The Oberwolfach Problem:
A History and Some New Results

by

Peder A. Bolstad

B.A. *cum laude*, St. Olaf College, 1974

THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE

in the Department
of
MATHEMATICS

SIMON FRASER UNIVERSITY

August 1990

Approval

Name: Peder Allan Bolstad

Degree: Master of Science

Title of thesis: The Oberwolfach Problem: A History and Some New Results

Examining Committee:

Chair: Dr. A. Lachlan

Dr. Brian Alspach
Senior Supervisor

Dr. Katherine Heinrich

Dr. Pavol Hell

Dr. Luis Goddyn
External Examiner

Date Approved: August 7, 1990

PARTIAL COPYRIGHT LICENSE

Title of Thesis/Project/Extended Essay

The Oberwolfach Problem: A History and Some New Results

Author:

(signature)

Peder A. Bolstad

(name)

July 26, 1990

(date)

Abstract

The Oberwolfach Problem asks whether it is possible to decompose the complete graph on $2n+1$ vertices (or the complete graph on $2n$ vertices with a spanning set of independent edges removed) into isomorphic factors each comprising a set of cycles whose combined length is $2n+1$ (or $2n$, respectively). We trace the history of the investigation of this problem, giving results that are known and noting questions that remain open. Solutions (or reasons why no solution exists) are given for all variations of the problem for small n. Some of the solutions are new and others have not been published previously. A new computer-assisted proof is given for the nonexistence of a decomposition of the complete graph on eleven vertices into factors comprising a 5-cycle and two 3-cycles. In the final section we consider each of the cases of the problem that are known to have no solution, and ask whether multiple copies of the complete graph can be 2-factored in the desired way.

This work is dedicated to Dennis and Marie Bolstad, my parents, who started my thinking processes and who have always been loving, supportive and encouraging in whatever serious or silly projects I have decided to undertake, and to the memory of Otis and Dora Trodahl, my maternal grandparents, who led lives driven by caring, and sharing, and who would have been tickled to see this.

Nobody is smarter than all of us.

Acknowlegements

Thanks is due to Brian Alspach for suggesting the topic for this thesis, for being an advocate and guide through various difficulties over the past five years, and for allowing the use of his office and computer during the final writing process. Thanks also to Kathy Heinrich for her exceptionally careful reading of the manuscript drafts. In addition I am also indebted to Patty Johann for helping to make my first year at S.F.U. survivable and to Sylvia Holmes for her tireless and often thankless efforts at keeping things running smoothly.

In addition, I wish to acknowledge Arthur Seebach who has been my most consistent source of intellectual stimulation and irritation since the early 1970's., Ted Vessey who first asked me to solve a case of the Oberwolfach Problem and Loren Larson who first got me interested in discrete mathematics.

Finally, I want to thank Linda Hunter for her encouragement and flexibility, without which the probability of completing this project would have been $\approx \varepsilon$.

Table of Contents

List of Figures

The Oberwolfach Problem: A History and Some New Results

1. The Oberwolfach Problem

Is it possible to partition the edge-set of the complete graph on n vertices (K_n) into isomorphic 2-factors (a 2-factor is set of disjoint cycles whose vertex set spans the graph being factored)? Such a partitioning is also often referred to as a *factorization* or *decomposition* of the graph. It is immediately apparent that each vertex of K_n is of degree n-1 and that since each cycle removed from the graph decreases the degree of each vertex used by 2, n-1 must be even if a cycle decomposition is to exist. Thus the question makes sense only if n is odd and this is the original Oberwolfach Problem (*OP*) mentioned in 1967 by Ringel at a graph theory meeting at the Oberwolfach conference center in Germany (hence the name), and first seen in the literature as part of a list of unsolved problems presented by Guy [6].

If we let n be even and consider the graph K_n-F where F is a 1-factor (a set of disjoint edges whose vertices span the vertex set of the graph), we have a graph that is regular of even degree which allows us to consider the question above for these graphs as well. This variation on the *OP* was originally worked on as a separate problem under the rubric '*NOP*' (for 'Nearly Oberwolfach Problem'), but is now accepted as part of the *OP*. The notation for the Oberwolfach Problem used in this thesis is as follows: $OP(n; a_1, a_2, \dots, a_t)$) represents the problem of decomposing K_n into isomorphic 2-factors where each of the 2-factors comprises one cycle of each length a_i, for $i = 1, 2, \dots, t$ and $a_1 + a_2 + \dots + a_t = n$. When there are cycles of the same length in a 2-factor, the above notation may be abbreviated by including each length only once in the list with an exponent that indicates the number of cycles of that particular length to be included.

We will review the history of this problem; indicating the techniques used to approach it. Then, for each K_{2n} on fewer than 19 vertices and for each K_{2n+1} on fewer than 16 vertices, we will consider all possible cycle combinations into which it might be isomorphically 2-factored and give such a decomposition if one has been found or a reason for its non-existence if that has been established. The discussion will include several new factorizations and a proof of the non-existence of a decomposition of K_{11} into five isomorphic 2-factors each comprising a 5-cycle and two 3-cycles (i.e., no solution for $OP(11; 3^2, 5)$ exists).

We will conclude by considering the possibility that the cases of the Oberwolfach Problem for which no solution exists in K_n might have a solution in λK_n - a complete multigraph on n vertices where every edge has multiplicity $\lambda > 1$.

1.1 Kirkman's Schoolgirl Problem

The quest for 2-factorizations of complete graphs is not new. In the *Lady's and Gentleman's Diary* of 1850, T.P. Kirkman asked whether it was possible for fifteen schoolgirls to be arranged in five lines of three girls on each of seven days in such a way that each girl was in a line with each of the other girls exactly once during those seven days. This problem is equivalent to asking if K_{15} can be decomposed into seven 2-factors, each comprising five 3-cycles. Current notation for the problem would be $OP(15; 3^5)$. According to Ball [4], solutions for this problem and the analogous problems where there are 9 and 27 girls in lines of three were found in the same year by unnamed authors through largely empirical methods.

The literature of the years following Kirkman's query contains solutions for various examples of what have now become known as *Kirkman Triple Systems*. In 1892 Ball [4] collected work done by several separate authors to give a listing of all known solutions for cases of the problem from K_9 to K_{99}, inclusive. Since we are considering rows of three children, the total number of children must be a multiple of three and since each child is in line with two other children in each arrangement, the total number of children must be odd. Thus the only numbers for which the problem has exact solutions are those that are odd multiples of three (i.e., those of the form $6m+3$).

Ball reports that solutions were found by different investigators in cases where the number of children is $12m+3$ when $6m+1$ is prime, $18m+3$, $18m+9$, $18m+15$, $24m+3$, and $24m+9$ where m is a positive integer. In all, solutions collected in [4] settle the question for every number of children from 9 to 99, inclusive, that is of the form $6m+3$.

Solutions were arrived at by methods ranging from trial and error to constructing a "base factor" (i.e., an arrangement of the children for the first day) which can be used to generate a full set of 2-factors by applying a permutation to the vertices of that original 2-factor and to those of each successive 2-factor until a complete set of factors is obtained.

It was not until 1971 that a general solution was found for the Kirkman problem. Any number $n = 6m+3$ of children can be arranged in rows of three on $3m+1$ days in such a way that each child is in the same row with each other child exactly once. The proof was done from the point of view of the theory of balanced incomplete block designs (*BIBD*'s). This gives us our first theorem on solutions to the Oberwolfach Problem.

Theorem 1.1.1: (Ray-Chaudhuri and Wilson [17]) *A solution exists for* $OP(6m+3;\ 3^{2m+1})$ *for all positive integers* m.

The best we could do with an even number of children is to find arrangements where each child is in a line with each other child *except one* exactly once during the sequence of walks. Solutions for this variation of the problem have became known as *Nearly Kirkman Triple Systems (NKTS).*

Kotzig and Rosa [14] showed the non-existence of $NKTS(6)$ and $NKTS(12)$, the existence of $NKTS(tv)$ given $NKTS(v)$ for any $t \equiv 3 \pmod 6$, and the existence of $NKTS(6t)$ when $6t$ is the product of two integers r and s where $r \equiv 1 \pmod 3$, $r \geq 4$ and $s \equiv 1 \pmod 2$. Baker and Wilson [3] showed $NKTS(6t)$ exists for $t > 2$, except possibly for $t = 14$, 17 or 29. Brouwer [5] constructed solutions for two of these three unsolved cases leaving only $t = 14$ in question. The final case was reported solved in [12], but the solution was incorrect. The description of a correct construction is given by Rees and Stinson [19]. Throughout these papers the tools, notation and terminology of design theory were employed to obtain the given results. In *OP* notation we have

Theorem 1.1.2: *A solution for* $OP(6t;\ 3^{2t})$ *exists for all* $t \geq 3$.

1.2 Hamilton Cycle Decompositions

Another variation on the Kirkman problem might be to have n children sit around a circular table on $\lfloor (n-1)/2 \rfloor$ consecutive days arranged in such a way that each child sits next to each other child (except one, if n is even) exactly once. In other words, can K_n (or K_n-F for even n) can be partitioned into $\lfloor (n-1)/2 \rfloor$ Hamilton cycles (i.e., each 2-factor is a single cycle containing all vertices of the original graph).

Letter arrangements and a diagram appear (attributed to Walecki) in Lucas' *Récréations Mathématiques* [15] in 1884 showing base factors for the Hamilton decomposition of K_{11} and K_{12}-F which are easily generalizable into base factors for decomposing any K_{2n+1} or K_{2n}-F into Hamilton cycles. Figure 1.2.1 and Figure 1.2.2 below show the generalized base factors for these two infinite classes of *OP* cases.

The base factor (notated as R below) will become a powerful tool as we proceed. We will use α to stand for a permutation and will write $\alpha(R)$ to indicate the application of α to the vertices of R to obtain another factor. By writing $\alpha^i(R)$ we indicate the result of applying the permutation α to the vertices of R and to each resultant factor until α has been applied i different times.

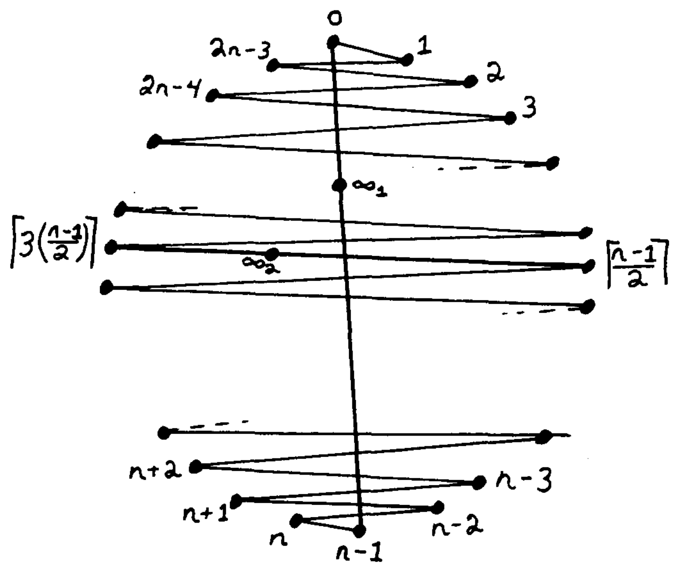


Figure 1.2.1

Figure 1.2.1 shows the first 2-factor of the Hamilton decomposition of K_{2n}-F. We have $2n$-2 vertices on the circumference of a circle labled consecutively from 0 to

$2n$-3 . We join vertex 0 to vertex 1, vertex 1 to vertex $2n$-3, vertex $2n$-3 to vertex 2, vertex 2 to vertex $2n$-4, and so on until we reach vertex n-1 which is then joined to vertex 0. Place a vertex labeled ∞_1 on the edge between vertex 0 and vertex n-1, and a vertex labeled ∞_2 on the edge joining vertex $\lceil (n-1)/2 \rceil$ with vertex $\lceil 3(n-1)/2 \rceil$.

Let this Hamilton cycle be R, and let $\alpha = (\infty_1)(\infty_2)(0\ 1\ 2\ \ldots\ 2n\text{-}3)$ be a permutation of the vertices of R. It is easy to check that the set of cycles generated by α, $\{\alpha^i(R) \mid i = 0, 1, 2, \ldots, n\text{-}2\}$, is a complete Hamilton decomposition of K_{2n}-F where $F = \{[\infty_1, \infty_2], [i, i+n\text{-}1] : i = 0, 1, 2, \ldots, n\text{-}1\}$. This construction gives us

Theorem 1.2.1: *A solution for OP(2n; 2n) exists for all n > 1.*

Figure 1.2.2 shows a base factor for the Hamilton decomposition of K_{2n+1} by a very similar construction to the one above. We start with $2n$ vertices labeled from 0 to $2n$-1 consecutively around the circumference of a circle. Vertex 0 is joined to vertex 1, vertex 1 to vertex $2n$-1, vertex $2n$-1 to vertex 2 and so on until we join vertex n+1 to vertex n. Vertex n is then joined to vertex 0 and a vertex labeled ∞ is placed on this last edge. Let this Hamilton cycle be R and let $\alpha = (\infty)(0, 1, 2, \ldots, 2n\text{-}1)$. Checking shows that $\{\alpha^i(R) : i = 0, 1, 2, \ldots, n\text{-}1\}$ gives a Hamilton decomposition of K_{2n+1}.

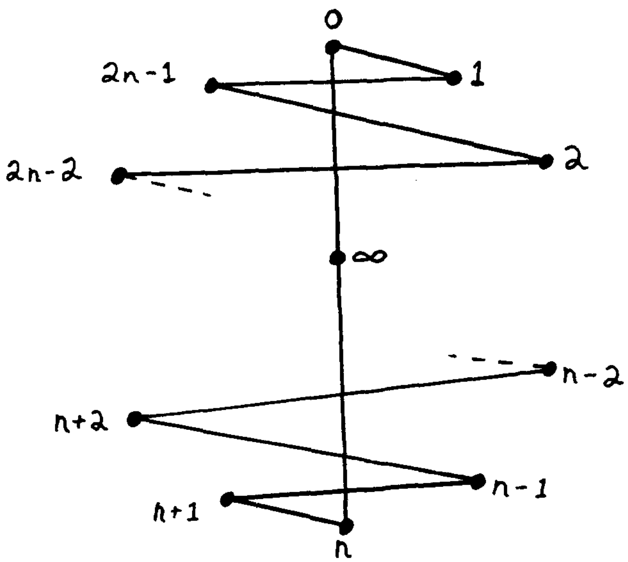


Figure 1.2.2

This construction yields

Theorem 1.2.2: *A solution for OP(2n+1; 2n+1) exists for all n > 0.*

1.3 Uniform Cycle Decompositions

The two parts of the Oberwolfach problem mentioned above are extremes between which lie a number of solved and a large number of unsolved cases. The Kirkman problem asks for a decomposition into the smallest cycle lengths possible and the Hamilton decompositions are decompositions into the longest possible cycle length. In both of these situations we were looking for what are now referred to as decompositions into *uniform* 2-factors (i.e., all cycles are the same length).

Several authors from the middle 1970's to the middle 1980's obtained results on uniform 2-factorizations. Hell, Kotzig and Rosa [8] introduced some notation that has become standard in these questions. $D(s)$ is defined as the set of all integers v such that K_v can be decomposed into uniform 2-factors of s-cycles. That paper included several results. If k is odd and $k \geq 3$ and there exists a resolvable $(v, k, 1)$-*BIBD* then $v \in D(k)$. This theorem immediately yields two corollaries. Since for any prime p and positive integer α, there exists a resolvable $(p^{2\alpha}, p^{\alpha}, 1)$-*BIBD*, it follows that if p is an odd prime and the integer $\alpha \geq 1$, then $p^{2\alpha} \in D(p^{\alpha})$. It is established in [18] that for any integer $k \geq 2$, there exists a constant $c(k)$ such that for every $v > c(k)$ where $v \equiv k \pmod{k(k-1)}$ there exists a resolvable $(v, k, 1)$-*BIBD*. Thus if k is odd and $k \geq 3$, there exists a constant $c(k)$ such that for all $v \geq c(k)$ where $v \equiv k$ (mod $k(k-1)$), $v \in D(k)$.

Hell, Kotzig and Rosa also show that "$3s \in D(s)$ if and only if s is odd, $s > 1$," by way of a construction. This theorem seems to contradict a theorm in [10] where Horton, Roy, Schellenberg and Stinson note that "*For* v *a positive integer,* v $\in$ D(4) *if and only if* v *is a multiple of* 4," which implies that $12 \in D(4)$. This confusion is easily resolved by realizing that in the ten years between these papers, the two parts of the Oberwolfach Problem had become one and thus the meaning of the $D(s)$ notation had changed to accommodate that newly modified understanding of the problem. Thus, in the current literature it is understood that $v \in D(s)$ means that K_v (if v is odd) or K_v-F (if v is even) can be uniformly 2-factored into s-cycles. The same problem occurs when earlier authors state results in terms of '*NOP*'. Modern notation would be '*OP*' and the restrictions on the parity of v would be either modified or dropped. Throughout this thesis we will use the more modern notation and phrasing, which will occasionally appear to be slightly different from the original statements of the results being reported.

Back to the results. Hell, Kotzig and Rosa also show in [8] that if $m \in D(s)$ and $n \in D(s)$, then $mn \in D(s)$ by observing that $K_{mn} = K_m \times K_n \cup K_m \otimes K_n$ and showing that $K_m \times K_n$ and $K_m \otimes K_n$ can be 2-factored into s-cycles whenever K_m and K_n can be. Given two graphs G and H, the graph $G \times H$ has vertex set $V(G) \times V(H)$ and an edge $[(g, h), (g', h')]$ if and only if $[g, g'] \in E(G)$ and $h = h'$, or $[h, h'] \in E(H)$ and $g = g'$. $G \otimes H$ also has vertex set $V(G) \times V(H)$, but has an edge $[(g, h), (g', h')]$ if and only if $[g, g'] \in E(G)$ and $[h, h'] \in E(H)$. This latest theorem yields the corollary: $s^n \in D(s)$ for odd s and every integer $n \geq 1$. The final theorem in this paper states that $rs \in D(s)$ when $r = 3^k s^{n-1}$, s is odd, $s \geq 3$, $k \geq 0$ and $n \geq 1$, but the arguments only support the claim when $0 \leq k \leq n-1$.

Five years later, Huang, Kotzig and Rosa [11] focused on the even cases (decomposition of K_{2n} into uniform isomorphic 2-factors) showing that $v \in D(4)$ whenever $v \equiv 0 \pmod 4$, $2k \in D(k)$ for $k \geq 4$, and $6k \in D(2k)$ for $k > 1$. These proofs were done by direct construction of a base factor and the results were reported in *NOP* notation. They also give a specific solution for $OP(10; 5)$.

In 1985, Horton, Roy, Schellenberg and Stinson [10] collected known results and added a few more of their own. For any positive integers s and t, $8ts \in D(4t)$. If $m \equiv 2 \pmod 4$ then $4m \in D(m)$. If n is a multiple of 3 other than 6 and 12, then we have $mn \in D(m)$. For positive s and t, $20ts \in D(10t)$ and $28ts \in D(14t)$. For $m > 2$ and t any positive integer except 2 or 4, $3tm \in D(m)$. Most of these results are derived from known results about *BIBD*'s, abelian groups and complete bipartite and tripartite graphs.

Alspach and Häggkvist [1] settled all cases of uniform 2-factorizations into even length cycles in the same year. For $m \geq 2$, $2mn \in D(2m)$ for all positive integers n. The proof of this theorem rests on visualizing K_{2mn} as various wreath products of appropriate size graphs so that the decomposition into $2m$-cycles follows directly from previously known results. The wreath product *GwrH* is obtained by replacing each vertex of G with a copy of H, joining two vertices in different copies of H only if the vertices of G corresponding to those copies of H are adjacent. See the solution given for $OP(9; 3^3)$ in Section 2 of this thesis to see an application of the wreath product idea. So we have, in OP notation,

Theorem 1.3.1: *If m is even and $m \geq 2$, a solution for $OP(mn; m)$ exists for every natural number n.*

The remaining cases of uniform 2-factorizations into odd length cycles for all complete graphs (except those of the form K_{4m} where m is the cycle length) were settled by Alspach, Schellenberg, Stinson and Wagner [2] four years later. The proof of this theorem also relies on visualizing complete graphs as wreath products and showing that decompositions must exist for the various pieces and therefore also for the complete graph.

The last remaining question regarding uniform 2-factorizations has been answered by Hoffman and Schellenberg [9]. It is now established that $4m \in D(m)$ and we have

Theorem 1.3.2: *For m odd and $m \geq 5$, a solution for $OP(mn; m)$ exists for every positive integer n. For $m = 3$, a solution for $OP(mn; m)$ exists for every positive integer n except 2 and 4.*

Taken together, Theorem 1.3.1 and Theorem 1.3.2 settle all cases of decomposition into uniform 2-factors.

1.4 Non-uniform Decompositions

What remains largely an open question in the Oberwolfach Problem is the existence of decompositions of K_n into non-uniform 2-factors. What follows is a collection of results that represent the progress to date.

Köhler [13] has shown that solutions for both $OP(8k+3; 3, 4k, 4k)$ and $OP(8k+1; 3, 8k-2)$ exist. Huang, Kotzig and Rosa [11] constructed solutions for $OP(k+3; 3, k)$ whenever k is odd and $k \geq 5$, and for $OP(k+4; 4, k)$ whenever k is even and $k \geq 4$. They also show that a solution exists for $OP(6k+4; 2k+2, 2k+1, 2k+1)$ when $k \geq 1$ and that a solution for $OP(2k+2\lceil k/2 \rceil+2c; k, k, 2\lceil k/2 \rceil+2c)$ exists for all positive integers c except 1.

This is the extent of the general cases that are solved and, though there are solutions to other specific cases, this leaves the Oberwolfach Problem whenever each 2-factor is to comprise several cycles of different length pretty much wide open.

2. $OP(n; a_1, a_2, \ldots, a_t)$ Solutions for Small n

In this section we will give a 2-factorization (if it is known) or reason for the non-existence of one for each possible combination of cycles into which each K_{2n} on fewer than 19 vertices and each K_{2n+1} on fewer than 16 vertices might be decomposed. As we go along we will use different solution techniques so the reader can get a feel for them. Unless otherwise noted, these are decompositions generated by the author, but only the existence of most of the decompositions of K_{18} is new.

The graphs K_1 and K_2 contain no cycles, so K_3 is the first complete graph where the Oberwolfach Problem makes sense. Since K_3 is a single cycle, it is in itself the solution for $OP(3; 3)$

$OP(4; 4)$ is the only possible case involving K_4. Removing any 1-factor from K_4 yields a 4-cycle and thereby a solution.

$OP(5; 5)$ and $OP(6; 6)$ are solved using the Walecki constructions used earlier to obtain Theorems 1.2.1 and 1.2.2. All Hamilton decompositions in this section will be accomplished by use of this construction.

For writing the solutions in base factor situations we will adopt the notation used by Huang, Kotzig and Rosa in [11]. V is the vertex-set , R, as above, is the base 2-factor in cycle notation, F is the 1-factor to be deleted (if appropriate), and α, as in the previous chapter, is the permutation on the vertex-set that is used to generate successive 2-factors to complete the decomposition. In addition we will denote by F_i a 2-factor of the decomposition which is usually the result of $\alpha^i(R)$. The symbol '∞' will be used to identify vertices that are fixed points of the permutation α. The solutions for $OP(5; 5)$ and $OP(6; 6)$ in this notation are as follows:

OP(5; 5)

$V = \mathbf{Z}_4 \cup \{\infty\}$
$\alpha = (\infty)\ (0\ 1\ 2\ 3)$
$F_{i+1} = \alpha^i(R),\ i = 0, 1$
$R = \{(\infty, 0, 1, 3, 2)\}$

$OP(6; 6)$	$V = \mathbf{Z}_4 \cup \{\infty_1, \infty_2\}$ $\alpha = (\infty_1)\ (\infty_2)\ (0\ 1\ 2\ 3)$ $F_{i+1} = \alpha^i(R),\ i = 0, 1$ $F = \{[\infty_1, \infty_2], [0, 2], [1, 3]\}$ $R = \{(\infty_1, 0, 1, \infty_2, 3, 2)\}$

The first possibility of a 2-factorization that is not into Hamilton cycles is $OP(6; 3^2)$, the decomposition of K_6 into 3-cycles, but no solution is possible. As soon as the first 2-factor is selected, the edge set remaining is isomorphic to $K_{3,3}$ (the complete bipartite graph with three vertices in each part) which contains no triangles from which to fashion further 2-factors.

The decomposition of K_7 can be done in two ways. The solution for $OP(7; 7)$ is a Hamilton decomposition and $OP(7; 3, 4)$ is accomplished with a permutation that adds 2 to each vertex number to get successive 2-factors, unlike the permutation for the Hamilton decomposition which adds 1. The solutions are listed below, but notice that when the same V is used or when the F_i's have the same designation for more than one case, we will show them only once at the beginning of the list of base factors. As we go on, the same will be true for α, F and the F_i's.

	$V = \mathbf{Z}_6 \cup \{\infty\}$ $F_{i+1} = \alpha^i(R),\ i = 0, 1, 2$
$OP(7; 7)$	$\alpha = (\infty)\ (0\ 1\ 2\ 3\ 4\ 5)$ $R = \{(\infty, 0, 1, 6, 2, 5, 3, 4)\}$
$OP(7; 3, 4)$	$\alpha = (\infty)\ (0\ 2\ 4)\ (1\ 3\ 5)$ $R = \{(\infty, 0, 1)\ (2, 4, 3, 5)\}$

In addition to the Hamilton decomposition of K_8, there are two other possibilities. Their base factors are shown schematically below because they represent another way of thinking about the vertex-set that is helpful in many upcoming cases. The labeling system is that used in [11]. The factorizations follow Figure 2.1. (Note that when V includes a copy of $\mathbf{Z}_n$ any addition done in specifying F is done modulo n.)

$OP(8;\, 3, 5)$ $\qquad\qquad$ $OP(8;\, 4^2)$

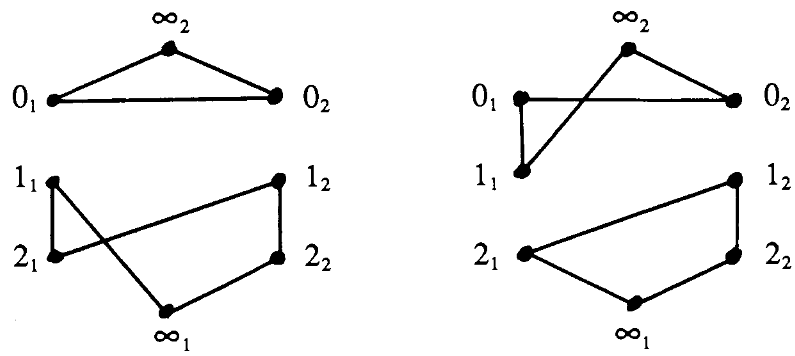


Figure 2.1

$$V = \mathbf{Z}_3 \times \{1,2\} \cup \{\infty_1, \infty_2\}$$
$$\alpha = (\infty_1)(\infty_2)(0_1\, 1_1\, 2_1)(0_2\, 1_2\, 2_2)$$
$$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2$$
$$F = \{[\infty_1, \infty_2],\ [i_1, (i+1)_2] : i = 0,1,2\}$$

$OP(8;\, 3, 5)$ $\quad R = \{(\infty_2, 0_1, 0_2), (\infty_1, 1_1, 2_1, 1_2, 2_2)\}$

$OP(8;\, 4^2)$ $\quad R = \{(\infty_1, 2_1, 1_2, 2_2), (\infty_2, 0_2, 0_1, 1_1)\}$

$$V = \mathbf{Z}_6 \cup \{\infty_1, \infty_2\}$$
$$\alpha = (\infty_1)(\infty_2)(0\ 1\ 2\ 3\ 4\ 5)$$
$$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2$$
$$F = \{[\infty_1, \infty_2], [0, 3], [1, 4], [3, 5]\}$$

$OP(8;\, 8)$ $\quad R = \{(\infty_1, 0, 1, 5, \infty_2, 2, 4, 3)\}$

Köhler [13] has shown that there is no solution for $OP(9;\, 4, 5)$. He finds that there are only four non-isomorphic ways to choose the first two 2-factors. He then considers the complements of these graphs. Since none of the complements is isomorphic to any of the original four graphs, they cannot contain two disjoint 2-factors and the result follows.

A solution for $OP(9;\, 3^3)$ is our first opportunity to visualize a solution in a wreath product. Visualize K_9 as $K_3 wr K_3$ (i.e., think of a copy of K_3 being inserted into each of the three vertices of another K_3 and then join all vertices that are from different copies of K_3). The three inserted K_3's form the first 2-factor and the other three are shown in Figure 5. Following the figure, a solution is given for each OP situation of K_9

decomposition that exists. Note that since this solution to $OP(9;\ 3^3)$ does not use a base factor, the 2-factors F_i are listed rather than R and a permutation α.

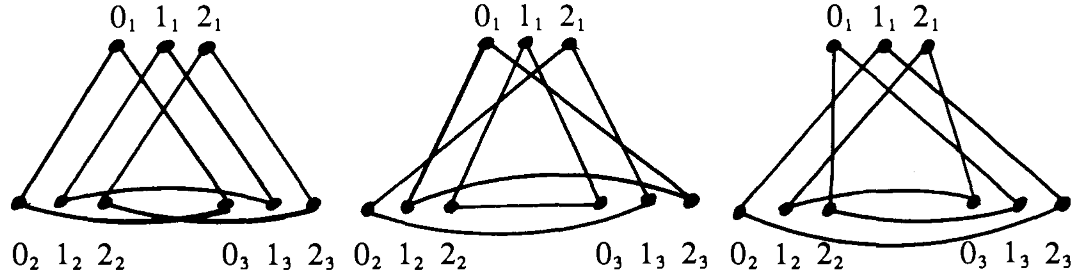


Figure 2.2

	$V = \mathbf{Z}_3 \times \{1, 2, 3\}$
$OP(9;\ 3^3)$	$F_1 = \{(0_i, 1_i, 2_i) : i = 1, 2, 3\}$
	$F_2 = \{(j_1, j_2, j_3) : j = 0, 1, 2\}$
	$F_3 = \{(j_1, (j+1)_2, (j+2)_3) : j = 0, 1, 2\}$
	$F_4 = \{(j_1, (j+2)_2, (j+4)_3) : j = 0, 1, 2\}$
	$V = \mathbf{Z}_8 \cup \{\infty\}$
	$\alpha = (\infty)(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7)$
	$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3$
$OP(9;\ 3, 6)$	$R = \{(\infty, 0, 4)\ (1, 2, 7, 5, 6, 3)\}$
$OP(9;\ 4, 5)$	Not Possible [12]
$OP(9;\ 9)$	$R = \{(\infty, 0, 1, 7, 2, 6, 3, 5, 4)\}$

Solutions for decomposing K_{10} are similar to those above. The solution for $OP(10;\ 5^2)$ is due to Huang, Kotzig and Rosa [11]. As with $OP(9;\ 3^3)$, it does not use a base factor with a permutation, but rather stipulates each 2-factor.

	$V = \mathbf{Z}_8 \cup \{\infty_1, \infty_2\}$
	$\alpha = (\infty_1)(\infty_2)(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7)$
	$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3$
	$F = \{[\infty_1, \infty_2], [0, 4], [1, 5], [2, 6], [3, 7]\}$
$OP(10;\ 10)$	$R = \{(\infty_1, 0, 1, 7, 2, \infty_2, 6, 3, 5, 4)\}$

	$V = \mathbf{Z}_4 \times \{1,2\} \cup \{\infty_1, \infty_2\}$
	$\alpha = (\infty_1)(\infty_2)(0_1\, 1_1\, 2_1\, 3_1)(0_2\, 1_2\, 2_2\, 3_2)$
	$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3$
	$F = \{[\infty_1, \infty_2], [i_j, (i+2)_j] : i = 0, 1; j = 1, 2\}$
$OP(10; 3, 7)$	$R = \{(\infty_2, 1_1, 0_2), (\infty_1, 3_2, 2_2, 2_1, 3_1, 1_2, 0_1)\}$
$OP(10; 4, 6)$	$R = \{(\infty_1, 3_1, 2_2, 3_2), (\infty_2, 0_1, 1_2, 1_1, 2_1, 0_2)\}$
$OP(10; 3^2, 4)$	$R = \{(1_2, 2_1, 3_1), (\infty_2, 0_1, 0_2), (\infty_1, 1_1, 2_2, 3_2)\}$

	$V = \mathbf{Z}_5 \times \{1,2\}$
	$F = \{[i_1, i_2] : i = 0, 1, 2, 3, 4\}$
$OP(10; 5^2)$	$F_1 = \{(0_1, 1_1, 2_1, 3_1, 4_1), (0_2, 1_2, 2_2, 3_2, 4_2)\}$
	$F_2 = \{(0_1, 2_1, 4_1, 3_2, 1_2), (0_2, 2_2, 4_2, 3_1, 1_1)\}$
	$F_3 = \{(0_1, 3_1, 0_2, 4_1, 2_2), (1_1, 3_2, 2_1, 1_2, 4_2)\}$
	$F_4 = \{(0_1, 3_2, 0_2, 2_1, 4_2), (1_1, 4_1, 1_2, 3_1, 2_2)\}$

With decomposing K_{11} there is only one problematic case, that of $OP(11; 3^2, 5)$. This case has defied all attempts at a proof of non-existence short of an exhaustive computer search for solutions. This case is dealt with in Section 3 of this thesis. The other decompositions of K_{11} are possible and examples of solutions follow.

	$V = \mathbf{Z}_{10} \cup \{\infty\}$
	$\alpha = (\infty)\ (0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9)$
	$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4$
$OP(11; 11)$	$R = \{(\infty, 0, 1, 9, 2, 8, 3, 7, 4, 6, 5)\}$

	$V = \mathbf{Z}_5 \times \{1,2\} \cup \{\infty\}$
	$\alpha = (\infty)(0_1\, 1_1\, 2_1\, 3_1\, 4_1)(0_2\, 1_2\, 2_2\, 3_2\, 4_2)$
	$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4$
$OP(11; 3, 8)$	$R = \{(\infty, 3_1, 4_2), (0_1, 0_2, 2_1, 1_1, 4_1, 3_2, 1_2, 2_2)\}$
$OP(11; 4, 7)$	$R = \{(0_2, 1_1, 3_1, 2_1), (\infty, 4_1, 4_2, 3_2, 1_2, 0_1, 2_2)\}$
$OP(11; 5, 6)$	$R = \{(\infty, 4_2, 2_2, 3_2, 4_1), (0_1, 1_2, 1_1, 2_1, 0_2, 3_1)\}$
$OP(11; 3^2, 5)$	Not Possible [15]
$OP(11; 3, 4^2)$	$R = \{(\infty, 0_2, 4_1), (0_1, 2_1, 3_1, 2_2), (1_1, 1_2, 3_2, 4_2)\}$

Once again with K_{12} there is one exceptional case. As mentioned once before, $OP(12; 3^4)$ has no solution. Kotzig and Rosa claim in [14] that there are only three non-isomorphic sets of four 2-factors, but in none of these cases do the remaining edges form a fifth 2-factor of 3-cycles. The solutions in the following list that are marked with '*' are also presented in [11].

	$V = \mathbf{Z}_{10} \cup \{\infty_1, \infty_2\}$
	$\alpha = (\infty_1)(\infty_2)(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9)$
	$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4$
	$F = \{[\infty_1, \infty_2], [0, 5], [1, 6], [2, 7], [3, 8], [4, 9]\}$
$OP(12; 12)$	$R = \{(\infty_1, 0, 1, 9, 2, 8, \infty_2, 3, 7, 4, 6, 5)\}$

	$V = \mathbf{Z}_5 \times \{1, 2\} \cup \{\infty_1, \infty_2\}$
	$\alpha = (\infty_1)(\infty_2)(0_1\ 1_1\ 2_1\ 3_1\ 4_1)(0_2\ 1_2\ 2_2\ 3_2\ 4_2)$
	$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4$
	$F = \{[\infty_1, \infty_2], [i_1, (i+1)_2] : i = 0, 1, 2, 3, 4\}$
$OP(12; 3, 9)$	$R = \{(\infty_1, 4_1, 2_2), (\infty_2, 0_1, 0_2, 1_1, 3_1, 2_1, 4_2, 3_2, 1_2)\}$
$OP(12; 4, 8)$	$R = \{(\infty_1, 4_1, 3_2, 4_2), (\infty_2, 0_1, 2_2, 0_2, 2_1, 3_1, 1_1, 1_2)\}$
$OP(12; 6^2)$	$R = \{(\infty_1, 4_1, 3_2, 0_1, 2_2, 4_2), (\infty_2, 0_2, 1_2, 1_1, 3_1, 2_1)\}$
$OP(12; 4^3)$	$R = \{(\infty_1, 1_1, \infty_2, 1_2), (4_1, 2_2, 4_2, 3_2), (0_1, 0_2, 3_1, 2_1)$
$OP(12; 3^4)$	Not Possible [13]

	$\alpha = (\infty_1)(\infty_2)(0_1\ 1_1\ 2_1\ 3_1\ 4_1)(0_2\ 1_2\ 2_2\ 3_2\ 4_2)$
	$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4$
	$F = \{[\infty_1, \infty_2], [i_1, (i+3)_2] : i = 0, 1, 2, 3, 4\}$
$*OP(12; 5, 7)$	$R = \{(\infty_1, 4_1, 3_2, 2_1, 4_2), (\infty_2, 3_1, 1_1, 0_1, 0_2, 2_2, 1_2)\}$
$*OP(12; 3^2, 6)$	$R = \{(\infty_1, 0_1, 1_2), (\infty_2, 1_1, 0_2), (2_1, 4_1, 3_1, 3_2, 2_2, 4_2)\}$
$*OP(12; 3, 4, 5)$	$R = \{(\infty_1, 3_1, 0_2), (4_1, 3_2, 2_2, 4_2), (\infty_2, 1_1, 2_1, 0_1, 1_2)\}$

All possible cycle combinations for decomposing K_{13} have been accomplished and an example of each follows. Though the notation has been adjusted to match the rest of this section, the solution given for $OP(13; 6, 7)$ is due to Köhler [13], and the five solutions marked with '+' are due to Piotrowski [16]. Notice that the Piotrowski solutions have *two* base factors and a more complicated permutation.

	$V = \mathbf{Z}_{12} \cup \{\infty\}$
	$\alpha = (\infty)(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11)$
	$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4, 5$
$OP(13; 13)$	$R = \{(\infty, 0, 1, 11, 2, 10, 3, 9, 4, 8, 5, 7, 6)\}$
$OP(13; 5, 8)$	$R = \{(\infty, 0, 11, 5, 6), (1, 3, 8, 4, 7, 9, 2, 10)\}$
$OP(13; 6, 7)$	$R = \{(0, 1, 3, 6, 7, 9), (\infty, 5, 10, 2, 8, 4, 11)\}$
$OP(13; 3^2, 7)$	$R = \{(1, 2, 10), (4, 8, 7), (\infty, 0, 5, 3, 9, 11, 6)\}$
$OP(13; 4^2, 5)$	$R = \{(2, 4, 9, 5), (8, 10, 3, 11)(\infty, 0, 1, 7, 6)\}$

$V = \mathbf{Z}_3 \times \{1, 2, 3, 4\} \cup \{\infty\}$
$\alpha = (\infty)(0_1\ 1_1\ 2_1)(0_2\ 1_2\ 2_2)(0_3\ 1_3\ 2_3)(0_4\ 1_4\ 2_4)$
$F_{i+1} = \alpha^i(R_1),\ i = 0, 1, 2$
$F_{i+1} = \alpha^i(R_2),\ i = 3, 4, 5$

+$OP(13; 3, 10)$ $R_1 = \{(1_4, 2_2, 2_3), (\infty, 0_3, 0_4, 1_1, 1_3, 0_1, 2_1, 2_4, 1_2, 0_2)\}$
$R_2 = \{(1_1, 2_2, 2_4), (\infty, 0_4, 1_4, 0_3, 1_3, 2_1, 1_2, 2_3, 0_2, 0_1)\}$

+$OP(13; 4, 9)$ $R_1 = \{(1_1, 0_4, 0_3, 1_3), (\infty, 0_1, 2_1, 1_2, 2_4, 2_2, 1_4, 2_3, 0_2)\}$
$R_2 = \{(\infty, 0_3, 1_4, 0_4), (0_1, 0_2, 1_2, 2_3, 2_2, 1_1, 2_4, 2_1, 1_3)\}$

+$OP(13; 3, 5^2)$ $R_1 = \{(0_4, 2_1, 2_3), (\infty, 0_1, 1_3, 0_3, 0_2), (1_1, 1_2, 2_4, 1_4, 2_2)\}$
$R_2 = \{(0_1, 2_1, 2_4), (\infty, 0_3, 1_1, 0_2, 0_4), (1_2, 2_2, 1_3, 1_4, 2_3)\}$

+$OP(13; 3, 4, 6)$ $R_1 = \{(1_4, 2_2, 2_3), (0_3, 0_4, 1_1, 1_3), (\infty, 0_1, 2_1, 2_4, 1_2, 0_2)\}$
$R_2 = \{(1_1, 2_2, 2_4), (\infty, 0_3, 1_4, 0_4), (0_1, 0_2, 2_3, 1_2, 2_1, 1_3)\}$

+$OP(13; 3^3, 4)$ $R_1 = \{(\infty, 0_1, 0_2), (1_2, 2_1, 2_4), (1_4, 2_2, 2_3), (0_3, 0_4, 1_1, 1_3)\}$
$R_2 = \{(0_1, 2_1, 1_3), (0_2, 1_2, 2_3), (1_1, 2_2, 2_4), (\infty, 0_3, 1_4, 0_4)\}$

There being nothing particularly special about the decompositions of K_{14}, we simply list them. Again, solutions marked with '*' also appear in [11]. It is perhaps worth noting that the solution for $OP(14; 4^2, 6)$ in [11] is incorrect.

$V = \mathbf{Z}_{12} \cup \{\infty_1, \infty_2\}$
$\alpha = (\infty_1)(\infty_2)(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11)$
$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4, 5$
$F = \{[\infty_1, \infty_2], [0, 6], [1, 7], [2, 8], [3, 9], [4, 10], [5, 11]\}$

$OP(14; 14)$ $R = \{(\infty_1, 0, 1, 11, 2, 10, 3, \infty_2, 9, 4, 8, 5, 7, 6)\}$

$V = \mathbf{Z}_6 \times \{1, 2\} \cup \{\infty_1, \infty_2\}$
$\alpha = (\infty_1)(\infty_2)(0_1\ 1_1\ 2_1\ 3_1\ 4_1\ 5_1)(0_2\ 1_2\ 2_2\ 3_2\ 4_2\ 5_2)$
$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4, 5$
$F = \{[\infty_1, \infty_2], [i_j, (i+3)_j] : i = 0, 1, 2; j = 1, 2\}$

$OP(14; 3, 11)$ $R = \{(\infty_1, 5_1, 4_2), (\infty_2, 2_1, 3_1, 1_1, 5_2, 4_1, 0_2, 0_1, 3_2, 1_2, 2_2)\}$
$OP(14; 4, 10)$ $R = \{(\infty_1, 5_1, 3_2, 5_2), (\infty_2, 3_1, 0_2, 1_2, 2_1, 4_2, 4_1, 0_1, 1_1, 2_2)\}$
*$OP(14; 5, 9)$ $R = \{(\infty_1, 5_1, 5_2, 4_2, 2_2), (\infty_2, 0_1, 4_1, 1_2, 3_1, 2_1, 3_2, 1_1, 0_2)\}$
*$OP(14; 6, 8)$ $R = \{(\infty_1, 0_1, 5_1, 5_2, 4_2, 2_2), (\infty_2, 0_2, 1_1, 3_2, 2_1, 4_1, 1_2, 3_1)\}$
$OP(14; 7^2)$ $R = \{(\infty_1, 5_1, 4_2, 1_1, 2_2, 3_2, 5_2), (\infty_2, 4_1, 0_2, 0_1, 2_1, 3_1, 1_2)\}$
*$OP(14; 3^2, 8)$ $R = \{(\infty_1, 2_1, 0_2), (\infty_2, 3_1, 2_2), (0_1, 1_2, 5_2, 4_2, 4_1, 5_1, 1_1, 3_2)\}$
*$OP(14; 3, 4, 7)$ $R = \{(3_1, 4_1, 1_2), (\infty_1, 5_1, 5_2, 4_2), (\infty_2, 0_1, 2_1, 3_2, 1_1, 0_2, 2_2)\}$
*$OP(14; 3, 5, 6)$ $R = \{(3_1, 4_1, 1_2), (\infty_1, 5_1, 5_2, 4_2, 2_2), (\infty_2, 0_1, 2_1, 3_2, 1_1, 0_2)\}$
$OP(14; 4^2, 6)$ $R = \{(\infty_2, 0_2, 2_2, 3_1), (0_1, 1_1, 1_2, 4_1), (\infty_1, 5_1, 3_2, 2_1, 4_2, 5_2)\}$
*$OP(14; 4, 5^2)$ $R = \{(\infty_1, 3_1, 1_1, 4_2), (\infty_2, 4_1, 5_1, 1_2, 2_2), (0_1, 0_2, 2_1, 3_2, 5_2)\}$
*$OP(14; 3^3, 5)$ $R = \{(\infty_1, 5_1, 3_2), (\infty_2, 4_1, 1_2), (3_1, 4_2, 5_2), (0_1, 1_1, 0_2, 2_2, 2_1)\}$
*$OP(14; 3^2, 4^2,)$ $R = \{(\infty_1, 5_1, 3_2), (\infty_2, 4_1, 5_2), (0_1, 1_1, 3_1, 0_2), (2_1, 1_2, 2_2, 4_2)\}$

The decompositions of K_{15} are also routine, but for $OP(15;\, 3^5)$, the original Kirkman problem, we give "an explicit solution of Kirkman's Problem in its original form" from Ball [4].

	$V = \mathbf{Z}_{14} \cup \{\infty\}$
	$\alpha = (\infty)\,(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13)$
	$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4, 5, 6$
$OP(15;\, 15)$	$R = \{(\infty, 0, 1, 13, 2, 12, 3, 11, 4, 10, 5, 9, 6, 8, 7)\}$
	$V = \mathbf{Z}_7 \times \{1,2\} \cup \{\infty\}$
	$\alpha = (\infty)(0_1\ 1_1\ 2_1\ 3_1\ 4_1\ 5_1\ 6_1)(0_2\ 1_2\ 2_2\ 3_2\ 4_2\ 5_2\ 6_2)$
	$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4, 5, 6$
$OP(15;\, 3, 12)$	$R = \{(0_2, 1_2, 3_2), (\infty, 6_2, 3_1, 2_1, 5_1, 0_1, 2_2, 1_1, 5_2, 6_1, 4_2, 4_1)\}$
$OP(15;\, 4, 11)$	$R = \{(\infty, 6_1, 5_2, 6_2), (0_1, 2_2, 4_2, 3_1, 3_2, 0_2, 2_1, 4_1, 1_2, 5_1, 1_1)\}$
$OP(15;\, 5, 10)$	$R = \{(\infty, 6_1, 5_2, 3_2, 6_2), (0_1, 0_2, 2_1, 4_2, 1_1, 4_1, 3_1, 5_1, 2_2, 1_2)\}$
$OP(15;\, 6, 9)$	$R = \{(\infty, 6_1, 4_2, 5_2, 3_2, 6_2), (0_1, 1_2, 4_1, 0_2, 5_1, 2_1, 2_2, 3_1, 1_1)\}$
$OP(15;\, 7, 8)$	$R = \{(\infty, 6_1, 6_2, 3_2, 2_2, 4_2, 5_1, 1_2), (0_1, 1_1, 4_1, 5_2, 3_1, 0_2, 2_1)\}$
$OP(15;\, 3^2, 9)$	$R = \{(0_1, 3_1, 4_2), (0_2, 2_2, 3_2), (\infty, 5_1, 6_1, 4_1, 6_2, 1_1, 1_2, 2_1, 5_2)\}$
$OP(15;\, 3, 4, 8)$	$R = \{(0_2, 1_2, 3_2), (\infty, 6_2, 3_1, 4_1), (0_1, 2_2, 1_1, 5_2, 5_1, 4_2, 6_1, 2_1)\}$
$OP(15;\, 3, 5, 7)$	$R = \{(0_2, 4_1, 5_1), (\infty, 6_1, 5_2, 3_2, 4_2), (1_1, 1_2, 3_1, 0_1, 2_1, 6_2, 2_2)\}$
$OP(15;\, 3, 6^2)$	$R = \{(2_1, 3_1, 3_2), (\infty, 6_2, 1_2, 4_2, 5_2, 0_1), (0_2, 1_1, 4_1, 6_1, 2_2, 5_1)\}$
$OP(15;\, 4^2, 7)$	$R = \{(\infty, 5_2, 3_1, 1_1), (0_2, 4_1, 2_2, 3_2), (6_1, 6_2, 4_2, 0_1, 1_2, 2_1, 5_1)\}$
$OP(15;\, 4, 5; 6)$	$R = \{(5_1, 2_1, 5_2, 3_1), (\infty, 6_1, 6_2, 3_2, 4_2), (0_1, 1_1, 0_2, 2_2, 4_1, 1_2)\}$
$OP(15;\, 5^3)$	$R = \{(\infty, 5_2, 3_1, 3_2, 4_1), (0_1, 1_1, 6_2, 2_1, 5_1), (0_2, 1_2, 4_2, 2_2, 6_1)\}$
$OP(15;\, 3^3, 6)$	$R = \{(0_1, 1_1, 1_2), (2_1, 4_1, 0_2), (3_1, 6_1, 5_2), (\infty, 5_1, 2_2, 3_2, 6_2, 4_2)\}$
$OP(15;\, 3, 4^3)$	$R = \{(0_1, 5_1, 4_1), (\infty, 6_1, 3_2, 4_2), (0_2, 1_1, 2_2, 2_1), (1_2, 6_2, 3_1, 5_2)\}$
$OP(15;\, 3^2, 4, 5)$	$R = \{(0_1, 1_1, 0_2), (6_1, 1_2, 4_2), (2_1, 5_1, 3_1, 6_2), (\infty, 4_1, 5_2, 3_2, 2_2)\}$
	$V = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
$OP(15;\, 3^5)$	$F_1 = \{(1, 2, 3), (4, 8, 12), (5, 10, 15), (6, 11, 13), (7, 9, 14)\}$
	$F_2 = \{(1, 4, 5), (2, 8, 10), (3, 13, 14), (6, 9, 15), (7, 11, 12)\}$
	$F_3 = \{(1, 6, 7), (2, 9, 11), (3, 12, 15), (4, 10, 14), (5, 8, 13)\}$
	$F_4 = \{(1, 8, 9), (2, 12, 14), (3, 5, 6), (4, 11, 15), (7, 10, 13)\}$
	$F_5 = \{(1, 10, 11), (2, 13, 15), (3, 4, 7), (5, 9, 12), (6, 8, 14)\}$
	$F_6 = \{(1, 12, 13), (2, 4, 6), (3, 9, 10), (5, 11, 14), (7, 8, 15)\}$
	$F_7 = \{(1, 14, 15), (2, 5, 7), (3, 8, 11), (4, 9, 13), (6, 10, 12)\}$

The decompositions of K_{16} are routine and once again solutions from [11] are marked with an asterisk.

	$V = \mathbf{Z}_{14} \cup \{\infty_1, \infty_2\}$
	$\alpha = (\infty_1)(\infty_2)(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13)$
	$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4, 5, 6$
	$F = \{[\infty_1, \infty_2], [0, 7], [1, 8], [2, 9], [3, 10], [4, 11], [5, 12], [6, 13]\}$
$OP(16; 16)$	$R = \{(\infty_1, 0, 1, 13, 2, 12, 3, 11, \infty_2, 4, 10, 5, 9, 6, 8, 7)\}$

	$V = \mathbf{Z}_7 \times \{1,2\} \cup \{\infty_1, \infty_2\}$
	$\alpha = (\infty_1)(\infty_2)(0_1\ 1_1\ 2_1\ 3_1\ 4_1\ 5_1\ 6_1)(0_2\ 1_2\ 2_2\ 3_2\ 4_2\ 5_2\ 6_2)$
	$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4, 5, 6$
	$F = \{[\infty_1, \infty_2], [i_1,(i\text{-}1)_2] : i = 0, 1, 2, 3, 4, 5, 6\}$
$OP(16; 3, 13)$	$R = \{(\infty_1, 6_1, 4_2), (\infty_2, 2_1, 3_1, 1_1, 4_1, 6_2, 5_2, 3_2, 0_2, 0_1, 1_2, 5_1, 2_2)\}$
$OP(16; 4, 12)$	$R = \{(\infty_1, 6_1, 4_2, 6_2), (\infty_2, 2_1, 5_1, 1_2, 4_1, 5_2, 3_1, 1_1, 0_1, 0_2, 3_2, 2_2)\}$
$OP(16; 8^2)$	$R = \{(\infty_1, 4_1, 0_2, 6_1, 4_2, 5_2, 3_2, 6_2), (\infty_2, 0_1, 2_2, 5_1, 2_1, 3_1, 1_1, 1_2)\}$
$OP(16; 4^2, 8)$	$R = \{(\infty_1, 5_1, 3_2, 6_2), (0_1, 0_2, 1_2, 4_1), (\infty_2, 2_1, 1_1, 6_1, 2_2, 4_2, 3_1, 5_2)\}$
$OP(16; 5^2, 6)$	$R = \{(\infty_1, 5_1, 5_2, 3_2, 6_2), (0_2, 1_2, 0_1, 4_2, 2_1), (\infty_2, 3_1, 4_1, 1_1, 6_1, 2_2)\}$
$OP(16; 4^4)$	$R = \{(\infty_1, 6_1, 4_2, 6_2), (\infty_2, 2_1, 5_1, 1_2), (0_1, 0_2, 3_2, 2_2), (1_1, 3_1, 4_1, 5_2)\}$

	$F = \{[\infty_1, \infty_2], [i_1, i_2] : i = 0, 1, 2, 3, 4, 5, 6\}$
$*OP(16; 5, 11)$	$R = \{(\infty_1, 6_1, 3_1, 1_2, 6_2), (\infty_2, 5_1, 2_2, 3_2, 0_1, 1_1, 0_2, 4_2, 2_1, 4_1, 5_2)\}$
$*OP(16;, 6, 10)$	$R = \{(\infty_1, 1_1, 0_2, 1_2, 3_1, 4_2), (\infty_2, 5_1, 6_1, 3_2, 5_2, 2_1, 4_1, 0_1, 2_2, 6_2)\}$
$*OP(16; 7, 9)$	$R = \{(\infty_1, 5_1, 2_2, 4_2, 2_1, 4_1, 5_2), (\infty_2, 6_1, 3_1, 1_2, 0_2, 1_1, 0_1, 3_2, 6_2)\}$
$*OP(16; 3^2, 10)$	$R = \{(\infty_1, 5_1, 2_2), (0_1, 1_1, 3_1), (\infty_2, 2_1, 4_2, 6_1, 5_2, 4_1, 0_2, 1_2, 3_2, 6_2)\}$
$*OP(16; 3, 4, 9)$	$R = \{(0_1, 1_1, 3_1), (\infty_1, 5_1, 6_2, 2_2), (\infty_2, 4_1, 0_2, 1_2, 2_1, 4_2, 6_1, 3_2, 5_2)\}$
$*OP(16; 3, 5, 8)$	$R = \{(0_1, 1_1, 3_1), (4_1, 1_2, 4_2, 5_1, 6_2), (\infty_1, 6_1, \infty_2, 2_2, 3_2, 5_2, 2_1, 0_2)\}$
$*OP(16; 3, 6, 7)$	$R = \{(1_2, 3_2, 4_2), (\infty_1, 0_1, 5_1, 6_1, 2_1, 5_2), (\infty_2, 3_1, 0_2, 1_1, 2_2, 4_1, 6_2)\}$
$*OP(16; 4, 5, 7)$	$R = \{(2_1, 0_2, 4_1, 1_2), (\infty_1, 6_1, 3_1, 5_1, 6_2), (\infty_2, 0_1, 1_1, 3_2, 4_2, 2_2, 5_2)\}$
$*OP(16; 4, 6^2)$	$R = \{(2_1, 0_2, 4_1, 1_2), (\infty_1, 0_1, 6_1, 3_1, 5_1, 6_2), (\infty_2, 1_1, 3_2, 4_2, 2_2, 5_2)\}$
$*OP(16; 3^3, 7)$	$R = \{(\infty_1, 4_1, 1_2), (\infty_2, 5_1, 3_2), (0_1, 1_1, 3_1), (2_1, 5_2, 6_1, 0_2, 6_2, 2_2, 4_2)\}$
$*OP(16; 3^2, 4, 6)$	$R = \{(\infty_1, 4_1, 5_2), (\infty_2, 5_1, 2_2), (1_1, 3_1, 1_2, 0_2), (0_1, 6_1, 2_1, 4_2, 6_2, 3_2)\}$
$*OP(16; 3^2, 5^2)$	$R = \{(\infty_1, 4_1, 5_2), (\infty_2, 1_1, 0_2), (3_1, 6_1, 5_1, 2_2, 1_2), (0_1, 2_1, 4_2, 6_2, 3_2)\}$
$*OP(16; 3, 4^2, 5)$	$R = \{(\infty_1, 4_1, 5_2), (0_1, 1_1, 0_2, 3_2), (3_1, 5_1, 2_2, 1_2), (\infty_2, 6_1, 2_1, 4_2, 6_2)\}$
$*OP(16; 3^4, 4)$	$R = \{(\infty_1, 6_1, 5_2), (0_1, 1_1, 3_1), (4_1, 0_2, 1_2), (5_1, 3_2, 6_2), (\infty_2, 2_1, 4_2, 2_2)\}$

The decompositions of K_{17} are not all known and, like those of K_{13}, appear to be more difficult to produce. Piotrowski [16] gives solutions for $OP(17; 3^4, 5)$, $OP(17; 3^2, 5, 6)$, $OP(17; 3, 5, 9)$, $OP(17; 4, 5, 8)$, $OP(17; 5^2, 7)$, $OP(17; 5, 6^2)$, and $OP(17; 5, 12)$. Of course, the Hamilton decomposition is also known.

Specific decompositions for some K_{18} cases have been given elsewhere, but this is the first complete set of solutions documented. The solution to $OP(18; 3^6)$ is the solution for *NKTS*(18), the smallest *Nearly Kirkman Triple System* to have a solution. The solution we present for this case is from Kotzig and Rosa [14].

$V = \mathbf{Z}_{16} \cup \{\infty_1, \infty_2\}$
$\alpha = (\infty_1)(\infty_2)(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15)$
$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4, 5, 6, 7$
$F = \{[\infty_1, \infty_2], [0, 8], [1, 9], [2, 10], [3, 11], [4, 12], [5, 13], [6, 14], [7, 15]\}$

$OP(18; 18)$ $R = \{(\infty_1, 0, 1, 15, 2, 14, 3, 13, 4, \infty_2, 12, 5, 11, 6, 10, 7, 9, 8)\}$

$V = \mathbf{Z}_8 \times \{1,2\} \cup \{\infty_1,\infty_2\}$
$\alpha = (\infty_1)(\infty_2)(0_1\ 1_1\ 2_1\ 3_1\ 4_1\ 5_1\ 6_1\ 7_1)(0_2\ 1_2\ 2_2\ 3_2\ 4_2\ 5_2\ 6_2\ 7_2)$
$F_{i+1} = \alpha^i(R),\ i = 0, 1, 2, 3, 4, 5, 6, 7$
$F = \{[\infty_1,\infty_2], [i_j,(i+4)_j] : i = 0, 1, 2, 3; j = 1, 2\}$

$OP(18; 3, 15)$ $R = \{(\infty_1, 7_1, 6_2), (\infty_2, 2_1, 1_1, 3_1, 6_1, 7_2, 5_1, 0_2, 3_2, 1_2, 2_2, 4_1, 4_2, 0_1, 5_2)\}$
$OP(18; 4, 14)$ $R = \{(\infty_1, 7_1, 6_2, 7_2), (\infty_2, 4_1, 1_1, 0_1, 2_1, 0_2, 3_2, 1_2, 6_1, 2_2, 5_1, 5_2, 3_1, 4_2)\}$
$OP(18; 5, 13)$ $R = \{(\infty_1, 7_1, 7_2, 6_2, 4_2), (\infty_2, 3_1, 1_2, 6_1, 2_2, 5_2, 0_1, 5_1, 4_1, 2_1, 3_2, 1_1, 0_2)\}$
$OP(18; 6, 12)$ $R = \{(\infty_1, 7_1, 7_2, 6_2, 3_2, 5_2), (\infty_2, 0_1, 1_1, 4_1, 6_1, 1_2, 5_1, 2_2, 3_1, 4_2, 2_1, 0_2)\}$
$OP(18; 7, 11)$ $R = \{(\infty_1, 7_1, 7_2, 6_2, 4_2, 1_1, 5_2), (\infty_2, 0_2, 3_2, 5_1, 2_2, 0_1, 1_2, 2_1, 3_1, 6_1, 4_1)\}$
$OP(18; 8, 10)$ $R = \{(\infty_1, 7_1, 7_2, 6_2, 3_2, 5_2, 1_1, 4_2), (\infty_2, 0_2, 2_1, 1_2, 0_1, 2_2, 5_1, 4_1, 6_1, 3_1)\}$
$OP(18; 9^2)$ $R = \{(\infty_1, 4_1, 1_1, 0_2, 6_1, 1_2, 4_2, 5_2, 7_2), (\infty_2, 7_1, 3_2, 3_1, 2_1, 0_1, 6_2, 5_1, 2_2)\}$
$OP(18; 3^2, 12)$ $R = \{(0_1, 5_2, 6_2), (0_2, 7_1, 2_2), (\infty_1, 6_1, 1_1, 3_2, 4_1, 4_2, 1_2, 5_1, 3_1, 2_1, \infty_2, 7_2)\}$
$OP(18; 3, 4, 11)$ $R = \{(0_2, 4_1, 5_1), (7_1, 4_2, 6_2, 7_2), (\infty_1, 5_2, 2_2, 1_1, 3_2, \infty_2, 0_1, 2_1, 1_2, 3_1, 6_1)\}$
$OP(18; 3, 5, 10)$ $R = \{(0_2, 4_1, 5_1), (\infty_1, 7_2, 6_2, 4_2, 7_1), (\infty_2, 0_1, 2_1, 3_2, 1_1, 1_2, 3_1, 6_1, 5_2, 2_2)\}$
$OP(18; 3, 6, 9)$ $R = \{(0_2, 5_1, 6_1), (\infty_1, 7_2, 5_2, 2_2, 3_2, 7_1), (\infty_2, 0_1, 2_1, 1_2, 3_1, 4_2, 4_1, 1_1, 6_2)\}$
$OP(18; 3, 7, 8)$ $R = \{(2_2, 6_1, 7_1), (\infty_1, 7_2, 6_2, 3_2, 5_2, 5_1, 3_1), (\infty_2, 0_1, 1_2, 4_1, 1_1, 0_2, 2_1, 4_2)\}$
$OP(18; 4, 6, 8)$ $R = \{(\infty_2, 4_1, 3_1, 6_2), (\infty_1, 7_2, 5_2, 4_2, 1_2, 7_1), (0_1, 0_2, 1_1, 2_2, 6_1, 3_2, 5_1, 2_1)\}$
$OP(18; 4, 7^2)$ $R = \{(\infty_2, 5_1, 7_1, 2_2), (\infty_1, 7_2, 5_2, 4_2, 0_1, 0_2, 6_1), (1_1, 4_1, 3_1, 1_2, 2_1, 3_2, 6_2)\}$
$OP(18; 4^2, 10)$ $R = \{(\infty_1, 7_2, 5_2, 3_1), (0_1, 1_1, 0_2, 3_2), (\infty_2, 6_1, 4_2, 7_1, 4_1, 2_1, 2_2, 1_2, 5_1, 6_2)\}$
$OP(18; 4, 5, 9)$ $R = \{(\infty_2, 0_2, 3_2, 0_1), (\infty_1, 7_2, 6_2, 4_2, 7_1), (1_1, 1_2, 5_1, 2_1, 3_1, 5_2, 6_1, 4_1, 2_2)\}$
$OP(18; 5^2, 8)$ $R = \{(\infty_1, 7_2, 5_2, 4_2, 6_1), (0_2, 3_2, 1_1, 1_2, 4_1), (\infty_2, 2_2, 7_1, 6_2, 5_1, 0_1, 2_1, 3_1)\}$
$OP(18; 5, 6, 7)$ $R = \{(\infty_1, 7_2, 5_2, 4_2, 7_1), (\infty_2, 6_1, 4_1, 3_1, 0_1, 6_2), (0_2, 3_2, 1_1, 2_2, 2_1, 1_2, 5_1)\}$
$OP(18; 6^3)$ $R = \{(\infty_1, 6_1, 3_2, 2_2, 4_2, 7_2), (\infty_2, 2_1, 1_1, 3_1, 0_1, 1_2), (0_2, 4_1, 6_2, 7_1, 5_2, 5_1)\}$
$OP(18; 3^3, 9)$
$R = \{(7_1, 5_2, 6_2), (\infty_2, 0_2, 5_1), (0_1, 3_1, 2_1), (\infty_1, 2_2, 6_1, 7_2, 4_2, 4_1, 1_2, 3_2, 1_1)\}$
$OP(18; 3^2, 4, 8)$
$R = \{(0_1, 1_1, 3_1), (5_1, 4_2, 7_2), (4_1, 0_2, 2_2, 1_2), (\infty_1, 6_2, 6_1, \infty_2, 3_2, 2_1, 5_2, 7_1)\}$
$OP(18; 3^2, 5, 7)$
$R = \{(3_1, 4_1, 6_1), (\infty_2, 0_1, 6_2), (\infty_1, 7_2, 5_2, 4_2, 7_1), (0_2, 3_2, 1_1, 2_2, 2_1, 1_2, 5_1)\}$
$OP(18; 3^2, 6^2)$
$R = \{(1_1, 4_1, 2_1), (\infty_2, 6_1, 4_2), (\infty_1, 7_1, 7_2, 6_2, 3_2, 5_2), (0_1, 1_2, 5_1, 0_2, 3_1, 2_2)\}$

$OP(18; 3, 4^2, 7)$

$$R = \{(\infty_2, 2_1, 1_2), (0_2, 4_1, 5_1, 3_2), (0_1, 6_1, 3_1, 5_2), (\infty_1, 7_1, 7_2, 6_2, 4_2, 1_1, 2_2,)\}$$

$OP(18; 3, 4, 5, 6)$

$$R = \{(\infty_2, 0_1, 0_2), (\infty_1, 7_2, 6_2, 5_1), (1_1, 3_1, 1_2, 3_2, 4_1), (2_2, 5_2, 2_1, 4_2, 7_1, 6_1)\}$$

$OP(18; 3, 5^3)$

$$R = \{(\infty_2, 1_2, 4_1), (\infty_1, 7_2, 5_1, 5_2, 2_1), (0_2, 7_1, 6_2, 3_2, 2_2), (0_1, 1_1, 3_1, 6_1, 4_2)\}$$

$OP(18; 4^3, 6)$

$$R = \{(2_2, 5_1, 6_1, 5_2), (0_1, 2_1, 0_2, 1_2), (1_1, 4_1, 6_2, 4_2), (\infty_1, 7_1, 3_2, 3_1, \infty_2, 7_2)\}$$

$OP(18; 4^2, 5^2)$

$$R = \{(0_1, 3_1, 5_2, 6_2), (1_2, 3_2, 7_1, 6_1), (\infty_1, 7_2, 4_2, 4_1, 2_1), (\infty_2, 0_2, 1_1, 2_2, 5_1)\}$$

$OP(18; 3^4, 6)$

$$R = \{(\infty_1, 2_1, 7_2), (\infty_2, 1_1, 2_2), (6_1, 7_1, 5_2), (4_1, 4_2, 6_2), (0_2, 1_2, 5_1, 3_1, 0_1, 3_2)\}$$

$OP(18; 3^3, 4, 5)$

$$R = \{(1_1, 2_1, 3_2), (5_1, 5_2, 7_1), (4_2, 7_2, 6_2), (\infty_1, 0_1, 3_1, 2_2), (\infty_2, 6_1, 1_2, 4_1, 0_2,)\}$$

$OP(18; 3^2, 4^3)$

$$R = \{(\infty_1, 7_2, 1_1), (\infty_2, 2_1, 2_2), (6_2, 4_1, 6_1, 7_1), (0_1, 3_1, 4_2, 5_2), (5_1, 0_2, 3_2, 1_2)\}$$

$$V = \{1_1, 2_1, 3_1, 4_1, 5_1, 6_1, 7_1, 8_1, 9_1, 1_2, 2_2, 3_2, 4_2, 5_2, 6_2, 7_2, 8_2, 9_2\}$$

$OP(18; 3^6)$

$$F_1 = \{(1_1, 4_1, 7_1), (2_1, 5_1, 8_1), (3_1, 6_1, 9_1), (1_2, 4_2, 7_2), (2_2, 5_2, 8_2), (3_2, 6_2, 9_2)\}$$
$$F_2 = \{(1_1, 5_1, 9_1), (2_1, 6_1, 7_1), (3_1, 4_1, 8_1), (1_2, 5_2, 9_2), (2_2, 6_2, 7_2), (3_2, 4_2, 8_2)\}$$
$$F_3 = \{(1_1, 2_1, 6_2), (4_1, 5_1, 9_2), (7_1, 8_1, 3_2), (1_2, 2_2, 6_1), (4_2, 5_2, 9_1), (7_2, 8_2, 3_1)\}$$
$$F_4 = \{(1_1, 3_1, 5_2), (4_1, 6_1, 8_2), (7_1, 9_1, 2_2), (1_2, 3_2, 5_1), (4_2, 6_2, 8_1), (7_2, 9_2, 2_1)\}$$
$$F_5 = \{(1_1, 6_1, 3_2), (2_1, 9_1, 8_2), (5_1, 7_1, 4_2), (1_2, 6_2, 3_1), (2_2, 9_2, 8_1), (5_2, 7_2, 4_1)\}$$
$$F_6 = \{(1_1, 8_1, 7_2), (3_1, 5_1, 2_2), (4_1, 9_1, 6_2), (1_2, 8_2, 7_1), (3_2, 5_2, 2_1), (4_2, 9_2, 6_1)\}$$
$$F_7 = \{(8_1, 9_1, 1_2), (2_1, 3_1, 4_2), (5_1, 6_1, 7_2), (8_2, 9_2, 1_1), (2_2, 3_2, 4_1), (5_2, 6_2, 7_1)\}$$
$$F_8 = \{(2_1, 4_1, 1_2), (6_1, 8_1, 5_2), (3_1, 7_1, 9_2), (2_2, 4_2, 1_1), (6_2, 8_2, 5_1), (3_2, 7_2, 9_1)\}$$

So with the exception of K_{17} we know whether or not solutions exist for all possible Oberwolfach questions for complete graphs on fewer than 19 vertices. Of all these cases, the only questions that are known to have no solution are $OP(6; 3^2)$, $OP(9; 4, 5)$, $OP(11; 3^2, 5)$ and $OP(12; 3^4)$. In Section 4 we will consider whether decompositions that are not possible in K_n might be possible in λK_n.

3. $OP(11; 3^2, 5)$

This is the smallest case of the Oberwolfach Problem that has defied all manual attempts at a solution. We will confirm the non-existence of a solution established by Piotrowski [16] and go on to show that even though a single copy of K_{11} cannot be decomposed into isomorphic 2-factors each comprising a 5-cycle and a 3-cycle, any other number of copies can be decomposed in this manner.

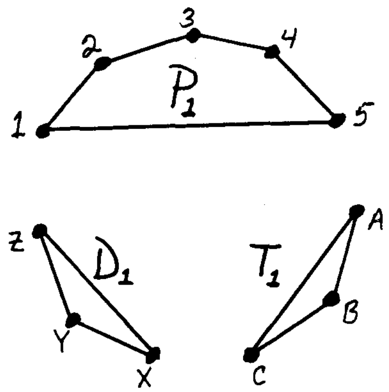


Figure 3.1

Given one copy of K_{11}, the first 2-factor (F_1) can be chosen arbitrarily without loss of generality. We call its pentagon P_1 and its triangles T_1 and D_1 with vertices labeled as shown in Figure 3.1.

As implied above, the notation F_n (n = 1, 2, 3, 4 or 5) will represent the nth 2-factor of a decomposition which comprises P_n, T_n and D_n.

Proposition 3.1: *Each 2-factor (except F_1) must contain a diagonal of P_1.*

Proof: Suppose there exists a factor that does not contain a diagonal of P_1. Each triangle in this factor must have exactly one vertex from each of P_1, T_1 and D_1. Two vertices from P_1 in a triangle would mean use of a diagonal of P_1 (not allowed by assumption) or use of an edge already used in F_1 (not allowed by definition of partition). Two vertices from either F_1 triangle would mean using the same edge in two different factors which is not allowed in partitioning. Thus, having used two P_1 vertices and four triangle vertices, the pentagon for this factor uses three vertices from P_1 and two triangle vertices. Of the three P_1 vertices used in this new pentagon, two of them must be adjacent. But this is impossible since adjacent P_1 vertices in the cycle means either a second use of an edge of P_1 or a diagonal of P_1. Therefore, no such factor can exist and we know all factors (except F_1) include a diagonal of P_1. ■

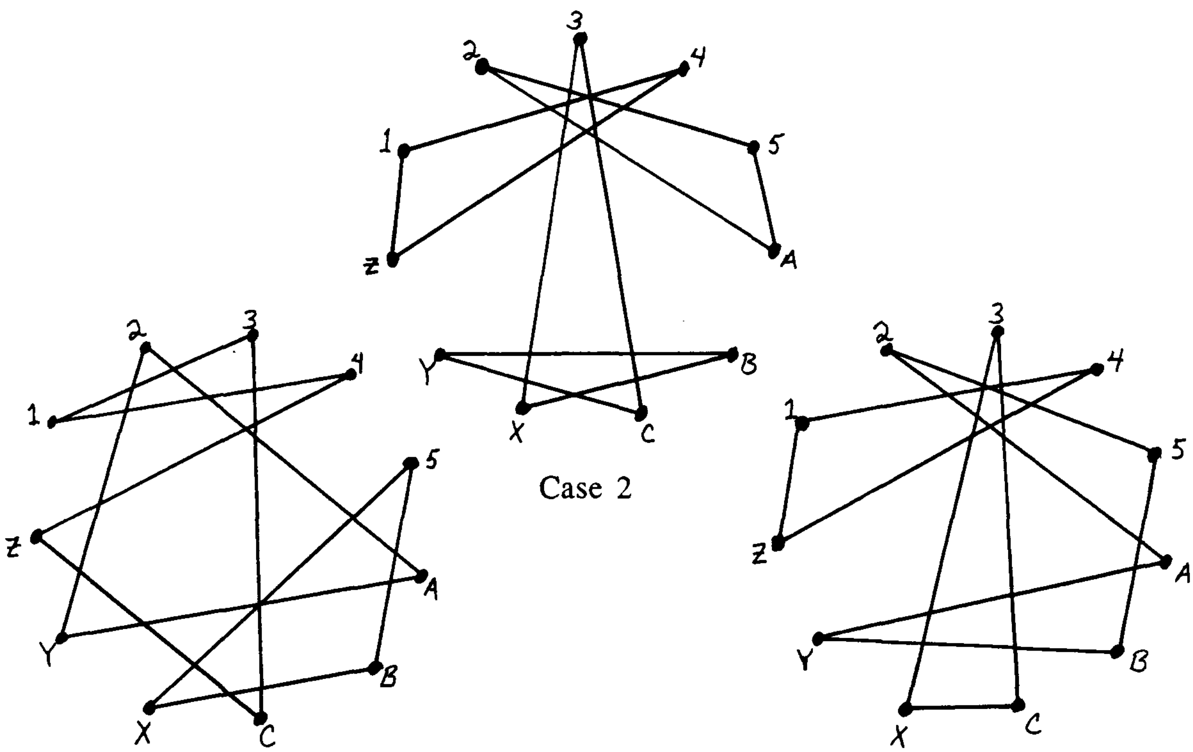
Proposition 3.2 : *There exists exactly one factor in $\{F_2, F_3, F_4, F_5\}$ which contains two diagonals of P_1.*

Proof: By Proposition 3.1, each of the F_i's contains a diagonal of P_1 which accounts for four of the five diagonals. The fifth P_1 diagonal must appear in one of the F_i's making that factor the only one containing two P_1 diagonals.■

Notation 3.3: Let F_2 be the name of the factor with exactly two P_1 diagonals.

Theorem 3.4: *The only three non-isomorphic possibilities for* F_2 *are those shown inFigure* 3.2.

Proof: We assume that F_1 has been removed from an unlabeled K_{11} and show that the vertex set of any F_2 can be labeled in such a way that the structure and labeling of F_1 is identical to Figure 3.1 and the structure and labeling of F_2 is identical to one of the drawings in Figure 3.2. The reason that many F_2's with different labelings can be isomorphic stems from the rotations and reflections of the dihedral groups for the triangles and pentagon of F_1.



Case 1

Case 3

Figure 3.2 : Possible F_2 structures

Clearly the two P_1 diagonals in F_2 either share a common vertex (call this Case 1) or they are disjoint. If they are disjoint, they cannot both be in the same triangle and they cannot both be edges of P_2. Thus we are left with the fact that the two P_1 diagonals in F_2 must occur in different cycles - one in each triangle (call this Case 2) or one in P_2 and the other in a triangle (call this Case 3).

Figure 3.2 establishes that at least one example of each case exists. In each of the three cases we will begin with F_1 and an arbitrary F_2 of the type in question and show that the eleven vertices can be labeled in such a way that the structure is identical with the corresponding drawing in Figure 3.2.

CASE 1: The P_1 diagonals in F_2 share a common vertex.

Choose an arbitrary F_2 whose P_1 diagonals share a common vertex. That common vertex must lie in P_2 and we label it as vertex 1.

The two P_1 diagonals account for three of the five vertices of P_2. The remaining two vertices of P_2 must be one each from T_1 and D_1 and they must be adjacent. Label the vertex from T_1 as vertex C and the vertex from D_1 as vertex Z.

Note that starting at vertex 1 and traversing the cycle P_2, the shortest path to vertex C has exactly one vertex between vertex 1 and vertex C. Label it as vertex 3. Continuing around the cycle there is exactly one vertex between vertex Z and vertex 1. Label it as vertex 4.

The remaining unlabeled P_1 vertices are adjacent to vertex 1 on the cycle P_1. Label them as vertex 2 and vertex 5 such that the vertices of P_1 are labeled in numerical order around the cycle.

Now vertex 2 lies in a triangle of F_2 whose other two vertices come one each from T_1 and D_1 (since any other possibility requires the use of another diagonal of P_1 or the re-use of an edge of F_1). Label the T_1 vertex in this triangle as vertex A and the D_1 vertex as vertex Y. There are now only two unlabeled vertices remaining. They are part of the F_2 triangle that includes vertex 5. Label the unlabeled vertex in T_1 as vertex B and the one in D_1 as vertex X.

Thus any F_2 containing adjacent P_1 diagonals has the same structure as the Case 1 diagram in Figure 3.2.

CASE 2: F_2 has one P_1 diagonal in each of its triangles.

Choose an arbitrary F_2 whose P_1 diagonals lie one each in its two triangles. The two diagonals contain four P_1 vertices. Label the fifth P_1 vertex as vertex 3. This vertex is in P_2. The other four vertices of P_2 are from T_1 and D_1. It is clear that as we traverse the cycle P_2, the vertices are alternately from T_1 and D_1. Thus vertex 3 is adjacent to one vertex of T_1 and one of D_1. Label the former as vertex C and the latter as vertex X. Label the remaining two P_2 vertices as vertex Y and vertex B such that Y is adjacent to C, B is adjacent to X and, of course, B is adjacent to Y.

Label the remaining T_1 vertex as vertex A and the remaining D_1 vertex as vertex Z. Vertex A is adjacent to two P_1 vertices in an F_2 triangle (the ends of one of the P_1 diagonals). One of these vertices is adjacent to vertex 3 in P_1. Label it as vertex 2 and the other as vertex 5. Similarly, vertex Z is adjacent to two P_1 vertices. Again, one of these two vertices is adjacent to vertex 3 in P_1. Label it as vertex 4 and the other as vertex 1.

We have now labeled the vertices of an arbitrary F_2 from Case 2 in such a way that it has the same structure as the Case 2 diagram in Figure 3.2.

CASE 3: F_2 has one P_1 diagonal in P_2 and one in a triangle.

Choose an arbitrary F_2 containing one P_1 diagonal in its pentagon and the other in a triangle. As in Case 2, the two P_1 diagonals contain four of the five vertices of P_1. Label the fifth vertex as vertex 3.

Consider the triangle in F_2 which contains a P_1 diagonal. One vertex of that diagonal must be adjacent to vertex 3 in P_1. Label that vertex as vertex 4 and the other as vertex 1. Clearly the third vertex of the triangle was a vertex in T_1 or D_1. Since the naming of T_1 and D_1 was arbitrary, we can assume without loss of generality that this third triangle vertex in F_2 is contained in D_1 and label it as vertex Z.

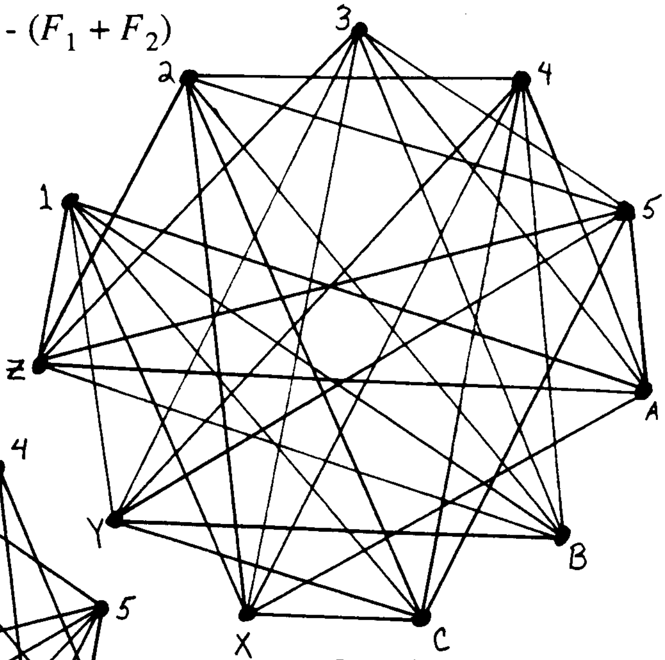
The F_2 triangle containing vertex 3 must also contain one vertex from T_1 and one vertex from D_1. Label the former as vertex C and the latter as vertex X. The vertices on the ends of the P_1 diagonal in P_2 can be labeled as vertex 2 and vertex 5 so that the five vertices of P_1 are labeled with consecutive integers as the cycle is traversed.

Three vertices remain unlabeled - two from T_1 and one from D_1 - all three of which are contained in P_2 along with vertex 2 and vertex 5. Label the remaining D_1 vertex as vertex Y. The two vertices from T_1 cannot be adjacent in P_2 and thus one must be adjacent to vertex 2 while the other is adjacent to vertex 5, and they must both be adjacent to vertex Y. Label the T_1 vertex adjacent to vertex 2 as vertex A and the last remaining unlabeled vertex as vertex B.

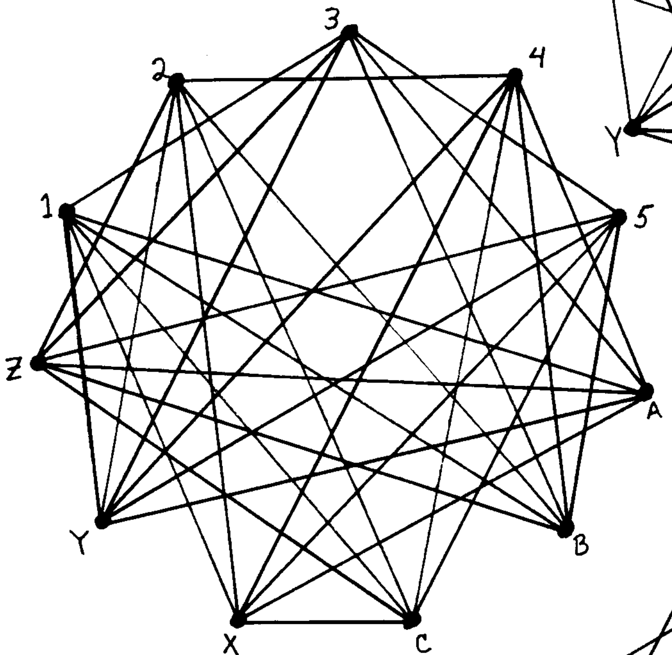
We have now labeled the vertices of an arbitrary F_2 from Case 3 in such a way that it clearly has the same structure as the Case 3 diagram in Figure 3.2.

Thus we have shown that there are only three non-isomorphic ways to choose the first two factors. Figure 3.3 shows the set of edges in K_{11}-(F_1+F_2) for each of the three cases. ■

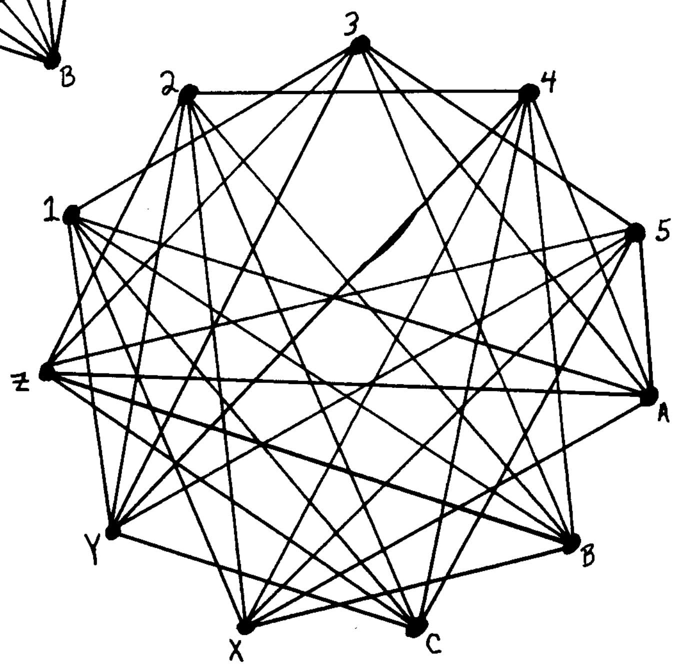
$K_{11} - (F_1 + F_2)$



Case 1



Case 2



Case 3

Figure 3.3

3.1 The Problem

How to proceed from here is not clear. In 1979 Wolf Piotrowski [16] reached this point and decided to write a computer program to find whether there existed three compatible 2-factors in the edge-set left by each of the three cases discussed above. The strategy he chose was to construct a list of all possible 2-factors from the edges of K_{11}-(F_1+F_2) and then to try to find three edge-disjoint factors from that list. His program (in FORTRAN run on a TR 440 computer) found roughly 200 possible factors and 200 edge-disjoint pairs of those factors, but no edge-disjoint triad of factors in any of the three cases. This proved (assuming no logical or mechanical problems) that the partition we seek does not exist. Other than establishing the answer to the basic question, the program provided no insight as to why there is no such partition, how close one could actually get to completing the final factor, or how one might establish the result without using a machine.

The strategy in the program included in this paper (OBRWLFCH.PRG) searches for a solution in a significantly different way from Piotrowski's. Where his approach was to generate complete factors and check their compatibility, the one here builds up all three factors simultaneously keeping track of how close the process gets to a complete set of factors.

The purpose of OBRWLFCH.PRG is twofold: (1) to check Piotrowski's result using a different strategy so as to minimize the possibility of repeating any errors that might exist in his program and (2) to keep track of what happens as the program tries to build factors in the hope that further light might be shed on exactly what makes this factorization impossible and how the result might be arrived at without computer assistance.

In each of the three cases we are dealing with between 233 and 272 pentagon possibilities and 25 or 26 triangle possibilities which yield approximately 10^{16} possible factor combinations for each of the three cases. This is an improvement over the roughly 10^{38} possible sets of five 2-factors of K_{11} that we started with, but the problem is still clearly too large to expect a microcomputer to resolve it in any reasonable amount of time.

We can further reduce the number of possibilities to be checked by being careful to keep track of the fact that each of the last three factors contains exactly one P_1 diagonal. This is not quite as easy as it seems since the P_1 diagonals can just as easily show up in a triangle as in a pentagon, but doing so reduces the number of possible 2-factor triads to about 10^5.

Though the problem is still clearly too large to do by hand it is small enough for a microcomputer to do an exhaustive search for the three final factors while keeping track of how close we get to a solution.

3.2 The Computing

Given the above argument, we have three sets of edges (see Figure 3.3) each left by the removal of F_1 and an F_2 from K_{11}. In each case we will attempt to extract three edge-disjoint 2-factors, each comprising a pentagon and two triangles. The labeling of the vertices will be as in Figure 3.3 where the vertices of P_1 are numbered 1 through 5 and the vertices of the triangles T_1 and D_1 are labeled A, B, C and X, Y, Z, respectively.

In each case we will construct databases containing all possible pentagons and triangles from the set of edges remaining. Since we know that each 2-factor must contain exactly one of the remaining P_1 diagonals we distinguish (by placing in separate databases) cycles that contain a P_1 diagonal (Class 1) and those that do not (Class 2).

Observing that the P_1 diagonals 24 and 35 are never used in F_1 or F_2, we can arbitrarily name the factors containing them as F_4 and F_5, respectively. This means that F_3 will be the factor that contains the P_1 diagonal 25 in Case 1, and the P_1 diagonal 13 in Cases 2 and 3.

Databases containing pentagons and triangles are named as follows: The first two characters (C1, C2 or C3) indicate Case 1, Case 2, or Case 3 depending on which F_2 is assumed. The second two characters (C1 or C2) indicate Class 1 if the cycles include a P_1 diagonal or Class 2 if they do not. The next character is either a P (for pentagon) or a T (for triangle). If the cycles in the database are class 1, there is one more character (3, 4 or 5) that indicates to which factor it must belong. The extension is always "DBF" (for DataBase File). Thus C3C1P5.DBF is the database file containing Case 3, Class 1 pentagons that are possible for F_5 (i.e., that contain the P_1 diagonal 35).

The possible triangles in each case are few and easily identified without using the machine. Therefore all databases containing possible triangles were constructed by hand. Possible pentagons, however, are many and were therefore generated by the program.

The dBase III Plus programming language is used because of its suitability, its availability and the author's familiarity with it.

3.3 The Setup Programs

GENC1P.PRG and GENC2P.PRG are the PRoGrams used to GENerate Class 1 and Class 2 Pentagons, respectively. In order that the main program can more quickly determine whether a particular edge is already in use when checking possible combinations, the databases are modified by PENTEDGE.PRG and TRIEDGE.PRG so that the database includes not only the vertices (in cycle notation) of the pentagons and triangles, but also the list of edges used in each. So that each edge has a unique label we adopt the convention that an edge is named by listing the two vertices with which it is incident in ascending order (note that the computer sees digits as "smaller" than letters so that the edge joining vertex Y with vertex 3 will be referred to as edge $3Y$).

GENC1P.PRG and GENC2P.PRG can be found in the appendix beginning on pages 52 and 54, respectively. PENTEDGE.PRG AND TRIEDGE.PRG are on page 56 of the appendix. The complete set of databases generated by these programs and used in the computing for Cases 1, 2 and 3 is provided beginning on pages 57, 64 and 71 of the appendix, respectively.

3.4 OBRWLFCH.PRG

The main program that searches for three edge-disjoint 2-factors (each comprising a pentagon and two triangles) in K_{11}-(F_1+F_2) is OBRWLFCH.PRG. Since there are three distinct choices for F_2, the program was run three different times. The programs used on the three runs were identical except for the names of the databases called into use. The following description assumes that we are running Case 1.

We have nine cycles to find (three pentagons and six triangles, though not necessarily in that order) so the program is written in nine levels.

In levels 1, 2 and 3 we are choosing the class 1 cycles (pentagon or triangle) to be used in factors 3, 4 and 5, respectively. In level n, $3 \leq n \leq 5$, we are searching for D_n (a class 2 triangle for F_{n}.). In level n, $7 \leq n \leq 9$, we are choosing (for factors 3, 4 and 5, respectively), a Class 2 triangle if a Class 1 pentagon has already been chosen at level n-6, or a Class 2 pentagon if a Class 1 triangle has already been chosen at level n-6.

On reaching level 8 (having found 7 of the 9 required cycles) it prints out the set of cycles found so far so that we can see how close we get to complete solutions.

The program starts in level 1 with the first record of C1C1P3.DBF (the first possible Case 1, Class 1 pentagon for F_3) and records its edge-set as used for P_3. In level 2 we then search sequentially through the records of C1C1P4.DBF to find a Class 1 pentagon (edge-disjoint from P_3) to be used as P_4. If found, its edge-set is recorded as used and we proceed to level 3 to search C1C1P5.DBF for a P_5 candidate.

If we find three compatible Class 1 pentagons for F_3, F_4 and F_5, we go sequentially to levels 4 through 9 looking through C1C2T.DBF (the set of Class 2 triangles) to find an edge-disjoint set of six triangles among the remaining available edges to complete the three factors. The factors are filled in the following order: D_3, D_4, D_5, T_3, T_4, T_5.

Whenever we reach the end of a database at level $n > 3$ (meaning that there are no more options left at that level with the choices made so far) we back up to level n-1 to look at the next record (next possibility) at that level.

If we reach the end of a pentagon database at level $n < 4$ we stay at that level and begin choosing Class 1 triangles from C1C1Tn.DBF. If found, a compatible triangle is stored as T_n and we proceed to the next level (always starting at the top of the appropriate database regardless of whether we have been at that level before).

Since there can only be one Class 1 cycle in any given factor and the T_n chosen at level n is Class 1, if we get as far as level n+6 we open C1C2P.DBF to look for a compatible Class 2 pentagon rather than looking for a triangle as described above.

The entire search can be shown as a digraph (Figure 3.4.1) whose vertices are databases and whose edges represent moving between databases. Movement to the right represents moving to the next level when a compatible cycle has been found in the current database. Movement to the left represents backing up to the previous level when the end of a database is reached without finding a compatible cycle. Movement down represents staying within the same level (n < 4) when the end of a Class 1 pentagon database is encountered without finding a compatible cycle and we move to the corresponding Class 1 triangle database. Paths to the possible end results of the program are also shown.

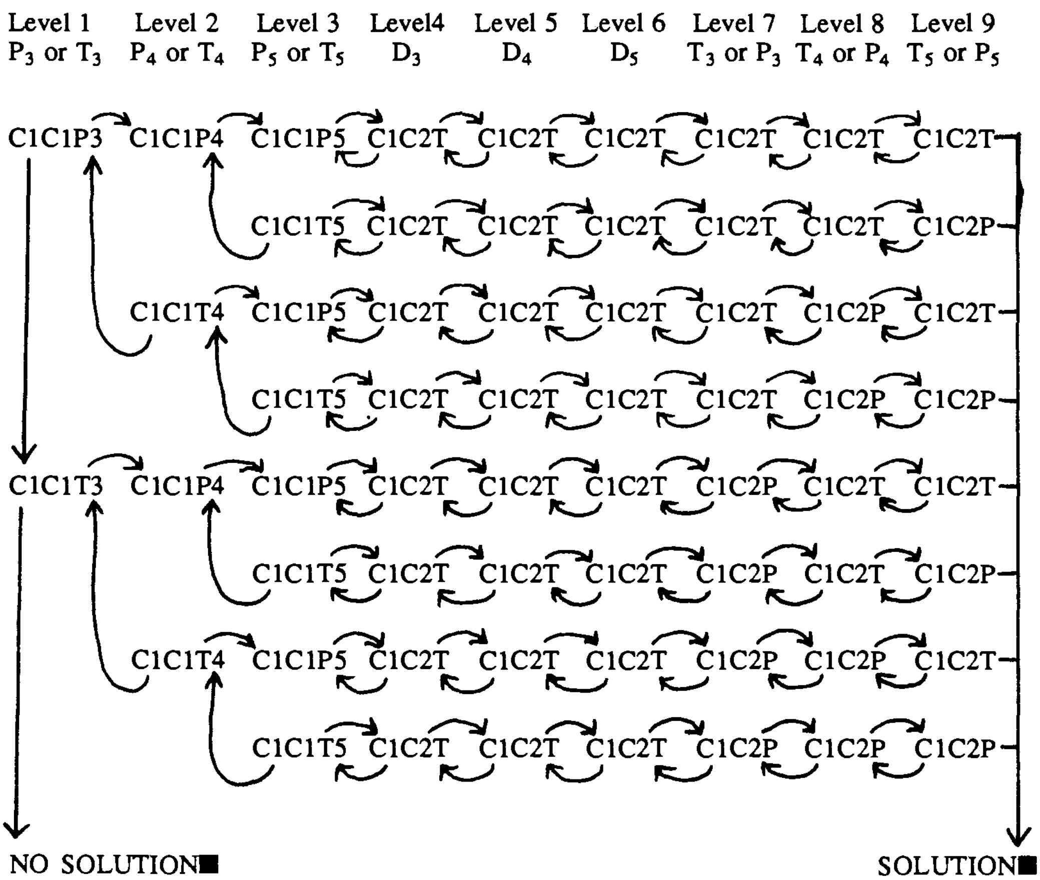


Figure 3.4.1

PROCFILE.PRG is the set of procedures that is invoked at appropriate times by OBRWLFCH.PRG. The FINDP and FINDT procedures do the search through the database in use to find a compatible pentagon or triangle, respectively. CPYRCRDS is the procedure that copies current node, edge and factor information to the next record so that another cycle can be added to the current information while still allowing us to return to the current situation if we need to back up. The BACKUP procedure is invoked when the end of a database is reached and we need to return to the previous level.

The first version of OBRWLFCH.PRG took about 9.5 days to do Case 1 on a Heathkit H100 with dBase III. This was a little too long due to the possibility of lightning or people accidentally turning the machine off during the run, so a switch was made to dBase IV on an Epson Equity II+ where it took about 2.5 days to run each of the three cases.

The final versions of OBRWLFCH.PRG and PROCFIL.PRG can be found in the appendix beginning with pages 41 and 47, respectively. The information generated by the programs for Cases 1, 2 and 3can be found beginning on pages 78, 83 and 88 respectively.

3.5 Data Anaysis

The running of OBRWLFCH.PRG confirmed the findings reached by Piotrowski. There do not exist three edge-disjoint, isomorphic 2-factors, each comprising a pentagon and two triangles, in any of the three possible cases. Consequently, $OP(11; 3^2, 5)$ has no solution.

In addition to confirming earlier work, there is further information from this program. One might have wondered whether it is possible to argue that one cannot find six edge-disjoint triangles or three edge-disjoint pentagons under the constraints of the three cases. It is clear from the output data that in each of the three cases there are several sets of six appropriate triangles. The program also found sets of three appropriate pentagons in each of the cases. Thus no machine-free argument could be made on that basis.

It is also interesting that in all three cases, there were many instances where seven of the nine requisite cycles could be found, but never more than seven. In fact it is easy to show that it is impossible to find an eighth without also finding the ninth.

All eleven vertices are degree 10 to begin with and are of even degree throughout the entire process. If we ever found an eighth appropriate cycle, we would have either three or five edges remaining. The only way of having either three or five edges in a graph where all vertices are of even degree is if they form a cycle. Indeed, since the vertices would be exactly the vertices as yet not used in the final factor, the cycle would be the one we need to complete the factorization.

Though it is frustrating to be so close, it is worth knowing that in each case we miss a complete factorization by the smallest margin possible.

The output data shows that among the partial factorizations when we are two triangles short of a complete factorization three possibilities occur: 1) the edges remaining form a 6-cycle, 2) they form two 3-cycles with one common vertex and 3) they form two disjoint 3-cycles. This last instance leads to a new result which is reported in Section 4.

4. Solutions in λK_n

Clearly, given any $\lambda > 1$, any 2-factorization of K_n can be used to 2-factor λK_n by simply decomposing each of the λ copies separately. However, for the cases of the Oberwolfach Problem where no 2-factorization of a particular type is possible we will now consider whether that type of decomposition is possible in λK_n. The first case with no solution is $OP(6, 3^2)$. We address below only the case where λ is even.

Theorem 4.1: Given an even integer λ, λK_6 *can be partitioned into* 2-*factors each comprising two* 3-*cycles if and only if* $\lambda \equiv 0$ (*mod* 4).

Proof: Given λK_6 we label the vertices 1, 2, 3, X, Y, and Z and designate the first 2-factor as $\{(1, 2, 3), (X, Y, Z)\}$ without loss of generality. We call an edge Type 1 if the vertices with which it is incident are either both labeled with numbers or both labeled with letters. Type 2 edges are incident with one numbered vertex and one vertex labeled with a letter. We call a 3-cycle Class 1 if all its edges are Type 1 and Class 2 otherwise. Any 3-cycle that is Class 2 comprises two Type 2 edges and one Type 1 edge. Note that no 3-cycle is possible using only Type 2 edges. Also note that any 2-factor always comprises two 3-cycles of the same type.

Consider $\lambda = 4t+2$ for any positive integer t. The number of Type 2 edges in λK_6 is $9(4t+2)$. Since all of these edges must be used in the 2-factorization and since they must be used 2 at a time in 3-cycles that are Class 2, we will need $9(4t+2)/2$ Type 1 edges to complete these 3-cycles. Note that this number is always odd. The number of Type 1 edges after the first 2-factor is removed is $6(4t+1)$. This number is even. Since it is always six of these edges that would be removed with any 2-factors containing 3-cycles that are Class 1, the number of remaining edges will always be even. Thus it is not possible to fashion a set of 2-factors comprising 3-cycles that will use the entire edge-set of λK_6.

Consider $\lambda = 4t$. It suffices to show that $4K_6$ can be 2-factored into 3-cycles. The following 2-factors accomplish the decomposition:

$F_1 = \{(1, 2, 3), (X, Y, Z)\}$
$F_2 = \{(1, 2, X), (3, Y, Z)\}$
$F_3 = \{(1, 2, Y), (3, X, Z)\}$
$F_4 = \{(1, 2, Z), (3, X, Y)\}$
$F_5 = \{(2, 3, X), (1, Y, Z)\}$
$F_6 = \{(2, 3, Y), (1, X, Z)\}$
$F_7 = \{(2, 3, Z), (1, X, Y)\}$
$F_8 = \{(1, 3, X), (2, Y, Z)\}$
$F_9 = \{(1, 3, Y), (2, X, Z)\}$
$F_{10} = \{(1, 3, Z), (2, X, Y)\}$

This yields the stated result.■

The next two cases with no solution are $OP(9; 4, 5)$ and $OP(11; 3^2, 5)$. To preface the next two theorems we note that if $2K_n$ and $3K_n$ can be decomposed into any particular type of 2-factor, then for any $\lambda > 1$, so can λK_n since $\lambda = 2s+3t$ for some pair of non-negative integers s and t. It is therefore sufficient to give 2-factorizations for $2K_n$ and $3K_n$ to establish the result for λK_n.

Theorem 4.2: *For any integer $\lambda > 1$, λK_9 can be partitioned into 2-factors, where each 2-factor comprises a 4-cycle and a 5-cycle.*

Proof: Let
$V = \mathbf{Z}_4 \times \{1, 2\} \cup \{\infty\}$
$\alpha = (\infty)\,(0_1\, 1_1\, 2_1\, 3_1)(0_2\, 1_2\, 2_2\, 3_2)$
$F_{i+1} = \alpha^i(R_1),\ i = 0, 1, 2, 3$
$F_{i+1} = \alpha^i(R_2),\ i = 4, 5, 6, 7$
$R_1 = \{(0_1, 2_1, 1_1, 1_2), (\infty, 3_1, 0_2, 2_2, 3_2)\}$
$R_2 = \{(\infty, 3_1, 2_2, 1_2), (0_1, 3_2, 1_1, 2_1, 0_2)\}$

This decomposes $2K_9$ as required

Now let
$V = \mathbf{Z}_3 \times \{1, 2, 3\}$
$\alpha = (0_1\, 1_1\, 2_1)(0_2\, 1_2\, 2_2)(0_3\, 1_3\, 2_3)$
$F_{i+1} = \alpha^i(R_1),\ i = 0, 1, 2$
$F_{i+1} = \alpha^i(R_2),\ i = 3, 4, 5$
$F_{i+1} = \alpha^i(R_3),\ i = 6, 7, 8$
$F_{i+1} = \alpha^i(R_4),\ i = 9, 10, 11$
$R_1 = \{(0_2, 0_3, 2_1, 1_2), (0_1, 1_3, 2_3, 2_2, 1_1)\}$
$R_2 = \{(0_1, 0_3, 1_1, 0_2), (1_2, 2_3, 2_1, 1_3, 2_2)\}$
$R_3 = \{(1_1, 2_1, 2_2, 2_3), (0_1, 0_2, 1_3, 0_3, 1_2)\}$
$R_4 = \{(0_1, 1_2, 0_2, 1_1), (0_3, 2_2, 1_3, 2_1, 2_3)\}$

This decomposes $3K_9$ as required and the result follows.■

Theorem 4.3: *For any integer $\lambda > 1$, λK_{11} can be partitioned into 2-factors, where each 2-factor comprises a 5-cycle and two 3-cycles.*

Proof: The column headed "1^{st} K_{11}" below is a Case 2 partial solution generated by OBRWLFCH.PRG. The unused edges for this partial solution form two disjoint triangles; $3AZ$ and $1CX$. Unfortunately, the triangles that are needed to complete F_4 and F_5 are $3CZ$ and $1AX$, respectively. The column headed "2^{nd} K_{11}" was generated from the first in such a way that it is a partial solution whose "extra" triangles complete the fourth and fifth factors of the first partial factorization and it can use the two "extra" triangles from the first to complete its fourth and fifth factors. Thus we have the desired decomposition of $2K_{11}$.

		1^{st} K_{11}	2^{nd} K_{11}
	P_1	12345	12345
F_1	T_1	ABC	CBA
	D_1	XYZ	XYZ
	P_2	$3CYBX$	$3AYBX$
F_2	T_2	$14Z$	$14Z$
	D_2	$25A$	$25C$
	P_3	$CZ5Y2$	$AZ5Y2$
F_3	T_3	$13B$	$13B$
	D_3	$4AX$	$4AX$
	P_4	$24B5X$	$24B5X$
F_4	♠T_4		
	D_4	$1AY$	$1CY$
	P_5	$35C4Y$	$35A4Y$
F_5	♦T_5		
	D_5	$2BZ$	$2BZ$
♠T_4	needs:	$3CZ$	$3AZ$
	has:	$3AZ$	$3CZ$
♦T_5	needs:	$1AX$	$1CX$
	has:	$1CX$	$1AX$

Similarly, the following are modifications of three partial solutions generated by OBRWLFCH.PRG which with their "extra" edges exchanged constitute the required decomposition of $3K_{11}$.

		$1^{st}\ K_{11}$	$2^{nd}\ K_{11}$	$3^{rd}\ K_{11}$
	P_1	12345	12345	42315
F_1	T_1	ABC	CBA	CBA
	D_1	XYZ	XYZ	YXZ
	P_2	$3CYBX$	$3AYBX$	$3AXBY$
F_2	T_2	$14Z$	$14Z$	$41Z$
	D_2	$25A$	$25C$	$25C$
	P_3	$CZ5Y2$	$AZ5Y2$	$AZ5Y2$
F_3	T_3	$13B$	$13B$	$43B$
	D_3	$4AX$	$4CX$	$1CX$
	P_4	$24B5X$	$24B5X$	$21B5X$
F_4	♠T_4			
	D_4	$1AY$	$3CZ$	$4CY$
	P_5	$35C4Y$	$35A4Y$	$35A4X$
F_5	♦T_5			
	D_5	$2BZ$	$2BZ$	$2BZ$

♠T_4	needs:	$3CZ$	$1AY$	$3AZ$
	has:	$3AZ$	$1CY$	$3CZ$
♦T_5	needs:	$1AX$	$1CX$	$1CY$
	has:	$1CX$	$1AX$	$1AY$

The desired result follows.■

The fourth and final case with no solution is $OP(12; 3^4)$. Hanani [7] establishes that there is a resolvable $(v, 3, 2)$-*BIBD* which is equivalent to the decomposition we seek for $2K_{12}$. This obviously settles the question for λK_{12} whenever λ is even. The case where λ is odd has not been studied.

We conclude with a note that may be of some interest. Even though $2K_6$ cannot be 2-factored into 3-cycles, it is possible for $2(K_6\text{-}F)$ to be decomposed in this way. The following is such a 2-factorization:

$$\begin{aligned}
F_1 &= \{(1, 2, 3), (X, Y, Z)\} \\
F_2 &= \{(1, 2, X), (3, Y, Z)\} \\
F_3 &= \{(1, 3, Y), (2, X, Z)\} \\
F_4 &= \{(1, X, Y), (2, 3, Z)\}
\end{aligned}$$

References

[1] B. ALSPACH AND R. HÄGGKVIST, Some observations on the Oberwolfach problem, *J. Graph Theory* 9 (1985), 177-187.

[2] B. ALSPACH, P. J. SCHELLENBERG, D. R. STINSON AND D. WAGNER, The Oberwolfach problem and factors of uniform odd length cycles, *J. Combin. Theory Ser. A* 52 (1989), 20-43.

[3] R. D. BAKER AND R. M. WILSON, Nearly Kirkman triple systems, *Utilitas Math. Vol. 11* (1977), 315-317.

[4] W. W. R. BALL, *Mathematical Recreations and Essays*, (revised by H. S. M. Coxeter), Macmillan, New York (1962), 267-298.

[5] A. E. BROUWER, Two new nearly Kirkman triple systems, *Utilitas Math. Vol. 13* (1978), 311-314.

[6] R. K. GUY, Unsolved Combinatorial Problems, *in* "Combinatorial Mathematics and Its Applications, Proceedings Conf. Oxford 1967" (D. J. A. Welsh, Ed.), p. 121, Academic Press, New York, (1971).

[7] H. HANANI, On resolvable balanced incomplete block designs, *J. Combin. Theory Ser A* 17 (1974), 275-289.

[8] P. HELL, A. KOTZIG AND A. ROSA, Some results on the Oberwolfach problem, *Aequationes Math.* 12 (1975), 1-5.

[9] D. G. HOFFMAN AND P. J. SCHELLENBERG, The existence of C_k-factorizations of K_{2n} *-F, Discrete Math.*, to appear.

[10] J. D. HORTON, B. K. ROY, P. J. SCHELLENBERG AND D. R. STINSON, On decomposing graphs into isomorphic uniform 2-factors, *Ann. Discrete Math.* 27 (1985), 297-319.

[11] C. HUANG, A. KOTZIG AND A. ROSA, On a variation of the Oberwolfach problem, *Discrete Math.* 27 (1979), 261-277.

[12] C. HUANG, E. MENDELSOHN AND A. ROSA, On partially resolvable *t*-partitions, *Ann. Discrete Math* 12 (1982), 169-183

[13] E. KÖHLER, Über der Oberwolfacher Problem, *Beiträge Geom. Algebra Basel* (1977), 189-201.

[14] A. KOTZIG AND A. ROSA, Nearly Kirkman systems, *in* "Proc. 5th S-E Conf. Combinatorics, Graph Theory, and Computing, Boca Raton. Florida, 1974" *Utilitas Math.*, Winnipeg, (1974), 607-614.

[15] É. LUCAS, *Récréations Mathématiques,* Vol. 2, Gauthier-Villars, Paris, (1884), 161-164

[16] W. PIOTROWSKI, Untersuchungen über das Oberwolfacher Problem, working papers, Hamburg, (1979).

[17] D. K. RAY-CHAUDHURI AND R. M. WILSON, Solution of Kirkman's schoolgirl problem, *in* Proc. Symp. Pure Math. Vol 19, pp.187-204, Amer. Math. Soc., Providence, RI, (1971).

[18] D. K. RAY-CHAUDHURI AND R. M. WILSON, The existence of resolvable block designs, in *A Survey of Combinatorial Theory* (edited by J. N Srivastava *et al)*, North Holland, Amsterdam, (1973), 361-375.

[19] R. REES AND D. R. STINSON, On resolvable group-divisible designs with block size 3, *Ars Comb. XXIII*, (1987), 107-120.

Appendix

OBRWLFCH.PRG

```
*                                                                *
* This is the main program for the Oberwolfach problem - case K11  *
* looking for five disjoint 2-factors each comprising a pentagon  *
* and two triangles. The first factor can be chosen arbitrarily   *
* and the second can be chosen in only three essentially different ways. *
* This program looks for the third, fourth and fifth factors given the *
* first and a second factor. Thus the program was run three times on *
* three sets of data - one for each of the three possibilities for the *
* second factor. The program here is the one run for Case 1, but the *
* changes necessary for the last two cases are included in brackets *
* to the right of the statement that was changed. Variables of the *
* form CnCmPo stand for Case n, Class m, Pentagon (or T for Triangle) *
* from factor o. Class 1 Pentagons and Triangles include a diagonal *
* of the factor 1 pentagon while Class 2 Pentagons and Triangles do not *
*                                                                *
CLEAR ALL
SET TALK OFF
SET ALTERNATE TO G:OBRUN
SET ALTERNATE ON
? "OPENING PROCEDURES FILE AND BOOKKEEPING DATABASES"
SET PROCEDURE TO I:PROCFILE
  * Opening databases for keeping track of edges and nodes that have been *
  * used and remembering which factor pieces have been filled in         *
SELECT 7
 USE I:USEDNODE ALIAS NODES   * This database keeps track of which nodes *
 GO TOP                       * are already used (in the current factor) *
SELECT 8
 USE I:USEDEDGE ALIAS EDGES   * This database keeps track of which edges *
 GO TOP                       * are already used (in any factor)         *
SELECT 6
 USE I:FACTOR ALIAS FACTORS   * This database keeps track of which pieces *
 GO TOP                       * of which factors have been found already *
STORE 0 TO COUNT1
STORE 0 TO COUNT2
STORE 0 TO COUNT3
STORE 0 TO COUNT4
STORE 0 TO COUNT5
STORE 0 TO COUNT6
STORE 0 TO COUNT7
STORE 0 TO COUNT8
STORE 0 TO COUNT9
PUBLIC FOUND, PENT, TRI, FACTOR, TRITYPE, STIME, FTIME
PUBLIC P3, P4, P5, T3, T4, T5, D3, D4, D5
* The first three levels step through all possible class 1 pentagons and tri-
* angles (for factors 3, 4 and 5 respectively) which contain a diagonal of the
* pentagon in the first factor (since each of these factors must contain exactly
* one such diagonal). The fourth through seventh levels step through all
* possible class 2 pentagons and triangles trying to fill in the rest of the
* remaining pieces for each factor.
? "  * LEVEL 1 *"
SELECT 1
```

```
USE I:C1C1P3 ALIAS C1P3                    *[ C2C1P3, C3C1P3 ]*
GO TOP
STORE '3' TO FACTOR
STORE 'C1P3' TO PENT
STORE 1 TO P3CHOSEN
DO WHILE .NOT. EOF()
 STORE COUNT1+1 TO COUNT1
 ? COUNT1
 IF P3CHOSEN = 1
  DO FINDP
  IF .NOT. FOUND
   SELECT 1
   USE I:C1C1T3 ALIAS C1T3                 *[ C2C1T3, C3C1T3 ]*
   STORE 'C1T3' TO TRI
   STORE 'T' TO TRITYPE
   STORE 0 TO P3CHOSEN
   GO TOP
   LOOP
  ENDIF
 ELSE
  DO FINDT
  IF .NOT. FOUND
   ? "NO SOLUTION FOUND - RESULTS OF PIOTROWSKI CONFIRMED FOR THIS
      CASE."
   DISPLAY MEMORY
   WAIT
   CLOSE DATABASES
   SET ALTERNATE OFF
   RETURN
  ENDIF
 ENDIF
      * LEVEL 2 *
 SELECT 2
 USE I:C1C1P4 ALIAS C1P4                   *[ C2C1P4, C3C1P4 ]*
 GO TOP
 STORE '4' TO FACTOR
 STORE 'C1P4' TO PENT
 STORE 1 TO P4CHOSEN
 DO WHILE .NOT. EOF()
  STORE COUNT2+1 TO COUNT2
  IF P4CHOSEN = 1
   DO FINDP
   IF .NOT. FOUND
    SELECT 2
    USE I:C1C1T4 ALIAS C1T4                *[ C2C1T4, C3C1T4 ]*
    STORE 'C1T4' TO TRI
    STORE 'T' TO TRITYPE
    STORE 0 TO P4CHOSEN
    GO TOP
    LOOP
   ENDIF
  ELSE
   DO FINDT
   IF .NOT. FOUND
    DO BACKUP
    EXIT
   ENDIF
```

```
ENDIF
      * LEVEL 3 *
SELECT 3
USE I:C1C1P5 ALIAS C1P5                    *[ C2C1P5, C3C1P5 ]*
GO TOP
STORE '5' TO FACTOR
STORE 'C1P5' TO PENT
STORE 1 TO P5CHOSEN
DO WHILE .NOT. EOF()
  STORE COUNT3+1 TO COUNT3
  IF P5CHOSEN = 1
    DO FINDP
    IF .NOT. FOUND
     SELECT 3
     USE I:C1C1T5 ALIAS C1T5               *[ C2C1T5, C3C1T5 ]*
     STORE 'C1T5' TO TRI
     STORE 'T' TO TRITYPE
     STORE 0 TO P5CHOSEN
     GO TOP
     LOOP
    ENDIF
  ELSE
    DO FINDT
    IF .NOT. FOUND
     DO BACKUP
     EXIT
    ENDIF
  ENDIF
        * LEVEL 4 *
  SELECT 4
  USE I:C1C2T ALIAS C2T                    *[ C2C2T, C3C2T ]*
  GO TOP
  STORE '3' TO FACTOR
  STORE 'C2T' TO TRI
  STORE 'D' TO TRITYPE
  DO WHILE .NOT. EOF()
   STORE COUNT4+1 TO COUNT4
   DO FINDT
   IF FOUND
     STORE RECNO() TO D3RECNO
   ELSE
     DO BACKUP
     EXIT
   ENDIF
          * LEVEL 5 *
   GO TOP
   STORE '4' TO FACTOR
   DO WHILE .NOT. EOF()
    STORE COUNT5+1 TO COUNT5
    DO FINDT
    IF FOUND
      STORE RECNO() TO D4RECNO
    ELSE
      DO BACKUP
      EXIT
    ENDIF
            * LEVEL 6 *
```

```
GO TOP
STORE '5' TO FACTOR
 DO WHILE .NOT. EOF()
  STORE COUNT6+1 TO COUNT6
  DO FINDT
  IF FOUND
   STORE RECNO() TO D5RECNO
 ELSE
   DO BACKUP
   EXIT
  ENDIF
                    * LEVEL 7 *
  STORE '3' TO FACTOR
  SELECT C2T
  GO TOP
  STORE 'T' TO TRITYPE
  SELECT 5
  USE I:C1C2P ALIAS C2P           *[ C2C2P, C3C2P ]*
  STORE 'C2P' TO PENT
  GO TOP
   DO WHILE .NOT. EOF()
    STORE COUNT7+1 TO COUNT7
    IF P3CHOSEN = 1
     SELECT C2T
     DO FINDT
   ELSE
     SELECT C2P
     DO FINDP
    ENDIF
    IF FOUND
      ?FACTORS->P3,FACTORS->T3,FACTORS->D3,FACTORS->P4, ;
        FACTORS->T4,FACTORS->D4,FACTORS->P5,FACTORS->T5, ;
        FACTORS->D5,TIME()
     STORE RECNO() TO F3RECNO
   ELSE
     DO BACKUP
     EXIT
    ENDIF
                      * LEVEL 8 *
   STORE '4' TO FACTOR
   SELECT C2T
   GO TOP
   SELECT C2P
   GO TOP
    DO WHILE .NOT. EOF()
     STORE COUNT8+1 TO COUNT8
     IF P4CHOSEN = 1
      SELECT C2T
      DO FINDT
    ELSE
      SELECT C2P
      DO FINDP
     ENDIF
     IF FOUND
       ?FACTORS->P3,FACTORS->T3,FACTORS->D3,FACTORS->P4, ;
         FACTORS->T4,FACTORS->D4,FACTORS->P5,FACTORS->T5, ;
         FACTORS->D5,TIME()
```

```
    STORE RECNO() TO F4RECNO
  ELSE
    DO BACKUP
    EXIT
  ENDIF
                      * LEVEL 9 *
  STORE '5' TO FACTOR
  SELECT C2T
  GO TOP
  SELECT C2P
  GO TOP
  DO WHILE .NOT. EOF()
   STORE COUNT9+1 TO COUNT9
   IF P5CHOSEN = 1
    SELECT C2T
    DO FINDT
   ELSE
    SELECT C2P
    DO FINDP
   ENDIF
   IF FOUND
    ? "SOLUTION FOUND DESPITE PIOTROWSKI"
   ?
   ?FACTORS->P3,FACTORS->T3,FACTORS->D3,FACTORS->P4, ;
     FACTORS->T4,FACTORS->D4,FACTORS->P5,FACTORS->T5, ;
     FACTORS->D5,TIME()
    WAIT
    CLOSE DATABASES
    SET ALTERNATE OFF
    RETURN
   ELSE
    DO BACKUP
    EXIT
   ENDIF
  ENDDO
   * BACK TO LEVEL 8 *
   STORE '4' TO FACTOR
   IF P4CHOSEN = 1
    SELECT C2T
    STORE 'C2T' TO TRI
   ELSE
    SELECT C2P
    STORE 'C2P' TO PENT
   ENDIF
   GO F4RECNO
 ENDDO
  * BACK TO LEVEL 7 *
  STORE '3' TO FACTOR
  IF P3CHOSEN = 1
   SELECT C2T
   STORE 'C2T' TO TRI
  ELSE
   SELECT C2P
   STORE 'C2P' TO PENT
  ENDIF
  GO F3RECNO
ENDDO
```

```
        * BACK TO LEVEL 6 *
        STORE '5' TO FACTOR
        STORE 'D' TO TRITYPE
        STORE 'C2T' TO TRI
        SELECT C2T
        GO D5RECNO
      ENDDO
       * BACK TO LEVEL 5 *
       STORE '4' TO FACTOR
       SELECT C2T
       GO D4RECNO
     ENDDO
      * BACK TO LEVEL 4 *
      STORE '3' TO FACTOR
      SELECT C2T
      GO D3RECNO
      STORE 'C2T' TO TRI
      STORE 'D' TO TRITYPE
    ENDDO
     * BACK TO LEVEL 3 *
     STORE '5' TO FACTOR
     SELECT 3
     IF P5CHOSEN = 1
      STORE 'C1P5' TO PENT
     ELSE
      STORE 'C1T5' TO TRI
      STORE 'T' TO TRITYPE
     ENDIF
   ENDDO
    * BACK TO LEVEL 2 *
    STORE '4' TO FACTOR
    SELECT 2
    IF P4CHOSEN = 1
     STORE 'C1P4' TO PENT
    ELSE
     STORE 'C1T4' TO TRI
    ENDIF
  ENDDO
   * BACK TO LEVEL 1 *
   STORE '3' TO FACTOR
   SELECT 1
   IF P3CHOSEN = 1
    STORE 'C1P3' TO PENT
   ELSE
    STORE 'C1T3' TO TRI
   ENDIF
ENDDO
RETURN
```

PROCEDURE FILE FOR OBRWLFCH.PRG

PROCEDURE FINDP

```
* This procedure finds a pentagon (if it exists) in the current pentagon
* database that is compatible with the pieces of factors already selected

STORE .F. TO FOUND
SKIP
DO WHILE .NOT. EOF()
  STORE "N"+FACTOR+V1 TO NODE1
  STORE "N"+FACTOR+V2 TO NODE2
  STORE "N"+FACTOR+V3 TO NODE3
  STORE "N"+FACTOR+V4 TO NODE4
  STORE "N"+FACTOR+V5 TO NODE5
  IF NODES->&NODE1=0 .AND. NODES->&NODE2=0 .AND. NODES->&NODE3=0;
    .AND. NODES->&NODE4=0 .AND. NODES->&NODE5=0
   STORE "E"+E1 TO EDGE1
   STORE "E"+E2 TO EDGE2
   STORE "E"+E3 TO EDGE3
   STORE "E"+E4 TO EDGE4
   STORE "E"+E5 TO EDGE5
   IF EDGES->&EDGE1=0 .AND. EDGES->&EDGE2=0 .AND. EDGES->&EDGE3=0 ;
     .AND. EDGES->&EDGE4=0 .AND. EDGES->&EDGE5=0
     STORE V1+V2+V3+V4+V5 TO MP
    DO CPYRCRDS
    SELECT NODES
      REPLACE &NODE1 WITH 1
      REPLACE &NODE2 WITH 1
      REPLACE &NODE3 WITH 1
      REPLACE &NODE4 WITH 1
      REPLACE &NODE5 WITH 1
    SELECT EDGES
      REPLACE &EDGE1 WITH 1
      REPLACE &EDGE2 WITH 1
      REPLACE &EDGE3 WITH 1
      REPLACE &EDGE4 WITH 1
      REPLACE &EDGE5 WITH 1
    SELECT FACTORS
     STORE "P"+FACTOR TO SLOT
      REPLACE &SLOT WITH MP
    SELECT &PENT
    STORE .T. TO FOUND
    EXIT
   ENDIF
  ENDIF
  SKIP
ENDDO
  IF EOF()
   SKIP -1
  ENDIF
  STORE TIME() TO FTIME
RETURN
```

PROCEDURE FINDT

```
* This procedure finds a triangle (if it exists) in the current triangle
* database that is compatible with the pieces of factors already selected

STORE .F. TO FOUND
SKIP
DO WHILE .NOT. EOF()
  STORE "N"+FACTOR+V1 TO NODE1
  STORE "N"+FACTOR+V2 TO NODE2
  STORE "N"+FACTOR+V3 TO NODE3
  IF NODES->&NODE1=0 .AND. NODES->&NODE2=0 .AND. NODES->&NODE3=0
   STORE "E"+E1 TO EDGE1
   STORE "E"+E2 TO EDGE2
   STORE "E"+E3 TO EDGE3
   IF EDGES->&EDGE1=0 .AND. EDGES->&EDGE2=0 .AND. EDGES->&EDGE3=0
     STORE V1+V2+V3 TO MT
     DO CPYRCRDS
    SELECT NODES
      REPLACE &NODE1 WITH 1
      REPLACE &NODE2 WITH 1
      REPLACE &NODE3 WITH 1
    SELECT EDGES
      REPLACE &EDGE1 WITH 1
      REPLACE &EDGE2 WITH 1
      REPLACE &EDGE3 WITH 1
    SELECT FACTORS
      STORE TRITYPE+FACTOR TO SLOT
      REPLACE &SLOT WITH MT
    SELECT &TRI
    STORE .T. TO FOUND
    EXIT
   ENDIF
  ENDIF
  SKIP
ENDDO
IF EOF()
  SKIP -1
ENDIF
STORE TIME() TO FTIME
RETURN
```

PROCEDURE CPYRCRDS

```
* This procedure copies to the next record information about nodes and
* edges that are currently in use so that nodes and edges from a newly
* found pentagon or triangle can be added while still preserving the
* current state in the event that we have to backtrack if the new
* choice proves to be unworkable

SELECT NODES
  STORE N31 TO MN31
  STORE N32 TO MN32
  STORE N33 TO MN33
  STORE N34 TO MN34
  STORE N35 TO MN35
  STORE N3A TO MN3A
  STORE N3B TO MN3B
  STORE N3C TO MN3C
  STORE N3X TO MN3X
  STORE N3Y TO MN3Y
  STORE N3Z TO MN3Z
  STORE N41 TO MN41
  STORE N42 TO MN42
  STORE N43 TO MN43
  STORE N44 TO MN44
  STORE N45 TO MN45
  STORE N4A TO MN4A
  STORE N4B TO MN4B
  STORE N4C TO MN4C
  STORE N4X TO MN4X
  STORE N4Y TO MN4Y
  STORE N4Z TO MN4Z
  STORE N51 TO MN51
  STORE N52 TO MN52
  STORE N53 TO MN53
  STORE N54 TO MN54
  STORE N55 TO MN55
  STORE N5A TO MN5A
  STORE N5B TO MN5B
  STORE N5C TO MN5C
  STORE N5X TO MN5X
  STORE N5Y TO MN5Y
  STORE N5Z TO MN5Z
 APPEND BLANK
   REPLACE N31 WITH MN31, N32 WITH MN32, N33 WITH MN33, ;
     N34 WITH MN34, N35 WITH MN35, N3A WITH MN3A, N3B WITH MN3B, ;
     N3C WITH MN3C, N3X WITH MN3X, N3Y WITH MN3Y, N3Z WITH MN3Z
   REPLACE N41 WITH MN41, N42 WITH MN42, N43 WITH MN43, ;
     N44 WITH MN44, N45 WITH MN45, N4A WITH MN4A, N4B WITH MN4B, ;
     N4C WITH MN4C, N4X WITH MN4X, N4Y WITH MN4Y, N4Z WITH MN4Z
   REPLACE N51 WITH MN51, N52 WITH MN52, N53 WITH MN53, ;
     N54 WITH MN54, N55 WITH MN55, N5A WITH MN5A, N5B WITH MN5B, ;
     N5C WITH MN5C, N5X WITH MN5X, N5Y WITH MN5Y, N5Z WITH MN5Z
SELECT EDGES
  STORE E1A TO ME1A
  STORE E1B TO ME1B
  STORE E1C TO ME1C
  STORE E1X TO ME1X
```

```
 STORE E1Y TO ME1Y
 STORE E1Z TO ME1Z
 STORE E24 TO ME24
 STORE E25 TO ME25
 STORE E2B TO ME2B
 STORE E2C TO ME2C
 STORE E2X TO ME2X
 STORE E2Z TO ME2Z
 STORE E35 TO ME35
 STORE E3A TO ME3A
 STORE E3B TO ME3B
 STORE E3X TO ME3X
 STORE E3Y TO ME3Y
 STORE E3Z TO ME3Z
 STORE E4A TO ME4A
 STORE E4B TO ME4B
 STORE E4C TO ME4C
 STORE E4X TO ME4X
 STORE E4Y TO ME4Y
 STORE E5A TO ME5A
 STORE E5C TO ME5C
 STORE E5Y TO ME5Y
 STORE E5Z TO ME5Z
 STORE EAX TO MEAX
 STORE EAZ TO MEAZ
 STORE EBY TO MEBY
 STORE EBZ TO MEBZ
 STORE ECX TO MECX
 STORE ECY TO MECY
APPEND BLANK
  REPLACE E1A WITH ME1A, E1B WITH ME1B, E1C WITH ME1C, ;
    E1X WITH ME1X, E1Y WITH ME1Y, E1Z WITH ME1Z, E24 WITH ME24, ;
    E25 WITH ME25, E2B WITH ME2B, E2C WITH ME2C, E2X WITH ME2X, ;
    E2Z WITH ME2Z
  REPLACE E35 WITH ME35, E3A WITH ME3A, E3B WITH ME3B, ;
    E3X WITH ME3X, E3Y WITH ME3Y, E3Z WITH ME3Z, E4A WITH ME4A, ;
    E4B WITH ME4B, E4C WITH ME4C, E4X WITH ME4X, E4Y WITH ME4Y, ;
    E5A WITH ME5A
  REPLACE E5C WITH ME5C, E5Y WITH ME5Y, E5Z WITH ME5Z, ;
    EAX WITH MEAX, EAZ WITH MEAZ, EBY WITH MEBY, EBZ WITH MEBZ, ;
    ECX WITH MECX, ECY WITH MECY
SELECT FACTORS
 STORE P3 TO MP3
 STORE T3 TO MT3
 STORE D3 TO MD3
 STORE P4 TO MP4
 STORE T4 TO MT4
 STORE D4 TO MD4
 STORE P5 TO MP5
 STORE T5 TO MT5
 STORE D5 TO MD5
APPEND BLANK
  REPLACE P3 WITH MP3
  REPLACE T3 WITH MT3
  REPLACE D3 WITH MD3
  REPLACE P4 WITH MP4
  REPLACE T4 WITH MT4
```

```
  REPLACE D4 WITH MD4
  REPLACE P5 WITH MP5
  REPLACE T5 WITH MT5
  REPLACE D5 WITH MD5
RETURN
```

PROCEDURE BACKUP

```
* This procedure backs us up to the previous working level whenever there
* are no more possibilities to try with the current configuration.

SELECT NODES
  STORE RECNO()-1 TO POINTNOD
  DELETE
  PACK
  GO POINTNOD
SELECT EDGES
  STORE RECNO()-1 TO POINTEDG
  DELETE
  PACK
  GO POINTEDG
SELECT FACTORS
  STORE RECNO()-1 TO POINTFAC
  DELETE
  PACK
  GO POINTFAC
RETURN
```

DATA-GENERATING PROGRAMS FOR OBRWLFCH.PRG

*GENC1P.PRG

```
* THIS IS A PROGRAM TO GENERATE CLASS 1 PENTAGONS FROM AVAILABLE
* EDGES FOR THE OBRWLFCH PROBLEM OP(11;3,3,5)

CLEAR
CLEAR ALL
SET TALK OFF
SELECT 1
 USE C:C1PPEDGE ALIAS PPEDGE     [ C:C2PPEDGE and C:C3PPEDGE, resp. ]
 GO TOP
SELECT 2
 USE C:TDVERT1 ALIAS TD1
 GO TOP
SELECT 3
 USE C:PVERT1 ALIAS P
 GO TOP
SELECT 4
 USE C:TDVERT2 ALIAS TD2
 GO TOP
SELECT 5
 USE C:EDGEUSED ALIAS USED
 GO TOP
SELECT 6
 USE C:C1C1P ALIAS PENTS         [ C:C2C1P and C:C3C1P, resp. ]
 GO TOP
SELECT PPEDGE
DO WHILE .NOT. EOF()
 STORE V1 TO MV1
 STORE V2 TO MV2
 SELECT TD1
 DO WHILE .NOT. EOF()
  STORE V TO MV3
  SELECT P
  DO WHILE .NOT. EOF()
   IF V = MV1 .OR. V = MV2
    SKIP
    LOOP
   ELSE
    STORE V TO MV4
   ENDIF
   SELECT TD2
   DO WHILE .NOT. EOF()
    IF V = MV3
     SKIP
     LOOP
    ELSE
     STORE V TO MV5
    ENDIF
    STORE "E"+MV2+MV3 TO EDGE2
    STORE "E"+MV4+MV3 TO EDGE3
    STORE "E"+MV4+MV5 TO EDGE4
```

```
      STORE "E"+MV1+MV5 TO EDGE5
      IF USED->&EDGE2=1 .OR. USED->&EDGE3=1 .OR. USED->&EDGE4=1 .OR. ;
        USED->&EDGE5=1
        SKIP
        LOOP
      ELSE
        SELECT PENTS
         APPEND BLANK
           REPLACE V1 WITH MV1
           REPLACE V2 WITH MV2
           REPLACE V3 WITH MV3
           REPLACE V4 WITH MV4
           REPLACE V5 WITH MV5
           REPLACE E1 WITH MV1+MV2
           REPLACE E2 WITH MV2+MV3
           REPLACE E3 WITH MV4+MV3
           REPLACE E4 WITH MV4+MV5
           REPLACE E5 WITH MV1+MV5
        SELECT TD2
        SKIP
       ENDIF
     ENDDO
     GO TOP
     SELECT P
     SKIP
   ENDDO
   GO TOP
   SELECT TD1
   SKIP
  ENDDO
  GO TOP
  SELECT PPEDGE
  SKIP
ENDDO
CLOSE DATABASES
RETURN
```

*GENC2P.PRG

```
* THIS IS A PROGRAM TO GENERATE CLASS 2 PENTAGONS FROM AVAILABLE
* EDGES FOR THE OBRWLFCH PROBLEM OP(11;3,3,5)

CLEAR
CLEAR ALL
SET TALK OFF
SELECT 1
 USE C:C1TDEDGE ALIAS TDEDGE       [ C2TDEDGE and C3TDEDGE, resp.]
 GO TOP
SELECT 2
 USE C:PVERT1 ALIAS P1
 GO TOP
SELECT 3
 USE C:TDVERT1 ALIAS TD
 GO TOP
SELECT 4
 USE C:PVERT2 ALIAS P2
 GO TOP
SELECT 5
 USE C:EDGEUSED ALIAS USED
 GO TOP
SELECT 6
 USE C:C1C2P ALIAS PENTS            [ C2C2P and C3C2P, resp.]
 GO TOP
SELECT TDEDGE
DO WHILE .NOT. EOF()
  STORE V1 TO MV1
  STORE V2 TO MV2
  SELECT P1
  DO WHILE .NOT. EOF()
    STORE V TO MV3
    SELECT TD
    DO WHILE .NOT. EOF()
      IF V = MV1 .OR. V = MV2
        SKIP
        LOOP
      ELSE
        STORE V TO MV4
      ENDIF
      SELECT P2
      DO WHILE .NOT. EOF()
        IF V = MV3
          SKIP
          LOOP
        ELSE
          STORE V TO MV5
        ENDIF
        STORE "E"+MV3+MV2 TO EDGE2
        STORE "E"+MV3+MV4 TO EDGE3
        STORE "E"+MV5+MV4 TO EDGE4
        STORE "E"+MV5+MV1 TO EDGE5
        IF USED->&EDGE2=1 .OR. USED->&EDGE3=1 .OR. USED->&EDGE4=1 .OR. ;
          USED->&EDGE5=1
          SKIP
```

```
        LOOP
      ELSE
        SELECT PENTS
         APPEND BLANK
           REPLACE V1 WITH MV1
           REPLACE V2 WITH MV2
           REPLACE V3 WITH MV3
           REPLACE V4 WITH MV4
           REPLACE V5 WITH MV5
           REPLACE E1 WITH MV1+MV2
           REPLACE E2 WITH MV3+MV2
           REPLACE E3 WITH MV3+MV4
           REPLACE E4 WITH MV5+MV4
           REPLACE E5 WITH MV5+MV1
        SELECT P2
        SKIP
       ENDIF
     ENDDO
     GO TOP
     SELECT TD
     SKIP
   ENDDO
   GO TOP
   SELECT P1
   SKIP
  ENDDO
  GO TOP
  SELECT TDEDGE
  SKIP
ENDDO
CLOSE DATABASES
RETURN
```

PENTEDGE.PRG

```
* THIS PROGRAM FILLS IN THE EDGES (SMALLEST VALUE NODE FIRST) IN THE*
* PENTAGON DATABASES WHEN THE VERTICES ARE ALREADY ENTERED. *
GO TOP
DO WHILE .NOT. EOF()
 IF V1 < V2
   REPLACE E1 WITH V1+V2
 ELSE
   REPLACE E1 WITH V2+V1
 ENDIF
 IF V2 < V3
   REPLACE E2 WITH V2+V3
 ELSE
   REPLACE E2 WITH V3+V2
 ENDIF
 IF V3 < V4
   REPLACE E3 WITH V3+V4
 ELSE
   REPLACE E3 WITH V4+V3
 ENDIF
 IF V4 < V5
   REPLACE E4 WITH V4+V5
 ELSE
   REPLACE E4 WITH V5+V4
 ENDIF
 IF V1 < V5
   REPLACE E5 WITH V1+V5
 ELSE
   REPLACE E5 WITH V5+V1
 ENDIF
 SKIP
ENDDO
```

TRIEDGE.PRG

```
*THIS PROGRAM FILLS IN THE EDGES (SMALLEST VALUE NODE FIRST) IN THE*
*TRIANGLE DATABASES WHEN VERTICES HAVE ALREADY BEEN ENTERED*
GO TOP
DO WHILE .NOT. EOF()
 IF V1 < V2
   REPLACE E1 WITH V1+V2
 ELSE
   REPLACE E1 WITH V2+V1
 ENDIF
 IF V2 < V3
   REPLACE E2 WITH V2+V3
 ELSE
   REPLACE E2 WITH V3+V2
 ENDIF
 IF V1 < V3
   REPLACE E3 WITH V1+V3
 ELSE
   REPLACE E3 WITH V3+V1
 ENDIF
 SKIP
ENDDO
```

DATABASES FOR OBRWLFCH.PRG - CASE 1

C1C1T3.DBF

Record#	V1	V2	V3	E1	E2	E3
1						
2	2	5	C	25	5C	2C
3	2	5	Z	25	5Z	2Z

C1C1T4.DBF

Record#	V1	V2	V3	E1	E2	E3
1						
2	2	4	B	24	4B	2B
3	2	4	C	24	4C	2C
4	2	4	X	24	4X	2X

C1C1T5.DBF

Record#	V1	V2	V3	E1	E2	E3
1						
2	3	5	A	35	5A	3A
3	3	5	Y	35	5Y	3Y
4	3	5	Z	35	5Z	3Z

C1C2T.DBF

Record#	V1	V2	V3	E1	E2	E3
1						
2	1	A	X	1A	AX	1X
3	1	A	Z	1A	AZ	1Z
4	3	A	X	3A	AX	3X
5	3	A	Z	3A	AZ	3Z
6	4	A	X	4A	AX	4X
7	5	A	Z	5A	AZ	5Z
8	1	B	Y	1B	BY	1Y
9	1	B	Z	1B	BZ	1Z
10	2	B	Z	2B	BZ	2Z
11	3	B	Y	3B	BY	3Y
12	3	B	Z	3B	BZ	3Z
13	4	B	Y	4B	BY	4Y
14	1	C	X	1C	CX	1X
15	1	C	Y	1C	CY	1Y
16	2	C	X	2C	CX	2X
17	4	C	X	4C	CX	4X
18	4	C	Y	4C	CY	4Y
19	5	C	Y	5C	CY	5Y

C1C1P3.DBF

Record#	V1	V2	V3	V4	V5	E1	E2	E3	E4	E5
1										
2	2	5	A	1	X	25	5A	1A	1X	2X
3	2	5	A	3	X	25	5A	3A	3X	2X
4	2	5	A	3	Z	25	5A	3A	3Z	2Z
5	2	5	A	4	X	25	5A	4A	4X	2X
6	2	5	C	1	X	25	5C	1C	1X	2X
7	2	5	C	4	X	25	5C	4C	4X	2X
8	2	5	Y	1	B	25	5Y	1Y	1B	2B
9	2	5	Y	1	C	25	5Y	1Y	1C	2C
10	2	5	Y	3	B	25	5Y	3Y	3B	2B
11	2	5	Y	4	B	25	5Y	4Y	4B	2B
12	2	5	Y	4	C	25	5Y	4Y	4C	2C
13	2	5	Z	1	B	25	5Z	1Z	1B	2B
14	2	5	Z	1	C	25	5Z	1Z	1C	2C
15	2	5	Z	3	B	25	5Z	3Z	3B	2B

C1C1P4.DBF

Record#	V1	V2	V3	V4	V5	E1	E2	E3	E4	E5
1										
2	2	4	A	1	X	24	4A	1A	1X	2X
3	2	4	A	1	Z	24	4A	1A	1Z	2Z
4	2	4	A	3	X	24	4A	3A	3X	2X
5	2	4	A	3	Z	24	4A	3A	3Z	2Z
6	2	4	A	5	Z	24	4A	5A	5Z	2Z
7	2	4	B	1	X	24	4B	1B	1X	2X
8	2	4	B	1	Z	24	4B	1B	1Z	2Z
9	2	4	B	3	X	24	4B	3B	3X	2X
10	2	4	B	3	Z	24	4B	3B	3Z	2Z
11	2	4	C	1	X	24	4C	1C	1X	2X
12	2	4	C	1	Z	24	4C	1C	1Z	2Z
13	2	4	C	5	Z	24	4C	5C	5Z	2Z
14	2	4	X	1	B	24	4X	1X	1B	2B
15	2	4	X	1	C	24	4X	1X	1C	2C
16	2	4	X	3	B	24	4X	3X	3B	2B
17	2	4	Y	1	B	24	4Y	1Y	1B	2B
18	2	4	Y	1	C	24	4Y	1Y	1C	2C
19	2	4	Y	3	B	24	4Y	3Y	3B	2B
20	2	4	Y	5	C	24	4Y	5Y	5C	2C

C1C1P5.DBF

Record#	V1	V2	V3	V4	V5	E1	E2	E3	E4	E5
1										
2	3	5	A	1	X	35	5A	1A	1X	3X
3	3	5	A	1	Y	35	5A	1A	1Y	3Y
4	3	5	A	1	Z	35	5A	1A	1Z	3Z
5	3	5	A	4	X	35	5A	4A	4X	3X
6	3	5	A	4	Y	35	5A	4A	4Y	3Y
7	3	5	C	1	X	35	5C	1C	1X	3X
8	3	5	C	1	Y	35	5C	1C	1Y	3Y
9	3	5	C	1	Z	35	5C	1C	1Z	3Z
10	3	5	C	2	X	35	5C	2C	2X	3X
11	3	5	C	2	Z	35	5C	2C	2Z	3Z
12	3	5	C	4	X	35	5C	4C	4X	3X
13	3	5	C	4	Y	35	5C	4C	4Y	3Y
14	3	5	Y	1	A	35	5Y	1Y	1A	3A
15	3	5	Y	1	B	35	5Y	1Y	1B	3B
16	3	5	Y	4	B	35	5Y	4Y	4B	3B
17	3	5	Z	1	A	35	5Z	1Z	1A	3A
18	3	5	Z	1	B	35	5Z	1Z	1B	3B
19	3	5	Z	2	B	35	5Z	2Z	2B	3B

C1C2P.DBF

Record#	V1	V2	V3	V4	V5	E1	E2	E3	E4	E5
1										
2	1	A	3	B	Y	1A	3A	3B	BY	1Y
3	1	A	3	B	Z	1A	3A	3B	BZ	1Z
4	1	A	3	X	C	1A	3A	3X	CX	1C
5	1	A	3	Y	B	1A	3A	3Y	BY	1B
6	1	A	3	Y	C	1A	3A	3Y	CY	1C
7	1	A	3	Z	B	1A	3A	3Z	BZ	1B
8	1	A	4	B	Y	1A	4A	4B	BY	1Y
9	1	A	4	B	Z	1A	4A	4B	BZ	1Z
10	1	A	4	C	X	1A	4A	4C	CX	1X
11	1	A	4	C	Y	1A	4A	4C	CY	1Y
12	1	A	4	X	C	1A	4A	4X	CX	1C
13	1	A	4	Y	B	1A	4A	4Y	BY	1B
14	1	A	5	C	X	1A	5A	5C	CX	1X
15	1	A	5	C	Y	1A	5A	5C	CY	1Y
16	1	A	5	Y	B	1A	5A	5Y	BY	1B
17	1	A	5	Y	C	1A	5A	5Y	CY	1C
18	1	A	5	Z	B	1A	5A	5Z	BZ	1B
19	1	B	2	C	X	1B	2B	2C	CX	1X
20	1	B	2	C	Y	1B	2B	2C	CY	1Y
21	1	B	2	X	A	1B	2B	2X	AX	1A
22	1	B	2	X	C	1B	2B	2X	CX	1C
23	1	B	2	Z	A	1B	2B	2Z	AZ	1A
24	1	B	3	A	X	1B	3B	3A	AX	1X
25	1	B	3	A	Z	1B	3B	3A	AZ	1Z
26	1	B	3	X	A	1B	3B	3X	AX	1A
27	1	B	3	X	C	1B	3B	3X	CX	1C
28	1	B	3	Y	C	1B	3B	3Y	CY	1C
29	1	B	3	Z	A	1B	3B	3Z	AZ	1A
30	1	B	4	A	X	1B	4B	4A	AX	1X
31	1	B	4	A	Z	1B	4B	4A	AZ	1Z
32	1	B	4	C	X	1B	4B	4C	CX	1X
33	1	B	4	C	Y	1B	4B	4C	CY	1Y
34	1	B	4	X	C	1B	4B	4X	CX	1C
35	1	B	4	X	A	1B	4B	4X	AX	1A
36	1	B	4	Y	C	1B	4B	4Y	CY	1C
37	1	C	2	B	Y	1C	2C	2B	BY	1Y
38	1	C	2	B	Z	1C	2C	2B	BZ	1Z
39	1	C	2	X	A	1C	2C	2X	AX	1A
40	1	C	2	Z	A	1C	2C	2Z	AZ	1A
41	1	C	2	Z	B	1C	2C	2Z	BZ	1B
42	1	C	4	A	X	1C	4C	4A	AX	1X
43	1	C	4	A	Z	1C	4C	4A	AZ	1Z
44	1	C	4	B	Y	1C	4C	4B	BY	1Y
45	1	C	4	B	Z	1C	4C	4B	BZ	1Z
46	1	C	4	X	A	1C	4C	4X	AX	1A
47	1	C	4	Y	B	1C	4C	4Y	BY	1B
48	1	C	5	A	X	1C	5C	5A	AX	1X
49	1	C	5	A	Z	1C	5C	5A	AZ	1Z
50	1	C	5	Y	B	1C	5C	5Y	BY	1B
51	1	C	5	Z	B	1C	5C	5Z	BZ	1B
52	1	C	5	Z	A	1C	5C	5Z	AZ	1A
53	1	X	2	B	Y	1X	2X	2B	BY	1Y

54	1	X	2	B	Z	1X	2X	2B	BZ	1Z
55	1	X	2	C	Y	1X	2X	2C	CY	1Y
56	1	X	2	Z	A	1X	2X	2Z	AZ	1A
57	1	X	2	Z	B	1X	2X	2Z	BZ	1B
58	1	X	3	A	Z	1X	3X	3A	AZ	1Z
59	1	X	3	B	Y	1X	3X	3B	BY	1Y
60	1	X	3	B	Z	1X	3X	3B	BZ	1Z
61	1	X	3	Y	B	1X	3X	3Y	BY	1B
62	1	X	3	Y	C	1X	3X	3Y	CY	1C
63	1	X	3	Z	A	1X	3X	3Z	AZ	1A
64	1	X	3	Z	B	1X	3X	3Z	BZ	1B
65	1	X	4	A	Z	1X	4X	4A	AZ	1Z
66	1	X	4	B	Y	1X	4X	4B	BY	1Y
67	1	X	4	B	Z	1X	4X	4B	BZ	1Z
68	1	X	4	C	Y	1X	4X	4C	CY	1Y
69	1	X	4	Y	B	1X	4X	4Y	BY	1B
70	1	X	4	Y	C	1X	4X	4Y	CY	1C
71	1	Y	3	A	X	1Y	3Y	3A	AX	1X
72	1	Y	3	A	Z	1Y	3Y	3A	AZ	1Z
73	1	Y	3	B	Z	1Y	3Y	3B	BZ	1Z
74	1	Y	3	X	A	1Y	3Y	3X	AX	1A
75	1	Y	3	Z	A	1Y	3Y	3Z	AZ	1A
76	1	Y	3	Z	B	1Y	3Y	3Z	BZ	1B
77	1	Y	4	A	X	1Y	4Y	4A	AX	1X
78	1	Y	4	A	Z	1Y	4Y	4A	AZ	1Z
79	1	Y	4	B	Z	1Y	4Y	4B	BZ	1Z
80	1	Y	4	C	X	1Y	4Y	4C	CX	1X
81	1	Y	4	X	A	1Y	4Y	4X	AX	1A
82	1	Y	4	X	C	1Y	4Y	4X	CX	1C
83	1	Y	5	A	X	1Y	5Y	5A	AX	1X
84	1	Y	5	A	Z	1Y	5Y	5A	AZ	1Z
85	1	Y	5	C	X	1Y	5Y	5C	CX	1X
86	1	Y	5	Z	A	1Y	5Y	5Z	AZ	1A
87	1	Y	5	Z	B	1Y	5Y	5Z	BZ	1B
88	1	Z	2	B	Y	1Z	2Z	2B	BY	1Y
89	1	Z	2	C	X	1Z	2Z	2C	CX	1X
90	1	Z	2	C	Y	1Z	2Z	2C	CY	1Y
91	1	Z	2	X	A	1Z	2Z	2X	AX	1A
92	1	Z	2	X	C	1Z	2Z	2X	CX	1C
93	1	Z	3	A	X	1Z	3Z	3A	AX	1X
94	1	Z	3	B	Y	1Z	3Z	3B	BY	1Y
95	1	Z	3	X	A	1Z	3Z	3X	AX	1A
96	1	Z	3	X	C	1Z	3Z	3X	CX	1C
97	1	Z	3	Y	C	1Z	3Z	3Y	CY	1C
98	1	Z	3	Y	B	1Z	3Z	3Y	BY	1B
99	1	Z	5	A	X	1Z	5Z	5A	AX	1X
100	1	Z	5	C	X	1Z	5Z	5C	CX	1X
101	1	Z	5	C	Y	1Z	5Z	5C	CY	1Y
102	1	Z	5	Y	C	1Z	5Z	5Y	CY	1C
103	1	Z	5	Y	B	1Z	5Z	5Y	BY	1B
104	2	B	3	A	X	2B	3B	3A	AX	2X
105	2	B	3	A	Z	2B	3B	3A	AZ	2Z
106	2	B	3	X	C	2B	3B	3X	CX	2C
107	2	B	3	Y	C	2B	3B	3Y	CY	2C
108	2	B	4	A	X	2B	4B	4A	AX	2X
109	2	B	4	A	Z	2B	4B	4A	AZ	2Z
110	2	B	4	C	X	2B	4B	4C	CX	2X

111	2	B	4	X	C	2B	4B	4X	CX	2C
112	2	B	4	Y	C	2B	4B	4Y	CY	2C
113	2	C	4	A	X	2C	4C	4A	AX	2X
114	2	C	4	A	Z	2C	4C	4A	AZ	2Z
115	2	C	4	B	Z	2C	4C	4B	BZ	2Z
116	2	C	4	Y	B	2C	4C	4Y	BY	2B
117	2	C	5	A	X	2C	5C	5A	AX	2X
118	2	C	5	A	Z	2C	5C	5A	AZ	2Z
119	2	C	5	Y	B	2C	5C	5Y	BY	2B
120	2	C	5	Z	B	2C	5C	5Z	BZ	2B
121	2	X	3	A	Z	2X	3X	3A	AZ	2Z
122	2	X	3	B	Z	2X	3X	3B	BZ	2Z
123	2	X	3	Y	C	2X	3X	3Y	CY	2C
124	2	X	3	Y	B	2X	3X	3Y	BY	2B
125	2	X	3	Z	B	2X	3X	3Z	BZ	2B
126	2	X	4	A	Z	2X	4X	4A	AZ	2Z
127	2	X	4	B	Z	2X	4X	4B	BZ	2Z
128	2	X	4	Y	C	2X	4X	4Y	CY	2C
129	2	X	4	Y	B	2X	4X	4Y	BY	2B
130	2	Z	3	A	X	2Z	3Z	3A	AX	2X
131	2	Z	3	X	C	2Z	3Z	3X	CX	2C
132	2	Z	3	Y	C	2Z	3Z	3Y	CY	2C
133	2	Z	3	Y	B	2Z	3Z	3Y	BY	2B
134	2	Z	5	A	X	2Z	5Z	5A	AX	2X
135	2	Z	5	C	X	2Z	5Z	5C	CX	2X
136	2	Z	5	Y	C	2Z	5Z	5Y	CY	2C
137	2	Z	5	Y	B	2Z	5Z	5Y	BY	2B
138	3	A	4	B	Z	3A	4A	4B	BZ	3Z
139	3	A	4	B	Y	3A	4A	4B	BY	3Y
140	3	A	4	C	X	3A	4A	4C	CX	3X
141	3	A	4	C	Y	3A	4A	4C	CY	3Y
142	3	A	4	Y	B	3A	4A	4Y	BY	3B
143	3	A	5	C	X	3A	5A	5C	CX	3X
144	3	A	5	C	Y	3A	5A	5C	CY	3Y
145	3	A	5	Y	B	3A	5A	5Y	BY	3B
146	3	A	5	Z	B	3A	5A	5Z	BZ	3B
147	3	B	4	A	X	3B	4B	4A	AX	3X
148	3	B	4	A	Z	3B	4B	4A	AZ	3Z
149	3	B	4	C	X	3B	4B	4C	CX	3X
150	3	B	4	C	Y	3B	4B	4C	CY	3Y
151	3	B	4	X	A	3B	4B	4X	AX	3A
152	3	X	4	A	Z	3X	4X	4A	AZ	3Z
153	3	X	4	B	Y	3X	4X	4B	BY	3Y
154	3	X	4	B	Z	3X	4X	4B	BZ	3Z
155	3	X	4	C	Y	3X	4X	4C	CY	3Y
156	3	X	4	Y	B	3X	4X	4Y	BY	3B
157	3	Y	4	A	X	3Y	4Y	4A	AX	3X
158	3	Y	4	A	Z	3Y	4Y	4A	AZ	3Z
159	3	Y	4	B	Z	3Y	4Y	4B	BZ	3Z
160	3	Y	4	C	X	3Y	4Y	4C	CX	3X
161	3	Y	4	X	A	3Y	4Y	4X	AX	3A
162	3	Y	5	A	X	3Y	5Y	5A	AX	3X
163	3	Y	5	A	Z	3Y	5Y	5A	AZ	3Z
164	3	Y	5	C	X	3Y	5Y	5C	CX	3X
165	3	Y	5	Z	A	3Y	5Y	5Z	AZ	3A
166	3	Y	5	Z	B	3Y	5Y	5Z	BZ	3B
167	3	Z	5	A	X	3Z	5Z	5A	AX	3X

168	3	Z	5	C	X	3Z	5Z	5C	CX	3X
169	3	Z	5	C	Y	3Z	5Z	5C	CY	3Y
170	3	Z	5	Y	B	3Z	5Z	5Y	BY	3B
171	4	A	5	C	X	4A	5A	5C	CX	4X
172	4	A	5	C	Y	4A	5A	5C	CY	4Y
173	4	A	5	Y	B	4A	5A	5Y	BY	4B
174	4	A	5	Y	C	4A	5A	5Y	CY	4C
175	4	A	5	Z	B	4A	5A	5Z	BZ	4B
176	4	C	5	A	X	4C	5C	5A	AX	4X
177	4	C	5	Y	B	4C	5C	5Y	BY	4B
178	4	C	5	Z	A	4C	5C	5Z	AZ	4A
179	4	C	5	Z	B	4C	5C	5Z	BZ	4B
180	4	Y	5	A	X	4Y	5Y	5A	AX	4X
181	4	Y	5	C	X	4Y	5Y	5C	CX	4X
182	4	Y	5	Z	A	4Y	5Y	5Z	AZ	4A
183	4	Y	5	Z	B	4Y	5Y	5Z	BZ	4B

USEDEDGE.DBF (for Case 1)

Record#	E1A	E1B	E1C	E1X	E1Y	E1Z	E24	E25	E2B	E2C	E2X	E2Z	E35	E3A	E3B	E3X
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

E3Y	E3Z	E4A	E4B	E4C	E4X	E4Y	E5A	E5C	E5Y	E5Z	EAX	EAZ	EBY	EBZ	ECX	ECY
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

DATABASES FOR OBRWLFCH.PRG - CASE 2

C2C1T3.DBF

Record#	V1	V2	V3	E1	E2	E3
1						
2	1	3	A	13	1A	3A
3	1	3	B	13	1B	3B
4	1	3	Y	13	1Y	3Y

C2C1T4.DBF

Record#	V1	V2	V3	E1	E2	E3
1						
2	2	4	B	24	2B	4B
3	2	4	C	24	2C	4C
4	2	4	X	24	2X	4X
5	2	4	Y	24	2Y	4Y

C12C1T5.DBF

Record#	V1	V2	V3	E1	E2	E3
1						
2	3	5	B	35	3B	5B
3	3	5	Y	35	3Y	5Y
4	3	5	Z	35	3Z	5Z

C2C2T.DBF

Record#	V1	V2	V3	E1	E2	E3
1						
2	1	A	X	1A	1X	AX
3	4	A	X	4A	4X	AX
4	1	A	Y	1A	1Y	AY
5	3	A	Y	3A	3Y	AY
6	4	A	Y	4A	4Y	AY
7	3	A	Z	3A	3Z	AZ
8	2	B	Z	2B	2Z	BZ
9	3	B	Z	3B	3Z	BZ
10	5	B	Z	5B	5Z	BZ
11	1	C	X	1C	1X	CX
12	2	C	X	2C	2X	CX
13	4	C	X	4C	4X	CX
14	5	C	X	5C	5X	CX
15	2	C	Z	2C	2Z	CZ
16	5	C	Z	5C	5Z	CZ

C2C1P3.DBF

Record#	V1	V2	V3	V4	V5	E1	E2	E3	E4	E5
.1										
.2	1	3	A	4	B	13	3A	4A	4B	1B
.3	1	3	A	4	C	13	3A	4A	4C	1C
.4	1	3	A	4	X	13	3A	4A	4X	1X
.5	1	3	A	4	Y	13	3A	4A	4Y	1Y
.6	1	3	B	2	C	13	3B	2B	2C	1C
.7	1	3	B	2	X	13	3B	2B	2X	1X
.8	1	3	B	2	Y	13	3B	2B	2Y	1Y
.9	1	3	B	4	A	13	3B	4B	4A	1A
10	1	3	B	4	C	13	3B	4B	4C	1C
11	1	3	B	4	X	13	3B	4B	4X	1X
12	1	3	B	4	Y	13	3B	4B	4Y	1Y
13	1	3	B	5	C	13	3B	5B	5C	1C
14	1	3	B	5	X	13	3B	5B	5X	1X
15	1	3	B	5	Y	13	3B	5B	5Y	1Y
16	1	3	Y	2	B	13	3Y	2Y	2B	1B
17	1	3	Y	2	C	13	3Y	2Y	2C	1C
18	1	3	Y	2	X	13	3Y	2Y	2X	1X
19	1	3	Y	4	A	13	3Y	4Y	4A	1A
20	1	3	Y	4	B	13	3Y	4Y	4B	1B
21	1	3	Y	4	C	13	3Y	4Y	4C	1C
22	1	3	Y	4	X	13	3Y	4Y	4X	1X
23	1	3	Y	5	B	13	3Y	5Y	5B	1B
24	1	3	Y	5	C	13	3Y	5Y	5C	1C
25	1	3	Y	5	X	13	3Y	5Y	5X	1X
26	1	3	Z	2	B	13	3Z	2Z	2B	1B
27	1	3	Z	2	C	13	3Z	2Z	2C	1C
28	1	3	Z	2	X	13	3Z	2Z	2X	1X
29	1	3	Z	2	Y	13	3Z	2Z	2Y	1Y
30	1	3	Z	5	B	13	3Z	5Z	5B	1B
31	1	3	Z	5	C	13	3Z	5Z	5C	1C
32	1	3	Z	5	X	13	3Z	5Z	5X	1X
33	1	3	Z	5	Y	13	3Z	5Z	5Y	1Y

C2C1P4.DBF

Record#	V1	V2	V3	V4	V5	E1	E2	E3	E4	E5
.1										
2	2	4	A	1	B	24	4A	1A	1B	2B
3	2	4	A	1	C	24	4A	1A	1C	2C
4	2	4	A	1	X	24	4A	1A	1X	2X
5	2	4	A	1	Y	24	4A	1A	1Y	2Y
6	2	4	A	3	B	24	4A	3A	3B	2B
7	2	4	A	3	Y	24	4A	3A	3Y	2Y
8	2	4	A	3	Z	24	4A	3A	3Z	2Z
9	2	4	B	1	C	24	4B	1B	1C	2C
10	2	4	B	1	X	24	4B	1B	1X	2X
11	2	4	B	1	Y	24	4B	1B	1Y	2Y
12	2	4	B	3	Y	24	4B	3B	3Y	2Y
13	2	4	B	3	Z	24	4B	3B	3Z	2Z
14	2	4	B	5	C	24	4B	5B	5C	2C
15	2	4	B	5	X	24	4B	5B	5X	2X
16	2	4	B	5	Y	24	4B	5B	5Y	2Y
17	2	4	B	5	Z	24	4B	5B	5Z	2Z
18	2	4	C	1	B	24	4C	1C	1B	2B
19	2	4	C	1	X	24	4C	1C	1X	2X
20	2	4	C	1	Y	24	4C	1C	1Y	2Y
21	2	4	C	5	B	24	4C	5C	5B	2B
22	2	4	C	5	X	24	4C	5C	5X	2X
23	2	4	C	5	Y	24	4C	5C	5Y	2Y
24	2	4	C	5	Z	24	4C	5C	5Z	2Z
25	2	4	X	1	B	24	4X	1X	1B	2B
26	2	4	X	1	C	24	4X	1X	1C	2C
27	2	4	X	1	Y	24	4X	1X	1Y	2Y
28	2	4	X	5	B	24	4X	5X	5B	2B
29	2	4	X	5	C	24	4X	5X	5C	2C
30	2	4	X	5	Y	24	4X	5X	5Y	2Y
31	2	4	X	5	Z	24	4X	5X	5Z	2Z
32	2	4	Y	1	B	24	4Y	1Y	1B	2B
33	2	4	Y	1	C	24	4Y	1Y	1C	2C
34	2	4	Y	1	X	24	4Y	1Y	1X	2X
35	2	4	Y	3	B	24	4Y	3Y	3B	2B
36	2	4	Y	3	Z	24	4Y	3Y	3Z	2Z
37	2	4	Y	5	B	24	4Y	5Y	5B	2B
38	2	4	Y	5	C	24	4Y	5Y	5C	2C
39	2	4	Y	5	X	24	4Y	5Y	5X	2X
40	2	4	Y	5	Z	24	4Y	5Y	5Z	2Z

C2C1P5.DBF

Record#	V1	V2	V3	V4	V5	E1	E2	E3	E4	E5
.1										
2	3	5	B	1	A	35	5B	1B	1A	3A
3	3	5	B	1	Y	35	5B	1B	1Y	3Y
4	3	5	B	2	Y	35	5B	2B	2Y	3Y
5	3	5	B	2	Z	35	5B	2B	2Z	3Z
6	3	5	B	4	A	35	5B	4B	4A	3A
7	3	5	B	4	Y	35	5B	4B	4Y	3Y
8	3	5	C	1	A	35	5C	1C	1A	3A
9	3	5	C	1	B	35	5C	1C	1B	3B
10	3	5	C	1	Y	35	5C	1C	1Y	3Y
11	3	5	C	2	B	35	5C	2C	2B	3B
12	3	5	C	2	Y	35	5C	2C	2Y	3Y
13	3	5	C	2	Z	35	5C	2C	2Z	3Z
14	3	5	C	4	A	35	5C	4C	4A	3A
15	3	5	C	4	B	35	5C	4C	4B	3B
16	3	5	C	4	Y	35	5C	4C	4Y	3Y
17	3	5	X	1	A	35	5X	1X	1A	3A
18	3	5	X	1	B	35	5X	1X	1B	3B
19	3	5	X	1	Y	35	5X	1X	1Y	3Y
20	3	5	X	2	B	35	5X	2X	2B	3B
21	3	5	X	2	Y	35	5X	2X	2Y	3Y
22	3	5	X	2	Z	35	5X	2X	2Z	3Z
23	3	5	X	4	A	35	5X	4X	4A	3A
24	3	5	X	4	B	35	5X	4X	4B	3B
25	3	5	X	4	Y	35	5X	4X	4Y	3Y
26	3	5	Y	1	A	35	5Y	1Y	1A	3A
27	3	5	Y	1	B	35	5Y	1Y	1B	3B
28	3	5	Y	2	B	35	5Y	2Y	2B	3B
29	3	5	Y	2	Z	35	5Y	2Y	2Z	3Z
30	3	5	Y	4	A	35	5Y	4Y	4A	3A
31	3	5	Y	4	B	35	5Y	4Y	4B	3B
32	3	5	Z	2	B	35	5Z	2Z	2B	3B
33	3	5	Z	2	Y	35	5Z	2Z	2Y	3Y

C2C2P.DBF

Record#	V1	V2	V3	V4	V5	E1	E2	E3	E4	E5
.1										
2	A	X	1	B	3	AX	1X	1B	3B	3A
3	A	X	1	B	4	AX	1X	1B	4B	4A
4	A	X	1	C	4	AX	1X	1C	4C	4A
5	A	X	1	Y	3	AX	1X	1Y	3Y	3A
6	A	X	1	Y	4	AX	1X	1Y	4Y	4A
7	A	X	2	B	1	AX	2X	2B	1B	1A
8	A	X	2	B	3	AX	2X	2B	3B	3A
9	A	X	2	B	4	AX	2X	2B	4B	4A
10	A	X	2	C	1	AX	2X	2C	1C	1A
11	A	X	2	C	4	AX	2X	2C	4C	4A
12	A	X	2	Y	1	AX	2X	2Y	1Y	1A
13	A	X	2	Y	3	AX	2X	2Y	3Y	3A
14	A	X	2	Y	4	AX	2X	2Y	4Y	4A
15	A	X	2	Z	3	AX	2X	2Z	3Z	3A
16	A	X	4	B	1	AX	4X	4B	1B	1A
17	A	X	4	B	3	AX	4X	4B	3B	3A
18	A	X	4	C	1	AX	4X	4C	1C	1A
19	A	X	4	Y	1	AX	4X	4Y	1Y	1A
20	A	X	4	Y	3	AX	4X	4Y	3Y	3A
21	A	X	5	B	1	AX	5X	5B	1B	1A
22	A	X	5	B	3	AX	5X	5B	3B	3A
23	A	X	5	B	4	AX	5X	5B	4B	4A
24	A	X	5	C	1	AX	5X	5C	1C	1A
25	A	X	5	C	4	AX	5X	5C	4C	4A
26	A	X	5	Y	1	AX	5X	5Y	1Y	1A
27	A	X	5	Y	3	AX	5X	5Y	3Y	3A
28	A	X	5	Y	4	AX	5X	5Y	4Y	4A
29	A	X	5	Z	3	AX	5X	5Z	3Z	3A
30	A	Y	1	B	3	AY	1Y	1B	3B	3A
31	A	Y	1	B	4	AY	1Y	1B	4B	4A
32	A	Y	1	C	4	AY	1Y	1C	4C	4A
33	A	Y	1	X	4	AY	1Y	1X	4X	4A
34	A	Y	2	B	1	AY	2Y	2B	1B	1A
35	A	Y	2	B	3	AY	2Y	2B	3B	3A
36	A	Y	2	B	4	AY	2Y	2B	4B	4A
37	A	Y	2	C	1	AY	2Y	2C	1C	1A
38	A	Y	2	C	4	AY	2Y	2C	4C	4A
39	A	Y	2	X	1	AY	2Y	2X	1X	1A
40	A	Y	2	X	4	AY	2Y	2X	4X	4A
41	A	Y	2	Z	3	AY	2Y	2Z	3Z	3A
42	A	Y	3	B	1	AY	3Y	3B	1B	1A
43	A	Y	3	B	4	AY	3Y	3B	4B	4A
44	A	Y	4	B	1	AY	4Y	4B	1B	1A
45	A	Y	4	B	3	AY	4Y	4B	3B	3A
46	A	Y	4	C	1	AY	4Y	4C	1C	1A
47	A	Y	4	X	1	AY	4Y	4X	1X	1A
48	A	Y	5	B	1	AY	5Y	5B	1B	1A
49	A	Y	5	B	3	AY	5Y	5B	3B	3A
50	A	Y	5	B	4	AY	5Y	5B	4B	4A
51	A	Y	5	C	1	AY	5Y	5C	1C	1A
52	A	Y	5	C	4	AY	5Y	5C	4C	4A
53	A	Y	5	X	1	AY	5Y	5X	1X	1A
54	A	Y	5	X	4	AY	5Y	5X	4X	4A
55	A	Y	5	Z	3	AY	5Y	5Z	3Z	3A

56	A	Z	2	B	1	AZ	2Z	2B	1B	1A
57	A	Z	2	B	3	AZ	2Z	2B	3B	3A
58	A	Z	2	B	4	AZ	2Z	2B	4B	4A
59	A	Z	2	C	1	AZ	2Z	2C	1C	1A
60	A	Z	2	C	4	AZ	2Z	2C	4C	4A
61	A	Z	2	X	1	AZ	2Z	2X	1X	1A
62	A	Z	2	X	4	AZ	2Z	2X	4X	4A
63	A	Z	2	Y	1	AZ	2Z	2Y	1Y	1A
64	A	Z	2	Y	3	AZ	2Z	2Y	3Y	3A
65	A	Z	2	Y	4	AZ	2Z	2Y	4Y	4A
66	A	Z	3	B	1	AZ	3Z	3B	1B	1A
67	A	Z	3	B	4	AZ	3Z	3B	4B	4A
68	A	Z	3	Y	1	AZ	3Z	3Y	1Y	1A
69	A	Z	3	Y	4	AZ	3Z	3Y	4Y	4A
70	A	Z	5	B	1	AZ	5Z	5B	1B	1A
71	A	Z	5	B	3	AZ	5Z	5B	3B	3A
72	A	Z	5	B	4	AZ	5Z	5B	4B	4A
73	A	Z	5	C	1	AZ	5Z	5C	1C	1A
74	A	Z	5	C	4	AZ	5Z	5C	4C	4A
75	A	Z	5	X	1	AZ	5Z	5X	1X	1A
76	A	Z	5	X	4	AZ	5Z	5X	4X	4A
77	A	Z	5	Y	1	AZ	5Z	5Y	1Y	1A
78	A	Z	5	Y	3	AZ	5Z	5Y	3Y	3A
79	A	Z	5	Y	4	AZ	5Z	5Y	4Y	4A
80	B	Z	2	C	1	BZ	2Z	2C	1C	1B
81	B	Z	2	C	4	BZ	2Z	2C	4C	4B
82	B	Z	2	C	5	BZ	2Z	2C	5C	5B
83	B	Z	2	X	1	BZ	2Z	2X	1X	1B
84	B	Z	2	X	4	BZ	2Z	2X	4X	4B
85	B	Z	2	X	5	BZ	2Z	2X	5X	5B
86	B	Z	2	Y	1	BZ	2Z	2Y	1Y	1B
87	B	Z	2	Y	3	BZ	2Z	2Y	3Y	3B
88	B	Z	2	Y	4	BZ	2Z	2Y	4Y	4B
89	B	Z	2	Y	5	BZ	2Z	2Y	5Y	5B
90	B	Z	3	A	1	BZ	3Z	3A	1A	1B
91	B	Z	3	A	4	BZ	3Z	3A	4A	4B
92	B	Z	3	Y	1	BZ	3Z	3Y	1Y	1B
93	B	Z	3	Y	2	BZ	3Z	3Y	2Y	2B
94	B	Z	3	Y	4	BZ	3Z	3Y	4Y	4B
95	B	Z	3	Y	5	BZ	3Z	3Y	5Y	5B
96	B	Z	5	C	1	BZ	5Z	5C	1C	1B
97	B	Z	5	C	2	BZ	5Z	5C	2C	2B
98	B	Z	5	C	4	BZ	5Z	5C	4C	4B
99	B	Z	5	X	1	BZ	5Z	5X	1X	1B
100	B	Z	5	X	2	BZ	5Z	5X	2X	2B
101	B	Z	5	X	4	BZ	5Z	5X	4X	4B
102	B	Z	5	Y	1	BZ	5Z	5Y	1Y	1B
103	B	Z	5	Y	2	BZ	5Z	5Y	2Y	2B
104	B	Z	5	Y	3	BZ	5Z	5Y	3Y	3B
105	B	Z	5	Y	4	BZ	5Z	5Y	4Y	4B
106	C	X	1	A	4	CX	1X	1A	4A	4C
107	C	X	1	B	2	CX	1X	1B	2B	2C
108	C	X	1	B	4	CX	1X	1B	4B	4C
109	C	X	1	B	5	CX	1X	1B	5B	5C
110	C	X	1	Y	2	CX	1X	1Y	2Y	2C
111	C	X	1	Y	4	CX	1X	1Y	4Y	4C
112	C	X	1	Y	5	CX	1X	1Y	5Y	5C

113	C	X	2	B	1	CX	2X	2B	1B	1C
114	C	X	2	B	4	CX	2X	2B	4B	4C
115	C	X	2	B	5	CX	2X	2B	5B	5C
116	C	X	2	Y	1	CX	2X	2Y	1Y	1C
117	C	X	2	Y	4	CX	2X	2Y	4Y	4C
118	C	X	2	Y	5	CX	2X	2Y	5Y	5C
119	C	X	2	Z	5	CX	2X	2Z	5Z	5C
120	C	X	4	A	1	CX	4X	4A	1A	1C
121	C	X	4	B	1	CX	4X	4B	1B	1C
122	C	X	4	B	2	CX	4X	4B	2B	2C
123	C	X	4	B	5	CX	4X	4B	5B	5C
124	C	X	4	Y	1	CX	4X	4Y	1Y	1C
125	C	X	4	Y	2	CX	4X	4Y	2Y	2C
126	C	X	4	Y	5	CX	4X	4Y	5Y	5C
127	C	X	5	B	1	CX	5X	5B	1B	1C
128	C	X	5	B	2	CX	5X	5B	2B	2C
129	C	X	5	B	4	CX	5X	5B	4B	4C
130	C	X	5	Y	1	CX	5X	5Y	1Y	1C
131	C	X	5	Y	2	CX	5X	5Y	2Y	2C
132	C	X	5	Y	4	CX	5X	5Y	4Y	4C
133	C	X	5	Z	2	CX	5X	5Z	2Z	2C
134	C	Z	2	B	1	CZ	2Z	2B	1B	1C
135	C	Z	2	B	4	CZ	2Z	2B	4B	4C
136	C	Z	2	B	5	CZ	2Z	2B	5B	5C
137	C	Z	2	X	1	CZ	2Z	2X	1X	1C
138	C	Z	2	X	4	CZ	2Z	2X	4X	4C
139	C	Z	2	X	5	CZ	2Z	2X	5X	5C
140	C	Z	2	Y	1	CZ	2Z	2Y	1Y	1C
141	C	Z	2	Y	4	CZ	2Z	2Y	4Y	4C
142	C	Z	2	Y	5	CZ	2Z	2Y	5Y	5C
143	C	Z	3	A	1	CZ	3Z	3A	1A	1C
144	C	Z	3	A	4	CZ	3Z	3A	4A	4C
145	C	Z	3	B	1	CZ	3Z	3B	1B	1C
146	C	Z	3	B	2	CZ	3Z	3B	2B	2C
147	C	Z	3	B	4	CZ	3Z	3B	4B	4C
148	C	Z	3	B	5	CZ	3Z	3B	5B	5C
149	C	Z	3	Y	1	CZ	3Z	3Y	1Y	1C
150	C	Z	3	Y	2	CZ	3Z	3Y	2Y	2C
151	C	Z	3	Y	4	CZ	3Z	3Y	4Y	4C
152	C	Z	3	Y	5	CZ	3Z	3Y	5Y	5C
153	C	Z	5	B	1	CZ	5Z	5B	1B	1C
154	C	Z	5	B	2	CZ	5Z	5B	2B	2C
155	C	Z	5	B	4	CZ	5Z	5B	4B	4C
156	C	Z	5	X	1	CZ	5Z	5X	1X	1C
157	C	Z	5	X	2	CZ	5Z	5X	2X	2C
158	C	Z	5	X	4	CZ	5Z	5X	4X	4C
159	C	Z	5	Y	1	CZ	5Z	5Y	1Y	1C
160	C	Z	5	Y	2	CZ	5Z	5Y	2Y	2C
161	C	Z	5	Y	4	CZ	5Z	5Y	4Y	4C

C2USDEDG.DBF

Record#	E13	E1A	E1B	E1C	E1X	E1Y	E24	E2B	E2C	E2X	E2Y	E2Z	E35	E3A	E3B	E3Y	E3Z
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

E4A	E4B	E4C	E4X	E4Y	E5B	E5C	E5X	E5Y	E5Z	EAX	EAY	EAZ	EBZ	ECX	ECZ
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

DATABASES FOR OBRWLFCH.PRG - CASE 3

C3C1T3.DBF

Record#	V1	V2	V3	E1	E2	E3
1						
2	1	3	A	13	1A	3A
3	1	3	B	13	1B	3B
4	1	3	Y	13	1Y	3Y

C3C1T4.DBF

Record#	V1	V2	V3	E1	E2	E3
1						
2	2	4	B	24	2B	4B
3	2	4	C	24	2C	4C
4	2	4	X	24	2X	4X
5	2	4	Y	24	2Y	4Y

C3C1T5.DBF

Record#	V1	V2	V3	E1	E2	E3
1						
2	3	5	A	35	3A	5A
3	3	5	Y	35	3Y	5Y
4	3	5	Z	35	3Z	5Z

C3C2T.DBF

Record#	V1	V2	V3	E1	E2	E3
1						
2	1	A	X	1A	1X	AX
3	4	A	X	4A	4X	AX
4	5	A	X	5A	5X	AX
5	3	A	Z	3A	3Z	AZ
6	5	A	Z	5A	5Z	AZ
7	1	B	X	1B	1X	BX
8	2	B	X	2B	2X	BX
9	4	B	X	4B	4X	BX
10	2	B	Z	2B	2Z	BZ
11	3	B	Z	3B	3Z	BZ
12	1	C	Y	1C	1Y	CY
13	2	C	Y	2C	2Y	CY
14	4	C	Y	4C	4Y	CY
15	5	C	Y	5C	5Y	CY
16	2	C	Z	2C	2Z	CZ
17	5	C	Z	5C	5Z	CZ

C3C1P3.DBF

Record#	V1	V2	V3	V4	V5	E1	E2	E3	E4	E5
1										
2	1	3	A	4	B	13	3A	4A	4B	1B
3	1	3	A	4	C	13	3A	4A	4C	1C
4	1	3	A	4	X	13	3A	4A	4X	1X
5	1	3	A	4	Y	13	3A	4A	4Y	1Y
6	1	3	A	5	C	13	3A	5A	5C	1C
7	1	3	A	5	X	13	3A	5A	5X	1X
8	1	3	A	5	Y	13	3A	5A	5Y	1Y

9	1	3	B	2	C	13	3B	2B	2C	1C
10	1	3	B	2	X	13	3B	2B	2X	1X
11	1	3	B	2	Y	13	3B	2B	2Y	1Y
12	1	3	B	4	A	13	3B	4B	4A	1A
13	1	3	B	4	C	13	3B	4B	4C	1C
14	1	3	B	4	X	13	3B	4B	4X	1X
15	1	3	B	4	Y	13	3B	4B	4Y	1Y
16	1	3	Y	2	B	13	3Y	2Y	2B	1B
17	1	3	Y	2	C	13	3Y	2Y	2C	1C
18	1	3	Y	2	X	13	3Y	2Y	2X	1X
19	1	3	Y	4	A	13	3Y	4Y	4A	1A
20	1	3	Y	4	B	13	3Y	4Y	4B	1B
21	1	3	Y	4	C	13	3Y	4Y	4C	1C
22	1	3	Y	4	X	13	3Y	4Y	4X	1X
23	1	3	Y	5	A	13	3Y	5Y	5A	1A
24	1	3	Y	5	C	13	3Y	5Y	5C	1C
25	1	3	Y	5	X	13	3Y	5Y	5X	1X
26	1	3	Z	2	B	13	3Z	2Z	2B	1B
27	1	3	Z	2	C	13	3Z	2Z	2C	1C
28	1	3	Z	2	X	13	3Z	2Z	2X	1X
29	1	3	Z	2	Y	13	3Z	2Z	2Y	1Y
30	1	3	Z	5	A	13	3Z	5Z	5A	1A
31	1	3	Z	5	C	13	3Z	5Z	5C	1C
32	1	3	Z	5	X	13	3Z	5Z	5X	1X
33	1	3	Z	5	Y	13	3Z	5Z	5Y	1Y

C3C1P4.DBF

Record#	V1	V2	V3	V4	V5	E1	E2	E3	E4	E5
1										
2	2	4	A	1	B	24	4A	1A	1B	2B
3	2	4	A	1	C	24	4A	1A	1C	2C
4	2	4	A	1	X	24	4A	1A	1X	2X
5	2	4	A	1	Y	24	4A	1A	1Y	2Y
6	2	4	A	3	B	24	4A	3A	3B	2B
7	2	4	A	3	Y	24	4A	3A	3Y	2Y
8	2	4	A	3	Z	24	4A	3A	3Z	2Z
9	2	4	A	5	C	24	4A	5A	5C	2C
10	2	4	A	5	X	24	4A	5A	5X	2X
11	2	4	A	5	Y	24	4A	5A	5Y	2Y
12	2	4	A	5	Z	24	4A	5A	5Z	2Z
13	2	4	B	1	C	24	4B	1B	1C	2C
14	2	4	B	1	X	24	4B	1B	1X	2X
15	2	4	B	1	Y	24	4B	1B	1Y	2Y
16	2	4	B	3	Y	24	4B	3B	3Y	2Y
17	2	4	B	3	Z	24	4B	3B	3Z	2Z
18	2	4	C	1	B	24	4C	1C	1B	2B
19	2	4	C	1	X	24	4C	1C	1X	2X
20	2	4	C	1	Y	24	4C	1C	1Y	2Y
21	2	4	C	5	X	24	4C	5C	5X	2X
22	2	4	C	5	Y	24	4C	5C	5Y	2Y
23	2	4	C	5	Z	24	4C	5C	5Z	2Z
24	2	4	X	1	B	24	4X	1X	1B	2B
25	2	4	X	1	C	24	4X	1X	1C	2C
26	2	4	X	1	Y	24	4X	1X	1Y	2Y
27	2	4	X	5	C	24	4X	5X	5C	2C
28	2	4	X	5	Y	24	4X	5X	5Y	2Y
29	2	4	X	5	Z	24	4X	5X	5Z	2Z

30	2	4	Y	1	B	24	4Y	1Y	1B	2B
31	2	4	Y	1	C	24	4Y	1Y	1C	2C
32	2	4	Y	1	X	24	4Y	1Y	1X	2X
33	2	4	Y	3	B	24	4Y	3Y	3B	2B
34	2	4	Y	3	Z	24	4Y	3Y	3Z	2Z
35	2	4	Y	5	C	24	4Y	5Y	5C	2C
36	2	4	Y	5	X	24	4Y	5Y	5X	2X
37	2	4	Y	5	Z	24	4Y	5Y	5Z	2Z

C3C1P5.DBF

Record#	V1	V2	V3	V4	V5	E1	E2	E3	E4	E5
1										
2	3	5	A	1	B	35	5A	1A	1B	3B
3	3	5	A	1	Y	35	5A	1A	1Y	3Y
4	3	5	A	4	B	35	5A	4A	4B	3B
5	3	5	A	4	Y	35	5A	4A	4Y	3Y
6	3	5	C	1	A	35	5C	1C	1A	3A
7	3	5	C	1	B	35	5C	1C	1B	3B
8	3	5	C	1	Y	35	5C	1C	1Y	3Y
9	3	5	C	2	B	35	5C	2C	2B	3B
10	3	5	C	2	Y	35	5C	2C	2Y	3Y
11	3	5	C	2	Z	35	5C	2C	2Z	3Z
12	3	5	C	4	A	35	5C	4C	4A	3A
13	3	5	C	4	B	35	5C	4C	4B	3B
14	3	5	C	4	Y	35	5C	4C	4Y	3Y
15	3	5	X	1	A	35	5X	1X	1A	3A
16	3	5	X	1	B	35	5X	1X	1B	3B
17	3	5	X	1	Y	35	5X	1X	1Y	3Y
18	3	5	X	2	B	35	5X	2X	2B	3B
19	3	5	X	2	Y	35	5X	2X	2Y	3Y
20	3	5	X	2	Z	35	5X	2X	2Z	3Z
21	3	5	X	4	A	35	5X	4X	4A	3A
22	3	5	X	4	B	35	5X	4X	4B	3B
23	3	5	X	4	Y	35	5X	4X	4Y	3Y
24	3	5	Y	1	A	35	5Y	1Y	1A	3A
25	3	5	Y	1	B	35	5Y	1Y	1B	3B
26	3	5	Y	2	B	35	5Y	2Y	2B	3B
27	3	5	Y	2	Z	35	5Y	2Y	2Z	3Z
28	3	5	Y	4	A	35	5Y	4Y	4A	3A
29	3	5	Y	4	B	35	5Y	4Y	4B	3B
30	3	5	Z	2	B	35	5Z	2Z	2B	3B
31	3	5	Z	2	Y	35	5Z	2Z	2Y	3Y

C3C2P.DBF

Record#	V1	V2	V3	V4	V5	E1	E2	E3	E4	E5
1										
2	A	X	1	B	3	AX	1X	1B	3B	3A
3	A	X	1	B	4	AX	1X	1B	4B	4A
4	A	X	1	C	4	AX	1X	1C	4C	4A
5	A	X	1	C	5	AX	1X	1C	5C	5A
6	A	X	1	Y	3	AX	1X	1Y	3Y	3A
7	A	X	1	Y	4	AX	1X	1Y	4Y	4A
8	A	X	1	Y	5	AX	1X	1Y	5Y	5A
9	A	X	2	B	1	AX	2X	2B	1B	1A
10	A	X	2	B	3	AX	2X	2B	3B	3A
11	A	X	2	B	4	AX	2X	2B	4B	4A

12	A	X	2	C	1	AX	2X	2C	1C	1A
13	A	X	2	C	4	AX	2X	2C	4C	4A
14	A	X	2	C	5	AX	2X	2C	5C	5A
15	A	X	2	Y	1	AX	2X	2Y	1Y	1A
16	A	X	2	Y	3	AX	2X	2Y	3Y	3A
17	A	X	2	Y	4	AX	2X	2Y	4Y	4A
18	A	X	2	Y	5	AX	2X	2Y	5Y	5A
19	A	X	2	Z	3	AX	2X	2Z	3Z	3A
20	A	X	2	Z	5	AX	2X	2Z	5Z	5A
21	A	X	4	B	1	AX	4X	4B	1B	1A
22	A	X	4	B	3	AX	4X	4B	3B	3A
23	A	X	4	C	1	AX	4X	4C	1C	1A
24	A	X	4	C	5	AX	4X	4C	5C	5A
25	A	X	4	Y	1	AX	4X	4Y	1Y	1A
26	A	X	4	Y	3	AX	4X	4Y	3Y	3A
27	A	X	4	Y	5	AX	4X	4Y	5Y	5A
28	A	X	5	C	1	AX	5X	5C	1C	1A
29	A	X	5	C	4	AX	5X	5C	4C	4A
30	A	X	5	Y	1	AX	5X	5Y	1Y	1A
31	A	X	5	Y	3	AX	5X	5Y	3Y	3A
32	A	X	5	Y	4	AX	5X	5Y	4Y	4A
33	A	X	5	Z	3	AX	5X	5Z	3Z	3A
34	A	Z	2	B	1	AZ	2Z	2B	1B	1A
35	A	Z	2	B	3	AZ	2Z	2B	3B	3A
36	A	Z	2	B	4	AZ	2Z	2B	4B	4A
37	A	Z	2	C	1	AZ	2Z	2C	1C	1A
38	A	Z	2	C	4	AZ	2Z	2C	4C	4A
39	A	Z	2	C	5	AZ	2Z	2C	5C	5A
40	A	Z	2	X	1	AZ	2Z	2X	1X	1A
41	A	Z	2	X	4	AZ	2Z	2X	4X	4A
42	A	Z	2	X	5	AZ	2Z	2X	5X	5A
43	A	Z	2	Y	1	AZ	2Z	2Y	1Y	1A
44	A	Z	2	Y	3	AZ	2Z	2Y	3Y	3A
45	A	Z	2	Y	4	AZ	2Z	2Y	4Y	4A
46	A	Z	2	Y	5	AZ	2Z	2Y	5Y	5A
47	A	Z	3	B	1	AZ	3Z	3B	1B	1A
48	A	Z	3	B	4	AZ	3Z	3B	4B	4A
49	A	Z	3	Y	1	AZ	3Z	3Y	1Y	1A
50	A	Z	3	Y	4	AZ	3Z	3Y	4Y	4A
51	A	Z	3	Y	5	AZ	3Z	3Y	5Y	5A
52	A	Z	5	C	1	AZ	5Z	5C	1C	1A
53	A	Z	5	C	4	AZ	5Z	5C	4C	4A
54	A	Z	5	X	1	AZ	5Z	5X	1X	1A
55	A	Z	5	X	4	AZ	5Z	5X	4X	4A
56	A	Z	5	Y	1	AZ	5Z	5Y	1Y	1A
57	A	Z	5	Y	3	AZ	5Z	5Y	3Y	3A
58	A	Z	5	Y	4	AZ	5Z	5Y	4Y	4A
59	B	X	1	A	3	BX	1X	1A	3A	3B
60	B	X	1	A	4	BX	1X	1A	4A	4B
61	B	X	1	C	2	BX	1X	1C	2C	2B
62	B	X	1	C	4	BX	1X	1C	4C	4B
63	B	X	1	Y	2	BX	1X	1Y	2Y	2B
64	B	X	1	Y	3	BX	1X	1Y	3Y	3B
65	B	X	1	Y	4	BX	1X	1Y	4Y	4B
66	B	X	2	C	1	BX	2X	2C	1C	1B
67	B	X	2	C	4	BX	2X	2C	4C	4B
68	B	X	2	Y	1	BX	2X	2Y	1Y	1B

69	B	X	2	Y	3	BX	2X	2Y	3Y	3B
70	B	X	2	Y	4	BX	2X	2Y	4Y	4B
71	B	X	2	Z	3	BX	2X	2Z	3Z	3B
72	B	X	4	A	1	BX	4X	4A	1A	1B
73	B	X	4	A	3	BX	4X	4A	3A	3B
74	B	X	4	C	1	BX	4X	4C	1C	1B
75	B	X	4	C	2	BX	4X	4C	2C	2B
76	B	X	4	Y	1	BX	4X	4Y	1Y	1B
77	B	X	4	Y	2	BX	4X	4Y	2Y	2B
78	B	X	4	Y	3	BX	4X	4Y	3Y	3B
79	B	X	5	A	1	BX	5X	5A	1A	1B
80	B	X	5	A	3	BX	5X	5A	3A	3B
81	B	X	5	A	4	BX	5X	5A	4A	4B
82	B	X	5	C	1	BX	5X	5C	1C	1B
83	B	X	5	C	2	BX	5X	5C	2C	2B
84	B	X	5	C	4	BX	5X	5C	4C	4B
85	B	X	5	Y	1	BX	5X	5Y	1Y	1B
86	B	X	5	Y	2	BX	5X	5Y	2Y	2B
87	B	X	5	Y	3	BX	5X	5Y	3Y	3B
88	B	X	5	Y	4	BX	5X	5Y	4Y	4B
89	B	X	5	Z	2	BX	5X	5Z	2Z	2B
90	B	X	5	Z	3	BX	5X	5Z	3Z	3B
91	B	Z	2	C	1	BZ	2Z	2C	1C	1B
92	B	Z	2	C	4	BZ	2Z	2C	4C	4B
93	B	Z	2	X	1	BZ	2Z	2X	1X	1B
94	B	Z	2	X	4	BZ	2Z	2X	4X	4B
95	B	Z	2	Y	1	BZ	2Z	2Y	1Y	1B
96	B	Z	2	Y	3	BZ	2Z	2Y	3Y	3B
97	B	Z	2	Y	4	BZ	2Z	2Y	4Y	4B
98	B	Z	3	A	1	BZ	3Z	3A	1A	1B
99	B	Z	3	A	4	BZ	3Z	3A	4A	4B
100	B	Z	3	Y	1	BZ	3Z	3Y	1Y	1B
101	B	Z	3	Y	2	BZ	3Z	3Y	2Y	2B
102	B	Z	3	Y	4	BZ	3Z	3Y	4Y	4B
103	B	Z	5	A	1	BZ	5Z	5A	1A	1B
104	B	Z	5	A	3	BZ	5Z	5A	3A	3B
105	B	Z	5	A	4	BZ	5Z	5A	4A	4B
106	B	Z	5	C	1	BZ	5Z	5C	1C	1B
107	B	Z	5	C	2	BZ	5Z	5C	2C	2B
108	B	Z	5	C	4	BZ	5Z	5C	4C	4B
109	B	Z	5	X	1	BZ	5Z	5X	1X	1B
110	B	Z	5	X	2	BZ	5Z	5X	2X	2B
111	B	Z	5	X	4	BZ	5Z	5X	4X	4B
112	B	Z	5	Y	1	BZ	5Z	5Y	1Y	1B
113	B	Z	5	Y	2	BZ	5Z	5Y	2Y	2B
114	B	Z	5	Y	3	BZ	5Z	5Y	3Y	3B
115	B	Z	5	Y	4	BZ	5Z	5Y	4Y	4B
116	C	Y	1	A	4	CY	1Y	1A	4A	4C
117	C	Y	1	A	5	CY	1Y	1A	5A	5C
118	C	Y	1	B	2	CY	1Y	1B	2B	2C
119	C	Y	1	B	4	CY	1Y	1B	4B	4C
120	C	Y	1	X	2	CY	1Y	1X	2X	2C
121	C	Y	1	X	4	CY	1Y	1X	4X	4C
122	C	Y	1	X	5	CY	1Y	1X	5X	5C
123	C	Y	2	B	1	CY	2Y	2B	1B	1C
124	C	Y	2	B	4	CY	2Y	2B	4B	4C
125	C	Y	2	X	1	CY	2Y	2X	1X	1C

126	C	Y	2	X	4	CY	2Y	2X	4X	4C
127	C	Y	2	X	5	CY	2Y	2X	5X	5C
128	C	Y	2	Z	5	CY	2Y	2Z	5Z	5C
129	C	Y	3	A	1	CY	3Y	3A	1A	1C
130	C	Y	3	A	4	CY	3Y	3A	4A	4C
131	C	Y	3	A	5	CY	3Y	3A	5A	5C
132	C	Y	3	B	1	CY	3Y	3B	1B	1C
133	C	Y	3	B	2	CY	3Y	3B	2B	2C
134	C	Y	3	B	4	CY	3Y	3B	4B	4C
135	C	Y	3	Z	2	CY	3Y	3Z	2Z	2C
136	C	Y	3	Z	5	CY	3Y	3Z	5Z	5C
137	C	Y	4	A	1	CY	4Y	4A	1A	1C
138	C	Y	4	A	5	CY	4Y	4A	5A	5C
139	C	Y	4	B	1	CY	4Y	4B	1B	1C
140	C	Y	4	B	2	CY	4Y	4B	2B	2C
141	C	Y	4	X	1	CY	4Y	4X	1X	1C
142	C	Y	4	X	2	CY	4Y	4X	2X	2C
143	C	Y	4	X	5	CY	4Y	4X	5X	5C
144	C	Y	5	A	1	CY	5Y	5A	1A	1C
145	C	Y	5	A	4	CY	5Y	5A	4A	4C
146	C	Y	5	X	1	CY	5Y	5X	1X	1C
147	C	Y	5	X	2	CY	5Y	5X	2X	2C
148	C	Y	5	X	4	CY	5Y	5X	4X	4C
149	C	Y	5	Z	2	CY	5Y	5Z	2Z	2C
150	C	Z	2	B	1	CZ	2Z	2B	1B	1C
151	C	Z	2	B	4	CZ	2Z	2B	4B	4C
152	C	Z	2	X	1	CZ	2Z	2X	1X	1C
153	C	Z	2	X	4	CZ	2Z	2X	4X	4C
154	C	Z	2	X	5	CZ	2Z	2X	5X	5C
155	C	Z	2	Y	1	CZ	2Z	2Y	1Y	1C
156	C	Z	2	Y	4	CZ	2Z	2Y	4Y	4C
157	C	Z	2	Y	5	CZ	2Z	2Y	5Y	5C
158	C	Z	3	A	1	CZ	3Z	3A	1A	1C
159	C	Z	3	A	4	CZ	3Z	3A	4A	4C
160	C	Z	3	A	5	CZ	3Z	3A	5A	5C
161	C	Z	3	B	1	CZ	3Z	3B	1B	1C
162	C	Z	3	B	2	CZ	3Z	3B	2B	2C
163	C	Z	3	B	4	CZ	3Z	3B	4B	4C
164	C	Z	3	Y	1	CZ	3Z	3Y	1Y	1C
165	C	Z	3	Y	2	CZ	3Z	3Y	2Y	2C
166	C	Z	3	Y	4	CZ	3Z	3Y	4Y	4C
167	C	Z	3	Y	5	CZ	3Z	3Y	5Y	5C
168	C	Z	5	A	1	CZ	5Z	5A	1A	1C
169	C	Z	5	A	4	CZ	5Z	5A	4A	4C
170	C	Z	5	X	1	CZ	5Z	5X	1X	1C
171	C	Z	5	X	2	CZ	5Z	5X	2X	2C
172	C	Z	5	X	4	CZ	5Z	5X	4X	4C
173	C	Z	5	Y	1	CZ	5Z	5Y	1Y	1C
174	C	Z	5	Y	2	CZ	5Z	5Y	2Y	2C
175	C	Z	5	Y	4	CZ	5Z	5Y	4Y	4C

C3USDEDG.DBF

Record#	E13	E1A	E1B	E1C	E1X	E1Y	E24	E2B	E2C	E2X	E2Y	E2Z	E35	E3A	E3B	E3Y	E3Z
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

E4A	E4B	E4C	E4X	E4Y	E5A	E5C	E5X	E5Y	E5Z	EAX	EAZ	EBX	EBZ	ECY	ECZ
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

USEDNODE.DBF (used in all three cases)

Record#	N31	N32	N33	N34	N35	N3A	N3B	N3C	N3X	N3Y	N3Z	N41	N42	N43	N44	N45
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

N4A	N4B	N4C	N4X	N4Y	N4Z	N51	N52	N53	N54	N55	N5A	N5B	N5C	N5X	N5Y	N5Z
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

OUTPUT FROM OBRWLFCH.PRG - CASE 1

1									
2									
25A3X	4CY	1BZ	24B3Z		1CX		35Y	4AX	10:20:01
25A3X	1BZ	4CY	24B3Z		1CX		35Y	4AX	10:21:43
25A3X	4CY	1BZ		24B	1CX	35C2Z		4AX	10:48:44
25A3X	1BZ	4CY		24B	1CX	35C2Z		4AX	10:51:02
25A3X	4CY	1BZ		24B	1CX		35Y	4AX	10:57:06
25A3X	1BZ	4CY		24B	1CX		35Y	4AX	10:59:58
25A3X	4CY	1BZ		24B	1CX		35Z	4AX	11:03:32
25A3X	1BZ	4CY		24B	1CX		35Z	4AX	11:06:59
3									
25A3Z	1CX	4BY		24C	1AZ	35Y1B		4AX	12:12:41
25A3Z	4BY	1CX		24C	1AZ	35Y1B		4AX	12:13:50
25A3Z	1CX	4BY		24C	1AZ		35Y	4AX	12:18:25
25A3Z	1CX	4BY		24C	1BZ		35Y	4AX	12:19:41
25A3Z	4BY	1CX		24C	1AZ		35Y	4AX	12:20:41
25A3Z	4BY	1CX		24C	1BZ		35Y	4AX	12:22:04
4									
25A4X	1CY	3BZ	24C5Z		3AX		35Y	1AZ	12:49:43
25A4X	3BZ	1CY	24C5Z		3AX		35Y	1AZ	12:50:57
25A4X	1CY	3BZ		24B	3AX	35C4Y		1AZ	13:18:41
25A4X	3BZ	1CY		24B	3AX	35C4Y		1AZ	13:20:43
25A4X	1CY	3BZ		24B	3AX		35Y	1AZ	13:28:48
25A4X	3BZ	1CY		24B	3AX		35Y	1AZ	13:30:57
25A4X	1CY	3BZ		24C	3AX		35Y	1AZ	13:55:23
25A4X	3BZ	1CY		24C	3AX		35Y	1AZ	13:57:31
5									
25C1X	4BY	3AZ		24C	1BZ	35Z2B		4AX	14:42:42
25C1X	3AZ	4BY		24C	1BZ	35Z2B		4AX	14:44:10
25C1X	4BY	3AZ		24C	1BZ		35Y	4AX	14:47:23
25C1X	3AZ	4BY		24C	1BZ		35Y	4AX	14:51:06
6									
25C4X	3BY	1AZ	24Y1B		3AX		35Z	1CX	15:22:04
25C4X	1AZ	3BY	24Y1B		3AX		35Z	1CX	15:22:32
25C4X	3BY	1AZ		24B	3AX		35Z	1CX	15:57:45
25C4X	3BY	1AZ		24B	3AX		35Z	1CY	15:58:38
25C4X	1AZ	3BY		24B	3AX		35Z	1CX	16:02:31
25C4X	1AZ	3BY		24B	3AX		35Z	1CY	16:03:25
7									
8									
25Y1C	3BZ	4AX		24B	1AZ		35A	4CY	17:36:28
25Y1C	4AX	3BZ		24B	1AZ		35A	4CY	17:38:00
9									
25Y3B	4CX	1AZ	24A3X		1CY	35C2Z		4BY	17:54:07
25Y3B	1AZ	4CX	24A3X		1CY	35C2Z		4BY	17:55:43
25Y3B	4CX	1AZ	24A3X		1CY		35Z	4BY	17:58:09
25Y3B	1AZ	4CX	24A3X		1CY		35Z	4BY	18:00:11
25Y3B	4CX	1AZ	24A3Z		1CY	35C2X		4BY	18:03:06
25Y3B	1AZ	4CX	24A3Z		1CY	35C2X		4BY	18:04:21
25Y3B	4CX	1AZ	24A5Z		1CY	35C2X		4BY	18:08:48
25Y3B	1AZ	4CX	24A5Z		1CY	35C2X		4BY	18:10:03
10									
25Y4B	1CX	3AZ	24A5Z		3BY	35C2X		1BZ	19:20:25

25Y4B	3AZ	1CX	24A5Z		3BY	35C2X		1BZ	19:21:46
25Y4B	1CX	3AZ	24A5Z		3BY	35C4X		1BZ	19:23:51
25Y4B	3AZ	1CX	24A5Z		3BY	35C4X		1BZ	19:25:13
25Y4B	1CX	3AZ	24C5Z		3BY	35A4X		1BZ	19:37:52
25Y4B	3AZ	1CX	24C5Z		3BY	35A4X		1BZ	19:38:33
25Y4B	1CX	3AZ		24C	3BY	35A4X		1BZ	19:50:27
25Y4B	3AZ	1CX		24C	3BY	35A4X		1BZ	19:52:13
11									
25Y4C	1BZ	3AX	24A1X		3BY		35Z	1CY	20:31:05
25Y4C	3AX	1BZ	24A1X		3BY		35Z	1CY	20:31:33
25Y4C	1BZ	3AX		24X	3BY		35Z	1CX	21:53:26
25Y4C	1BZ	3AX		24X	3BY		35Z	1CY	21:54:08
25Y4C	3AX	1BZ		24X	3BY		35Z	1CX	21:55:16
25Y4C	3AX	1BZ		24X	3BY		35Z	1CY	21:55:56
12									
25Z1B	4CY	3AX		24X	3BZ		35Y	1CX	22:56:48
25Z1B	3AX	4CY		24X	3BZ		35Y	1CX	22:58:28
13									
25Z1C	4BY	3AX	24A1X		3BZ		35Y	4CX	23:02:44
25Z1C	3AX	4BY	24A1X		3BZ		35Y	4CX	23:03:12
25Z1C	3BY	4AX	24B1X		5CY		35A	2BZ	23:11:32
25Z1C	4AX	3BY	24B1X		5CY		35A	2BZ	23:12:00
25Z1C	3BY	4AX		24B	3AZ	35A1X		4CY	23:27:07
25Z1C	4AX	3BY		24B	3AZ	35A1X		4CY	23:28:18
25Z1C	4BY	3AX		24X	5CY	35A1Y		2BZ	23:36:37
25Z1C	3AX	4BY		24X	5CY	35A1Y		2BZ	23:37:53
14									
25Z3B	1CY	4AX		24C	1BZ	35A1X		4BY	00:31:04
25Z3B	4AX	1CY		24C	1BZ	35A1X		4BY	00:31:58
25Z3B	1CY	4AX		24C	1AZ		35A	4BY	00:40:17
25Z3B	1CY	4AX		24C	1BZ		35A	4BY	00:41:47
25Z3B	4AX	1CY		24C	1AZ		35A	4BY	00:44:03
25Z3B	4AX	1CY		24C	1BZ		35A	4BY	00:45:06
15									
16									
BZ1Y4	25C	3AX	24A1X		3BY		35Z	4CX	01:16:27
BZ3Y4	25C	1AX	24A3X		1CY	35Z1B		4CX	01:32:22
BZ3Y4	25C	1AX	24A3X		1BY	35Z2B		4CX	01:34:21
BZ3Y4	25C	1AX	24A3X		1CY	35Z2B		4CX	01:38:10
AZ3X1	25C	4BY	24A5Z		1CY		35Y	1BZ	03:16:51
AX1Z3	25C	4BY	24A5Z		1CY		35Y	4CX	03:17:36
AX3Z1	25C	4BY	24A5Z		1CY		35Y	4CX	03:17:46
AZ1X3	25C	4BY	24A5Z		1CY		35Y	4CX	03:17:56
AZ3X1	25C	4BY	24A5Z		1CY		35Y	4CX	03:18:06
BY1Z3	25C	4AX	24B1X		5AZ		35Y	2BZ	03:32:19
BY3Z1	25C	4AX	24B3X		5AZ	35Y1A		2BZ	03:51:30
BZ3Y1	25C	4AX	24B3X		1AZ		35A	1CX	03:58:51
BZ3Y1	25C	4AX	24B3X		1AZ		35A	4CY	04:00:18
BY3Z1	25C	4AX	24B3X		1CY		35A	2BZ	04:01:46
BY4X1	25C	3AZ	24B3X		1CY		35Y	2BZ	04:14:25
AX3Y4	25C	1BZ	24B3Z		1CY		35A	4CX	05:02:46
BY3Z1	25C	4AX	24C1X		5AZ	35Y1A		2BZ	05:32:18
BY1Z3	25C	4AX	24C1X		5AZ		35Y	2BZ	05:57:58
AX3Z1	25C	4BY	24C1X		5AZ		35Y	2BZ	06:08:05
BZ1Y3	25C	4AX		24B	3AZ	35A1X		4CY	07:48:34
AX3Y4	25C	1BZ		24B	5AZ	35Y1A		4CX	08:35:32
BZ3Y1	25C	4AX		24B	1CX	35Z1A		4CY	08:46:37

AZ3X4	25C	1BY		24B	1CX	35Z1A		4CY	08:48:10
BZ3Y1	25C	4AX		24B	1AZ		35A	1CX	09:07:04
BZ3Y1	25C	4AX		24B	1AZ		35A	4CY	09:09:09
BY1Z3	25C	4AX		24B	1CX		35A	4CY	09:11:53
BY3Z1	25C	4AX		24B	1CX		35A	4CY	09:12:30
BZ1Y3	25C	4AX		24B	1CX		35A	4CY	09:13:11
BZ3Y1	25C	4AX		24B	1CX		35A	4CY	09:13:50
AZ3X4	25C	1BY		24B	1AX		35A	4CY	09:16:31
AZ3X4	25C	1BY		24B	1CX		35A	4CY	09:20:05
AX3Y4	25C	1BZ		24B	1CY		35A	4CX	09:23:55
AX1Y4	25C	3BZ		24B	1AZ		35A	4CX	09:35:50
BY1Z3	25C	4AX		24B	5AZ		35Y	1CX	10:03:00
AX1Y4	25C	3BZ		24B	1AZ		35Y	4CX	10:27:41
AX4Y1	25C	3BZ		24B	5AZ		35Y	1CX	10:31:41
AX1Y4	25C	3BZ		24B	5AZ		35Y	4CX	10:33:08
BZ1Y3	25C	4AX		24B	1CX		35Z	4CY	10:54:35
AX3Y4	25C	1BZ		24B	1CY		35Z	4CX	11:08:49
AZ1X4	25C	3BY		24B	3AX		35Z	1CY	11:15:19
AZ1X4	25C	3BY		24B	3AX		35Z	4CY	11:17:24
AX3Y4	25C	1BZ		24X	3AZ	35Z2B		1CX	12:50:56
AX3Y4	25C	1BZ		24X	3AZ	35Z2B		1CY	12:52:06
AZ3X1	25C	4BY		24X	1BZ	35Z2B		1CY	12:58:27
AZ3X1	25C	4BY		24X	1BZ		35A	1CY	13:39:06
AZ3X1	25C	4BY		24X	1CY		35A	1BZ	13:41:50
AX3Z1	25C	4BY		24X	1CY		35A	2BZ	13:43:11
AZ3X1	25C	4BY		24X	1CY		35A	2BZ	13:43:58
BZ1Y4	25C	3AX		24X	5AZ		35Y	1CX	13:57:10
AX1Z3	25C	4BY		24X	5AZ		35Y	2BZ	14:28:52
AX3Z1	25C	4BY		24X	5AZ		35Y	2BZ	14:29:43
AX3Z1	25C	4BY		24X	5AZ		35Y	1CX	14:31:02
AZ3X1	25C	4BY		24X	1CY		35Y	1BZ	14:36:12
AX1Z3	25C	4BY		24X	1CY		35Y	2BZ	14:37:31
AX3Z1	25C	4BY		24X	1CY		35Y	2BZ	14:38:17
AZ1X3	25C	4BY		24X	1CY		35Y	2BZ	14:39:01
AZ3X1	25C	4BY		24X	1CY		35Y	2BZ	14:39:46
BZ1Y4	25C	3AX		24X	3BY		35Z	1CX	14:58:46
17									
CY4B1	25Z	3AX	24A1X		3BZ		35Y	4CX	15:37:56
BY1C4	25Z	3AX	24X1B		5CY	35A1Z		2CX	17:53:32
BY1C4	25Z	3AX	24X1B		5CY	35A4Y		2CX	17:56:29
AX3B4	25Z	1CY	24X1B		3AZ		35Y	2CX	18:06:09
CY1B4	25Z	3AX	24X1C		3BZ		35Y	1AZ	18:18:32
CX3A4	25Z	1BY	24X1C		3BZ		35Y	1AZ	18:19:43
CY3B4	25Z	1AX	24Y1C		3AZ	35A4X		1BZ	19:32:36
CY3B4	25Z	1AX	24Y5C		3AZ	35A4X		1BY	20:32:18
CY3B4	25Z	1AX	24Y5C		3AZ	35A4X		1BZ	20:33:13
CX1A4	25Z	3BY	24Y5C		1BZ		35A	1CY	21:04:47
AX3B4	25Z	1CY	24Y5C		1AZ		35A	4CX	21:08:47
AX3B4	25Z	1CY	24Y5C		1BZ		35A	4CX	21:09:33
BY4C1	25Z	3AX		24B	5CY	35A1Y		2CX	21:21:16
BY4C1	25Z	3AX		24B	5CY	35A1Z		2CX	21:29:52
AX1C4	25Z	3BY		24B	5CY	35A1Z		2CX	21:42:29
BY1A3	25Z	4CX		24B	5CY	35A4Y		1BZ	22:34:51
AX1C4	25Z	3BY		24B	3AZ	35C2X		1BZ	22:54:14
AX4C1	25Z	3BY		24B	3AZ	35C2X		1BZ	22:55:03
CX1A4	25Z	3BY		24B	3AZ	35C2X		1BZ	22:56:08
CX4A1	25Z	3BY		24B	3AZ	35C2X		1BZ	22:56:56

CX1A4	25Z	3BY	24B	1CY	35C2X		1BZ	22:59:53
CY3B1	25Z	4AX	24B	1AZ		35A	2CX	00:04:06
AX1C4	25Z	3BY	24B	1AZ		35A	2CX	00:25:07
CX1A4	25Z	3BY	24B	1CY		35A	1BZ	00:28:50
AX1C4	25Z	3BY	24B	5CY		35A	1BZ	00:30:45
AX4C1	25Z	3BY	24B	5CY		35A	1BZ	00:31:22
CX1A4	25Z	3BY	24B	5CY		35A	1BZ	00:32:14
CX4A1	25Z	3BY	24B	5CY		35A	1BZ	00:32:50
AX1C4	25Z	3BY	24B	5CY		35A	2CX	00:34:05
AX4C1	25Z	3BY	24B	5CY		35A	2CX	00:34:42
BY4C1	25Z	3AX	24B	1AZ		35Y	2CX	00:57:46
AX1B3	25Z	4CY	24B	1AZ		35Y	2CX	01:43:46
AX3B1	25Z	4CY	24B	3AZ		35Y	1CX	01:48:40
AX3B1	25Z	4CY	24B	3AZ		35Y	2CX	01:50:09
CY3B1	25Z	4AX	24C	3AZ	35A1X		4BY	01:54:12
CY3B1	25Z	4AX	24C	1AZ		35A	4BY	03:24:38
CY4B1	25Z	3AX	24C	3BZ		35Y	1AZ	04:00:39
CX3A1	25Z	4BY	24C	1BZ		35Y	4AX	04:26:12
CX3A1	25Z	4BY	24C	3BZ		35Y	4AX	04:29:47
AX3B4	25Z	1CY	24C	3AZ		35Y	1BZ	04:41:55
CY1B4	25Z	3AX	24X	1AZ	35A4Y		1CX	05:06:21
CY1B4	25Z	3AX	24X	3BZ	35A4Y		1CX	05:09:22
BY1C4	25Z	3AX	24X	5CY	35A4Y		1BZ	05:11:23
AX3B4	25Z	1CY	24X	3AZ	35C4Y		1BZ	05:37:13
CX1A4	25Z	3BY	24X	1BZ		35A	1CY	06:12:38
CX1A4	25Z	3BY	24X	1CY		35A	1BZ	06:14:31
AX1C4	25Z	3BY	24X	5CY		35A	1BZ	06:15:38
CX1A4	25Z	3BY	24X	5CY		35A	1BZ	06:16:30
CY1B4	25Z	3AX	24X	1AZ		35Y	1CX	06:44:25
BY1C4	25Z	3AX	24X	3BZ		35Y	1AZ	06:47:49
BY4C1	25Z	3AX	24X	3BZ		35Y	1AZ	06:48:38
CY1B4	25Z	3AX	24X	3BZ		35Y	1AZ	06:49:41
CY4B1	25Z	3AX	24X	3BZ		35Y	1AZ	06:50:31
CY1B4	25Z	3AX	24X	3BZ		35Y	1CX	06:51:51
CX3A4	25Z	1BY	24X	3BZ		35Y	1AX	06:57:13
CX3A4	25Z	1BY	24X	3BZ		35Y	1AZ	06:58:46
AX3B4	25Z	1CY	24X	3AZ		35Y	1BZ	07:13:12
AX3B1	25Z	4CY	24X	3AZ		35Y	1CX	07:17:54

18

NO SOLUTION FOUND - RESULTS OF PIOTROWSKI CONFIRMED FOR CASE 1.

```
. DISPLAY MEMORY
STIME      priv C "09:33:31"                        A:obrwlfch.prg
COUNT1     priv N      18 (       18.00000000)  A:obrwlfch.prg
COUNT2     priv N     233 (      233.00000000)  A:obrwlfch.prg
COUNT3     priv N    1758 (     1758.00000000)  A:obrwlfch.prg
COUNT4     priv N    4734 (     4734.00000000)  A:obrwlfch.prg
COUNT5     priv N    7865 (     7865.00000000)  A:obrwlfch.prg
COUNT6     priv N    8424 (     8424.00000000)  A:obrwlfch.prg
COUNT7     priv N    4125 (     4125.00000000)  A:obrwlfch.prg
COUNT8     priv N     188 (      188.00000000)  A:obrwlfch.prg
COUNT9     priv N       0 (        0.00000000)  A:obrwlfch.prg
FOUND      pub  L .F.
PENT       pub  C "C2P"
TRI        pub  C "C1T3"
FACTOR     pub  C "3"
TRITYPE    pub  C "T"
```

```
P3          pub  L  .F.
P4          pub  L  .F.
P5          pub  L  .F.
T3          pub  L  .F.
T4          pub  L  .F.
T5          pub  L  .F.
D3          pub  L  .F.
D4          pub  L  .F.
D5          pub  L  .F.
P3CHOSEN    priv  N      0  (      0.00000000)    A:obrwlfch.prg
P4CHOSEN    priv  N      0  (      0.00000000)    A:obrwlfch.prg
P5CHOSEN    priv  N      0  (      0.00000000)    A:obrwlfch.prg
D3RECNO     priv  N     18  (     18.00000000)    A:obrwlfch.prg
D4RECNO     priv  N     12  (     12.00000000)    A:obrwlfch.prg
D5RECNO     priv  N     14  (     14.00000000)    A:obrwlfch.prg
F3RECNO     priv  N     20  (     20.00000000)    A:obrwlfch.prg
   31 variables defined,     191 bytes used
  225 variables available,  5809 bytes available
```

OUTPUT FROM OBRWLFCH.PRG - CASE 2

1									
2									
3									
4									
5									
6									
13B2X	5CZ	4AY	24B1C		3AZ		35Y	4CX	12:43:44
13B2X	4AY	5CZ	24B1C		3AZ		35Y	4CX	12:44:05
13B2X	5CZ	4AY	24B1Y		3AZ		35Y	4CX	12:45:38
13B2X	4AY	5CZ	24B1Y		3AZ		35Y	4CX	12:46:20
7									
13B2Y	5CZ	4AX	24B5X		3AZ		35Y	1CX	13:24:20
13B2Y	4AX	5CZ	24B5X		3AZ		35Y	1CX	13:25:05
13B2Y	5CZ	4AX		24C	3AZ	35B4Y		1CX	13:44:28
13B2Y	4AX	5CZ		24C	3AZ	35B4Y		1CX	13:45:47
13B2Y	5CZ	4AX		24C	3AZ		35Y	1CX	13:48:48
13B2Y	4AX	5CZ		24C	3AZ		35Y	1CX	13:50:35
8									
9									
10									
11									
12									
13									
13B5X	2CZ	4AY	24B1Y		3AZ		35Y	4CX	17:07:46
13B5X	4AY	2CZ	24B1Y		3AZ		35Y	4CX	17:08:28
13B5X	2CZ	4AY		24B	3AZ		35Y	4CX	17:19:01
13B5X	4AY	2CZ		24B	3AZ		35Y	4CX	17:20:21
14									
13B5Y	2CZ	4AX	24C5X		3AY		35Z	1CX	17:44:44
13B5Y	4AX	2CZ	24C5X		3AY		35Z	1CX	17:45:08
13B5Y	2CZ	4AX		24B	3AY		35Z	1CX	17:58:24
13B5Y	4AX	2CZ		24B	3AY		35Z	1CX	18:00:37
15									
13Y2B	5CZ	4AX	24C1X		3AZ		35B	1AY	18:25:44
13Y2B	4AX	5CZ	24C1X		3AZ		35B	1AY	18:26:34
13Y2B	5CZ	4AX	24Y5X		3AZ		35B	1AY	18:46:07
13Y2B	5CZ	4AX	24Y5X		3AZ		35B	1CX	18:46:16
13Y2B	4AX	5CZ	24Y5X		3AZ		35B	1AY	18:47:04
13Y2B	4AX	5CZ	24Y5X		3AZ		35B	1CX	18:47:13
13Y2B	5CZ	4AX		24C	1AY	35Y4B		1CX	18:54:16
13Y2B	5CZ	4AX		24C	3AZ	35Y4B		1CX	18:55:06
13Y2B	4AX	5CZ		24C	1AY	35Y4B		1CX	18:56:42
13Y2B	4AX	5CZ		24C	3AZ	35Y4B		1CX	18:57:34
13Y2B	5CZ	4AX		24C	1AY		35B	1CX	18:58:31
13Y2B	5CZ	4AX		24C	3AZ		35B	1AY	18:59:20
13Y2B	5CZ	4AX		24C	3AZ		35B	1CX	19:00:05
13Y2B	4AX	5CZ		24C	1AY		35B	1CX	19:01:52
13Y2B	4AX	5CZ		24C	3AZ		35B	1AY	19:02:47
13Y2B	4AX	5CZ		24C	3AZ		35B	1CX	19:03:36
16									
13Y2C	5BZ	4AX	24Y5X		3AZ	35C4B		1AY	19:36:09
13Y2C	4AX	5BZ	24Y5X		3AZ	35C4B		1AY	19:36:30
17									
18									

19									
20									
21									
22									
13Y5B	2CZ	4AX	24C5X		1AY		35Z	1CX	00:02:58
13Y5B	4AX	2CZ	24C5X		1AY		35Z	1CX	00:03:18
13Y5B	2CZ	4AX		24B	1AY		35Z	1CX	00:17:20
13Y5B	2CZ	4AX		24B	1CX		35Z	1AY	00:18:13
13Y5B	2CZ	4AX		24B	5CX		35Z	1AY	00:18:50
13Y5B	4AX	2CZ		24B	1AY		35Z	1CX	00:20:29
13Y5B	4AX	2CZ		24B	1CX		35Z	1AY	00:21:26
13Y5B	4AX	2CZ		24B	5CX		35Z	1AY	00:22:17
13Y5B	2CZ	4AX		24Y	3AZ	35C4B		1AY	00:29:26
13Y5B	2CZ	4AX		24Y	1CX	35C4B		1AY	00:30:10
13Y5B	4AX	2CZ		24Y	3AZ	35C4B		1AY	00:31:21
13Y5B	4AX	2CZ		24Y	1CX	35C4B		1AY	00:32:05
13Y5B	2CZ	4AX		24Y	3AZ	35X2B		1AY	00:34:06
13Y5B	2CZ	4AX		24Y	1CX	35X2B		1AY	00:34:50
13Y5B	4AX	2CZ		24Y	3AZ	35X2B		1AY	00:36:13
13Y5B	4AX	2CZ		24Y	1CX	35X2B		1AY	00:36:56
13Y5B	2CZ	4AX		24Y	1CX		35Z	1AY	00:41:32
13Y5B	2CZ	4AX		24Y	5CX		35Z	1AY	00:42:13
13Y5B	4AX	2CZ		24Y	1CX		35Z	1AY	00:43:26
13Y5B	4AX	2CZ		24Y	5CX		35Z	1AY	00:44:13
13Y5C	2BZ	4AX	24B1X		3AZ		35B	1AY	00:48:37
13Y5C	4AX	2BZ	24B1X		3AZ		35B	1AY	00:49:02
13Y5C	2BZ	4AX	24B1Y		3AZ		35B	2CX	00:50:55
13Y5C	4AX	2BZ	24B1Y		3AZ		35B	2CX	00:51:28
13Y5C	2BZ	4AX		24C	3AZ		35B	1AY	01:05:39
13Y5C	4AX	2BZ		24C	3AZ		35B	1AY	01:06:56
13Y5C	2BZ	4AX		24Y	3AZ		35B	1AY	01:13:51
13Y5C	2BZ	4AX		24Y	3AZ		35B	2CX	01:14:32
13Y5C	4AX	2BZ		24Y	3AZ		35B	1AY	01:15:43
13Y5C	4AX	2BZ		24Y	3AZ		35B	2CX	01:16:22
24									
25									
13Z2B	5CX	4AY	24C1Y		5BZ		35Y	1AX	02:09:16
13Z2B	4AY	5CX	24C1Y		5BZ		35Y	1AX	02:09:37
13Z2B	5CX	4AY		24C	5BZ		35Y	1AX	02:34:45
13Z2B	4AY	5CX		24C	5BZ		35Y	1AX	02:35:39
13Z2B	5CX	4AY		24X	5BZ		35Y	1AX	02:40:29
13Z2B	4AY	5CX		24X	5BZ		35Y	1AX	02:41:42
26									
27									
28									
29									
30									
31									
32									
33									
34									
CX5Y4	13A	2BZ	24B1X		5CZ		35B	4AX	07:26:25
CX5Y4	13A	2BZ	24B1Y		5CZ		35B	4AX	07:39:35
BZ2Y5	13A	4CX		24B	5CZ	35X1B		4AY	11:42:41
35									
CZ5Y2	13B	4AX	24B5X		3AZ	35C1Y		2BZ	15:04:12
CZ5Y2	13B	4AX	24B5X		1AY	35C4Y		2BZ	15:10:30

CZ5Y2	13B	4AX	24B5X		3AZ	35C4Y		2BZ	15:11:16
CZ2Y5	13B	4AX	24B5X		1AY		35Z	1CX	15:43:30
CZ2Y5	13B	4AX	24B5X		3AY		35Z	1CX	15:44:17
CZ5X2	13B	4AY	24B5Y		1AX	35C1Y		2BZ	15:50:37
CZ5X2	13B	4AY	24B5Y		3AZ	35C1Y		2BZ	15:51:21
CZ2X5	13B	4AY	24B5Y		1AX		35Z	4CX	16:18:29
AY5X4	13B	2CZ	24C1Y		3AZ	35B4Y		1AX	17:02:31
CX2Z5	13B	4AY	24C1Y		3AZ		35Y	1AX	17:14:17
CX5Z2	13B	4AY	24C1Y		3AZ		35Y	1AX	17:14:24
CZ2X5	13B	4AY	24C1Y		3AZ		35Y	1AX	17:14:30
CZ5X2	13B	4AY	24C1Y		3AZ		35Y	1AX	17:14:38
CZ5X2	13B	4AY	24C1Y		3AZ		35Y	2BZ	17:15:18
CZ2X5	13B	4AY	24C1Y		5BZ		35Y	1AX	17:17:10
CX5Z2	13B	4AY	24C5B		3AZ		35Y	1AX	17:49:00
CZ5X2	13B	4AY	24C5B		3AZ		35Y	1AX	17:49:08
CZ5X2	13B	4AY	24C5B		3AZ		35Y	1CX	17:49:48
AX5Y4	13B	2CZ	24C5B		1AY		35Z	1CX	18:02:22
AX5Y4	13B	2CZ	24C5B		3AY		35Z	1CX	18:03:49
CZ5Y2	13B	4AX	24C5X		1AY	35B2Z		1CX	18:10:16
CZ5Y2	13B	4AX	24C5X		3AY	35B2Z		1CX	18:11:02
CZ5Y2	13B	4AX	24C5X		1AY	35B4Y		1CX	18:12:38
CZ5Y2	13B	4AX	24C5X		3AZ	35B4Y		1CX	18:14:01
AX2Y4	13B	5CZ	24X1C		3AZ		35Y	2BZ	19:02:44
CZ5X2	13B	4AY	24X1Y		3AZ		35Y	2BZ	19:11:24
CX2Z5	13B	4AY	24X5B		3AZ		35Y	1AX	19:30:21
AX2Y4	13B	5CZ	24X5B		1AY		35Y	1CX	19:34:47
AX2Y4	13B	5CZ	24X5B		3AZ		35Y	1CX	19:36:11
AY2X4	13B	5CZ	24Y1C		3AZ		35Y	1AX	20:11:44
AY2X4	13B	5CZ	24Y1C		3AZ		35Y	2BZ	20:12:49
AY5X4	13B	2CZ		24B	3AZ	35C4Y		1AX	21:25:58
CZ5Y2	13B	4AX		24B	1CX	35X2Z		1AY	21:33:39
CZ5X2	13B	4AY		24B	1AX	35Y2Z		4CX	21:49:33
CZ2X5	13B	4AY		24B	1AX		35Y	4CX	22:25:43
CZ5X2	13B	4AY		24B	1AX		35Y	4CX	22:26:26
CX2Z5	13B	4AY		24B	3AZ		35Y	1AX	22:28:15
CX5Z2	13B	4AY		24B	3AZ		35Y	1AX	22:28:54
CZ2X5	13B	4AY		24B	3AZ		35Y	1AX	22:29:31
CZ5X2	13B	4AY		24B	3AZ		35Y	1AX	22:30:10
CZ2X5	13B	4AY		24B	3AZ		35Y	1CX	22:31:18
CZ5X2	13B	4AY		24B	3AZ		35Y	1CX	22:31:58
CZ2X5	13B	4AY		24B	3AZ		35Y	4CX	22:33:41
CZ5X2	13B	4AY		24B	3AZ		35Y	4CX	22:34:21
AX2Y4	13B	5CZ		24B	1AY		35Y	1CX	22:56:16
AX2Y4	13B	5CZ		24B	1AY		35Y	4CX	22:57:46
AY2X4	13B	5CZ		24B	3AZ		35Y	1AX	22:58:59
AX2Y4	13B	5CZ		24B	3AZ		35Y	1CX	23:00:33
AY2X4	13B	5CZ		24B	3AZ		35Y	1CX	23:01:15
AX2Y4	13B	5CZ		24B	3AZ		35Y	4CX	23:02:47
CZ2Y5	13B	4AX		24B	1AY		35Z	1CX	23:04:56
CZ2Y5	13B	4AX		24B	1AY		35Z	2CX	23:06:15
CZ2Y5	13B	4AX		24B	3AY		35Z	1CX	23:07:39
CZ2Y5	13B	4AX		24B	3AY		35Z	2CX	23:08:57
CZ2Y5	13B	4AX		24B	1CX		35Z	1AY	23:10:21
CZ2X5	13B	4AY		24B	1AX		35Z	4CX	23:12:51
AZ2Y4	13B	5CX		24B	3AY		35Z	1AX	23:19:48
AX5Y4	13B	2CZ		24B	1AY		35Z	1CX	23:22:58
AX5Y4	13B	2CZ		24B	1AY		35Z	4CX	23:24:07

AX5Y4	13B	2CZ		24B	3AY		35Z	1CX	23:26:11
AX5Y4	13B	2CZ		24B	3AY		35Z	4CX	23:27:19
AX5Y4	13B	2CZ		24B	1CX		35Z	1AY	23:28:57
CX2Z5	13B	4AY		24C	3AZ	35B2Y		1AX	23:34:16
CZ2X5	13B	4AY		24C	3AZ	35B2Y		1AX	23:35:00
CZ2X5	13B	4AY		24C	3AZ	35B2Y		1CX	23:36:19
AX2Y4	13B	5CZ		24C	1AY	35B2Z		1CX	23:46:08
AX2Y4	13B	5CZ		24C	3AY	35B2Z		1CX	23:48:06
CZ2Y5	13B	4AX		24C	1AY	35B4Y		1CX	23:50:43
CZ2Y5	13B	4AX		24C	3AZ	35B4Y		1CX	23:52:07
AY2X4	13B	5CZ		24C	3AZ	35B4Y		1AX	23:54:29
AY2X4	13B	5CZ		24C	3AZ	35B4Y		1CX	23:55:47
AX2Y4	13B	5CZ		24C	3AZ	35X1Y		2BZ	00:02:00
AY2X4	13B	5CZ		24C	3AZ	35X1Y		2BZ	00:02:46
CX2Z5	13B	4AY		24C	3AZ		35Y	1AX	00:30:39
CZ2X5	13B	4AY		24C	3AZ		35Y	1AX	00:31:25
CZ2X5	13B	4AY		24C	3AZ		35Y	1CX	00:33:18
CZ2X5	13B	4AY		24C	5BZ		35Y	1AX	00:34:38
CZ2X5	13B	4AY		24C	5BZ		35Y	1CX	00:35:39
AZ2Y4	13B	5CX		24C	5BZ		35Y	1AX	00:39:45
AY2X4	13B	5CZ		24C	1AX		35Y	2BZ	00:41:18
AX2Y4	13B	5CZ		24C	1AY		35Y	2BZ	00:42:54
AX2Y4	13B	5CZ		24C	1AY		35Y	1CX	00:44:02
AY2X4	13B	5CZ		24C	3AZ		35Y	1AX	00:45:16
AX2Y4	13B	5CZ		24C	3AZ		35Y	2BZ	00:46:56
AY2X4	13B	5CZ		24C	3AZ		35Y	2BZ	00:47:42
AX2Y4	13B	5CZ		24C	3AZ		35Y	1CX	00:48:55
AY2X4	13B	5CZ		24C	3AZ		35Y	1CX	00:49:41
CZ2Y5	13B	4AX		24C	1AY		35Z	1CX	00:51:35
CZ2Y5	13B	4AX		24C	3AY		35Z	1CX	00:52:59
AZ2Y4	13B	5CX		24C	3AY		35Z	1AX	00:55:17
CX5Z2	13B	4AY		24X	3AZ	35B2Y		1AX	00:57:29
AX5Y4	13B	2CZ		24X	1AY	35B2Y		1CX	01:01:02
AX5Y4	13B	2CZ		24X	3AZ	35B2Y		1CX	01:02:38
CX5Z2	13B	4AY		24X	3AZ		35Y	1AX	01:39:22
AZ2Y4	13B	5CX		24X	5BZ		35Y	1AX	01:48:33
AZ2Y4	13B	5CX		24X	3AY		35Z	1AX	01:55:20
AX5Y4	13B	2CZ		24X	1AY		35Z	1CX	01:56:33
AX5Y4	13B	2CZ		24X	3AY		35Z	1CX	01:58:08
36									
CZ5B2	13Y	4AX	24C5X		3AZ	35Y4B		1CX	06:47:51
AX2B4	13Y	5CZ	24X5Y		3AZ		35B	1CX	08:16:22
BZ5C2	13Y	4AX	24Y5X		3AZ		35B	1CX	08:54:39
CZ2B5	13Y	4AX		24C	3AZ	35Y4B		1CX	10:38:46
AX2B4	13Y	5CZ		24C	3AZ		35B	1CX	10:54:18
AX5B4	13Y	2CZ		24X	3AZ	35Y2B		1CX	11:14:11
AX5C4	13Y	2BZ		24X	3AZ	35Y4B		1CX	11:22:12
AX5C4	13Y	2BZ		24X	3AZ		35B	1CX	11:32:57
AX2B4	13Y	5CZ		24Y	3BZ	35B1A		4CX	11:47:15
BZ5C2	13Y	4AX		24Y	3AZ		35B	1CX	12:39:30
AX5C4	13Y	2BZ		24Y	3AZ		35B	1CX	12:45:38
AX5C4	13Y	2BZ		24Y	3AZ		35B	2CX	12:46:52
AX2B4	13Y	5CZ		24Y	3AZ		35B	1CX	13:04:50
AX2B4	13Y	5CZ		24Y	3AZ		35B	4CX	13:06:28
37									

NO SOLUTION FOUND - RESULTS OF PIOTROWSKI CONFIRMED FOR CASE 2.

```
. DISPLAY MEMORY
STIME      pub  C "07:47:22"
COUNT1     priv N        37 (        37.00000000)   C:OBRWLC2.prg
COUNT2     priv N       866 (       866.00000000)   C:OBRWLC2.prg
COUNT3     priv N      8520 (      8520.00000000)   C:OBRWLC2.prg
COUNT4     priv N     16531 (     16531.00000000)   C:OBRWLC2.prg
COUNT5     priv N     16895 (     16895.00000000)   C:OBRWLC2.prg
COUNT6     priv N     11792 (     11792.00000000)   C:OBRWLC2.prg
COUNT7     priv N      4687 (      4687.00000000)   C:OBRWLC2.prg
COUNT8     priv N       187 (       187.00000000)   C:OBRWLC2.prg
COUNT9     priv N         0 (         0.00000000)   C:OBRWLC2.prg
FOUND      pub  L .F.
PENT       pub  C "C2P"
TRI        pub  C "C1T3"
FACTOR     pub  C "3"
TRITYPE    pub  C "T"
FTIME      pub  C "13:13:08"
P3         pub  L .F.
Press any key to continue...
P4         pub  L .F.
P5         pub  L .F.
T3         pub  L .F.
T4         pub  L .F.
T5         pub  L .F.
D3         pub  L .F.
D4         pub  L .F.
D5         pub  L .F.
P3CHOSEN   priv N         0 (         0.00000000)   C:OBRWLC2.prg
P4CHOSEN   priv N         0 (         0.00000000)   C:OBRWLC2.prg
P5CHOSEN   priv N         0 (         0.00000000)   C:OBRWLC2.prg
D3RECNO    priv N        15 (        15.00000000)   C:OBRWLC2.prg
D4RECNO    priv N        14 (        14.00000000)   C:OBRWLC2.prg
D5RECNO    priv N         3 (         3.00000000)   C:OBRWLC2.prg
F3RECNO    priv N         9 (         9.00000000)   C:OBRWLC2.prg
   32 variables defined,     201 bytes used
  224 variables available,  5799 bytes available
```

OUTPUT FROM OBRWLFCH.PRG - CASE 3

1									
2									
3									
4									
13A4Y	5CZ	2BX		24C	3BZ		35Y	1AX	19:49:55
13A4Y	2BX	5CZ		24C	3BZ		35Y	1AX	19:52:07
5									
6									
13A5X	4CY	2BZ	24A1Y		5CZ		35Y	4BX	20:39:12
13A5X	2BZ	4CY	24A1Y		5CZ		35Y	4BX	20:39:49
13A5X	4CY	2BZ	24B1Y		5CZ		35Y	4AX	20:42:54
13A5X	2BZ	4CY	24B1Y		5CZ		35Y	4AX	20:43:11
7									
8									
9									
10									
13B2Y	5CZ	4AX		24C	3AZ		35Y	1BX	23:45:36
13B2Y	4AX	5CZ		24C	3AZ		35Y	1BX	23:47:48
11									
12									
13									
14									
13B4Y	2CZ	5AX		24X	3AZ		35Y	1BX	03:08:35
13B4Y	5AX	2CZ		24X	3AZ		35Y	1BX	03:10:01
13B4Y	2CZ	5AX		24X	5CY		35Z	1BX	03:11:37
13B4Y	5AX	2CZ		24X	5CY		35Z	1BX	03:12:24
15									
13Y2B	5CZ	4AX	24Y5X		3BZ		35A	1CY	03:48:04
13Y2B	4AX	5CZ	24Y5X		3BZ		35A	1CY	03:48:29
13Y2B	5CZ	4AX		24C	3BZ		35A	1CY	03:56:49
13Y2B	4AX	5CZ		24C	3BZ		35A	1CY	03:57:45
16									
17									
18									
13Y4A	5CZ	2BX	24X5Y		3BZ		35A	1CY	05:54:48
13Y4A	2BX	5CZ	24X5Y		3BZ		35A	1CY	05:55:43
13Y4A	5CZ	2BX		24C	3AZ	35X4B		1CY	06:03:13
13Y4A	2BX	5CZ		24C	3AZ	35X4B		1CY	06:04:38
13Y4A	5CZ	2BX		24C	3BZ		35A	1CY	06:07:33
13Y4A	2BX	5CZ		24C	3BZ		35A	1CY	06:08:50
19									
13Y4B	2CZ	5AX	24X1Y		3BZ	35C1A		2BX	06:49:36
13Y4B	5AX	2CZ	24X1Y		3BZ	35C1A		2BX	06:50:14
13Y4B	2CZ	5AX	24X1Y		3BZ	35C4A		2BX	06:51:07
13Y4B	5AX	2CZ	24X1Y		3BZ	35C4A		2BX	06:51:45
20									
13Y4C	2BZ	5AX	24A1X		5CY		35Z	4BX	07:17:28
13Y4C	5AX	2BZ	24A1X		5CY		35Z	4BX	07:17:46
13Y4C	2BX	5AZ	24B3Z		1AX	35X4A		2CY	07:38:21
13Y4C	5AZ	2BX	24B3Z		1AX	35X4A		2CY	07:38:42
13Y4C	2BZ	5AX		24X	5CY		35Z	1BX	08:00:11
13Y4C	5AX	2BZ		24X	5CY		35Z	1BX	08:00:59
21									

22									
13Y5A	2CZ	4BX		24Y	3AZ	35X2B		1CY	09:27:50
13Y5A	4BX	2CZ		24Y	3AZ	35X2B		1CY	09:28:46
23									
13Y5C	2BZ	4AX	24B1Y		5AZ	35X1A		4CY	09:41:35
13Y5C	4AX	2BZ	24B1Y		5AZ	35X1A		4CY	09:42:14
13Y5C	2BZ	4AX	24Y1X		3AZ	35A1B		2CY	09:50:58
13Y5C	4AX	2BZ	24Y1X		3AZ	35A1B		2CY	09:51:37
24									
25									
13Z2B	4CY	5AX	24A1X		5CZ		35Y	4BX	10:49:59
13Z2B	5AX	4CY	24A1X		5CZ		35Y	4BX	10:50:16
13Z2B	4CY	5AX	24A1Y		5CZ		35Y	4BX	10:52:03
13Z2B	5AX	4CY	24A1Y		5CZ		35Y	4BX	10:52:20
26									
27									
28									
29									
30									
31									
32									
33									
34									
CZ5Y2	13A	4BX	24A5X		1CY	35C4Y		2BZ	16:22:56
CZ5Y4	13A	2BX	24B3Z		1CY	35C2Y		4AX	18:47:42
CY4X5	13A	2BZ	24C1Y		5AZ		35Y	1BX	20:14:29
BZ2X4	13A	5CY	24C1Y		5AX		35Z	1BX	20:28:26
BZ2X4	13A	5CY	24Y1C		5AX		35Z	1BX	22:34:29
CZ2Y5	13A	4BX	24Y3B		5AX		35Z	1CY	23:07:53
BZ2X4	13A	5CY	24Y3B		5AX		35Z	1BX	23:10:31
CZ2Y5	13A	4BX		24C	5AZ	35X2B		1CY	00:47:13
CY4X5	13A	2BZ		24C	5AZ		35Y	1BX	01:16:57
BX2Y4	13A	5CZ		24C	5AX		35Y	2BZ	01:20:56
BX2Y4	13A	5CZ		24C	3BZ		35Y	4AX	01:23:17
CZ2Y5	13A	4BX		24C	5AX		35Z	1CY	01:27:09
BZ2X4	13A	5CY		24C	5AX		35Z	1BX	01:31:48
BZ2X4	13A	5CY		24Y	5AX		35Z	1BX	04:02:29
35									
CZ2Y4	13B	5AX	24A1X		5CY		35Z	4BX	04:26:26
AZ2X5	13B	4CY	24A1Y		5CZ		35Y	4BX	04:33:44
CZ5Y2	13B	4AX	24C1X		3AZ	35A1Y		2BZ	05:02:54
AX5Y4	13B	2CZ	24C1X		3AZ	35A1Y		4BX	05:06:38
CZ5Y2	13B	4AX	24C5X		3AZ	35A1Y		2BZ	06:11:15
AX2Y4	13B	5CZ	24X1C		3AZ	35A1Y		2BZ	06:45:03
CZ2Y4	13B	5AX	24X1C		3AZ		35Y	2BX	06:54:37
AX2Y4	13B	5CZ	24X1C		3AZ		35Y	2BZ	06:58:11
AX5Y4	13B	2CZ	24X1Y		5AZ	35C1A		2BX	07:03:59
CY2Z5	13B	4AX	24Y1C		3AZ		35Y	2BX	07:47:41
AZ5X4	13B	2CY	24Y3Z		1AX		35A	2BX	08:36:51
CZ2Y5	13B	4AX		24B	5AZ	35X1A		4CY	09:27:53
CY2X4	13B	5AZ		24B	1AX	35Y4A		2CZ	10:13:44
CY2X4	13B	5AZ		24B	1AX		35Y	2CZ	10:31:34
AZ2X5	13B	4CY		24B	5CZ		35Y	1AX	10:41:51
AZ2X5	13B	4CY		24B	5CZ		35Y	4AX	10:42:57
CY2Z5	13B	4AX		24C	3AZ	35A1Y		2BX	10:55:01
CZ2Y5	13B	4AX		24C	3AZ	35A1Y		2BX	10:55:41
AX2Y4	13B	5CZ		24C	3AZ	35A1Y		4BX	10:59:14

```
AX2Y4  13B  5CZ          24C  3AZ  35A1Y         2BZ   11:00:30
AX2Y4  13B  5CZ          24C  3AZ  35X1Y         2BZ   11:19:01
CY2Z5  13B  4AX          24C  3AZ           35Y  2BX   11:47:28
AX2Y4  13B  5CZ          24C  3AZ           35Y  4BX   11:58:34
AX2Y4  13B  5CZ          24C  3AZ           35Y  2BZ   11:59:50
CY5Z2  13B  4AX          24Y  3AZ  35A1Y         2BX   13:21:42
CY5Z2  13B  4AX          24Y  3AZ  35C1Y         2BX   13:33:53
  36
BZ2C4  13Y  5AX  24A3B        5CY           35Z  1BX   15:31:17
AZ5C4  13Y  2BX  24B3Z        1AX           35A  2CY   16:42:44
AX2B4  13Y  5CZ  24X1C        3AZ  35A1B         4CY   18:31:08
AX2B4  13Y  5CZ  24X1C        3BZ           35A  4CY   18:42:04
CZ5A4  13Y  2BX  24X5C        3AZ  35Y4B         1AX   18:47:23
AX2B4  13Y  5CZ  24X5Y        3AZ  35A1B         4CY   18:51:55
AZ5C4  13Y  2BX  24X5Y        3BZ           35A  2CZ   19:02:09
AX2B4  13Y  5CZ  24X5Y        3BZ           35A  1BX   19:04:45
AX2B4  13Y  5CZ  24X5Y        3BZ           35A  4CY   19:05:59
AX2B4  13Y  5CZ          24C  3BZ           35A  1BX   21:34:58
AX5C4  13Y  2BZ          24X  3AZ  35A1B         2CY   21:40:09
BX5A4  13Y  2CZ          24X  3AZ  35Y2B         1AX   22:08:58
CZ2B4  13Y  5AX          24X  3BZ  35Y4A         1BX   22:12:52
BZ2C4  13Y  5AX          24X  5CZ  35Y4A         1BX   22:14:37
BZ2C4  13Y  5AX          24X  5CY           35Z  1BX   22:31:33
CZ2B4  13Y  5AX          24X  5CY           35Z  1BX   22:32:18
BX5A4  13Y  2CZ          24X  5CY           35Z  1AX   22:34:26
AZ5C4  13Y  2BX          24Y  3BZ  35X1A         2CZ   23:27:11
BX2C4  13Y  5AZ          24Y  1AX  35X4A         2BZ   23:38:51
AZ5C4  13Y  2BX          24Y  1AX  35X4B         2CZ   23:47:29
AZ5C4  13Y  2BX          24Y  1AX           35A  2CZ   23:57:39
AZ5C4  13Y  2BX          24Y  3BZ           35A  2CZ   23:59:09
AX2B4  13Y  5CZ          24Y  3BZ           35A  1BX   00:08:58
  37
NO SOLUTION FOUND - RESULTS OF PIOTROWSKI CONFIRMED FOR CASE 3.

. DISPLAY MEMORY
STIME     pub  C "16:42:21"
COUNT1     priv  N      37  (     37.00000000)    C:OBRWLC3.prg
COUNT2     priv  N     819  (    819.00000000)    C:OBRWLC3.prg
COUNT3     priv  N    7651  (   7651.00000000)    C:OBRWLC3.prg
COUNT4     priv  N   15994  (  15994.00000000)    C:OBRWLC3.prg
COUNT5     priv  N   17709  (  17709.00000000)    C:OBRWLC3.prg
COUNT6     priv  N   12448  (  12448.00000000)    C:OBRWLC3.prg
COUNT7     priv  N    4685  (   4685.00000000)    C:OBRWLC3.prg
COUNT8     priv  N     105  (    105.00000000)    C:OBRWLC3.prg
COUNT9     priv  N       0  (      0.00000000)    C:OBRWLC3.prg
FOUND      pub  L .F.
PENT      pub  C "C2P"
TRI      pub  C "C1T3"
FACTOR     pub  C "3"
TRITYPE    pub  C "T"
FTIME     pub  C "00:17:20"
P3       pub  L .F.
Press any key to continue...
P4       pub  L .F.
P5       pub  L .F.
T3       pub  L .F.
T4       pub  L .F.
```

T5	pub	L	.F.		
D3	pub	L	.F.		
D4	pub	L	.F.		
D5	pub	L	.F.		
P3CHOSEN	priv	N	0 (	0.00000000)	C:OBRWLC3.prg
P4CHOSEN	priv	N	0 (	0.00000000)	C:OBRWLC3.prg
P5CHOSEN	priv	N	0 (	0.00000000)	C:OBRWLC3.prg
D3RECNO	priv	N	16 (	16.00000000)	C:OBRWLC3.prg
D4RECNO	priv	N	7 (	7.00000000)	C:OBRWLC3.prg
D5RECNO	priv	N	3 (	3.00000000)	C:OBRWLC3.prg
F3RECNO	priv	N	11 (	11.00000000)	C:OBRWLC3.prg

32 variables defined, 201 bytes used
224 variables available, 5799 bytes available