

BERTRAND RUSSELL'S THEORIES OF DENOTATION
FROM AN ARISTOTELIAN PERSPECTIVE

by

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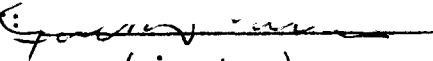
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RUSSELL'S THEORIES OF DENOTATION
FROM AN ARISTOTELIAN PERSPECTIVE

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ABSTRACT

This thesis deals with issues that came to mind when I studied Russell's work on sentences that meet three tests: first, that the surface grammar contains definite descriptions; second, that the subject of the proposition in question does not exist; and, third, that the proposition in question contains a denial of a predicate, e.g., 'The present King of France is not bald'.

Russell's approach requires the solving of a puzzle regarding the law of excluded middle. His solution involves a propositional function, primary and secondary occurrences, and denial.

I use Aristotle's approach to shed light on what Russell is doing and how he is doing it. Key differences in the approaches of the two philosophers include, first, the nature and genesis of abstract knowledge, second, how the mind functions in the composition of propositions, and, third, how to evaluate the truth conditions of denials when a predicate is denied of a non-existent subject.

In the exposition of his 1905 theory of denotation, Russell poses a puzzle about the law of excluded middle. Russell formulates this law thus: 'A is B' or 'A is not B' must be true. I argue that, in light of an Aristotelian analysis, Russell's treatment of this puzzle manifests an understanding of the law of excluded middle thus: in propositions opposed as 'A is B' or 'A is not-B', if one of the disjunctions is analyzed as true, then the other disjunct must be false.

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My greatest philosophical gratitude goes to my friend Drake O'Brian whose help through Russell's paradoxes, theory of types, and the first volume of *Principia Mathematica* was indispensable to my sanity. I do not directly consider Russell's paradoxes or theory of types in my thesis, yet the perspective from which my analysis proceeds was crystallized as a result of confrontation with aspects of them in the light of Drake's work on 'Set Theory from an Aristotelian Ground'.

Finally, I thank my monastic family for so patiently supporting and putting up with my absences from home. Very special thanks to Father Alban for his help in my final revisions; and to Brother Thomas, my "humble nit-picker", whose tireless efforts expunged the majority of my grammatical lapses. Brother Thomas is also responsible for the presence of Greek text in my thesis.

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INTRODUCTION

Even though I came from a philosophical environment where Aristotle's works were prominent, it was the study of Bertrand Russell's philosophy that crystallized certain aspects of Aristotle's philosophy for me. When I began writing this thesis I did not intend to make any mention of Aristotle. If for no better reason, this was because of my lack of understanding of Aristotle's teachings on denotation. Reflection on Russell's paradoxes, his treatment of denotation, and the insights of my friend Drake O'Brien, led to certain intuitions about Aristotle's teachings. I returned to Aristotle's writings: they confirmed many of my intuitions and triggered others about Russell's teachings. This thesis is the result of following through on some of these intuitions.

I see Russell's theory of denotation as having a broad base and coming to a point — as a kind of pyramid. Its broad base is Russell's reflection on the issues involved in denotation and his unsatisfactory attempts at formulating a theory to meet his requirements. The primary issues which Russell saw as connected with denotation were those of generalization and the relation of denoting phrases to their objects. How is one term able to stand in for many individuals? and how do such phrases as 'all men', 'every man', 'a man', 'some man', 'the man', relate to their objects? Many of Russell's reflections on these issues and an attempt at formulating a theory of denotation are recorded in [1903]. Russell's reflections come to maturity in his [1905b] theory of denotation. I see this [1905b] theory of denotation as culminating in its analysis of certain propositions. These propositions contain, according to surface grammar, a definite description which has no denotation. These propositions also deny a predicate of the said definite description. Russell's favorite example is 'The present King of France is not bald.'

My thesis, too, develops from a broad base and comes to a point in an examination of Russell's analysis of 'The present King of France is not bald'. I begin by presenting a non-technical or commonsensical Aristotelian account of denotation to serve as a background to examine Russell's theories of denotation. Next, I give a short sketch of the philosophical environment from which Russell's theories of denotation arose — namely, the views of T.H. Green, F.H. Bradley, and G.E. Moore. Understanding the philosophical milieu in which and out of which Russell was working sheds light on why Russell considered denotation such an important issue. My historical understanding of these men and this period relies heavily upon Peter Hylton's *Russell, Idealism, and the Emergence of Analytic Philosophy*.¹ I then examine Russell's [1903] theory of denotation and try to present his own account of the problems and issues connected with denotation. Finally, Russell's [1905b] theory of denotation is analyzed.

In his exposition of the [1905b] theory of denotation Russell poses a puzzle about the law of excluded middle which he hopes any legitimate theory of denotation should be able to solve. Russell formulates the law of excluded middle as

"A is B' or 'A is not B' must be true"(110).

The statements that form Russell's puzzle about this law are 'The present King of France is bald' and 'The present King of France is not bald'. The puzzle is that both statements seem to be false, so how can the law of the excluded middle be universally valid? In the course of giving a solution to this puzzle Russell says that 'The present King of France is not bald' must be interpreted as 'It is false that there is an entity which is now King of France and is bald'. In this interpretation Russell takes a statement that contains a single denial of the predicate bald and interprets it as a statement containing two affirmations — a principal affirmation of falsity and a secondary affirmation of the predicate bald.

¹ (Oxford: Clarendon, 1990). I am greatly indebted to Greg Sherkoske for recommending this work.

I argue that in view of how Russell analyses denials his formulation of the law of excluded middle should read:

For two propositions understood as 'A is B' and 'A is not-B', if one is true the other will necessarily be false.

These two formulations of the law of excluded middle may not seem significantly different. From an Aristotelian perspective these two formulations have important consequences in the determination of the truth and falsity of sentences having non-existent subjects. From an Aristotelian perspective, these two formulations represent two different ways in which statements are opposed to each other. The former, 'A is B' or 'A is not B' must be true, represents two statements opposed as proper affirmation and denial; such an opposition necessitates that one of the statements must be true and the other false — even if the subjects of the statements do not exist. The latter formulation expresses an opposition which is of two affirmations both of which will be false if the subject does not exist.

I also attempt to demonstrate that an Aristotelian perspective offers a valuable tool for taking Russell's analysis of denials apart and shows why Russell's solution to his puzzle about the law of excluded middle seems to make sense — despite the apparent inconsistency of interpreting a single denial as a double affirmation.

My presentation of Aristotle responds to a request made by Bertrand Russell. Russell realizes his [1905b] theory may seem complex. He asks his reader not to be too quick in dismissing it on account of its apparent complexity:

I will only beg the reader not to make up his mind against the view — as he might be tempted to do, on account of its apparently excessive complication — until he has attempted to construct a theory of his own on the subject of denotation.
([1905b],119)

ARISTOTLE²

The problem of the one and the many occupies a central place in the philosophical systems of Aristotle as well as others of Russell's predecessors. Some philosophers approach this problem by beginning with the one and accounting for the many by the many's participating in the one. Aristotle's approach begins with the many and attempts to give an account of how the many can be one (Owens, 292). He attempts to account for the way many individuals can in some way be one in the areas of knowledge and predication. He seeks to do this without losing the primacy of the many individuals as the basis of knowledge of the oneness among them.

For Aristotle, experience of reality consists in interaction with existing individuals. Individuals are confronted through the senses and *as* individuals. The particular properties³ experienced in an individual are experienced as inhering as parts of an ordered whole, parts of "a definite abiding something" (Owens, 236). In general, when I speak of ordered combinations of properties, I take the *ordered* to be fundamental; it signifies a whole, a substance,⁴ which unifies, situates, and gives existence to the properties. Properties may be considered individually but they never actually exist by themselves.

² References to Aristotle are in general from:
THE WORKS OF ARISTOTLE, (Chicago: Encyclopedia Britannica, 1952)
Categoriae and De Interpretatione translator E. M. Edghill.
Analytica Priora, translator A. J. Jenkinson.
Analytica Posteriora, translator G. R. G. Mure.
Sophistical Refutations, translator W. A. Pickard-Cambridge.
Topics, translator W. A. Pickard-Cambridge.
Physics, translators R. P. Hardie, R. K. Gaye.
Metaphysics, translator W. D. Ross.
On The Soul, translator J. A. Smith.

These will be referred to respectively, as: Cat., de In., APr., APo., SR., Top., Phys., Meta., de An.. Text references are rounded off to the nearest 5 lines.

At times I quote from De Anima, books II and III, trans. D. W. Hamlyn, (Clarendon: Oxford, 1993), Posterior Analytics, trans. Jonathan Barnes, (Clarendon: Oxford, 1994). These offer more literal translation and often, I think, convey Aristotle's thought with greater clarity. When I do quote from these texts I will flag the reference with C.

³ I am using 'properties' in a very broad non-technical way.

⁴ For Aristotle "Substance, in the truest and primary and most definite sense of the word" simply indicates an existent individual (Cat.2a.10-15).

For Aristotle all human knowledge is rooted in the sense experience of actual individuals.⁵ Fundamental to this approach is the difference between sense knowledge and intellectual knowledge. Each of these has its respective memory and each deals with properties and ordered combinations of properties in distinctly different ways.

Sense knowledge results from interaction with individuals in the world. In sense knowledge individuals are known by being separated, in the knower, *from the physical matter of the individuals*. In sense knowledge the individual, as an individual, impresses itself in the knower and leaves an impression of its ordered properties: "as wax receives the imprint of the ring without the iron or gold, and it takes the imprint which is of gold or bronze, but not *qua* gold or bronze" (*de An.* 424a.15-20). In sense knowledge we attain a *this*, an actual individual which is perceptible through the senses. It is impossible to perceive universals in this way — "for no universal is a *this*" (*APo.* 87b.30-35_c). To say something is a universal is to say that it is predicable: individuals, whether substances or properties (and terms which specify them *qua* individuals) can not be predicates in the way Aristotle's philosophy deals with predication. Individuals can only be identified. In 'This is John Smith', the individual's proper name is in the predicate spot but this is not predication. Individuals can only be the targets of predication. It is only by the separation which occurs in intellectual knowledge that substances and properties are considered as predicable.⁶

In intellectual knowledge properties and ordered combinations of properties are separated not only from the physical matter of the individuals which have these properties but *from all individuals* themselves. This degree of separation allows for predication: 'John Smith is a man', 'John Smith is black'. When speaking of intellectual knowledge, I refer to properties (e.g., red, round, flat, etc.) and ordered combinations of properties (e.g., man, pig,

⁵ "Since there is no actual thing which has separate existence, apart from, as it seems, magnitudes which are objects of perception....unless one perceived things one would not learn or understand anything..." (*DeAn.* 432a.1-10_c) See also (*APo.* 99b.35-100a.1-10).

⁶ Aristotle calls an individual man a "primary substance". As a predicate, 'man' is a universal and is called a "secondary substance". (*Cat.* 2a.10-15).

butterfly) as 'concepts'. Aristotle sees the establishment of concepts as occurring through repeated confrontations with external things: from confrontation with many particulars a universal is established in the mind.

If a child who has never experienced a pig, directly through the senses or indirectly through discourse and pictures, sensibly experiences such an individual, he would be unable to know what kind the individual is specifically — since such an ordered whole of properties has never entered his experience. The child would recognize, generically, that it is an animal if he knows what other animals are. Before the child can recognize and predicate what kind of animal it is, he requires a concept of *pig*. The child would register in his sensible memory a likeness of the actual individual, i.e. a likeness of its individual characteristics considered as characteristics of a *this*.

Through repeated experiences of the actions of pigs,⁷ the child's mind stabilizes⁸ a concept containing intellectual representations of certain properties such as four-footed, curly-tailed, flat-nosed, able to run, grunt, sniff, to eat corn, to wallow and so on.⁹ Once the concept is established the child is able to recognize and predicate¹⁰ what this kind of thing is. The ordered unity of properties, present in individuals and separated in intellectual knowledge, is expressed in our language when we say *what* something is. In sense knowledge the content is individual. In intellectual knowledge the *what* is considered as predicable of any individuals possessing this whatness. Intellectual knowledge is about whatnesses — humanity, pigness, redness, flatness, etc.

⁷ Or experiences of the same pig, for that matter.

⁸ Through a process of picking up and separating determinations. This process collects determinations which are predicable of many and separates determinations that are particular to the individuals themselves.

⁹ I take intellectual representations as differing fundamentally from sensible representations somewhat as a verbal definition differs from a picture. A picture may accompany a verbal definition and they both may refer to the same individual but their manner of representations is nevertheless of a different order. Certain intellectual representations are by their very nature unrepresentable sensibly such as capacities: the best a picture could do is to represent an action.

¹⁰ If he has the capacity of language use.

Aristotle describes the establishment of certain concepts as "a rout in battle". The rout is first stopped by one's taking a stand, then another until "the original formation" comes to a stand in a concept (*APo.* 100a.15-20). I see this "battle" as being between the mind and the many individuals or individual acts. By its natural orientation, the mind seeks to discern the unity of common properties present in the many instances. It seeks this unity so as to form concepts that will serve as standards of recognition and predication of the many instances. The mind records the union of those properties that are common to many individuals. It separates those properties that are not common. It thus begins to bring the multitude to a unity. The "original formation" is the unity in a concept at which the mind finally arrives through experience of the unity of common properties present in many individuals.

The intellect continues the process of collecting common properties and separating uncommon ones from what is known; by this process the mind forms concepts of higher abstraction: as a species is to individuals, so is a genus to its species (*Cat.* 2b.5-25). The various levels of abstraction allow us to recognize and predicate similarities among actual individuals in the world. They allow us to recognize that many individuals are of the same kind generically and specifically. The process of abstraction provides us with the stable standards of measure for the sake of scientific investigation and categorization of things and events in the world.

The process of abstraction I take as being primarily a passive process. A passive process does not eliminate activities on the part of the patient. When one billiard ball strikes another the activity of the ball struck is its passive response. By the very nature of the causal interaction with things through sense knowledge, the intellect responds in such-and-such a way — this response is passive. The physical interaction between two billiard balls can serve as a metaphor for the passive intellectual activity that individuals known through sense knowledge activate in the mind. The mind is, in a sense, being activated by the external individual and its properties; the mind is responding according to its *passive* potentialities. The various levels

of abstraction are an objective response to individuals and their properties rather than a subjective *active* construction. Billiard ball A objectively sets in motion billiard ball B; billiard ball B only comes to rest¹¹ after the motion ceases. Similarly,¹² the processes of the mind are objectively set into motion by its interaction with individuals. The mind's passive activity comes to rest when a concept, a one over the many, is established.

Nevertheless, the mind is able to compose subjective constructions of concepts. It does this by combining known properties and making judgments about its combination. The combination and the judgment are both subjective activities. The mind can join properties in ways that may or may not correspond to actual individuals in the world. The mind can also compose unities of properties which are never experienced in individuals in the world — as for example, the conceptions of unicorns and golden mountains. Affirmation is a conjunctive activity of the mind. In affirmation the mind joins something to a subject. Denial is a separating activity of the mind. In denial the mind separates, takes away, something from a subject. Affirmations and denials are expressed in propositions. Propositions are true or false as the affirmation or denial corresponds or does not correspond to what is.

Linguistic signs denote by objectively communicating properties (e.g., red, bald), or ordered complexes of properties (Socrates, man, animal). By "objectively communicating" I mean that properties, and ordered complexes of properties, are referred to by linguistic signs in such a way that the speaker/writer and hearer/reader are able to have the same consideration of the object of reference — commonsensically it means that we are able to mean the same things by our words. This is easy to see in reference to actual individuals, such as to Mike Harcourt. At the same time, we often speak about things "on the basis of abstraction" (*APo.* 81b.1-5_c). We do this by means of such terms as 'man', 'animal', and 'red'. When we use and understand such terms, we presuppose a common ability to respond to individuals

¹¹ That is, potentialities collapse into actuality, or, to use more conventional Aristotelian language, it is actualized with respect to place.

¹² Though the passivity of the mind is not seen as physical in Aristotle's philosophy.

and their properties. We have the common ability to consider individuals and their properties in various levels of separation *from the physical matter of actual individuals* and *from all actual individuals* — i.e. various degrees of separation from immediate sense perception (*APo.* 72a.1-5).¹³

This does not mean that we all have identical sense experiences. It means that we have the same kinds of abilities to respond to actual things. One of the products of this capacity is the ability to refer to those things individually, specifically, and generically and in various manners of combination with other things of the same or of different kinds.

This common ability to consider ordered combinations of properties in different levels of separation from actual individuals is essential for understanding how Aristotle's philosophy analyzes propositions like "The present King of France is not bald".

¹³ The degrees of separation from sense perception are the degrees of abstraction and determine the order of predication. In sense perception we only confront definite individuals and properties of individuals. In sense knowledge we know these individuals perceived immediately through the senses through a mechanism of separation from the actual physical matter of the individuals. As the processes of the mind are activated by the knowledge of individuals the process of abstraction establishes concepts of species from which are established concepts of genera. The greater the abstraction the further away from sense perception the concepts are. As the common properties of the individuals of sense knowledge are stabilized in concepts of greater abstraction, the mind considers these properties, and combinations of these properties, in manners removed from the experience of these properties in the actual subjects from which they were originally encountered. The greater the abstraction the further removed are the objects of consideration removed from sense experience.

The process described with respect to the child's establishing the concept 'pig' is continued in the establishment of genus concepts such as 'animal'. As the actual pigs are related to the establishment of the species concept 'pig', so the many species concepts are related to the genus concept 'animal'. As there is a hierarchy of abstraction so there is a hierarchy of predication. Just as an individual pig can not be predicated of the species pig so a species (e.g., pig, horse, cow, man, etc.) cannot be predicated of the genus 'animal': "for the species is to the genus as subject is to predicate, since the genus is predicated of the species, whereas the species cannot be predicated of the genus." (*Cat.* 2b.15-25).

RUSSELL'S ANALYTIC BEGINNINGS

Russell began his philosophical career as an idealist. Later, through the influence of G.E. Moore, Russell abandoned his idealism. Moore led Russell into what he later described as "a revolt into pluralism" [1959, 54]. To understand the implications of this revolt for Russell's theories of denotation we must have a general understanding of that against which he revolted.

The idealism that Russell abandoned was primarily that of T.H. Green and F.H. Bradley. According to Hylton, both these philosophers maintained theological or at least mystical convictions (35, 59). The philosophies of Green and Bradley put forward versions of holism: they portray reality as a unity. Reality is viewed as a unity in which the apparent multitude of individuals of experience finds its explanation.

T.H. Green (1836-82)

For Green, reality is an expression of the self-conscious activity of a single eternal Mind (Hylton, 34, 41).¹⁴ Reality and Mind, in this vision, are not identical: reality is an activity of Mind. As an activity of Mind reality is viewed as relational. Reality is dependent upon, and unified by, the activity of Mind. Our sense experience of, and thoughts, about individuals in the world are seen as partial comprehensions of the expression of Mind's relational activity (41).

Through the process of knowledge, the human mind imposes a structure and form upon what is given through the senses. Such notions as space, time, causality, and substance are "formal conceptions" (33). The phenomenal world comprises the way things appear *to us*.

¹⁴ In footnote 31 on page 35 I cite Hylton [1984]; this is the only reference to this work. All references to Hylton in the text are to [1990].

Human beings are said to be able to participate in the eternal Mind's activity. They do this by means of the constitutive, unifying, role human thought plays in its knowledge of the apparent individuals of the world:

We are related to this being, not merely as parts of the world which is its expression, but as partakers in some inchoate measure of the self-consciousness through which it at once constitutes and distinguishes itself from the world. (35)

In thought and language, Green sees us as able to recognize and predicate a unity of a multitude of apparent individuals. Thus, thought and language are viewed as presupposing generalization. Since generalization in the acts of recognition and predication involves a one-to-many and many-to-one relation, thought and language are said to presuppose relations (30-31).

Since all reality has its true identity within the unity of Mind's activity, Green maintains that there are degrees of truth. Our mind's compositions of propositions are said to be true insofar as they accord with the activity of Mind, i.e., with the whole of reality (41).

This notion of reality, the presuppositions of knowledge, and degrees of truth are some of the things against which Moore and Russell are reacting.

F.H. Bradley (1846-1924)

F.H. Bradley's philosophy portrays reality somewhat differently from Green's philosophy. For Bradley, reality is not an expression of the activity of a universal and relational Mind. For him, reality is a single non-relational unity. Bradley speaks of the Absolute rather than of Mind. The Absolute is not separate from or related to reality. Reality and the Absolute are identical and non-relational.

To say that something is real, according to Bradley, is to say that it is not dependent upon any other being: "it exists absolutely and unconditionally" (Hylton, 54). This presents problems about the validity of knowledge. All knowledge is relational. Knowledge has its source outside the knower, so knowledge is not real.

Thought is seen to be defective because of its presuppositions. All objects of experience are experienced as unities; yet thought presupposes a separation. Thought presupposes the separation of *what* the object of thought is from *the fact that* it is. In thought we are aware of the distinction between *what* the thing we are thinking about is and *the fact that* it actually exists. How could we think if thought did not make such a separation? "Thought seems essentially to consist in their division" (Bradley, 143). Actually existent objects cannot be *in my mind*. My mind must separate *what* things are from their actual existence. It is only in acts of judgment that I unite what has been separated in thought (144-45). In contrast, reality is a unity. In reality *that* things are is not separate from *what* they are. Since thought presupposes such a separation, thought is, by its very nature, defective.

For Bradley, in contrast to Green, reality cannot be composed of thought, even divine thought. For Bradley, there is no composition at all: "For him, Reality or the Absolute is not beyond all phenomena or totally distinct from them. On the contrary, all phenomena find their place within the Absolute, but are there so changed as to form a unified whole in which all things are harmoniously combined ..." (Hylton, 57).

Thought is also seen as presupposing generality. Thought presupposes many-to-one and one-to-many relations. Like thought's separation of *what* and *is*, thought's presupposition of generality is seen as demonstrative of the defective nature of thought. Thought presupposes generality and relations because it is fundamentally defective. Given the inherent presuppositions of thought, it cannot serve as an adequate means of attaining or conveying the truth. Still, truth is said to have degrees.

Bradley's idealism is usually characterized as putting forth a doctrine of internal relations. According to the doctrine of internal relations, a thing's relations to everything else are essential elements of the being of that thing. If it lacked any of its relations it would be a different thing. Relations, in this interpretation of Bradley, are internally essential components of each thing. This is how Moore interprets Bradley's philosophy.

MOORE AND RUSSELL

Both Moore and Russell disagree with the idealists. For the purpose of this thesis, what is important is the positions they adopt in their reactions against Green and Bradley.

The forms of idealism against which Moore and Russell react rely very heavily on "the nature of mind or of thought [and] the presuppositions of experience or of knowledge" (Hylton, 106). These idealists avoid being labeled "psychologistic" by clinging to certain metaphysical commitments which undergird their philosophical systems. They are not overly concerned with the presuppositions of thought and knowledge. Their purpose is not to offer an account of the way Reality is known, that is, of how we pass from sense experience to knowledge of Reality. These philosophies start with a commitment to Reality and attempt to give an account of the way sense experience and knowledge relate to it.

If one is inclined to reject their metaphysical starting points, it is natural to consider their philosophical approaches as being too concerned with psychological matters (106-7). This seems to be the line of revolt that Moore and Russell assumed. Their revolt was, as Russell said, a "revolt into pluralism" [1959, 54] — a revolt into a metaphysical commitment to reality as composed of a plurality of individuals.

Against the holism of Green and Bradley, Moore adopts a pluralistic commitment to reality. For Moore, reality is composed of discrete individuals. Contrary to what Moore sees as Bradley's doctrine of internal relations, he maintains that all relations are external. Each individual in the universe is seen as independent of every other — in no way internally dependent on any other.

Moore's initial pluralism included propositions among its discrete individuals. Propositions are objective entities in no way dependent upon any subjective composition. By virtue of the independence of each entity in the universe (including propositions), truth and falsity

are treated as qualities of propositions and do not have degrees. Each proposition is true or false absolutely. Things are externally related in such-and-such a way or they are not (112-13).

Contrary to what is seen as the idealist's emphasis on psychological matters, Moore treats all such issues as outside the subject matter of philosophy, whose concern is truth (108). Moore maintains that all knowledge is immediate and presuppositionless (109-10). In knowledge, even of abstract things (such as judgments and propositions), our minds are said to be in direct contact with objects external to us. This is a rigid doctrine of a two-place interaction between knower and known. The mind is in direct contact with its object; just as in sense perception, the perceiver is in direct contact with the object of perception (127, 129). There are no presuppositions in knowledge. There is no formal activity of the mind of the knower that affects the objects of knowledge. The idealist's psychologistic approach can be seen as a three-place interaction. The interaction between the knower and known involves mediatorial actions on the part of the knower through which the object of knowledge is known.

Moore does not maintain that there is no distinction between kinds of objects; nor does he maintain that all the objects of knowledge exist. For him (as for Russell afterwards) the "fundamental and inclusive ontological category was being. All things are, or have being. Some things, those which are temporal, also exist. ... Among the atemporal non-existent entities which Moore explicitly recognizes are numbers, logical relations, and, paradigmatically, propositions" (Hylton, 130-132).

Hylton describes the metaphysics of Moore, adopted by Russell, as "Platonic Atomism". The 'atomism' part is easy to understand in view of Moore's commitments to the discreteness of each individual in the universe. Hylton adopts the term 'Platonism' as a label "for any view that freely accepts abstract objects (especially in the philosophy of mathematics)" (112). In a letter to Desmond MacCarthy, in August of 1898, Moore writes of his new philosophical

position: "I am pleased to believe that this is the most Platonic system of modern times" (137).

As I have mentioned on page 13, propositions are seen as objective entities. The contents of propositions, that is, concepts, are also seen as having this objective ontological status (109). The mind has an entirely passive relation to reality. Truth and falsehood are seen as objective properties of propositions in no way dependent upon any activity of the human mind.¹⁵ In his 1898 article "The Nature of Judgment", Moore writes:

A proposition is constituted by any number of concepts together with a specific relation between them ... And this description will also apply to those uses where there appears to be a reference to existence. Existence is itself a concept; it is something which we mean; and the great body of propositions, in which existence is joined to our concepts or syntheses of concepts, are simply true or false according to the relation in which it stands to them ... But if [the proposition that this paper is white] is true, it means only that the concepts, which are combined in specific relations in the concept of this paper, are also combined in a specific manner with the concept of existence. That specific manner is something immediately known, like red (pp. 180-1). (Hylton, 138)

In [1903] Russell's acceptance of Moore's Platonic Atomism is unconditional. Further, his endeavors with respect to mathematics are seen as inextricably bound to such a metaphysics. In the preface to [1903] he writes:

On fundamental questions of philosophy, my position, in all its chief features, is derived from Mr G.E. Moore. I have accepted from him the non-existential nature of propositions (except such as happen to assert existence) and their independence of any knowing mind: also the pluralism which regards the world, both that of existents and that of entities, as composed of the indefinite number of mutually independent entities, with relations which are ultimate, and not reducible to adjectives of their terms or of the whole which these compose. Before learning these views from him, I found myself completely unable to construct any philosophy of arithmetic, whereas their acceptance brought about an immediate liberation from a large number of difficulties which I believe to be otherwise insuperable. The doctrines just mentioned are, in my opinion, quite indispensable to any even tolerably satisfactory philosophy of mathematics, as I hope the following pages will show. ... Formally, my premisses are simply assumed; but the fact that they allow mathematics to be true, which most current philosophies do not, is surely a powerful argument in their favour. (xviii).

¹⁵ Moore says of truth, that it is a simple concept that "cannot be further defined, but must be immediately recognized" (Hylton, 135).

Russell, following Moore, assumes the immediacy of knowledge. Our minds are seen as completely passive with respect to the objects of knowledge — be these objects abstract or concrete. Russell calls this immediacy 'acquaintance'. In [1903] the notion of acquaintance receives little explicit attention. According to Hylton, Russell for the most part merely assumes it. Knowledge is not seen by Moore and Russell as in any way problematic. Philosophy does not have to concern itself with *how* we know — "we just do know" (110). When Russell discusses, in his preface, the indefinables of his logic and the clarity of vision he hopes to impart through his work, he says:

The discussion of indefinables — which forms the chief part of philosophical logic — is the endeavour to see clearly, and to make others see clearly the entities concerned, in order that the mind may have that kind of acquaintance with them which it has with redness or the taste of pineapple. ([1903], xv)

The relation of acquaintance is "one that holds between a mind and an object, equally without regard to anything else" (Hylton, 113-14).

On the non-metaphysical side of influence on Russell's budding analytic philosophy were Peano and later Frege. At the International Congress of Philosophy of July 1900 Russell became acquainted with the person and writings of Giuseppe Peano:

It became clear to me that his notation afforded an instrument of logical analysis such as I had been seeking for years, and that by studying him I was acquiring a new and powerful technique for the work that I had long wanted to do. ...

September 1900 was the highest point of my life. I went about saying to myself that now at last I had done something worth doing, and I had the feeling that I must be careful not to be run over in the street before I had written it down. I sent a paper to Peano for his journal, embodying my new ideas. With the beginning of October I sat down to write *The Principles of Mathematics*, at which I had already made a number of unsuccessful attempts. Parts III, IV, V, and VI of the book as published were written that autumn. I wrote also Parts I, II, and VII at that time, but had to rewrite them later, so that the book was not finished in its final form until May 1902. Every day throughout October, November and December, I wrote my ten pages, and finished the MS on the last day of the century ...([1951], 218-19)

The impact Peano's notation and logic had on Russell's philosophy is described in Russell's [1959]. The alterations that result from this contact are seen as giving a new foundation to Russell's philosophical project. Future developments are described as an evolution of the initial revolution caused by Peano (11). Soon Russell also became acquainted with the works

of Frege. Much to Russell's surprise, Frege had independently been developing a system of mathematical logic that bore striking resemblances to his own [1902b]. Frege's writings influenced the development of Russell's logical work as well.

Between 1900 and 1910 Russell and Whitehead devoted much of their energy to the construction of *Principia Mathematica*. Their aim "was to show that all pure mathematics follows from purely logical premises and uses only concepts definable in logical terms" ([1959], 74). Philosophically, the results were found to be both rewarding and disappointing. Russell discovered that the logical apparatus required for the project was smaller than expected (ibid.). It was also discovered that classes were unnecessary. These were the rewarding discoveries. The disappointment resulted from the realization that, through the application of Cantor's arguments concerning logical classes, contradictions could be produced from premises that had been held to be indisputable by all logicians since the time of Aristotle (ibid.)¹⁶. Russell's discovery of the logical impasse of these contradictions led Frege to abandon his attempt to deduce arithmetic from logic (ibid.).

Russell saw the apparent impasse in the paradoxes' being connected with the issue of denotation. His [1905a] theory of denotation was seen as the fundamental breakthrough out of the impasse of the paradoxes. More specifically, Russell saw the breakthrough as being accomplished in the [1905b] theory's ability to handle sentences containing grammatical subjects that do not exist.¹⁷

¹⁶ The contradictions which Cantor discovered, and Russell made famous, are about certain classes and their members. Such classes are said to be self-reflexive with their members: i.e., these classes are said to be contained within their own extension. The puzzles that arise from considerations of these classes Russell terms paradoxes. In [1908,136] Russell claims that there are an indefinite number of such paradoxes. The paradox which is given Russell's name goes something like this: the class of classes which are not members of themselves, is it a member of itself? If it is a member of itself then it cannot be the class of all classes which are not members of themselves since it would then contain a class, itself, which is a member of itself - which is a contradiction. If it is not a member of itself then it cannot be the class of all classes which are not members of themselves since it would then fail to contain itself. Either answer leads to a contradiction.

¹⁷ "... My work during 1905 was certainly better in quality and quantity than any I have done in the years before, unless perhaps in 1900. The difficulty [the paradoxes] which I came upon in 1901, and was worrying over all the time you were in Europe, has come out at last, completely and finally, so far as I can judge. It all came from considering whether the King of France is bald - a question which I decided in the same article in which I proved that George IV was interested in the Law of Identity. The result of this is that Whitehead and I expect to have a comparatively easy time from now to the publication of our book [*Principia Mathematica*], which we may hope will happen within four or five years." (From a letter to Lucy, 1906) ([1951], 277)

Russell makes similar claims about the relation of the solution to the paradoxes and his theory of definite descriptions in [1910a]. The connection between the two is not obvious since he also claims that his theory of types offers the solution.

As sketchy as it is, I hope this suffices as an introduction to the background of Russell's theories of denotation. Henceforth, my procedure is to give a detailed presentation, together with an Aristotelian analysis, of Russell's theories of denotation as presented in *Principles of Mathematics* [1903] and *On Denoting* [1905b]. I present the latter work in light of the developments of the issues surrounding denotation as articulated in Russell's later works on symbolic logic.

PROPOSITIONAL FUNCTIONS¹⁸

I begin with propositional functions because of their importance to both of Russell's theories of denotation. Russell describes a propositional function as an expression containing a real variable; the expression becomes a proposition once the real variable has been replaced by an apparent variable or by a constant.¹⁹ By 'real' variable Russell means one which is not bound by an existential, ($\exists x$), or universal, (x), quantifier. When variables are bound by a quantifier Russell calls them 'apparent' variables.

The general notion of *propositional function*, according to Russell, is indefinable (83.80). Propositional functions are objects of immediate acquaintance. That an object is indefinable or is an object of immediate acquaintance does not mean that we have a clear vision of it. It is the purpose of Russell's [1903] analysis to clarify the immediacy of our acquaintance with the indefinables. Russell does this for propositional functions by drawing our attention to a quality of propositions of which we are aware. Propositional functions are recognized by the feature of propositions which permits propositionhood to survive the replacement of terms by other terms (19.22). The replacement of one term for another yields propositions differing only with respect to the variable term: e.g. "Herbert is a pig" may become "Goldy is a pig", "the number 2 is a pig" etc. (ibid.). Given this property of propositions, the reality of a propositional function becomes clear when we replace a term with a variable, x : e.g., ' x is a pig'. We thus become acquainted with a propositional function which "expresses the type of all such propositions" (20.22). In this example the propositions resulting from the values we assign to the variable are said to be true or false depending on the replacement term. Of

¹⁸ All references to [1903] are indicated according to page and paragraph number respectively.

¹⁹ "... where there are one or more real variables, and for all values of the variables the expression involved is a proposition, I shall call the expression a propositional function." [1903, 12.13]

Almost identical descriptions of propositional functions can be found in [1903], 19.22; [1905b], 104; [1910a], 38; [1918], 230; [1919], 155-6.

propositional functions which are true for every value assigned we say they "all express implication such as 'x is a man implies x is mortal'" (20.22).

According to Russell [1903], the traditional analysis of propositions into subject and predicate is destructive of the proposition, which he sees as an undivided whole: "a proposition is ... essentially a unity" (50.54). On account of the traditional analysis's focus on subjects and predicates, it is said to have the "defect of omitting the verb" from its proper consideration in analysis (39.43.). Russell suggests a less destructive analysis of propositions. He divides propositions into *term* (subject) and *assertion* (something said about the term): "Thus 'Socrates is a man' may be divided into *Socrates* and *is a man*. The verb, which is "the distinguishing mark of propositions, remains with the assertion ..." (ibid.). When "robbed of its subject" the assertion is neither true nor false (ibid.).

By replacing a given term in a proposition with another term, and leaving the rest of the terms of the proposition unchanged, we obtain a class of propositions with constancy of form. The constancy of form, given by a propositional function, Russell sees as "a primitive idea", i.e. unanalyzable (89.86). Constancy of form is more fundamental "than the general notion of *class*" since the latter can be defined in terms of the former but not *vice versa* (ibid.).²⁰ This constancy of form relies upon the constancy of the assertion which, like the subject term, can itself be replaced by a variable. The assertion 'is a man' in the propositional function "x is a man" Russell replaces with a Greek letter: 'x is a man' becomes ϕx . In such propositional functions either variable can be replaced by a constant or be bound by a quantifier.

A *term* is "Whatever may be an object of thought, or may occur in any true or false proposition or can be counted as *one* ..." (43.47). Russell distinguishes two kinds of terms which he refers to as *things* and *concepts* respectively (44.48). The former are "indicated by

²⁰ As will be explained on page 36, Russell attempts to account for the generation of classes by means of propositional functions. E.g. The class of men is generated from the individuals of which the propositional function ' \underline{x} is a man' is true.

proper names", the latter by "all other words" (ibid.).²¹ In propositions, proper names function as subjects but never as predicates. Proper names are what "the proposition or some subordinate constituent proposition is about" (43,46). What is said about the subject in a proposition is a concept since concepts comprise "all other words" (44,48). This does not preclude concepts from functioning as subjects of predicates.

Nothing more is said of proper names, except that they are "to be understood in a somewhat wider sense than is usual, and things also are to be understood as embracing all particular points and instances, and many other entities not commonly called things" (44,48).

At least two kinds of concepts are distinguished: "those indicated by adjectives and those indicated by verbs" (44,48). The former are referred to as "predicates or class-concepts", the latter usually referred to as relations. Little elaboration is given of this division of concepts. Predicates "occur in propositions having only one term or subject"; they are characterized by their connection with denoting (45,48). Concepts which are indicated by adjectives, which he calls "predicates or class-concepts", would be illustrated in 'Socrates was a snub-nosed philosopher', 'snub-nosed' being an adjective and 'philosopher' being a class concept. An example of a concept functioning as a verb would be something like 'living' in 'Sally is living at S.F.U.' or 'running' in 'Herbert is running to greet Bertha'.

As mentioned on page 15, the contents of propositions, for Moore and Russell, are not words. *Words* are, in a simple sense, seen as symbols standing for things other than themselves. Propositions, unless they have merely linguistic reference, contain "the entities indicated by words" (47,51). Propositions are independent entities.²²

²¹ The ontological status of each term for Russell is worth keeping in mind. In accordance with Moore's all-encompassing notion of being, Russell states: "Being is that which belongs to every conceivable term, to every possible object of thought - in short to everything that can possibly occur in any proposition, true or false, and to all such propositions themselves. ...Numbers, Homeric gods, relations, chimeras, and four-dimensional spaces all have being, for, if they were not entities of a kind, we could make no propositions about them. Thus being is a general attribute of everything, and to mention anything is to show that it is." (427,449)

²² See footnote 54 on page 58.

An apparent difficulty in most of Russell's early philosophy is his usages of 'proposition'. As Hylton comments, it seems that much of Russell's work is mired in a confusion between use and mention (217).²³ 'Proposition' seems in one line to refer to a declarative sentence and in another to the objects of a declarative sentence. It seems as if Russell is unable to distinguish use and mention. If Hylton is right, Russell simply does not worry about the distinction. A declarative sentence and its object are seen as inextricably connected. On the one hand we have a proposition and its contents as objective entities; on the other hand we have the linguistic or mental counterparts. The relation between objective entities and their linguistic or mental counterparts is seen as so immediate that it is not of any significant concern.

Terms and propositions, as expressions which refer to entities, are seen by Russell as mediums completely transparent to their objects. A window is a medium between a perceiver and what is seen through it. The mediatorial role of the window in perception is not considered so long as it does not distort one's vision — though it is recognized that the window and what is seen through it are not the same. The window it is simply a *medium of direct acquaintance*. This is how Russell sees linguistic terms and propositions, namely, as so transparent that they are ignored (171,269). It is not that Russell is unclear about the distinction: "[i]t is, rather, because the assumed symmetry between the linguistic and the non-linguistic means that it is not important to keep the distinction clear in practice" (171).

Hylton speaks of Moore's and Russell's terms as mediums perfectly transparent to their objects. Passmore mentions that Moore was also reacting against any sort of Lockean theory of judgment in which the objects of thought were ideas, be the ideas mediums or not (Passmore, 202-3). As mentioned on page 14, during this period Moore and Russell see judgments as objective entities not dependent upon any active mental composition. Insofar as we consider Russell's terms as mediums, it must not be as mediums of *indirect reference* but

²³ Quine was the first to draw attention to Russell's use/mention confusion; as an example see Quine [1940], p.31 ff.

rather as mediums of *direct acquaintance* (i.e., of direct reference). (More will be said on this distinction shortly, on page 25.)

After 1905 Russell begins to alter certain features of his [1903] theory of judgment and truth. He begins to give greater attention to the divergence between a sentence's grammatical and logical form. In [1903] such a concern is not to the fore. Grammatical components, for the most part, are seen as indicative of the contents of propositions:

The study of grammar, in my opinion, is capable of throwing far more light on philosophical questions than is commonly supposed by philosophers. Although a grammatical distinction cannot be uncritically assumed to correspond to a genuine philosophical difference, yet the one is *prima facie* evidence of the other, and may often be most useful employed as a source of discovery. (46.42)

For Russell's theory of denotation this view does present problems. Admittedly, if this position were correct we would have an account of propositions and truth that is of the utmost simplicity. Following Moore, Russell sees truth as a property of propositions — a property with which we are acquainted when we are acquainted with propositions. In [1904a] Russell says that truth and falsity are properties of propositions as red and white are properties of roses (Passmore, 204). It is certainly simpler if we can do without any mental mediums of reference and judgment. Moore's position in this regard is summarized in his article 'Truth' in Baldwin's *Dictionary* (as quoted in Passmore):

Once it is definitely recognized that the proposition *is to denote* not a belief (in the psychological sense), nor a form of words, but the object of belief, it seems plain that it differs in no respect from the reality to which it is supposed merely to correspond, i.e. the truth that "I exist" differs in no respect from the corresponding reality "my existence". (Passmore, 203; emphasis added)

At least "officially", Russell in [1903] maintains that denotation is a single relation between concepts and the objects connected with them "in a certain peculiar way" (53.56). In [1903] Russell is at pains to emphasize that denotation is a single relation. The objects of denotation are certainly peculiar and at times are seen by Russell as identical with their terms.

In his zeal for objectivity of thought, Moore, according to Hylton, deliberately dismisses any activity of the mind in the composition of propositions. Moore and Russell soon alter their positions on this matter.

Russell may be aware of the difference between a declarative sentence and its corresponding proposition, but I will purposely avoid the use of 'proposition' when I mean a declarative sentence.

DENOTING

For Russell, the notion of denoting is "obtained by a kind of logical genesis from subject-predicate propositions, upon which it seems more or less dependent" (54.57). A denoting concept is said to denote the term "connected in a certain peculiar way with" it (53.56).

Denotation is a special relation for Russell. It is a single relation that holds between certain concepts and their objects. Russell does not speak of denotation as a relation holding between all concepts and their objects.

Russell's usage of 'denotation' is not as general as Moore's. In the quotation cited above on page 23 from Baldwin's dictionary, Moore speaks of a declarative sentence's denoting its object and being identical with what it denotes. I take this sense of 'denotes' to indicate a *medium of direct acquaintance*.

By a *medium of direct acquaintance* I mean that for Russell the word and its object are treated, for all practical purposes, as identical — as my perception of a tree through a window is considered as identical to my perception of the tree. When treated as such, we see declarative sentences, and their objects (which includes their truth or falsity) standing in a two-place relation of immediate acquaintance. This is akin to the two-place relation of perceiver and perceived in sense perception. In the act of knowing, the mind is entirely passive. The mind is in no way considering the proposition, the ontological object (which is expressed by the declarative sentence), as differing in any significant way from the declarative sentence. The declarative sentence is seen as transparent to the actual acquaintance. The truth or falsity of a declarative sentence is not seen as a correspondence with a proposition. Rather, a declarative sentence's truth or falsity is seen as a quality of the proposition with which we are acquainted.

When words and declarative sentences are considered as *mediums of indirect reference* the mind is actively considering objects *through, by means of*, words and declarative

sentences. Declarative sentences do not necessarily stand in a one-to-one relation with their objects.²⁴ According to this perspective a declarative sentence, its object(s), and its truth or falsity stand in a three-place relation. A declarative sentence is seen as referring to a fact.²⁵ Declarative sentences may or may not correspond to the facts to which they purport to refer. It is their correspondence, or lack of correspondence, which determines their truth and falsity.

In the quotation from Baldwin's Dictionary, Moore seems to be adhering to a view of declarative sentences as mediums of *direct acquaintance*. He speaks of a declarative sentence's denotation and its truth being identical to the reality "to which it supposedly merely corresponds".

Russell does not speak of all declarative sentences or terms as denoting their objects. For Russell denotation is a special relation between certain terms and their objects. Denotation is a relation between terms which function as indirect mediums of reference. Ordinary terms, by contrast, are direct mediums of reference.

Given Russell's and Moore's metaphysics, there is an inherent problem with one thing's indirectly denoting another. Reality is composed of nothing but discrete objects. Knowledge is an immediate acquaintance with such objects. There is no room for objects or mental processes that function as indirect mediums of reference — be they declarative sentences or terms. How can one object of knowledge, in a universe of discrete atomic entities, function as an indirect medium through which some other entity, or entities, is known or referred to? Platonic Atomism rejects the possibility of such objects being psychological; that would be to place one's foot on the slippery slope of the psychologism of the idealists.

Russell's account of denotation only covers what he calls 'denoting phrases'. Denoting phrases are composed of class-concepts preceded by one of the following six words *all, every,*

²⁴ In a direct acquaintance model they would be seen as standing in a one-to-one relation.

²⁵ It is proposing how something actually is or is not.

any, a, some, the. Denoting phrases are connected with their objects in "a certain peculiar way" (53.56). Such phrases always denote (56.58), but their objects and manner of denotation vary significantly.²⁶

What is the object of 'all men' or 'some man' in such declarative sentences as 'All men are mortal' or 'I met some man'? Russell recognizes that such denoting phrases do not have objects of immediate acquaintance.

The general truths of mathematics are characterized by denoting phrases. Russell must find a way of accounting for the denotation of such phrases if he is to establish the principles of mathematics on a logical basis.

These problems are connected with the problem of generality. If terms and their objects are immediate and indefinable, what is the object of general terms? How can a single term refer to many individuals at the same time? General terms have an indirect element; they can be predicated of many individuals. They seem somehow to stand apart from and be present in many individuals at the same time.

Central to how he deals with generality is Russell's notion of variables. How are variables able to function? Russell says that "The notion of the variable is one of the most difficult with which Logic has to deal ..." (5.6; 89.86). How can a variable stand for any individual in the universe?

Russell attempts to solve these problems by his theory of denotation (Hylton, 206). By means of this theory, Russell attempts to explain how one term can indirectly refer to another.

Russell gives a number of examples to illustrate how the various kinds of denoting phrases function.²⁷ In fact, few of these examples do contain instances of the kind of denoting phrases they are supposed to illustrate.

²⁶ This is a key point in distinguishing Russell's theories of denotation. In [1905b] it is fundamental to his theory that such phrases do not always denote.

²⁷ Geach makes clear that many of the notions Russell uses were developed by Medieval logicians with whose works Russell was obviously familiar: see Geach [1962].

Russell attempts to give an account of how each kind of denoting phrase denotes. In each of the various kinds of denoting phrase, Russell argues, it is the *object* of the denoting phrase that determines how the phrase denotes.

I give individual treatments of each of the first five kinds of denoting phrases, and Russell's summary of them, before offering any analysis. I treat separately denoting phrases of the *the* type, just as Russell does.

Denoting phrases of the *all* type are said to denote in the collective sense: i.e. the elements denoted are to be treated as a numerical conjunctive whole (58.60).²⁸

Denoting phrases that contain *every* denote in a distributive sense. This is said to be similar to the collective sense; a conjunction is denoted, but not merely a conjunction of individuals, rather a conjunction of statements. The illustrations he gives of how *all* and *every* function are, respectively: 'Brown and Jones are two of Miss Smith's suitors' and 'Brown and Jones are paying court to Miss Smith'. The latter statement denotes in a distributive or propositional-conjunctive sense. It is understood as the conjunction of two propositions — 'Brown is a suitor of Miss Smith' and 'Jones is a suitor of Miss Smith'. The collective denotation of 'Brown and Jones' is a numerical conjunction of elements of the previous statement, not the conjunction of propositions.

Denoting phrases of the *any* type are said to denote in an ambiguous manner. *Any* is said to denote ambiguously since its denotation "seems half-way between a conjunction and a disjunction" (57.59). Russell's example is 'If it was Brown or Jones you met, it was a very ardent lover.' In this statement it seems that Brown and Jones are denoted in a purely disjunctive manner — one or the other of the disjuncts makes the statement true. That is, 1) 'If it was Brown you met, it was a very ardent lover' or 2) 'If it was Jones you met, it was a very ardent lover.' Russell claims it is not so simple. The denotation is different from a mere dis-

²⁸ In a footnote on page 45, Russell states "I use all men as collective, i.e. as nearly synonymous with the human race, but differing therefrom by being many and not one. I shall always use all collectively, confining myself to every for the distributive sense. Thus I shall say "every man is mortal," not "all men are mortal.""

junction since "it [the denotation] implies and is implied by a statement concerning *both*" (ibid.) and it is irrelevant which one we should choose. It is not a matter of either 1) or 2)'s being true, but rather of 1) and 2)'s both being true — but neither 1) or 2) is specified in the original statement. Since *any*'s manner of denotation is in between a disjunction and a conjunction, its manner of denotation is called a "*variable conjunction*" (57.59).²⁹

The fourth manner of denotation is that signified by the presence of *a (an)*. Russell defines this manner of denotation as a "*variable disjunction*" (ibid.). His illustration is 'If it was one of Miss Smith's suitors, it must have been Brown or Jones.' Russell calls this manner of denotation a *variable disjunction* since, though it definitely has a disjunctive character — 'it must have been Brown or Jones', it does not necessitate either one of them. It is not true that it must have been Brown or it must have been Jones. It is not equivalent to the disjunction or the conjunction of these statements, except, as Russell notes, in a "very roundabout form: 'if it was not Brown, it was Jones, and if it was not Jones, it was Brown,'" which becomes rather difficult to enunciate when we have more than two terms in our disjunction (58.59). Russell concludes that this manner of disjunction "denotes a variable term"; i.e., whichever of the terms in the disjunction we pick out it does not necessarily denote this term or the other but either in a variable, ambiguous, manner (57-8.59). If we choose one of the individuals and apply the assertion merely to him, the resulting statement will be true. Still, Russell maintains that the original disjunction did not denote him or the other separately (59.60).

The last kind of denoting phrase to be presented in this section is *some*. Russell calls *some*'s manner of denoting *constantly disjunctive*. This is illustrated in 'Miss Smith will marry Brown or Jones.' One or other of the pair is said to be denoted which makes the statement equivalent to a disjunction of statements: Miss Smith will marry Brown or she will marry Jones. That is, she will marry *some* one of the two. Though it is silent as to which one will be

²⁹ Russell's analysis is apparent in the French equivalent of 'any', *n'importe quel*; literally 'no matter what', 'no matter which one'.

married, it denotes that one and only one will be married. This is why it is called a *constant disjunction* (58.59).

The constructions of each of these illustrations could be seen as picking out a two-membered class. Russell sees these same manners of denotation being applicable to infinite classes. Russell gives a clear summary of how each denoting phrase denotes a finite class, a , containing $a_1, a_2, a_3, \dots a_n$:

- (1) *all a's* denotes a_1 and a_2 and \dots and a_n . [collectively as a conjunctive whole, "the terms of a taken all together"] (58.60)
- (2) *Every a* denotes a_1 and denotes a_2 , and \dots and denotes a_n . [divisively, each is denoted individually]
- (3) *Any a* denotes a_1 or a_2 or \dots or a_n where *or* has the meaning that it is irrelevant which we take.
- (4) *An a* denotes a_1 or a_2 or \dots or a_n , where *or* has the meaning that no one in particular must be taken, just as in *all a's* we must not take any one in particular.
- (5) *Some a* denotes a_1 or denotes a_2 or \dots or denotes a_n , where it is not irrelevant which is taken, but on the contrary some one particular a must be taken. (59.61)

With some difficulty, and if one forms a mental picture of a collective set regarding (1) and (4), the various manners of denotation seem to make sense. We have examples for (1) through (5) if we take Russell's examples of Brown, Jones, and Miss Smith. Replace the conjunction or disjunction of Brown and Jones by the sequences of $a_1 a_2 \dots a_n$: as an example, for the *all* type of denoting phrase his example was 'Brown and Jones are two of Miss Smith's suitors,' this could be replaced by 'Brown and Jones \dots and Ward [where Ward is the n th in the enumeration] are n of Miss Smith's suitors'.

Despite Russell's account of the mechanisms by which these phrases denote, he has not yet broached the issue of how they are able to do so within his metaphysical and logical schema. The notion of denotation is introduced precisely to act as a tool within his metaphysics and his logic. This is where Russell's presentation becomes difficult to follow.

To situate the distinctions between the denoting phrases within his metaphysics, Russell says that each of the denoting phrases has a distinct object (61.62), rather than being distinguished by *the way* in which each kind of phrase relates to a single object.

Following Moore, Russell is opposing any philosophical view that excessively multiplies mediatorial entities or mental activities. He rejects any view whereby what distinguishes the different denoting phrases is the *relation* between the (unchanging) object and the denoting phrase, as opposed to where the different denoting phrases differ because each has a different object. In Russell's view, the objects, not the relation with an unchanging object, differentiate the various denoting phrases. Also, Russell opposes any view which would see the difference of these phrases as dependent upon how the mind considers a single object (e.g., the class of men). If this were true, it would mean that the denoting phrases' manner of denotation is somehow dependent upon subjective constructions. Further, if each had a different relation to the same object, then there would be several relations, that is, several distinct relations of denotation. Russell concludes that "denoting is a perfectly definite relation, the same in all six cases, and that it is the nature of the denoted object and the denoting concept which distinguishes the cases" (65.65). The nature of these objects, Russell admits, poses a number of difficult problems.

All men Russell identifies with the class of men: "*All men*, which I shall identify with the class of men, seems to be an unambiguous object, although grammatically it is plural" (62.62).³⁰ In this case it is impossible coherently to distinguish between denotation and identity. How can a denoting phrase *be* what it denotes and how can an object be grammatically plural especially within Platonic Atomism? More importantly, the theory of denotation is a response to the problems arising in the consideration of terms which are not, for all practical purposes, identical with their objects but somehow stand between one term and another or between one term and many things.

³⁰ Russell says that a concept in inverted commas or in italics signifies the concept itself rather than denoting what the concept is about, i.e., its proper object [1903, 53.56]. This is simply the distinction between use and mention. *All men*, or 'all men', signifies the concept not the object of the concept. Nevertheless he frequently uses italics merely as emphasis. This seems to be the way he is using italics here since he is focusing on the objects of these concepts, i.e., what the concepts are about.

It would seem that Russell means that the phrase *all men* denotes a collection of the extension of the concept *man*; but he in fact does not mean this.

An account of the objects of the other kinds of denoting phrases is "not so simple" as that of *all men*: "we may doubt whether an ambiguous object is unambiguously denoted, or a definite object is ambiguously denoted" (62.62). Russell gives a single account of the *a* kind of denoting phrase and leaves his reader to apply it to the other kinds.

The reader is asked to consider the statement 'I met *a* man.' What is implied by this statement is certainly "that what I met was an unambiguous perfectly definite man" (62.62). Russell says that "in the technical language which is here adopted, the proposition is expressed by 'I met some man' (ibid.)." The actual man met is said to be "specially denoted by *some man*". Nevertheless, this man does not form a part of this proposition. Neither is the concrete event of the meeting asserted in this declarative sentence:

What is asserted is merely that some one of a class of concrete events took place. The whole human race is involved in my assertion if any man who ever existed or will exist had not existed or been going to exist, the purport of my proposition would have been different. Or, to put the same point in a more intensional language, if I substitute for *man* any of the other class-concepts applicable to the individual whom I had the honour to meet, my proposition is changed, although the individual in question is just as much denoted as before. What this proves is that *some man* must not be regarded as actually denoting Smith and actually denoting Brown, and so on: the whole procession of human beings through the ages is always relevant to every proposition in which *some man* occurs, and what is denoted is essentially not each separate man, but a kind of combination of all men. This is more evident in the case of *every*, *any*, and *a*. There is, then, a definite something, different in each of the five cases, which must, in a sense, be an object, but is characterized as a set of terms [individuals] combined in a certain way, which something is denoted by *all men*, *every man*, *any man*, *a man* or *some man*; and it is with this very paradoxical object that propositions are concerned in which the corresponding concept is used as denoting. (62.62; emphasis added)

Before I give Russell's analysis of the *the* kind of denoting phrases I will offer three observations on this quotation.

First, Russell is supposed to be giving an analysis of "I met *a* man." He says that in his "technical language" this is expressed by "I met *some* man." Yet he does not explain what this technical language is. Nor does he explain why any instance of a declarative sentence con-

taining a denoting phrase of the *a* type is expressed by one containing a denoting phrase of the *some* type. Earlier he demonstrated "that it may often happen that there is a mutual implication ... of corresponding propositions concerning *some* and *a* ..." (61.61). Yet, according to his previous analysis, each of these different kinds of denoting phrases denotes differently. This difference of denotation is said to be on account of their denoting different objects. How can a denoting phrase of the *a* type be expressed by one of the *some* type if they are distinguished as having different objects?

True, both *a* and *some* were said to denote in disjunctive manners, but in distinctly different ways (57-8.59). The above quotation states that "what is denoted [by 'some man'] is ... a kind of combination of all men" and that "This is more evident in the case of *every*, *any*, and *a*." But the original proposition being analyzed was an instance of the *a* type of denoting phrase and this was said to be expressed by one of the *some* type. How can it be more evident for one of the *a* type?

It seems that what Russell is saying, in the above quotation, is that all of the denoting phrases denote the class of men but each in a different way. It would make sense if we interpret the "paradoxical object" of each denoting phrase as signifying *the different way* in which each of the phrases denotes the class of men: he says that this object "is characterized as a set of terms combined in a certain way" (62.62). But Russell explicitly rejects the interpretation that the phrases all have one object, (say) the class of all men, which they denote in different ways. He says that "denoting is a perfectly definite relation, the same in all six cases, and that it is the nature of the denoted object and the denoting concept which distinguishes the cases" (64-5.65).

Second, the contents of Russell's class of *men* supposedly affects the proposition containing *some man* in a peculiar way. The class includes every man who is, was, and ever will be, each of whom is said to determine the purport of the proposition. This conclusion leads him to the further conclusion that "*some man* must not be regarded as actually denoting

Smith and actually denoting Brown" since "what is denoted is essentially ... a kind of combination of all men." But previously, on the same page, he says that "the actual man whom I met [say Smith] ... is specially denoted by *some man*" when used in a statement. The *some* type of denoting phrase was characterized as a *constant* disjunction whose distinguishing mark is that "this disjunction denotes a particular one of them" (58.59).

Finally, his extension of the class *man* and the conclusion he draws from this extension is difficult to imagine. Russell claims that every past, present, and future man enters into "the purport of my proposition" containing "some man" — but how? It is possible that I could be mistaken about who the individual I met was. My mistake could include present and past men. So the purport of my statement, 'I met some man', may be altered by these two groups of men. But how could the existence or non-existence of future men actually enter into any consideration of this statement? It is possible to think of an example of 'some man' including present and future men in its scope: e.g., 'Some man will be the first to walk on Mars'. But when I make the statement 'I met a man' how can I be including future men within the purport of my statement?

Lastly, I turn to Russell's analysis of the *the* type of denoting phrase. The notion of *the* is of the utmost importance for Russell because the use of identity and the theory of definition depend on it (62.63). Russell's treatment of this notion also sheds light on his representation of class-concepts in general.

Russell's account of the relevance of *the* for identity and definition is short but important. The way in which *the* functions in definitions is said to account for "the adequacy of [denoting] concepts to deal with things" (63.63). Russell defines identity as a relation in which the referent and relatum are not distinct (64.64). Identity statements such as 'Edward VII is the King' are said to be significant since in one case the "actual term [the person himself] occurs, while in the other a denoting concept takes its place" (64.64). 'Edward VII is the King' is said to contain both the actual object and something that takes its place. In this case

we have a proposition containing both an actual object and a denoting term. What accounts for the unity of the proposition if not the mind of a composer? Propositions are objective entities for Russell. They are in no way dependent upon any subjective actions on the part of knowers: "in judgment the mind stands in a single and presuppositionless relation to one of these entities" (Hylton [1980], 129). Hylton notes that a general problem with Platonic Atomism is its inability to offer any justification for the unity of propositions (114, 258-9).³¹

To return to Russell [1903], *the* (when correctly employed in the singular) is said to denote a class-concept with "one and only one instance" — e.g., *the King*, *the Prime Minister*. This manner of denotation also includes the notion *at the present time*: "in such cases there is a method of denoting one single definite term [object] by means of a concept [e.g. *the King*], which is not given us by any other of the five words [kinds of denoting phrases]" (62.63).

By means of the individuating denotation of *the* Russell attempts to determine the object(s) denoted by denoting phrases or terms in general:

Every term [object of a denotation] is the only instance of *some* class-concept, and thus every term, theoretically, is capable of definition, provided we have not adopted a system in which the said term is one of our undefinables. ...

In most actual definitions of mathematics, what is defined is a *class* of entities, and the notion of *the* does not then explicitly appear. But even in this case, what is really defined is *the* class [treated as a single object] satisfying certain conditions; for a class...is always a term or conjunction of terms and never a concept. Thus the notion of *the* is always relevant in definitions; and we may observe generally that the adequacy of concepts to deal with things is wholly dependent upon the unambiguous denoting of a single term which this notion gives. (63.63)

Russell claims that by means of his definitions he is able to determine the objects³² of the other denoting phrases. This brings into relief how Russell wishes to consider what class-concepts are. Class-concepts are said to represent a collection *qua* a collection (69.71). They are not conceptual specifications having extension to many individuals. Russell is here treat-

³¹ See also Hylton [1984], p. 382: "If everything is, so to speak, object-like, what could be the source of the unity of the proposition? Anything one might put forward as an answer would turn out to be just one more item in need of unification. ... Russell's anti-Kantianism forbids an appeal to what is in any sense an act of unification or synthesis; and his metaphysics forbids any other kind of answer."

³² Objects are specified by "the class satisfying certain conditions" (63.63).

ing class-concepts as representational rather than extensional without offering any coherent account of how the members of classes, within his atomistic universe, are gathered together. It may seem commonsensical that we simply admit that many individuals happen to have the same properties: after all, we are acquainted with the individuals and with the properties. But how are we able to recognize that they are of the same kind, if each individual is discrete and we do not have a single stable standard by which to measure the sameness? Russell says that the members of classes "satisfy certain conditions," but how do we measure, how are we able to judge, that many individuals do satisfy these conditions unless we have some stable condition-specifying standard by which to measure and judge?

Russell does attempt to account for the membership of a class (and the generation of class-concepts) by means of propositional functions. The class of pigs is seen as that collection of entities of which the propositional function 'x is a pig' yields a true proposition. Yet it is even suspect to account for the membership of classes by propositional functions. Of propositional functions containing denoting phrases, Russell says that the term in the assertion (e.g., 'pig' in 'is a pig') must *be* a class-concept (as opposed to a predicate) if its value is to be a proposition. The difference between a class-concept and a predicate is that between *man* and *human* respectively.

The characteristic of a class-concept, as distinguished from terms in general, is that "x is a u" is a propositional function when, and only when u is a class-concept. It must be held that when u is not a class-concept, we do not have a false proposition, but simply no proposition at all, whatever value we may give to x. This enables us to distinguish a class-concept belonging to the null-class, for which all propositions of the above form are false, from a term which is not a class-concept at all, for which there are no propositions of the above form. Also it makes it plain that a class-concept is not a term in the proposition "x is a u" for u has a restricted variability if the formula is to remain a proposition (56.58).

Russell has already said that the notion of propositional function is more fundamental than that of class (40.44). Later, he claims that classes are generated by means of propositional functions (89.86). The presence of a class-concept is here said to be a necessary condition of propositional functions containing denoting phrases. Without a class-concept the

propositional function is said not to be able to have a value: in other words, without a class-concept it would not be a propositional function. It is fundamental to Russell's description of a propositional function that it become a proposition when its variable is replaced by a constant or by an apparent variable. Let us call this requirement that propositional functions which contain denoting phrases must also contain class concepts, *R*. In his chapter on classes, Chapter VI [1903], Russell explains class-concepts in a way that would make *R* circular: "*a* is a class-concept when '*x* is an *a*' is a propositional function" (74.74) — i.e., *a* is a class concept when it is an element of a denoting phrase in a propositional function. Yet *R* maintains that "*x* is an *a*" is a propositional function if and only if *a* is a class concept (56.58).

Russell is not blind to the problems in his treatment of classes. By his own admission he fails to decide whether a class is one or many (74.76). According to his Platonic Atomism and his theory of denotation, classes cannot be one or many according to the way they are subjectively considered. Russell admits in his preface that he is unsatisfied with the [1903] theory of classes and that "I have failed to perceive any concept fulfilling the conditions requisite for the notion of a class" (xv-xvi; Hylton, 233).

In summary, Russell's [1903] theory of denotation intended to define the function of denoting phrases within his system of metaphysics and logic.³³ Denoting phrases are so important for Russell precisely because they characterize the general formulations of mathematical formulae that he wishes to legitimate logically. Denotation is said to be a single definite relation, "the same in all six cases" (65.65). Denoting phrases are said to be distinguished by their respective objects, each object having the same relation of denotation between it and its denoting phrase. Russell's attempt to explicate the nature of "this paradoxical object" results in confusions that make his distinctions untenable. He appeals to the *the* type of denoting phrase as a necessary component in the specification and definition of the

³³ The two being for Russell inextricably combined (Hylton [1990], 205).

objects of the other denoting phrases. His explanations offer no aid to the difficulties surrounding the nature of the objects.

Before Russell begins to explicate the objects of the various denoting phrases, many of his explanations as to how they function in declarative sentences are compatible with an Aristotelian perspective. Russell's collective *all* and divisive *every* present no difficulty for Aristotle's philosophy, insofar as Russell's presentations limit themselves to how collections of individuals *are considered*. Similarly with Russell's presentations of *any*, *some*, and *a*. They can coherently be seen as specifying individuals or collections in various ambiguous manners. In each one of Russell's denoting phrases Aristotle would see the 'man', in a denoting phrase '... man', as specifying what kind of thing is being spoken of, and the *all*, *every*, *any*, *some*, *a* or *the* as indicating how the individual or individuals specified are to be considered within the context of the statement in which they occur.

My approach and method toward Russell and Aristotle thus far has been rather non-technical and commonsensical and of little interest in itself. For the purpose of this thesis, these considerations highlight problems encountered and how such problems are approached by Russell and Aristotle. Some of the more interesting philosophical implications of both Russell and Aristotle's approaches are brought into greater relief in the examination of Russell's [1905b] theory of denotation.

ON DENOTING

This is the principal theory of denoting I wish to advocate: that denoting phrases never have any meaning in themselves, but that every proposition in whose verbal expression they occur has a meaning. [1905b, 105]

Russell says that the [1903] theory "is quite different from the theory to be advocated" in this later work [1905b, 104]. Russell's new theory of denoting was developed and published in 1905.³⁴ In general, Russell develops and defends the position that a sentence's grammatical form does not always reflect its logical form and actual contents. In this respect, the 1905 theory is radically different from that enunciated in [1903]. Further, in [1903] the various types of denoting phrases are said always to denote. They differed from each other only with respect to their objects (56.58). In [1905b] a fundamental feature of the theory is that denoting phrases do not always denote.

As in [1903], phrases beginning with *a*, *some*, *any*, *every*, *all*, and *the* are called 'denoting phrases'. Also, as in [1903], after a general overview of denoting phrases, the first five are considered before the *the* type, which is given its special position.

Russell's [1905b] continues to maintain that denotation is a single notion. Instead of being a single relation between phrases and paradoxical objects, though, denoting phrases now denote "solely in virtue of [their] *form*" (103). He distinguishes three cases (i.e. three *forms*):

(1) A phrase may be denoting, and yet not denote anything; e.g. 'the present King of France'. (2) A phrase may denote one definite object; e.g. 'the present King of England' denotes a certain man. (3) A phrase may denote ambiguously; e.g. 'a man' denotes not many men, but an ambiguous man. (103)

The first two of these forms represent instances of the *the* type of denoting phrase; the third represents how the other five denote.

³⁴ Certain aspects of this theory appeared in some detail in [1905a], published three months before [1905b]. In [1905a] the theory of descriptions, specifically of negative existentials, is the same as that of [1905b]. Even the example of "the present King of France" is used ([1905a], 100-101).

Russell next deals with two notions, the first of which (being "ultimate and undefinable" (105)) gives the key to interpreting denoting phrases: "'C(x) is always true' and 'C(x) is sometimes true'". The latter is defined by means of the first: in its full articulation "'C(x) is sometimes true' is 'It is not true that "C(x) is false" is always true'" (104). The first of these may be symbolically representable as $(x)(Cx)$, the second as $\sim(x) \sim(Cx)$ or equivalently as $(\exists x)(Cx)$

In [1879, 27] Frege supplies the background for many of Russell's articulations and for a version of the symbolic notation. Russell adds the above notation to these considerations in his [1908] and [1910a]. It is helpful to keep in mind that, when Russell uses such articulations as 'is always true', this can be represented by a universal quantifier; and that 'it is not true that "... is false" is always true', 'not always false' and 'sometimes true' are each representable by an existential quantifier.³⁵

From this basis we can interpret "the most primitive of denoting phrases" — *everything*, *nothing* and *something*:

C(everything) means 'C(x) is always true';
 C(nothing) means "'C(x) is false" is always true';
 C(something) means 'it is false that "C(x) is false" is always true' [this latter being replaceable by the less complicated 'C(x) is not always false', or 'C(x) is sometimes true'] (104).

The 'C' in these statements is an assertion variable as in [1903], i.e., is "a statement about" its subject (107). These forms have no meaning in isolation "but every proposition in whose verbal expression they occur has a meaning" (105).

Russell takes up the same example as in [1903] to demonstrate how denoting phrases denote, namely, the interpretation of the statement 'I met a man.' According to his new theory of denoting, what is affirmed by the proposition (if it is true) is not that "I met some definite man" but "'I met x , and x is human" is not always false' (105).

Supposing that those objects with the predicate *human* constitute the class of men, Russell continues: 'C(a man)' means "'C(x) and x is human" is not always false'. Here Russell is

³⁵ In the conclusion I shall say more on the issue of existence and Russell's philosophical analysis of it.

giving the form by means of which the a type of denoting phrase is said to denote — if it denotes.

Contrast this with [1903]. In [1903] the denoting phrase 'a man', in 'I met a man', was said to be expressed by 'I met some man' whose analysis became lost in a confusion of paradoxical objects and whose purport contained the entire human race from start to finish. In [1905b] the denoting phrase is analyzed into a propositional function having at least one value — a vast simplification. The other four types of denoting phrases are dealt with in the same simplified manner.

Russell asks us next to consider 'All men are mortal' which is said to mean "'If x is human, x is mortal" is always true' (105). More generally, 'C(all men)' is said to mean "'If x is human, then C(x) is true" is always true' (106).

Contrary to [1903], the denoting phrase 'every man' in C(every man) has the same meaning as 'all men' in C(all men). Similarly, 'a man' and 'some men' both denote by means of the same form given above for 'C(a man)'.

Russell next turns to "the most interesting and difficult of denoting phrases" — those qualified by *the* (106).

Before turning to these "most interesting" cases, it is helpful to situate this new theory of denotation within Russell's metaphysics.

The opening and closing paragraphs of [1905b] express Russell's adherence to the fundamental tenets of his theory of acquaintance: "In perception we have acquaintance with objects of perception, and in thought we have acquaintance with objects of a more abstract and logical character" (104); "... in every proposition that we can apprehend ... all the constituents are really entities with which we have immediate acquaintance" (119).

Russell departs in [1905b] from his [1903] approach. In [1905b] Russell considers the part played by the process of thinking in relation to objects with which we are not acquainted: "All thinking has to start from acquaintance: but it succeeds in thinking *about* many things

with which we have no acquaintance" (ibid.). In [1905b] Russell does not in any way abandon the anti-psychologism assumed in [1903]; but he does begin to show interest in certain issues that he would then have considered unduly psychological — such as affirmation and denial (Hylton, 244).

What about the main reason for introducing the notion of denotation in the first place? How does Russell's 1905 theory differ in offering an account of the mediatorial role certain phrases play in some declarative sentences? According to Hylton, Russell does not, on this point, offer any elucidation. What he does is treat his new theory as a special case of the old theory. Here, Russell reduces his analysis of denotation to a consideration of propositional form. The issue of generality, i.e., how a single term is able to apply to many individuals, is not answered: it is simply accepted for variables. Still, how are variables able to function? What is it by virtue of which they are able to stand in for every individual in the universe? Within the overall dogmatic framework of Platonic Atomism³⁶ what is a variable? Is it something with which we are acquainted? If so, how does it mediate, stand in for, many entities?

After reading [1905b], Moore wrote to Russell concerning these points. On October 23, 1905 Moore wrote: "What I should chiefly like explained is this. You say 'all the constituents of propositions we apprehend are entities with which we have immediate acquaintance.' Have we, then, immediate acquaintance with the variable? And what sort of an entity is it?" (Hylton, 256). Two days later Russell replied:

"I am glad that you agreed to my main contentions in the article on Denoting. I admit that the question you raise about the variable is puzzling, as are all questions about it. The view I usually incline to is that we have immediate acquaintance with the variable, but it is not an entity. Then at other times I think it is an entity, but an indeterminate one. In the former view there is still a problem of meaning and denotation as regards the variable itself. *I only profess to reduce the problem of denoting to the problem of the variable.* This latter is horribly difficult, and there seem equally strong objections to all the views I have been able to think of." (ibid., emphasis added)

³⁶ To which Russell continues to adhere.

On the general issue of denotation, Russell offers no solution. He merely reduces "the problem of denoting to the problem of the variable" (Hylton, 256). As I have mentioned on page 39, there are, nevertheless, major differences in the two theories, most notably the difference that denoting phrases do not have to denote.

I now proceed with "the most interesting and difficult" of denoting phrases — Russell's theory of definite descriptions.

Objects denoted by *the* are said to have the property of uniqueness. Russell admits that we do at times use such phrases as "*the* son of So-and-so' even when So-and-so has several sons but", he concludes, "it would be more correct to say 'a son of So-and-so'" (1905b, 106). In [1910a] Russell is not so rigid regarding the "correctness" of the singular character of such denoting phrases. In any case, according to Russell [1905b], when we say 'x was *the* father of Charles II' what we assert is that 'x and only x begat Charles II'; and

'If y is other than x, y did not beget Charles II', or what is equivalent, 'If y begat Charles II, y is identical with x'. Hence, 'x is the father of Charles II becomes: 'x begat Charles II; and 'if y begat Charles II, y is identical with x' is always true of y' (106).

Whatever statement 'C' may be,

'C(the father of Charles II)' implies: 'It is not always false of x that "if y begat Charles II, y is identical with x" is always true of y', which is what is expressed in common language by 'Charles II had one father and no more' (107).

This uniqueness property of definite descriptions is representable symbolically as

$(\exists x)(Cx \ \& \ (y)(Cy \rightarrow y = x))$.

To demonstrate the applicability of his theory, Russell proposes three puzzles which he thinks any legitimate theory of denoting should be able to solve. I shall examine the first two puzzles.³⁷ These puzzles, especially the second, convey the heart of Russell's new theory of definite descriptions.

³⁷ The third puzzle is only given two paragraphs at the end of [1905b]; the analysis that it presents offers no new aspects of Russell's theory of denotation that are not given in the first two.

The first puzzle is about how the law of identity functions regarding the denotation of terms and phrases in statements: if *a* is identical to *b*, then either should be able to be substituted for the other in a declarative sentence without altering the truth of the sentence.

Russell presents an instance of a statement in which, according to surface grammar, there seem to be two terms with identical denotations, i.e., 'Scott' and 'the author of *Waverley*' in 'George IV wished to know whether Scott was the author of *Waverley*'. The puzzle is that these two terms, though identical in denotation, cannot be substituted for each other without altering the meaning and truth of the original sentence; e.g. 'George IV wished to know whether Scott was Scott' (110). Russell demonstrates that once the statement is analyzed into its full expression "any propositions in which [the author of *Waverley*] occurs ... do not contain the phrase, which has been broken up" (114). The sentence 'Scott was the author of *Waverley*', under analysis becomes 'There exists an entity, *x*, such that *x* wrote *Waverley*, and if *y* wrote *Waverley* then *y* is identical to *x*, and Scott is identical to *x*.' If *W* = wrote *Waverley* and *s* = Scott, 'Scott was the author of *Waverley*' can be represented as $(\exists x) [Wx \ \& \ (y) (Wy \rightarrow y = x) \ \& \ x = s]$. This sentence "does not contain any constituent, 'the author of *Waverley*' for which we could substitute 'Scott'" (114). The puzzle is solved.

Russell still maintains that "the truth inferences" of the statement are not affected by the *verbal* substitution of 'Scott' for 'the author of *Waverley*', so long as the denoting phrase has what Russell calls *primary* occurrence in the proposition (114).

Russell explains the distinction between *primary* and *secondary* occurrences as being the distinction between eliminating a denoting phrase from the subordinate proposition of which it is part or eliminating it from the entire proposition of which the subordinate proposition is part — the fact of eliminating it from the entire proposition being a *primary* occurrence and eliminating it from the subordinate proposition being a *secondary* occurrence (115). Different sentences result from the different *occurrences* that the denoting phrases have. From his examples it is clear that what Russell means by "eliminate this denoting phrase" (115)

refers to its disappearance under analysis into a proposition in which the phrase does not occur as a constituent:

...when we say 'George IV wished to know whether Scott was the author of *Waverley*', we normally mean 'George IV wished to know whether one and only one man wrote *Waverley* and Scott was that man' [*secondary* occurrence]; but we *may* also mean: 'One and only one man wrote *Waverley*, and George IV wished to know whether Scott was that man' [*primary* occurrence]. (114-115)

If we give parts of this quotation symbolic representation it makes things a bit clearer.

Let 'W' = 'wrote *Waverley*', 'Kg' = 'George IV wished to know whether', 's' = 'Scott'. Substituting, we obtain the following:

...when we say 'George IV wished to know whether Scott was the author of *Waverley*', we normally mean:
 $Kg(\exists x) [Wx \ \& \ (y) (Wy \rightarrow y = x) \ \& \ x = s]$
but we *may* also mean:
 $(\exists x) [Wx \ \& \ (y) (Wy \rightarrow y = x) \ \& \ Kg(x = s)].$

In both interpretations we are in fact dealing with what is meant by, what is contained in, what is communicated by means of, the surface grammar of the English sentence. This sentence, according to Russell, can be interpreted in two ways.

It seems that the key to understanding what Russell means by 'occurrences' is to note the scope of the existential quantifier in each occurrence. In the primary occurrence the quantifier ranges over the entire proposition; in a secondary occurrence it only has the subordinate proposition within its scope. In the primary occurrence 'Kg' is within the scope of the quantifier; in the secondary 'Kg' is outside its scope. This is vital to keep in mind when analyzing the second puzzle. There, occurrences are going to play an essential role in Russell's solution. 'Kg' is inside or outside the scope of the quantifier in the various occurrences given above; in the second puzzle, a negation, \sim , is going to be inside or outside the quantifier's scope. The position of the negation will determine whether the proposition has primary or secondary occurrence.

Russell poses his second puzzle to demonstrate how his theory of denotation enables us to maintain the law of excluded middle³⁸ for certain statements. The kind of statements that

³⁸ "'A is B' or 'A is not B' must be true." (110)

this puzzle applies to are such that seem to assert predications of non-existent subjects: e.g., "either 'the present King of France is bald' or 'the present King of France is not bald' must be true" (110). Russell sees these statements as puzzling because there is no such entity as 'the present King of France' among all the things in the world which are or are not bald (110). This being so, how is it possible to maintain the law of excluded middle for such statements? It would seem that both of the above statements should be false. But if this is so, the assumption of the universal applicability of the law of excluded middle is shown to be invalid.

Russell appeals to his *primary/secondary* occurrence distinction to solve the puzzle. This distinction is seen as giving us the key to understanding "the logical status of denoting phrases that denote nothing" (115).

Russell first offers a general procedure for analyzing denoting phrases which do not denote:

If 'C' is a denoting phrase, say 'the term having the property F',³⁹ then 'C has the property ϕ '⁴⁰ means 'one and only one'⁴¹ term has the property F, and that one has the property ϕ '.

If now the property F belongs to no terms, or to several, it follows that 'C has the property ϕ ' is false for all values of ϕ (115-16).

Let us call this general rule G. The application of G to 'the present King of France is bald' demonstrates that this statement is false since the property F belongs to no terms. If 'K' = 'is King of France' and 'B' = 'is bald', 'The present King of France is bald', when fully expressed becomes $(\exists x)[(Kx \ \& \ (y)(Ky \rightarrow y = x)) \ \& \ Bx]$.

Russell thus establishes that 'The present King of France is bald' is false. In order to maintain the validity of the law of excluded middle he must be able to interpret 'The present King of France is not bald' as true. But how? Given G, it seems that this statement is false, for the same reasons that 'The present King of France is bald' is false.

³⁹For example, the individual having the property of being King of France.

⁴⁰For example, the individual having the property of being King of France has the property of being bald.

⁴¹This 'one and only one' is the simplified version of his uniqueness and existence conditions mentioned above.

As mentioned, Russell appeals to the *primary/secondary* occurrence distinction to solve this puzzle. The statement under considerations is said to have two interpretations:

'There is an entity which is now King of France and is not bald', ...
[and]
'It is false that there is an entity which is now King of France and is bald'.⁴² (116)

Symbolically these are:

' $(\exists x)[(Kx \ \& \ (y)(Ky \rightarrow y = x)) \ \& \ \sim Bx]$ ',
and
' $\sim(\exists x)[(Kx \ \& \ (y)(Ky \rightarrow y = x)) \ \& \ Bx]$ '.

Russell offers another general rule: "all propositions in which 'the King of France' has primary occurrence are false; the denials of such propositions are true, but in them 'the King of France' has secondary occurrence" (116). In the first interpretation 'the King of France' has *primary* occurrence and so is false. In the second interpretation 'the King of France' has *secondary* occurrence and is true, and Russell has achieved his goal.

Observations

In comparison with the complexities and confusions of his [1903] theory, Russell's new theory of denoting is a breath of fresh air. In the [1903] theory, there was a great confusion about the objects of denoting phrases. There is no such confusion in [1905b]. His [1905b] theory advocates that what is communicated by means of statements containing denoting phrases is an ordered logical form which may or may not denote an actual object.

Aristotle's philosophy agrees with certain aspects of Russell's theory. For Aristotle, statements communicate ordered specifications of objects. These ordered specifications are said to be true or false insofar as they correspond to the objects and their properties which are specified. If 'Socrates is pug-nosed' is true, then the nose of Socrates has the determina-

⁴² In these interpretations Russell is assuming the uniqueness specification. I do the same while analyzing these sentences and Russell's analysis.

tion specified by 'pugnness.' Aristotle agrees with Russell that denoting phrases do not have to denote an object.

Also, Aristotle's philosophy is not averse to Russell's method of proposing general propositional forms for the various types of denoting phrases. Russell's method is to specify the logical contents of statements. He does this by offering general forms of certain statements. Russell's project is complimentary to Aristotle's philosophy insofar as Russell's method, and his general forms, correspond to the actual way in which the mind composes the intelligible contents of what is communicated.

Nevertheless, an Aristotelian analysis of these puzzles and of Russell's solutions sheds light on a number of points upon which the two philosophers differ.

ARISTOTLE AND THE FIRST PUZZLE

Russell's first puzzle about the apparently identical denotations of 'Scott' and 'the author of *Waverley*' would be treated quite differently from the perspective of Aristotle's philosophy. From this perspective, 'Scott' and 'the author of *Waverley*' may both specify the same individual. If they both specify the same individual, they do so in different ways. 'Scott', being a proper name of the individual, refers to *who* the object is, namely, the individual considered as an individual. A proper name is seen as a label which refers to an individual *qua* individual.⁴³ With a proper name, the individual is specified as such, rather than by means of predicates which specify the individual as to what kind he is or how he is in relation to other things. Since the same name can be given to different individuals, and to different kinds of things, it may be necessary to specify the kind of thing which has the name. A man and a dog with the same name, (say) Callias, may be equally worthy of consideration as the subject of a statement: e.g., 'Callias is chasing the cows.' In such cases one needs a further specification of which Callias is intended. This could be done by means of specifying *what* the individual of reference is, e.g., 'the man Callias'.

As mentioned on page 5, Aristotle does not consider proper names as predicates. For him, predicates are always universals. In Aristotle, 'The author of *Waverley*' would be treated as a definite description which refers to the individual named 'Scott', by means of specifying some accidental activity which the individual has accomplished (*Top.*, 103a, 30-40); just as 'the man' in 'the man Scott' is a definite description which refers to the individual by means of specifying *what* he is with respect to the genus *animal*. 'The author of *Waverley*' is seen from this perspective as a predicate on account of the specification of the universal 'author'. The uniqueness of the predication is supplied by the individual work *Waverley* — provided that there is only one work with this title.

⁴³ An individual may have more than one proper name and each of them will equally 'label' the individual.

Russell says that 'Scott' and 'the author of *Waverley*' are not substitutable, in 'George IV wished to know if Scott was the author of *Waverley*'. Under analysis, the latter is said not to be a component of the proposition; it could not be replaced by 'Scott' which is a component. Further, Russell maintains that, a substitution of these is possible *verbally* only in sentences in which the definite description has secondary occurrence.

From Aristotle's perspective, 'the author of *Waverley*', in 'George IV wished to know if Scott was the author of *Waverley*', is predicated of Scott. 'Scott' and 'The author of *Waverley*' are not substitutable without altering the meaning or the truth inferences because they do not specify the individual in the same manner.⁴⁴ The one refers to the individual by means of his name, the other by means of something he has accomplished. The question of George IV — 'whether Scott is the author of *Waverley*' — is of a subject predicate form while 'Scott is Scott' is an assertion of identity. These simply do not assert the same thing.

⁴⁴ When I say the truth inferences would be altered by the substitution, I mean that the substitution can not be made salva veritate. If it is true that George IV wished to know if Scott was the author of Waverley, it does not follow that it is true that George IV wished to know if Scott was Scott, or that he wished to know if the author of Waverley was the author of Waverley.

ARISTOTLE AND THE SECOND PUZZLE

The second puzzle considered the law of excluded middle for such propositions as 'The present King of France is bald' or 'It is not the case that the present King of France is bald'.

Russell's treatment of how uniqueness and existence function in relation to the truth of certain statements, specified by definite descriptions, also has some parallel in Aristotle's writings.

Russell's specification of existence in statements containing definite descriptions, from Aristotle's perspective, is connected with certain conditions of the truth of statements.⁴⁵ Aristotle also sees every statement containing some tense of the verb to be⁴⁶ (*de In.* 16a.15-20). The ways in which the two perspectives analyze this second puzzle demonstrates certain divergences. They diverge as to how existence functions with respect to truth and falsity, and how existence functions in relation to the subject of the statements analyzed.

Aristotle's perspective agrees with Russell's general rule,⁴⁷ G, for analyzing the truth and falsity of statements of affirmation that contain phrases that do not denote, for example, 'The present King of France is bald'. When analyzing 'The present King of France is not bald', however, Aristotle's perspective does not agree with the way Russell applies the clause "it follows that 'C has the property ϕ ' is false for all values of ϕ ".

In the implications of G's applicability to 'The present King of France is bald' Aristotle agrees with Russell. If the denoting phrase has no denotation, any affirmation of a property is false.⁴⁸

⁴⁵ "The fact of the being of a man carries with it the truth of the proposition that he is, and the implication is reciprocal: for if a man is, the proposition wherein we allege that he is is true, and conversely, if the proposition wherein we allege that he is is true, then he is." *Cat.* 14b.10-20.

⁴⁶ Though not necessarily in the present tense as Russell will maintain in [1918].

⁴⁷ "If 'C' is a denoting phrase, say 'the term having the property F', then 'C has the property ϕ ' means 'one and only one term has the property F, and that one has the property ϕ '.

If now the property F belongs to no terms, or to several, it follows that 'C has the property ϕ ' is false for all values of ϕ " (115-16).

⁴⁸ "Neither 'Socrates is ill' nor 'Socrates is well' is true, if Socrates does not exist at all." (*Cat.* 13b.15-20)

As for Russell, for Aristotle, the existence or non-existence of the object denoted by the statement is a determining factor in its truth or falsity — and, as for Russell, it is not the sole determining factor.

Aristotle's examples do not all use what Russell calls denoting phrases; some use what Russell calls proper names. This does not alter the import of Aristotle's analysis; Russell will eventually regard proper names as truncated definite descriptions.⁴⁹

What may be called Aristotle's version of the law of excluded middle is this: in statements that are opposed "as affirmation and negation [i.e., denial] ... it is necessary for the one opposite to be true and the other false" (*Cat.*, 13b.35-40). However, this 'law' does not always apply if two statements are contraries with respect to "the words that enter into the opposed statements":

'Socrates is ill' is the contrary of 'Socrates is well', but not even of such composite expressions is it true to say that one of the pair must be always true and the other false. For if Socrates exists, one will be true and the other false, but if he does not exist, both will be false; for neither 'Socrates is ill' nor 'Socrates is well' is true, if Socrates does not exist at all. (*Cat.* 13b.15-20)⁵⁰

This is almost a rewording of Russell's puzzle, but not quite. 'Is well' and 'is ill' are not opposed as the affirmation of 'is bald' and its denial. If the above examples, instead of 'is well' and 'is ill', had 'is ill' and 'is not ill', Aristotle would have considered it an instance of the statements' being opposed as affirmation and denial. In such an opposition "whether the subject exists or not, one is always false and the other true" (*ibid.* 25-30). Aristotle gives no explanation of this position since he considers it obvious:

For manifestly, if Socrates exists, one of the two propositions 'Socrates is ill', 'Socrates is not ill', is true, and the other false. This is likewise the case if he does not exist; for if he does not exist, to say that he is ill is false, to say that he is not ill is true. (*ibid.* 25-35)

⁴⁹ In [1910a] and [1918].

⁵⁰ Aristotle's focus here is not on existence but on the different kinds of oppositions. The issue of existence is brought in to demonstrate how the opposition of affirmation and denial differs from the opposition of contrary terms.

This last statement Russell would admit provisionally, after much analysis involving primary and secondary occurrences. Is Aristotle presupposing some sort of Greek version of Russell's analysis? If so, he must be supposing that 'Socrates is not ill' has something similar to Russell's secondary occurrence; in which case, what is asserted is not 'Socrates *is not ill*' but that 'it is false that there exists an entity which is Socrates and *is ill*' ([1905b], 116). But this has been affirmed in the context of Aristotle's example of 'Socrates *is ill*'; this is said to be false if Socrates does not exist. I do not think Aristotle is presupposing any form of analysis similar to Russell's in this instance.

For Aristotle, to affirm a predicate of an existing individual is to express a judgment concerning an actual individual. It is to judge that this individual is such and such — i.e., that it is determined in the way the predicate specifies. If Socrates exists, and I express my judgment 'Socrates is pug-nosed', then my statement is true if and only if the nose of Socrates has the determination specified by the concept of 'pugness'. If Socrates' nose does not have the determination specified by 'pugness' then 'Socrates is not pug-nosed' is true no matter what kind of determination his nose has. The denial of a determination, of a property, does not presuppose the assertion of any positive specification of another determination for the statement to be true. In a denial the determination denied is, as it were, taken away (*APo.* 72a.10-15). It is in no way attributed positively — to anything.⁵¹

However, when a denial is asserted of an existing individual it is the denials correspondence to the *positive* determinations of the existing individual which make the *denial* true. If Socrates' nose is actually pointed and I deny that it is pugged, my statement of denial is true⁵² on account of the *actual determination* of his nose being pointed, or any other actual determination which it has so long as it *is not* that of being pugged. Yet, the determinational

⁵¹ Allusions to this are made in SR.181b.30-35; Iop.136a.35-40; 136b.1-5.

⁵² If we presume that Socrates has only one nose.

content specified by my statement, of the denial, does not contain or presuppose the specification of the actual determination.⁵³

How does this apply if Socrates does not exist? According to Aristotle's account, 'Socrates is not bald' is true, plain and simple. To arrive at the truth of this statement does not require a translation in any way similar to a secondary occurrence. Aristotle's perspective sees this statement as evidently true on account of the very nature of predication — which is a specification of determinations (properties). The denial of a predicate (the denial of a determination) does not include an affirmation of any determination — if the subject of the statement does not exist then what is denied is denied of nothing and is true.

Russell would object. For Russell, in order for this statement to be true on these premises one has to suppose, contradictorily, that what does not exist, exists. The Aristotelian reply is that, if in establishing the truth or falsity of this statement it is established that the subject of the statement does not exist, then the denial of the predicate of such a subject must be true. Since a non-existent subject can have no determinations, the denial of any determination to such a subject will be true.

As does Russell, Aristotle admits that the supposed subject of a statement does not necessarily have to denote. But Aristotle puts the matter rather differently than does Russell. Aristotle makes a distinction between an "object of knowledge" and an "object of opinion": the former requires an actual object to be denoted, while the latter does not (*Top.* 121a.20-25; 121b.1-5). According to this distinction, 'The present King of France' is simply an object of opinion. It is simply the mind's ordered composition of properties which are not so ordered in reality. If I make a statement in which an object of opinion serves as a subject, the subject of my statement does not have to point to an actual object for that statement to

⁵³ If we consider the ordered whole of determinations which is the existing individual as a set of determinates, a denial is seen as asserting that such-and-such a determination is not present in the set of determinates. If the denial is true then the set of determinates does not contain the determination denied. It is the correspondence to the actual contents of the set that make the denial true.

be understood or to be true. In such cases no actual object is pointed at. What is presupposed is the common ability to consider ordered combinations of properties free from all actually existing individuals.

AN ANALYSIS OF A) B) AND C)

I shall briefly repeat the steps in Russell's analysis of the sentences in the second puzzle, then offer an Aristotelian analysis of each of these steps. From the two sentences of the puzzle Russell's analysis ends up with three propositions, two of which are false and the third true. I call Russell's interpretation of the first statement A) and his two interpretations of the second statement B) and C) respectively. I argue that Russell offers a solution to his puzzle at the expense of much complexity and by interpreting a denial of baldness as an affirmation of falsity.

Russell's line of argument is:

A) 'The present King of France is bald' under analysis becomes:
'There is an entity which is now King of France and is bald'.
When fully expressed symbolically this becomes:
' $(\exists x)[(Kx \ \& \ (y)(Ky \rightarrow y = x)) \ \& \ Bx]$ ',
This is proved to be false on the grounds that there is no King of France.

'The present King of France is not bald' under analysis is said to have two interpretations depending on the *occurrence* the denoting phrase has:

B) 'There an entity which is now King of France and is not bald'. That is
' $(\exists x)[(Kx \ \& \ (y)(Ky \rightarrow y = x)) \ \& \ \sim Bx]$ '.
The definite description has primary occurrence and the statement is said to be false for the same reasons that A) was said to be false.

C) 'It is false that there is an entity which is now King of France and is bald.' That is
' $\sim(\exists x)[(Kx \ \& \ (y)(Ky \rightarrow y = x)) \ \& \ Bx]$ '.
The definite description has secondary occurrence. This statement is true, and offers the solution to this puzzle.

Russell says that in general "all propositions in which 'the King of France' has primary occurrence are false; the denials of such propositions are true, but in them 'the King of France' has secondary occurrence"(116). For C), this means that the denial of the predicate is applied to the entire proposition instead of to the predicate alone, as it is in B).

Russell's analysis is of sentences that are asserted; sentences that are asserted with the conviction that the assertion is true. Throughout [1905b] Russell uses such phrases as 'this asserts that' (106), 'when we say' (106, 114), 'if we say' (108), and 'suppose we wish to

say'(114). His analysis of 'The present King of France' is of the sentence's being asserted *qua* true. Naturally the conditions necessary for the assertion to be true are supplied in the formalization.

From the perspective of Aristotle's philosophy, if I say 'the present King of France is bald', A) represents the composition in my mind of an ordered set of properties. This ordered set specifies a determinate way in which these properties are believed to stand in relation to each other. By asserting 'The present King of France is bald', I make a judgment concerning this composition's conformity to facts in the world: this is because of the specifications of the actual present time period and the actual country France. This statement's actual conformity or lack of conformity with reality determines its truth or falsity. When the components of my judgment are measured against reality, the subject of my judgment is found not to exist; it follows that it is impossible that such a subject have any determinations. Nothing can be truly predicated, affirmed, of it — the proposition is false.

Russell says that if we enumerate all the bald things in the world we would not find a King of France ([1905b], 110). But this enumeration only considers half of what is asserted by 'The present King of France is bald'. If we were to enumerate all things that exist in the world, we would not find a King of France: this is why we can not find such an individual among all the things that are bald. It seems that from Aristotle's standpoint, 'The present King of France is bald' is not about the things in the world that are or are not bald: 'The present King of France is bald' is about an object of opinion and not about what is predicated of such an object. If the object of opinion does not exist, then the statement which affirms something of it will be false — regardless of whether the predicate has any instances in the world or not.

Let us apply this approach to 'The present King of France is not bald'. Russell says B) is 'There is an entity which is now King of France and is not bald'. From Aristotle's standpoint B) accurately expresses the mental composition of properties expressed by me in this state-

ment. B) expresses a denial of a predicate. In a denial the mind separates a predicate from a subject. Upon empirical examination the subject of my assertion is found not to exist. Since I am not affirming any actual determination (property) of the subject, but am denying one, the proposition is true.

Here Russell would disagree. Russell asks us to enumerate all the things in the world which are not bald ([1905b], 110). The King of France is not among this enumeration; there cannot be a conformity of my statement to what is; therefore 'the present King of France is not bald' is false.⁵⁴

According to Aristotle's analysis, Russell treats 'is bald' and 'is not bald' as opposites in a way similar to the way 'is well' and 'is ill' are opposites (*Cat.* 13b.10-20). In statements which are opposed to one another by such contrary assertions it is not necessary that one be true and the other be false.

When the properties specified by the terms 'well' and 'ill' are affirmed, the mind is joining a predicate to a subject. Something is said *to be* well or *to be* ill. In denial a property is not affirmed of anything (*Top.* 136a.35-40-136b.1-5). A denial expresses the act of the mind that separates a predicate from a subject. Something may actually have the property; but such a fact is neither affirmed by my denial nor necessary for my denial to be true. It is true

⁵⁴ It should be noted that Russell's theory of truth in [1905b] is in a state of transition. Previously, Russell saw truth and falsity as properties of propositions, as -for example- red and white are properties of roses (Passmore, 204). Truth and falsity were seen as objects of acquaintance. Already in [1903] the problems encountered with denoting phrases presupposed some sort of correspondence theory of truth. By admitting that certain concepts mediate to other objects, and that the propositions which contain such concepts are true and false depending upon whether there is such an object, Russell is compelled to admit some sort of correspondence theory. Similarly in [1905b] his propositional functions and the forms of propositions presuppose some sort of correspondence theory of truth: if there is such and such an object then the proposition is true; if not, it is false. By 1906 Russell openly adopts a correspondence theory of truth but does not seem to see the metaphysical implications of such an adoption (Hylton, 282).

Russell not only alters his position regarding the nature of truth, but also regarding the nature of propositions. Propositions and judgments are no longer considered as objective entities. They are henceforth seen as subjective constructions regarding objective entities (333-4). Throughout Russell's shifts in emphasis and dogmatic commitments to certain issues, he continues to hold to some sort of object-based metaphysics; he also continues to maintain a theory of immediate acquaintance with these objects - even when these 'objects' become reduced to sense-data and subjective percepts as they do in [1918]. Despite this, these shifts do not seem to alter his theory of the nature of propositional functions. I know of no text in Russell that suggests that they are anything but indefinable objective entities of acquaintance as laid down in [1903] and maintained through [1910]. As will be mentioned, despite these shifts regarding propositions, judgments, and truth, Russell's theory of definite descriptions after 1905 remains unaltered.

that the president of the United States is not a Dodo bird; it is true that Michelangelo is not a Dodo bird — no positive specification is asserted by the statements themselves.

These considerations highlight a general characteristic of the way Russell seems to deal with denials. Russell's [1903] analysis of propositions into term and assertion sets the stage for this treatment. Russell joins the verb and predicate into a single assertion for the sake of preserving the "essential unity" of propositions.⁵⁵ This conjunction of verb and predicate causes difficulties in expressing denials.

When Russell symbolizes a positive assertion (e.g., 'is bald') he does so by a simple assertion variable ϕ . He does the same thing in his treatment of such assertions as 'is not bald'. As 'is bald' is a positive assertion so 'is not bald' is treated as a positive assertion: as something 'being not bald' ([1910a], 174) he also speaks of the enumeration of things which "are not bald" ([1905b], 110).⁵⁶ This seems harmless. From Aristotle's perspective, however, what Russell is doing is treating the denial of a predicate as a positive specification. According to an Aristotelian understanding of the functionality of predicates, 'is not bald' should be treated in English as 'is-not bald', rather than 'is not-bald'.⁵⁷ A denial specifies an act of mental separation. Russell treats 'is not bald' as 'is not-bald'; he treats 'is not bald' as an affirmation of a negation rather than as a denial of a predicate. Affirmations, for Aristotle, express the mental act of joining. From Aristotle's perspective, Russell is treating the statements of this puzzle as composed of two statements neither one of which must be true if the other is false: i.e., 'The present King of France is bald' and 'The present King of France is not-bald'.

Aristotle's philosophy recognizes the legitimacy of affirming negations such as 'not-bald'; but Aristotle is careful to point out that 'is bald' and 'is not-bald' are not related in the same

⁵⁵ A unity for which Russell's metaphysics can offer no justification.

⁵⁶ When Russell analyzes 'the present King of France is not bald' into B) he applies his general rule, G, to demonstrate its falsity. When he does this 'not bald' is treated as as ϕ .

⁵⁷ Such terms Aristotle calls "indefinite verbs". (DeIn. 16b.14-15) "... but let them be called indefinite verbs, since they apply equally well to that which exists and to that which does not exist".

ἀλλ' ἔστω ἀόριστον ῥῆμα, ὅτι ὁμοίως ἐφ' ὅτου ὄν ὑπάρχει καὶ ὄντος καὶ μὴ ὄντος.

The term rendered by Edghill in English as "indefinite" is ἀόριστον.

way as proper affirmations and denials are.⁵⁸ Further, "every affirmation has an opposite denial, and similarly every denial an opposite affirmation" (*de In.* 17a.30-35). Since 'is not-bald' is an affirmation it must have an opposite denial. For Aristotle, the corresponding denial of 'is not-bald' is not 'is bald': "for the denial must deny just that which the affirmation affirms concerning the same subject, and must correspond with the affirmation" (*de In.* 17b.35-40). The corresponding denial of 'is not-bald', then, is 'is not not-bald' (*de In.* 19b.25-30).

In his *Prior Analytics*, Aristotle offers a fuller picture. Having given a general overview of the distinction between the above mentioned affirmations and denials he asks us to line up the varying affirmations of denials. In the following quotation, one may substitute

$A = \text{is white}$ $B = \text{is not white}$
 $D = \text{is not not-white}$ $C = \text{is not-white.}$

Then either A or B will belong to everything, but they will never belong to the same thing; and either C or D will belong to everything, but they will never belong to the same thing. And B must belong to everything to which C belongs. For if it is true to say 'it is not-white' it is true also to say 'it is not white': for it is impossible that a thing should simultaneously be white and be not-white, or be a not-white log and be a white log; *consequently if the affirmation does not belong, the denial must belong.* But C does not always belong to B : for what is not a log at all, cannot be a not-white log either. On the other hand D belongs to everything to which A belongs. For either C or D belongs to everything to which A belongs. But since a thing cannot be simultaneously not-white and white, D must belong to everything to which A belongs. For of that which is white it is true to say that it is not not-white. But A is not true of all D . For of that which is not a log at all it is not true to say A , viz. that it is a white log. Consequently D is true, but A is not true, i.e. that it is a white log. It is clear also that A and C cannot together belong to the same thing, and that B and D may possibly belong to the same thing. (*APr.* 51b.35-52a.1-15; emphasis added)

Let us put this text in accord with Russell's example and explain in it relation to Russell's puzzle. Let

$A = \text{is bald}$ $B = \text{is not bald}$
 $D = \text{is not not-bald}$ $C = \text{is not-bald.}$

⁵⁸ "For a denial must always be either true or false" (*DeIn.* 20a.30-40) which, as will be explained immediately, is not the case when 'is not-bald' is treated as a denial of 'is bald'.

Aristotle opposes A with B and C with D : he says that in each opposition one of the assertions will belong to everything, but that they will never belong to the same thing. By 'belong' he means that it will be true of. By classifying each A or B and C or D as exclusive disjunctions, Aristotle is saying they are opposed as proper affirmations and denials. When he states that " B must belong to everything to which C belongs" he is saying that if C , as an affirmation, is true of something then B will be true of it: if 'The present King of France is not-bald' is true, then 'The present King of France is not bald' will be true. Since B is a proper denial of A , and what holds of C holds of B , it would seem that C can be said to be, or to entail, a proper denial of A . Not so, says Aristotle. For C and B are not convertible: " C does not always belong to B ". Why? Because B may be true of a non-existent subject while C cannot be. 'The present King of France is not bald' is true; 'The present King of France is not-bald' is false. The first is true and the second is false because there is no present King of France. For any affirmation to be true, even an affirmation of a negation such as 'not-bald', there must be "something underlying" the subject — i.e. there must *be* something of which the affirmation is true (the subject must have a denotation) (*APr.* 51b.25-35). Since 'is not-bald' is an affirmation, when it is used in a statement, it expresses the mind's action of joining. For a denial of a subject to be true there is no need that the subject actually exist; the subject can be purely fictive. A denial expresses the mental act of separation; denial does not in any way affirm (join) anything, it simply takes away.

In the law of excluded middle Aristotle sees the middle that is excluded as being between an affirmation and a denial *qua* privation of affirmation. Russell sees both the positive affirmation and the denial equally as affirmations. He naturally runs into serious difficulties in solving his puzzle. How does one find a true counterpart to the false affirmation A) 'The present King of France is bald'? By treating B) 'The present King of France is not bald' as 'The present King of France is not-bald' he quite naturally sees a problem in finding how to interpret it so as to become true. Aristotle's philosophy accords with Russell when he says

'The present King of France is not-bald' is false. For Aristotle the matter stops here. An affirmation, even of a negation, cannot be true if its subject does not exist.

What about C)? From Aristotle's perspective, C) does not represent a legitimate interpretation of what is asserted in 'The present King of France is not bald'. According to Aristotle's philosophy, one's mental composition and judgment which is expressed by the statement 'The present King of France is not bald' is expressed accurately by B). For Aristotle, to admit that it is expressed accurately by C) would be to confuse what the truth of a denial necessarily implies, from its corresponding affirmation, with the statement itself which is a denial. 'The present King of France is not bald' is the proper denial of 'The present King of France is bald'. Since the denial is true the affirmation must be false. For Aristotle a denial is a denial, rather than an affirmation that happens to deny.

Russell's analysis resulting in C) ends with an affirmation of falsity. From an Aristotelian perspective it is easy to arrive at the truth of an assertion of falsity from a true denial; all that is required is the citation of the denial's corresponding affirmation. Russell maintains that C) is meant by the assertion of 'The present King of France is not bald', if the assertion is to be interpreted as being true. Aristotle's philosophy accords with Russell, [1905b] onwards, in that it recognizes the difference between the conventions of language usage and the content of statements. In Aristotle's philosophy the content of a statement is determined by analyzing the statement into simple affirmations and denials. This process manifests the components and the mind's acts of conjunction or of separation. On this model of analysis it is difficult to justify the steps Russell sets out in arriving at C). On Aristotle's model of analysis, 'The present King of France is not bald' is a simple denial; Russell's C), 'It is false that there an entity which is present King of France and is bald', analyzes into two affirmations, the principal affirmation (which is one of falsity) and the subordinate affirmation (that of baldness).

Still, no matter how we slice up Russell's analysis, the fact still remains that his conclusion results in one statement's being true and the other false. Further, Russell's analysis

resulting in C) does seem to make sense. A) is false and C) is true, they are exclusively opposed to each other:

A) 'There is an entity which is now King of France and is bald'

C) 'It is false that there is an entity which is now King of France and is bald.'

From an Aristotelian perspective we can take either one of these statements and give its proper affirmation or denial which in truth and content seem on Russell's interpretation for all practical purposes to be identical. In Aristotle's philosophy, C) is an affirmation which has a proper denial, namely,

It is *not* false that there is an entity which now King of France and is bald.

This denial is false. To interpret C) as a denial would be to interpret it as:

It is not true that there is an entity which is now King of France and is bald.

This denial has as its corresponding affirmation

It is true that there is an entity which is now King of France and is bald.

This affirmation is false. Either way we interpret C), its affirmation or denial corresponds very closely with Russell's A).

However, in these interpretations of A) and C) what is affirmed or denied is truth or falsity. In the original sentences, from an Aristotelian standpoint, 'bald' is what was affirmed or denied. Despite the similarities between the opposition of A) and C) and the opposition of these proper affirmations and denials, there are some hidden assumptions about how negations are functioning in Russell's analysis that an Aristotelian method brings to light. Once brought to light, the difference between Aristotle's affirmations and denials on the one hand and A) and C) on the other hand (as well as why Russell gives such interpretations to these his statements) becomes clear.

Russell arrives at his conclusion of the puzzle about the King of France in three steps: first, by treating 'not bald' as 'not-bald' in B), second, by separating the 'not' from 'bald' in C),

and third, by having the 'not' of B) expressed by 'it is false that' in C). In B) 'not' functions as an aspect of a property; in C) it functions as an assertion of falsehood. In [1910a] Russell states as a primitive proposition

If p is any proposition, the proposition "not- p ," or " p is false," will be represented by " $\sim p$ " (93).

There is an important parallel in the predicate 'not-bald' and the proposition 'not- p '. It seems that, despite the apparent incongruity between how a negation functions in B) and C), there is an inherent consistency that is not immediately evident from the expressions 'not-bald' and 'it is false that ...'.

Russell's conclusion results in one statement of his puzzle's being true and the other false. My question is, why do Russell's conclusions, of the puzzle about the present King of France being bald or not bald, seem to make sense; and why does his method work despite the apparent incongruity between what appears as a denial in the surface grammar of the original sentence and the affirmation expressed in C)?

DENIALS AND NEGATIONS

I believe an Aristotelian approach to the distinction between denials and indefinite predications can aid in offering an answer to these questions. To my knowledge, Aristotle does not offer much explicitly on indefinite predicates (*de In.* 16b.10-20). It seems that such assertions as 'is not-white' are said to be indefinite since they do not positively specify anything about the subject of which they are true. When such assertions are true, what they do specify is an exclusion. If 'This is not-white' is true, then the act of affirmation, in a way, excludes whiteness from the subject. The exclusion indicated by 'not-' in such terms as 'not-white' and 'not-bald' I shall call 'negation'.⁵⁹ There are important similarities and differences between negation and denial. An affirmation of a negation differs from a denial, as 'is not white' (denial) differs from 'is not-white' (affirmation of a negation).

In denial, the mind *separates* a predicate from a subject — there is no affirmation, there is no joining. In the affirmation of a negation, the mind is affirming a predicate of a subject; the mind *joins* a predicate to a subject — even through the predicate joined is an exclusion. If an affirmation of a negation is true, the predicate joined to the negation is excluded from the subject. If 'This tree is not-white' is true, then 'whiteness' is excluded from this tree; i.e., the properties of this tree are such that whiteness is not one of them.

⁵⁹ In Greek the same word ἀπόφασις is sometimes rendered by the Oxford translators as 'negation' and sometimes as 'denial'. I am using 'negation' as opposed to denial, i.e., 'negation' specifies an indefinite predicate.

de In., 17b.37-40: "It is evident also that the denial corresponding to a single affirmation is itself single; for the denial must deny just that which the affirmation affirms."

φανερὸν δ' ὅτι καὶ μία ἀπόφασις μιᾶς καταφάσεως· τὸ γὰρ αὐτὸ δεῖ ἀποφῆσαι τὴν ἀπόφασιν ὅπερ κατέφησεν ἢ κατάφασις...

The term rendered by Edghill in English as "denial" is ἀπόφασις.

Cat., 13b.33-35: "Thus it is in the case of those opposites only, which are opposite in the sense in which the term is used with reference to affirmation and negation, that the rule holds good, that one of the pair must be true and the other false."

ὥστε ἐπὶ μόνων τούτων ἴδιον ἂν εἴη τὸ ἀεὶ θάτερον αὐτῶν ἀληθὲς ἢ ψεῦδος εἶναι, ὅσα ὡς κατάφασις καὶ ἀπόφασις ἀντίκειται.

The term rendered by Edghill in English as "negation" is ἀπόφασις.

If the affirmation of a negation, 'is not-white', results in a true statement, the denial, 'is not white', will always be true; but the converse does not hold. For Aristotle, a denial may be true of a non-existent subject, even though, the affirmation of a negation about a non-existent subject is always false: 'The present King of France is not bald' is true; 'The present King of France is not-bald' is false. The negation is affirmed; in the act of affirmation the mind is joining one term to another; thus, the affirmation of a negation requires a non-hypothetical subject.⁶⁰ Of course, negations can be denied, so negations can be denied of non-existent subjects and be true.

The element of a denial's separation is closely akin to the exclusion specified by a negation. Metaphorically I see a denial as the mind's *picking up, taking away*, a predicate; in the affirmation of a negation I see the mind's joining a subject to a negation, the negation *pushes out* what it negates, the negation *excludes* something from the subject. Negations are said to be indefinite because they do not specify anything positive of the subject of which they are true; negations only say that, whatever properties the subject actually does have, they exclude what is negated by the negation; negations *do specify* what the subject *is not*. This aspect of what negations *do specify* certainly makes them very similar in function to denials.

However, denial is an act of separation by the mind; *negation* is a universal, a concept, that specifies exclusion. How does the mind arrive at this universal? From an Aristotelian perspective, all universals are abstractions from particulars. What are the particulars from which the concept *negation* is abstracted? It seems that *negation* is an abstraction from particular denials. The mind abstracts from the particular instances and establishes a concept of *negation*. This concept is then joined with other concepts to form negations which are affirmed or denied in statements.

⁶⁰ There must be "something underlying" the subject of the assertion for the statement to be true. (APr.51b.25-30)

RUSSELL AND DENIALS

When Russell says that C) is the denial of A), it seems that by 'denial' Russell signifies negation ([1905b], 116).⁶¹ From an Aristotelian perspective, Russell's treatments of 'not bald' and 'it is false' and ' \sim ' seem to specify the exclusion that is proper to a negation rather than the separation that is proper to a denial. If the assertion of 'is not-bald' is true of x , then 'not' signifies that baldness is excluded from x . If ' $\sim p$ ' is true, ' \sim ' signifies that truth is excluded from ' p '; if ' $\sim p$ ' is false, ' \sim ' signifies that falsehood is excluded from ' p '.

We can represent A) and C) in a way that seems to clarify just how Russell is considering negation.

A) 'There is an entity which is now King of France and is bald'

C) 'It is false that there is an entity which is now King of France and is bald.'

If A) = p , then C) is 'it is false that p ' or 'not- p ' or ' p is false' or ' $\sim p$ ' [1910a, 93].

If (as I would hold) Russell is consistently using each of his 'not', and 'it is false' and ' \sim ', to specify exclusions, and if A) and C) are both affirmations, it seems that A) and C) should be represented as A) = ' p is true' and C) = ' p is not-true'. Such an interpretation maintains that the truth of one excludes the truth of the other leaving Russell's solution intact. This interpretation also clarifies how Russell is considering the law of excluded middle, denial, and the assertion of falsehood.

Russell's formulation of the law of excluded middle is "'A is B' or 'A is not B' must be true" [1905b, 110]. In light of an Aristotelian analysis, Russell's formulation should read

For two propositions understood as 'A is B' and 'A is not-B' if one is true the other will necessarily be false.

Because of Russell's manner of considering denials, his understanding of the law of excluded middle should be represented in a conditional form. It is possible that both of state-

⁶¹ Russell continues to use 'denial' in the same way while presenting his theory of definite descriptions in his later writings (see [1910a], pp. 68-70 and [1919], p. 179).

ments so opposed could be false. It is only on condition that one of them is true that the other is necessarily false. Contrary to Aristotle's understanding of the opposition of affirmation and denial, Russell's interpretation of denials as affirmations of negations cannot maintain

'A is B' or 'A is not B' must be true.

Russell's analysis does arrive at two propositions opposed in this way. Yet he does so along an arduous path of analysis in terms of negation which, from an Aristotelian standpoint, misunderstands the nature of denial and the syntax of natural language.

RUSSELLIAN REFUGE IN EXISTENCE

From an Aristotelian perspective, the main problem with Russell's theories of denotation is the failure of their underlying philosophies to account adequately for (a) a coherent link between universals and particulars, and (b) how the mind acts in the formation of statements. In Aristotle's philosophy there is a hierarchical relation between concepts and particulars. This hierarchy has its basis in sense perception and results from the mind's response to things it accesses through sense perception. A hierarchy of concepts results from the confrontation with things perceived through the senses. This hierarchy manifests itself in the ordered way we make predications. The mind is able actively to compose elements of sense and intellectual knowledge and make judgments concerning its composition's correspondence with actual things in the world.

From Aristotle's perspective, Russell's [1903] theory of denotation is problematic in its formulation of terms and the terms' relation to the objects of which they may be predicated. From Aristotle's perspective, this results from Russell's failure to see how the mind is working in the formulation of concepts through abstraction from particulars. In [1905b] Russell's analysis manifests, from the Aristotelian perspective, a failure to understand how the mind functions in the composition of propositions (particularly in affirmation and denial). This perspective also sees Russell as failing to understand how to determine the truth or falsity of propositions that deal with non-existent subjects.

It is highly likely that Russell would not be inclined to agree with my Aristotelian analysis of his theories of denotation. Russell could respond to my analysis by noting its reliance upon an understanding of existence as much as on an understanding of the nature of knowledge and predication. In fact, Russell could charge that this Aristotelian understanding of the nature of knowledge and predication presupposes a particular understanding of existence.

In the majority of the Aristotelian criticisms given above it is presupposed that existence is predicable of the subjects of the statements. For Aristotle, the existence of a thing is presupposed in the acquisition of knowledge of it.⁶² Things that we are acquainted with are said to exist.⁶³

Russell could easily evade my Aristotelian analysis by referring me to his complex theory of existence which was developing alongside his theory of denotation and had its sources in the works of his predecessors.

Already in [1903] Russell had an interpretation for the term 'existence' that specified that a class had at least one member (93.93),⁶⁴ but 1905 was the turning point in Russell's theory of denoting and his theory of existence. By [1910a] "there exists" and "sometimes true" are explicitly linked (pp. 20, 42, 127): "We shall denote " ϕx sometimes" by the notation $(\exists x)\phi x$ " (127). In [1905b] "it is sometimes true that x " is equivalent to "it is not always false that x " (114); in terms of definite descriptions these are equivalent to "there exists an x such that".

In [1905a] Russell explicitly makes the distinction between a philosophical understanding of existence and how 'existence' is used in symbolic logic. For Russell, the philosophical sense of existence sees 'existence' as a predicate applicable to individuals that have this property. In Russell's symbolic logic 'existence' signifies that a class has at least one member. Russell says that symbolic logic "does not care a pin whether its entities exist" in the philosophical sense [1905a, 98-99]. According to this distinction, Russell's understanding of existence in [1905b] is from the perspective of symbolic logic.

It seems that Frege supplies some of the background to Russell's analysis of existence. Frege [1879] does not have explicit notation for an existential quantifier, but it can be intro-

⁶² "It is true that if the object of knowledge does not exist there can be no knowledge: for there will be no longer anything to know." (Cat. 7b.25-30; see also APo. 92b.5-10).

⁶³ "... we plainly cannot grasp what it is to be something without grasping that it exists; for we cannot know what something is when we do not know whether it exists. ... Hence in so far as we grasp that it exists, to that existent we also have some grasp on what it is." (APo. 93a.15-25_c)

⁶⁴ According to Hylton this was borrowed from Peano (211).

duced by definition, in terms of the universal quantifier and negations. (The notation that I shall use to render Frege is equivalent to his own.) According to Frege when $(\alpha)\Lambda(\alpha)$ is denied by $\sim(\alpha)\sim\Lambda(\alpha)$ it is to be translated as "There are Λ " (27). Frege adds in a footnote "This must be understood in such a way as to include the case "There exists one Λ " as well if, for example $\Lambda(x)$ means the circumstance that x is a house, then $\sim(\alpha)\sim\Lambda(\alpha)$ reads "There are houses or there is at least one house"" (27). On the following page Frege adds to his interpretation of this combination of denials and a universal quantifier the notions of "some" and "it is possible"; he adds in a footnote "The word "some" must always be understood here in such a way as to include the case "one" as well. More explicitly we would say "some or at least one""(28).

Similarly Russell states in [1910a], " $(\exists x)(\phi x)$... states that *at least* one object satisfies ϕx "(68).

After 1905, Russell not only focuses on 'existence' as used in symbolic logic, but the philosophical use of 'existence' even begins to be absorbed into that of symbolic logic.

By [1918], when Russell's logical theory of existence is fully developed, to assert existence of something is to assert a description of something. "Officially" for Russell [1918] *esse est describi*, to be, to exist, is to be described. The foundation of this position is already laid in [1910a]. In [1910a] the distinction between the philosophical and symbolic logical uses of 'existence,' first enunciated in [1903] at 21.25, is destroyed in favor of the interpretation of symbolic logic: "when, in ordinary language or in philosophy, something is said to 'exist,' it is always something *described*" [1910a, 174]. From now on, to predicate existence is to speak of the truth values of propositional functions ([1918], 243).

In [1918] Russell maintains that it is nonsense to say that someone or something with which I am acquainted *exists* — it is only when we describe them that we can say they exist and make sense (252). In this essay Russell is expounding his theory of Logical Atomism. This theory makes a radical distinction between what we are acquainted with through im-

mediate sense perception and what we are acquainted with by means of descriptions. Since existence, according to this theory, can only be applied to a description it is excluded from any application to objects of acquaintance through sense perception. Wherever 'existence' seems to be used as a predicate in a proposition, it is said to be actually connected with a description and never with a genuine name — i.e., a 'this' or a 'that'. The description disappears into a propositional function containing a variable with a determining property which specifies an entity. If "there is" such an entity the proposition yielded by the propositional function is true; if not it is false (250).

In [1918] and [1919] all usages of 'existence' are said to derive exclusively from 'sometimes true' and 'is possible' which are said to give the "fundamental meaning" of 'existence'[1918, 232]: all "Other meanings are either derived from this, or embody mere confusion of thought" [1919, 164].

The connection of 'existence' with 'possible' is not quite as explicit in Russell's development as its connection with 'sometimes true'; but it is discernible before [1918]. In [1908] and [1910a] 'possible' is used in relation to variables of propositional functions. Arguments substitutable for the variable of a propositional function are said to be 'possible' when they do not result in paradoxes. Hence, 'possible' is limited to legitimate arguments and never with the notion of 'sometimes true'; but 'possible' is connected with 'existence' when a propositional function has only one value: as is the case when analyzing definite descriptions. Similarly, a propositional function which has no possible values is said not to exist: its class of possible arguments is empty.

Given such an understanding of what Russell means by 'existence' in and after 1905, he could neatly evade many of my Aristotelian criticisms. How Aristotle and Russell deal with existence is a major determinant of how each views the nature and project of philosophy. How Russell and Aristotle interpret actions and interactions with the world and with other language users pivots on how each one interprets existence.

For Aristotle, individuals of which 'existence' can be predicated (primary substances), serve as the foundation of all knowledge and predication:

... everything except primary substances is either predicated of primary substances, or is present in them, and if these last [i.e., primary substances] did not exist, it would be impossible for anything else to exist. (*Cat.*2b.1-10).

It is the existence of such substances, individuals known through sense experience, that is the basis from which and of which all predications are made. If Russell's understanding of 'existence' is correct, it cuts the bottom out of Aristotle's whole philosophical project. If there are no existent individuals then there is no knowledge and there are no predicates — as Aristotle's philosophy interprets them.

An Aristotelian analysis of Russell's developing theory of existence and that theory's effects on my criticisms of Russell's theories of denotation, are not, unfortunately, within the scope of this thesis.

CONCLUSION

Aristotelian criticisms notwithstanding, Russell's theories of denotation bring to light numerous philosophical problems — on the nature of predication, acquaintance, generalization, variables, negations, and truth and falsity. The speculations which lead to and surrounded Russell's theories of denotation highlight the importance of the relation of terms and propositions to their objects and how these function in relation to truth and falsity. Russell does not pretend to offer irrefutable conclusions on these matters. His repeated attempts at solutions bear witness to his awareness of their philosophical importance.

Russell's developing theory of existence notwithstanding, an Aristotelian perspective on Russell's theories of denotation, and surrounding issues, offers a valuable tool for pulling apart and analyzing these issues. Aristotle's theory of the hierarchical nature of abstraction and predication gives a perspective from which to analyze the issues of reference and generalization that initially generated Russell's theories of denotation. Russell's difficulties with the relation of statements and denoting phrases to their objects, from an Aristotelian perspective, arises from a misunderstanding the nature of abstraction and predication. For Aristotle, our minds naturally abstract concepts, universals, from particular instances of things. Concepts serve as objective standards of recognition and predication. We are able to subjectively compose conceptions in ways which may or may not have instances in reality.

Aristotle's philosophy also offers an interesting perspective on how negations and denials function in the syntax of natural and formal languages, and on how the truth conditions of negations and denials may be discerned. By offering a method of unpacking Russell's usages of denials and negations, Aristotle's philosophy shows the need for a reformulation to Russell's understanding of the law of excluded middle from his "'A is B' or 'A is not B' must be true" [1905b, 110], to: for two propositions understood as 'A is B' and 'A is not-B', if one is true the other will necessarily be false

Despite the apparent inconsistency of Russell's interpretation of the 'The present King of France is not bald' as 'It is false that there is an entity which is now King of France and is bald', an Aristotelian analysis brings to light how Russell interprets a denial as an affirmation of a negation. This fact accounts for the consistency of Russell's understanding of how 'not' is functioning in his interpretation of 'The present King of France is not bald'. This fact also accounts for why Russell's solution to his puzzle seems to make sense. Russell understands the law of excluded middle as — for two propositions understood as 'A is B' and 'A is not-B', if one is true the other will necessarily be false. If 'A is B' represents 'there is an entity which is now King of France and is bald' and this = p , then Russell's solution to his puzzle about the two statements 'The present King of France is bald' and 'The present King of France is not bald' should be read as ' p is true' and ' p is not-true' if one is true the other is necessarily false. Russell's solution seems to make sense on account of Russell's consistent interpretation of denials as the affirmation of negations, and his solution arrives at two propositions one of which is true and the other false — which is just what he was seeking to attain.

Like any philosophical perspective, Aristotle's does not offer a refutation or perspective which commands general assent. It does, nevertheless, offer a coherent systemization of philosophical issues connected with denotation that is worthy of serious consideration.

APPENDIX

Williams's essay "Aristotle's theory of Descriptions" approaches issues contained in my thesis from a slightly different, though complimentary, perspective. Williams looks at certain of Aristotle's analyses of statements containing definite descriptions: Aristotle sees some of these as instances of *per accidens* predication. Williams shows how, in the case of Russell's puzzle about the law of identity, both Russell's and Aristotle's theories of descriptions lead to the same conclusion. Williams concludes that Aristotle's analysis offers as sophisticated a solution to this puzzle as Russell's and offers insights into the nature of certain philosophical problems which continue to be "philosophically alive" (80).

Williams limits his analysis of Aristotle's theory of descriptions to its ability to solve the first puzzle posed by Russell in [1905b] — the puzzle of the law of identity (see page 44 above). Williams demonstrates how Aristotle's solution to this puzzle accords with that of Russell, albeit from a different perspective. Some of the statements used by Aristotle, which Williams uses to demonstrate his point, are 'The white thing is walking' and 'The white thing is a log' (Williams, 66-7). Aristotle says that the mode of predication indicated by these statements is *per accidens* since 'white' requires an underlying, substantial, subject to which to adhere. This mode of predication is contrasted with *per se* or *simpliciter* predications such as 'The log is white': the *per se* predication 'white' is said to be situated *in* a substance.

Williams cites Aristotle's puzzle of the hooded man as a close parallel to Russell's puzzle about 'Scott' and 'author of *Waverley*' (72). Williams summarizes Aristotle's puzzle *thus*: 'Coriscus' and 'the hooded man' both refer to the same individual. "I do not know who the hooded man is; Coriscus is known by me to be Coriscus; Coriscus is the same as the hooded man; but *ex hypothesi*, the hooded man is not known by me to be Coriscus"(72). This is Aristotle's solution:

For in all cases it is clearly not necessary that what is true of the accident is true also of the thing; for it is only to those things which are indistinguishable and one in substance that all the same things are thought to belong. (*De Sophisticis Elenchis*, 179a,36-b1)⁶⁵ (Williams, 72)

In this puzzle, "the thing" is Coriscus, and "the accident" is the hooded man. 'The hooded man' specifies "an accidental unity" (72), while the name 'Coriscus' refers to a substance.

After showing Aristotle's understanding of *per accidens* predication, Williams concludes that Aristotle would be able to solve Russell's puzzle about identity. For Aristotle 'the author of *Waverley*' is not substitutable for 'Scott' in 'George wished to know if Scott was the author of *Waverley*' since 'the author of *Waverley*' is a *per accidens* predication. It is the predication of accidental features that the man Scott has accomplished; in contrast, 'Scott' specifies the individual substantially as an individual.

Some of Williams' conclusions are very similar to some of my own: for example, his conclusions about whether Aristotle could solve Russell's puzzle about identity are almost identical with my own. Both Williams and I see 'the author of *Waverley*', on the one hand, as the predication of a feature accidental to its subject, and 'Scott', on the other hand, as indicating the individual as an individual, i.e. substantially.

Despite similarities between Williams' and my conclusions on the solution to Russell's puzzle of identity, I see our overall approaches to Aristotle and Russell as being somewhat different, because our starting points are different. I start with Aristotle and approach Russell's theory of descriptions from an Aristotelian perspective⁶⁶; to me, Williams starts from a contemporary analytic—and hence at least partially Russellian—perspective: from this modern and very different starting point Williams then approaches Aristotle. The main difference that our different starting points produce, in our approaches to Aristotle, is how each

⁶⁵As cited in Williams, *loc. cit.*

⁶⁶Admittedly, my approach to Aristotle's philosophy is not one universally used by contemporary scholars of Aristotle.

of us sees Aristotle's theory of descriptions. In his presentation of what he sees as Aristotle's theory of descriptions, Williams focusses in on certain cases of definite descriptions. This sharp focus approach is quite similar to Russell's insistence on the importance of definite descriptions.

From my perspective, the very core of Aristotle's theory of descriptions is his *Categories*. Aristotle offers his ten categories as the general ways that we describe things — we describe things under these aspects:

1. being substance,
2. quantity,
3. quality (e.g., 'is white'),
4. relation,
5. time,
6. place,
7. position (e.g., sitting),
8. state (e.g., 'wearing a coat'),
9. acting, and
10. being acted on (*Cat.*1b.25-2a.5).

I see Aristotle's use of definite descriptions as one small consideration within his overall understanding of descriptions⁶⁷.

Williams mentions that Aristotle "held a pretty low view of 'accidental being'" (71). From my perspective of Aristotle's theory of descriptions, accidental being is given neither a low nor a high place: rather, it is simply give its proper place — i.e., as inhering in and determining substantial beings.

⁶⁷ It is possible that Williams would agree with me on this point. His focus on definite descriptions, despite his title, could be seen as reflecting contemporary philosophical parlance in which a theory of descriptions means tout simple a theory of definite descriptions.

Williams focuses only on Russell's first puzzle: he shows how, even though Aristotle's method differs from that of Russell, Aristotle's conclusions accord with Russell's. My analysis in contrast focuses on Russell's second puzzle. I show that in analyzing the first and second puzzles there are, to be sure, certain similarities, but there are also very significant dissimilarities between Russell and Aristotle. These dissimilarities are not limited merely to method, but also extend to conclusion.

Williams also maintains that there is a similarity between Aristotle's understanding of *per accidens* predications and Russell's logical fictions (63).

I am not familiar with all of Russell's early works, but the only place I have come across the phrase "logical fictions" is in his [1918]. In [1918] Russell uses the phrase "logical fictions" to indicate the subjective composition of sense data into logical fictions, which in their turn are then composed into judgments and expressed as propositions. These logical fictions of Russell's are entirely subjective, that is, they do not have objective reference or communicability. In [1918] Russell argues that, because of the subjective nature of each person's percepts and the logical fictions composed out of them, where two persons use the same words, they cannot mean the same thing. He concludes that language functions precisely through the mechanism of ambiguity (196).

It would be absolutely fatal if people meant the same things by their words, it would make all intercourse impossible, and language the most hopeless and useless thing imaginable, because the meaning you attach to your words must depend upon the nature of the objects you are acquainted with, and since different people are acquainted with different objects, they would not be able to talk to each other unless they attached quite different meanings to their words. (195, emphasis added)

By "the nature of the objects" Russell means the logical fictions created from subjective sense data. If Russell meant the objective determinations (properties) of the objects of acquaintance, it could not be the case that the meaning of our words for such objects would be different for each person.

It is not my task to question Russell's conclusions on this matter; yet, Russell's logical fictions, thus understood, and their implications in language usage are not⁶⁸ (*pace* Williams) "near relations" (Williams, 63) to aspects of Aristotle's theory of accidental predications..

William's use of 'logical fictions' could refer to Russell's theory of denotation in which certain phrases are shown not to have a denotation: in this case 'logical fictions' are seen as subjective compositions of concepts; these compositions are seen as not necessarily corresponding with any actual individuals (e.g., 'The present king of France') or as specifying an accidental unity (e.g. 'the author of *Waverley*'). If this is how Williams understands Russell's logical fictions then there is no divergence in our understandings on this point.

On Russell's first puzzle, William's analysis chimes in with mine and fleshes it out. By citing Aristotle's own puzzle which contains definite descriptions, and by offering references to Aristotle's distinction between *per se* and *per accidens* predications, Williams fills out my treatment. I had not come across this definite description puzzle in Aristotle's writings: my treatment of the accidental nature of the predication of 'the author of *Waverley*' is based on my own overall understanding of Aristotle's philosophy rather than on any explicit examples I have discovered in Aristotle's writings.

⁶⁸ As I understand Aristotle.

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