εij is a stochastic residual. To estimate the parameters with the ordinary least squares method, equation (2.8) is transformed through the logarithmic function.

$$\log T_{ij} = \log k + \log O_i D_j - n \log d_{ij} + \log \varepsilon_{ij} \qquad (2.9)$$

With the maximization entropy approach a method similar to the microcanonical ensemble technique in statistical mechanics is used. The details are not discussed in this work and the reader is referred to (Wilson, 1971, p.4 and Haggett, 1977, p.40).

The Gravity Model as a Spatial Model

The Newtonian gravity model is undoubtedly extremely simple. Its geographic counterpart is also simple. Social processes such as migration and journey-to-work trips are predicted from the relationship between two entities: "mass" and "distance".

Depending on the purpose of the application, mass and distance represent different quantities. Their relationship is clearly established in each one of the postulated models. For example, in equation (2.1) for a fixed mass the flow of population increases as the distance decreases (the inverse is also true). This is a well-known geographic relation which has been stated in various ways: "Towns attract more trade from near than from far locations" (Taylor, 1977, p.207); "Everything is related with everything else but near things are more related than distant ones" (Tobler, 1970). Despite their simplicity, gravity models allow researchers to work with important aspects of spatial structure.

It is important to note that by means of the law of universal gravitation it is possible to calculate the force exerted between two particles. However, if more than two particles are involved in the analysis the problem becomes more complicated. Although the motions of the planets in the solar system have been calculated through numerical solutions (Symon, 1969, p.185), there is no general solution to the problem involving the motion of any number of particles under the forces exerted on one another.

Analogously, some of the gravity models postulated in geography describe interaction exclusively between two entities. For example, in the prediction of migration between two cities as represented in equation (2.1), it is assumed that there is no other interaction between either of these two cities and the rest of the universe of study. There are, however, geographic gravity models that have been designed to account for the effect of other cities in the interaction between two cities. For example, in the study of transportation it is assumed that the interaction between any two cities is reduced due to the presence of a third one. This assumption is based on the concept of "intervening opportunity" which according to Taafe and Gauthier (1973, p.95) was first formulated by Stenffer in a study of intra-urban migration. This concept is derived from the following reasoning:

"The number of migrants from any point within a city to a zone at the periphery of the city was directly related to the number of opportunities or vacancies in that zone and inversely related to the number of opportunities between the originating point within the city and the zone in the periphery."

In the gravity model that includes this notion it is assumed that the relationship of intervening opportunity is similar to that of a distance. The model can be expressed as follows:

$$Tij = k\ Oi\ Dj\ (dij)^{-n}\ Pij \qquad (2.10)$$

where Pij is the intervening opportunity.

2.1.3 Networks

One of the branches of mathematics that has had a wide range of application is graph theory. It has been used in physics, chemistry, computer technology, architecture, sociology, anthropology, linguistics, geography and other disciplines. According to Harary (1972, p.1), graph theory has been independently discovered several times. In the 18th century Euler gave a solution to the Köningsberg Bridge Problem with the aid of graphs. In the 19th century Kirchoff and Cayley solved problems in physics and chemistry respectively using elements of graph theory (Harary, 1972, p.2).

Graphs are commonly represented through diagrams that greatly facilitate their interpretation. The diagram is composed of a set of points representing different entities and a set of lines joining these points, representing a pre-established relationship among the points.

Formally a graph can be defined as follows:

> A graph consists of a finite set V=V(G) of p points together with a prescribed set X of q ordered pairs of distinct points of V. Each pair x=(u,v) of points in X is a line of G, and x is said to join u and v.
> (Harary, 1972, p.9)

This simple mathematical structure has allowed researchers to solve a wide variety of problems and at the same time has encouraged the development of this branch of mathematics. In particular, the use of the model to represent geographic entities and their relationships is very common. For example, graph theory as it relates to the concept of connectivity has been used to establish the definition of routes between two or more population centers. In a similar context, transport networks such as railway and road systems have been modeled with graphs and compared spatially and temporally through different measures. Since the economic development of various countries has been related to the connectivity of railway networks, the technique has allowed researchers to establish interesting relationships. Studies have been carried out at different levels (urban, regional, international) using the appropriate measures in each case (Beta index, density, etc.)

(Haggett, 1977, pp. 86-92). Additionally, temporal comparisons in the growth of transport networks permit Taaffe, Morrill and Gould to identify four phases of development in various countries. In other cases graphic simulation models have been developed to predict network growth (Haggett, 1977, p.301).

Since graph theory is closely related to other branches of mathematics including linear programming, combinatorics, matrix theory, topology and probability, it is common to find many other instances where graphs are used in a geographic context. Whenever any of these types of models is applied, graphs are often used either as part of the analysis or as an aid in the presentation and interpretation of results.

Networks as Spatial Models

Among the existing mathematical tools, graphs appear to be one of the most appropriate means to model spatial relationships among geographic entities. They have been used as descriptive and predictive tools and for hypothesis testing.

The diagramatic representation of a graph gives the researcher the opportunity to obtain a complete image of the relationship among the entities under study. This is extremely important for spatial analysis purposes since it allows the researcher to perceive given relationships spatially and, if necessary, to correlate them with other spatial relationships. The model is flexible enough to allow various spatial relationships to be represented. Metric relations, such as distances, either measured in the Euclidean plane or in a geographic space (e.g. traffic flows, route distances) can be represented in the model by attaching a weight and/or a direction to the corresponding link. Non-metric or qualitative relations such as contiguity are also easily modeled.

2.2 The Neighborhood Approach

During the first stage of development of quantitative geography there was a strong tendency to use models that had been designed in other branches of knowledge. However, as a consequence of the awareness of spatial analysts of the need to represent the notion of spatial dependence mathematically, several models that incorporate this concept have been developed in different branches of geography.
Neighborhoods have been commonly used as a tool in the modeling of spatial dependence. This concept has different meanings in geographical and mathematical terms. Both meanings are fundamental to the development of "neighborhood models" which is the major aim of the present work.

In the first part of the following section the geographical and mathematical meanings of neighborhood are presented, and in the second part three models developed in the latter stage of the quantitative revolution which include the notion of neighborhood are discussed.

2.2.1 Geographical and Topological Neighborhoods

Geographical Neighborhoods.

The notion of neighborhood appears frequently in geographical theory. For example, as cited by Taylor (1977), in the central place theory proposed by Christaller (1933) and Lösch (1940) it is assumed that settlements provide specialized functions for other settlements. The size and shape of the areas served by each of the "central places" has been one of the main topics studied in this branch of geography. One of the best known hypotheses is that under certain conditions, including aspects such as demand for central goods, purchasing power, flow of consumers and other factors, the shape of the trade regions of the central places is hexagonal (Haggett, 1977, p.146). Similarly, in applied branches of geography such as school location planning (see chapter 5), the areas surrounding a population center play an important role in educational planning. The areas served by a school or group of schools are called catchment areas. The distribution, size and shape of these areas allow planners to evaluate and to optimize the design of school districts, among other things. Similar neighborhood concepts have been useful in the planning of health services, shopping centers and banking facilities.

These and other geographic notions of neighborhood have been formalized using different mathematical concepts including geometric entities and graphs. Commonly, the point of departure is a set of geographic units which are often identified with either points or areas in the Euclidean plane.

Geometric Entities.-

When the Euclidean plane is the model involved, it is common to use regular geometric entities such as circles, rectangles or hexagons to delimit neighborhoods. Other geometric entities such as Thiessen polygons are also used to define neighborhoods. In this case the polygons are constructed so that given a set of data points in the real plane, all points inside a polygon centered on a data point are closer to that point than to any other data point (Peucker et al., 1976, p.26).

Geometric entities that satisfy a geographic condition are also used to define neighborhoods. For example, in the delimitation of catchment areas, traveling distances or existing physical barriers may determine the shape of the neighborhood (see Chapter 5).

Two common characteristics of the neighborhoods described above are: a) the unit of interest (either a point or a line) belongs to the neighborhood and b) the resulting Euclidean subspace (the neighborhood) is connected in mathematical terms.

It should be mentioned that in geographical applications it is common to deal with either point or areal units. In both cases the treatment is very similar. When points are the units of interest they are often assumed to be in the center of the geometric entity. When dealing with areal units, each one is identified with a point (e.g. the centroid) and the neighborhood is defined exactly as it is in the case of point units.

Contiguity.-

Another way of defining the neighbors of an areal unit is through a contiguity relation. Among areal units it is said that two units are contiguous if they share a common boundary. In more formal terms, two areal units are neighbors if they have at least one segment in common, and the Euclidean subspace formed by the set of neighbors of a fixed areal unit "a" is called the neighborhood of "a".

Graphs.-

When other models such as networks are used, two points are said to be neighbors if there is a line connecting them. For a given point "p" in the graph, the neighborhood of "p" is defined as the set of neighbors of "p". When areal units are involved, it is possible to identify each one with a point and to draw a line between any two points, provided the corresponding areal units are neighbors. The original structure involving areal units is known in graph theory as the dual of the graph (Harary, p.113), and the definition of neighborhood is very similar to the case where the units are points.

The relations established in these graphs are sometimes represented in matrix form. Given "n" units, a nxn matrix is defined as follows:

$$mij = \begin{cases} 1 \text{ if the units are neighbors} \\ 0 \text{ otherwise} \end{cases}$$

In some applications weights are given to the neighboring relation. These quantities represent factors that are considered important for the phenomena under study. Examples might be the length of the boundary between two counties or the size of flows between two population centers. In this case the elements of the matrix are the values of the weights attached to each pair of units.

Orders of Neighborhoods.-

It has sometimes been useful in geographical analysis to define different orders of neighborhoods and neighbors. Neighborhoods like those defined in previous paragraphs are called first order neighborhoods and their elements first order neighbors. The set of points which are neighbors of the first order neighbors and are not first order neighbors themselves are called second order neighbors. The resulting set is called a second order neighborhood. Third, fourth and successive orders of neighborhoods can be defined in a similar manner.

Topological Neighborhoods

The concept of neighborhood plays a fundamental role in the development of topological theory. Some of this theory's basic concepts which will be used in subsequent chapters are discussed in the following paragraphs.

First of all, it should be said that the definition of topological space is based on the notion of open set. A topology with which most readers are familiar is the "usual topology" in the real line.

The Usual Topology.-

Intervals are sets of real numbers commonly used in calculus and mathematical analysis. An interval is determined by two real numbers a, b where $a < b$. It is said that the interval is open if it does not contain its "extreme points a and b." Haaser et. al., (1959, p.23) define an open interval as follows:

The open interval determined by two numbers a and b, where $a < b$, is the set of all real numbers x for which $a<x<b$. This open interval is denoted by (a,b). Another way of writing this definition is

$$(a,b) = \{ x ; a<x<b \}$$

An open set in the real line can be defined as follows:

A set O is open if for every point x in O there is an open interval I such that x belongs to the interval I and the interval is contained in O. The open intervals are examples of open sets (Royden, 1968, p.39).

A more formal definition is given by Hu (1964, p.39).

A subset U of R is said to be an open set if for an arbitrarily given point u in U there exists a positive real number d such that a real number x is in U if $|x-u| < d$.

In the real plane the open sets are defined in a similar way. Consider a disk surrounded by a tight ribbon. The ribbon is the "border" or limit of the disk. A circle (disk) that does not contain its border (ribbon) is called an open circle. Open circles are also examples of open sets in the usual topology of the real plane RxR. There are of course other definitions of open set which vary according to the topological space under consideration.

Definitions.-

The open set concept is crucial for establishing the concept of topological neighborhood.

It is said that a set is a neighborhood of a point if it contains an open set that contains the point.

Let X be a given space and p be a given point in X. A set $N \subset X$ is said to be a neighborhood of the point p in the space X iff there exists an open set U of X such that

$$p \in U \subset N$$

(Hu, 1964, p.19)

This definition comprises the intuitive notion of neighborhood. The point of interest belongs to the neighborhood and the neighborhood is formed by the "space" that is near or proximal to the point. For example, in the real plane an open circle with center in "a" is a neighborhood of point "a" (see figure 2.1).

Other topological concepts which have been useful in interpreting some of the results obtained in this study are those of interior, exterior and boundary points. Hu (1964, p.21) gives the following definitions:

The point p is said to be an interior point of the set E provided that there exists a neighborhood N of p in X contained in E. The point p is said to be an exterior point of E if there exists a neighborhood N of p in which X contains no point of E. Finally, the point p is said to be a boundary point of E in case every neighborhood N of p in X contains at least one point in E and at least one point not in E.

By examining a particular case in the Euclidean plane it is possible to acquire a more intuitive grasp of this abstract concept. Consider a circle in the real plane RxR. The points in the circumference that limit the circle are border points while those in the circle are interior points (see figure 2.1).

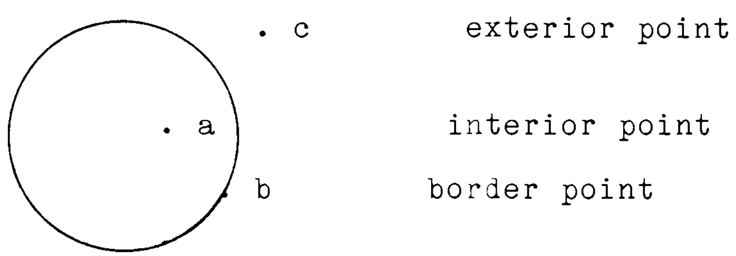


Figure 2.1

In other words, an open circle is defined as the set $C = \{x \in RxR \; ; \; |x-r'| < d\}$. All the points belonging to the open circle are interior points of C. On the other hand, the points lying on the circumference of the open circle, that is $\{x \in RxR \; ; \; |x-r'| = d\}$, are boundary points of C. Finally, the set of points that are neither in the open circle nor in its boundary are exterior points of C i.e. $\{x \in RxR \; ; \; |x-r'| > d\}$.

2.2.2 Spatial Autocorrelation

A mathematical technique that was specifically designed for the study of spatial dependence is that of spatial autocorrelation. The concept of spatial autocorrelation can be summarized as follows: it is said that a set of areas exhibits positive spatial autocorrelation if high values of a variable in one area are associated with high values of the same variable in neighboring areas. In brief, spatial autocorrelation is a statistical technique that allows the researcher to test hypotheses on spatial dependence.

The entity under study is assumed to be a two-dimensional area which has been partitioned into non-overlapping regions that are exhaustive of the area. The basic areal units are called counties, but the technique is equally valid if the objects of interest are point units.

Since in the study of spatial dependence the relationship between an entity and its surrounding is fundamental, the concept of neighborhood has a key role in the spatial autocorrelation model. It is common practice to represent the neighborhood relationship in this type of analysis through the use of a matrix. In some cases this permits the relations to be weighted and different orders of neighbors to be taken into consideration.

In order to test hypotheses on spatial autocorrelation various statistical techniques have been designed. One of the best known is the one proposed by Geary (Cliff and Ord, 1973, p.8):

$$C = \frac{(n-1) \sum_{i=1}^{n} \sum_{j=1}^{n} dij \, (xi-xj)^{2}}{4 \, A \sum_{i=1}^{n} Zi^{2}} \quad ; \; i \neq j \qquad (2.11)$$

where n is the number of units,

xi is the value associated to the ith unit,

$Zi = xi - \overline{x}$; the deviation with respect to the mean,

$$dij = \begin{cases} 0 & \text{if units i and j are not linked} \\ 1 & \text{if units i and j are linked} \end{cases}$$

A= 1/2 Li ; the total number of links in the county system

$Li = \sum_{j=1}^{n} Wij$; the number of units linked to unit i.

Clearly, this measure is sensitive to the spatial pattern induced by the neighboring relation. However, it ignores other spatial characteristics of the units such as shape and size (Cliff an Ord, 1973, p.272).

There are many situations in which this type of technique has proven useful. Among them are map comparisons with applications to diffusion processes and the analysis of regression residuals (Cliff and Ord, 1973, p.69, 105).

2.2.3 Geostatistics

Geostatistics is a field which was developed for, and mainly applied to, mining problems. Its relevance is not limited to mining however. Again the concept of neighborhood is a fundamental part of the model.

According to Clark (1979, p.1) geostatistics began in the early 1960's with the work of George Matheron and was then introduced as "The Theory of Regionalized Variables." The basic problem it addresses is the estimation of a sample at a particular location in space or time. A well-known application of these statistical techniques is the estimation of ore reserves.

The method is designed to permit local estimation. Given a relatively small number of samples in an area, how can the value of a fixed point belonging to that same area be estimated?

The relative position of the point with respect to the samples is assumed to determine its value. This factor is accounted for in the model by means of the concept of distance. In fact it is often assumed that the difference in value between two points depends only on the distance between them and their relative orientation (Clark, 1979, p.5).

A basic concept in geostatistics is that of the variogram. Given the set of differences between the values of all the sample points, the variogram is defined as its standard deviation.

The experimental variogram is expressed as follows:

$$2\,\gamma(h) = 1/n \sum_{1}^{n} [\; g(x) - g(x+h) \;]^{2} \qquad (2.12)$$

Where h describes the distance and the relative orientation, g is the grade (value) associated with the point, x denotes the position of one sample and x+h the position of the other, and n is the number of possible pairs in the sample set. $\gamma(h)$ is called the semi-variogram (Clark, 1979, p.5).
For a given distance and orientation (e.g. 100m and north-south) the values of the experimental variogram are calculated. The resulting set of values is plotted and used to calculate "expected" values of the difference between the grade values of two samples (Clark, 1979, p.18).

According to Clark (1979, p.6) several semi-variogram models have been designed, but only a few are regularly used. These include the spherical and exponential models (Clark, 1979, p.6)..

2.2.4. Topological Data Structures

Another branch of geography in which the concept of geographical neighborhood has been especially important is that of Geographic Information Systems (GIS). There are in fact several areas where these applications have proven fruitful. Examples are image processing, digital terrain models, computer cartography and census data bases.

A similar process to mathematical modeling must be followed in systems design. A set of entities along with their characteristics and relationships has to be identified with formal structures that are representable in computer systems. Fortunately, there are several well-established computer representations for those mathematical structures such as graphs and matrices which are often used in geographic applications.

In the design of a GIS it is particularly common to find spatial relationships that are easily manipulated through the use of graphs. Two examples are street structure in urban areas and transportation networks. Consequently, data structures that allow efficient manipulation of graphs have been designed in the past. These types of structures have been referred to in geographic literature as topological data structures.

An example of an application of a topological data structure is the one Peucker et. al. proposed (1976) for the treatment of three-dimensional surfaces.

The data is a set of irregularly distributed points of the surface. Each of the points is selected so that it has a high content of information and is significant for the digital terrain model (Peucker and Chrisman, 1975, p.64). The data set is assumed to be "triangulated" so that every point is a vertex of a triangle. This triangular irregular network (TIN) is composed by triangular facets that cover the study area. The neighborhood of each point in the TIN is defined as the set of points that are connected to it by an edge of a triangle. The data structure is designed so that the neighborhood of each point is explicitly stored.

This type of structure is important because it adequately represents a graph such as the TIN. However, its real value is that this representation of geographical data effectively allows the user to manipulate information using spatial criteria. The idea behind this type of structures is similar to the one proposed in this study and applied in a different context (Peucker and Chrisman, 1975).

Many other GIS based on this type of structures are found in the literature. The reader is referred to Dutton, (1978) and Peucker and Chrisman (1975).

The concept of neighborhood is also found in other areas of geoprocessing. For example, in image processing when the purpose is texture discrimination, it is common to replace the grey level of each point by the average grey level of its neighborhood (Rosenfeld, 1978, p.3), and in the manipulation of polygonal data the concept of local processing allows the user to work with an amount of data which would be impossible to consider if the whole data set were involved.

2.3 Discussion

Mathematical modeling has been used in geographical studies for the past thirty five years. The first stage of development of geographical quantitative techniques is characterized by the adaptation of existing techniques and models in other areas of knowledge, while a second stage can be identified by the development of models and techniques specifically designed for geographical purposes. Presently both the classical models and those that incorporate the notion of neighborhood continue to be widely used.

It should be mentioned that besides those models that have been described there is a considerable amount of mathematical models and techniques that have been applied in a geographical context. Such is the case of regression analysis (Brouwer and Nijkamp, 1984, Rogerson, 1984), discriminant analysis (Fotheringham and Reeds, 1979, Yupa and Mayfield, 1978), probability theory (Morley and Thornes, 1972, Burnett, 1978, Muckay, 1983), simulation (Phipps and Laverty, 1983, Morrill and Kelly, 1970) and linear programming (Cromley and Hanink, 1985, Garfinkel and Nemhauser, 1970, Maxfield, 1972).

Currently, three main tendencies of research are found in the area of quantitative geography: the application of existing models or techniques to real-world problems, the examination of the mathematical properties and characteristics of existing models and the modification of existing tools so that they overcome criticisms.

Examples of the application, and in some cases adaptation of models to specific situations are the works of Brouwer and Nijkamp (1984) where a regression model is applied to the study of the regional quality-of-life and residential preferences in Holland and in that of Mulligan and Gibson (1984) where the purpose is to calibrate an economic base model for small communities. On the other hand, further studies of the characteristics of models are found in the research undertaken by Smith (1984) where the main purpose is to characterize the gravity model in theoretical terms and in that of Jong, Sprenger and Van Veen (1984) where the extreme values of two spatial autocorrelation indices are derived. Finally, efforts to adapt existing models to conditions that had not been considered in the first design are found in the works of Bodson and Peeters (1975) and Bivand (1984), regarding modifications of the linear regression model and the spatial dependence effect and in that of Schwab and Smith (1985) and Slater (1984) where the question of the form of spatial interaction models regarding the level of spatial resolution is addressed.

Since the initial stage of development of quantitative geography criticisms have been made at two different levels. At the more general level the criticisms are directed to the general use of quantitative techniques. The main argument is based on the idea that the quantitative approach is a positivist one (Bennett and Wrigley, 1981, p.10, Johnston, 1981). At a second level comments are made around either the use of the models or in the mathematical characteristics of specific models and techniques. Most of the criticisms made in this second level are related with the statistical methods that are commonly found in geographical studies (Gould, 1970, Martin, 1974, Sheppard, 1979, Bennett and Wrigley, 1981, p.8).

Quantitative geography is at a mature stage were the initial enthusiasm provoked by early results has faded, and the complete rejection of its benefits is not a current tendency. Mathematical modeling is viewed as a tool for geographical studies accepting that in some cases the quantitative methods have proven to be a fruitful approach and at the same time that their limitations are such that the search for better models is far from having come to an end. This last statement is particularly true in relation with the modeling of spatial structures.

As mentioned in the description of the classical models, widely used tools such as the factor models, were not designed for the representation of spatial structures. It is a fact that in the modeling of the geographical landscape two competing components are often found. On one hand the geographer is interested in studying the characteristics of a phenomena that are a consequence of the site itself but on the other hand geographical studies are focussed in the spatial relationships between a site and its surrounding. Berry (1968, p.226) describes this fact as the dichotomy within Geography, the dual concepts of site and situation: "Site is vertical referring to local, man-made relations, to form and morphology. Situation is horizontal and functional, referring to regional interdependencies and the connections between places, or what Ullman calls spatial interactions". These two competing components are present in the design of models. In the case of the factor models the "site component" is completely dominant over the "situation or Galton's" one, while in other models such as that of spatial autocorrelation the relationship is reversed. There is no doubt that to adequately represent spatial structures it is necessary to design models with a dominant "Galton's component" without obliterating the other one. In this thesis, models that incorporate Galton's component through the topological concept of neighborhood are presented.

Chapter 3.

NEIGHBORHOOD MODELS

Among the models presented in Chapter 2, those that use the notion of neighborhood to model spatial structure are clearly distinguishable. In each one of them geographical neighborhoods are represented through different abstract entities. However, a global treatment of the use of neighborhoods to represent spatial structures is not found in the literature.

In the first sections of this chapter a quasi-mathematical structure is proposed as a general framework for the design of models that follow a neighborhood approach, and the concept of neighborhood model is presented.

Different characteristics of neighborhoods are of interest for the spatial analyst. In the second section of the chapter, the notion of local variation of a neighborhood is formalized through various indices, and its geographical and mathematical interpretations are discussed.

3.1 General Framework

Since the use of mathematical models in human geography is relatively new, the accumulated experience in formalizing (in a mathematical sense) geographical concepts is also relatively small. In the natural sciences it is a common practice to establish abstract models based on the scientist's knowledge of the phenomena under study. An analogous procedure has been followed in geographical models.

It is, however, possible to establish explicitly, intermediate stages in the formalization process. This makes the modeler's task of selecting appropriate mathematical representations of the geographical landscape easier.

In this section two quasi-mathematical structures (geo-spaces and geo-subspaces) are defined as an aid in the design of models that incorporate neighborhoods. Both geo-spaces and geo-subspaces must be defined prior to designing the mathematical model.

3.1.1. Intuitive Ideas

Whenever the geographical landscape is modeled, geographical entities are commonly identified with mathematical entities such as points, lines, areas or surfaces. There are, however, other elements of importance to geographical analysis, such as the relative position of a entity with respect to its surrounding or neighborhood, that have seldom been dealt with mathematically.

Maps are excellent examples of the fact that geographers are usually not interested in the study of isolated entities. Undoubtedly, the map is the most successful of geographical models. It allows the geographer to represent the most relevant spatial relationships including distance, contiguity, connectivity and shape. However, its most outstanding characteristic is that the relative position of all its elements with respect to their neighborhood is explicitly represented. This fact allows geographers to manipulate the information content of maps using spatial criteria which focus on spatial relationships among the entities rather than on the entities themselves, although it should be mentioned that for geographers spatial relationships are often implicit in maps, and in many cases they deal with them in an intuitive manner.

3.1.2 Spaces and Subspaces

The major aim of this study is to present the development of formal models with characteristics similar to the ones mentioned above for maps. The first step in the design of such models is to formalize the concept of "geographical neighborhood." Since it is a very broad concept, discussion in the following paragraphs is limited to the case where the "geographical landscape" has been modeled representing its entities and relations in a Euclidean space. Since Euclidean spaces are subject to very intuitive geometrical interpretations, their correspondence with geographical space becomes very natural.

In crude terms a geographical neighborhood is either a geographical area limited by physical features or administrative boundaries or an area that surrounds a geographical entity such as a city, a school or an airport.

In Euclidean space the concept of "geographical neighborhood" can be identified with that of subspace, a concept which is often used in mathematical analysis. In general terms, a subspace of a Euclidean space is simply a subset of the original one. There are, however, other definitions of subspace depending on the mathematical structure in question. For example, Royden (1968, pp.127, 137) gives the following definitions of metric space and subspace:

A metric space (X,p) is a nonempty set X of elements (which we call points) together with a real-valued function p defined on XxX such that for all x,y and z in X:

i) $p(x,y) \geq 0$;

ii) $p(x,y) = 0$ if and only if $x = y$;

iii) $p(x,y) = p(y,x)$;
iv) $p(x,y) \leq p(x,z) + p(z,y)$.

The function p is called a metric.

If (X,p) is a metric space and S is a subset of X, then S becomes a metric space if we restrict p to S, that is to say, if we take as the distance between two points of S their distance as points of X. When we consider S as a metric space with this metric, we call S a subspace of X.

In other words, a set of the space is a subspace if it inherits the mathematical structure defined in the space. In many other mathematical spaces, such as vector and topological, subspaces play an important theoretical role.

Considered intuitively and transferred to a map context, the concept of subspace might be expressed as follows: for a given map, a piece of map is still a map. However, closer scrutiny of the analogy makes it clear that this statement is not always true. If too small a piece of map is taken, it ceases to satisfy the function of a map. In the same way, if isolated elements of a map are cut out, the result will probably not be a map.

This observation clearly indicates that if the concept of geographical neighborhood is to be formalized, precautions should be taken so that the proposed model preserves certain features that are essential for spatial analysis.

The conditions imposed on a set of a space to be a subspace must be analogous to the conditions imposed on the abstract entities selected to represent a geographical neighborhood regarding certain spatial conditions such as maximum distance, minimum area or contiguity constraints.

3.1.3 Geo-Spaces

Having established a resemblance between the concept of neighborhood in geographical terms and the mathematical concept of subspace, in order to proceed with the formalization process it is necessary to introduce the concept of geo-space.

It is possible to ascribe roles to entities in geographical theory that are similar to these roles played by space and subspace in mathematics. These entities are named geo-spaces and geo-subspaces. They are discussed in the the following paragraphs, and although their definition is intuitive and general, they have proved to be useful for the purposes of this study.

In the geographic modeling process it is common to find a set of entities under study (such as rivers, roads, cities or census tracts) that are identified with mathematical entities (such as points and lines in a Euclidean space, nodes or links in a graph or elements of a matrix). One or more spatial relations are established among the entities. These spatial relations are represented mathematically through equations or specific mathematical structures. In the formalization process it is common to assume that the mathematical entities of interest are immersed in a mathematical space. Some of the most commonly used mathematical spaces in geographic applications are the Euclidean, matrix and topological spaces.

A quasi-mathematical structure of this type is called a geo-space.

To come back to the example in Section 2.1.2, in the modeling of migration it is common to find a set of population centers identified with a set of points in the Euclidean plane. The basic spatial relation is established through distance and is expressed in an equation such as equation 2.1. In this case we say that we have a Euclidean geo-space.

Clearly, a geo-space is not really a mathematical structure since it contains elements of the geographical landscape. Rather it is the quasi-mathematical product of an intermediate step in the modeling process. The purpose of using such an intermediate structure in the formalization process instead of a purely mathematical one is to ensure the inclusion of all the spatial relationships that have been identified as relevant.

3.1.4 Geo-subspaces

A natural way of conceptualizing a subspace of a geo-space is to consider a subset of the entities under study along with their mathematical counterpart, where the spatial relations established in the geo-space are preserved along with their mathematical expressions. The subsets considered have to be part of a subspace of the mathematical space under consideration whenever the latter is part of the geo-space.

For example, a subspace of the Euclidean geo-space described in the previous section is simply a subset of the set of cities considered and the points in the Euclidean space with which they are identified. The spatial relation established among the points is preserved, since in the Euclidean space it is always possible to calculate the distance between any two points.

In mathematical metric spaces the constraint imposed on the subspace is the preservation of the distance function. Similarly, in the case of geo-subspaces the main constraint is related to the preservation of the spatial relationships established in the geo-space.

3.2 Definition of Neighborhood Model

Once the spatial relationships of interest have been explicitly expressed either verbally or mathematically in the definition of the geo-space and the corresponding geo-subspaces have been identified, the next stage is to formally establish the mathematical model to be used.

As will be seen in the following chapters the concept of geo-subspace permits the design of formal tools to model the concept of geographical neighborhood.

Formal tools that are designed to represent mathematically the spatial structure through the notion of neighborhood are called Neighborhood Models.

These models are suitable whenever the interest of the study resides in the characteristics or behavior of sub-spaces rather than in single or isolated entities. Autocorrelation, geostatistics and topological data structures are examples of neighborhood models.

3.3. The Heterogeneity Index

Section 2.2 shows that the notion of "geographical neighborhood" figures in the most recent geographical models.

It is vital here to achieve the previously stated objective of modeling the "geographical landscape" through the concept of geo-subspace. One of the characteristics of a subspace that interests the geographer is "variation." The similarity (or difference) between an entity and its surrounding is a measure of this variation. Throughout this chapter several measures of variation of a geo-subspace are proposed and possible interpretations are indicated. These measures will be called heterogeneity indices

3.3.1 Formal Definition

In order to begin formalizing the idea of local variation it is assumed that the study area is partitioned into non-overlaping areal units that completely cover it and that the variable of interest associated with each areal unit is only one and it is of interval scale type. Additionally, it is assumed that the geo-subspace of each areal unit is well-defined. Thus, the number of geo-subspaces (in this case neighborhoods) is equal to the number of areal units. The definitions would also be valid if the units of study were points.

The heterogeneity index associated with the neighborhood of unit "a", Ia is defined as follows:

$$Ia = \sum_{i=1}^{k} \frac{(Xi - Xa)^2}{k} \qquad (3.1)$$

where Xi is the value associated with the ith areal unit, Xa is the value of unit "a" and k is the number of neighbors of unit "a." Ia is therefore the sum of squares of deviations between unit "a" and its k neighbors.

Clearly, this index is highly dependent on the units of measurement. Therefore, in order to make comparisons between neighborhoods easier a new index is defined as follows:

$$Ha = \frac{Imax - Ia}{Imax - Imin} \qquad (3.2)$$

where Ia is the heterogeneity index associated with the neighborhood of "a", and Imin and Imax are the maximum and minimum value of the set of heterogeneity indices associated with the neighborhoods of the areal units under study. Ha takes values between zero and one. The higher the degree of variation of the neighborhood, the closer the value of Ha to zero.

Another measure of variation which appears natural under the same assumptions is that of local variance defined as follows:

$$Va = \sum_{i=1}^{k+1} \frac{(Xi - \overline{Xa})^2}{k+1} \qquad (3.3)$$

where k is the number of neighbors of unit "a", Xi is the value associated to the ith unit and $\overline{Xa}$ is the mean value of the values associated with unit "a" and its neighbors i.e.

$$\sum_{i=1}^{k+1} \frac{Xi}{k+1} \quad ; \quad \text{where} \quad Xa = X\ k+1$$

Va is therefore the sum of squares of deviations towards the mean.

A way to standardize this method has been previously proposed (Silk, p.20). Since comparisons are essential in this type of analysis, a similar standardization is proposed for Va as follows:

$$Va = \frac{\sum_{i=1}^{k+1} (Xi - \overline{Xa})^2}{(k+1)\ \overline{Xa}} = \frac{Va}{\overline{Xa}} \qquad (3.4)$$

In statistical terms, the index as defined in equation 3.3. corresponds to the variance of the geo-subspace. Usually, for a given sample a mean and a variance are associated to it. In this case, for a given geo-space a set of variances and means are associated to it, one for each of the geo-subspaces under study. It is, however, also possible to calculate the mean and variance of the set of heterogeneity indices associated to a geo-space. For example, consider a geo-space represented via a graph where each point is connected to all the other points of the graph (i.e. it is a complete graph) as shown in figure 3.1. In this case, for each point its neighbors are the remaining points in the graph, and as will be show in the following paragraph, the value of the heterogeneity index as defined in equation 3.3 is the same for every one of the points.

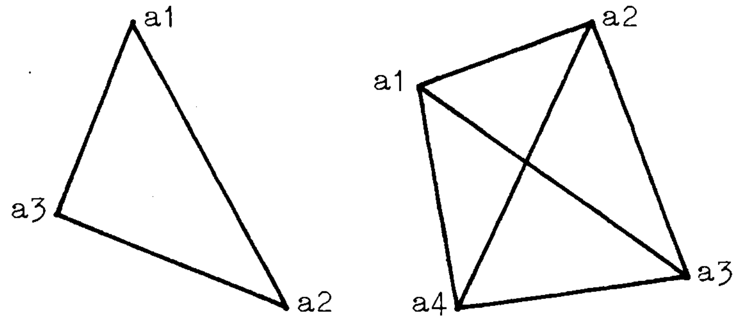
Let a1, a2, a3,...an be the points of the graph and Xa1, Xa2,...Xan the values associated to each one of them. The heterogeneity index of the geo-subspace of each point ai is:

$$Vai = \frac{(Xai-\overline{X})^2 + (Xa2-\overline{X})^2 + \dots + (Xan-\overline{X})^2}{n}$$

$$\text{where} \quad \overline{X} = \frac{Xa1 + Xa2 + \dots + Xan}{n}$$

In this case, since all the heterogeneity indices of the geo-subspaces have the same value, the variance of the indices for this particular geo-space is always zero.

However, as shown in section 4.3.2, the most common case in geographical studies is that of a geo-space represented via a non-complete graph, so that the variance of its heterogeneity indices will differ from zero in most cases.



Complete graphs of three and four points.
Figure 3.1.

The Multivariate Case

In spatial analysis the researcher often deals with several traits which characterize each of the units. Therefore, it becomes necessary to extend the definition of the heterogeneity index to the multivariate case.

The purpose of such an index is to summarize for the different values associated to each of the spatial units the relationship between each of the units and its neighbors. A feasible way of doing this is by obtaining the heterogeneity index separately for each one of the variables of interest and then adding them.

The multivariate heterogeneity index for unit "a" is defined as follows:

$$Ia = \sum_{j=1}^{p} \sum_{i=1}^{k} \frac{(Xij - Xaj)^2}{k} \qquad (3.5)$$

where Xij is the value of the jth variable for the ith neighbor of "a", Xaj is the value of the jth variable associated with unit "a", p is the number of variables and k is the number of neighbors of unit "a".

Similarly, as the heterogeneity index was defined as a measure of the local variance in equation 3.3, it is feasible to define a multivariate index adding the variances associated to each variable for a given geo-subspace as follows:

$$\sum_{j=1}^{p} \sum_{i=1}^{k+1} \frac{(Xij - \overline{Xj})^2}{k+1} \qquad (3.6)$$

where $\overline{Xj}$ is the mean value of the jth variable associated to unit "a" and its neighbors.

Analogously to the univariate case, the variance of the set of multivariate heterogeneity indices can be calculated for the geo-space under study.

There are various problems involved in the multivariate case.

The most obvious one is the fact that the variables are measured in different units which are often non-comparable. The usual procedure to overcome this restriction is to apply a transformation to the original variables. An example of a transformation used to equalize the variables is to force them to have unit variance. There is no complete agreement on the merits of these methods, and the discussion of whether to ignore the problem or to apply a transformation is left to the judgment of the analyst of the problem at hand.

Nevertheless, it is possible to redefine the index so that comparisons among the various indices associated with the different variables become easier. Let Haj be the index associated to regional unit "a" according to variable j. The new index is defined as follows:

$$Ha = \sum_{j=1}^{p} Haj \qquad (3.7)$$

The value of this index is between zero and p. The smaller the variation of the neighborhood with respect to the p variables, the closer the value of Ha to p.

Another problem that arises when several variables are included in the analysis is related to the definition of neighborhood.

The geo-spaces generated by the study of two variables are not necessarily the same. As a consequence a geo-subspace of one of them is not necessarily a geo-subspace of the other. In terms of neighborhoods this means that for a given unit "a" its neighbors with respect to one variable are not necessarily the same with respect to another one. If the neighborhood relation for each variable were represented by means of a graph, the generated graphs could be different.

The definition of the multivariate heterogeneity index has to be altered as follows to consider this contingency:

$$Ia = \sum_{j=1}^{p} \sum_{i=1}^{kj} \frac{(Xij - Xaj)^2}{kj} \qquad (3.8)$$

where kj is the number of neighbors of unit "a" induced by the jth variable and kj = k1,...,kp.

As with previous indices, the deviation with respect to the value of unit "a" is calculated for each one of the "p" variables. However, in this case the neighbors and their number can vary from one variable to the other.

3.3.2 Interpretations

In the previous section measures of local variation were proposed for univariate and multivariate cases. However, no geographical meaning was given to their values. Two possible interpretations of the heterogeneity indices are described in the following paragraphs.

The Heterogeneity Index as a Topological Measure

In the interpretation of the heterogeneity index as a topological measure, knowledge of two branches of mathematics is combined. Concepts that are a traditional part of topological studies such as the interior and boundary of a region, are combined with concepts from the relatively new area of fuzzy sets, such as the degree of membership of an element to a set.

In order to fully understand the role of the heterogeneity index it is necessary to establish the assumptions upon which the interpretation rests. Thus, in the first part of this section some basic mathematical concepts are mentioned prior to interpreting the index.

Topological Concepts.- First of all, it is assumed that the point of departure is a connected graph G that forms part of a geo-space. That is, the entities under study have been identified with the nodes and links between them that represent a spatial relationship. Additionally, the neighbors of a node are defined as its first order neighbors.

For convenience, a topology is defined on the graph so that every subgraph formed by a node and its neighbors is a topological neighborhood. The mathematical details of this definitions are discussed in section 3.4.

Fuzzy Set Concepts.- In the classic concept of membership of an element to a set, the element either belongs or does not belong to the set. This concept was expanded by Zadeh (1965) to reflect more accurately situations which often arise in the real world. Whether an element belongs to a class is often a matter of degree. To model these situations mathematically Zadeh proposed an entity which he called "fuzzy set." Zadeh's definition of fuzzy set (1965) follows:

Given X a space of points, a fuzzy set A in X is characterized by a membership function fa(x) which associates with each point in X a real number in the interval (0,1), The nearer the value of fa(x) to unity, the higher the grade of membership of x in A.

Based on this definition, operations and concepts similar to those studied in ordinary sets have been applied to the study of fuzzy sets. Examples of such operations include union and intersection, convexity and algebraic operations.

The concept of fuzzy sets has been widely applied in areas that include metamathematics, numerical taxonomy and pattern recognition. A large amount of research has been undertaken since the concept was formulated by Zadeh in the 1960's.

A Geographical Interpretation.- One of the problems that has traditionally worried geographers is the definition of classes among a set of entities. It is in this context that the heterogeneity index becomes meaningful. In this thesis the idea is to define the degree of membership in a class for each one of the nodes of the graph.

Since in this initial process there are no pre-defined classes, the degree of membership is better understood as the potential for becoming an interior point of a hypothetical class.

For a fixed point p in the graph G the heterogeneity index Hp can be interpreted as a measure of the potential of membership of p to the interior of a hypothetical region. According to the topological definition of the interior of a region, a point is in the interior if there exists a neighborhood of p that belongs to the region. In this case the topological neighborhood of point p is the set of its first order neighbors. It is at this stage that the concept of fuzzy set becomes relevant.

Although it can not be established at this point whether the neighborhood belongs to the region or not, it is possible to measure the degree of membership of the neighborhood to the interior of the region. This measure is given by the heterogeneity index associated with the neighborhood of p. The closer the value of Hp to one, the higher the degree of membership of the neighborhood to the interior of the region. Inversely, the closer the value of Hp to zero, the lower the degree of membership of the neighborhood to the interior. This relationship corresponds to our intuitive conception of the interior of a region. If a geo-subspace tends to be homogeneous; that is, if the similarity between an entity and its surrounding is high, then it must be in the interior of a region. As expected, in this case the value of the index is close to one. In the inverse case, if the subspace is highly heterogeneous, it must belong to the border of a region and the value of the heterogeneity index is close to zero.

It should be noted that in our problem the notion of the exterior of a region is meaningless since there are no defined regions. Therefore, it is possible to distinguish only between interior and boundary points.

This interpretation of the heterogeneity index as a measure of the potential of a point to be either in the interior or border of a region will be applied in a classification context in the following chapter.

The Heterogeneity Index as a Geographical Measure

The homogeneity of a geo-subspace has also been a traditional problem for geographers. While the number of geographical studies related to regionalization (see Chapter 4) is quite large, the study of the heterogeneity of a geo-subspace has not received much attention. Intuitively however, the concept seems to be very important for spatial analysis studies.

For example, in issues related to mapmaking the cartographer sometimes views the areas of "heterogeneity" as an indicator of the scales that should be used. In such cases, the assumption is that for areas that show a "uniform landscape" there is, in general terms, less interest in producing larger scale maps. A second example comes from social geography, where the study of urban spatial patterns has received much attention, particularly regarding the distribution of social groups (Silk, 1979, p.100). Urban areas have been differentiated by the social characteristics of their population. Nevertheless, spatial patterns of the boundaries of the "city neighborhoods" are intuitively equally important. Two contiguous city neighborhoods most likely interact significantly through their boundaries. If this is so, "highly heterogeneous" boundaries must play a different role in the study of social interaction than "less heterogeneous" ones. A high income residential neighborhood surrounded by a low income one must interact in a different manner with its surroundings than a similar high income residential neighborhood would with a middle-class one.

Heterogeneity indices similar to the ones proposed in this chapter could be used in the study of geographical heterogeneity. An example of an application of this concept is found in the educational planning problem presented in Chapter 5.

3.4. Fuzzy Topology

A close examination of the intuitive ideas behind the mathematical theory of topology clearly points out the strong resemblance between the geographical problem posed in this thesis and one commonly encountered in this branch of mathematics. Firby and Gardiner (1982) give an excellent overview of the main ideas that are the basis for the development of topological theory. The term "topology" was originally introduced in the 19th century by one of Gauss' students and was used in addition to "analysis situs" to refer to this new branch of mathematics. Two parallel developments of topological theory can be identified: point-set or general topology and algebraic topology. Point-set topology was first inspired by Cantor's work (1880) on the general theory of sets, but its major advancement occurred only in this century in the work of Frechet (1906) and Hausdorff (1912).

In general topology, concepts that are usually defined in a Euclidean space such as "limit" and "continuity" are generalized to abstract sets through the notion of neighborhood.

For example, the definition of continuity of a function defined in the real plane (RXR) is based on the notion of open interval as can be appreciated from the following formal definition given by Haaser et.al. (1959, p.327).

The function f is continuous at the point Xo in Df if for each $c > 0$ there exists a $d > 0$ such that

$$|f(X) - f(Xo)| < c$$

whenever $X \in Df$ and $|X - Xo| < d$. Df denotes the domain of the function f.

In a similar manner the concept of continuity is generalized to abstract sets using the concept of open set and neighborhood.

For example, a function from a metric space X to a metric space Y is continuous if and only if for each open set O in Y, the set $f^{-1}(O)$ is an open set in X (Royden, 1968, p.132).

In summary, the space surrounding a point or, in other words, the notion of nearness to a point is formalized in general topology by means of the concepts of open set and neighborhood and is used to generalize ideas that had been developed when the set of interest was the real numbers.

In contrast, algebraic topology, inspired by more geometrical problems, was introduced by Poincare between 1895 and 1905. It should be mentioned that this thesis focusses on the application of general rather than algebraic or surface topology. Nevertheless, it is recognized that concepts developed in areas where there is a geometrical approach, such as surface topology, can be of interest for certain geographical studies.

3.4.1 Definition of Fuzziness

As in many other branches of mathematics, general topology is based on the traditional concept of membership to a set where an element either belongs or does not belong to it. As mentioned in section 3.3.2 the concept of fuzzy set has been used in various branches of mathematics to generalize theories. For example in traditional systems of formal logic a proposition is either true or false. However, the application of the notion of fuzziness has permitted the development of a multi-valued logic which has been found useful in the design of the so-called artificial intelligence expert systems.

In the case of general topology the idea of fuzziness in a geographical context appears in a natural manner. In the same way as the bivalent notion of membership to a set does not provide an adequate model for some real problem-solving situations in applied mathematics, in geography the bivalent notion of the interior of a set as defined in topology is not always adequate for regionalization purposes (see section 4.3.4).

With the aid of the heterogeneity index it is possible to formalize the idea of fuzziness in topological terms. For example, once a geo-space under study has been identified with a graph as defined in section 3.3.2, a topology can be defined on this set.

Since the operations among graphs are implicit in the concept of topology, the definitions of union and intersection between graphs have to be established.

Union: Given two graphs G1 and G2, with their corresponding sets of nodes V1 and V2, and of links X1 and X2, the union between G1 and G2 (G1 U G2) is the graph G with V = V1 U V2 and X = X1 U X2 (Harary, 1972, p.21).

Intersection: The intersection G1 ∩ G2 is defined through the links as follows: X = X1 ∩ X2 and V is the set of all the nodes represented in X.

For convenience the following definition of a topology on G is given:

U is an open set of G if:

i) U is a subgraph of G and

ii) for every point $p \in V(U)$, there exists a non-empty connected subgraph N of U such that $V(N) \neq \{p\}$ and $p \in V(N)$.

It can be proven that these open sets satisfy the conditions required to be a topology of G (see Appendix A).

In particular for every point p in V(G) the subgraph formed by p and its neighbors is a topological neighborhood. This can also be proven (see Appendix A).

It should be remembered that other topologies can be defined on G. The convenience of this particular definition is that entities which have been previously used to model the notion of "geographical neighborhood" such as the subgraph formed by a node and its first neighbors are also topological neighborhoods.

As a result, the heterogeneity index associated to a geographical neighborhood becomes part of a topological space. The heterogeneity index can be interpreted topologically as the degree of membership of a neighborhood to the interior of a set.

3.5 Other Indices

In the definition of the heterogeneity index it was assumed that the variables involved were of interval scale. At this point the question of whether it is possible to define equivalent indices for other types of variables is considered, and an equivalent index is proposed for those cases in which the variables are of nominal type.

The class to which a particular unit belongs can be determined by a nominal variable. These types of variables are encountered in geographical problems in which characteristics that can only be described through classes are involved. Such is the case of spatial analysis problems where variables such as sex (female, male), income (low, medium, high), religion, or nationality characterize the spatial units under study.

There are several measures used to assess the similarity between units with respect to nominal variables. The comparison is made in terms of whether the units have the same or different scores on the variables (Andenberg, 1973, p.123).

The following "matching coefficient" is one of those similarity measures:

$$Sab = \frac{Nab}{T} \qquad (3.9)$$

where Sab is the similarity between units "a" and "b", Nab is the number of variables on which the units match and T is the total number of variables. The more similar two units are, the closer to 1 the value of Sab.

The particular objective at this point is to define a similarity index that reflects the relationship between a unit and its neighbors. The index should be defined so that the more similar a unit is to its neighbors, the larger the value of the index; the more heterogeneous a unit is with respect to its neighbors, the closer to zero the value of the index. The proposed index follows:

$$Ia = \frac{1}{k} \sum_{i=1}^{k} \frac{Nia}{T} \qquad (3.10)$$

where Nia is the number of variables on which units "a" and "i" match, T is the total number of variables and k is the number of neighbors of "a".

If unit "a" and its k neighbors match in all the variables then the index Ia equals 1. If unit "a" does not match in any of the variables with any neighbor, the value of the index is zero.

There are many other matching coefficients. Therefore, the heterogeneity index has to be redefined in every case depending on the measure used. Whether a particular index is appropriate or not depends on the problem at hand.

3.6 Conclusions

A general framework for the design of models that represent spatial structure mathematically using the notion of neighborhood was established in the first part of this chapter. The benefits of this approach can be appreciated in the design of the two neighborhood models in the following chapters.

As a first step in the development of neighborhood models measures of local variation of a geo-subspace were defined and through them the geographical notion of neighborhood was related to the topological concept of neighborhood.

These measures, heterogeneity indices, are meaningful in both geographical and mathematical terms. From a geographical point of view, this formalization is important for the modeler since the geographical entity of neighborhood is identified with an element of a mathematical structure which has been broadly studied during this century. On the other hand, in mathematical terms the indices allow the definition of fuzziness in a topological space. A development which to the best of the knowledge of the author has not been explored before and could lead to the development of a new topology, a fuzzy topology.

Chapter 4.

A TOPOLOGICAL APPROACH TO REGIONALIZATION

Regionalization is probably one of the best known branches of geography. One of the central issues in regionalization problems is homogeneity. It seems natural, therefore, to apply a concept like local variation of a geo-subspace to the process of region building. In this chapter an application of the heterogeneity index in the design of classification algorithms is presented. In the first and second sections an overview of regionalization is given, and several existing spatial algorithms are discussed. In the final sections two regionalization algorithms that use a heterogeneity index are presented, and a hypothetical case is included.

4.1 Regionalization as a Classification Problem

The identification of areal groups that show a homogeneous distribution of one or more characteristics but differ from other groups is one of the central issues in regional geography (Zobler, 1958, p.140).

Regionalization is the process by which regions are identified and classified. Bunge (1966) clearly recognizes the definition of regions as a classification or taxonomic problem. In taxonomic terminology, a uniform region is equivalent to an areal class, a single feature region is a classification using a single category, etc. From this point of view a regionalization is a classification of geographic units.

A whole body of classification techniques have been developed as an inquiry tool for other sciences such as biology and botany. The methods developed in classification or cluster analysis are in essence formal; that is, they employ a mathematical frame. The intent of such methods is to find a solution to a classification problem similar to the one produced by a specialist. Decision rules for classifications are usually designed in the form of algorithms. Various disciplines use algorithms that are in essence equal but have been adapted to different circumstances. Regional geography shares universally accepted methods such as "central agglomerative procedures." Geographic studies which adopt a classification methodology to various contexts have been reported. Some examples are the studies of areal patterns in cities (Jones, 1977), the partition of an area into adequate zones for the optimal location of service centers such as hospitals and schools (Scott, 1969) and political districting (Garfinkel and Nemhauser, 1968).

According to Haggett et al. (1977, p.451) there are three classificatory approaches that have been used by geographers: uniform regions, nodal regions and planning or programming regions. Uniform regions are those in which places located within the regions are homogeneous with respect to one or more properties. The regions are disjoint, contiguous and completely exhaust the study area. Nodal regions measure interactions between units such as migration and number of telephone calls. Planning regions are created to satisfy specific needs of an institution, to implement policy decisions or for administrative purposes. The criteria selected to define these regions reflect the objectives for which they were created. Such is the case of the definition of enumeration areas for a census. The resulting regions are not necessarily contiguous and might not exhaust the study area.

In addition to the clear differences among types of regions it should be emphasized that the classification of locations into regions also serves different purposes. The main ones are: hypothesis testing, administration and programming. In the case of nodal and uniform regions, classification is often undertaken as an exercise to substantiate spatial theories. The definition of programming regions serves specific purposes. This does not mean that the types of regions are not closely related. In fact, the definition of programming regions is often constrained by previously defined nodal or uniform regions.

4.1.1 Elements of a Regionalization

Depending on the purpose of regionalization, different choices are available to the analyst. Each one has an impact on the result of the process. Therefore, the appropriate selection of units, algorithms, etc is of vital importance. In some cases choices are almost equivalent, while others differ drastically. These decisions often depend on the analyst's understanding of the problem itself. It could therefore be argued that this introduces a subjective factor to the regionalization process.

When a classification exercise is carried out, the first stage consists in defining its elements. A regionalization has the same basic elements as other classifications, but it also adds spatial constraints. The elements of regionalization are:

- the units for the regionalization
- the properties that characterize the regions
- a measure of homogeneity or similarity

- spatial constraints such as contiguity and compactness
- a grouping criterion
- the algorithm to create the regions
- the number of regions.

At this point it is worth mentioning the principal factor that singularizes region-building in comparison to other classification schemes: the data units have an implicit locational characteristic (Bunge, 1966). In numerical taxonomy similarity and nearness are equivalent; however in spatial applications it is important to draw a distinction between the two terms. Similarity measures are commonly applied with a nearness or contiguity constraint.

4.1.2 The Geographic Units

The two main limitations which the analyst faces in the selection of units are the availability and the level of aggregation of the data. According to Sawicki (1973), the availability of locational data for urban and regional researchers is very limited. In most cases it is obtained from secondary sources such as census and administrative offices. Data is often compiled for fixed administrative areas such as school districts or for street blocks. The accessibility of census data and availability of statistical packages has increased, but spatial analysts have become increasingly aware that existing data is not always compatible with the hypothesis under study (Sawicki, 1973, p.146).

The two most common regionalization units are areal and point types. It appears that the most popular areal level used in urban studies has been the census tract (Sawicki, 1973, p.110). Tracts are delineated so that they are homogeneous with respect to characteristics such as income and topographical features, as well as constraints such as population size and contiguity. It seems to be the case that analysts have a vast range of levels of aggregation from which an appropriate selection may be made. However, spatial analysis done at different levels of aggregation has shown not only different but contradictory results (Sawicki, 1973, p.110). The fact that the selection of units determines to a great extent the results of spatial analysis severely restricts the researcher since s/he often does not have control of the definition of units used to compile data.

4.1.3 Measures of Homogeneity

In geographic studies regions are considered "areal systems based on levels of similarities and differences in spatially distributed traits" (Zobler, 1958, p.140). Homogeneity is identified with low areal variance and heterogeneity with high areal variance (Bunge, 1966, p.22). Regions must be internally homogeneous and differentiated from other regions. It is in this context that the grouping of areas into regions has been approached as a classification problem.

Identifying homogeneity with similarity as it is understood in numerical taxonomy has made it possible to define regions using the same techniques as in cluster analysis. To carry out a regionalization it is therefore necessary to establish the significance of homogeneity or similarity among areas and regions.

In the following paragraphs some of the measures of similarity and dissimilarity that have been used for grouping purposes are presented. The point of departure is a set of units and a set of variables characterizing them. Depending on the scale of measurement the variables can be classified in four groups: nominal, ordinal, interval and ratio.

1. A nominal scale allows distinctions to be made between classes.
2. An ordinal scale induces an ordering of the objects.
3. An interval scale allows comparisons of two objects by means of the differences between them.
4. A ratio allows comparisons of two objects by both a difference and a ratio.

(Andenberg, 1973, p.27)

Some of the similarity measures used for interval scale data follow:

Minkowski Metric

The distance between units "i" and "j" according to the Minkowski metric is:

$$Dq(Uk,Uj) = \left(\sum_{i=1}^{p} |Xij - Xik|^{q} \right)^{1/q} \qquad (4.1)$$

where q>1 and p is the number of variables. In particular for q = 2, Dq is the Euclidean distance.

$$D2(Uk,Uj) = \left(\sum_{i=1}^{p} |Xij - Xik|^{2} \right)^{1/2} \qquad (4.2)$$

The greater the dissimilarity between units "k" and "j", the larger the value of Dq. The measure increases with decreasing similarity and decreases with increasing similarity. In geographic applications the Euclidean distance is the most commonly used metric.

When the Minkowski metric is used, it is assumed that the variables are immersed in an orthogonal space. This poses some limitations in geographic applications, since the variables are often not orthogonal. Principal component analysis has been used to overcome this restriction (Byfulgen and Nordgard, 1973).

Correlation Analysis

The product moment correlation coefficient can be used as a measure of association between units. In geographic terms the degree of association has been interpreted as a measure of "regional bonds" (Haggett et. al., 1977, p.476). One of the central problems of this method is that the variables associated to a unit involve different measurement units. This renders mean and variance meaningless (Andenberg, 1973, p.113). The correlation between data units "j" and "k" is defined as:

$$Rjk = \frac{\sum_{i=1}^{p} (Xij - \overline{Xj})(Xik - \overline{Xk})}{\left[\sum_{i=1}^{p} (Xij - \overline{Xj})^2 \sum_{i=1}^{p} (Xik - \overline{Xk})^2\right]^{1/2}} \quad (4.3)$$

where $\overline{Xi} = 1/p \sum_{i=1}^{p} Xi$ and p is the number of variables.

Analysis of Variance

There are at least four measures of similarity that have been used in terms of analysis of variance. The first two quantities (a and b below) are used in univariate cases while the last two (c and d below) are used in multivariate cases.

a) In grouping procedures the following quantity is used as an objective function:

$$D = \sum_{i=1}^{n} Wi \ (Xi - \overline{Xi})^2 \qquad (4.4)$$

where Wi is the weight assigned to each data unit, n is the number of units and $\overline{Xi}$ denotes the weighted arithmetic mean of those Xi that are assigned to the subset to which element "i" belongs (Fisher, 1958, p.789). D decreases as the groups become more homogeneous. D is known as the sum of squares within groups in the sense of analysis of variance. In grouping procedures the objective is to minimize D.

b) In building regions it is desirable to have internal differences minimized and differences between regions maximized. That is, homogeneity within regions and heterogeneity between regions should characterize the grouping.

The following measure shows these inter and intra-regional differences.

$$H = \frac{\text{external variation (between regions)}}{\text{internal variation (within regions)}} \qquad (4.5)$$

The closer the grouping fits the desired requirements, the higher the value of "H". It should be noted that this quantity is used as a type of objective function rather than as a crude measure of homogeneity.

c) The Ward Method

Ward defined the following measure based on the idea that whenever there is a grouping there is a loss of information:

$$E = \sum_{k=1}^{m} Ek$$ total within region error sum of squares (4.6)

$$Ek = \sum_{i=1}^{p} \sum_{j=1}^{mk} (Xijk-\overline{Xik})^2$$ error sum of squares for region k (4.7)

$$\overline{Xik} = 1/mk \sum_{j=1}^{mk} Xijk$$ mean of the ith variable for areas in region k (4.8)

Where Xijk = value of the ith variable for the jth area in the kth region,

n = number of areas,

m = number of regions,

mk= number of areas in region k,

p = number of variables.

In this case E is used as an objective function that has to be minimized. The more information that is lost in a regionalization, the larger the value of "E".

d) Cliff and Haggett (1970) defined a similar homogeneity measure as:

$$B = \frac{1 - E}{\max E} \qquad (4.9)$$

where E is the total within-region error sum of squares, and the maximum is taken over all the possible values of E. In fact the maximum value is obtained when the resulting region is only one; that is, when all the units are grouped together. Since B is equal to zero when all areas are grouped into one region and equal to one when each area is a region, the closer B is to one the better the regional system performs in terms of homogeneity.

It should be added that there are many other similarity measures that have been used for grouping purposes. The reader is referred to Andenberg (1973), Hartigan (1974) and Cormack (1971).

4.1.4 Regionalization Constraints

In addition to homogeneity there are other constraints that are sometimes imposed on regionalizations. Among the criteria that have been used for both districting and region building are:

1. Equality of population
2. Contiguity
3. Compactness
4. Preservation of political or administrative boundaries
5. Region boundaries should follow geographic features such as rivers and mountains (Thoresson, p. 237).

Even though all of these constraints are in essence spatial, the contiguity constraint is particularly interesting for this work since it is strongly related to the notion of geographical neighborhood.

Contiguity

Homogeneity, as understood in classification analysis, means either similarity or nearness. Regional homogeneity, however, refers to both similarity and geographical nearness.

There are two manners in which the contiguity constraint can be interpreted:

a) When the units are of areal type, a region is contiguous if for any two units, a1 and a2, that belong to it, it is possible to travel from a1 to a2 through a path wholly contained in the region. In mathematical terms this is called a connected set.

b) In some other instances the need for contiguity does not necessarily imply a physical border-to-border relation but simply a neighborhood one as described in Chapter 2.

There has been some disagreement among geographers about the necessity of imposing a contiguity constraint on a regionalization. In some instances, such as the definition of administrative zones or electoral districts, there can be no doubt concerning the need for such a constraint. However, the requirement is less clear in the use of grouping for research purposes.

There are two basic reasons researchers carry out regionalizations: as an exploratory tool or to test a hypothesis. However, it should be remembered that there is an important difference between using the grouping itself to test a hypothesis and testing the hypothesis of whether there are clusters or not. Classification or cluster analysis can only be used as such in the first case.

Two approaches to the building of uniform regions are possible:

a) A classification without a contiguity constraint (called a typification) is undertaken, followed by the mapping of results.

b) A classification with a contiguity constraint is undertaken.

According to Byfulgien and Nordgard (1974) these two approaches are "not necessarily conflicting but complementary." However, this is not a universally accepted position. For example, Johnston (1970) argues that "regionalization with contiguity constraints over-simplifies and operates against efficient hypothesis testing."

The problems arises in the interpretation of results. The product of a typification is a set of groups that satisfy a condition of homogeneity, but are not necessarily contiguous. These results have a value in themselves. However, when the resulting groups are mapped, the units are implicitly classified by another variable, that of location. Sets of units that belong to the same group and are contiguous seem to form regions. However, it can not be ascertained that these newly formed regions satisfy the same homogeneity condition as the original grouping.

4.1.5 The Number of Regions

In a regionalization problem the main task is to find a grouping of the units that "best" satisfies the needs of the analyst. Therefore, the first idea that comes to mind is to select from all the possible groupings the one that best satisfies the constraints. Cliff and Haggett (1970) have looked into some combinatorial aspects of the regionalization problem. They were able to calculate the number of different aggregations to form "m" regions given "n" areas without a contiguity constraint.

$$N = \sum \frac{n!\ (g1!\ \ldots\ gj!)}{\prod_{i=1}^{m} fi!} \qquad (4.10)$$

where gj is the number of regions which comprise j units, fi is the number of areas combined to form region i and the summation is over all m element partitions of n. For example, if the number of units is four (n=4) and the number of regions is two (m=2), the m element partitions of n are two; (3,1) and (2,2).

They also calculated the number for the case where the areas are under a strong contiguity constraint, i.e. when they form a chain.

$$A = \sum \frac{m!}{g1! \ldots gj!} \qquad (4.11)$$

As Cliff and Haggett (1970, p.288) have shown, in both cases the number is too large to permit approaching the regionalization problem by exhausting the possibilities. This is the reason why it has become necessary to design heuristic algorithms to find solutions which approximate the "best" result.

When a classification approach is undertaken the number of regions to which "n" areas should be aggregated often has to be defined by the analyst. As will be seen in the following section, hierarchical methods group "n" units in any number between one and m. It is the task of the analyst to decide on the "best" level of aggregation. In non-hierarchical methods the number of seeds determines the number of regions. In other instances, such as some of the algorithms described in section 4.3, the number of regions is a result of the algorithm.

4.1.6 The Algorithms

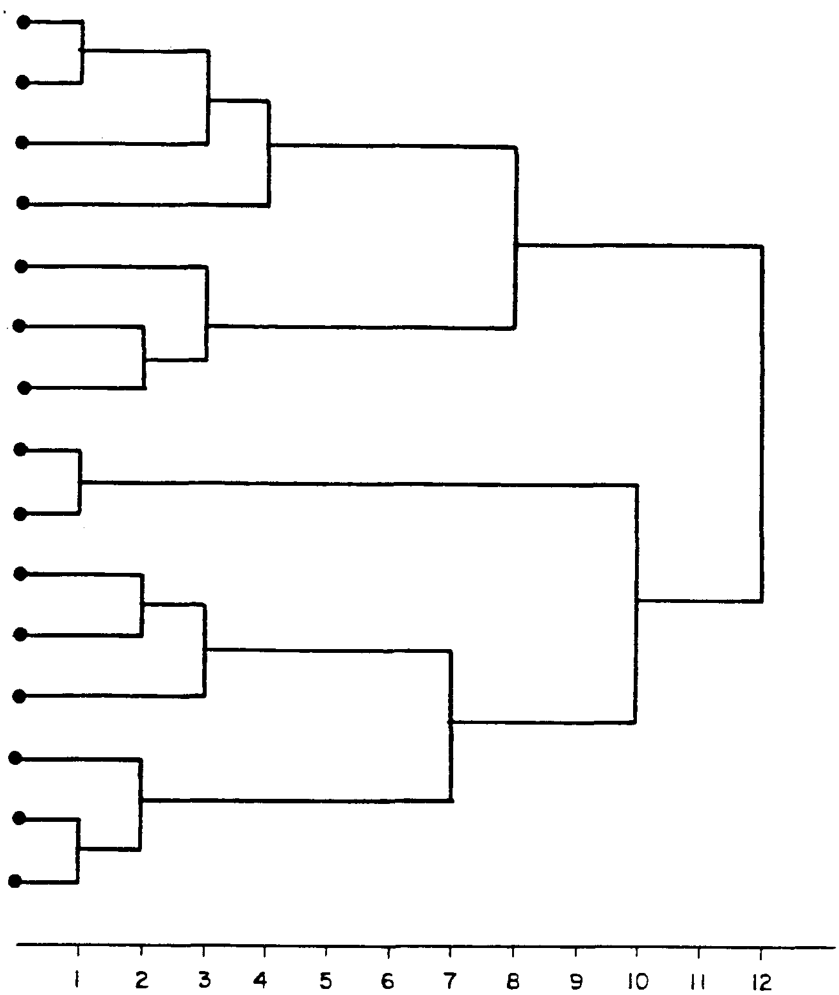
Once a similarity measure as well as the constraints for the grouping have been established, it is necessary, in order to actually obtain the regions, to define a procedure by which the areas are to be clustered. The existing methods can be classified as hierarchical or non-hierarchical.

Hierarchical Methods

In a hierarchical method the starting point is a set of "n" data units, and it ends with the universal region in which all the units are grouped in one region. In some cases the procedure is divisive because it starts from the universal region. What is common to the hierarchies, whether they are divided or grouped is that they remain as such throughout the entire remaining process.

An easy way to visualize this process is by means of tree diagrams as shown in figure 4.1. Each node represents a region, and the stages of the procedure are shown in the axis below the tree. In the first step the two most similar units are merged, and the number of regions (or units) left is reduced to "n-1". After the ith step the number of regions is "n-i". The process involves "n-1" steps.

According to Andenberg (1973, p.132) there are three major hierarchical clustering methods: linkage, centroid and variance. Briefly, in the single linkage method, clusters are merged using the shortest distance (similarity) between their elements as a criterion. In the centroid method, the



Distance Between Groups
Dendogram

Figure 4.1

similarity between clusters is given by the similarity between their means. Finally, in the Ward method the clusters that produce the minimum increase in the total within-group error sum of squares (as defined in section 4.1.3) are merged. Specific hierarchical methods used in geographical studies will be presented in the next section.

Non-hierarchical Methods

The difference between hierarchical and nonhierarchical methods is that in the latter two units that belong to the same region, at any stage of the process, do not necessarily remain joined. In fact, nonhierarchical methods are based on the assumption that given an initial partition of the units, subsequent improvements are feasible. Usually the first step is the selection of a set of units called seeds. An initial partition is defined by joining each unit to its most similar seed. In the following stages each new partition is defined by taking the previous one as a point of departure. The process ends when the "best" partition is found.

4.2 Spatial Algorithms

The algorithms that geographers have used for regionalization purposes can be divided into two groups. The first is composed of those shared with other disciplines, such as the

Ward and Singe linkage methods. The second group is composed of those algorithms that include specific spatial constraints such as contiguity and compactness.

The first group has been extensively described in the literature (see Andenberg, 1973, Cormack, 1971, Hartigan, 1975) and will not be discussed in further detail here. The second group is of more interest. It will be referred to throughout this study as "spatial algorithms."

From a methodological point of view we can distinguish three types of spatial algorithms:

a) those in which the contiguity of groups can only be assured by checking if the units have a common border;
b) those that use the notion of neighborhood to identify contiguous groups;
c) those in which contiguity is assured together with other constraints imposed on the resulting regions.

For comparative purposes some "typical" algorithms of the first two types will be described. The third type of spatial algorithms which is not described here, uses techniques such as integer programming and is usually applied to districting problems (Garfinkel, 1970).

4.2.1 Byfulgien and Nordgard Algorithm

The following method was originally introduced by McQuitty and later transformed into a spatial algorithm by Byfulgien and Nordgard (1973). This is an example of a hierarchical algorithm of the first type. The similarity measure is the Euclidean distance, and the clustering criterion is of the single linkage type. The number of resulting regions is determined by the algorithm. The main characteristic of the resulting regions is that "all basic units have their most similar contiguous unit within the same region."

Byfulgien and Nordgard applied this method in eastern Norway to agricultural data and concluded that it can produce regions with very dissimilar units. This is because the condition required to add a unit to a region is its similarity to just one of the other units of the region.

The Algorithm

Def. 1. "A" is the set of "n" areal units in which the area of study is subdivided. That is, A = {a1,...an}.

Def. 2. Dij is the distance between areal units ai and aj.

Def. 3. M is a nxn matrix that contains all the distances between areal units. That is $m_{ij}=D_{ij}; i,j=1,...n$

Step 1. Find the two most similar areal units. Let ai and aj be these two units.

Step 2. Check if the areal units ai and aj have a border in common. If they are contiguous continue with Step 3. Otherwise let Dij be a "large number" and continue with Step 1.

Step 3. Merge units ai and aj to form region R.

Step 4. Let N be the set of areal units that have a border in common with R. For each element of N check if it is closer to one of the units that belong to R than to any other of its contiguous units.

Step 5. Let F be the set of units that satisfy the condition stated in step 4. If there is no such unit, i.e. $F = \emptyset$, continue to Step 7.

Step 6. Form a new region merging region R and the previously defined set F. That is, take $R \cup F$ and call it R. Continue with Step 4.

Step 7. Region R is one of the resulting regions.

Step 8. Take the set of areal units that do not belong to any region and obtain its distance matrix M.

Step 9. If all areal units belong to a region, then end the grouping process. Otherwise repeat the procedure starting from step 1.

4.2.2 Berry's Algorithm

Berry (1961) modified a central agglomerative procedure to include a contiguity constraint and used it for an economic regionalization. Most of the central agglomerative procedures follow the general scheme given by Andenberg (1973, p.133). The modified or "spatial" general procedure is as follows:

Def. 1 "A" is the set of "n" areal units in which the area of study is subdivided. That is A = {a1, a2, ..., an}

Def. 2 Dij is the distance between areal units ai and aj

Def. 3 M' is a n X n matrix where:

$$mij = \begin{cases} Dij & \text{if ai and aj are contiguous;} \\ \infty & \text{otherwise (or a very large number);} \end{cases}$$

In this case the matrix M' reflects not only the similarity between regions but also their contiguity relation.

Step 1. Begin with n areal units.

Step 2. Search the matrix M' for the two most similar pairs of contiguous regions. Let the chosen regions or units be labeled ai and aj.

Step 3. Reduce the number of regions (or units) by one merging regions ai and aj. Label the resulting region ai and update matrix M' to reflect the similarities between ai and all other existing regions (or units). Delete the row and column of M' that corresponds to region (or unit) aj.

Step 4. Perform Steps 2 and 3 a total of (n-1) times.

Both areal and point units can be used with this type of algorithm. The range of similarity measures and clustering criteria that can be incorporated into this algorithm is the same as that of a non-spatial central agglomerative procedure. Other constraints such as compactness can be included without changing the basic structure of the algorithm.

One of the disadvantages of this type of algorithm is that sometimes it produces a chaining process. This occurs when in every stage a unit is merged to the same (or to a few) region(s). The result is that at any stage there are a few "large" regions together with ungrouped units.

In this type of algorithm the user decides on the number of resulting regions, since the procedure starts with n regions and ends with one.

4.2.3 Lankford Algorithm

An example of an algorithm that uses the notion of neighborhood to identify regions is described by Lankford (1969). This algorithm was not specifically developed for cases where there are contiguity constraints and the units themselves are not necessarily spatial.

Given the set of variables which represent the attributes with which the units are characterized, it is assumed that they are immersed in a m-dimensional orthogonal space. A similarity measure designed to detect zones of "high density" in the m-dimensional space is introduced.

The Algorithm

Def. Let the density W(a) for areal unit "a" be defined as follows:

$$W(a) = (1/n) \sum_{x \in N(a)} d(a,x)^{-1} \qquad (4.12)$$

where n is the number of neighbors of unit "a", d(a,x) denotes the Euclidean distance between units "a" and "x" and N(a) is the neighborhood of "a."

This measure was designed to detect the high density zones. A unit immersed in a dense zone has a high "W" value associated.

Def. Let f(a,b) be the association between units "a" and "b":

$$f(ab) = \frac{W(a)\ W(b)}{d^2\ (a,b)} \qquad (4.13)$$

This expression resembles the one for gravitational attraction.

The following definitions are necessary to extend the concepts previously given to the case of units already grouped.

Def. Two groups G and H are called neighbors if there is an element g in G such that its neighborhood N(g) intersects H i.e. $N(g) \cap H \neq \emptyset$.

Def. The interface I(G,H) between two groups G and H is the subset of elements of G (or H) such that its neighborhood intersects H (or G).

$$I(G,H) = \{x \in G \mid N(x) \cap H \neq \emptyset\} \cup \{x \in H \mid N(x) \cap G \neq \emptyset\} \qquad (4.14)$$

Def. The association between neighbor groups G and H is the average of the associations between all pairs (g,h) in the interface:

$$f(G,H) = (1/m) \sum_{(g,h)} f(g,h) \qquad (4.15)$$

where m is the number of pairs in the interface, and $(g,h) \in I(G,H)$.

The algorithm itself is a central agglomerative procedure using "f" as the measure of similarity. This procedure was applied in a two-dimensional space. Apparently, no attempt has been made to use it in a more general case.

4.2.4 Brantingham Algorithm

Brantingham (1978) has presented an algorithm that uses topological concepts not only intuitively but in a formal sense as well. The procedure is designed in such a way that at each stage it is decided whether two units should be separated by a border. In this sense it differs radically from previously described algorithms, in which the main decision leads to the grouping of units.

City blocks were used as basic units in the regionalization. The similarity measure is based on the difference of absolute values, and it is a multivariate grouping.

The Algorithm

Def. 1 "A" is the set of "n" areal units in which the area of study is subdivided. That is A = {a1, a2, ..., an}

Def. 2 Let N(aj) be the set of units contiguous to aj.

Def. 3 Let fi(aj) be the value of the ith variable associated to aj.

Def. 4 A basis of a topology T in X is a subcollection B of T such that every open set U in T is a union of some open sets in B (Hu, 1964, p. 17).

Def. 5 A basis set Bi of the topology is the set of all contiguous units such that the interunit variation of the variable of interest is less than some fixed percentage.

Step 1. Fix a maximum percentage of interunit variation, call it b.

Step 2. Let ak be an element of the basis Bi (i=1 the first time), where ak is an arbitrary unit.

Step 3. For each element aj in Bi (which has not been tested before) perform steps 4 to 7.

Step 4. For each element aj in Bi check among the neighbors of aj that do not belong to Bi, if they exceed the maximum percentage of interunit variation with respect to aj. That is:

$$\left| f(aj) - f(ai) \right| > \max \{bf(ai), bf(aj)\}$$

where $aj \in Bi$, $ai \in N(aj)$, $ai \notin Bi$ and $0 < b < 1$. Call F the set of units that exceed the maximum percentage of internal variation and I the complement. That is:

$$F \cup I = N(aj) - Bi$$

Step 5. Draw a border between every element of F and aj.

Step 6. Those units that do not exceed the interunit variation are added to Bi. Redefine Bi as Bi U I.

Step 7. If there are elements of Bi that have not been tested for the percentage of internal variation, continue with step 3.

Step 8. The resulting set Bi is a basis set.

Step 9. If there are still units that do not belong to any basis set, then choose any arbitrary element of this set and continue to step 2 to create a new basis set.

In this case the basis sets are interpreted as the resulting regions. If the pattern that appears from the above procedure is not satisfactory, a new value for the internal variation is fixed and the procedure is repeated. The basis sets are defined so that the newly defined variation is smaller than the previous variation used. This guarantees that once a border is drawn between two units it will remain as such through the whole process.

4.3 The Design of Algorithms

From the previous sections it can be seen how cluster analysis techniques have been used for regionalization purposes. In some cases the techniques have been applied without any modifications, while in others they have been adapted for spatial analysis purposes.

Considering that one of the interpretations of the heterogeneity index is as a measure of the degree of membership of a point to a set, it seems natural to use it in a region-building process.

In this case the heterogeneity index is assumed to be an indicator of the potential of an element to belong to the interior of a region. The closer the value of the index to 1, the higher the potential of the element to be an interior point.

As is common in a regionalization problem, it is assumed that the elements under consideration are clustered into homogeneous groups. That is, every element is either an interior point of a homogeneous region or is on its border.

4.3.1 A Topological Algorithm

The next objective is to define a procedure that integrates the heterogeneity index and the grouping process. A hierarchical method is proposed in which the clustering criterion is determined by the heterogeneity index. The algorithm is designed so that the order in which the elements are grouped is determined by the index. Intuitively, those elements that have a higher potential to be in the interior of a region should be clustered before the ones that have a high potential to be on the borders. The proposed algorithm is described below:

Def. 1 "A" is the set of "n" areal units in which the area of study is subdivided. That is: $A = \{a1, a2, \ldots, an\}$.

Def.2 A unit that does not belong to any region is called an "elementary unit."

Def. 3 A region is the union of at least two (elementary) units.

Def. 4 A unit is an entity that originally was elementary but now constitutes part of a region.

Def. 5 N(ai) is the set of neighbors of entity "ai". The possible entities are: region, unit and elementary unit.

Def. 6 The heterogeneity index associated with a region is similar to the one defined for elementary units as follows:

$$Ir = \sum_{j=1}^{p} \sum_{i=1}^{k} \frac{(Xij - Xrj)^2}{k} \qquad (4.16)$$

where Xij is the value of the jth variable for the "k" neighboring entities of the region and Xrj is the value of the variable associated with the region. The manner in which these quantities are calculated depends on the problem at hand.

Step 1. Select the elementary unit with the highest

value in the heterogeneity index.
Call it ai.

Step 2. Search for the most similar neighbor to ai.
Call it aj.

Step 3. If aj is an elementary unit then group ai and aj. Call the region Rk.

Step 4. If aj is a unit then group ai and the region that contains aj. Note that ai and aj are no longer elementary units.

Step 5. If there are more elementary units left, return to step 1; otherwise continue with step 6.

Step 6. If the number of resulting regions is less than desired then finish the procedure. Otherwise continue with step 7.

Step 7. Rename the regions as elementary units. Establish the neighborhood relations for the new elementary units.

Step 8. Start again with step 1.

General Characteristics of the Algorithm

This type of algorithm can be used when the units of interest are either areas such as census tracts, counties and provinces of a country or points such as airports in a transportation network. The selection of the similarity measure depends on the problem at hand, but it has to be consistent with the definition of the heterogeneity index. That is, if a neighborhood N "tends" to be in the interior of a region, it should also be homogeneous with respect to the similarity measure used.

The heterogeneity index can be redefined to consider this restriction as follows: Assume S to be the function of similarity between any two elements. The heterogeneity index associated to the neighborhood of unit "a" is of the form:

$$Ia = \sum_{i=1}^{k} \frac{S\ (Xi,Xa)}{k} \qquad (4.17)$$

where S(Xi,Xa) is a measure of similarity between the value associated to the ith neighbor and unit "a" and k is the number of neighbors.

This algorithm was designed to assure that the resulting regions satisfied a contiguity constraint. However, other constraints such as that of compactness could be introduced without altering the basic structure of the algorithm. The algorithm is hierarchical, but it differs basically from others in its clustering criterion. In this case there are two clustering criteria: a spatial and a non-spatial one. The heterogeneity index determines the order in which the grouping is going to take place, while the similarity measure determines which units are to be grouped. Any of the several criteria used in hierarchical algorithms as described in section 4.1.6 could be adapted.

The number of resulting regions obtained by this type of algorithm is not the same as in a typical hierarchical one. The number of regions in each stage is determined by the data. While in typical cases the analyst can choose any number desired between 1 and n, in this case it can only be selected from the results. This constraint can actually be an advantage, if the analyst has no way of anticipating the number of regions there may be.

4.3.2 The Regions as Graphs

The structure of the units in a clustering problem can be viewed as a graph (Andenberg, 1973, p.150). The nodes of the graph are the units themselves, and the lengths of the edges are given by the similarity between the units. The graph is complete since all its nodes are adjacent. The single linkage method finds the minimal spanning tree; that is, the shortest tree with (n-1) edges that connects all the nodes (Andenberg, 1973, p.150). In a regionalization problem the structure of the data units can also be viewed as a graph. Again, the nodes of the graph represent the units. There is an edge between two nodes if the units are neighbors, and the length of the edges is given by the similarity measure. This graph is in fact a subgraph of the complete graph G' generated in a classification problem without a contiguity constraint.

When a single linkage criterion is used in the proposed algorithm, the resulting regions obtained in a first stage generate a set of graphs where the nodes represent the elements that belong to each region and the edges are defined through the clustering criterion. Each of them is a subgraph of G'. Moreover, each one of these subgraphs is a minimal spanning tree of the subgraph of G' formed by the nodes that belong to the regions together with their neighboring relations.

If the algorithm is repeated until all the units are grouped in a single region, the resulting graph will also be a minimal spanning tree of G'.

Both the topological algorithm and the original single linkage method will generate a minimal spanning tree. The basic difference is that at any given stage the order in which the units are grouped is not necessarily the same. A hypothetical case that exemplifies this is presented in the following paragraphs.

A Hypothetical Example

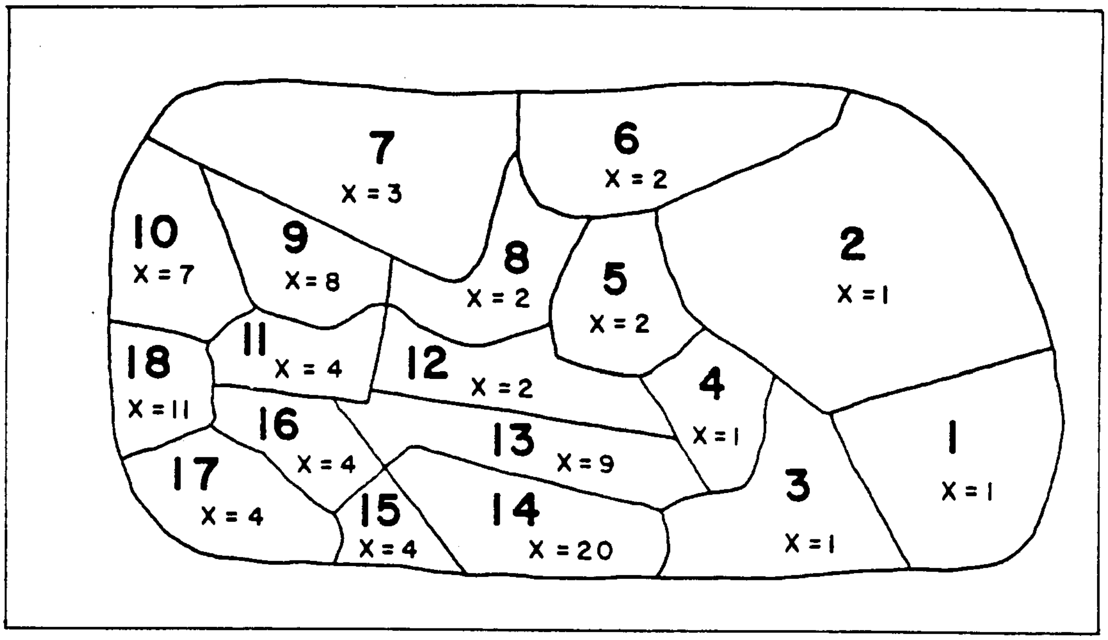
To illustrate the issues discussed in this section concerning the concept of minimal spanning tree, a brief example is presented. The purpose of the exercise is to regionalize the 18 areal units shown in figure 4.2. The areas were grouped according to two different algorithms: a single linkage method with a contiguity constraint and the topological one presented in section 4.3.1. In both cases two areas are said to be contiguous if they have at least one segment in common. The similarity between two areas is given by the absolute difference. That is:

$$dij = |Xi - Xj|$$

where Xi is the value associated to the ith area.

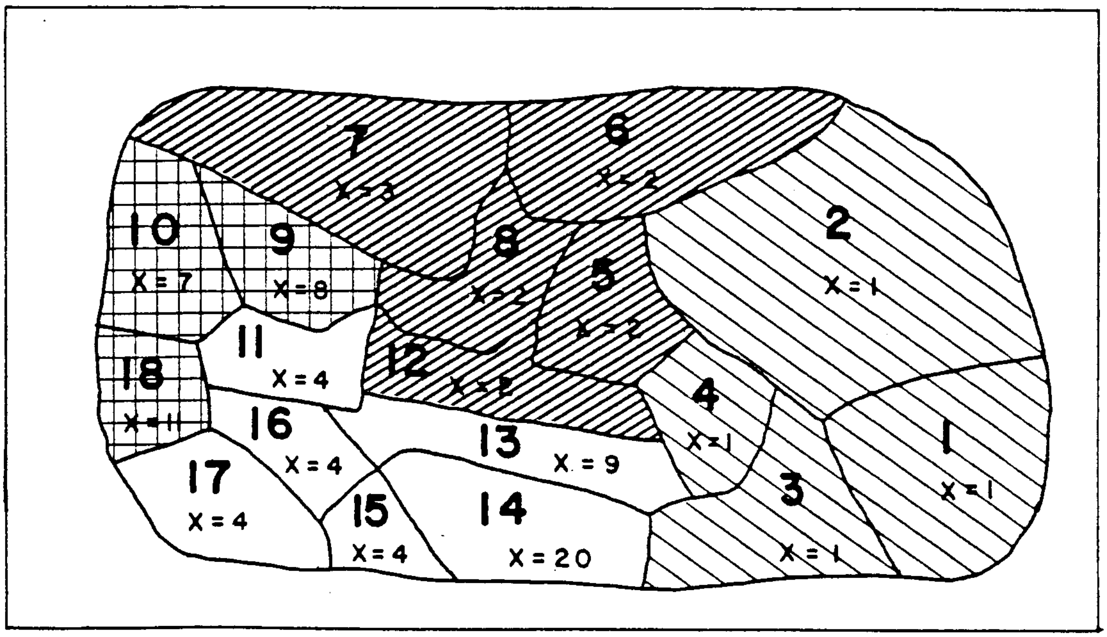
Figure 4.3 shows the resulting regions obtained after the first stage of the topological algorithm. Each one of the subgraphs (one for each region) shown in figure 4.4 is a minimal spanning tree. In the second stage of the procedure all the areal units are grouped into one region. The minimal spanning tree generated is shown in figure 4.5.

The dendogram generated by the usual linkage method is shown in figure 4.6. Since the procedure is hierarchical, in the final stage all the units are grouped into one region. The minimal spanning tree generated by this procedure is shown in



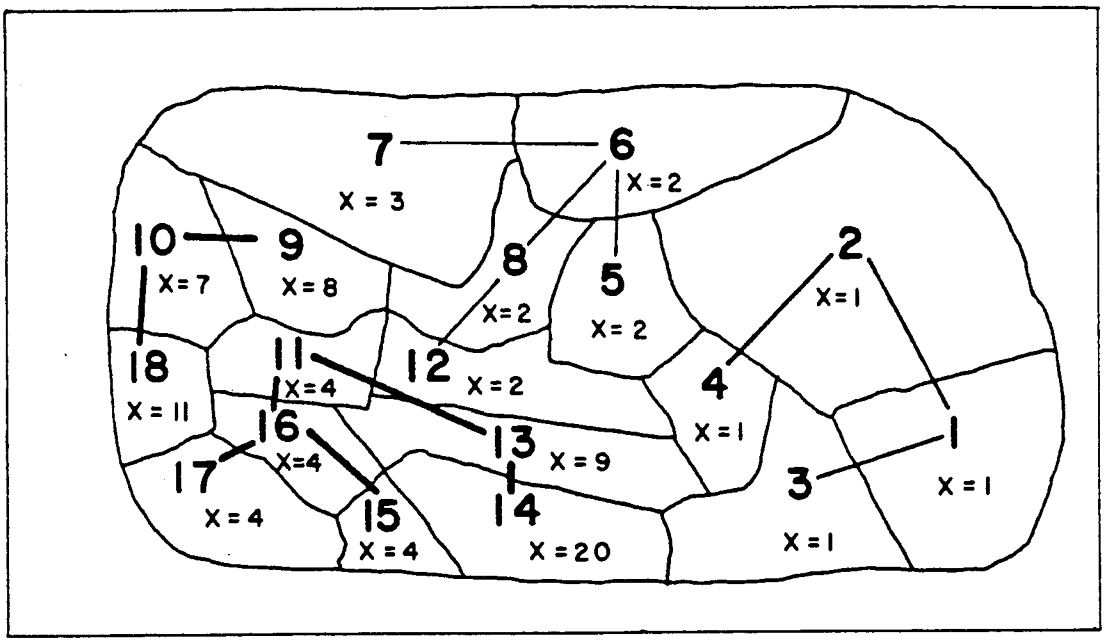
Original Data

Figure 4.2



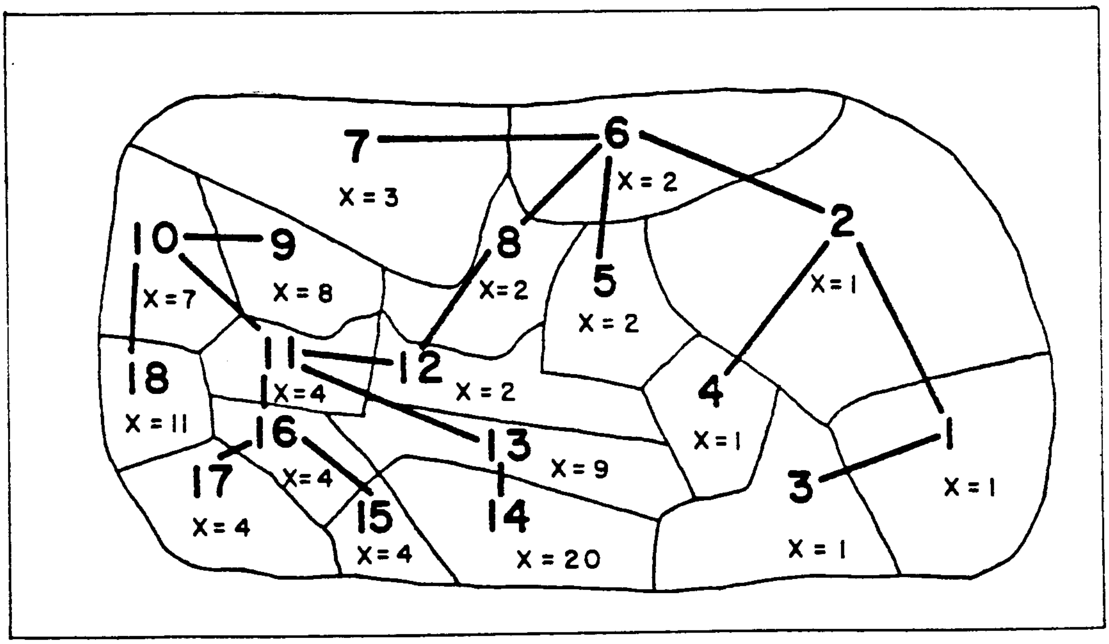
Resulting Regions (First Stage)

Figure 4.3



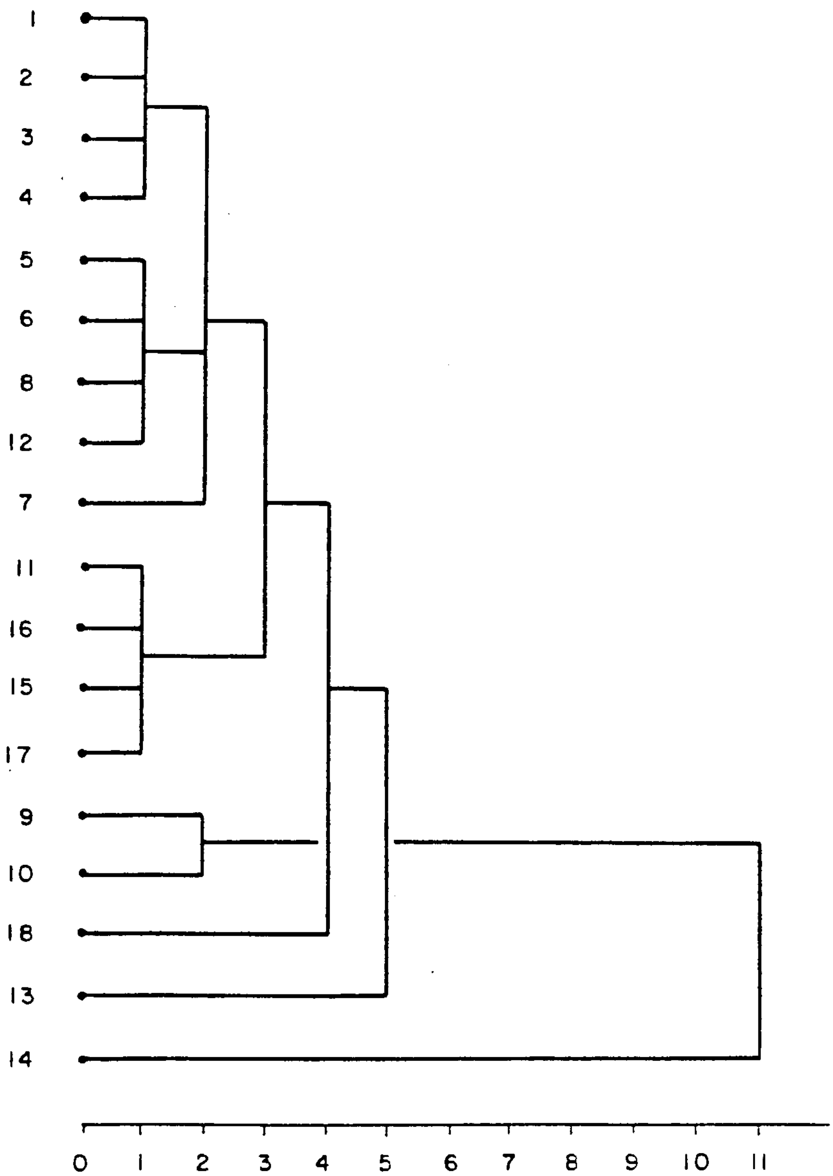
Subgraphs and Minimal Spanning Trees

Figure 4.4



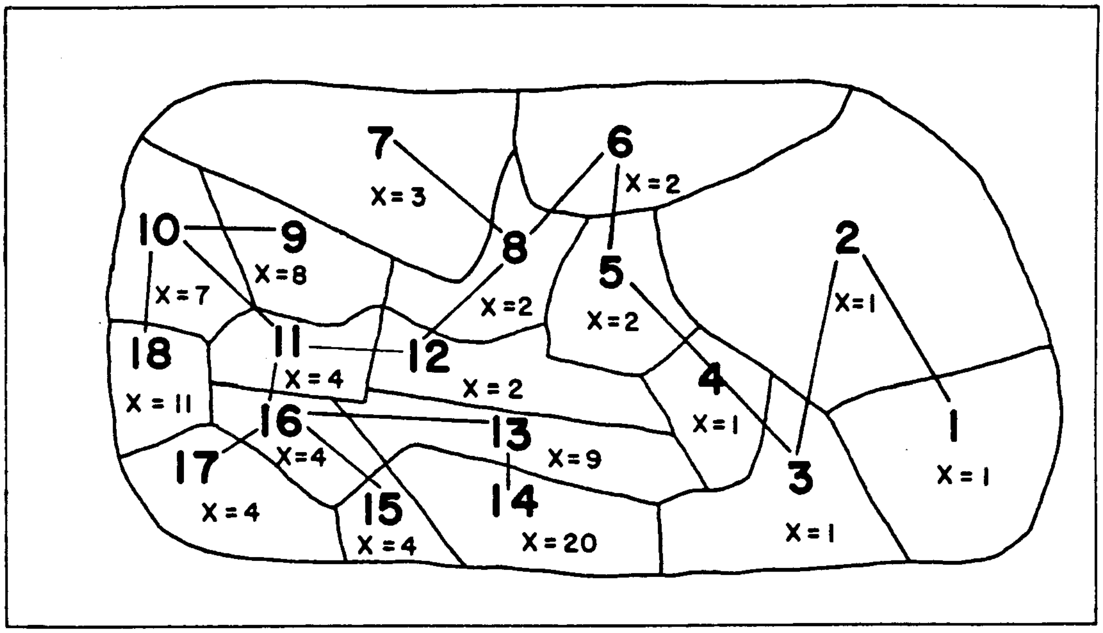
Minimal Spanning Tree (Second Stage)

Figure 4.5



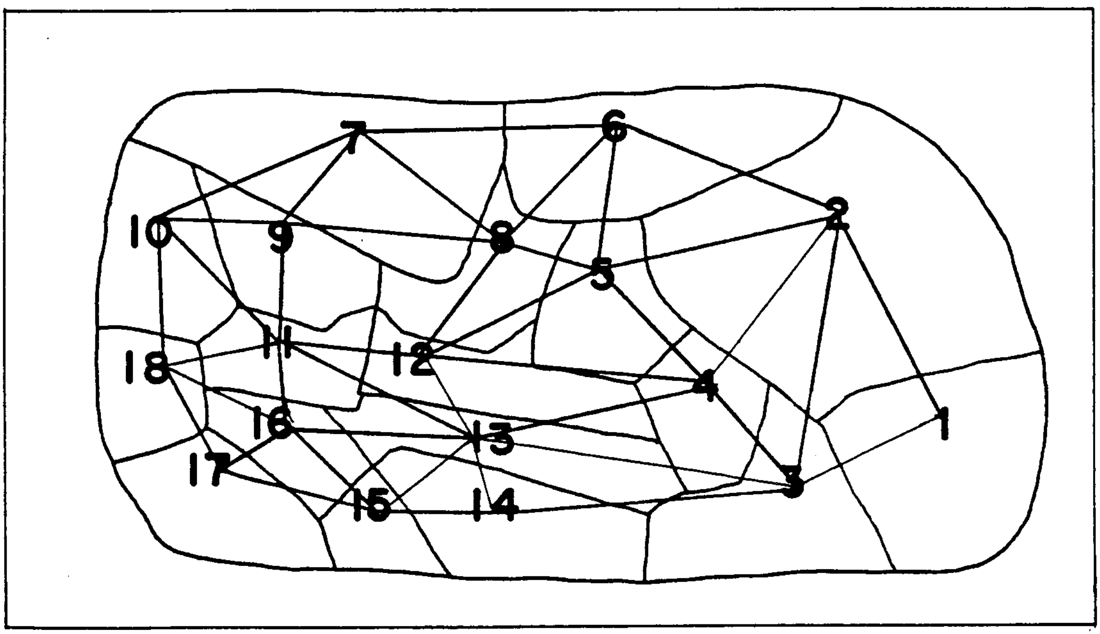
Dendogram

Figure 4.6



Minimal Spanning Tree. Single Linkage Method

Figure 4.7



First Order Neighbors

Figure 4.8

figure 4.7.

As can be appreciated by comparing graph G' (figure 4.8) to the graphs formed after the first stage of the topological algorithm (figure 4.4), each one of the latter is a subgraph of G'. Analogously, the minimal spanning tree (figure 4.5) is also a subgraph of G'. Finally, it should be noted that the minimal spanning trees generated by the two algorithms (topological and single linkage) are not necessarily the same.

4.3.3 Heterogeneous Regions

Like homogeneity, heterogeneity may also be used as a constraint in the definition of regions. In some instances researchers seek to identify groups of elements that are heterogeneous and spatially clustered. In these cases measures of dissimilarity and the heterogeneity index can be used in the design of algorithms, just as similarity and the homogeneity index were used in the cases mentioned previously.

4.3.4 Fuzzy Regions

In section 4.3.1 a hierarchical algorithm incorporating the concept of heterogeneity index was proposed. The index was used as an indicator of the order in which the units were to be merged. There are other ways in which the index can be used in the design of algorithms. In this section an alternative hierarchical algorithm which uses the notion of fuzzy sets is proposed.

The heterogeneity index associated with the neighborhood of unit "a" is interpreted as the degree of membership of the unit to a hypothetical region. When the decision to group two units is made, it becomes natural to ask what the degree of membership of the resulting group is to the hypothetical region. Intuitively, the two units that have the higher degree of membership to a region are the ones that should be clustered first. The concept of fuzzy set makes it possible to assign a value to the degree of membership of the set of two units to a region.

Given two contiguous units ai and aj, their corresponding neighborhoods N(ai) and N(aj) and associated heterogeneity indices Hai and Haj, the degree of membership of the set {ai,aj} to a hypothetical region R can be defined as follows:

$$H(ai,aj) = \min \{ Hai, Haj \} \qquad (4.18)$$

The above definition can be extended to the case where the units are entities as defined in the preceding algorithm, since both the heterogeneity index and the concept of neighborhood have been defined for regions (see Def.6, section

4.3.1).

The terms and assumptions under which the proposed algorithm is described are exactly the same as the ones presented for the preceding algorithm except for an additional matrix M1 defined as follows:

$$mij = H(ai,aj)$$

where ai and aj are either regions or elementary units as defined in section 4.2.1.

Step 1. Find the two contiguous entities ai and aj with the maximum value of H. That is, find the maximum value in M1;

$$H(ai,aj) = \max \{ H(am,an) : \text{for all pairs of contiguous units} \}$$

Step 2. Group entities ai and aj and name it ai (i<j).

Step 3. Update the ith row and column of matrix M1, and delete the jth row and column of matrix M1.

Step 4. If there are elementary units left in M1, then start again with step 1. Otherwise the remaining elements of M1 represent the resulting regions.

This procedure,like the preceding algorithm, can be repeated until all the units are clustered in one region.

Given the resulting regions R1,...Rk, the heterogeneity index associated to an element $ai \in Ri$ can be interpreted in fuzzy set terms as the degree of membership of "ai" to the interior of Ri. Its complement (1-Hai) can be interpreted as the degree of membership of "ai" to the border of Ri. Therefore, it can be said that the resulting groups are fuzzy regions.

As in many other areas of knowledge, in regionalization studies, regions are defined so that for every element of the universe of study it is clearly distinguishable whether or not it belongs to the region. There are however, certain geographical problems that can benefit from a "fuzzy" definition of the degree of membership of an element to a region. For example, consider an ecological study of an urban area such as Mexico City. Although there are no previous studies of this type for this particular area, an obvious characteristic of the city is its lack of clear-cut differences in residential areas as well as in land use. That is, it is common to find "border areas" between urban neighborhoods where middle and low income families or even high and low income families dwell on contiguous pieces of land. Similarly, it is common to find areas with various simultaneous uses. Such is the case of zones that are residential, providers of public services, educational, medical and industrial.

In a study area with these characteristics a method that incorporates the definition of fuzzy regions seems to be the most appropriate choice.

There are precedents for the application of the concept of fuzzy set in the design of classification algorithms (Dunn, 1974). In the cases Dunn mentions, the result is a set of regions where each of the elementary units has assigned a degree of membership to each region. On the other hand, the proposed algorithm differs from previous ones, since the degree of membership is assigned only to the interior and the border of a given region.

4.3.5 The Heterogeneity Surface

In the design of the previously discussed classification algorithms it was assumed that the units under study were clustered in homogeneous or heterogeneous groups. There are, however, some instances where researchers do not know the number of regions or have no previous information on the spatial patterns of the data. In these cases they can use the heterogeneity index to increase their knowledge.

To illustrate the manner in which the heterogeneity index can be applied in these situations, a heterogeneity "surface" is defined as follows:

$$F(x,y) = 1 - Ha \ ; \ F : RxR \longrightarrow [0,1]$$

where p is a point inside the areal unit "a" and Ha is its associated index. F is a step function; therefore its graph is not a continuous surface. However, for illustrative purposes it can be said that a "basin" in the graph is a set of contiguous areal units that have a value of F close to zero, and "ridges" are areal units with values close to one.

This graph can provide some significant information. For example, basins indicate the presence of the interior of a region, and ridges indicate boundary points. If it is assumed that the elements are clustered into regions, then the graph must be composed of basins surrounded by ridges. In such cases F can be used to estimate the number of resulting regions.

F may also be used in cases where the area under study is composed of both homogeneous and heterogeneous regions. The "highlands" in F indicate the presence of heterogeneous regions in the same manner as "lowlands" indicate the existence of homogeneous ones. The graph of F can therefore be used in the identification of such regions.

In other cases the lack of pattern in the heterogeneity surface might indicate an absence of regions in the study area.

Finally, the possibility of using the heterogeneity surface in multivariate cases is worth mentioning. When the objective of regionalization is to obtain homogeneous regions with respect to more than one characteristic, the first question that arises is whether it is possible to obtain homogeneous regions with respect to all the variables simultaneously.

For each one of the variables involved, let Fi be the function associated with the ith variable. A quick look at a pair of graphs can aid the analyst in deciding on the compatibility of the variables. If the two graphs show that basins and ridges coincide, then it may be concluded that the variables are in fact compatible. However, if for one variable there "tend" to be basins where there are ridges in the other, grouping the areas using both variables simultaneously is intuitively outruled.

4.4 A Comparative Example

In section 4.3.1 a modified agglomerative single linkage algorithm was presented. As mentioned before, the main difference between the usual single linkage method and the topological algorithm resides on the order in which the units are aggregated. Besides this obvious difference there are other ones that are derived from the use of a neighborhood approach. To exemplify these ideas as well as to test the performance of the topological algorithm the method was applied to a hypothetical case and the results were compared with those of a contiguity-constrained single linkage method.

4.4.1 A Hypothetical Case

The hypothetical case is defined on a regular area subdivided into 240 units as shown in figure 4.9. The classification of the units is based on three variables and the values of each variable were assigned under the assumption that the area is formed by homogeneous regions considering each of them separated and simultaneously. That is, if each one of the variables were to be represented on a map, homogeneous regions would be present. Moreover if the three above-mentioned maps were combined, the resulting map would also be formed by homogeneous regions. The values were assigned so that variable "A" was used as a basis. Therefore in the first

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135
136	137	138	139	140	141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160	161	162	163	164	165
166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195
196	197	198	199	200	201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220	221	222	223	224	225
226	227	228	229	230	231	232	233	234	235	236	237	238	239	240

Study Area
Figure 4.9

stage the values of variable "A" were defined as shown in figure 4.10. The values of variable "B" were defined so that the regions formed under it, were sub-regions of variable "A" (see figure 4.10). Finally, the values of variable "C" were assigned in such a way that its "border zones" did not necessarily coincide with those of variable "A" and "B" (see figure 4.10).

One of the central issues in the application of a topological algorithm is the definition of the neighborhood relationship. The advantage of using a regular structure is the ease of the implementation of the algorithm since the neighborhoods for each of the areal units is explicitly defined. In this case it was decided to define the neighborhood of each of the units as the set of bordering units together with the unit itself. The areas that are connected to a unit solely by a point were excluded to avoid regions linked through points.

Two regionalization algorithms were applied to this hypothetical problem, the topological algorithm as described in section 4.3.1 and a contiguity-constrained single linkage method that follows the general format as described in section 4.2.2. Both algorithms are hierarchical and use the same grouping criterion but differ in other aspects. The topological algorithm is based on the neighborhood approach which is reflected in the algorithm through the inclusion of the heterogeneity index concept, while the other algorithm follows the traditional approach were the area is assumed to be formed by units rather than by geo-subspaces or neighborhoods.

The Topological Algorithm

The first step in the application of the algorithm to the multivariate hypothetical case is the calculation of the standardized heterogeneity index using equations 3.1 and 3.2. In this case, the heterogeneity index can be interpreted as the degree of membership of a unit to the interior of a region. The closer the value to one, the higher the degree of membership to the region. The multivariate heterogeneity index was calculated according to equation 3.7 as shown in figure 4.11. Again, the closer the value of the index to three, the higher the degree of membership of a unit to a region formed according to the three variables.

The heterogeneity index plays a fundamental role at this stage of the analysis in the definition of the "grouping pattern" . Assume for example, that one of the variables of interest is the column-wise position of each of the units. That is, all the units that are positioned in the ith column have assigned a value of "i". The value of the heterogeneity index for an interior unit is a constant. That is, the degree of

33 33 4	32 32 4	33 33 4	33 33 4	33 33 3	33 33 4	33 1 4	33 1 3	33 2 5	33 1 2	33 1 1	33 2 3	33 0 3	33 1 3	33 3 3
33 33 4	33 33 3	33 33 4	33 33 3	33 33 3	33 33 4	33 2 3	33 1 2	33 3 1	33 7 4	33 8 1	33 3 3	33 1 3	33 1 3	33 2 3
33 33 3	33 33 3	33 33 3	33 33 3	33 33 4	33 33 5	33 1 1	33 3 3	33 2 4	33 8 1	33 7 5	33 1 3	33 2 4	33 2 3	33 2 3
33 33 4	33 33 4	33 33 3	33 33 3	33 33 3	33 33 4	33 1 4	33 2 4	33 3 4	33 0 4	33 1 4	33 0 4	33 1 3	33 1 3	34 1 3
33 33 32	33 33 32	33 33 32	33 33 32	33 33 33	33 33 34	33 0 31	33 1 33	33 2 28	33 0 28	33 2 29	33 1 30	33 2 22	33 1 23	34 1 22
85 85 31	85 85 30	86 86 29	88 88 31	87 87 29	89 89 34	90 90 32	90 90 34	90 90 29	90 90 32	15 15 31	19 19 33	18 18 17	16 16 15	19 19 16
86 86 33	86 86 34	89 89 33	90 90 32	88 88 30	90 90 33	90 90 30	91 91 33	91 91 33	90 90 33	16 16 29	19 19 29	19 19 15	19 19 17	19 19 16
86 86 34	88 88 35	88 88 33	87 87 32	89 89 31	91 91 32	91 91 31	92 92 33	91 91 32	91 91 31	18 18 29	19 19 28	19 19 28	19 19 27	18 18 16
87 87 15	88 88 17	87 87 18	88 88 19	89 89 19	91 91 21	92 93 20	93 93 21	91 91 20	91 91 24	18 18 25	19 19 25	19 19 26	19 19 26	17 17 17
87 75 17	87 73 17	87 74 18	89 89 19	90 90 19	90 90 19	93 93 20	94 94 22	93 93 21	92 92 24	19 19 26	18 18 25	17 17 25	17 17 25	17 17 16
88 74 19	86 73 20	89 75 19	88 88 20	89 89 20	91 91 21	94 94 20	93 132 22	95 132 21	93 93 25	93 93 26	96 96 25	17 17 23	18 18 28	19 19 17
88 73 19	87 74 19	88 74 18	90 90 20	90 90 20	93 93 20	95 95 20	92 127 21	92 120 21	96 96 25	97 97 23	98 98 24	19 10 27	18 11 29	17 10 17
87 70 19	87 72 17	89 75 19	91 91 21	91 91 22	96 96 20	96 96 20	94 94 21	95 95 20	97 97 24	95 95 22	93 93 22	20 13 21	17 14 23	19 14 17
88 71 18	88 72 16	88 75 19	90 90 20	93 93 20	98 98 21	96 96 20	96 96 20	97 97 21	97 97 28	96 96 29	99 99 26	18 10 25	19 11 24	17 13 18
88 70 18	87 71 18	88 75 17	92 92 21	85 85 21	99 99 20	93 93 20	95 95 20	98 98 21	99 99 26	97 97 25	99 99 21	18 14 22	18 12 24	19 11 17
88 72 18	88 72 18	87 75 17	96 96 21	83 83 21	87 87 21	88 88 21	99 99 20	98 98 20	98 98 27	99 99 24	98 98 23	20 13 23	17 12 25	20 10 17

Hypothetical Case. Variables A, B and C.

Figure 4.10

membership of all these units is exactly the same, so that there is no grouping pattern. Since the algorithm is designed to be applied to problems were a grouping pattern exists, it is not advisable to include in the classification a variable such as the one previously described.

An additional criterion was added to the algorithm to solve cases were two or more neighbors satisfy the grouping criterion. If two or more neighbors are at the same distance from a unit, then the one selected to be grouped is the one that satisfies the following conditions: a) it already belongs to a region and b) is the first clock-wise neighbor.

First Stage.-

Under the assumptions described above, the algorithm was applied obtaining as a result the regions shown in figure 4.12. It should be remembered that the number of resulting regions in this type of algorithm is part of the results of the procedure. That is, contrary to a usual hierarchical method were the analyst has to decide on the number of resulting regions, in this method the number of regions is determine through the heterogeneity index. In this application, the number of resulting regions from the first stage is 69.

Since the heterogeneity index can be interpreted as the degree

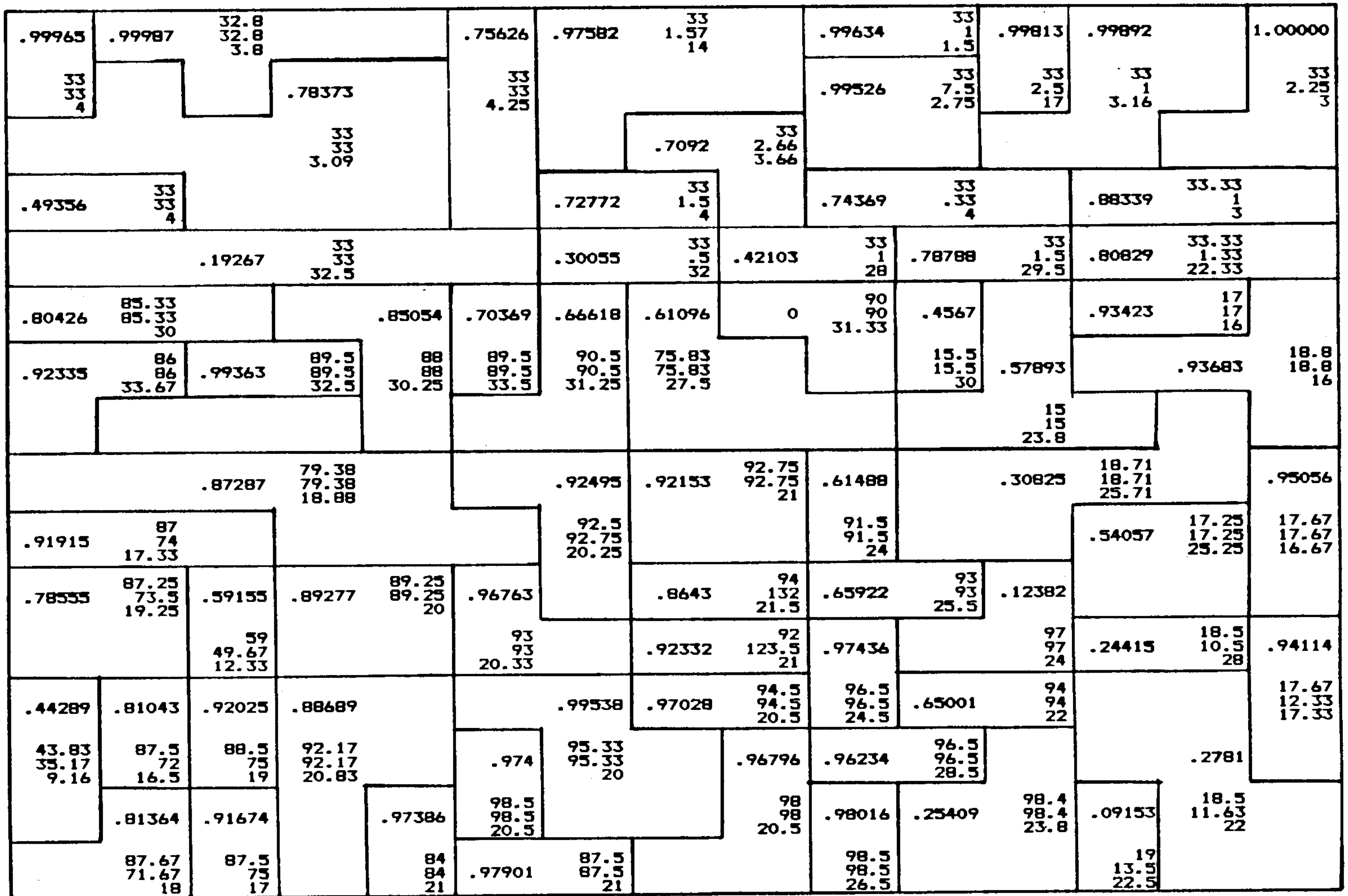
2.99969	2.99811	2.99979	2.99745	2.99745	2.89925	2.89788	2.99226	2.96277	2.97857	2.98877	2.99432	2.99942	2.99951	2.99927
2.99745	2.99698	2.99713	2.99809	2.99809	2.92712	2.92308	2.99552	2.96496	2.96645	2.96676	2.99370	2.99861	2.99985	2.99981
2.99745	2.99904	2.99904	2.99904	2.99618	2.90724	2.89350	2.99258	2.97901	2.95149	2.95926	2.99121	2.99596	2.99890	2.99979
2.00000	2.24904	2.19554	2.19650	2.13822	2.06361	2.22015	2.19525	2.44881	2.43569	2.39907	2.35194	2.65297	2.61768	2.53981
1.44510	1.83096	1.75556	1.72901	1.67413	1.56857	1.35714	1.32716	1.59684	1.57936	2.35812	2.24373	2.53200	2.51481	2.43710
2.43725	2.56250	2.53685	2.52348	2.50633	2.48775	2.13393	2.12977	2.12356	1.25496	2.08064	2.68560	2.68607	2.88705	2.89523
2.99215	2.97969	2.97972	2.99150	2.98413	2.97997	2.97772	2.98998	2.98345	2.13682	2.13592	2.79511	2.64348	2.89447	2.99852
2.53655	2.68348	2.77979	2.83432	2.85817	2.88076	2.87798	2.85703	2.85940	2.12424	2.15898	2.98934	2.83376	2.78679	2.84397
2.51554	2.66889	2.77035	2.83696	2.85768	2.87465	2.88158	2.85855	2.84399	2.11660	2.16231	2.99014	2.99365	2.91947	2.89324
2.97523	2.97353	2.96763	2.98091	2.99858	2.98971	2.99345	2.88852	2.87742	2.16598	1.33096	2.06219	2.99445	2.91229	2.89345
2.99280	2.98688	2.98221	2.98477	2.99701	2.99054	2.88824	2.78146	2.74992	2.87012	2.14283	1.09532	1.98858	2.84692	2.83526
2.99872	2.99359	2.97395	2.97548	2.99449	2.99504	2.92278	2.83761	2.87944	2.93498	2.98228	1.91099	1.86305	2.81863	2.80607
2.99205	2.98594	2.97121	2.97403	2.98296	2.98937	2.99828	2.91608	2.93420	2.96326	2.94641	2.07171	2.04480	2.92359	2.94992
2.99323	2.98184	2.96917	2.97826	2.97857	2.99095	2.99704	2.99717	2.95035	2.93235	2.92650	1.83545	1.86301	2.96179	2.94942
2.99931	2.99351	2.95626	2.95078	2.94992	2.93911	2.98766	2.99443	2.97251	2.96884	2.96552	1.89710	1.92563	2.94773	2.93507
2.99942	2.99754	2.92566	2.89006	2.96121	2.96569	2.96729	2.97041	2.93610	2.92443	2.98476	1.63386	1.63267	2.90956	2.87536

Multivariate Heterogeneity Index
Figure 4.11

of membership of a unit to a region, the result of the classification procedure is not restricted to the definition of each of the regions but, allows the analyst to gain information on the "interiority" of a unit. In this sense, the resulting regions are fuzzy. For example, both units 19 and 48 belong to the same region. However, according to their heterogeneity indices, unit 19 has a higher degree of membership to the region than unit 48. To ease the reading of this measure, the multivariate heterogeneity index was standardized as shown in figure 4.13. In it, the degree of membership of each of the units to the defined regions is clearly appreciated. The closer the value to one, the higher the degree of membership i.e. the more interior to the region is the unit. Analogously, the closer the value of the unit to zero, the lower the degree of membership i.e. the unit is characterized more as a "border" element. It should be noted that "border" units are not necessarily always in the actual border of a region.

Second Stage.-

It is possible to iterate this type of algorithm in order to obtain a smaller number of regions, as described in section 4.3.1. The 69 resulting regions from the first stage, were re-named, values for the three variables were assigned to the new elementary units and a neighborhood relationship was established. This task can be accomplished in several ways.



Topological Algorithm (First Stage)
Variables A, B and C; Standardized Heteregeneity Index
Figure 4.12

In this case, the chosen procedure was to assign to the new units the mean value of the elementary units that belonged to it. This procedure was repeated for the three variables as shown in figure 4.12.

The topological algorithm was applied to the 69 new units and as a result 21 regions were formed as shown in figure 4.14. Again, in this case, the heterogeneity index associated to each elementary unit can be interpreted as the degree of membership to the region. For example, the units formed with original units {2, 3, 4, 5, 18} and {17, 19, 20, 31, 32, 33, 34, 35, 48, 49, 58} belong to the same region however, it can be stated that the first unit is more an interior element of the region than the second one as can be appreciated from the heterogeneity index values as shown in figure 4.12.

There are other manners in which the values of the variables and the neighborhood relationship can be defined. For example, instead of using the mean value, the minimum, or the maximum or a weighted mean could have been considered and even the original values of the elementary units could have been preserved. In this last case, the neighbors of a region could be defined as the set of original elementary units that have a border in common with it, and the distance between the regions could be calculated considering the elementary units in the border of each region.

Standardized Multivariate Heterogeneity Index

.999915	.999084	.999969	.998739	.998739	.947175	.946456	.996012	.980526	.988825	.994182	.997095	.999770	.999821	.999694
.998739	.998491	.998572	.999073	.999073	.961808	.959686	.997726	.981677	.982459	.982621	.996769	.999345	1	.999974
.998739	.999575	.999575	.999575	.998070	.951371	.944157	.996182	.989053	.974603	.978684	.995460	.997955	.999498	.999969
.475013	.605777	.577684	.578186	.547585	.508411	.590606	.577531	.710665	.703778	.684550	.659803	.817862	.799334	.758450
.183656	.386256	.346668	.332724	.303909	.248483	.137470	.121729	.263331	.254150	.663050	.602986	.754347	.745319	.704515
.704599	.770361	.756893	.749874	.740866	.731114	.545334	.543152	.539890	.083820	.517356	.834995	.835245	.940771	.945065
.995956	.989414	.989430	.995612	.991744	.989559	.988377	.994818	.991387	.546850	.546382	.892497	.812879	.944669	.999300
.756738	.833884	.884454	.913082	.925606	.937467	.936009	.925007	.926253	.540249	.558487	.994478	.912789	.888127	.918151
.745702	.826221	.879493	.914472	.925347	.934262	.937897	.925806	.918162	.536236	.560234	.994898	.996743	.957793	.944021
.987069	.986176	.983082	.990053	.999332	.994673	.996637	.941540	.935716	.562160	.123724	.507668	.997164	.954022	.944129
.996295	.993189	.990737	.992081	.998508	.995110	.941395	.885329	.868771	.931881	.550008	0	.469018	.919702	.913577
.999402	.996709	.986397	.987200	.997181	.997474	.959533	.914812	.936773	.965936	.990774	.428277	.403106	.904848	.898250
.995905	.992692	.984960	.986440	.991131	.994497	.999171	.956013	.965525	.980788	.971937	.512666	.498537	.959956	.973780
.996522	.990541	.983889	.988662	.988824	.995322	.998524	.998588	.974007	.964557	.961483	.388616	.403084	.980014	.973519
.999714	.996671	.977108	.974235	.973779	.968105	.993595	.997150	.985643	.983717	.981970	.420983	.435964	.972630	.965986
.999770	.998784	.961042	.942350	.979712	.982060	.982899	.984538	.966525	.960398	.992072	.282764	.282140	.952591	.934631

Figure 4.13

Topological Algorithm (Second Stage). 21 Resulting Regions

33 33 4	32 32 4	33 33 4	33 33 4	33 33 3	33 33 4	33 1 4	33 1 3	33 2 5	33 1 2	33 1 1	33 2 3	33 0 3	33 1 3	33 3 3
33 33 4	33 33 3	33 33 4	33 33 3	33 33 3	33 33 4	33 2 3	33 1 2	33 3 1	33 7 4	33 8 1	33 3 3	33 1 3	33 1 3	33 2 3
33 33 3	33 33 3	33 33 3	33 33 3	33 33 4	33 33 5	33 1 1	33 3 3	33 2 4	33 8 1	33 7 5	33 1 3	33 2 4	33 2 3	33 2 3
33 33 4	33 33 4	33 33 3	33 33 3	33 33 3	33 33 4	33 1 4	33 2 4	33 3 4	33 0 4	33 1 4	33 0 4	33 1 3	33 1 3	34 1 3
33 33 32	33 33 32	33 33 32	33 33 32	33 33 33	33 33 34	33 0 31	33 1 33	33 2 28	33 0 28	33 2 29	33 1 30	33 2 22	33 1 23	34 1 22
85 85 31	85 85 30	86 86 29	88 88 31	87 87 29	89 89 34	90 90 32	90 90 34	90 90 29	90 90 32	15 15 31	19 19 33	18 18 17	16 16 15	19 19 16
86 86 33	86 86 34	89 89 33	90 90 32	88 88 30	90 90 33	90 90 30	91 91 33	91 91 33	90 90 33	16 16 29	19 19 29	19 19 15	19 19 17	19 19 16
86 86 34	88 88 35	88 88 33	87 87 32	89 89 31	91 91 32	91 91 31	92 92 33	91 91 32	91 91 31	18 18 29	19 19 28	19 19 28	19 19 27	18 18 16
87 87 15	88 88 17	87 87 18	88 88 19	89 89 19	91 91 21	92 93 20	93 93 21	91 91 20	91 91 24	18 18 25	19 19 25	19 19 26	19 19 26	17 17 17
87 75 17	87 73 17	87 74 18	89 89 19	90 90 19	90 90 19	93 93 20	94 94 22	93 93 21	92 92 24	19 19 26	18 18 25	17 17 25	17 17 25	17 17 16
88 74 19	86 73 20	89 75 19	88 88 20	89 89 20	91 91 21	94 94 20	93 132 22	95 132 21	93 93 25	93 93 26	96 96 25	17 17 23	18 18 28	19 19 17
88 73 19	87 74 19	88 74 18	90 90 20	90 90 20	93 93 20	95 95 20	92 127 21	92 120 21	96 96 25	97 97 23	98 98 24	19 10 27	18 11 29	17 10 17
87 70 19	87 72 17	89 75 19	91 91 21	91 91 22	96 96 20	96 96 20	94 94 21	95 95 20	97 97 24	95 95 22	93 93 22	20 13 21	17 14 23	19 14 17
88 71 18	88 72 16	88 75 19	90 90 20	93 93 20	98 98 21	96 96 20	96 96 20	97 97 21	97 97 28	96 96 29	99 99 26	18 10 25	19 11 24	17 13 18
88 70 18	87 71 18	88 75 17	92 92 21	85 85 21	99 99 20	93 93 20	95 95 20	98 98 21	99 99 26	97 97 25	99 99 21	18 14 22	18 12 24	19 11 17
88 72 18	88 72 18	87 75 17	96 96 21	83 83 21	87 87 21	88 88 21	99 99 20	98 98 20	98 98 27	99 99 24	98 98 23	20 13 23	17 12 25	20 10 17

Figure 4.14

The Single Linkage Method

To illustrate the difference between the results obtained by using the topological algorithm with other existing methods, a contiguity-constrained single linkage procedure was applied to the same problem. Since the main purpose was to evaluate the differences due to the inclusion of the heterogeneity index, the similarity measure used was also the euclidean distance and the contiguity relations and grouping criterion were preserved.

The single linkage method is a hierarchical procedure where given "n" elementary units there are "n-i" regions in the ith step. To have results comparable to those obtained via the topological algorithm, the procedure was stopped at the 171th and 219 steps. The 69 and 21 resulting regions are shown in figure 4.15 and 4.16 respectively.

A Comparison

Comparison of regionalization algorithms can be undertaken focussing on different aspects and at various levels. Murtagh (1985) for example, compares contiguity-constrained algorithms under two main aspects, their computational performance and the differences derived from the inclusion of a contiguity

33 33 4	32 32 4	33 33 4	33 33 4	33 33 3	33 33 4	33 1 4	33 1 3	33 2 5	33 1 2	33 1 1	33 2 3	33 0 3	33 1 3	33 3 3
33 33 4	33 33 3	33 33 4	33 33 3	33 33 3	33 33 4	33 2 3	33 1 2	33 3 1	33 7 4	33 8 1	33 3 3	33 1 3	33 1 3	33 2 3
33 33 3	33 33 3	33 33 3	33 33 3	33 33 4	33 33 5	33 1 1	33 3 3	33 2 4	33 8 1	33 7 5	33 1 3	33 2 4	33 2 3	33 2 3
33 33 4	33 33 4	33 33 3	33 33 3	33 33 3	33 33 4	33 1 4	33 2 4	33 3 4	33 0 4	33 1 4	33 0 4	33 1 3	33 1 3	34 1 3
33 33 32	33 33 32	33 33 32	33 33 32	33 33 33	33 33 34	33 0 31	33 1 33	33 2 28	33 0 28	33 2 29	33 1 30	33 2 22	33 1 23	34 1 22
85 85 31	85 85 30	86 86 29	88 88 31	87 87 29	89 89 34	90 90 32	90 90 34	90 90 29	90 90 32	15 15 31	19 19 33	18 18 17	16 16 15	19 19 16
86 86 33	86 86 34	89 89 33	90 90 32	88 88 30	90 90 33	90 90 30	91 91 33	91 91 33	90 90 33	16 16 29	19 19 29	19 19 15	19 19 17	19 19 16
86 86 34	88 88 35	88 88 33	87 87 32	89 89 31	91 91 32	91 91 31	92 92 33	91 91 32	91 91 31	18 18 29	19 19 28	19 19 28	19 19 27	18 18 16
87 87 15	88 88 17	87 87 18	88 88 19	89 89 19	91 91 21	92 93 20	93 93 21	91 91 20	91 91 24	18 18 25	19 19 25	19 19 26	19 19 26	17 17 17
87 75 17	87 73 17	87 74 18	89 89 19	90 90 19	90 90 19	93 93 20	94 94 22	93 93 21	92 92 24	19 19 26	18 18 25	17 17 25	17 17 25	17 17 16
88 74 19	86 73 20	89 75 19	88 88 20	89 89 20	91 91 21	94 94 20	93 132 22	95 132 21	93 93 25	93 93 26	96 96 25	17 17 23	18 18 28	19 19 17
88 73 19	87 74 19	88 74 18	90 90 20	90 90 20	93 93 20	95 95 20	92 127 21	92 120 21	96 96 25	97 97 23	98 98 24	19 10 27	18 11 29	17 10 17
87 70 19	87 72 17	89 75 19	91 91 21	91 91 22	96 96 20	96 96 20	94 94 21	95 95 20	97 97 24	95 95 22	93 93 22	20 13 21	17 14 23	19 14 17
88 71 18	88 72 16	88 75 19	90 90 20	93 93 20	98 98 21	96 96 20	96 96 20	97 97 21	97 97 28	96 96 29	99 99 26	18 10 25	19 11 24	17 13 18
88 70 18	87 71 18	88 75 17	92 92 21	85 85 21	99 99 20	93 93 20	95 95 20	98 98 21	99 99 26	97 97 25	99 99 21	18 14 22	18 12 24	19 11 17
88 72 18	88 72 18	87 75 17	96 96 21	83 83 21	87 87 21	88 88 21	99 99 20	98 98 20	98 98 27	99 99 24	98 98 23	20 13 23	17 12 25	20 10 17

Single Linkage Method. 69 Resulting Regions
Figure 4.15

Single Linkage Method. 21 Resulting Regions

33 33 4	32 32 4	33 33 4	33 33 4	33 33 3	33 33 4	33 1 4	33 1 3	33 2 5	33 1 2	33 1 1	33 2 3	33 0 3	33 1 3	33 3 3
33 33 4	33 33 3	33 33 4	33 33 3	33 33 3	33 33 4	33 2 3	33 1 2	33 3 1	33 7 4	33 8 1	33 3 3	33 1 3	33 1 3	33 2 3
33 33 3	33 33 3	33 33 3	33 33 3	33 33 4	33 33 5	33 1 1	33 3 3	33 2 4	33 8 1	33 7 5	33 1 3	33 2 4	33 2 3	33 2 3
33 33 4	33 33 4	33 33 3	33 33 3	33 33 3	33 33 4	33 1 4	33 2 4	33 3 4	33 0 4	33 1 4	33 0 4	33 1 3	33 1 3	34 1 3
33 33 32	33 33 32	33 33 32	33 33 32	33 33 33	33 33 34	33 0 31	33 1 33	33 2 28	33 0 28	33 2 29	33 1 30	33 2 22	33 1 23	34 1 22
85 85 31	85 85 30	86 86 29	88 88 31	87 87 29	89 89 34	90 90 32	90 90 34	90 90 29	90 90 32	15 15 31	19 19 33	18 18 17	16 16 15	19 19 16
86 86 33	86 86 34	89 89 33	90 90 32	88 88 30	90 90 33	90 90 30	91 91 33	91 91 33	90 90 33	16 16 29	19 19 29	19 19 15	19 19 17	19 19 16
86 86 34	88 88 35	88 88 33	87 87 32	89 89 31	91 91 32	91 91 31	92 92 33	91 91 32	91 91 31	18 18 29	19 19 28	19 19 28	19 19 27	18 18 16
87 87 15	88 88 17	87 87 18	88 88 19	89 89 19	91 91 21	92 93 20	93 93 21	91 91 20	91 91 24	18 18 25	19 19 25	19 19 26	19 19 26	17 17 17
87 75 17	87 73 17	87 74 18	89 89 19	90 90 19	90 90 19	93 93 20	94 94 22	93 93 21	92 92 24	19 19 26	18 18 25	17 17 25	17 17 25	17 17 16
88 74 19	86 73 20	89 75 19	88 88 20	89 89 20	91 91 21	94 94 20	93 132 22	95 132 21	93 93 25	93 93 26	96 96 25	17 17 23	18 18 28	19 19 17
88 73 19	87 74 19	88 74 18	90 90 20	90 90 20	93 93 20	95 95 20	92 127 21	92 120 21	96 96 25	97 97 23	98 98 24	19 10 27	18 11 29	17 10 17
87 70 19	87 72 17	89 75 19	91 91 21	91 91 22	96 96 20	96 96 20	94 94 21	95 95 20	97 97 24	95 95 22	93 93 22	20 13 21	17 14 23	19 14 17
88 71 18	88 72 16	88 75 19	90 90 20	93 93 20	98 98 21	96 96 20	96 96 20	97 97 21	97 97 28	96 96 29	99 99 26	18 10 25	19 11 24	17 13 18
88 70 18	87 71 18	88 75 17	92 92 21	85 85 21	99 99 20	93 93 20	95 95 20	98 98 21	99 99 26	97 97 25	99 99 21	18 14 22	18 12 24	19 11 17
88 72 18	88 72 18	87 75 17	96 96 21	83 83 21	87 87 21	88 88 21	99 99 20	98 98 20	98 98 27	99 99 24	98 98 23	20 13 23	17 12 25	20 10 17

Figure 4.16

constraint. The purpose of the comparison between the topological algorithm and the single linkage one is to look into the differences derived from applying a neighborhood approach to the design of regionalization procedures.

Both algorithms are hierarchical and follow a single linkage grouping criterion, however the order in which the units are grouped is not necessarily the same, therefore the spatial structures portrayed in the resulting regions are not necessarily equal.

This fact points to a fundamental issue in the formalization of regionalization procedures. It was mentioned before that one of the advantages of using a mathematical framework in the classification problems is that once an algorithm is established, the solution becomes reproducible. However, it should also be considered that the design of the algorithm depends on the analyst's knowledge and understanding of the problem at hand. Therefore, the resulting regionalization depends on the assumptions made in the design of the algorithm and the different regionalizations obtained from different procedures can be explained under these considerations.

Comparison of the regionalizations obtained by both algorithms shows that different aspects of the spatial structure of the data emerges from the two procedures. While the topological algorithm regions have a tendency to be small and compact and there are no isolated elementary units, the single linkage method tends to produce larger regions as well as single-unit regions. On the other hand, while in the single linkage method the regions are well-defined, the resulting groups from the topological algorithm are "fuzzy regions".

Finally, it should be remembered that since both algorithms are hierarchical it is possible to continue the process until all the units are grouped into one single region. However, it was considered for this particular example, that the comparison of results at the two presented stages satisfied the purpose of the exercise and therefore no further stages were implemented.

4.4.2 A Topological Ward Algorithm

Two of the most commonly used algorithms in regionalization problems are the contiguity-constrained Single Linkage and Ward Methods. The Ward Method is a hierarchical procedure where the grouping criterion as described in section 4.1.3 is defined so that the increase in the within group variance as defined in equation 4.6 is minimized. Similarly, as the neighborhood approach was applied to a single linkage method, it is possible to include the heterogeneity index concept in the design of a Ward-type algorithm.

The algorithm is similar to the one proposed in section 4.3.1 except that in this case step 2 has to be modified as follows:

Step 2. Search for the neighbor of ai such that the increment of the error sum of squares as defined in equation 4.7, is the smallest. Call it aj.

Again the main difference between the usual contiguity-constrained Ward method and the topological one, is the order in which the units are grouped.

4.4.3 Conclusions

There are many classification algorithms that can be applied for regionalization purposes and the selection of the appropriate procedure depends on the knowledge and understanding the analyst has on the problem at hand. This knowledge is reflected in every decision made regarding the different elements that are involved in the design of the algorithm. For example, in the case of the two algorithms that were compared, the single linkage and the topological, the introduction of the heterogeneity index had a significant effect in the results. The conclusions that can be drawn from the exercise are: the algorithms were designed to discover different aspects of the spatial structure and the single linkage method is a "sensitive model" to changes in the grouping order. Besides the abovementioned, it should be added that the different manners in which the two algorithms group the units clearly points to the importance of an adequate selection of parameters. Although the most evident border lines are identified by both procedures, the final geometric patterns are clearly different. If the analyst is interested in avoiding single-area units and regions dissimilar in size, then a topological algorithm would be an appropriate approach in a similar problem as the one presented here. Moreover, if the analyst needs to measure the degree of interiority within a region, a topological algorithm would have to be applied since it is the only existing procedure that provides this type of information.

To better understand the issues involved in the use of a classification scheme with regionalization purposes, it should be remembered that the design of a regionalization algorithm can be viewed as a modeling process. In the case of the neighborhood approach the point of departure is the intuitive notion that certain aspects of the geographical landscape can be adequately represented through the topological concept of neighborhood. These notions are formalized through the heterogeneity indices and applied in the design of algorithms. In this case the form of the algorithms is intimately related with that of a more general model, where the main elements of study of the spatial structure are geo-subspaces. It can therefore be stated that the application of a topological algorithm is of interest where the Galton's component is an important factor in the analysis of the problem.

Chapter 5.

AN APPLICATION TO EDUCATIONAL PLANNING

5.1 Introduction

In general, the heterogeneity index may be interpreted as a measure of the local variation of the geographical landscape. In particular, the study of the heterogeneity of geo-subspaces may be used as an aid to solve planning problems. In this chapter an application of a neighborhood model in an educational planning environment is presented.

Background

This application is part of a major project of the Mexican government to provide planning agencies with cartographic products and technical support in all aspects related to the geographic information required for their activities.

The 25 million student education system was selected for two reasons. First it is one of the current Mexican administration's highest priorities. Second, geographic information has not yet been systematically applied to educational planning in Mexico.

One of the main concerns of Mexican educational planners is the location of present and future school services. In the past, the decisions made by the government regarding the location of schools have not included spatial criteria. However, a geographic information system for educational planning purposes is currently being developed by the Ministry of Education. It is expected that 1986 will be the first year when the spatial criteria is incorporated into the decision making process.

Some methods to solve school location problems have been developed by the World Bank, the International Institute for Educational Planning. However, spatial models and methods such as those presented in this thesis are believed to improve the solutions provided by previously used methods.

5.2 School Location Planning

Many different models and methods have been developed for educational planning purposes. Most of them have been applied to regional or national planning, but little emphasis has been given to spatial aspects. For example, models and methods for projecting school enrollments and manpower requirements are often presented at a national level (Davis, 1980), although maps and some spatial criteria have been included for local planning purposes.

School location planning and area planning are the names that several authors (Davis and Schefelbein, 1980, Gould, 1978) have given to the set of administrative policies, models and methods that are used "to plan the distribution, size and spacing of schools" (Gould, 1978, p.2).

According to Davis and Schefelbein (1980), the basic purposes of educational planning for areas are:

- To assess the outreach, or coverage, and distribution of educational services to population in areas within a nation state.
- To compare the coverage between and among the areas, usually on the basis of the percentages of the relevant population receiving service.
- To compare the coverage of the area with national norms, standards or plan targets.
- To inventory facilities and resources allocated to programs in the areas.
- To plan the provision of educational services so as to expand coverage, enhance equity in the coverage, and to improve the efficiency and effectiveness of educational services in the areas.

5.2.1 Models and Methods

Maps are the spatial models that are most commonly used in area planning. Usually, various indicators are mapped to study their spatial distribution.

Which indicators or variables are represented on a map depends on the depth of the analysis. Indicators and variables can be classified into three major groups: 1) basic indicators such as population, enrollments and school services; 2) efficiency and effectiveness indicators such as the enrollment ratios, percentage of enrollees who graduate; and 3) complementary variables such as topography, highways and trails, and potential usage of soil.

The first group of inventory indicators aids the planner in gaining knowledge of the spatial location of educational services. The second group allows the planner to make comparisons among areas. The third group of complementary variables provides information necessary to understand the spatial behavior of the previous two. Another group of indicators and spatial variables provides guidelines for the location of new school services. Examples are threshold population density and range measurements. Threshold population density represents the minimum total population necessary for establishing a school, and range is the maximum distance children are expected to travel to school (Gould, 1978).

5.2.2 Administrative Policies

In addition to the specific procedures developed for school location planning, strong emphasis has been given to the administrative policies that would lead to successful implementation of a plan.

Gould (1978) gives a detailed description of the role of both central authorities and local officials according to the World Bank's guidelines for school location.

Central authorities, through their national ministries, are expected to do the following: provide norms for the sizes and costs of schools and classrooms; establish construction standards; ensure that adequate data is compiled for area diagnosis; administer the allocation of resources among the various regions; and analyse spatial patterns of the services schools provide. Local officials are expected to apply the norms established by the ministries and to provide the data required by the central authorities.

School location planning has often been applied in Third World countries. UNESCO and the International Institute for

Educational Planning, for example, have undertaken studies of school location planning in Costa Rica (Hallak, 1975), Sri Lanka (Guruge and Ariyadasa, 1976) and Uganda (Gould, 1973).

5.3 Educational Planning in Mexico

5.3.1 Historical Background

In order to understand the relevance of school location planning in the overall context of Mexican educational planning, historical perspective is important. The promulgation of the 1917 Constitution and the Lopez Mateos Eleven-Year Plan are two events of this century considered by experts as crucial in the development of Mexican education .

When the 1910 Mexican Revolution ended in 1917, a new Constitution was promulgated. Article 3 stipulated that education be compulsory, secular and free for all Mexicans. More than 40 years later, during the administration of Adolfo Lopez Mateos (1958-64), an eleven-year plan was developed and partially carried out.

The main goal of the plan was to completely satisfy the demand for elementary education throughout the country. To achieve this there were several major programs. First, there was massive construction of classrooms. Between 1958 and 1964, 21,000 classrooms were built at the rate of "... one classroom every two hours..." (Solana et al. 1981). Second, a National Commission in charge of publishing free textbooks for all elementary school students was organized. In addition, important curricular changes were carried out, and Finally, special attention was given to programs for the in-service training of elementary school teachers (Solana et al., 1981).

Like Lopez Mateos virtually all post-Revolutionary Mexican administrations have dedicated considerable human and financial resources to elementary education. This certainly does not mean that other levels of education have been abandoned. However, the main targets have been the lower levels.

5.3.2 Planning Experiences

Before 1970, an educational planning agency did not exist within the Mexican government. It was not until the administration of Luis Echeverria (1970-76) that an organization for educational planning was formally established within the Ministry of Education (Secretaría de Educación Pública).

As would be expected, one of the main goals of the planning agency was to assure every school-age child access to elementary education.*

For this purpose various quantitative analyses were undertaken, and in some cases sophisticated mathematical models were used to predict, among other things, the flow of students through the lower levels of education.

In order to use quantitative techniques it was necessary to develop an information system. This system would contain reliable, up-to-date data on the number of students and teachers at the various educational levels as well as data describing the schools' physical resources.

During the following administration of President José López Portillo (1976-82), the general tendencies in educational planning remained the same. It has only been in the last three years that planners have started looking more closely into the quality of education. This is probably a natural consequence of the considerable progress the country has made in quantitative terms. (see Figure 5.1).

As Figure 5.1 shows, the number of schools has increased

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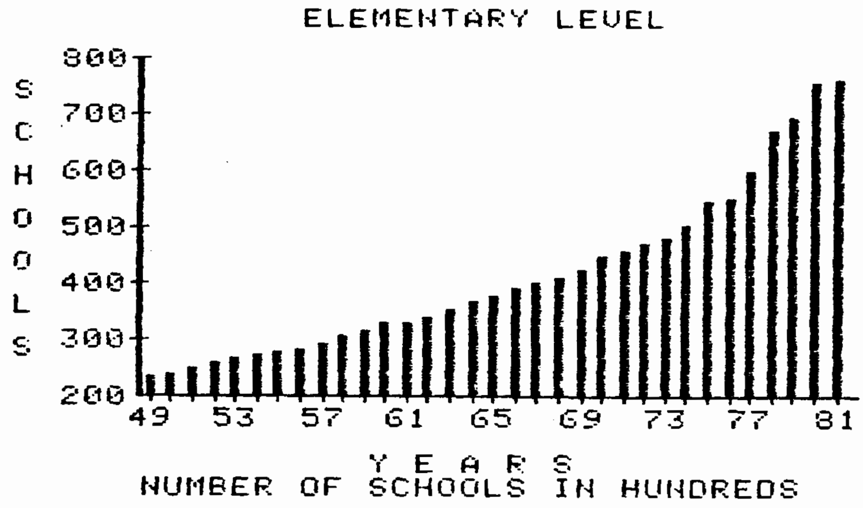
* The Mexican educational system comprises various levels. Among them are elementary, secondary and preparatory levels. Children are expected to enter elementary school at the age of 6 or 7 and remain there for six years. The following two levels are secondary and preparatory with a duration of three years each. There are different types of secondary and preparatory schools (technical, general, etc).


ELEMENTARY LEVEL
SCHOOLS
800
700
600
500
400
300
200
49
53
57
61
65
69
73
77
81
YEARS
NUMBER OF SCHOOLS IN HUNDREDS

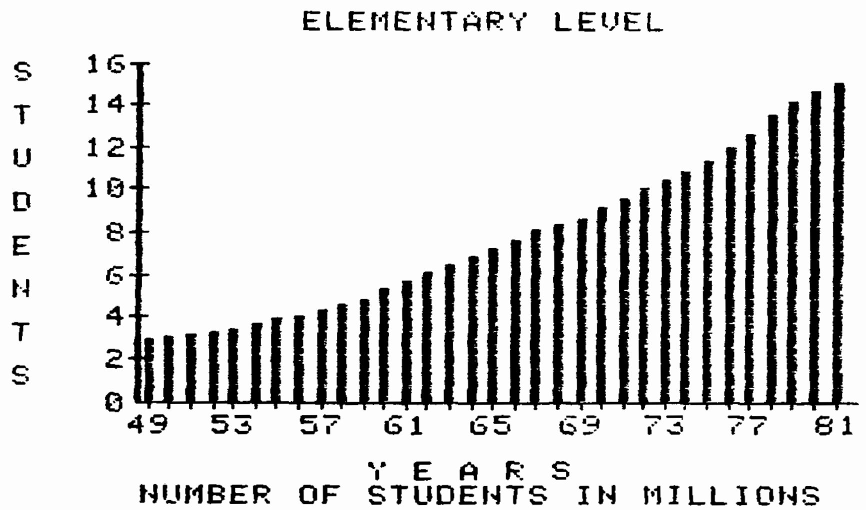

ELEMENTARY LEVEL
STUDENTS
16
14
12
10
8
6
4
2
0
49
53
57
61
65
69
73
77
81
YEARS
NUMBER OF STUDENTS IN MILLIONS

Figure 5.1

throughout the century. The early demand for elementary schools was so great that establishing one almost anywhere was beneficial to the local community and to the country. Now however, the precise location of a school has become particularly important. Today's density of schools obliges the planner to make a detailed and accurate study of the geographic distribution of resources before making decisions.

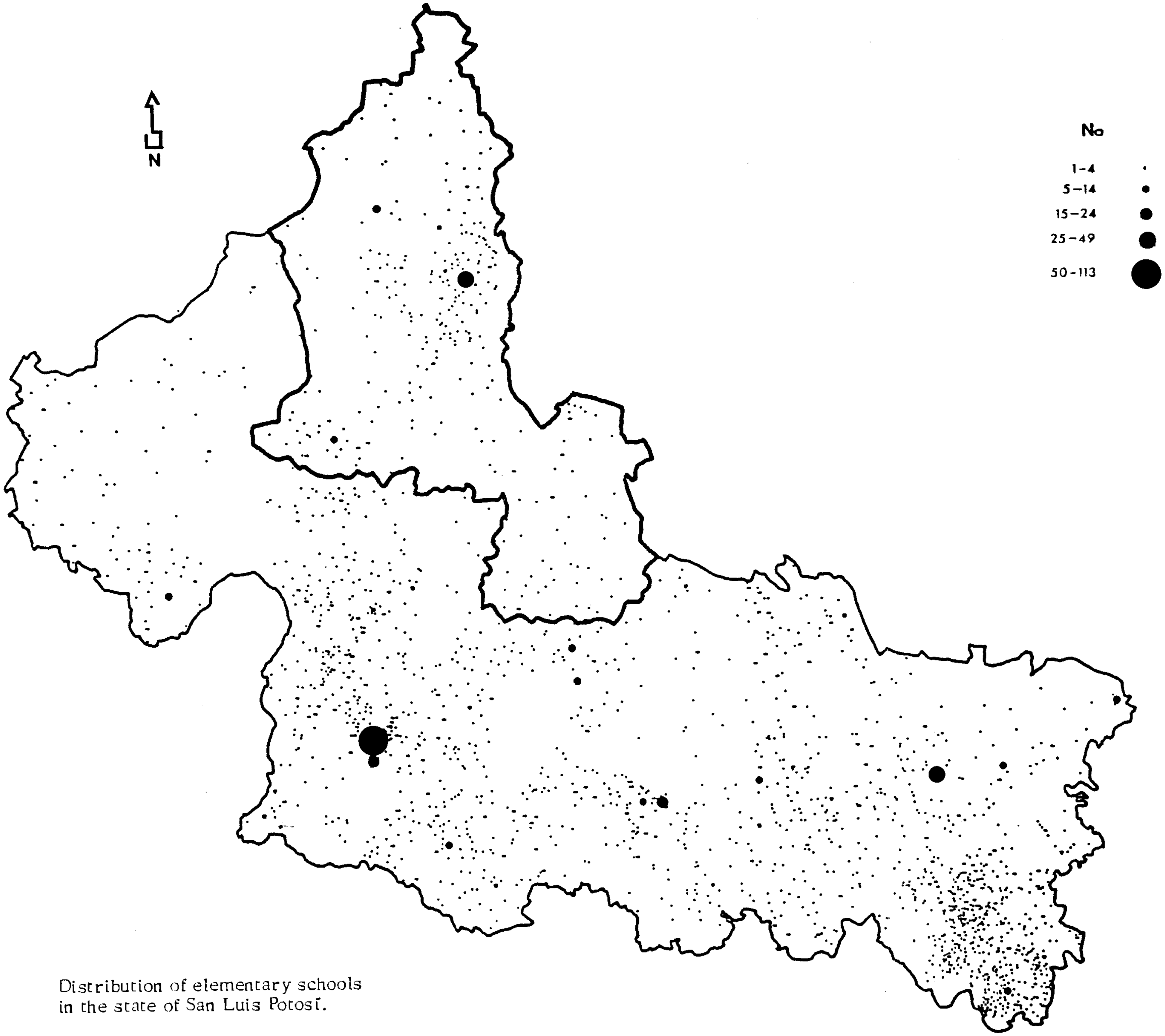
Another factor which has become important is the need to ensure the coordinated growth of the educational levels. There is no point in building a secondary school where there is an insufficient flow of students from the elementary schools or if migration affects the school-age population significantly. Geographic information is essential to permit rational planning of these educational issues.

The use of geographic information in educational planning in Mexico has not been systematic, but there is increasing awareness of the need to support educational planning with spatial analysis methods.

5.4 A Case Study of the State of San Luis Potosi

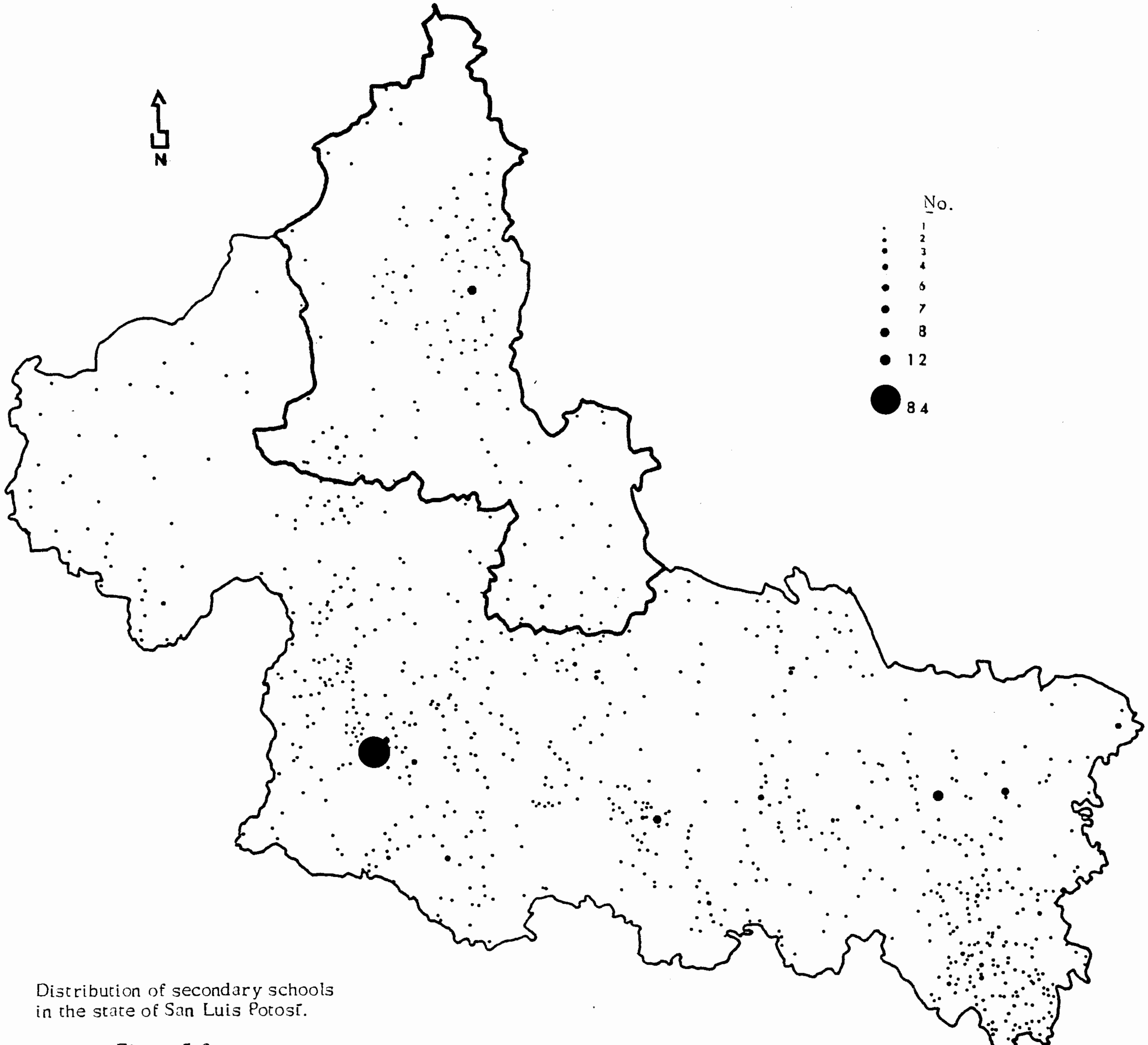
As mentioned in the previous section, the Mexican government's investment in education has been mainly directed to the elementary level. However, the government has dedicated considerable attention to the development of the secondary system during the last six years, especially in the state of San Luis Potosi. As can be observed in figures 5.2 and 5.3, the distribution of both elementary and secondary schools throughout the area is reasonably uniform; that is, the growth of both systems has apparently been coordinated. This pattern is not maintained at the next level of education. The number of preparatory schools capable of receiving the flow of secondary graduates is evidently insufficient, as can be seen in figure 5.4. The Central Planning Office has become aware of this problem and has decided to alleviate it through the allocation of additional resources.

In this section a neighborhood model is proposed as an aid in the selection of sites for the location of preparatory schools and in school location planning in general. Although the technique is presented within a specific context, an analysis of the different alternatives for the allocation of additional resources (such as the establishment of a new school) is not undertaken and no particular solutions are proposed. However, in the presentation of the model, some possible interpretations are indicated to point out potential uses of this tool in an educational planning environment.



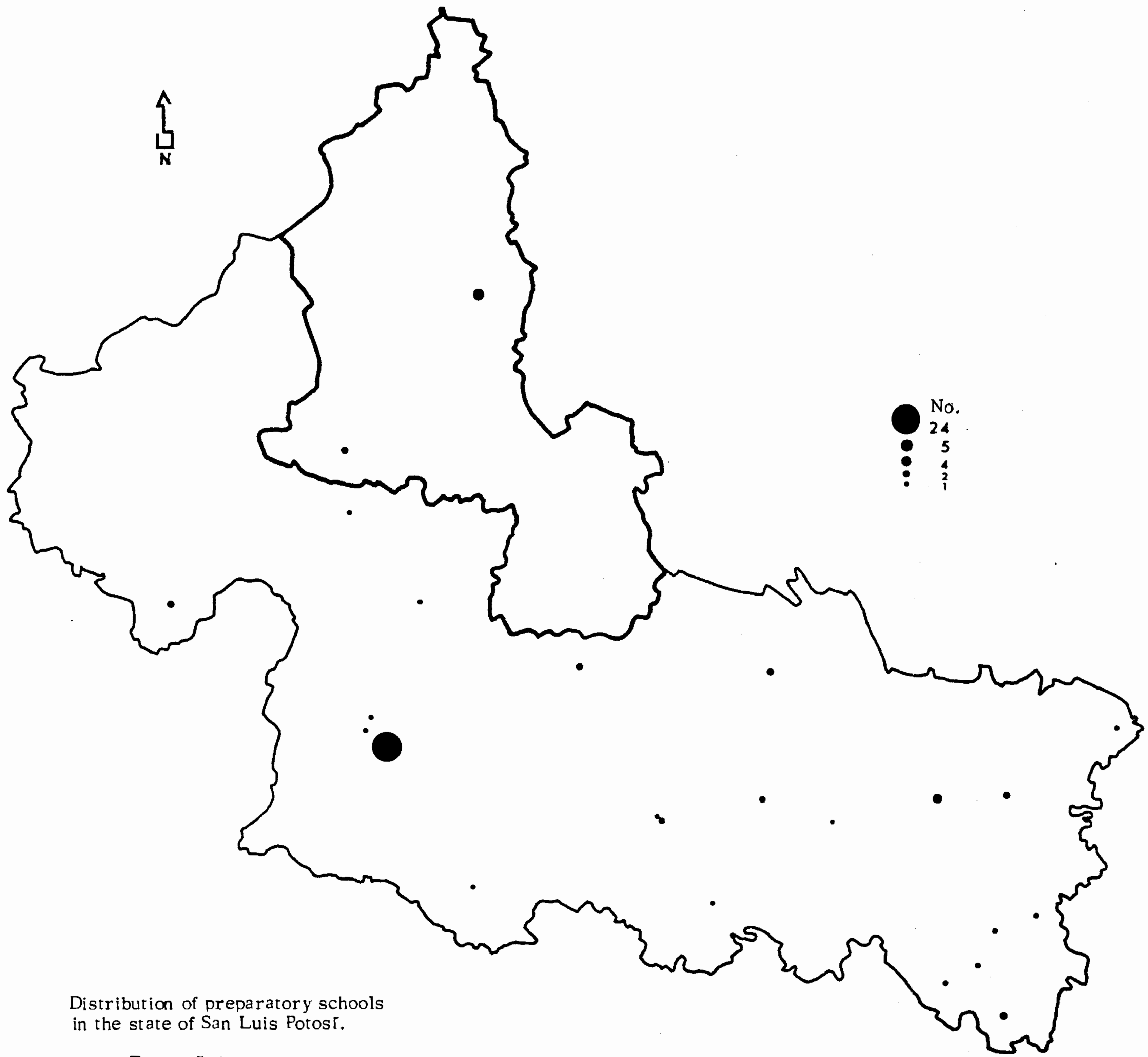
Distribution of elementary schools
in the state of San Luis Potosí.

Figure 5.2



Distribution of secondary schools
in the state of San Luis Potosí.

Figure 5.3



Distribution of preparatory schools
in the state of San Luis Potosí.

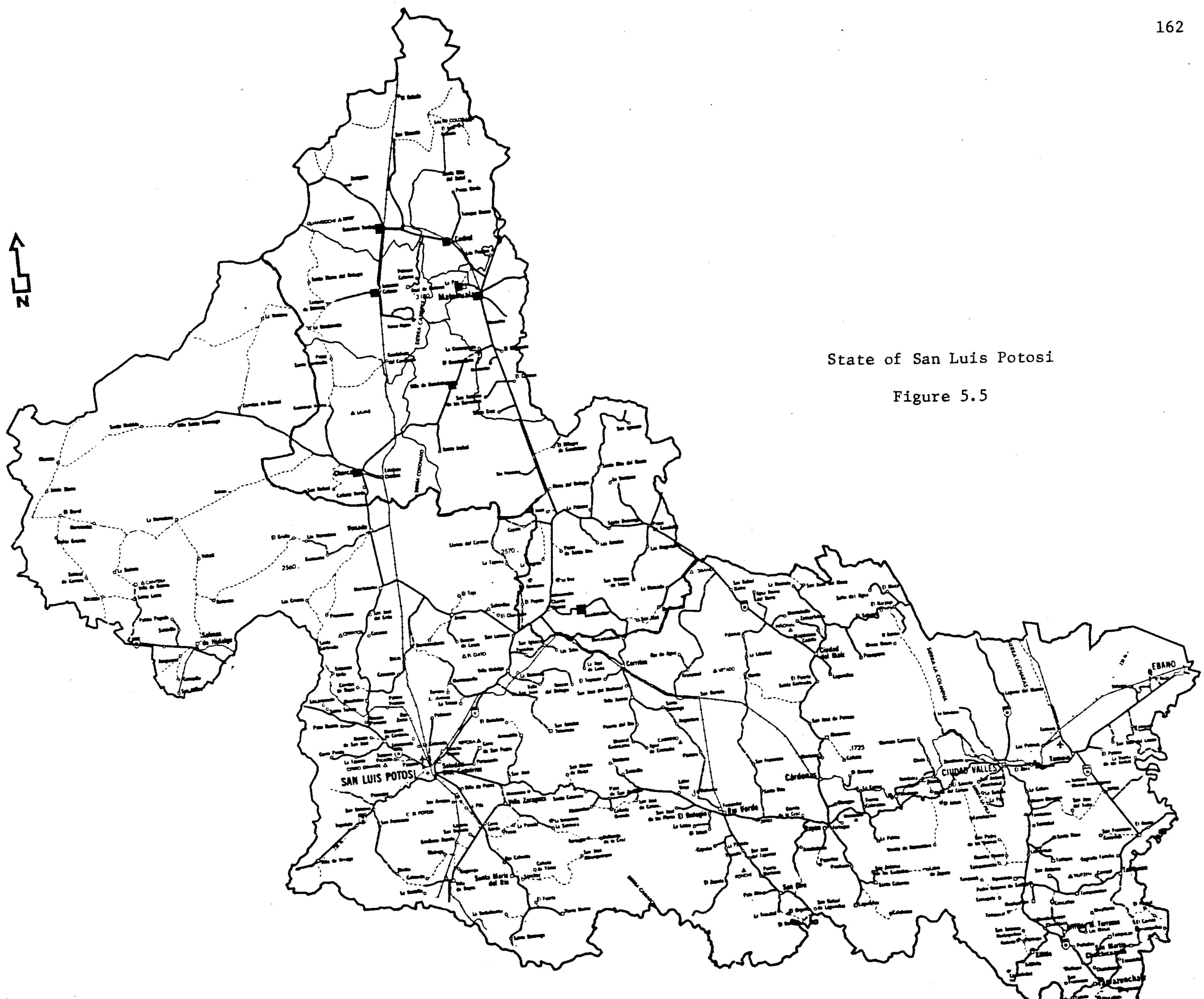
Figure 5.4

5.4.1 The Data

Originally the state of San Luis Potosi had been selected as the area of study on the basis of the availability of data. Studies at the local level require disaggregated data. At a settlement level, census data has only been processed for a few areas, among them San Luis Potosi.

In order to test the proposed model in a reasonable period of time only a portion of the study area was selected. This area includes eight counties in the northern part of San Luis Potosi as shown on figure 5.5. This area is interesting because of its apparently "heterogeneous landscape".

The two main sources of information were the Ministry of Education and the National Institute of Statistics, Geography and Informatics. The Ministry has established a nationwide information system with detailed data on its own human and material resources as well as on the country's students. Every elementary, secondary and preparatory school in the country is registered, and information is stored on the number of enrollees per group; the total number of groups, teachers and classrooms; the estimated capacity and location of schools; the number of students that have graduated, passed to the following grade, failed or dropped out.



State of San Luis Potosi

Figure 5.5

The National Institute of Statistics, Geography and Informatics is in charge of the national census and of the production of diverse cartographic products at a national level.

5.4.2 The Geo-Space

A set of entities of interest to the present study are the settlements inside the selected area that have a secondary and/or a preparatory school. The flow of graduates among these entities is assumed to depend on their spatial relationship. In this case the spatial factor considered decisive was the distance a student has to travel to attend school ("traveling distance").

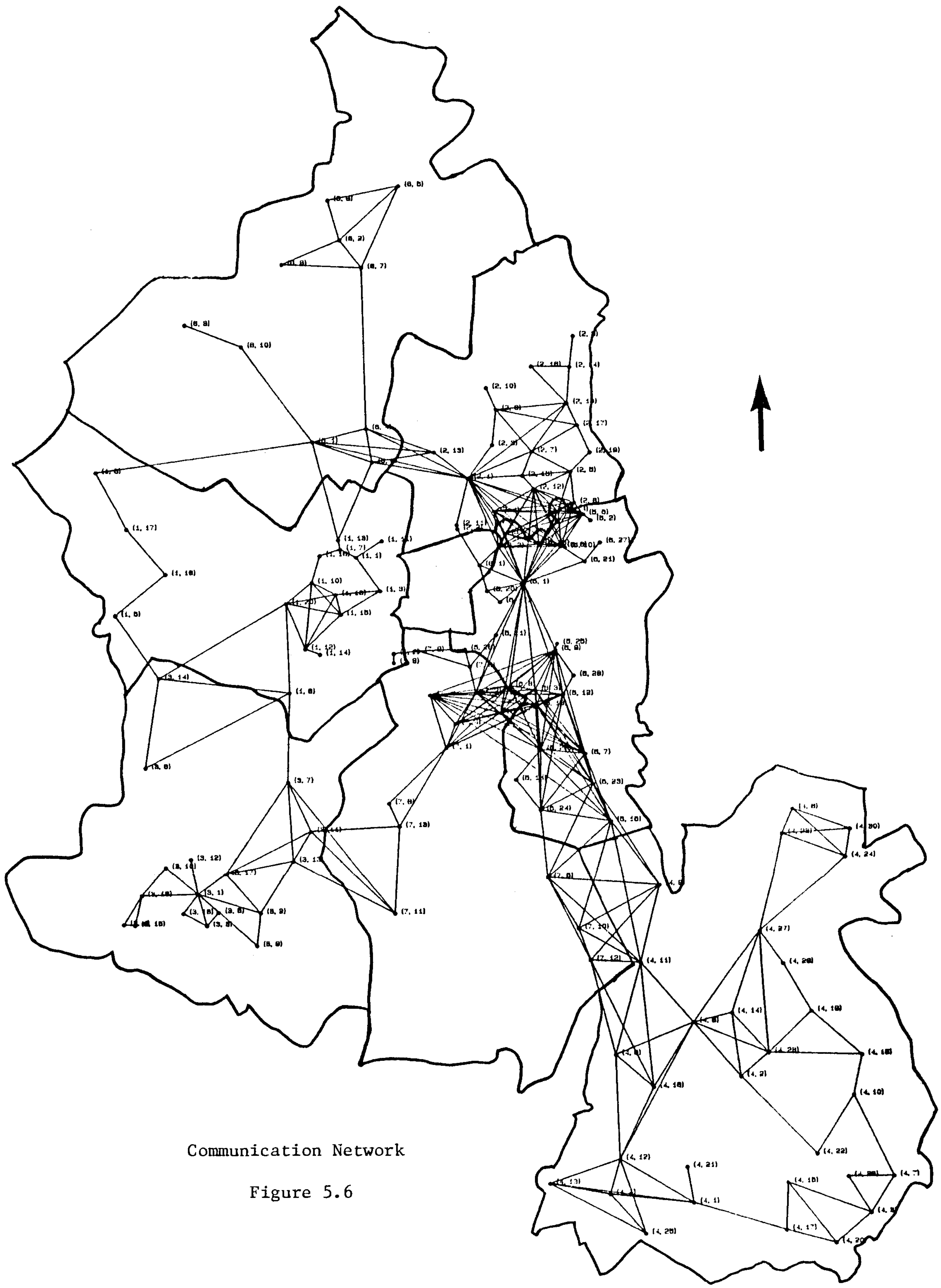
Graphs were the mathematical models that were considered appropriate to represent both the entities and their relationships. Each node in the graph represents a settlement, and the links between them are defined through the spatial relation of traveling distance.

The Traveling Distance Network

In order to determine the traveling distance between any two settlements of the geo-space, a network was defined through the existing communication network. Although the feasibility of traveling from one town to another depends on the physical characteristics of the terrain, it was assumed that the highway and trail system could provide enough information to compensate for these differences.

The network was defined with the aid of topographic maps (scale 1:250,000) as follows: a link was established between any two settlements whenever they were connected by any type of road without passing through a third settlement. The resulting network is shown on figure 5.6.

According to the definition of the network, each link represents a road connecting two settlements. Since the type of road is an important factor to be considered in terms of the ease of transportation, each road was divided in at most two representative sections. For example, the road between two settlements could be composed of a section of highway together with a section of trail. Two weights at most were attached to each link according to the type of road of its sections. The five types of roads together with their weights are: paved (1), unpaved (1.5), trail (2), unpaved road or trail in a mountainous area (3) and footpath (4). These weights were determined by a reliable informant familiar with the area of study so the values assigned to the roads are, to some extent, subjective.



Communication Network

Figure 5.6

The traveling distance between two settlements is calculated using the network as follows:
Let ti and tj be two settlements that are connected through a link and the distance between them, Dij is:

$$Dij = wik\ dik + wkj\ dkj \qquad (5.1)$$

where wik, wkj and dik, dkj are the two weights and distances attached to the link between settlements ti and tj.

If ti and tj are not linked, the traveling distance is calculated by the sum of distances of the shortest path between ti and tj.

5.4.3 The Geo-Subspaces

From a spatial point of view the flow of secondary school graduates between settlements is one of the relevant factors to be considered in the allocation of new schools. The subsets of settlements that have the possibility of interacting through the flow of students are therefore part of the geo-subspaces of interest in the present problem. With this idea in mind a neighborhood of each of the nodes was defined through the traveling distance.

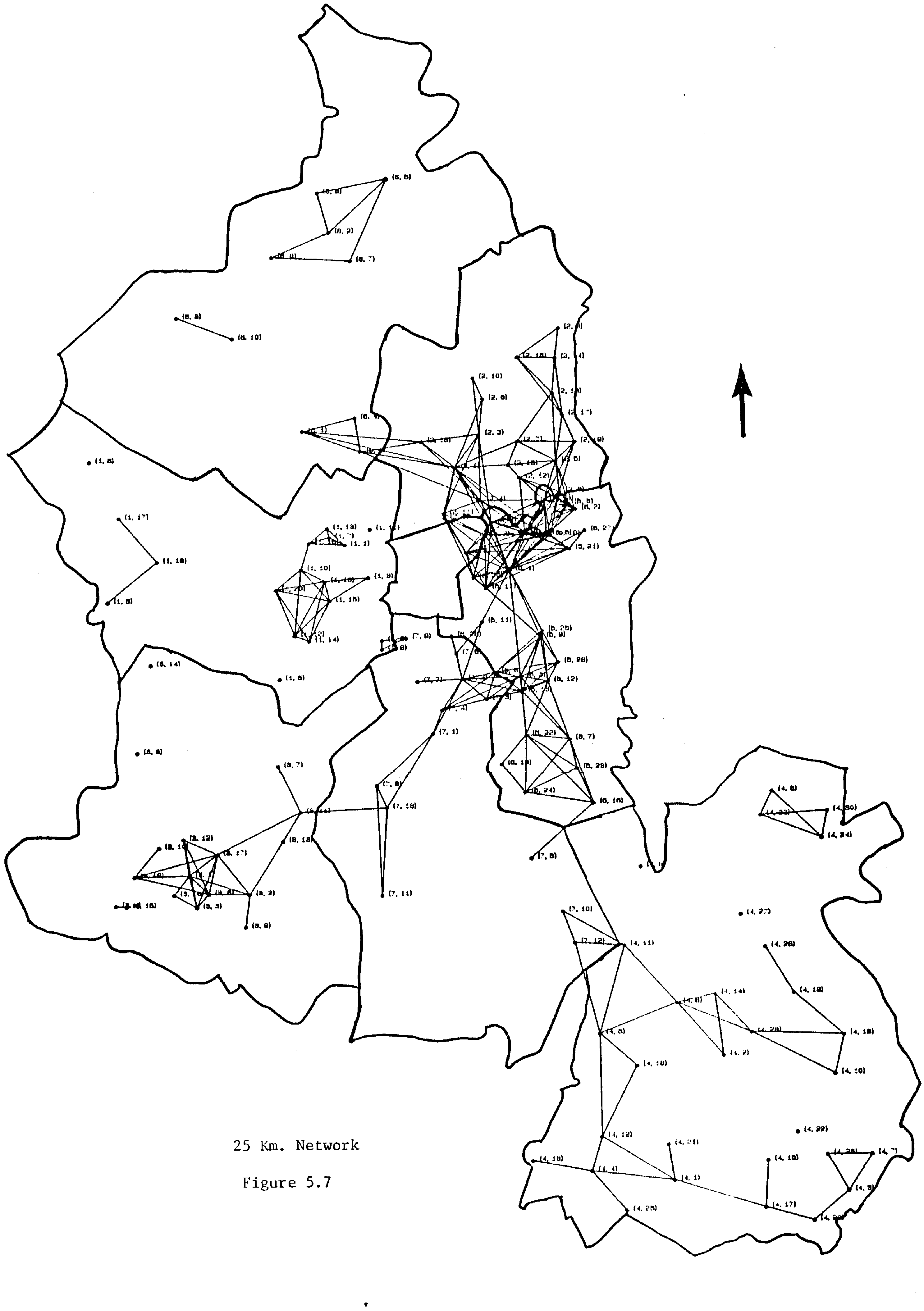
For a given maximum traveling distance dm, the neighborhood of the node N(ti) is the set of nodes where the traveling distance to ti is less or equal to dm.

$$N(ti) = \{ tj \; ; \; Dij < dm \}$$

The maximum traveling distance can be fixed at different values. Consequently, a network can be associated with each one of them as follows: the nodes of the network are the set of settlements of the geo-space, and a link exists between any two of them if they are neighbors.

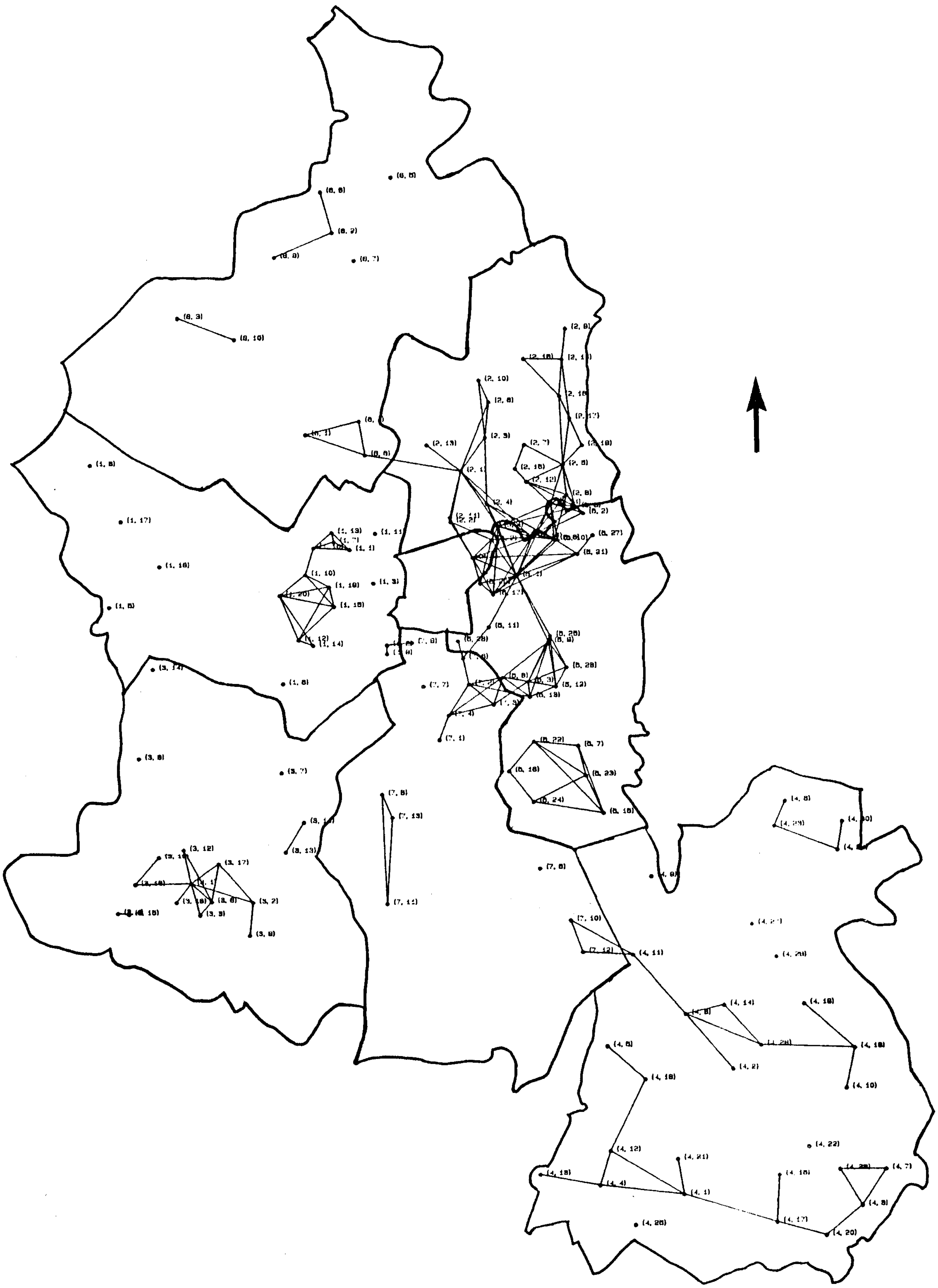
In order to test different maximum traveling distances a small computer system that allows the user to postulate different distances and to plot the resulting networks was implemented. Figures 5.7, 5.8, 5.9 and 5.10 show the results obtained by testing different values for the maximum traveling distance for the study area of San Luis Potosi.

Each of the networks shows a different spatial pattern according to the fixed distance. For example, for the 25 km threshold, most of the settlements are clustered in two large networks and only a few of them appear isolated. In contrast, in the 10 km case most of the settlements are isolated .The percentage of settlements without neighbors for each of the distances considered are: 60%, 26.42%, 13.57% and 5.71% for the 10, 15, 20 and 25 km networks respectively. These observations allow the planner to better understand the spatial dispersion of the units in the area.



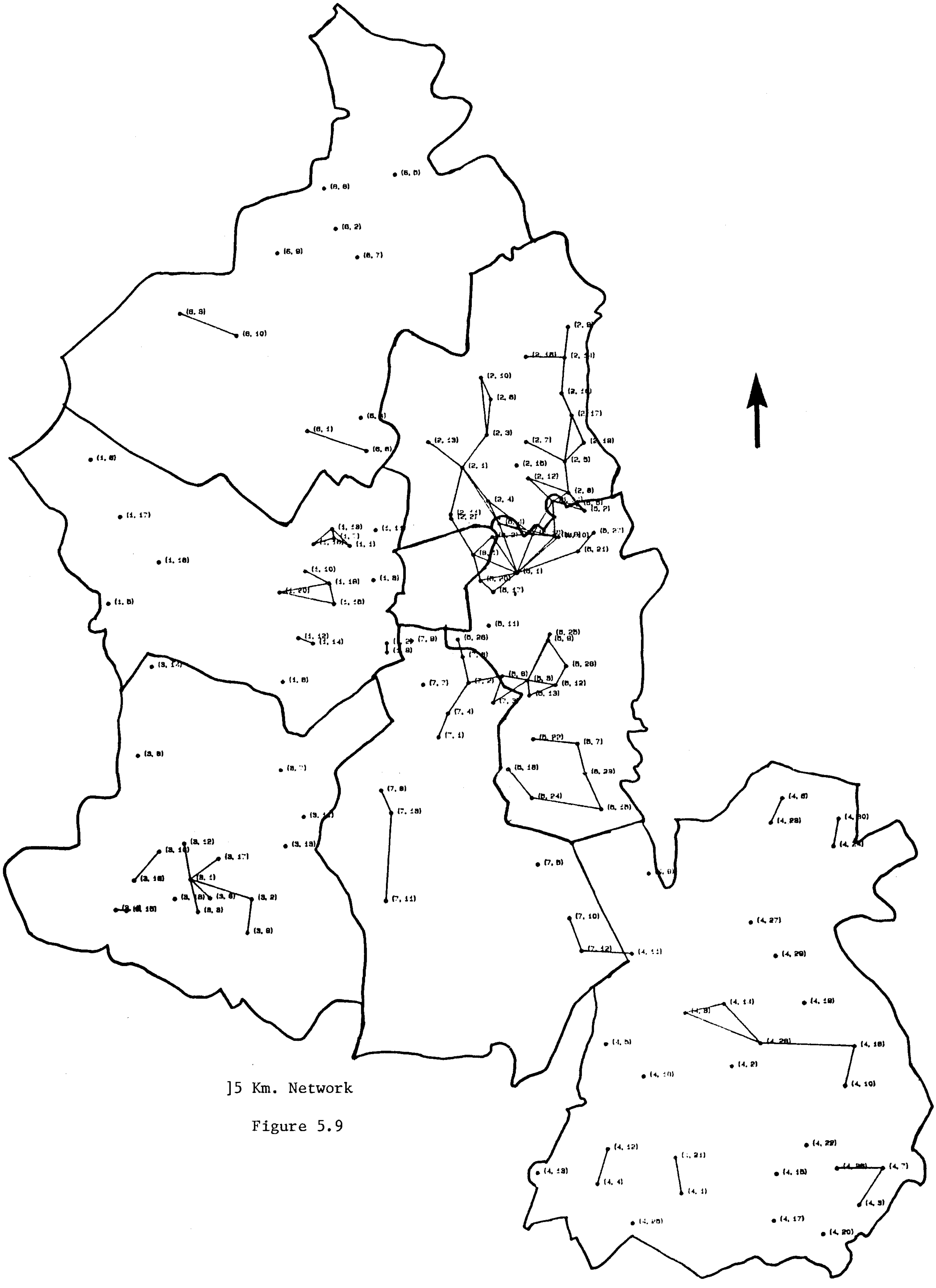
25 Km. Network

Figure 5.7



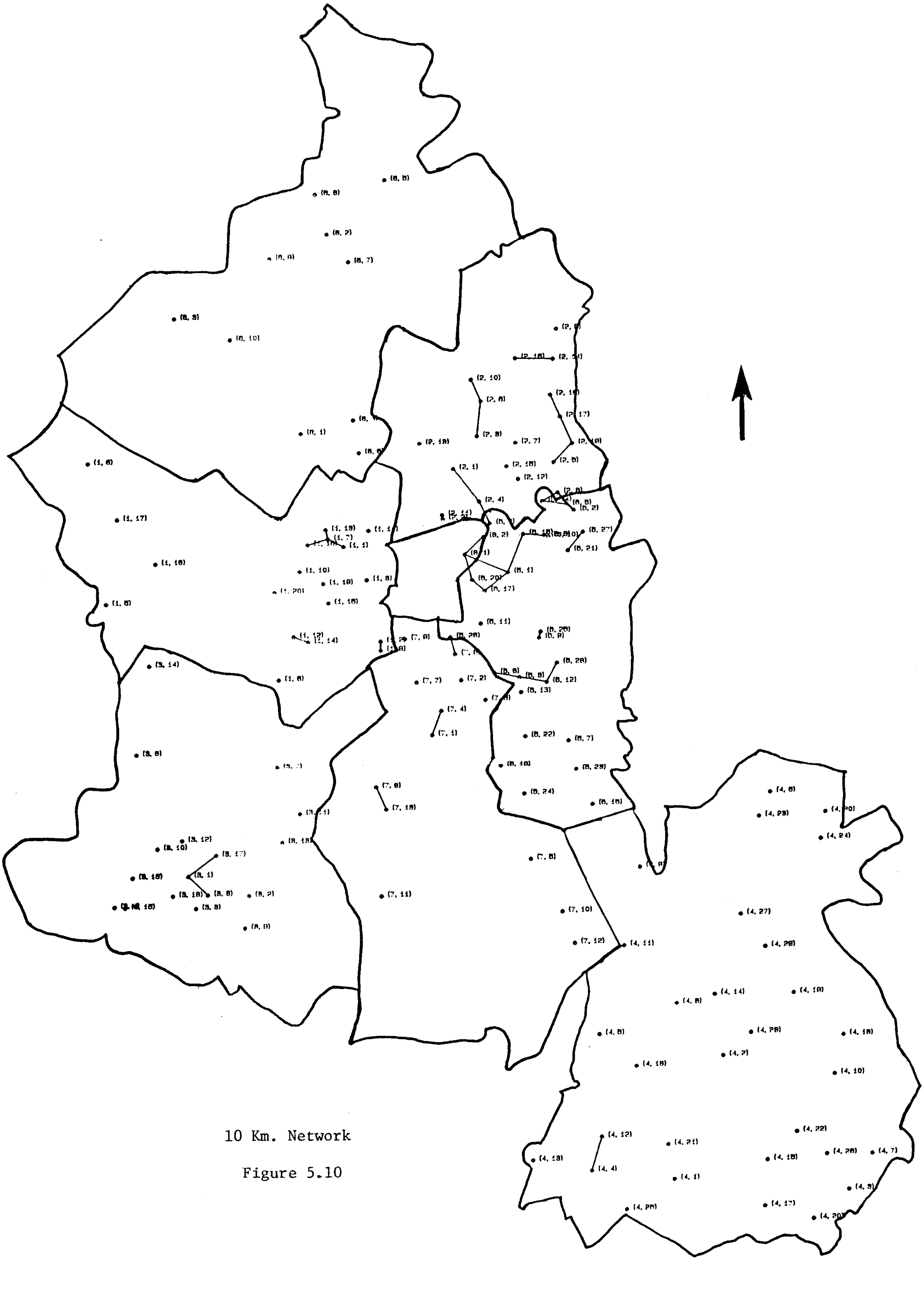
20 Km. Network

Figure 5.8



]5 Km. Network

Figure 5.9



10 Km. Network

Figure 5.10

Besides the measures commonly used in area planning, (number of students per classroom, enrollment ratios, etc), some of the specific characteristics of these networks can be included as criteria for the location of schools. The percentage of isolated units and the number of neighbors of each node are indicators of the degree of communication of each of the settlements. Table 5.1 shows the values of these quantities associated with each settlement and for the different distances. The settlements are identified in the table by their code, as shown in table 5.3.

Catchment Areas

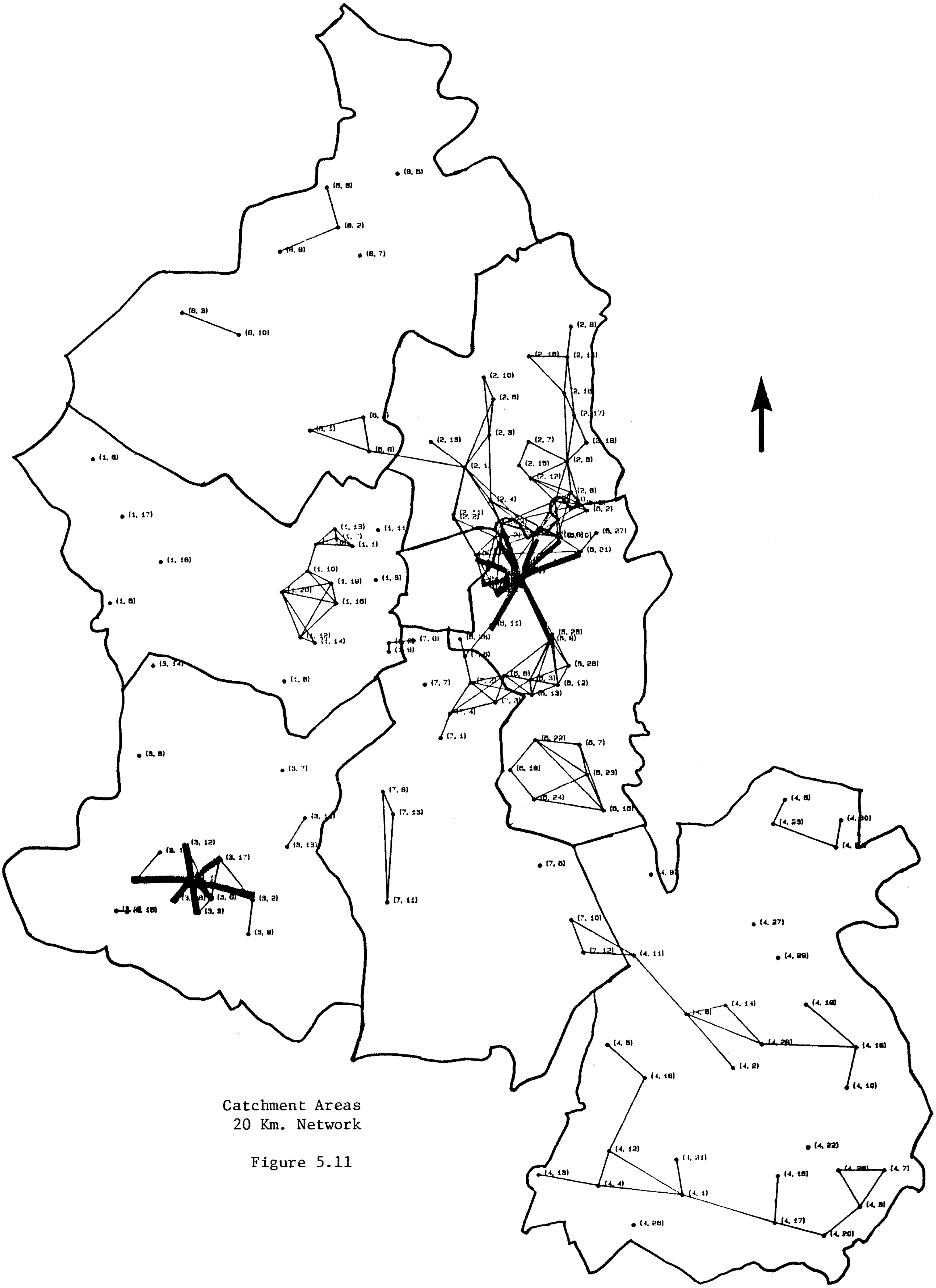
The catchment area associated to a school is simply the area served by it. In defining a catchment area factors such as transportation facilities and terrain are considered. In this case each of the networks can be used in the definition of these areas. Since the interaction relationships are explicitly represented through the set of links in this case, each of the networks can be used in the definition of the catchment areas. Figure 5.11 shows the catchment areas of each of the existing preparatory schools, assuming a maximum traveling distance of 20 km.

The lack of service at the preparatory level was quantified

CODE	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
10KM	1	1	0	8	0	0	3	0	1	0	0	1	1	1	0	1	0	0	0	0
15KM	2	1	0	8	0	0	3	0	1	1	0	1	3	1	2	2	0	0	3	2
20KM	3	2	0	8	0	0	3	0	1	4	0	4	3	2	4	4	0	0	4	5
25KM	3	2	2	8	1	0	3	0	2	5	0	5	3	4	6	4	1	2	6	5
CODE	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	301
10KM	1	1	1	2	1	2	0	3	0	1	1	0	0	1	0	1	2	1	2	2
15KM	5	2	3	3	4	2	1	5	1	2	2	2	1	3	0	1	3	3	3	3
20KM	9	4	4	7	8	3	2	7	1	2	3	5	1	4	2	2	4	5	3	7
25KM	14	6	7	11	9	3	5	11	2	2	6	7	5	4	4	4	5	6	6	7
CODE	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	401	402	403
10KM	0	0	0	10	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
15KM	2	1	1	10	1	0	0	1	1	0	1	0	0	1	1	1	0	1	0	1
20KM	3	2	1	10	4	0	0	1	1	1	2	1	0	1	2	3	1	4	1	3
25KM	5	5	1	10	6	1	0	1	1	4	4	2	0	1	4	7	2	4	2	3
CODE	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423
10KM	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
15KM	1	0	1	2	2	0	1	1	1	0	2	0	0	0	2	0	0	1	0	1
20KM	3	1	1	2	4	0	1	3	3	1	2	1	2	3	3	1	2	1	0	2
25KM	4	5	2	2	5	0	2	4	4	1	3	1	2	3	3	2	2	1	0	3
CODE	424	425	426	427	428	429	430	501	502	503	504	505	506	507	508	509	510	511	512	513
10KM	0	0	0	0	0	0	0	3	2	2	1	3	2	0	1	1	1	0	2	0
15KM	1	0	1	0	3	0	1	9	3	6	6	3	5	2	4	3	4	0	4	2
20KM	2	0	2	0	3	0	1	14	4	7	10	7	9	3	7	7	6	2	6	5
25KM	3	1	2	0	4	1	2	21	9	12	15	10	14	7	9	8	12	3	9	10
CODE	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	601	602	603	604	605
10KM	2	0	2	2	0	10	2	1	0	0	0	1	1	1	1	0	0	0	0	0
15KM	6	2	7	2	1	10	3	2	1	2	2	2	1	1	2	1	0	1	0	0
20KM	10	4	13	7	2	10	4	5	4	4	3	5	1	1	4	2	2	1	2	0
25KM	14	5	16	11	2	10	7	9	8	4	5	7	2	2	6	4	3	1	2	3
CODE	606	607	608	609	610	701	702	703	704	705	706	707	708	709	710	711	712	713	801	802
10KM	0	0	0	0	0	1	0	0	1	0	1	0	1	0	0	0	0	1	3	1
15KM	1	0	0	0	1	1	3	2	2	0	2	0	1	0	1	1	2	2	4	3
20KM	3	0	1	1	1	1	5	4	4	0	3	0	2	1	2	2	2	2	8	6
25KM	5	2	2	2	1	4	10	6	5	1	3	1	3	2	2	2	3	4	11	11

NUMBER OF NEIGHBORS BY DISTANCE

TABLE 5.1



Catchment Areas
20 Km. Network

Figure 5.11

using the number of secondary graduates, the capacity of each of the preparatory schools and the catchment areas. A measure of the coverage of each of the settlements with preparatory schools was calculated as follows:

$$Lj = \frac{\left(\sum_{i=1}^{k} Xi\right) + Xj - Cj}{\left(\sum_{i=1}^{k} Xi\right) + Xj} \qquad (5.3)$$

where Lj is the lack of service associated with the jth settlement, Xi is the number of secondary school graduates the ith settlement and a fixed year, k is the number of neighbors of the jth settlement and Cj is the capacity of its preparatory schools. Table 5.2 shows the results obtained using a traveling distance of 20 km and 1984 data. In this case the lack of service for Charcas is a negative number. This means that there is a surplus in the service for this particular settlement.

Settlement	Ci	Li	%
Charcas	C1=392	L1=-0.0481	-4.81 %
Matehuala	C2=1242	L2= 0.1573	15.73 %

Table 5.2 Service of preparatory schools for a 20 km traveling distance (1984 data).

5.4.4 Some Spatial Characteristics of the Demand for Preparatory Schools

One of the factors that is considered crucial in the location of preparatory school services is the spatial distribution of the demand. In this case the study of the demand was carried out using the number of secondary school graduates for three consecutive years (1983-85) (see table 5.3). It was additionally assumed that all graduates demand preparatory school services, and that there is no flow of students from neighboring counties outside the study area.

Interaction Spaces

Besides the local demand generated by a settlement, the possible flow of secondary graduates among settlements is another of the issues that has to be considered in the location of a preparatory school. In this case the space where a flow of students to attend a preparatory school is expected, is delimited by the neighborhood of each of the settlements. This space is called interaction space. A measure of the expected intensity of interaction among settlements in a geo-subspace is given by the heterogeneity index of the neighborhood. For the study area of San Luis Potosi, a univariate heterogeneity index (equation. 3.2) was

SETTLEMENT	CODE	1983	1984	1985
Real de Catorce	101	67	98	84
Las Adjuntas	102	10	4	15
Alamitos de los Diaz	103	2	5	10
La Cañada	104	8	6	8
Cardoncita	105	7	5	6
Castañon	106	8	2	4
Los Catorce	107	7	3	7
Guadalupe del Carnicero	108	6	5	8
La Maroma	109	11	6	12
El Mastranto	110	0	7	7
Potrero No.1	111	12	19	14
Ranchito de Coronados	112	7	11	12
El Salto y Anexos	113	6	4	7
San Antonio de Coronados	114	15	12	18
San Jose de Coronados	115	11	11	14
Santa Cruz de Carretas	116	8	8	10
Santa Maria del Refugio	117	21	12	14
Tanque de Dolores	118	7	4	10
Vigas de Coronado	119	7	3	7
Wadley	120	13	8	24
Cedral	201	144	159	161
El Blanco	202	0	11	13
Cerro de las Flores	203	10	11	12
La Cruz	204	9	7	8
Cuarejo	205	23	8	8
Hidalgo	206	6	3	10
Jesus Maria	207	11	4	3
Lagunillas	208	9	6	11
Palo Blanco	209	7	4	5
Presa Verde	210	19	7	12
Refugio de las Monjas	211	0	10	0
El Saladito	212	13	11	9
San Isidro	213	11	11	8
San Lorenzo	214	24	7	12
San Pablo	215	0	8	11
Santa Rita de Sotol	216	11	10	21
Tanque Nuevo	217	17	8	11
Zamarripa	218	11	9	8
Progreso	219	0	5	12
Charcas	301	269	319	346
Alvaro Obregon	302	17	6	14
Cañada Verde	303	22	12	13
El Capulin	304	0	17	9
El Cedazo	305	10	6	10

Number of Secondary Graduates
per settlement.

Table 5.3

Emiliano Zapata	306	10	5	9
Francisco I. Madero	307	5	5	15
Guadalupe Victoria	308	9	9	14
La Borcilla	309	14	9	9
Lo de Acosta	310	0	7	10
Miguel Hidalgo	311	10	8	11
Noria de Cerro Gordo	312	0	4	9
Pocitos	313	9	8	9
Presa Santa Gertrudis	314	5	4	0
San Rafael	315	0	8	9
El Terrero	316	0	14	8
Vicente Guerrero	317	12	4	8
La Zapatilla	318	0	10	10
Guadalcazar	401	90	69	79
Amoles	402	15	6	13
Buenavista	403	27	25	41
Charco Blanco	404	0	12	11
Charco Cercado	405	0	11	12
El Fraile	406	0	9	9
La Hincada	407	25	22	16
Huisache	408	10	22	21
Milagro de Guadalupe	409	12	12	16
Negritas	410	5	10	15
Noria del Refugio	411	23	7	11
Nuñez	412	8	9	16
Peyote	413	17	10	20
La Polvora	414	7	6	9
Potreritos	415	7	4	5
Pozas de Santa Ana	416	15	14	11
Pozo de Acuña	417	4	6	8
Presa de Guadalupe	418	5	6	7
Presa de Tepetate	419	7	7	6
Quelital	420	27	12	16
Realejo	421	0	16	13
San Antonio de Trojes	422	17	11	10
San Francisco del Tulillo	423	6	4	7
San Ignacio	424	12	6	8
San Jose de Cervantes	425	11	12	8
San Rafael de los Nietos	426	9	6	4
Santa Rita del Rocio	427	17	8	15
Santo Domingo	428	15	8	18
Ventana	429	9	7	8
La Rosita	430	16	10	11
Matehuala	501	1033	1272	1377
Arroyito de Agua	502	18	11	10
La Bonita	503	5	6	11
La Cabra	504	11	8	15
La Caja	505	11	7	10
La Carbonera	506	9	6	10

Table 5.3 (cont.)

El Carmen	507	7	5	9
Concepcion	508	12	18	19
Encarnacion de Abajo	509	12	12	13
Estanque de Agua Buena	510	16	11	18
Guerrero	511	6	13	17
El Mezquite	512	10	12	16
Pastoriza	513	9	6	7
Los Pocitos	514	11	12	17
Pozo de Santa Clara	515	14	10	7
Rancho Nuevo	516	8	8	11
Sacramento	517	18	8	11
San Antonio de las Barrancas	518	15	14	13
San Antonio de los Castillo	519	9	10	11
San Francisco Caleros	520	6	7	7
San Jose de la Viuda	521	16	8	12
San Jose de los Guajes	522	10	3	11
San Miguel	523	4	6	10
Santa Cruz	524	12	11	9
Santa Lucia	525	13	12	19
Tanque Colorado	526	20	13	18
El Vaquero	527	21	11	10
Los Cinco Señores	528	13	4	15
Vanegas	601	65	63	72
El Gallo	602	10	9	10
Huertecillas	603	0	8	0
La Punta	604	0	13	11
El Salado	605	4	7	10
San Juan de Vanegas	606	9	4	6
San Vicente	607	11	9	13
Tanque de Lopez	608	0	11	10
El Tepetate	609	16	8	12
Zaragoza	610	8	9	10
Villa de Guadalupe	701	35	40	34
Biznaga	702	6	5	9
Guadalupito	703	16	11	16
Llano de Jesus Maria	704	7	5	6
La Masita	705	4	7	8
La Presita	706	24	10	23
Puerto de Magdalenas	707	0	9	11
Rancho Alegre	708	9	16	11
San Bartolo	709	10	6	10
San Francisco	710	9	4	14
Santa Isabel	711	9	2	13
Santa Teresa	712	4	4	12
Zaragoza de Solis	713	8	10	13
La Paz	801	68	82	98
San Antonio de las Trojes	802	9	8	13

Table 5.3 (cont.)

calculated using the number of secondary school graduates for each settlement and the neighborhood relation established in the 20 km network.

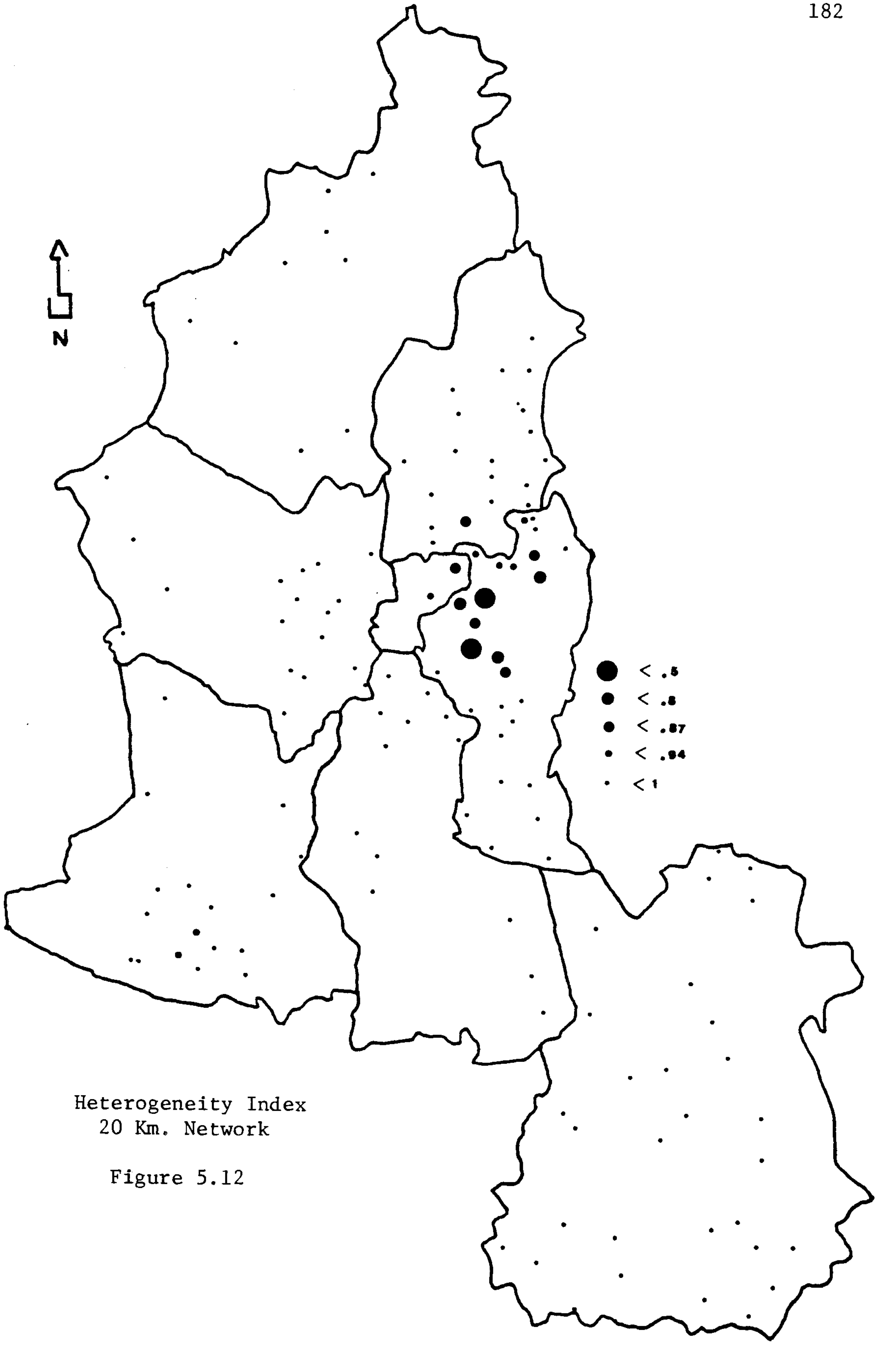
As can be observed in table 5.4, the pattern of the demand is very similar in all three cases. The spatial distribution for 1983 is shown in figure 5.12. Two areas are distinguished by "less homogeneous" subspaces. These are the area surrounding Matehuala, the largest settlement inside the study area, and a smaller area around Charcas, the second most important urban center. The heterogeneity index therefore indicates that the geo-subspaces that form the area around Matehuala and Charcas are characterized by relative heterogeneity in the demand. The standardized heterogeneity index associated with each settlement shows that the city of Matehuala's variation is much more significant than that of the rest of the settlements. The settlement of Guerrero is the only other one where the value of the index is smaller than 0.5.

In fact, the values associated with most of these settlements is close to 1 which means that the geo-subspaces associated to them are homogeneous in comparison with Matehuala's.

In those areas that are formed by heterogeneous geo-subspaces, greater interaction among the settlements can be expected than in areas where homogeneity prevails. Thus, the impact of the

CODE	101	102	103	104	105	106	107	108	109	110
1983	.99652	.99999	1	1	1	1	.99884	1	.99999	.9999
1984	.99452	.99999	1	1	1	1	.99809	1	.99999	.99999
1985	.99686	.99999	1	1	1	1	.99892	1	.99999	.99995
CODE	111	112	113	114	115	116	117	118	119	120
1983	1	.99997	.9988	.99996	.99996	.99914	1	1	.99997	.99995
1984	1	.99998	.99813	.999999	.99998	.99871	1	1	.99997	.99999
1985	1	.99997	.99892	.99998	.99997	.99925	1	1	.99995	.9999
CODE	201	202	203	204	205	206	207	208	209	210
1983	.9819	.99386	.99564	.85292	.99982	.99381	.99987	.99995	.99972	.99987
1984	.9856	.99573	.99652	.85339	.99999	.99485	.99998	.99999	.99999	.99998
1985	.98749	.99602	.99698	.85298	.99999	.99587	.99999	.99999	.9997	.99999
CODE	211	212	213	214	215	216	217	218	219	301
1983	.99184	.99994	.98292	.99983	.99986	.99991	.9999	.9999	.99969	.93455
1984	.99422	.99999	.98615	.99999	.99999	.99999	.99999	.99999	.99999	.93879
1985	.99354	.99999	.9873	.99998	.99999	.99993	.99999	.99997	.99999	.93908
CODE	302	303	304	305	306	307	308	309	310	311
1983	.97956	.97049	1	1	.98375	1	1	.99999	1	.99999
1984	.97935	.97019	.99994	1	.9894	1	1	.99999	.99996	1
1985	.98006	.9708	1	1	.98459	1	1	.99998	.99999	.99999
CODE	312	313	314	315	316	317	318	401	402	403
1983	.96503	.99999	1	1	.96508	.97874	.93017	.99268	.99997	.99989
1984	.96863	1	1	.99994	.97058	.97908	.93963	.99784	.99994	.99988
1985	.9692	.99999	1	1	.96902	.97934	.93877	.99756	.99996	.99952
CODE	404	405	406	407	408	409	410	411	412	413
1983	.99728	.99978	.99996	.99987	.99994	1	1	.99976	.9978	.99972
1984	.99931	.99999	.99998	.99991	.99988	1	.99998	.99994	.99923	.99999
1985	.99914	.99999	.99999	.99979	.99995	1	.99996	.99998	.99927	.99995
CODE	414	415	416	417	418	419	420	421	422	423
1983	.99996	.99999	.99986	.99744	.99996	.99999	.99974	.99918	1	.99996
1984	.99991	.99999	.99998	.99915	.99999	.99999	.99993	.99822	1	.99999
1985	.99993	.99999	.99999	.99907	.99996	.99999	.99981	.99763	1	.99999
CODE	424	425	426	427	428	429	430	501	502	503
1983	.99997	1	.99972	1	.99993	1	.99998	0	.99993	.99994
1984	.99999	1	.9998	1	.99995	1	.99998	0	.99999	.99997
1985	.99999	1	.99958	1	.99996	1	.99999	0	.99999	.99998
CODE	504	505	506	507	508	509	510	511	512	513
1983	.89749	.99997	.88754	.99997	.99998	.85628	.83361	.49093	.99999	.99999
1984	.89755	.99999	.88741	.99999	.99993	.8566	.83245	.49897	.99998	.99997
1985	.89824	.99999	.88739	.99999	.99995	.85586	.83305	.49849	.99998	.99996
CODE	514	515	516	517	518	519	520	521	522	523
1983	.89918	.99995	.92034	.85757	.99998	1	.74458	.79984	.99997	.99994
1984	.89962	.99998	.92092	.85521	.99995	1	.7962	.7973	.99997	.99999
1985	.89968	.99999	.9209	.85485	.99999	1	.74441	.79711	.99999	.99999
CODE	524	525	526	527	528	601	602	603	604	605
1983	.99997	.79918	.99998	.99997	.99998	.99644	.99993	.99993	.99792	1
1984	.99999	.79925	.99999	.99999	.99996	.9981	.99999	.99999	.99918	1
1985	.99999	.79997	.99998	.99999	.99999	.99781	.99999	.99994	.99898	1
CODE	606	607	608	609	610	701	702	703	704	705
1983	.9931	1	.9999	.99996	.99993	.99724	.9999	.99992	.99978	1
1984	.99418	1	.99999	.99999	.99999	.99922	.99997	.99997	.99977	1
1985	.99489	1	1	.99999	.99994	.99957	.99996	.99997	99985	1
CODE	706	707	708	709	710	711	712	713	801	802
1983	.99978	1	.99999	1	.99989	.99999	.99981	.99999	.8846	.83077
1984	.99999	1	.99992	.99999	.99999	.99991	.9999	.99996	.88511	.83108
1985	.99995	1	.99999	.99998	.99999	.99999	.99999	.99999	.88539	.83119

STANDARDIZED HETEROGENEITY INDEX
20 KM. NETWORK
TABLE 5.4



Heterogeneity Index
20 Km. Network

Figure 5.12

location of a school on a settlement immersed in a heterogeneous environment should be analysed in greater detail. For example, the settlement of Guerrero is in the catchment area of two settlements with very different demands: Matehuala and La Presita. In this case, before a decision is taken regarding the location of a school, several alternatives related to the possible flow of students have to be analysed. For example, Matehuala could become a point of attraction for the secondary school graduates of Guerrero. On the other hand, the location of a school in La Presita could satisfy the demand of Guerrero and avoid the overcrowding of Matehuala.

In summary, the measure of the local variation of interaction spaces allows the planner to identify the degree of expected interaction in settlements. This aids in the delimitation of zones where "intense" interactions are expected. Similarly, the index can serve in the definition of zones formed by geo-subspaces of homogeneous demand.

Finally, to test the impact of a change in the criterion of maximum traveling distance on the spatial pattern of variation, the heterogeneity index was calculated using the 15 and 25 km network for the year 1983. As can be seen in table 5.5, the pattern presented is similar to the one obtained for the 20 km network. There are however, differences in the values associated with particular settlements. This indicates

CODE	101	102	103	104	105	106	107	108	109	110
15KM	.99644	.99999	1	1	1	1	.99883	1	99999	.99995
25KM	.99649	.99999	.99994	1	1	1	.99883	1	.99999	.99991
CODE	111	112	113	114	115	116	117	118	119	120
15KM	1	.99993	.99879	.99993	.99999	.99999	1	1	.99996	.99998
25KM	1	.99996	.99879	.99996	.99995	.99913	.9998	.9999	.99996	.99995
CODE	201	202	203	204	205	206	207	208	209	210
15KM	.98209	.99775	.99416	.9941	.99978	.99991	.99986	.99994	.99971	.99987
25KM	.92892	.82271	.99747	.90556	.99978	.99375	.99648	.90715	.99985	.99987
CODE	211	212	213	214	215	216	217	218	219	301
15KM	.98994	.99999	.98283	.99979	1	.99983	.99988	.99989	.99969	.93597
25KM	.99584	.99995	.99598	.99983	.99475	.99994	.99991	.99992	.99979	.93397
CODE	302	303	304	305	306	307	308	309	310	311
15KM	.96919	.94081	1	1	.93492	1	1	.99999	1	1
25KM	.9876	.98788	1	1	.98905	.99997	1	.99999	1	.99999
CODE	312	313	314	315	316	317	318	401	402	403
15KM	.92979	1	1	1	1	.93592	1	.99214	1	.99999
25KM	.98221	.99996	1	1	.98232	.99075	.96453	.99262	.99995	.99989
CODE	404	405	406	407	408	409	410	411	412	413
15KM	.99993	1	.99996	.99987	.99998	1	1	.99964	.99993	1
25KM	.99791	.99981	.99991	.99987	.99993	1	.99995	.99969	.99832	.99971
CODE	414	415	416	417	418	419	420	421	422	423
15KM	.99996	1	1	1	.99995	1	1	.99214	1	.99996
25KM	.99995	.99999	.99986	.99742	.99996	.99999	.99974	.99211	1	.99994
CODE	424	425	426	427	428	429	430	501	502	503
15KM	.99998	1	.99975	1	.99993	1	.99998	0	.99994	.99994
25KM	.99993	.99988	.99971	1	.99992	.99999	.99994	0	.8885	.91422
CODE	504	505	506	507	508	509	510	511	512	513
15KM	.82825	.99998	.79653	.99999	.99997	.99998	.74911	1	.99999	.99999
25KM	.93083	.89828	.92554	.99998	.99998	.87312	.91583	.65759	.99998	.99998
CODE	514	515	516	517	518	519	520	521	522	523
15KM	.99998	.99994	.85438	.50019	.99999	1	.65763	.49828	.99999	.99994
25KM	.92733	.99994	.9347	.90854	.99998	1	.85272	.88779	.99998	.99994
CODE	524	525	526	527	528	601	602	603	604	605
15KM	.99999	.99996	.99998	.99997	.99999	.99695	1	.99993	1	1
25KM	.99997	.85528	.99989	.50143	.99998	.99597	.99994	.99993	.9979	.99996
CODE	606	607	608	609	610	701	702	703	704	705
15KM	.99695	1	1	1	.99993	.99923	.99988	.99993	.99961	1
25KM	.99582	.99996	.99994	.99997	.99993	.99926	.99984	.99993	.99982	.9999
CODE	706	707	708	709	710	711	712	713	801	802
15KM	.99983	1	.99999	1	.99997	.99999	.99981	.99999	.77124	.65978
25KM	.99978	.99996	.99978	.99999	.99989	.99999	.99986	.99982	.91449	.90685

STANDARDIZED HETEROGENEITY INDEX
15 AND 25 KM NETWORKS (1983)

TABLE 5.5

that although no significant change should be expected in the general interaction pattern if any of the three (15,20,25 km) maximum traveling distances is taken as a threshold, in the analysis of individual settlements attention has to be given to the heterogeneity values associated in each case.

A Temporal Analysis

Up to this point the whole analysis has focussed on the state of the educational system at a fixed point in time. However, the analysis of the evolution of the system is important both to understand its present state and to evaluate the impact of planning actions.

Temporal Stability

At this point local variation of temporal subspace was studied insofar as it could be expected to indicate temporal "stability" of the demand in each of the settlements.

A heterogeneity index similar to the spatial heterogeneity index was used to calculate the temporal variation of the demand for each settlement. The temporal heterogeneity index is defined as follows:

$$TH(a) = (1/k) \sum_{i=1}^{k} (Xk - \bar{X})^2 \qquad (5.4)$$

and

$$\bar{X} = (1/k) \sum_{i=1}^{k} Xi$$

where k is the number of school years considered, $\bar{X}$ is the mean value of the demand and Xi is the demand in the ith year. This index can be standardized in a similar manner to the spatial case.

The index was applied to the study of the temporal stability of the demand for a specific type of secondary schools "telesecundarias."

Three different types of secondary schools can be distinguished in the school system: general, technical and TV-secondary. TV-secondary is the type of school that has been established in most of the settlements in San Luis Potosi. In fact, the only places where there are technical and general schools are the county seats. TV-secondaries are designed to serve communities where the size of the population is too small to establish a regular school, and the settlement can not be serviced by neighboring ones. Televised classes keep the number of teachers required to a minimum.

Besides the deficit of preparatory school service in those settlements that have regular secondary schools, there is also a lack of preparatory schools for the students graduating from a TV-secondary in the area. The number of students in each of these schools is in the interval [1,50]. There are, however, variations in the number of students from one school year to the next. The temporal heterogeneity index was used as a tool to quantify this variation (see table 5.6).

With the aid of this index it is possible to identify those settlements where temporal variation is significant. The value of the index can be interpreted as a measure of "temporal stability" of demand for preparatory services. For planning purposes a preparatory school in a settlement or catchment area which has an "unstable" demand is not advisable.

Degree of temporal stability is a measure that has been associated to each settlement as an isolated entity. There is, however, an interaction space related to each settlement that must also be considered. The spatial distribution of temporal stability as shown in figure 5.13 presents a "heterogeneous pattern." A heterogeneity index was applied using the value of the temporal heterogeneity index as a variable to quantify this spatial variation and the 20 km network as a basis to define the neighborhoods (see table 5.7).

The value of the index associated with a settlement is that it provides a measure of the spatial variation of its neighborhood according to the temporal variation of the settlements. For example, values close to zero indicate that the spatial neighborhood is "highly" heterogeneous with respect to "temporal stability." On the other hand, values close to 1 indicate that the spatial neighborhood is "much less" heterogeneous with respect to the "temporal stability" of the settlements inside it.

The map in figure 5.14 shows the spatial distribution of the spatial heterogeneity index of temporal stability. Both maps (figures 5.13 and 5.14) can be used to identify neighborhoods where temporal stability is high and spatial heterogeneity is low. Assuming that the temporal trend is maintained, this characteristic of a neighborhood indicates to the planner that demand in the catchment area of a settlement where a school is to be located will not have a large temporal variation.

5.4.5 Additional Considerations

Besides the use of the heterogeneity index as an aid in the study of the spatial characteristics of demand, this same

CODE	101	102	103	104	105	106	107	108	109	110
INDEX	XXX	.60262	.78602	.98253	.98689	.87772	.93013	.96943	.86462	.78602
CODE	111	112	113	114	115	116	117	118	119	120
INDEX	.82969	.90829	.96943	.88209	.96069	.98253	.70742	.88209	.93013	.12227
CODE	201	202	203	204	205	206	207	208	209	210
INDEX	XXX	.35807	.98689	.98689	.01746	.83842	.70742	.91703	.96943	.52401
CODE	211	212	213	214	215	216	217	218	219	301
INDEX	.56331	.94759	.96069	0	.57641	.51528	.72489	.96943	.52401	XXX
CODE	302	303	304	305	306	307	308	309	310	311
INDEX	.57641	.66812	.0524	.93013	.90829	.56331	.89082	.89082	.65502	.96943
CODE	312	313	314	315	316	317	318	401	402	403
INDEX	.73362	.99563	.90829	.68622	.35371	.79039	.56331	XXX	.70742	.00436
CODE	404	405	406	407	408	409	410	411	412	413
INDEX	.41921	.41921	.64628	.72489	.41921	.93013	.67248	.0917	.75109	.65502
CODE	414	415	416	417	418	419	420	421	422	423
INDEX	.96943	.96943	.94323	.94759	.98689	.99563	.2096	.0524	.81222	.96943
CODE	424	425	426	427	428	429	430	501	502	503
INDEX	.87772	.94323	.91703	.70742	.65502	.98689	.86462	XXX	.75109	.86462
CODE	504	505	506	507	508	509	510	511	512	513
INDEX	.83842	.94323	.94323	.94759	.81222	.99563	.82969	.59388	.87772	.96943
CODE	514	515	516	517	518	519	520	521	522	523
INDEX	.86462	.83842	.96069	.65502	.98689	.98689	.99563	.79039	.75109	.87772
CODE	524	525	526	527	528	601	602	603	604	605
INDEX	.96943	.81222	.82969	.51528	.55021	XXX	.99563	.72052	.35807	.88209
CODE	606	607	608	609	610	701	702	703	704	705
INDEX	.91703	.94759	.51528	.79039	.98689	XXX	.94323	.89082	.98689	.94323
CODE	706	707	708	709	710	711	712	713	801	802
INDEX	.20087	.55021	.82969	.93013	.67248	.70742	.72052	.97703	XXX	.90829

STANDARDIZED TEMPORARAL HETEROGENEITY INDEX

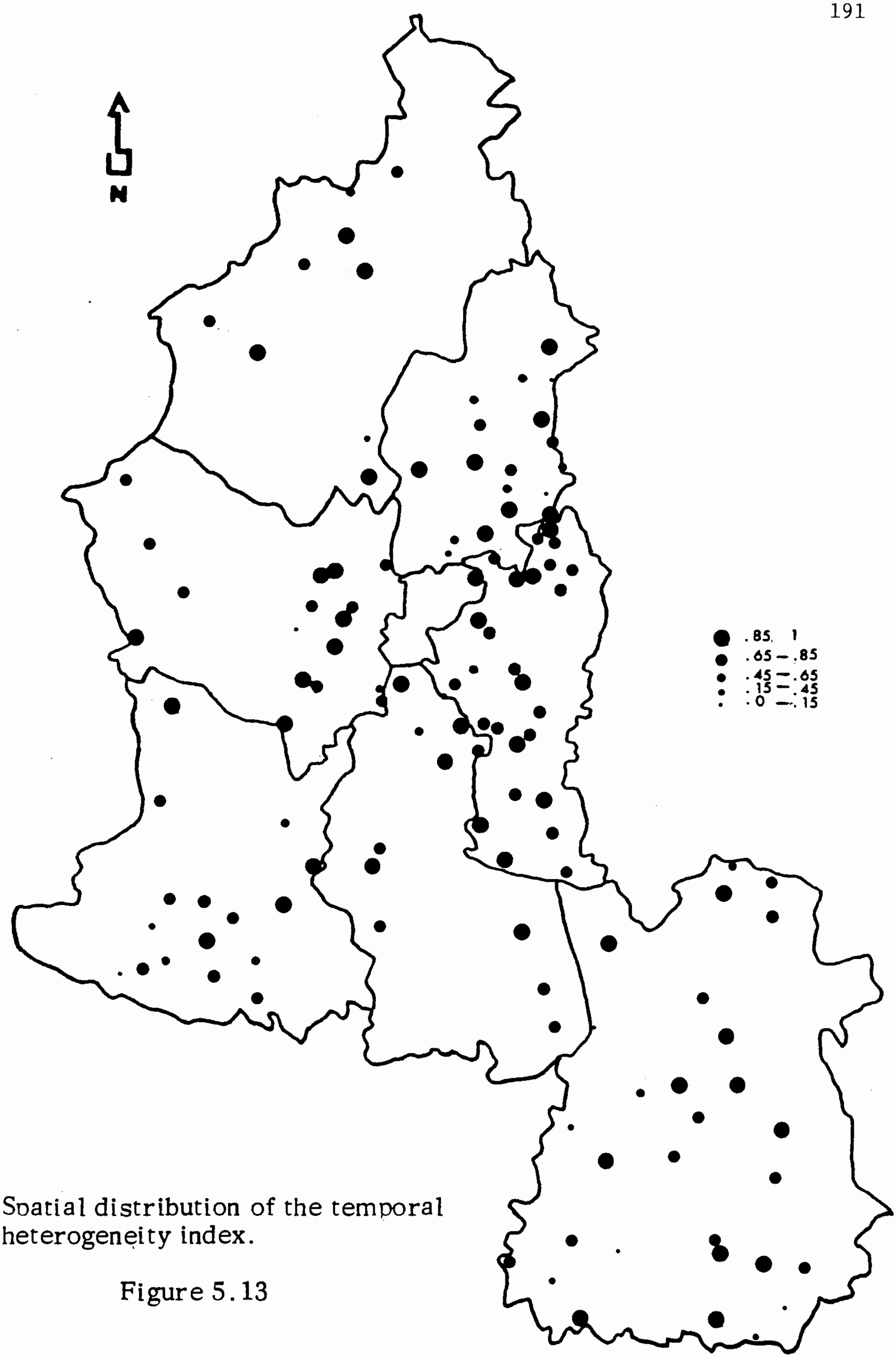
(EXCLUDING COUNTY SEATS)

TABLE 5.6

CODE	101	102	103	104	105	106	107	108	109	110
INDEX	XXX	.9064	1	1	1	1	.99707	1	.92695	.85888
CODE	111	112	113	114	115	116	117	118	119	120
INDEX	1	.8346	.99908	.69247	.8039	.72284	1	1	.82048	.3634
CODE	201	202	203	204	205	206	207	208	209	210
INDEX	XXX	.78438	.92854	.95657	.2833	.92567	.7376	.87174	0	.83341
CODE	211	212	213	214	215	216	217	218	219	301
INDEX	.91946	.78489	1	.28958	.91756	.749	.70045	.5083	.82431	XXX
CODE	302	303	304	305	306	307	308	309	310	311
INDEX	.92304	.94722	.57925	1	.97444	1	1	.89481	.90339	.99926
CODE	312	313	314	315	316	317	318	401	402	403
INDEX	.92695	.99926	1	.57925	.90339	.96589	1	XXX	.91161	.50548
CODE	404	405	406	407	408	409	410	411	412	413
INDEX	.91181	.70781	.88888	.70415	.85404	1	.89481	.70206	.97986	.94083
CODE	414	415	416	417	418	419	420	421	422	423
INDEX	.78634	.99949	.83426	.70998	.92584	.99991	.68782	1	1	.93997
CODE	424	425	426	427	428	429	430	501	502	503
INDEX	.99543	1	.5372	1	.90614	1	.99981	XXX	.96773	.97972
CODE	504	505	506	507	508	509	510	511	512	513
INDEX	.98803	.86134	.99104	.98034	.98187	.94667	.99306	.87014	.97546	.99032
CODE	514	515	516	517	518	519	520	521	522	523
INDEX	.90983	.98982	.8875	.9137	.97025	1	.99991	.95837	.96864	.99178
CODE	524	525	526	527	528	601	602	603	604	605
INDEX	.99082	.97187	.57925	.91946	.87413	XXX	.85483	.92449	.57925	1
CODE	606	607	608	609	610	701	702	703	704	705
INDEX	.66755	1	.75448	.95517	.92449	XXX	.87793	.99498	.9485	1
CODE	706	707	708	709	710	711	712	713	801	802
INDEX	.6095	1	.98798	.88586	.81931	.96867	.78839	.97256	XXX	.91711

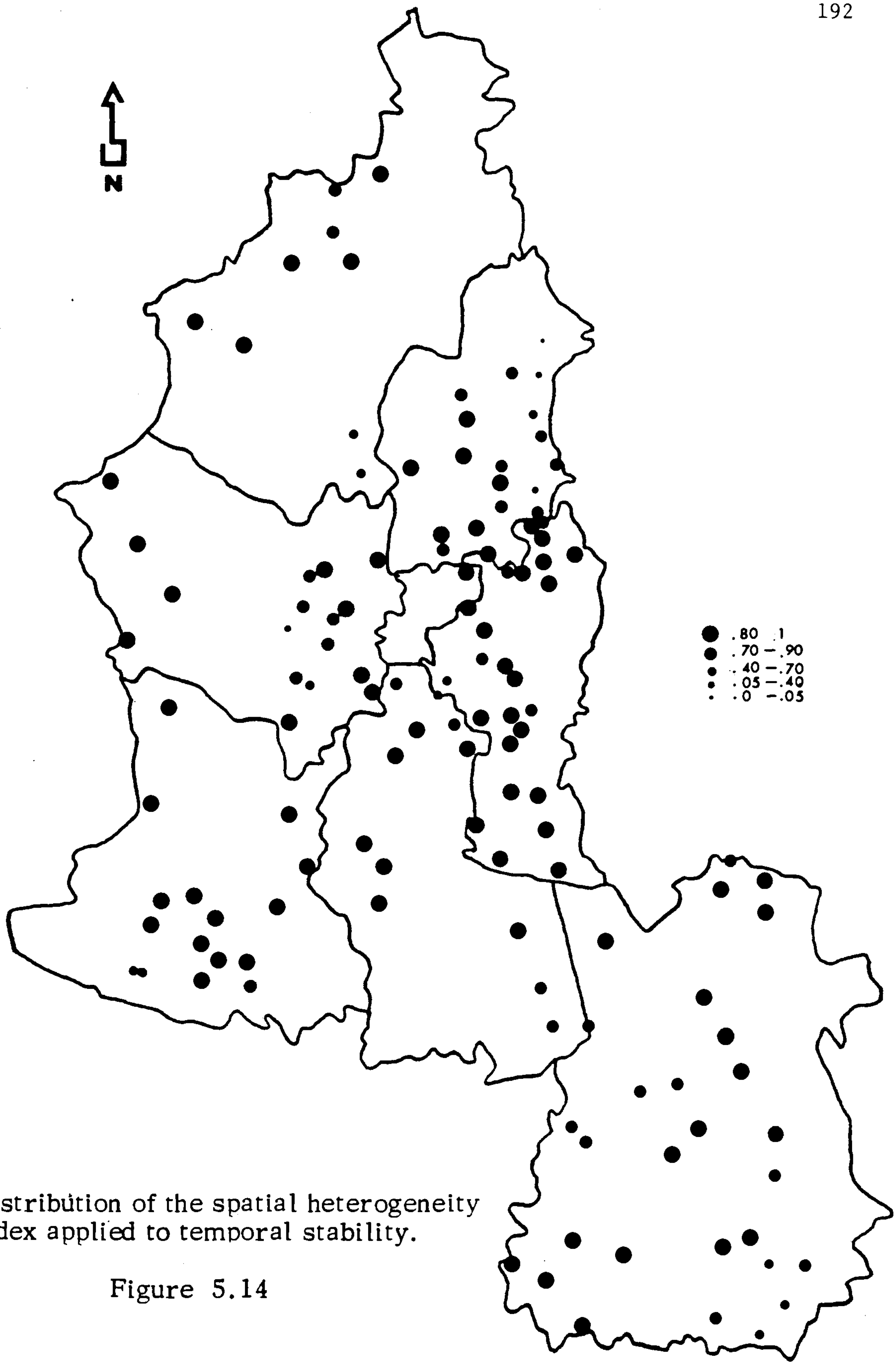
STANDARDIZED HETEROGENEITY INDEX
OF THE TEMPORAL STABILITY

TABLE 5.7



Spoatial distribution of the temporal heterogeneity index.

Figure 5.13



Distribution of the spatial heterogeneity index applied to temporal stability.

Figure 5.14

measure can be used to study other aspects of educational planning. Some examples follow:

1. Various indicators can be used to compare efficiency in the settlements studied. The rate of students graduating from elementary and secondary schools for a fixed cohort may be an indicator of the quality of the educational services.

The heterogeneity index can be used to test the uniformity of the services provided. Settlements with high values indicate anomalous conditions either superior or inferior to those of surrounding settlements. Homogeneous zones receive similar services, while heterogeneous zones show disparate services.

2. The number of inhabitants per school in the different age groups and at different educational levels is an indicator of the distribution of the resources among the areas.

The interpretation of the heterogeneity index in this case is similar to the previous example. Other variables that indicate the amount of resources given to a settlement can receive a similar treatment.

3. If two or more indicators of the equity or efficiency of the system have been defined, a multivariate heterogeneity index can aid the planner in identifying zones or settlements

by equity/inequity or efficiency/inefficiency measures.

4. Finally, a time series analysis of the evolution of school services in such aspects as equity and efficiency can give the planner important information in order to better understand the present state of the educational service and to anticipate future developments.

5.5 Summary

The heterogeneity index as a measure of the degree of membership of an areal unit to a region was applied in the design of regionalization algorithms. There are however other possible interpretations of a measure of local variation of a geo-space. Planning was selected for the application of the heterogeneity index because areas of "high contrast" are of special interest for planners. In particular in school location planning various indicators are used in the characterization of the spatial distribution of human resources and material assets of the school systems. The usual procedure in school location problems has two stages, first a definition of measures of interest for the planner, such as efficiency and effectiveness is done, and second, a representation of these indicators is made in maps. As mentioned before, one of the main differences between the neighborhood approach presented in this work and previous ones is that while in the most traditional models the representation of the geographical landscape is made through isolated entities such as lines,points and areas, in neighborhood models the basic units of study are geo-subspaces.

In the first part of the chapter a "school location problem scenario" is established. The area of study is the northern part of the state of San Luis Potosi in central Mexico. The Mexican school system has reached a point where there is a need to assure a coordinated growth among the different levels of education. Although since the 1970's a planning system was established within the government, very little emphasis has been done on the spatial aspects in the different school systems models that have been implemented. Currently, the information system that supports the decision making is being transformed into a geographic information system. That is, for the first time, location variables are being included into the planning system at a national level.

The area of study is characterized by a significant secondary school system growth, as a consequence there is a greater demand on the higher levels of education. The central authorities are aware of this phenomena and have decided to satisfy the demand establishing new schools.

It is a common practice to use various indicators as well as their spatial distribution and catchment areas in the decision process for the location of schools. There are however besides the above-mentioned spatial aspects of the school location problem, other ones that have not been studied. In the second part of the chapter several indicators based on the notion of "local variation" are proposed as tools in a further study of the spatial characteristics of the problem. However, it is important to mention that the basic goal is to present the tools rather than specific solutions for the location of new schools. It is clear that in a problem of the complexity of the one presented here, in order to reach the best feasible solution it is necessary to include in the analysis social, cultural, administrative and financial factors besides the geographical aspects.

The geo-space of interest is defined as the set of settlements inside the study area that have a secondary and/or a preparatory school, together with the spatial relationships that are relevant to the problem, the geo-subspaces are defined as sub-sets of settlements. The size and shape of the geo-subspaces is not necessarily fixed. It is in fact a parameter that the planner can use at the decision making stage. Such is the case of the traveling distance, that can be used in the definition of the "optimum" number of schools to be located if there is a limitation on the number of schools to be established, since it allows the planner to determine the size of the catchment areas. For example, if it is assumed that the "optimum" traveling distance for a secondary graduate is less than 10km (figure 5.10) it is clear that the number of schools is larger than if the optimum is fixed at 25 km. (figure 5.7).

Two indices were applied in the study of the demand of preparatory schools in the area of interest. In the first case the heterogeneity of the geo-subspaces was interpreted as a measure of the expected degree of interaction within a catchment area. Since the degree of interaction is a measure of the flow of secondary graduates, this characteristic of the geo-subspaces can be used as a factor in the analysis of the impact of the location of new schools. The second index is a measure of the local variation of the temporal stability of the demand. In this case, the demand's stability or unstability in a region can be used in similar studies.

In brief, the study of the local variation of the geo-subspaces that form the area of study allows the planner to include into the decision process, spatial aspects such as the degree of communication of the settlements, the level of interaction within a neighborhood and the local variation of the temporal stability, that were not previously considered.

Chapter 6.

CONCLUSIONS

6.1 Summary

A common criticism of the mathematical models used in human geography is that in many cases key features that are considered essential for geographical analysis purposes are not represented. This limitation has been discussed with regard to factor analytic models and statistical inference in section 2.1.1 (see also Haining, 1983). Neighborhoods is one of the geographical concepts that had received little attention by modelers until the latter part of the quantitative revolution. Although there are several mathematical models that that have been specifically designed to represent geographical neighborhoods, there has been no general attempt to establish an overall framework for the development of these tools. Therefore, as mentioned in the introduction, the principal objective of this work has been to present a general approach to the modeling of the notion of geographical neighborhood as well as to develop mathematical representations of it.

Three different levels of modeling are found in the thesis. In the first and most general, the notion of neighborhood model has been introduced through the mathematical concepts of space and subspace. Second, the local variation of geo-subspaces has been modeled through the heterogeneity index. Finally, two neighborhood models have been developed and applied to specific situations: the design of regionalization algorithms and the definition of spatial criteria for school location planning.

The first level of modeling is based on the intuitive notion that subjacent to the geographical landscape there are spaces and subspaces. In the development of mathematical theory, spaces and subspaces play a fundamental role.

Since the mathematical concepts of space and subspace satisfy certain requirements that make them appropriate as elements of representation of the geographical notion of neighborhood, two corresponding quasi-mathematical structures, geo-spaces and geo-subspaces, have been introduced to further the objective of establishing a general framework for the design of neighborhood models.

A general approach to the design of neighborhood models has been discussed, and tools have been developed to the study of geo-subspaces. Although other existing techniques such as autocorrelation and geostatistics have incorporated neighborhoods in models with predictive purposes, the present research is based on the assumption that the concept of geographical neighborhood has not been fully modeled previously.

With this idea in mind a measure of "local variation" of a geo-subspace has been defined. Although the measure itself resembles that of statistical variance, in this case the treatment is non-inferential. In fact, formalization has been carried out in topological rather than in statistical terms.

Applications of topological concepts in spatial analysis are found in several branches of geography including geomorphology (Mark,1977), geographic information systems (Dutton, 1968) and transportation (Haggett and Chorley, 1969). The topological entities on which these applications are based are graphs. There are however other topological concepts of interest for geographical purposes. One of the main sub-branches in topology is general, or set, topology. Based on the mathematical concept of neighborhood, in set topology, concepts such as that of limit and continuity are extended to abstract sets (Firby and Gardiner, 1982). Topological concepts such as the boundary or interior of a region, open set and neighborhood are used to model the geographical concept of neighborhood.

Here, the introduction of a topology to an specific element of a geospace, a graph, has allowed the identification between the geographical and topological concepts of neighborhood. This might be expected since both notions have their origin in the same conception of nearness to a point or entity.

The idea of fuzziness as developed by Zadeh (1965) has been incorporated to a topological space through the heterogeneity index. This result suggests the possible development of a new mathematical structure, Fuzzy Topology.

In the third level of modeling, neighborhood models have been applied in two instances: the design of regionalization algorithms and the definition of criteria for the location of schools.

The algorithms designed are presented as examples of the use of the neighborhood approach. As mentioned previously, algorithms have to be designed according to the problem at hand. That is, the algorithms that have been presented are not necessarily adequate for every regionalization problem, although for the specific case of a central agglomerative procedure the application of the heterogeneity index has been fully described.

The fuzzy set algorithm presented provides the analyst with information which is not available when a bivalent logic is used. A degree of membership of an element to a region has been given as a result of the classification procedure. It is important to note that geographical concepts have been previously modeled using fuzzy sets (Gale, 1972 and Leung, 1982). However, in the design of classification algorithms that include a contiguity constraint the use of fuzzy concepts has posed some special problems. Contiguity is a characteristic that has been considered essentially bivalent, although in some cases (Cliff and Ord, 1973) quantities such as the length of the border have been used as a measure of "contiguousness." In the algorithm discussed here the fuzziness refers, in topological terms, to the degree of "interiority" of a point to a set. As a result, the "membership" of an element to a region has not been described in terms of its "membership" to contiguous regions but rather with respect to its degree of membership to the border or the interior of the region.

In the second application a neighborhood model has been designed to aid in a school location problem. In this particular area of educational planning the use of spatial models has been scarce, although indicators that include some spatial criteria are generally used as an aid in the selection of sites to establish new schools. In this case the heterogeneity index has been applied to aid the study of some spatial and temporal aspects of the demand for preparatory schools. The spatial interaction among settlements and the temporal stability of demand are two spatial factors that have been pointed out as important indicators in the analysis of alternatives for the allocation of resources.

6.2 Discussion

It has been stated that three levels of modeling have been the concern of this study: 1) the establishment of a general framework for the design of neighborhood models; 2) the design of tools for the study of geo-spaces and 3) the application of neighborhood models to specific geographical problems.

This concluding section contains some brief and somewhat speculative remarks on the general importance of each level of analysis.

At the third and most detailed level of modeling two neighborhood models were applied to geographical problems: regionalization and school location planning. Although in neither case was the use of the technique exhaustive, the results obtained indicate that the mathematical modeling of geographical landscape through "neighborhoods" constitutes a fruitful avenue of inquire for both applied and basic research purposes.

At the second tool design level, it was the development of the heterogeneity index as a measure of local variation of a geo-subspace that made the third-level applications possible. It would seem to follow that the same conceptual tool could be applied not only to similar contexts in the future but also to the study of the set of neighborhoods that conform a geo-space. One obvious area of exploration would be to substitute neighborhoods for the entities used in existing models. For example, a measure of correlation could be applied to two or more sets of neighborhoods of one or more geo-spaces. Similarly, the representation of a geo-space through a topological space with fuzzy characteristics, raise the possibility of using topological and fuzzy set theory for a thorough study of the geographical landscape. In this case, even though formalization was achieved by identifying geographical entities with mathematical ones, the use of the mathematical models themselves was not extensive. It must therefore be acknowledged that the strengths and weaknesses of the application of topological and fuzzy set theory to geographical problems remains largely unexplored.

Finally, from a more general point of view, this thesis illustrates the kind of discoveries that can be expected from high-level communication and interaction among two or more fields of inquiry. In this particular case the geo-mathematician finds, on one hand, a previously unknown universe of applications of abstract mathematical theory, and on the other hand, the equally unsuspected possibility of modeling the fundamental notion of geographical neighborhoods.

APPENDIX A

MATHEMATICAL CONCEPTS

This appendix contains the mathematical details of the definitions of a topology in a connected graph, as presented in chapter 3. The definitions of some mathematical concepts necessary for the discussion are given in the first section.

A.1 Mathematical Definitions

1) A graph with m points and q lines is called a (m,q) graph.

2) Walk of a graph.
   A walk of a graph is an alternating sequence of points and lines V0,X1,V1,X2,...,Vn-1,Xn,Vn beginning and ending with points, in which each line is incident with the two points immediately preceding and following it.

3) Path of a graph.
   A path of a graph is a walk where all the points

(and thus all the lines) are distinct.

4) Connected graph.

A graph G is connected if every pair of points are joined by a path.

5) Subgraph.

A subgraph of G is a graph having all its points and lines in G.

(Harary, 1972, pp.11-13)

6) Difference.

For this particular application the difference between two graphs G1 and G2 is defined as follows:

The lines in G1 - G2 are all the lines that belong to G1 and do not belong to G2. That is:

$$X(G1-G2) = X(G1) - X(G2)$$

The points in G1-G2 are those that are represented in X(G1-G2).

7) Topology

Let X be a given set of objects called the points of X.

A topology in X is a non-empty collection of subsets of X called open sets satisfying the following four axioms:

Ax. 1 The empty set is open.

Ax. 2 The set X itself is open.

Ax. 3 The union of any family of open sets is open.

Ax. 4 The intersection of any (and hence of any finite number of) open sets is open.

A set is said to be topologized if a topology has been given in X. A topologized set X is called a topological space and the topology T is called the topology of the space X (Hu, 1964, p.16).

In this case the set of interest is a connected graph G with points V(G) and lines X(G). To apply the concept of topology to G, the concept of subset is identified with that of subgraph.

To exemplify some of these definitions assume that G1, G2, G3 and G4 are four graphs as shown in figure A.1.

By definition G5 = G1 U G2 is such that:

$V(G1 \cup G2) = V(G1) \cup V(G2) =$

$= \{ V1, V2, V3, V4, V5, V6\ V7 \}$ and

$X(G1 \cup G2) = X(G1) \cup X(G2) =$

$= \{(V1,V2),(V2,V3),(V3,V4),(V3,V5),(V2,V6),(V6,V7)\}$

G5 is shown diagrammatically in figure A.2.

The intersection between G1 and G3 ($G6 = G1 \cap G3$) as defined in Chapter 3 is such that:

$X(G1 \cap G3) = X(G1) \cap X(G3) =$

$= \{(v1,v2),(v2,v3)\}$

and it points are those represented in $X(G1 \cap G3)$ so that $V(G1 \cap G3) = \{v1, v2, v3\}$. G6 is shown in figure A.2.

As another example of the intersection of two graphs consider $G7 = G3 \cap G4$. In this case since $X(G7) = \emptyset$, G7 is the (0,0) graph.

The difference between G1 and G3 ($G8 = G1 - G3$) is such that:

$X(G8) = X(G1) - X(G3) =$

$= \{(v3,v4),(v3,v5)\}$

and $V(G8) = \{v3,v4,v5\}$

$= V(G4)$

Moreover, as expected $G1 - G4 = G3$, $G3 \cup G4 = G1$ and $G3 - G1$ is the (0,0) graph since $X(G3) \subset X(G1)$.

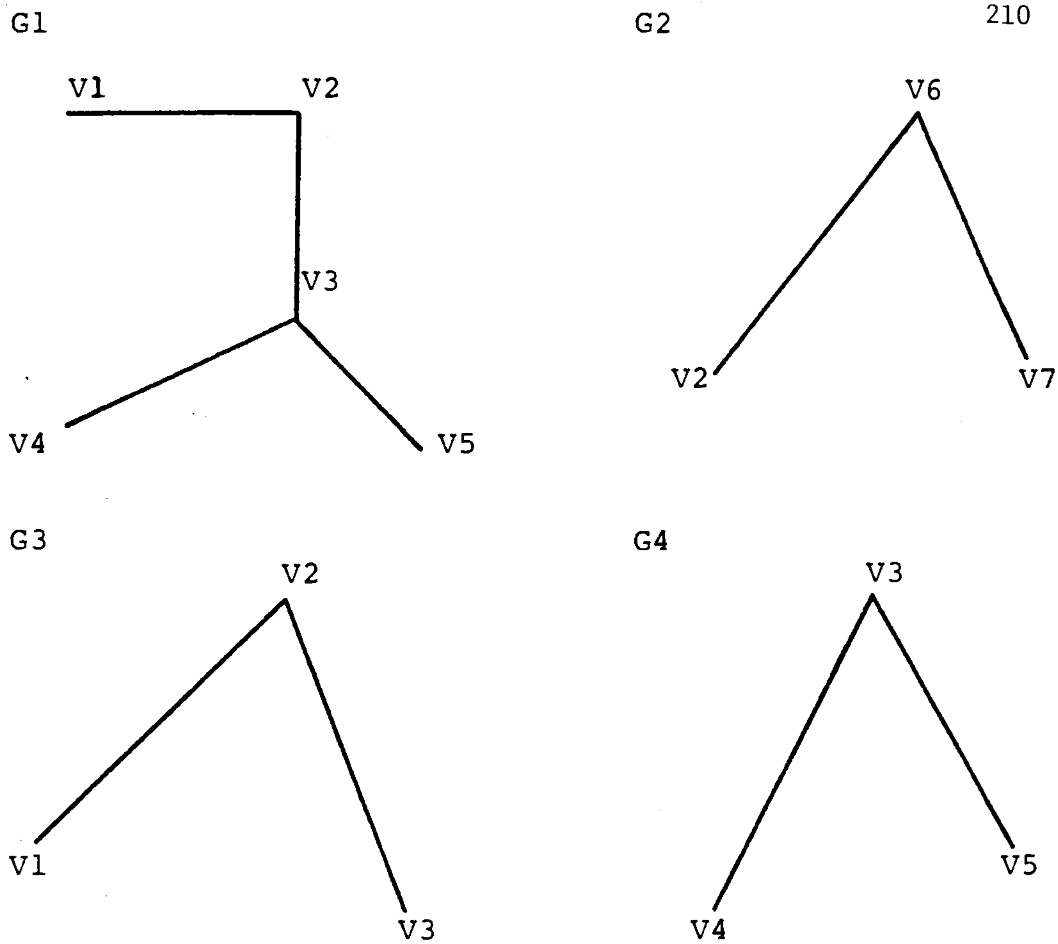


Figure A.1

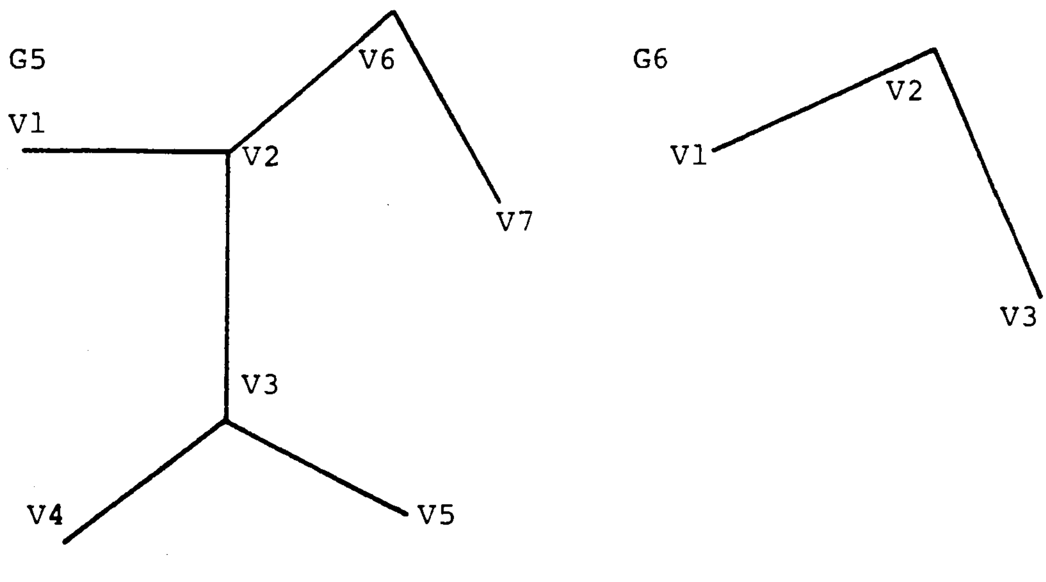


Figure A.2

A.2 Mathematical Discussion

Given (m,q) a connected graph G such that $q \neq 0$ then the following propositions are true:

Proposition 1. The collection of open sets as defined in section 3.4 is a topology of G

Proof:

Ax.1 The (0,0) graph is an open set. This is true by the empty condition.

Ax.2 i) G is an open set. By definition G is a subgraph of G.

ii) Let $p \in V(G)$. Since G is connected and $q \neq 0$, there exists a point p1 such that $(p,p1) \in X(G)$. Let subgraph N be defined as: $V(N) = \{p,p1\}$ and $X(N) = \{(p,p1)\}$. N is a non-empty connected subgraph of G such that $V(N) \neq \{p\}$ and $p \in V(N)$. Therefore G is an open set.

Ax.3 The union of any family of open sets is open.

Let 01, 02, ...On be a family of open sets of G and

$$O = \bigcup_{i=1}^{n} Oi$$

i) By definition $V(O)= \bigcup_{i=1}^{n} V(Oi)$ and $X(O)= \bigcup_{i=1}^{n} X(Oi)$.

Given $p \in V(O)$ and $(p1,p2) \in X(O)$, there exist open sets Oi and Oj such that $p \in V(Oi)$ and $(p1,p2) \in X(Oj)$. Since Oi and Oj are subgraphs of G, then $p \in V(G)$ and $(p1,p2) \in X(G)$. Therefore all the points and lines of O belong to G and O is a subgraph of G.

ii) Given $p \in V(O)$ there exists Oi such that $p \in V(Oi)$. Since Oi is an open set there exists a non-empty connected subgraph N of Oi such that $V(N) \neq \{p\}$ and $p \in V(N)$. However N is also a connected subgraph of O since $V(N)\subset V(Oi)\subset V(O)$ and $X(N)\subset X(Oi)\subset X(O)$. Therefore for every point p in O there is a non-empty connected subgraph N such that $V(N) \neq \{p\}$ and $p \in V(N)$. Therefore O is an open set

Ax.4 The intersection of any two open sets is open.

Let O1 and O2 be two open sets and $O = O1 \cap O2 = (m,q)$ where $q \neq 0$.

i) By definition $X(O) = X(O1)\cap X(O2)$ and $V(O)$ are all the points that belongs to at least one pair in $X(O)$. Let $(p1,p2) \in X(O)$, then $(p1,p2) \in X(O1)$ and (p1,p2) $\in$ X(02). Since 01 and 02 are subgraphs of G, (p1,p2) $\in$ X(G) and let p $\subset$ V(0), then p $\in$ V(01) and p $\in$ V(02). But 01 and 02 are subgraphs of G, then p $\in$ V(G). Therefore all the points and lines of 0 belong to G, so 0 is a subgraph of G.

ii) Let p1 be a point in 0, p1 $\in$ V(0). By definition there exists a line in 0, (p,p1) such that (p,p1) $\in$ X(01) and (p,p1) $\in$ X(02). Let N be defined as follows: V(N) = {p,p1} and X(N) = {(p,p1)}. N is a nonempty connected subgraph of 0 such that V(N) $\neq$ {p} and p $\in$ V(N).
Therefore 0 is an open set.

Proposition 2. For every point p $\in$ V(G) the subgraph formed by its first order neighbors and the lines joining them to p, is a topological neighborhood of p.

Proof:

Let p1,p2, ...pn be the set of neighbors of p. Define N as follows:
V(N) = {p, p1,p2, ...pn}. (V(N) $\neq$ {p} since G is connected) and X(N) = {(p,p1), (p,p2), ...(p,pn)}. Let pi be an arbitrary neighbor of p and U a subgraph of N such that V(U) = {p,p1} and X(U) = { (p,p1) }. U is an

open set such that p is a point of U and U is a subgraph of N. Therefore N is a topological neighborhood of p.

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