

AN EXPLORATORY STUDY OF TEACHERS' CONCEPTIONS OF
MATHEMATICS AND THE ROLE OF COMPUTERS IN THE
DEVELOPMENT AND
TEACHING OF MATHEMATICS

by

Harold Brochmann

B.A., University of British Columbia, 1959

B.Ed., Dalhousie University, 1961

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APPROVAL

Name: Harold Brochmann
Degree: Master of Arts (Education)
Title of Thesis: An Exploratory Study of Teachers' Conceptions of Mathematics and the Role of Computers in the Development and Teaching of Mathematics.
Examining Committee:
Chair: Gloria Sampson

Thomas O'Shea
Senior Supervisor

A.J. (Sandy) Dawson
Associate Professor

Dr. Walter Szetela
Associate Professor
Faculty of Education
University of British Columbia
External Examiner

Date Approved 1990-11-21

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An Exploratory Study of Teachers' Conceptions of Mathematics and the

Role of Computers in the Development and Teaching of Mathematics.

Author:

(signature)

Harold BROCHMANN

(name)

Nov. 21 '90

(date)

ABSTRACT

The purpose of this study was to explore the views, beliefs, conceptions and attitudes of practising secondary mathematics teachers regarding the nature of mathematics and how it should be taught. Of particular interest were the implications the findings might have for shifts in teaching methodology and curriculum reform in the light of increasing student and teacher access to computers. An inventory was also obtained describing teachers' experience and knowledge of the new technology, new topics in mathematics, and level of professional activities.

A series of nineteen one-session interviews was conducted in two stages with practising secondary mathematics teachers. The first six were open ended and all conversations were recorded and transcribed. On the basis of these interviews a number of paraphrased summary statements were constructed which were used to formulate specific questions which were put to the remaining thirteen participants. All responses and accompanying comments were recorded for later analysis.

The results are presented as a series of synthesized observations and speculations stimulated by the study. The teachers participating in the study tended to equate mathematics with the prescribed curriculum and were hesitant to discuss abstract and theoretical aspects of their discipline. They were concerned about the practical problems of teaching and viewed themselves as good teachers in the sense of being conscientious employees rather than mathematicians with an academic interest in the discipline. The participants in this study did not, in general, believe that mathematics courses develop transferable thinking skills. There was general agreement that computers have an important role to play in mathematics education, but this role was perceived to be one of support for a traditional curriculum rather than serving as the basis for content reform.

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CHAPTER ONE

INTRODUCTION

Background

My initial experience with computers was in 1975. At that time North Vancouver School District, where I had been teaching Junior Secondary Mathematics and Science since 1967, acquired a Hewlett Packard computer along with a card reader and plotter. This machine was labelled and sold under the rubric "calculator". I was told that this had been done by the manufacturer to make it more saleable to educational institutions that might have less difficulty in getting budget approval for calculators than for computers.

I spent many hours writing programs for the Hewlett Packard in BASIC that did what I considered to be fascinating things. Most of these programs were written by placing pencil marks on cards which were later fed into the card reader. This was a tedious business.

We had a surveyor's transit in the mathematics department of the school at the time. I set up three transit stations on the school grounds and triangulated a number of points on the school building, such as window edges and roof corners. Readings from the transit were used as data in a computer program I wrote that used the plotter to produce plans and perspective drawings of the school buildings as viewed from various vantage points. At the same time I became absorbed in the problem of simulating a French curve with a computer program so that my program could be used to draw plans of structures with curved surfaces. I never did succeed in this latter component of the enterprise.

At the time it puzzled me a great deal that I seemed unable to elicit any interest in these activities from my mathematics teaching colleagues. Was it not the case that designing computer programs to turn the data from a surveyor's transit into maps was an exciting mathematical activity? Why were the obviously intelligent students in our charge so stimulated by programming the computer, whereas their teachers were not? Was it due to teachers' inability or unwillingness to communicate, or to a fundamental lack of agreement of what mathematics was or ought to be, or was it that the mathematics teachers in my district were intellectually stultified?

About the same time I met Dr. Ted Edwards who lent me his prototype portable MCM computer which had APL in ROM. It also had cassette based virtual memory. Although Ted tried valiantly to explain to me the notion of APL as an *alternate mathematical notation system*, I did not understand. Through Ted I also met Kenneth Iverson, the inventor of APL. Neither of these people accepted what I perceived as critical; namely, the operational and practical difficulties of using APL to communicate and teach mathematics. I believed that the essential factor was to have students use computers for mathematics education, in which case there were more advantages to using BASIC as opposed to APL. Ted and Kenneth finally lost patience with me. The issue in their minds was not *computers* in mathematics. The issue was the notion that APL was a vastly superior *notation system*. The fact that the notation system was executable on a computer was almost incidental.

This whole experience was extremely frustrating for me. I realized that I did not quite understand what these very bright people were talking

about; however I couldn't quite identify *what* it was I didn't understand. At the same time I was equally frustrated by their apparent inability to follow *my* arguments. In the end I gave up and finally I also rejected the whole idea of using computers in the public school system because of the high cost of providing student access to computers at that time. For example, the Hewlett Packard cost \$25 000.

Ten years later I met Kenneth Iverson again in Waterloo. He had retired and intended to devote his time to publicizing the advantages of APL. By this time I understood and recognized the validity of what he had been saying. I think I also understood some of the reasons why APL was never accepted as *the* mathematics notation system for education. It was not a satisfactory meeting. It became obvious to me then that he found it impossible to see things from the perspective of the practising mathematics teacher because although he had talked *to* many of them he had apparently not talked *with* many. This made me realize that if you want to convince someone to see things from your point of view, you have to have considerable familiarity with and appreciation for *his* or *her* point of view and the background knowledge and experiences upon which that view is based, an approach he appeared to dismiss.

In 1980 I became involved with the use of computers at the district level. During the first two years I was heavily committed to the notion of using computers to deliver instruction, to which end I wrote a great deal of instructional software, including a PILOT interpreter. For a number of reasons I finally concluded that the Computer Assisted Instruction paradigm was flawed and that the appropriate use of computers in education was that of

a *tool* -- the most obvious variety of which was the word processor. My employers supported this notion and invested heavily (for that time) in the development of computer-based writing in the North Vancouver School District. I was provided with two sets of 30 PET microcomputers, each set of which was placed in a school for a period of one month. The idea was that teachers and students would learn to "word process" together. My function was to make sure everyone knew what he or she was doing as well as to see that the equipment worked.

When I was introduced to Logo in 1984, I became fascinated with the fact that one could define custom mathematical functions in a Logo workspace including APL-like primitives. This is when it occurred to me that there *were* alternate ways of expressing mathematical ideas -- and that computer languages in general, and Logo and APL in particular, might be superior alternatives to conventional systems for understanding, communicating and teaching mathematics.

Once again my employers supported my ideas and for a year I taught Logo full time to thousands of students in North Vancouver -- but again, there was no evidence that any of my colleagues saw the relevance of what I was trying to do.

For the past several years I have to some extent, been desk bound. Every school in my district now has a computer lab and this equipment is used almost exclusively for writing as part of the English program. Having finally established the writing program, I have switched the focus of my attempts to influence mathematics teaching from Logo to spreadsheets; but

though there are some encouraging signs, resistance to this approach as well is widespread.

How Computers Can Be Used In Mathematics Education

There is a variety of ways in which computers can be used in mathematics education. Not all teachers feel that all uses of computers are appropriate or practical. Many reservations are based on the real and perceived practical difficulties of scheduling large numbers of students into the limited resources characteristic of most public schools.

The computer as an audio-visual aid. The general idea is that demonstrations of, for example, graphing are carried out on a computer screen. The problem of economical large area display screens has recently been addressed by using Liquid Crystal Display (LCD) devices on overhead projectors. It should be noted that many mathematics teachers are familiar with this educational application of computers because it has been extensively used in several recent conferences, and equipment is available in many secondary schools.

Computer assisted instruction (CAI). Although this term is used by some to connote *all* educational use of computers, it is most useful when its reference is limited to some form of computer-based programmed learning. Articles in various trade journals seem to imply that many teachers are very enthusiastic about this use of computers. I have not been able to confirm this through personal observation in the schools I have visited. In my opinion it is not likely that CAI will be widely used in the public school system because it requires very significant hardware and software investments. Software

development costs, in particular, tend to be high. Much educational software that purports to be CAI turns out, on examination, to be of the "drill and practice" variety.

Assessment. In the assessment application the computer is used to administer a series of multiple-choice questions to the student. The main advantage is that questions can be selected at random from a large test bank, thereby ensuring that each student has a unique test and reducing the problems associated with testing security. In a variation of this computer use, the random questions are printed on paper rather than being displayed on the terminal screen.

Simulations. In this application, mathematical models of physical or biological systems are implemented in computer programs that may employ graphics, and sometimes sound, as well as text. In general, the idea in a simulation is for the student to provide inputs for the model and then make inferences on the basis of how the model responds. The most convenient way to implement exploratory models in mathematics is through the use of spreadsheets. Some teachers feel that this use of computers is a practical way to implement discovery learning in mathematics.

Programming. If computer programming is a legitimate mathematical activity, then consideration should be given to including programming in mathematics curricula. When teachers debate this issue they rarely seem to reach agreement on the details of how it should be done and which language should be used.

Implications of Computer Use for Mathematics Curriculum Developers

In addition to the educational uses of computers just discussed, there are two ways in which the relationship between computers and mathematics can be viewed. On one hand, the computer can be viewed as a kind of "full function calculator" that can be a convenient adjunct to the present mathematics curriculum. On the other hand, one may recognize that the sheer "number-crunching" power of the computer allows us to go beyond "... regarding computers as a means of mechanizing existing methods ... to seeing them as a means of reconceptualizing methods and contents, and, more broadly, for providing 'tools for thought'" (Pea & Kurland, 1984). Schwartz (1990) puts it rather more strongly: "It is insane and reprehensible to spend 200 hours of a kid's life teaching him to be an unreliable imitation of a \$5 machine."

Computer access allows us not only to shift the focus of mathematics courses away from computational techniques and on to other matters, but it also makes it possible for us to reconsider what ought to be presented as the nature of the subject itself. "Changes in mathematics education need to challenge fundamental assumptions about the nature of mathematics or else remain marginal in effect" (Lerman, 1990).

Crawford (1986) puts it this way, "The most basic flaw, in my judgment, is in the confusion over what the nature of mathematics is, and what the purposes and hence the content of school mathematics should be."
(p. 7)

Computers have provided mathematics curriculum developers with an opportunity to re-consider what ought to be presented as the essence of the subject. The question arises as to whether mathematics should be presented as a set of computational algorithms, or a descriptive language, or a logic system, or a versatile and convenient 'tool kit' for dealing with the world.

Clearly, a teacher's perspective of what the essence of his or her subject is must be heavily influenced by the curriculum perspective of curriculum designers. Equally clearly he or she passes this perspective on to students and participates in curriculum decision matters. According to Ernest (1989), "The argument is that such conceptions have a powerful impact on teaching through such processes as the selection of content and emphasis, styles of teaching and modes of learning." (p. 20)

In the last decade there have been many calls for reform in mathematics education. With some justification, critics charge that students and indeed the community at large, perceive mathematics as a series of rules and algorithms of doubtful value which obviate the need to develop understanding of underlying principles; this has been referred to as "bifurcated interpretation of school mathematics".

Purpose of the Study

Although there seems to be consensus among those calling for reform as to what the ideal outcome of mathematics education ought to be (see for example NCTM, 1989), little can be found in the way of specific suggestions

as to how this is to be accomplished. No one seems to be asking: "Precisely who should be doing exactly what?"

Whatever the answers to this question are, mathematics teachers will inevitably play a crucial role in determining the direction any evolution in mathematics education will actually take. It follows that it is crucial for interested parties to have access to direct information on the views and perceptions of practising teachers. What changes do *they* see as desirable? What are *teachers'* perceptions of their roles in curriculum development? How prepared are *they* to participate in the decision making and implementation? What are *their* conceptions and attitudes towards "school mathematics"? Do they know enough about computers and what they can do to make informed judgments?

If progress is to be made in mathematics education, it will, among other things, involve changes in teachers' conceptions and attitudes. To effect these changes the forces that will have to be brought into play must necessarily "...begin with an understanding of the conceptions held by the teachers and how these are related to their instructional practice" (Thompson, 1984, p 106).

This study is based on the premise that secondary mathematics teachers' beliefs, conceptions, and attitudes relative to school mathematics and the form its evolution should take, particularly as these relate to the use of computers in mathematics education to a considerable extent dictate the de facto curriculum. We must therefore determine what these beliefs, conceptions and attitudes are. Therefore the intent of this paper was to carry out a broad exploratory investigation to answer the following research

question: What aspects of secondary mathematics teachers' conceptions, beliefs and attitudes towards mathematics and the role of computers in mathematics education may be identified through teacher interviews? I had originally anticipated that this would lead to discussions on the *nature of mathematics*. It turned out that teachers found it very difficult to engage in this kind of discussion. Further, it was hoped the study would disclose unanticipated difficulties inherent in this type of investigation. The most interesting difficulty identified in this respect was the differences that existed between the interviewer and some participants in our understanding of certain terminology

Limitations of the Study

This study is based on nineteen one-session teacher interviews. Participants were not randomly selected; all reside and work in the vicinity of Vancouver, British Columbia. All are secondary teachers, all are male, and all but one has taught for more than fifteen years. The number of participants and their non-random selection precludes generalizing the results to the population of mathematics teachers.

Structure of the Study

Chapter Two contains a review of related literature. The amount of research done in the area of interest appears to be quite limited.

In Chapter Three the methodology is described. Interviews were conducted in two stages. The first stage interviews were unstructured and

exploratory. The second or main stage interviews were based on a series of specific questions, and specific options were presented for responses.

In Chapter Four each question posed to the second stage participants is presented together with some introductory discussion. The responses chosen for each question are also recorded. Each set of responses is discussed briefly.

Chapter Five consists of a number of observations and speculations that were stimulated by the study.

CHAPTER TWO

LITERATURE REVIEW

There is currently little literature available relative to the underlying rationale of the present curriculum, questions touching on the nature of mathematics, documentation of practising teachers' viewpoints, proposed constructs related to teaching and views on the potential impact of computing on mathematics education will be examined.

Why We Teach Mathematics

Although mathematics has been universally taught in public schools and by and large the curricula used have not differed radically from one jurisdiction to another, the justifications for teaching mathematics and the rationale for selecting particular items found in most curricula are not always clear.

Dörfler and McLone (1986) discussed the various proposed justifications for teaching mathematics to *the average pupil*. The authors listed these as preparation for employment, academic preparation, and generation of higher level thinking skills. With respect to preparation for employment Dörfler and McLone observed that:

In an advanced technological society today there is clearly a requirement for mathematics training at a very high level to cope with the requirements of science and technology. However, the requirements at a more general level are much more mundane. (p. 51)

With respect to academic preparation, the authors observed that, "...it is difficult to support an "academically inspired curriculum for *all* pupils" (p. 52).

On the question of developing higher order thinking skills,

...although mathematics can assist in this aim, it is by no means uniquely able to do so..... Moreover the simply repeated performance of routine tasks which sometimes passes for mathematical education hardly gives ground for optimism in the development of logical thought. (p. 53)

In any event, mathematics is uniquely characterized in that it has "...developed specific features which cannot be found elsewhere" (p. 62) and:

...there is no activity from which the contents of the teaching and learning process can be derived. The motivation to teach or to learn a certain part of mathematics in this setting does not come as a result of one's own mathematical or other activities or of one's own experiences. The abstract content itself is expected to carry the incentive for studying it. (p. 62)

In a study prepared for the British Columbia Ministry of Education, Robitaille and Dirks (1982) observed:

The allegation that no one knows why we teach the mathematics we do to so many students may seem somewhat exaggerated, but it does contain a certain amount of truth. An examination of documents describing school mathematics curricula makes it appear that, in some cases, little thought has been given to fundamental issues such as the goals of the mathematics curriculum. Where goals have been identified it is often difficult to see how the content selected for inclusion in the curriculum is related to those goals. (p. 1)

It would appear that the mathematics curriculum has been determined by some mechanism other than a rational examination of the real needs of students and society.

The National Council of Teachers of Mathematics (NCTM, 1989) took a definitive step towards clarifying this dilemma by stating that there is a need to produce "an informed electorate, mathematically literate workers, opportunities for all students, and problem-solving skills that serve lifelong learning" (p. 4). These statements include the suggestion that mathematics is connected with the process by which citizens in a democratic society become and stay informed. The association with the study of statistics is obvious. The reference to literate workers implies that mathematics should be useful and have applications related to people's economic welfare. The inclusion of the statement on problem-solving skills for lifelong learning is no less significant.

What appears to be a different view of mathematics is reflected in the Statement of Philosophy provided by the British Columbia Curriculum Guide for Mathematics 7 - 12 (British Columbia Ministry of Education, 1988)

Mathematics is an integral part of the human experience. The reasoning skills developed through the study of mathematics are necessary for all citizens to function productively in society. Also important is the human satisfaction that arises from understanding mathematics as an extension of the concrete world. For these reasons, mathematics is an important component of education, and therefore it should be the right of every student to receive a level of mathematics instruction appropriate to his or her needs and abilities.

Neither of the two reasons given seem to suggest that the things learned in mathematics need be *useful*. There is no suggestion that mathematics has any practical application. There is no reference to literate workers; there is no mention of economic welfare. Nothing is said about opportunity and

lifelong skills. The implications are that mathematics is taught because that part of it *which extends beyond the concrete world* is satisfying, and also because it develops reasoning skills in areas outside mathematics. The notion of using mathematics as a training mechanism for developing logical thinking is an old one and the extent to which this philosophy is shared by practising teachers is explored in this paper. One also wonders why the mathematics of the concrete world should be less satisfying than that which "extends beyond" it.

Theoretical Perspectives on the Nature of Mathematics

The literature of educational research contains some material that describes how differing perspectives of the nature of mathematics may be categorized. For example, Robitaille and Dirks (1982) proposed that a useful distinction be made between the *Pure vs Applied vs Creative Arts* perspectives. From the *pure* perspective mathematics is a set of highly abstract and formal conjectures, proofs and refutations. Although the medium through which this formalized view has found expression in the public school curriculum has traditionally been Euclidean Geometry, it may also influence how a variety of other topics is presented. From the perspective of the formalist, mathematical activities consist of discourse and search for consistency and truth. According to Robitaille and Dirks this paradigm continues to have a significant influence on mathematics curricula. Even today the Grade 10 mathematics curriculum for British Columbia suggests that 40% of the school year be devoted to a fairly traditional treatment of Euclidean Geometry -- presumably because of its salutary effect

on the development of reasoning skills. Valid though a *formal logic* view of mathematics may be, it hardly seems realistic to expect the public school system's typical client -- the average teenager -- to identify with it, and one questions what purpose is served by adopting such a guiding principle for curriculum design. Be that as it may, the matter of interest in this paper is the extent to which the deliverers of curriculum, the teachers, share this perspective.

The second perspective of mathematics described by Robitaille and Dirks is that of *Applied Mathematics*. The Applied Mathematics perspective, very generally, is one which views mathematics as a tool for solving *practical* problems. I suspect that neither of the terms "practical" and "problem" share universal definitions among mathematics educators, but Applied Mathematics implies mathematics which is useful for personal and vocational needs as well as the skills needed for "mathematization", a term intended to describe the process by which individuals use mathematics to build meaningful models of the world.

There is a certain amount of polarization of views between those who hold a pure, and those who hold an applied perspectives of mathematics. Whereas the pure and applied perspectives of mathematics tend to antagonistic, proponents of either view may in addition see mathematics as a creative art. Quite possibly this is the "human satisfaction" alluded to in the philosophy statement of the British Columbia Curriculum Guide.

Ernest (1989) outlined an analytical model for an inventory of teachers' perspective schema in which he suggested that research should distinguish between *Knowledge, Beliefs* and *Attitudes*. In Ernest's view,

knowledge pertaining to other subject matter, teaching (pedagogy and curriculum), organizational context, students and psychology as well as mathematical knowledge are of consequence in building a valid model. For example, knowledge of other subject matter, "provides justification and motivation for students for studying some of the content of mathematics by showing children its relevance" (p. 17).

In discussing teachers' beliefs, Ernest distinguished between conception of the nature of mathematics, models for teaching and learning, and principles of education. The nature of mathematics may, according to Ernest, may be perceived to be dynamic, static or unrelated. Models for teaching were given as investigational, conceptual understanding, mastery, and survival.

The suggested classifications and hypothetical descriptions proposed above are interesting, but they, like the work discussed at the beginning of this section, seem to be based on empirical observation only to a limited extent. Although the readings I encountered may contribute to providing a *framework*, I was more interested in material which was based on investigations of *real teachers*.

Empirical Investigations

Thompson (1984) studied three teachers extensively for four weeks each and found evidence to support the assumption that teachers' beliefs, views and preferences about mathematics play a significant role in shaping their behaviour. "Any attempt to improve the quality of mathematics

teaching must begin with an understanding of the conceptions held by the teachers and how these are related to their instructional practice" (p. 106).

Thompson found that mathematics teachers believed that mathematics

- is a subject of ideas and mental processes,
- has as its purpose to develop reasoning skills,
- is prescriptive in nature,
- is absolute, fixed, predictable, reliable, logical, procedural, prescriptive yet challenging and involving discovery, and
- is practical.

I find some of these results puzzling. For example, I would have thought that "challenging" and "discovery" were inconsistent with "predictable" and "prescriptive". Thompson also found inconsistencies between teachers' stated beliefs in the relevance of mathematics and the failure to discuss practical applications of the topics taught. This suggests that the research in this area is not yet definitive.

A few years later, Owens (1987) conducted seven one-hour interviews with each of four *pre-service* teachers with a view to learning something about their constructs related to mathematics and mathematics teaching. Owens found that his subjects viewed mathematics in terms of its perceived usefulness in the secondary curriculum and also in terms of *ease of learning to the individual and its anticipated ease of teaching* (italics mine). Owens went on to conclude:

The pre-service teachers' view of mathematics appears to be based more on the individual's prior academic success with the subject than with an involvement with or interest in the nature of mathematics. "Mathematics", primarily computation and

algorithms characterized by solving equations, has "come easy" to the individual. This construct of "easy mathematics", coupled with an anticipated use of mathematics as a secondary teacher gleaned primarily from student experiences, guides the individual's interpretation of mathematics experiences (p. 273).

Owens seems to be saying that deep personal interest and fascination with the subject was not the primary reason that the future mathematics teachers he studied entered the field. As the subsequent study will show, the teachers who I interviewed shared this characteristic.

Computers and Mathematics Curricula

In 1980 calculators were already commonplace in mathematics classrooms, the first *affordable* computers were just making their appearance and The National Council of Teachers of Mathematics in a major policy paper (NCTM, 1980) recommended that in the decade ahead that "mathematics programs take full advantage of the power of calculators and computers at all grade levels" (p. 13). In its *Computing and Mathematics* (Fey, 1984), the NCTM collected the views of a number of authors on the ways that computers might influence mathematics curriculum development. In the preface the editors note that:

The most common pattern in current efforts is the use of computers in clever ways to accomplish traditional educational objectives. But the implications of the microelectronic revolution extend far beyond the technology of teaching; they raise basic questions about the goals of mathematics education . (p. 3)

How computing may affect the teaching of algebra, geometry, calculus and discrete mathematics are examined in separate chapters. With respect to

algebra it is suggested that computing makes possible a reversal of the traditional sequence in which topics are taught, a shift in emphasis from symbolic to numeric solution processes and increased integration with other mathematics subjects. Access to dynamic visual models promises to alleviate long-standing pedagogical problems in geometry. In calculus, computing not only promises to provide unprecedented possibilities for enhancing student understanding but makes available methods for solving a broad range of problems that are at least as effective, and certainly different from, the traditional ones. Finally, it is proposed that curricula be modified so that topics commonly described with the label "discrete mathematics" get early and prominent attention.

Exhortations with respect to mathematics curriculum reform based on computing made earlier in the early 1980's seem not to have had as much effect as some may have wished. In one unpublished study, based on extensive reviews of articles appearing in *The Mathematics Teacher* over a ten-year period, Brochmann & O'Shea (1989) observe that

...there is little evidence to suggest that much in the way of fundamental changes in curriculum content or emphasis has occurred in the last ten years. Further, the amount of discussion [related to computing based curriculum change] seems, if anything, to be diminishing.... teachers are not yet generally familiar with functional mathematical languages.... commercial mathematical modelling software such as spreadsheets are not widely used. (p. 19)

British Columbia teachers are hardly encouraged to incorporate computing in their teaching. The most recent mathematics curriculum guide (British Columbia Ministry of Education, 1988), in its introduction states

that "Despite the fact that the microcomputer has had an impact on society and promises to be a tremendous learning aid in the mathematics classroom, *computer studies is not a mathematical topic.*" (Italics mine). This statement reflects, in my opinion, a lack of distinction between the "medium" and the "message".

Summary

The aims of public education and the means by which they ought to be achieved is a popular topic for discourse even among laymen. There are well-founded philosophical underpinnings for the various views expressed, and some considerable amount of research has been done with students. However, empirical investigations of teachers, particularly as they relate to their conceptions of mathematics has so far been limited. Those investigations that have been done characteristically involve only three or four subjects who tend to be pre-service, rather than practising teachers. This paucity of empirical data may be accounted for by the lack of a firm theoretical structure on which to base research.

There is evidence that teachers' beliefs influence the way they teach, but the relationship between teachers' professed beliefs and actual teaching behaviour is complex. It may also be that there is a divergence between teachers' professed beliefs and operational beliefs; and that research techniques employed to date have failed to elucidate these distinctions.

It appears very little, if any, research has been done to date on the views which teachers hold with respect to the use of computers in mathematics education. Such research may yield limited useful information

in view of the extent to which computing has so far found its way into curricula, the lack of clear emerging trends in computing in mathematics education, and the apparently limited insight of at least some curriculum determining bodies.

CHAPTER THREE

METHODOLOGY

The study involved two sets of interviews with practising secondary mathematics teachers, referred to here as the Preliminary Interviews and the Main Interviews. The Preliminary Interviews were open-ended and on the basis of the discussions that took place I constructed a set of *Paraphrased Summary Statements* which were intended to be representative of participants' views. These statements formed the basis for the questions posed to the participants in the Main Interviews.

Preliminary Interviews

During April to June and in August of 1989, I interviewed six teachers (all male) for approximately one and one half hours each. Because it was important that these interviews be as spontaneous as possible I asked for and obtained the cooperation of personal acquaintances. These six participants were all secondary mathematics teachers who had taught in North Vancouver School District, where I also teach. All six had taught in the district for twelve years or more and were relatively active professionally. Two taught Computer Science in addition to mathematics and three were heads of department. In later discussion these participants are referred to as Participant 0.1 to Participant 0.6.

The Preliminary Interviews were recorded on audio tape. The first four interviews were transcribed verbatim. The transcripts appear in Appendix A. Each conversational exchange is identified with the codes I-1

(Interviewer's statement number 1), P-1 (Participant's statement number 1), etc. for later reference.

The first four interviews were very open ended so that the topics touched on varied somewhat from one interview to the other. In some instances I made a statement and asked the participant to comment. For example, this is the opening exchange with Participant 0.1:

I-1: The purpose of this exercise is to build a picture of your conceptions of what mathematics is all about.

There is a difference between mathematics in the abstract sense, as it is mandated by curricula, math as it is reflected in the delivery of curriculum and as it is perceived by students. Please comment and elaborate from your perspective. [This was repeated for clarification.]

P-1: I can probably answer the latter ones first. I'm not sure what the abstract means. Let's start backwards. I don't think students perceive mathematics as being anything that has any connection....

I frequently paraphrased a point just made by the participant so as to elicit further comment. This example is from the interview with Participant 0.2:

I-8: Ok. So what you are saying here is that this is an example of where an abstract mathematical concept found an application.

P-8: Yeah ... and it probably goes both ways. I mean a lot of mathematics is developed that way, and the formal discipline, ah ...

At times I would ask a specific question and try to pin down a definitive answer. From the interview with Participant 0.3:

I-15: Do I understand you to say that in mathematics you cannot progress to the next level until this one is mastered?

P-15: That's, I guess, what I'm saying. Yes.

I-16: It is a highly structured, sequential thing?

P-16: Yeah. Yeah. More or less. Yeah.

I-17: And that in other subjects that does not apply as much?

P-17: Not as much. No. That is absolutely true.

Although several topics were common to the first four interviews, an attempt was made to ask a variety of suggestive questions and to be as informal and conversational as possible so as to encourage each participant to "open up" and express himself freely on whichever topics he felt were interesting and on which he had something to say. My perception was that my personal relationship with the participants was helpful in establishing an atmosphere conducive to expression of candid opinions.

It was by no means clear how I was going to extract useful information from the transcribed record of these interviews. The sheer bulk and richness of material was daunting. The procedure I adopted was to construct a series of *Paraphrased Summary Statements* for each participant. Each was my interpretation of a statement that would reflect a view held by a participant. For example, on the basis of the opening exchange with Participant 0.1 reproduced on the previous page, I constructed these two statements:

S-1: I have difficulty in talking about math in the abstract sense.

S-2: Students do not see the connectedness between different topics in math.

In this way I was able to construct a data base consisting of a series of statements reflecting a variety of views held by each participant.

A copy of the set of each participant's Paraphrased Summary Statements was then forwarded to him by mail for verification. In a subsequent telephone call he was asked to confirm that the statements were valid. Only one statement needed to be modified.

In keeping with the nature of this "emergent methodology", I decided to do two more interviews using a different approach. In these interviews the participants were asked to place themselves in *my position* and to formulate a series of questions which they would ask if our roles were reversed -- and to answer these questions. The first of these (Participant 0.5), found this an acceptable role and performed as requested. The second (Participant 0.6) did not seem able to assume the role of question poser and the interview took a form similar to that of the first four.

The transcribed record of the interviews with Participants 0.5 and 0.6 is contained in Appendix A. Paraphrased summary statements were also constructed for Participants 0.5 and 0.6 and verified by them as being valid.

The Paraphrased Summary Statements for all six participants in the Preliminary Interviews were entered into a computer data base that could be conveniently searched for specific key words. For example, it was possible to extract for comparison all statements containing such words as "programming" or "geometry" and so on. This data base is reproduced in Appendix B.

The Main Interviews

The main purpose of the Preliminary Interviews was to identify a suitable set of questions that could be used to provide a common structure for the Main Interviews. An examination of the Paraphrased Summary Statements data base suggested a number of questions under 26 headings which then became the questions included in the questionnaire which served to focus the second set of interviews. The sequence of the headings was scrambled so that questions relating to similar topics did not follow each other.

In the interviews the participants were provided with a copy of the questionnaire. Each question was preceded by an explanatory paragraph which I would refer to and which I read aloud. This was done in order to clarify the intent of each question. Participants were given only three options for their response to each of these questions. For example, Question 9a:

"Teaching for understanding is generally impractical. Choose 'impractical', 'maybe' or 'practical'."

The questionnaire used in the Main Interviews is reproduced in Appendix C. The contents of the questionnaire are also embedded in Chapter Four for ease of reference.

To obtain a sample of teachers to interview I obtained the cooperation of the heads of mathematics departments at two secondary schools in two different school districts. One school (identified in this paper as School A) was housed in an old facility located in a relatively economically disadvantaged neighbourhood. This school had had a laboratory facility with Commodore 64 computers for about five years. Two of the mathematics

teachers had been relatively active users of this equipment; the other five mathematics teachers had only limited exposure to the computing facilities.

The second school (identified in this paper as School B) is a newer facility in a much more affluent community. Although this school contained two newly installed labs of Macintosh computers, more limited computing facilities had been available previously. This school did not have a history of extensive use of computers by mathematics teachers.

The understanding was that I would be able to interview *everyone* who taught mathematics in these two secondary schools. School A had seven mathematics teachers and School B had six. The thirteen teachers participating in the Main Interviews therefore brought the total number of interviews conducted to nineteen. All participants were male. I do not believe there are any female secondary mathematics teachers in my district and certainly there were none in either of the two schools cooperating with the study.

The Main Interviews were conducted during October and November of 1989. Most took place after hours at the teacher's school, either in the staff room or in a classroom. Three took place at the teachers' homes. One took place in a pub. Initially the interviews took one and one half hours to conduct, but this was reduced to slightly over an hour for the later ones.

Each participant was provided with a printed copy of the questionnaire. After an introductory discussion, the participant was asked to read each item to himself. I would then summarize the introductory statements and elaborate on the points I sought responses to under each heading to be sure that there were no misunderstandings with respect to the

intent of the question or to the choices that were to be used. If the participant did not find the choices satisfactory, he was to provide an alternate response. Participants were told that in addition to the requested multiple choice response, they should add additional comments whenever they felt that this was desirable. All verbal responses were recorded by the interviewer on another copy of the questionnaire. All comments were written down and repeated aloud and the interviewee was asked to verify the accuracy of the written record. The Main Interviews were also recorded on audio tape. Due to malfunctioning equipment, two of these interviews failed to record.

I then compiled the written records of the interviews so as to produce a Summary of Responses and Comments which is to be found in Appendix C. Included also are the participants' comments.

RESULTS AND OBSERVATIONS

Introduction

In this chapter I discuss some background to each of the questions asked in the main study, examine the responses obtained, review the additional comments made by the participants and make some observations and comments.

The Main Interview section of the chapter is divided into twenty-six topic sections. Each topic section contains some background discussion on the topic based mainly on my experience with the Preliminary Interview participants. This is followed by the preamble and actual question(s) put to the participants. The numerical summary of the responses chosen is provided in brackets following each response option. The term NA indicates that the participant chose not to respond to one of the options available.

In a few instances I have *assigned* a response on the basis of my interpretation of the significance of the participant's accompanying comment. All such instances of assigned responses are indicated by asterisks (*).

Each topic section also contains my subsidiary observations and some comments on the results. The actual responses chosen by each participant together with their additional comments are to be found in Appendix C.

A characteristic which the participants shared was their willingness to cooperate in the study. Several remarked that they felt the questions I asked were important, that they felt positive about being given an opportunity to

express their views, and that they hoped I would make the results available to them.

Preliminary Interviews

The six interviews conducted in the preliminary stage of the study fulfilled their intended purpose by revealing a profusion of issues that concerned the participating teachers, and which they felt at ease talking about. Many of these, such as the question of sufficiency of time available and lack of student motivation, were anticipated at the outset of the study. Some were not.

What these initial interviews did more than anything else was to make it clear that teachers were very much concerned with practical problems of teaching mathematics as prescribed in the published curriculum, in the existing system, and that teachers were not inclined to discuss hypothetical situations or the nature of mathematics in an abstract sense. For example, in Appendix A, page A6 and page A55, there are illustrations of how I attempted to elicit the participants' views with respect to their abstracted perceptions of mathematics without reference to either applications or educational setting. ("Do mathematical laws preexist or are they discovered?") The sole attempt that was marginally successful was with Participant 0.2 who made repeated references to beauty, discipline, and logic. This participant also tended to qualify most of his statements with terms such as "perhaps" and "maybe".

In the initial visualization of this study I had imagined discussing in the interviews what I think of as fundamental reform in curriculum content. By

this I mean the notion that the implications of recent discoveries and developments in mathematics such as fractal geometry and chaos theory, coupled with the reality of virtual universal access to sophisticated computers could render much of traditional curriculum content obsolete. Could it not be that a totally new approach to curriculum design was in order? I made repeated attempts (see Appendix A, pages A9, A23 and A44) to raise this issue. None of the participants seemed interested in pursuing this topic.

The thrust of the study was of necessity therefore modified to focus on an examination of teachers' beliefs and perceptions of practical problems related to mathematics education as it exists under current conditions, as distinct from what it might, should or could be.

Main Interviews

Public Perception of Mathematics

Several participants in the Preliminary Interviews alluded to the perception that although many people feel that mathematics is an important subject, the general attitude towards it is more negative than that displayed towards other disciplines. One participant stated that when he is asked, in a social situation, what his profession is, he avoids answering the question because the response is invariably: "I was never any good at that".

One reason suggested for the apparently prevalent negative view of mathematics was that the students perceive mathematics as simply a collection of isolated skills with little obvious application and little apparent interconnectedness. Another reason suggested was that many people have had frustrating experiences in attempting to cope with school mathematics, either

because it was too difficult, or because it was badly presented. In any event, the question of student and public attitude towards the subject was deemed to be of interest.

Participants in the main study were asked (Question 1)

Is the general community and student view of mathematics, in comparison with other subject areas, a positive, neutral or a negative one? Reasons?

General community view of mathematics:

1: positive [6] 2: neutral [0] 3: negative [7] 0: NA [0]

Some comments alluded to perceived lack of community understanding of the aims of the mathematics program ("The public does not understand...") and supported the Preliminary Interviews results of a perceived generally negative public attitude towards mathematics. By and large the reasons cited by the Preliminary Interviews participants were confirmed.

An examination of the actual responses, however, revealed that half of them were positive. Further, with one exception either way, the positive responses were obtained from those who taught at School B, and the negative responses came from those who taught at School A.

As pointed out in Chapter 3, school A was situated in a generally economically depressed neighbourhood. The one positive response from this group was made by a teacher of oriental extraction and he referred to the high proportion of oriental students in his class. His comment also mentions the requirement of mathematics for further education. School B is located in

an affluent neighbourhood, and two of the positive comments from this group refer to the connection between mathematics and "getting ahead".

Finally, two participants who chose positive responses made less than flattering comments: "The public is misguided" and "I question relevance".

This item may have yielded more clear-cut responses if two contrasting questions had been asked, along the lines of:

What is the student and public feeling about the importance and usefulness of mathematics in achieving what is important in life?
and

Do students feel that they have had generally positive experiences with mathematics?

From the responses one might conclude that mathematics has a public relations problem which may have arisen from negative experiences students have had in the subject. Some of these experiences may result from its linear structure, while others may be related to students' difficulties in appreciating the inter-connectedness of topics studied in mathematics and perhaps the relevance of these topics outside the subject.

What Mathematics Marks Indicate

Four of the six participants in the Preliminary Interviews suggested that mathematics marks are used as a screening device by post secondary institutions. The implication was that although this practice has validity when the mathematics learned at secondary school is a necessary prerequisite for future courses, this is not the case in many instances. What other reasons might there be for such screening?

It occurred to me that there may be a perception that good marks in mathematics is proof of general learning ability, or perhaps that problem solving skills and thinking styles developed in mathematics are transferable to the study of other disciplines.

Participants in the main interviews were asked: (Question 2)

Mathematics examination results are often used as a screening device by post secondary educational institutions even if the course being applied for is not math related because 'good marks in math test results indicate an ability to learn effectively and to think logically'.

How do you feel about using math marks as an indicator of general learning ability? Indicator of logical thinking?

a) Math marks as indicator of general learning ability:

1: good indicator [4] 2: not particularly [9] 3: poor [0] 0: NA [0]

b) Math marks as indicator of logical thinking:

1: good indicator [8] 2: not particularly [4] 3: poor [1] 0: NA [0]

Most of the participants seemed to believe that though mathematics marks are a good indicator of logical thinking ability, they do not reflect general learning ability. The comments (reproduced in Appendix C) reflect some of the first six participants' misgiving about this use of mathematics because "it's unfair". "Not all bright students do well in math" (Participant 1.2).

One participant (0.3) explained his views on why mathematics marks are used for screening purposes this way:

P-7: I think in Social Studies and English there is a subjective kind of evaluation; while mathematics is so objective that ... it's pretty hard to argue against. So ...

P-146: You can BS your way into the next level in English or Socials. In math you just can't do it. ...

Secondary mathematics teachers who participated in the study could not be characterized as supporting the use of mathematics examination marks as a screening device for entry to post-secondary institutions.

The Benefits of Euclidean Geometry

When I first taught in British Columbia [1967] we devoted one entire year to Euclidean Geometry. The amount of formal geometry has been reduced in recent years and only fragments of the original remain. Some teachers feel that the inclusion of this topic is no longer justified. The traditional justification for teaching Euclidean Geometry is that its study develops a logical thinking style which is beneficial to the study of other branches of mathematics and other disciplines.

In the Preliminary Interviews, detailed discussions about the benefits to be derived from teaching Euclidean Geometry did not arise in the interviews with Participant 0.1 who did not place much credence in transfer, and 0.4 who did. Of the remaining four, 0.2 and 0.3 indicated that geometry had unique uses. 0.5 and 0.6 suggested that Euclidean Geometry was useful for teaching logic, but that there are other ways to do this.

The main participants were then asked: (Question 3)

Historically, the justification for teaching Euclidean geometry has been that it develops a person's ability to think in a logical manner and this 'thinking style' is transferred to other subject areas. Therefore learning Euclidean geometry has a generalized beneficial effect on a variety of disciplines. Others believe that the amount of transfer is either minimal or nonexistent. What is your view on the question?

Transfer of logical thinking style from Euclidean geometry to other disciplines.

1: yes [0] 2: maybe some [4] 3: little or none [7] 0: NA [1]

None of the Main Interview participants indicated support for the notion that there is significant transfer of logical thinking from Euclidean geometry to other disciplines. Particularly noteworthy is the opinion of Participant 2.2, one of seven who chose 'little or none' ("I did my M.A. thesis on this topic").

Alternative Ways to Teach Logic

Traditional programming languages such as Fortran and Pascal tend to encourage a structured "top down" as opposed to "bottom up" approach to problem solving. Advocates of the inclusion of computer programming in mathematics education often suggest that it is an effective method for teaching students to think logically by which I suspect they mean to think in a "convergent", as opposed to "divergent" style.

The six Preliminary Interview participants had a variety of views with respect to the value of computer programming as a mathematical activity. Some felt that it should be included in the curriculum, others did not.

Participants in Stage 2 were therefore asked: (Question 4)

Some teachers I have spoken with take the position that *if* there is a transfer of a 'logical thinking style' from mathematics to other disciplines, then computer programming is a preferable alternative to Euclidean geometry for achieving this objective.

Computer programming as a preferable alternative for teaching logical thinking style.

1: preferable [3] 2: possibly [6] 3: do not agree [2] 0: NA [2]

Only those Preliminary Interview participants who had experience with computers seemed to favour this use of computers, and only three Main Interviewees gave clear indications of agreement, while eight seemed uncertain by selecting "possibly" or not providing an answer. In retrospect I feel that this question was not well phrased because there may be a lack of consensus of exactly what "logical thinking" means and how desirable such a trait may be (see discussion in the previous section). For example it may be that some people may consider a logical style to be a restricting one, favouring divergent thinking as being more creative.

Computer Programming as a Mathematical Activity

One of the questions that arose in the Preliminary Interviews was whether or not computer programming qualified as a mathematical activity.

This question was put to the Main Interview participants as follows:

(Question 5)

It has been suggested that 'training for logical thinking notwithstanding, computer programming is a worthwhile mathematical activity in its own right. How do feel about this?

Computer programming is a worthwhile mathematical activity in its own right.

1: agree [6] 2: possibly [2] 3: do not agree [4] 0: NA [1]

Although it would appear that the majority were in favour, the general lack of comments contribute to my feeling that the notion of including

computer programming in the curriculum on the basis that it is a mathematical activity would not be favourably received by mathematics teachers.

In addition to the fact (as shown in later data) that many mathematics teachers have limited experience with computer programming, I have the sense that teachers tend to have a rather restricted definition of mathematics - that they tend to equate it with mathematics curriculum content. Either that or they dismiss mathematics that falls outside the curriculum as something that does not concern them because they are *teachers* in the sense of *conscientious employees* rather than *mathematics* teachers. This question would be very interesting to pursue further.

Teacher Dialogues

In the introduction to this section, I mentioned that I would like to have explored the extent to which *fundamental curriculum reform* was perceived as being desirable. The major problem associated with pursuing this thought seemed to be related to communicating to participants what *I* meant by this term. Although several Preliminary Interview participants alluded to the desirability of placing greater emphasis on applications, as opposed to a theoretical treatment of mathematics, only one made a specific suggestion along the lines I had in mind -- that curriculum content should be rebuilt from first principles; that on the secondary level at least, this might mean a totally new list of topics and a totally new paradigm for school mathematics.

Another factor which interfered in my attempts at investigating this issue was the attitude that some teachers seem to have towards change of any

kind. There appeared to be a certain degree of impatience towards all changes because curriculum changes are not perceived as always being well thought out. To paraphrase Preliminary Interview participant 0.3:

I am concerned with introduction of new topics and new curricula. Sometimes they are just fads.

Additionally, as noted earlier, I was not successful in eliciting participants' ideas about hypothetical situations in general. This led to

Question 6:

I have found that it is difficult to get math teachers to talk about fundamental issues related to what should be taught as school mathematics. A prevalent view seems to be that the traditional content is so firmly rooted that only cosmetic, as opposed to fundamental, curriculum changes are worth even talking about.

Do you think that it is worthwhile for math teachers to engage in a dialogue in which no prior assumptions are made about what should be taught as mathematics?

It would be worthwhile for teachers to engage in such a dialogue.

1: agree [6] 2: possibly [2] 3: I doubt it [4] 0: NA [1]

The results indicates a certain amount of polarity. As with question #1, comments made by some participants could be interpreted as implying a certain amount of cynicism ("I am too much of a realist. It can't happen"). A general conclusion might be that teachers do not foresee radical curriculum changes as occurring in the immediate future, and that this is at least partly because mathematics teachers do not talk to each other about the nature of their subject.

A Specific Curriculum Alternative

In Question 7a some specific suggestions were posed related to curriculum reform offered by Preliminary Interview Participant 0.5. Question 7b was presented simultaneously with 7a so as to deal with the anticipated objection that students would not be properly prepared for post-secondary education.

Question 7a was:

Several teachers that I interviewed stated that they would like to see the mathematics topics dealt with in the curriculum broadened so as to include for example, economics, topology, fractals, optimization theory, mathematical history and biographies of mathematicians. The general idea is to give students an *appreciation for the subject and its broad scope and social implications*. Would this be desirable?

a) It would be desirable to broaden curriculum and stress appreciation
1: agree [4] 2: possibly [2] 3: I doubt it [6] 0: NA [1]

And Question 7b:

It has been suggested that if one were to broaden the curriculum as discussed in the last question, then 'catching up' with the traditional requirements of post secondary institutions could be managed in a relatively short time for those students for whom this was necessary if the post secondary institutions cooperated. Do you think this is possible?

b) It would be possible for selected students to catch up
1: agree [4] 2: possibly [2] 3: I doubt it [7] 0: NA [0]

The results and certainly the comments suggest that the teachers found it difficult to accept that the mathematics curriculum could be broadened to include the topics and the emphasis suggested. ("Too many people would have trouble if topics are expanded..." and "Waste of time. Students do not

think the way adults do.") One participant in particular, was preoccupied by the point that he made repeatedly in the interview... that there has to be streaming and that each stream needs a different curriculum... even with the qualifying effect intended by Question 7b.

It would be fair to state that the participants were not uniformly enthusiastic about an *appreciation* approach to mathematics curriculum. Neither were they confident that academic students would be able to "catch up" later. ("Students would be deficient in skills")

Time Requirements

The question of the sheer volume of present curriculum content and the amount of time available to "cover" the prescribed material arose in most of the interviews. Among the Summary Statements of the preliminary set of interviews there are several references to this, including:

- There is insufficient time to deal with topics to the desirable depth.
- There are too many things that are to be covered.
- Math deserves relatively more time than some other subjects.
- There is too much in the curriculum.
- Our curriculum is far too full.

The question used in the study was: (Question 8)

Some people feel that the number of hours available in school is insufficient to cover the prescribed mathematics curriculum to the desired level. Do you find that this is so?

There is insufficient time

1: insufficient [9] 2: almost sufficient [0] 3: sufficient [3] 0: NA [1]

These results clearly indicate that the majority of teachers feel that the curriculum does not allow sufficient time to for all students to properly master the expected material.

Teaching for Understanding

The question of the extent to which it is necessary and/or desirable to teach for understanding arose in several of the exploratory interviews. Although it was recognized that understanding was a desirable instructional objective in any subject area, it was frequently seen as an impractical one because this kind of instruction necessarily required more time than was available, and sometimes led to a breakdown in class discipline. One participant stated that mathematics involved the mastering of algorithms and that these often seem to have little to do with understanding.

Question 9 was intended to establish how the Main Interview participants felt about this topic:

There is a divergence of opinion among mathematics teachers on the whole topic of *teaching for understanding*. Some teachers feel that though teaching for understanding is *desirable*, it is *impractical* to insist on as a primary goal of mathematics instruction. The reasons cited include time constraints, the inability to assess understanding as opposed to *performance* of algorithms, etc. In any event, understanding develops later, as the student matures and perhaps uses the math he has learned. The opposing view is that teaching that does not result in understanding is ineffective at best, certainly regrettable, and deplorable at the worst. Do you feel that teaching for understanding is generally impractical under present circumstances? Do you feel that teaching for understanding is sufficiently practical to make it a curriculum priority?

a) Teaching for understanding is generally impractical
1: impractical [5] 2: maybe [2] 3:practical [5] 0: NA [1]

b) Curriculum priority?

1: high [1] 2: greater than now [5] 3: not a priority [4] 0: NA [3]

Participants were approximately evenly divided about the practicality of teaching for understanding. A study of the comments, however, supports my feeling that some of those choosing "practical" were not really convinced. Among those choosing "practical", one made a comment to the effect that this is already being done to some degree. All of those choosing "impractical" made comments to reinforce their opinion ("No matter what you do, some students won't understand - they have to do it by algorithm").

Two drew attention to the very legitimate difficulties attendant upon *testing* for understanding.

When it came to the question of priorities five chose "greater than now" and four "not a priority." It appears then, that some teachers were ambivalent about teaching for understanding because of the practical difficulties involved ("Understanding is impractical if you have to cover the course..."), while others felt that they already did teach for understanding to a satisfactory degree.

Discovery Learning

Discovery Learning involves the notion that children learn things more effectively by exploring and making inferences than they do by being told things. This topic arose in only two of the exploratory interviews. Here the views were that although ideally good teaching should involve a certain amount of student discovery, it is not essential. In any event the practicalities of the classroom dictate that a high degree of structure be imposed on the

subject, that some considerable amount of guidance be provided, and that this is very much more difficult to achieve in a discovery learning environment.

Question 10 asked:

Advocates of Discovery Learning believe that much mathematics can be discovered and that this will lead to understanding. Again, opponents claim that this is *impractical*. What is your view on practicality and desirability?

a) Discovery Learning is desirable

1: Yes [3] 2: Possibly [6] 3: No [0] 0: NA [4]

b) Discovery Learning is practical

1: Yes [4] 2: Possibly [5] 3: No [3] 0: NA [1]

The data from part a of this question are noteworthy in that here we see the highest proportion of "No Answer" to any of the questions posed. This is likely related to the conflict that arises in participants' minds between desirability and impracticality. No one indicated that Discovery Learning is *undesirable* by selecting "No".

The comments reveal a degree of ambivalence towards Discovery Learning resulting from teachers' preoccupation with the day-to-day realities of the profession, particularly time limitations ("It has its place - but we have to be efficient.")

Problem Solving

Problem Solving is not a particularly well defined concept. It seems to be associated with logical thinking and with problem-solving heuristics. There is not universal agreement on the extent to which these heuristics are transferable within mathematics, and from mathematics to other disciplines.

The Summary Statements constructed on the basis of the Preliminary Interviews include the following:

- Universities assume that problem-solving skills and logic learned in math are transferable.
- We do not fit enough time to allow students to participate in cooperative learning approaches to problem solving.
- Problem-solving skills (heuristics) learned in mathematics are transferable to other areas.

Participants in the main interviews were presented with the issue in

Question 11:

A topic we have heard much about in the last few years is *problem solving skills*. The concept is similar, but not identical to the notion that one can teach a logical thinking style and that this is transferable to other disciplines. The general idea is that familiarity with problem solving *heuristics* affect learning in a generally beneficial way. The opposing view is that there is no evidence to support this contention. Problem solving skills in math are specific to solving math problems of the type used in the problem solving skill training. Do you feel that problem solving heuristics improve performance on math problems other than those practiced? Do problem solving heuristics learned in math class improve problem solving in other disciplines?

a) Heuristics are transferable within math

1: yes [10] 2: maybe [2] 3: I doubt it [1] 0: NA [0]

b) Heuristics are transferable to other disciplines.

1: yes [7] 2: maybe [5] : I doubt it [1] 0: NA [0]

The teachers interviewed did feel that problem solving heuristics were transferable within mathematics, but they were less certain that this transfer took place to other disciplines. The two doubtful responses came from the participant who has been mentioned elsewhere as the teacher who was most

active in computer use, and who was most vocal in his concerns with the need for streaming.

The question of how many of the participants in fact taught problem solving heuristics, or whether these heuristics could be taught in a setting other than mathematics was not asked. On reflection this latter question could have been included to good effect.

Curriculum Emphasis

Question 12 was intended to elicit the participants' attitudes towards the use of computers and calculators in an *indirect* manner and also to determine which topics teachers considered to be relatively more important than others. The first six parts (a to f) are *paired* so as to indicate the *relative emphasis* that should be placed on the use of calculating tools versus traditional approaches to computation. Parts g and h seek opinions on what emphasis should be placed on Statistics and Probability versus Number Theory. Parts i and j concern themselves with possible alternate topics.

Question 12:

If one were to establish a *core curriculum* - something that *every student* regardless of academic program should be required to have a degree of mastery of, should it include more, the same, or less of the following topics than is now the case?

a) Calculator based arithmetic

1: More [7] 2: The same [6] 3: Less [0] 0: NA [0]

b) Arithmetic algorithms/skills

1: More [2] 2: The same [7] 3: Less [4] 0: NA [0]

c) Computer based algebra

1: More [6] 2: The same [3] 3: Less [2] 0: NA [2]

d) Algebra algorithms/skills

1: More [0] 2: The same [8] 3: Less [3] 0: NA [2]

e) *Application* geometry

1: More [9] 2: The same [4] 3: Less [0] 0: NA [0]

f) Euclidean geometry (drafting, sketching, visualization)

1: More [0] 2: The same [9] 3: Less [3] 0: NA [1]

g) Probability and statistics

1: More [5] 2: The same [7] 3: Less [1] 0: NA [0]

h) Number theory

1: More [0] 2: The same [10] 3: Less [3] 0: NA [0]

i) Math history/Social aspects

1: More [10] 2: The same [3] 3: Less [0] 0: NA [0]

j) Personal finance

1: More [8] 2: The same [5] 3: Less [0] 0: NA [0]

These results indicate that in general, the teachers interviewed feel that relatively greater emphasis should be placed on:

- the use of computers and calculators in algebra as well as in arithmetic,
- applications of geometry as opposed to a more traditional treatment,
- probability and statistics, as opposed to number theory.
- Math history/Social aspects and Personal finance.

A summary of the number of "More's", "Less's", "Same's", for each participant follows:

Participant	More	Same	Less
1.1	5	3	2
1.2	5	5	0
1.3	1	7	0
1.4	3	4	0
1.5	5	7	4
1.6	3	8	0
1.7	1	8	1
2.1	5	5	0
2.2	4	4	2
2.3	3	5	2
2.4	3	7	0
2.5	5	0	1
2.6	3	6	2
Totals	43	63	12

No participant balanced the amount of material that should be added to the curriculum with a corresponding suggestion as to what should be deleted.

In fact, half of them made no suggestions for reduction of material at all

The comments included:

- Not familiar with calculator/computer based algebra.
- There should be less work on logarithms.
- Algebra and computers are mutually exclusive.

One participant emphasized that mathematics education should be applications oriented.

With the caveat that the small number of participants makes generalization open to debate, these results tend to support the following hypothesis about teachers' beliefs with respect to needed increases in time:

- *At least* as much time should be devoted to calculators and computers, vs. algorithm based arithmetic and algebra as is now the case.

- There should be more emphasis devoted to applications vs. Euclidean geometry as well as to the historical and social aspects of mathematics and to personal finance.
- There is already sufficient time assigned to number theory.
- A significant number of participants indicated that a greater amount of time should be allocated to probability and statistics.

Influence in Curriculum Design

One participant in the preliminary interviews made specific mention of his notion that *University Professors* influence curriculum design. He seemed to have a negative view of this, which I later realized, was due to his feeling of rejection for not having been invited to participate in the Mathematics Curriculum Revision Committee. This is developed in more detail in the next question. In any event, Question 13 asked:

A frequently held perception is that the requirements of University Professors influence the public school curriculum to some considerable degree. Some see this as desirable to maintain standards. Some see it as undesirable. Some see the role of secondary math teachers as preparing students for the requirements of post secondary institutions, some feel that we know better what is best for our students, that we have a broad mandate and that post secondary institutions should 'pick up where we left off'. Naturally, there is some influence, but is it unduly strong? Do you feel this influence is desirable? Do you feel our primary mandate is to meet this need?

a) Influence is unduly strong

1: yes [7] 2: not really [4] 3: I doubt it [2] 0: NA [0]

b) Influence is desirable

1: yes [2] 2: not really [3] 3: I doubt it [5] 0: NA [3]

c) Our mandate

1: meet post-sec. requirements 2: there is no conflict

3: we should determine students' needs

On reading the data I decided that part c of this question was too vague and it is therefore ignored in the data analysis. The use of the word "unduly" in question 13a seemed to convey a great number of unrelated value judgments. As a result, this word may have prejudiced the results.

Incidentally, when in a casual conversation, I asked a real university professor this question, his response was something like: "Certainly they influence the curriculum and so it should be. The problem is that it is the wrong university professors who do the influencing!".

The teachers interviewed seemed to feel that the influence of post secondary institutions on secondary curriculum is strong and not particularly desirable. On the other hand, it was my impression that this influence was accepted as a fact of life, and that there was no resentment about it.

The Mathematics Curriculum Committee

One of the participants in the preliminary interviews was noticeably more concerned with discovery learning, teaching for understanding and, in particular cooperative learning than the others. These concerns were definitely connected with the difficulties he was experiencing with a class of nonacademic students. He also discussed at great length the Mathematics Curriculum Revision Committee. The following are the Paraphrased Summary Statements attributed to him:

- The curriculum committee should investigate what is being done around the world.

- The curriculum committee should canvass opinions of people at all levels about views on essential and nonessential math.
- I would have liked to be on the curriculum committee.
- Some progress was made in the recent curriculum revision, for example, Data Analysis.
- I am not happy with the curriculum we have.
- The curriculum committee originally simply left everything in and added more to it.
- The curriculum committee did not do its job well.
- I feel a part of the curriculum designing process.
- The curriculum committee didn't listen to me.
- I was involved in writing letters etc. to the curriculum committee.
- It bothers me that I'm not more deeply involved in the curriculum design process.

Question 14 attempts to explore the issue of the Curriculum Committee.

The curriculum committee at the Ministry is seen by some as a body apart from practicing mathematics teachers. They appear as a conservative clique who makes changes with reluctance, preferring to preserve the status quo. Others perceive this group as being representative of mathematics teachers and that they do a competent job of providing a realistic and generally 'good' curriculum. How do you feel about this?

a) clique

1: yes [7] 2: somewhat [4] 3: no [2] 0: NA [0]

b) conservative

1: yes [2] 2: somewhat [3] 3: no [5] 0: NA [3]

c) competent

1: yes [3] 2: somewhat [3] 3: no [4] 0: NA [3]

Judging by the number of "No opinion" and the paucity of comments, it would appear that teachers are not very familiar with the Mathematics Curriculum Committee and its workings.

It would be fair to say that among the teachers interviewed there was a general perception that the Mathematics Curriculum Committee was cliquish. With respect to conservativeness and competence no significant statement of overall opinion can be made. One comment can be classified as decidedly negative, and one was decidedly positive.

Word Associations with Mathematics

Only one of the initial six participants responded positively to the request for 'associations' with mathematics. Here are some of the Summary Statements assigned to him:

- Math requires a highly developed intellect.
- Math involves a lot of abstraction.
- Many math applications developed from theoretical considerations.
- Math requires good work habits.
- Girls can handle abstract concepts earlier than boys.
- Intellectual readiness is important.
- Logic, rigour, discipline, attention to detail and carrying something through are related
- I identify with the intellectual-abstract view of math.
- I think of math as fun.
- I love the subject partly because of its 'purity'.

In Question 15 I attempted to elicit similar comments from the other teachers:

My discussions with mathematics teachers have revealed that some teachers find it possible to describe mathematics with words such as *difficult, disciplined, beautiful, delightful, intellectually rewarding* etc. Others find this awkward because all such descriptions are simply a matter of personal perspective. What words do you feel comfortable associating with mathematics, if any?

Responses:

- All of the above.
- Interesting, exciting.
- (none)
- Challenging, interesting, useful.
- Difficult, useful {I like it to be}, 'controlled' thinking {you can come to a conclusion}, acceptable universal definitions/conclusions {describes formal aspects of the world}.
- Main association: It is a language.
- Challenging, logical, objective.
- Practical, useful.
- Consistent, integrated, 'self-supporting'.
- Discipline, logical, delightful
- Discipline, delightful. You can see the Eureka syndrome. Popcorn popping. Kids get self-satisfaction when they pop.
- Difficult - but only for some. It hasn't got beauty if its difficult.
- There is beauty in it. The way things unfold. Discipline.
- Interesting. Fascinating.

I will not attempt to draw any conclusions from these responses beyond the observation already alluded to that many teachers did not seem to feel "comfortable" with this sort of request.

Relative Emphasis in Mathematics Instruction

Question 16 is another attempt to determine what relative emphasis different aspects of mathematics instruction should have in terms of time allocation.

There seems to be several schools of thought: For pedagogical reasons, all, or nearly all school mathematics should be applications oriented. It should largely be directed at dealing with real engineering, financial and social problems. The other school of thought suggests that mathematics should be taught from the theoretical perspective. Students should learn to appreciate math for its own sake and applications should be used primarily to illustrate mathematical concepts. A third school of thought advocates a 50 - 50 blend of these approaches. A fourth school stresses the importance of high levels of skill in performing mathematical operations. What proportion of time would you allocate to the applications versus abstract versus skills?

Applications - abstract - skills ratio:

_____ % vs _____ % vs _____ %

The data was difficult to deal with. I decided to assign each participant to an arbitrary category for each of *applications*, *abstract/theoretical* and *skills* on the basis of how time was allocated to this emphasis *vis a vis* the other two. According to this scheme, three participants assigned a greater value to Applications while two assigned to the Abstract/Theoretical category a shared high value, etc. These results are summarized here:

	Applications	Abs./Theo.	Skills
Greatest emphasis	3	1	5
Shared high	3	2	1
Middle	3	2	2
Shared low	3	4	3
Lowest	1	4	2

The average of the portions of time suggested by the participants that should be allocated to applications was 39%, for abstract/theoretical 26% and for skills 35%.

These numbers suggest that the participants felt that the majority of time should be devoted to applications, while relatively little time should be devoted to the theoretical aspects of mathematics. In the case of time that should be allocated to skills the numbers in the last column seem to indicate a certain amount of uncertainty. Possibly this is related to the ambivalence that some teachers feel about the relationship between skills in performance of mathematical algorithms and understanding of mathematics.

Some Problems Related to Teaching Mathematics

The preliminary interviews left no doubt that many teachers experience a high level of frustration in teaching mathematics. Aside from the problems imposed by time constraints particularly as they pertain to differing student needs, these frustrations seemed primarily to result from the difficulties associated with motivating nonacademic students. There were noticeable differences in emphasis placed on this issue by different participants, and it is suggested that a teacher's preoccupation with the problem of student motivation is very much a function of *his present teaching assignment*. One participant, in particular, elaborated on the topic of nonacademic student motivation and related several stories from his classroom experience to illustrate the point. Another problem hinted at by two participants was the difficulty they had in maintaining class discipline

when the instructional mode shifted from a teacher-centered to a student-centered one.

Questions 17:

It is generally conceded that mathematics is a difficult subject. This is frequently ascribed to its highly linear structure. 'You can't do this until you have that.' Other reasons cited are lack of perceived relevance by students, lack of quality teachers and the notion that a portion of students simply do not have the *aptitude* for the subject. Some teachers complain of lack of student motivation. What do you think? Math is difficult because:

a) Linear structure

1: definitely [3] 2: only partly [9] 3: not at all [1] 0: NA [0]

b) Lack of relevance

1: definitely [3] 2: only partly [9] 3: not at all [1] 0: NA [0]

c) Teacher quality

1: definitely [3] 2: only partly [9] 3: not at all [1] 0: NA [0]

d) Student aptitude

1: definitely [6] 2: only partly [5] 3: not at all [1] 0: NA [1]

e) Student motivation

1: definitely [6] 2: only partly [5] 3: not at all [1] 0: NA [1]

The recorded responses to question parts d and e all contain six 'definitely' entries, while those to parts a, b and c contain only three. This observation suggests that teachers tend to attribute difficulties associated with teaching mathematics more to student aptitude and motivation, than to difficulties inherent to the subject or teacher limitations. The use of 'only partly' as a response option clouded the inferences that could be drawn from the responses. In retrospect I realize that it would have been better if the

question had been presented as a series of comparisons so as to elicit *comparisons* of relative importance of various factors.

Information on Participants

I did not formally collect any personal information on the Preliminary Interview participants because I felt that this might be awkward in view of the fact that they were social as well as professional acquaintances. However I know that with the exception of one, they are professionally active, attending professional meetings and conferences regularly. All have taught for a considerable period of time. I also know that two have extensive experience with the use of computers, one much more so than the other; and that the others could not realistically be described as computer literate.

Also, these people indicated that they had not really learned much in the way of new mathematics since leaving school, but had, through their teaching, gained a much better understanding of previously learned mathematics.

In questions 18, 21, 22, 23 and 26 I collected various personal data on each main stage participant with the intent of building some sort of profile that might be compared to other data. This was prompted by the suggestion made by one of the preliminary participants that a useful question might be "Would you describe yourself as a curious person?"

Question 18:

It is appreciated that these questions are of a personal nature, and you may prefer not to answer some of them. Would you describe yourself as a curious person - interested in how things work and how to fix things? Are you excited about your job - do you look forward to going

to work? Do you feel that school administrative duties and discipline problems are 'getting you down'? Do you feel that you are a 'better than average' math teacher? Would you rather be doing something else? How do you feel about other math teachers?

a) Curious

1: more than most [11]

3: less than [1]

2: about the same as [1]

0: NA [0]

b) Enthusiastic

1: more than most [11]

3: less than [1]

2: about the same as [1]

0: NA [0]

c) Frustrated

1: more than most [11]

3: less than [1]

2: about the same as [1]

0: NA [0]

d) Good teacher

1: more than most [11]

3: less than [1]

2: about the same as [1]

0: NA [0]

e) I'd rather be doing something else

1: definitely [2] 2: would consider [4] 3: prefer to teach [7] 0: NA [0]

f) Other teachers

1: highly skilled [0] 2: mostly good [10] 3: many poor [2] 0: NA [1]

These results from parts a to e of this question indicate that teachers participating in the main study had a good self-image and like their jobs. The results from part f might indicate that the teachers were not quite as enthusiastic in their assessment of their colleagues.

Using Computers to Solve Numerical Problems

Calculators are now widely used in mathematics education. Initial objections and criticisms made by those teachers who still resist their use seem to be based on the twin issues of thinking and understanding. The

perception seems to be that doing algorithmic arithmetic involves thinking and leads to understanding, whereas using calculators inhibits thinking and does not allow understanding to develop. Similar objections will presumably arise with respect to algebra as students and teachers get access to equation solvers and similar symbol manipulating computer programs.

Question 19 asked how desirable participants view instruction in the use of computers to assist in the solution of numeric problems as part of their mathematics courses. My own view is that there is no operational distinction between calculators and computers. I originally assumed that this view was widely held. I now realize that this is not the case.

Question 19:

When computers are used to solve *real world* problems the techniques and algorithms employed are frequently quite different from traditional ones. This is because of the tremendous *number crunching* power of computers makes these alternate approaches more efficient. They may not be as *elegant* but they work! Do you feel that students should have access to computers and learn to use such techniques as part of the high schools math program or is this not consistent with what our curriculum should stress?

Students should have instruction in such techniques

1: definitely [9] 2: maybe a little [2] 3: no [2] 0: NA [0]

Nine participants endorsed the use of computers for numerical algorithms by choosing "definitely". Two said "no". One comment by one of the latter, ("Takes lots of time. We used to do "base 2" in grade 8. It doesn't last.") is revealing in so far as it suggests what the participant's underlying thoughts may have been when the statement was made. *The use of calculators and computers must be restricted because it is very time consuming to teach students to use these tools and also because it takes more*

time to use these tools. The reference to base 2 refers to an aspect of New Math of the late 1960's and early 1970's in which arithmetic algorithms were practised on numbers of bases other than 10. The idea was that this would lead to better understanding of how and why the algorithm worked. My interpretation is that the participant is saying here that *teachers should ensure that students are proficient in the execution of algorithms without being overly concerned with their understanding of the mechanisms involved because such understanding is only temporary.* In other words; rote operations are retained, understanding is not.

Using Computers in Algebra

Summary statements based on the preliminary interviews indicated that these teachers had some misgivings about the algebra content of the mathematics curriculum. Here are some sample statements:

- Curriculum designers have recognized the need for less emphasis on algebra.
- I did better in geometry than in algebra.
- Much of algebra can be left out, for example factoring trinomials with first coefficient other than 1.
- Some students who did not do well in algebra were pretty good when it came to deductive reasoning.
- Much of algebra is probably excluded from the fundamental set.
- Cooperative learning is one method of addressing the needs of individuals.
- A lot of algebra could be eliminated.
- Much routine algebra which can be done by machines should be left out.

None of these people mentioned the possibility that symbol manipulating computer programs might change the way in which algebra is taught or what is taught as algebra. This was addressed in Question 20:

Computers may be used as powerful calculators to solve problems numerically. Using calculators/computers for routine numerical calculations is generally felt to be an acceptable practice. There are also computer programs that will determine symbolic solutions to mathematical problems. They will integrate, differentiate, and so on. Do you feel that instruction in the use of these tools would be desirable in a high school?

Students should have instruction in the use of symbol manipulating computer programs.

1: definitely [8] 2: reservations [3] 3: probably not [1] 0: NA [1]

As with computer use for numerical calculations, there was a definite positive response with respect to students using symbol manipulating computer programs though not as strong as was the case with respect to numerical computations. I find this positive response rather surprising in view of the fact such computer programs are not widely available, and I would therefore suspect that few teachers have personal experience with them. My own very limited experience with symbol manipulating computer programs (MuMath and Mathematica) does not make it clear to me what purpose this kind of activity serves.

Two of the comments of those who expressed reservations or did not endorse this use of computers refer to inhibition of understanding. One makes the proviso that students must be able to do the algorithms manually first, and one makes the puzzling suggestion that using computers might lead to making it more difficult to motivate students.

The participant who made the reference to "base 2" and chose "no" in Question 19 (Participant 1.3) endorsed the use of symbol manipulating computer programs -- but, again, provided that understanding was achieved prior to their use.

Computer Experience Inventory

As mentioned previously, only two Preliminary Interviews participants could be described as being 'computer literate' according to *my* understanding of that word. This prompted Question 21:

a) I use a computer for my personal and professional needs.
0: no [5] 1: recently started [1] 2: have done so for some time [7]

b) I can program in Pascal
0: no [10] 1: cursory [1] 2: some facility [1] 3: versatile [1]

c) I can program in BASIC
0: no [4] 1: cursory [2] 2: some facility [3] 3: versatile [4]

d) I can program in Logo
0: no [9] 1: cursory [2] 2: some facility [1] 3: versatile [1]

e) I can program in Other language
0: no [10] 1: cursory [1] 2: some facility [0] 3: versatile [2]

I am familiar with the following application programs:

f) spreadsheets
0: no [3] 1: cursory [3] 2: some facility [4] 3: versatile [3]

g) graphing utility
0: no [1] 1: cursory [2] 2: some facility [6] 3: versatile [4]

h) word processor
0: no [5] 1: cursory [1] 2: some facility [3] 3: versatile [4]

i) graphics

0: no [5] 1: cursory [2] 2: some facility [3] 3: versatile [3]

j) other:

0: no [10] 1: cursory [0] 2: some facility [1] 3: versatile [2]

Half the participants reported having used a computer for their own needs for some time and having some facility or being versatile with spreadsheets, a word processor and a graphics program. Participant 2.1 stated that he had used a computer for his personal and professional needs, yet for part h) indicated that he did not know how to use a word processor. This turned out to be actually the case. He has a personal computer which he has programmed himself to record marks.

The data also shows that most participants had limited programming experience. At least five had virtually none. Only two had extensive programming experience. The most commonly known language was BASIC.

All but one claimed to have some experience with a graphing utility. Whether this experience included actual operation and experimentation with such software in a classroom setting, I did not, and in retrospect, should have, asked.

Participants in the main set of interviews rated themselves better than I would have anticipated after my experience with participants in the preliminary set of interviews.

Personal Mathematics Learning Inventory

When I attempted to steer the subject of the discussions with participants in the preliminary study onto what might be called *non-standard*

mathematics topics, such as Fractal Geometry and Discrete Mathematics, I detected a tendency on the part of the interviewee to avoid comment. It occurred to me that maybe some teachers may not be conversant with these topics. Thus, Question 22:

When asked what (new) math they have learned since leaving university, some teachers stated that they hadn't really learned any; but that they had gained a greater understanding of the math they had previously been taught. Others echoed the statement about increased understanding, but went on to list several mathematical topics that they have added to their repertoire more recently. What about you?

Which new math topics have you learned?

1: hardly any [7] 2: one or two [3] 3: yes - several [3]

Of those that mentioned specific topics three stated that they had learned "computer work". Two made reference to what might be thought of as novel topics (Quantum Mechanics and Linear Programming).

These results suggest that mathematics teachers may not be characterized by a high degree of curiosity about new developments in their discipline. In fact, very much the opposite seems to be the case. This lack of curiosity, if that is what it is, seems to strongly contrast with the responses given to the earlier self assessment question about how they rated themselves with respect to curiosity compared to other people.

Of interest in this connection are the reactions of several of the preliminary participants to the question "Do you consider yourself to be a mathematician?" Their responses tended to be along the lines of "I am primarily a teacher".

Professional Activities

I knew that all but one of the preliminary participants were active in professional organizations, attend workshops, and so on. What about mathematics teachers in general? Question 23 asks:

Some teachers make a habit of going to every conference or workshop they can. In addition they attend professional association meetings and read journals. Others feel that this sort of thing can be greatly over done. There are more important things in life. How many conferences/workshops did you attend in the last two years? Did you learn new pedagogy? Did you learn new mathematics? How heavily are you involved in your PSA? [Provincial Specialists' Association] Do you read professional journals?

a) How many conferences/workshops in past two years
0: none [2] 1: a couple [5] 2: several [6]

b) New pedagogy
0: not really [4] 1: some [8] 2: much [1]

c) New mathematics
0: not really [8] 1: some [4] 2: much [1]

d) PSA attendance
0: do not attend [10] 1: occasionally [2] 2: regularly [1]

e) Read Journals
0: hardly ever [4] 1: occasionally [2] 2: regularly [7]

f) (Number of journals mentioned)
0: zero [3] 1: one [5] 2: two [3] 3: three [0] 4: four [2]

All but two participants stated that they had attended one or more conferences/workshops in the past two years. Two of these had "not really" learned any new pedagogy or mathematics. Three of the thirteen attended Provincial Specialists Association meetings. Four "hardly ever" read

professional journals. Five cited *Vector* (the journal of the British Columbia Association Mathematics Teachers) and five, *The Mathematics Teacher* (a journal of the National Association of Teachers of Mathematics) as journals they read regularly.

I cannot point to any data that supports my feeling that the level of professional involvement was less than what might be desired. But I sensed that if I had, for example, asked specific questions about articles appearing in recent issues of the journals cited above, it might be revealed that these were not as heavily read as the responses indicated.

Appropriate Uses of Computers in Mathematics Instruction

Chapter One includes a discussion of the various uses to which computers can be put in mathematics education. Prior to Question 24 being administered, I first summarized this discussion so as to encourage maximum "common understanding" of the terminology.

Question 24:

I am interested in finding out just how much computers are being used in mathematics instruction in your school. There is another question; it has to do with appropriate use. Some people aren't sure that mathematics instruction should involve computers at all. Others, though endorsing using computers for some teaching/learning activities, are much less enthusiastic about other uses. How do you feel about these uses of computers, and to what extent have you tried them?

Computers used for:

a) Audio-visual aid

(i) Should: 1: yes [9] 2: maybe [3] 3: no [1]

(ii) I have: 1: yes, often [1] 2: once or twice [1] 3: no [11]

(iii) In the past year I have used this with my class(es) approximately... hours.

b) Programmed Instruction

- (i) Should: 1: yes [5] 2: maybe [3] 3: no [5]
 (ii) I have: 1: yes, often [2] 2: once or twice [2] 3: no [9]
 (iii) In the past year I have used this with my class(es) approximately... hours.

c) Assessment

- (i) Should: 1: yes [4] 2: maybe [5] 3: no [4]
 (ii) I have: 1: yes, often [0] 2: once or twice [1] 3: no [12]
 (iii) In the past year I have used this with my class(es) approximately... hours.

d) Provide Exploratory models

- (i) Should: 1: yes [10] 2: maybe [2] 3: no [1]
 (ii) I have: 1: yes, often [4] 2: once or twice [2] 3: no [7]
 (iii) In the past year I have used this with my class(es) approximately... hours.

e) Be used to learn programming

- (i) Should: 1: yes [4] 2: maybe [5] 3: no [4]
 (ii) I have: 1: yes, often [1] 2: once or twice [4] 3: no [8]
 (iii) In the past year I have used this with my class(es) approximately... hours.

School A has had a computer lab for several years. It was used to a considerable extent by a teacher of my acquaintance who has since left the school. I know he went out of his way to involve as many mathematics teachers as possible in the use of the equipment. The first seven teachers participating in the main set of interviews, with the exception of one who just joined the staff, not only had access to computing facilities and in-house assistance, but had been encouraged to use them for some time. School B has also had computing facilities for some time; but it is my understanding that the mathematics teachers at this school were not encouraged to the same degree.

Six of the seven teachers in School A, which does not have an LCD/overhead projector display system, said that the audio visual use of computers is a "should" application. Of these, four in their comments, made reference to their desire to make use of such a device.

In School B, where they *do* have a display unit, only three of the six picked "should", and one actually said "no". Of the three who did not pick "should", one commented on the need for student interaction ("Presumably it is no worse or better than any other teacher directed activity"), one said he hadn't seen any decent software and one suggested that he had tried it and found it cumbersome. Part (ii a) revealed that only two had actually done this. Thus the school that does not have it, wants to, while the school that does have it, does not want to.

Fewer teachers seemed to be in favour of computer-based programmed instruction than the audio visual aid application. Four claimed to have tried this to some degree; only one of these claimed to have done so "often". The comments were almost uniformly unenthusiastic; the main complaint was the perceived lack of good software.

Only four of the thirteen teachers responded that computers 'should definitely' be used to assess students. The comments revealed concerns with security as one reason for this.

The suggestion of using computers to explore mathematical models was enthusiastically received with ten of thirteen choosing "definitely". A surprising number claimed to have tried this.

The reaction to learning computer programming in mathematics was positive, but decidedly not enthusiastic. Approximately one-half of the

teachers had used a computer or computers at least "once or twice" in their mathematics teaching.

The data shows that as far as to what computers *should* be used for in mathematics education, there is considerably greater endorsement of the "audio-visual aid" and "exploratory models" categories than there are for the others. Only four teachers chose "yes" for including programming a mathematics course; these were all from School A. The only two that expressed hesitation about "exploratory models" were from School B.

In the "I have done this" column for each computer application:

- Audio-visual aid:** Two people from School B claimed to have done so to some extent. I happen to know that this school has equipment for this purpose (overhead projector display), while School A does not (see comments by 1.1, 1.2 and 1.3).
- Programmed Inst.:** Two people from each school claimed to have done some of this last year.
- Assessment:** The one person who claimed putting into use this computer application commented that security was a problem.
- Exploratory models:** This was apparently the most popular use with six teachers claiming to have made some use of this approach. It would appear, however, that only two (both from School A) engaged in this to a significant extent.

Programming: Five teachers claimed to have done at least some. Again it appears that the same two teachers as mentioned in the last paragraph had significant amounts of this activity with their classes.

Methodology

One of the preliminary interview participants made the suggestion that the new mathematics curriculum implied the use of manipulative materials to a greater extent than is now commonly practised, but according to another there simply isn't time for it. Question 25 asked:

To what extent is it desirable to make use of manipulative materials, models, films, videos etc. in teaching mathematics? Is it practical? Do you do it? What resources are available?

a) Desirability

1: very [6] 2: beneficial [4] 3: marginal [3]

b) Practicality

1: very [2] 2: somewhat [7] 3: marginal [4]

c) I do this

1: frequently [0] 2: occasionally [9] 3: infrequently/never [4]

d) Availability

1: inadequate [9] 2: sufficient [3] 3: much [1]

After reviewing the data and comments I conclude that the participants in this study, though they for the most part recognize the desirability of making use of manipulative materials and related techniques, do not consider them generally practical and rarely use them in their classroom practise.

They tend to cite "lack of availability of suitable materials" as the main reason for their lack of manipulative materials.

Teaching Assignment

Question 26 asked:

How many years have you taught? _____

Mostly math grades 8, 9 and 10

Mostly math grades 11 and 12

Math at all levels

Mostly math

Math as well as a significant amount in other subjects

I prefer teaching math to other subjects because _____

I'd rather teach something else because _____

I consider myself adequately prepared to teach math

I do not consider myself adequately prepared to teach math because _

a) Years taught

1: Less than 5 [1] 2: 5 to 10 [1] 3: 11 to 20 [4] 4: More than 20 [7]

b) Predominant level

1: junior high [3] 2: senior high [1] 3: all secondary levels [9]

c) Subject area

1: mostly mathematics [6] 2: a significant amount of other subjects [7]

d) Teaching preference

1: mathematics [8] 2: other subjects [5]

e) Preparation

1: I'm adequately prepared [13] 2: I'm not adequately prepared [0]

The average number of years taught was twenty, and seven had taught longer than this. The newest teacher had just returned from an overseas assignment with CUSO; the second-newest teacher had worked in industry

for some time. As a group, then, and with the exception of the newcomer, these people could hardly be described as novices in the field.

Half had taught a significant amount of other subjects, while half had taught mostly mathematics. Most preferred to teach mathematics and all of them considered themselves to be adequately prepared.

Unanticipated Difficulties

In several interviews it became apparent that my understanding of the terminology associated with a topic differed from that of the participant. It would seem that this kind of problem is likely to be encountered in most data gathering activities. The following is a discussion of some of the terminology of which I felt the participants and I lacked common understanding.

Computer Assisted Instruction

A profusion of similar terms are in common usage. In addition to Computer Assisted Instruction (CAI), there is Computer Based Instruction (CBI), Computer Managed Instruction (CMI), Computer Assisted Learning (CAL), etc. I do not believe that the use or definitions of any of these terms or their acronyms have been standardized and in any event it is unlikely that the majority of people have given a great deal of thought to precisely what the various terms imply. In my lexicon the term Computer Assisted Instruction is most useful when it is restricted to computer based tutorial or programmed learning activities, which does not include demonstrations of computer programs, the use of the computer for

assessments, or for drill and practise. I feel confident that such specificity of definition was not shared by the participants.

Exploratory models

Although it might, in theory, be possible to create exploratory models outside a computer environment, to me the term implies a computer-based "microworld" in which "the rules of the game" are discovered by trial and error and hypothesis testing. Conceptual understanding of this meaning of the term is probably only accessible to people who have themselves participated in "microworld" based activities such as Logo. It is possible to construct a kind of exploratory models on spreadsheet templates - but in reality such models are limited in their range of application; and limited also in that one must create artificial barriers to prevent the student from discovering "short cut methods" at solutions.

Discovery Learning

Presumably discovery learning is, or ought to be, the result of student interaction with exploratory models. Discovery learning is not restricted to computer based interactions in theory; though in practise it may be difficult to achieve without the use of computers on anything but a sporadic basis. There is probably a range of understandings associated with this term, differering by the extent to which the discovery is contrived.

Use of computers for personal and professional use

There are at least two ways in which this term can be interpreted when applied to teachers. It might be interpreted so as to mean entering of

marks and/or attendance data into a previously configured system. On the other hand it could be argued that this type of computer use is not very different from ringing up a sale on a computerized cash register. Some people, I suspect, would not consider a teacher as using computers for personal and professional use unless he or she at least had a reasonable level of competence with a word processor.

Public perception of mathematics

This term could be taken to mean perceptions about ease or difficulty attendant to taking a mathematics course in school, perceptions about the everyday usefulness of what one would learn in such a course, perceptions about the need for mathematics for further education, and many other interpretations.

Logical thinking

Does logical thinking imply formalized linear thinking or does it imply divergent thinking? Are there other thinking models? Which is the more "creative"? Which is the more useful? Which is associated with mathematics in the popular mind? Which characterizes successful mathematicians? Which is developed by studying mathematics? Is the type of logical thinking developed by the study of mathematics beneficial to the student pursuing a career in political science? I have come to realize that I know the answers to none of these questions.

CHAPTER FIVE

SYNTHESIS OF OBSERVATIONS AND SPECULATIONS STIMULATED BY THE STUDY

Despite the limited number of participants I have found it possible to synthesize some general observations that may characterize secondary mathematics teachers in general. In addition, a number of apparently relevant questions and speculations were stimulated by the study.

Gender of Mathematics Teachers

All of the participants in this study were male. This raises the question of what is the ratio of male to female mathematics teachers generally and how this ratio compares with that of other disciplines. Does the ratio vary between cultures? Is it changing? What are the implications for how teachers, students and the public view the subject? What, if any, are the curriculum implications?

Professionalism

The question of secondary mathematics teacher professionalism can be viewed from several perspectives. To what extent is professionalism characterized by a teacher's interest and competence in the subject -- in this case secondary level mathematics, and by interest and competence in delivering instruction? Does interest and competence in the discipline imply a dynamic concern with philosophical issues and a level of abstract academic interest as opposed to a more pragmatic and static interest in the subject?

With respect to interest and competence in delivering instruction, is a concern for pedagogical matters outside the immediate confines of the system and classroom the hallmark of the professional?

The interviews conducted in this study seemed to indicate that mathematics teachers may exhibit a certain reluctance to discuss the more abstract, theoretical aspects of their discipline. There was evidence that this reluctance was not confined to the interview setting, but extended to formal and informal interactions with other teachers. Is this so because such discussions are not considered to be useful, is it because teachers do not consider themselves qualified to engage in it, or are they simply not interested? Comments recorded in response to the question which asked if it would be worthwhile for teachers to engage in a dialogue on curriculum matters included, "I am too much of a realist. It can't happen" and "... if you had the right group. Math teachers as a group never agree."

When asked how they rated themselves as teachers, all but one chose "better than most"; but when several participants were asked if they considered themselves to be "mathematicians" they responded that they were first and foremost *teachers*. Despite the fact that most participants rated themselves "more curious than most", the majority (seven) indicated that they had learned "hardly any" new mathematics topics since leaving university. When asked if they had learned any new mathematics and/or pedagogy at conferences and workshops only one participant chose "much"; the majority chose "not really".

My sense is that the interests of and conversations between secondary mathematics teachers tends to focus on the day-to-day practicalities of the

working environment and the problems encountered in dealing with specific students particularly with respect to discipline -- and that their personal interest in mathematics *per se* is limited. Is it true of teachers in other disciplines? If this phenomenon is more marked in mathematics teachers, is it the result of the mathematics teacher selection process and/or pre-service training? Could it be that the tedium of teaching essentially the same material to thousands of students year after year has a stultifying effect?

Transfer Learning from Mathematics

As noted in Chapter Two, some mathematics curricula content are sometimes justified on the notion that "logical thinking" is developed through learning mathematics, particularly geometry, and that this attribute is transferred to the study of other disciplines. If one were to accept this tenet, a number of questions need to be answered. When people make an association between mathematics and logic, how do they define logic? Is logical thinking associated with convergent, divergent or lateral thinking style? Is ability to think logically seen as a necessary prerequisite for the study of mathematics or is it assumed that the study of mathematics develops it? Is logical thinking ability of whatever type we are talking about a desirable requisite for further education in *all* areas, and if not, in which areas is it desirable and which areas may it be a hindrance?

As detailed in Chapter Four (Question 3) there doesn't seem to be credence given to the notion of transfer of logic from mathematics to other areas. Comments solicited in response to Question 3 included "Belief - no

proof", "Questionable", and "It would be nice if this were so - but there is no evidence that it is so."

Concerns of Mathematics Teachers

The two recurring themes that emerged in discussions on teachers' concerns were *time* and the difficulties imposed by heterogeneous student groupings. This was particularly true when the conversation drifted on to such topics as teaching for understanding and discovery learning. Teachers felt that time constraints eliminate the possibility of allowing students to explore, and dictate a highly teacher-centered and structured teaching methodology. Further, when student-centered instructional methodologies were used with nonacademic students they often led to breakdown in discipline.

Time was also cited as the reason for the limited extent to which teachers made use of manipulative materials, models, films and techniques such as discovery learning in their instruction.

The question of time was also definitely related to the wide range of abilities and motivation level of the students with whom teachers had to work. The consensus seemed to be that although the required schedule was manageable for academically inclined students, it was unrealistic to impose this pace on all. Related factors were class size and streaming of students into ability levels. It was felt that the strain experienced by secondary mathematics teachers could only be alleviated by either increasing the amount of time allocated to mathematics, by increased streaming, by

ignoring teaching for understanding or by a severe reduction in the amount of core material that is required to be covered by all students.

How Time Should be Allocated

The study showed that teachers favour spending relatively greater amounts of time on mathematics which is more clearly *application* oriented, as opposed to that mathematics which is intended as preparation for future study in mathematics. It is supposed that one reason for this is the limited number of secondary students that proceed to advanced study. Another reason, it is suggested, is related to the perceived greater motivational value of mathematics which is clearly useful.

The study also indicated that teachers are inclined to not balance support for the inclusion of specific topics with support for exclusion of others. Given that any mathematics curriculum can contain only finite material, how do teachers prioritize the inclusion of existing and potential topics; in other words, if something *must be* cut, what would teachers cut and for what reason?

Proposals for including additional topics in a future curriculum must not only come to grips with what should be eliminated but must clearly identify and obtain consensus on the rationale for such new inclusion. For example if some form of computing is somehow to be incorporated into new curricula, fears that the addition of such topics might increase the time requirements for delivery must be addressed.

It is my sense that the views teachers express when they made references to the implications of streaming students into different categories

on the basis of their abilities and future educational plans are very much related to their own present and past experiences. For example the teacher who normally teaches only senior mathematics courses to academically gifted students will tend to respond quite differently to many questions than the teacher whose assignment includes work with lower grade students and those with little interest or ability in mathematics.

Use of Mathematics Marks

One of the questions in the study dealt with the use of mathematics marks as a device for screening students for entry into post secondary institutions. The study revealed that the teachers interviewed were not enthusiastic about this practice. Comments included, "Not all bright students do well in mathematics" and "The screening function dictates the curriculum. We make things difficult for this purpose. It is a poor scene when this is used."

Computing in Mathematics

The question of relevance of computing to mathematics is a complex one. Aspects of computing can be employed as an alternative or supplementary instructional delivery method of traditional curriculum, as in drill and practice programs and audio-visual presentations. It can be used as a tool to alleviate the tedium of repetitive calculations or it can form the basis for a restructured paradigm of what it means to "do mathematics".

Meaningful dialogue in this area can only take place if the participants in the dialogue have a common understanding. For example, teachers' lack

of experience with mathematics programs such as MuMath, Mathematica and Maple may result in them not being clear about the distinctions between the symbol manipulating functions of such computer programs and the capacity they share with less sophisticated software to determine numerical solutions to complex equations.

How many mathematics teachers have any experience with symbolic mathematics computer programs? What do those who do, visualize being done with them, and in what ways, if any, do they feel this would enhance students' understanding of and ability to perform mathematics and apply mathematics?

There are many problems associated with obtaining an inventory of a person's computer literacy. There are many interpretations and subtleties to this question; proficiency with the use of one or two specific kinds tool software, familiarity with a range of computer tools, the extent of any programming background and the nature of the language employed for programming all come into play. To illustrate, the school secretary would likely be described as being "computer literate" in that he or she is likely to be proficient in the use of a word processor, a student attendance program, and a marks gathering package. But he/she would not at all fall into the same category of computer literacy as a person who had programming experience. Similarly someone with a Fortran background would very likely have a totally different perspective on the relevance of computers for mathematics education than one whose programming experience was based on APL because each computer language has associated with it a computing paradigm. For these reasons any attempt at describing a person's computer

experience inventory must be designed so as to identify not only the extent of the experience, but also the subject's perception of what computing means in an abstracted sense.

The extent to which mathematics teachers endorse the use of computers in mathematics education, the degree to which they actually do use them, as well as the uses to which computers are put is likely to be evolving and very much dependent on the facilities available. In my view, convenience of access and the availability of colleague support and encouragement is a major factor. Much of the hesitation expressed by the participants with respect to computing originated in the perception that such activities are time consuming and that the accessibility of computing facilities makes scheduling difficult.

In any event, it is a fact that the curriculum delivered in the classroom is driven almost exclusively by that published by the Ministry -- and the present edition of this document states quite plainly "Computing is not a mathematical activity".

In summary, the present study revealed four characteristics of participating teachers with respect to computing in mathematics:

- a cautious, but generally receptive attitude towards discussing the subject,
- a diversity of personal experience ranging from the non-existent to the sophisticated,
- an apparent unawareness of the potential impact on curricula as outlined by the NCTM (1984), and

- outright skepticism of the prospects of there being significant changes in this regard in the near future.

Teacher Awareness of New Mathematics

In the thirty-odd hours I spent talking with the teachers participating in this study, I would occasionally make references to such topics as the Mandelbrot Set, fractal geometry, chaos theory, discrete mathematics, iterative functions, and difference equations. It would not be an understatement to say that the majority of the interviewees indicated lack of familiarity with many of these terms. This raises the question of the extent to which teachers, particularly those participating in curriculum building, are aware of and familiar with new developments in mathematics and their possible inclusion in future curricula. How does the familiarity of mathematics teachers with new developments in their subject matter compare to that of teachers in other areas?

I suspect that teachers in most subject areas are comparatively more dynamic in the acquisition of new knowledge than is the case with mathematics teachers. For example, the English teacher must familiarize himself with emerging authors, the Social Studies teacher must remain current with respect to world events, the Science teacher may be called upon to explain recent discoveries in genetic engineering. By contrast, until recently at least, very little in the way of new developments in mathematics, at least on a level accessible to high school students, has been drawn to the public's attention. It may therefore be that this perceived lack of academic curiosity, if it may be described that way, is peculiar to the subject of

mathematics. Alternately, may it be that people who are drawn to this profession bring with them a resistance to innovation?

APPENDIX A
TRANSCRIPTS OF EXPLORATORY INTERVIEWS

Participant 0.1

April 26 and continuing May 11

I-1: The purpose of this exercise is to build a picture of your conceptions of what mathematics is all about.

There is a difference between math is the abstract sense, math as it is mandated by curricula, math as it is reflected in the delivery of curriculum and as it is perceived by students. Please comment and elaborate from your perspective. This was repeated for clarification.

P-1: I can probably answer the latter ones first. I'm not sure what the abstract means. Let's start backwards. I don't think students perceive mathematics as being anything that has any connection. Most students deem mathematics as sort of a isolated series of skills. They don't relate the numbers and number operations that they learn early on in elementary school to the algebra. They don't really relate the algebra to the calculus that they may get on to; though so few students take calculus, so maybe you are getting your upper level who are making the connection... but I don't think that students in general view mathematics as a package of related material. And I guess what teachers attempt to do and from my experience I guess I didn't see the inter relatedness of a lot of the concepts until I had to start teaching it. You know - you know - you start teaching a concept and all of a sudden all of these things that you studied in school started to fall into their appropriate places in the whole scheme of mathematics. What I think that teachers try to do, or at least should try to so, is to take sort of the magic, the black box aspect out of the operation. They should be attempting to make connections when they are teaching algebra to the situations they have encountered previously with arithmetic. I think that if you look at the curriculum as it is set out the curriculum sort of specifies topics that are to be covered without actually specifying methodology. I think that in a lot of cases the curriculum that we see now assumes a methodology that isn't comfortable to a lot of teachers.

- I-2: I'm sorry - you said that there was an assumption of methodology in the curriculum...
- P-2: Well, I think the new curriculum assumes that we are going to make extensive use of manipulatives; that we are going to involve technology to a greater extent than we have in the past; that we are going to get away from the 'teacher lectures - kid listens, does homework, repetitive exercise. You know, I think there is an assumption among people who set curriculum that we are going to change methods - and I guess it will happen eventually, but I see it as a pretty slow process. I don't think - if we went into most classrooms in this province - particularly at the secondary level - I don't think you are going to see a whole lot of changes in methodology at this point.
- I-3: You said that there was an assumption of methodology inherent in the curriculum... to what extent do you think that the success of the curriculum as mandated is dependent on that methodology? I mean will it not be successful for lack of this methodology or does it not matter so much or...?
- P-3: Well, I think what happens is that if you don't change your methods you are going to have kids going away from school with the same negative - I think people generally view mathematics negatively - like I never could do math. and my mom never could do math and why do we do it anyway?
- I-4: That was one of the questions I wrote down here... Why do so many people claim they dislike math?
- P-4: Well, probably because it is a skill based course. Whereas if you don't have the skills at the first level there isn't much chance you are going to pick them up at the second level or the third - you know there is a lot of things that I think go into that. As students start to build their base in mathematics in elementary school perhaps their exposure to mathematics is through the eyes of a very good generalist and not someone who is extremely good in mathematics. It's the kind of course that if you get a poor start - that can carry through with you all the way. It's not like social studies where if you don't study Egypt this year, then next year you can't study - you know, Europe. The

study of Egypt doesn't have any real impact on the study of Europe. You know, sure, there may be some reading and research and writing skills and that sort of thing that are needed in writing reports - but you are not prohibited from understanding about Europe just because you didn't understand about Egypt. Whereas in mathematics if you don't understand about positive integers there isn't really too much sense attempting to teach anything about negative integers.

I-5: What you are saying is that nearly every other subject in school - the topics are independent - in the sense that none of them would be necessarily prerequisite for any other.

P-5: Yes. Yes.

I-6: While in mathematics everything that we do is built upon the assumption that something else has gone before.

P-6: Absolutely. Absolutely. So that if you are missing one of those basic blocks in the foundation then you are going to have difficulties building on that - that is what I see - certainly in secondary mathematics as the biggest problem that kids have. They have gaps in their foundation which is prohibiting them from moving on. So many times in the past - and this is where I see the new curriculum addressing this somewhat - if some could not work with fractions, say what did we do - we gave them more fractions in grade 8, and they still didn't do them so we gave them more fractions in grade 9 and more to do in grade 10 and all they succeeded in doing is proving to them that they couldn't do fractions.

I-7: I suspect that one of the reasons he couldn't do fractions is that they were introduced too early - in grade 4 or whatever it was.

P-7: Yes it could have been some re-arrangement of some of those topics too but...

I-8: Why do you teach fractions, by the way?

P-8: Why do you teach fractions? Well... I guess there's... I guess there is a lot of situations where you are going to encounter fractional

quantities - do you mean 'Why do we teach fractions as opposed to decimals?'?

I-9: Yes. I suppose I can conceive that the words 'three quarters', 'a half' and 'two thirds' are useful pieces of vocabulary for conversations. It would not follow from this that taking seven eighths of two and three quarters is something is a useful activity. So the question is 'Why is it that one is required to know how to multiply seven eighths by two and three quarters?'

P-9: I agree with you that the fraction is useful. You know I think that the concept of a fraction for most people is easier to visualize - to build a mind picture of a certain portion of certain portion of something...

I-10: But isn't that what a decimal fraction is... you know .5 is 5 of the ten pieces....

P-10: That's right...

I-11: What is the difference?

P-11: I don't know if..... I find it much easier to conceptualize - to build a mental image of a fraction... when someone says a half or something as opposed to someone saying 'point five'...

I-12: I'm going to argue about this separately!

P-12: Also, I guess, if you want to speak purely from a mathematics point of view; if you are going to move forward in mathematics - you know, the progression being from arithmetic to algebra into calculus, into trigonometry which involves algebra - that sort of thing - there are situations where you really find dealing with fractions is preferable to dealing with decimals - I mean when you are working with rational expressions in algebra which later on appear in trigonometry.....

I-13: Sure... we are now talking about at least grade 10.. or 11

P-13: Yes....

- I-14: You have mentioned that the difference between the curriculum as it is delivered and as it is mandated is that there is an assumption on the part of the curriculum about methodology. Can you think of any other differences in there?
- P-14: I think they may have addressed some of (...) that you have mentioned. They have said 'Yes we want the concept introduced before grade eight; but let's leave a lot of it - the manipulation with these - the add subtract, multiply, divide - the operations with fractions - let's leave that till secondary school; when perhaps kids are more ready to understand that concept... I guess we are now going to wait and see if that was a valid judgment on their part. That by grade seven the kids know about integers and now is the time for negative integers. That we are going to leave most of the operations on fractions - which, it seems to me was taking up a disproportionate amount of time in grade six and seven - to later on...
- I-15: Incidentally, you know, when I taught Logo... there was no difficulty in having a grade five student plot points on a coordinate plane in other than the first quadrant...
- P-15: And yet we don't allow them to until grade seven now...
- I-16: That didn't bother them at all.
- P-16: That's right. It's sort of like in grade two and three and we limit them to dealing with integers - whole numbers up to 50 or something like that - I'm not sure what it is - but if you look in the curriculum guide it'll tell you - when really all around them are numbers that are much greater than that so it seems to me that we could expect kids in the primary school to deal with numbers in the hundreds - I mean they are dealing with money all the time; they look at their television set at home and the numbers certainly go past 50 on their televisions... you look at the temperature outside, the amount of rainfall... they can handle that sort of thing. It seems there are some places that we don't expect enough of the kids.
- I-17: At the beginning of this session I suggested that I was interested in finding out about your perceptions of what mathematics is in the

abstract sense. your comment was something to the effect, as I recall, that you were not sure if you could.....

P-17: Could you give me some clarification on that one?

I-18: Some people feel that mathematics is a formal logic system. Other people feel that mathematics divorced from its application simply doesn't make sense ...

P-18: Well, all right. I probably fall into the second camp. I don't see that something is mathematics and unless you study its application it isn't really of much use to us. That's why at the conference in Florida they were talking about the Mandelbrot set and fractals and that sort of thing - and it's very interesting and it's certainly extremely involved with the theoretical area of mathematics... but I didn't see any use for it... you know, the diagrams and pictures all looked very nice, but I want to see some application - or some use for it other than just mental gymnastics.

I-19: What is the application, if I may ask this questions, of knowing the triangle congruencies - proving two triangles congruent - what is the application of that?

P-19: I'm not sure there is one other than it fits into the system of logic that deductive geometry is all about - and I think what we are hoping for, and that I think we miss in mathematics is we are hoping there will some transfer of these skills that are developed by studying Euclidian geometry and that sort of thing... and I'm not sure that transfer happens very often. You know, by studying a deductive system in geometry...

I-20: The justification for studying geometry is to become familiar with the notion of a formal deductive system so that we can transfer the rules of thinking and logic to other situations...

P-20: To other situations.... and I'm not convinced that that happens in very many situations. I don't know about you, but I (laugh) I have seldom seen a transfer.

- I-21: Is that also.... can you justify the teaching of mathematics in general on that basis? I was reading for example, last evening that some people..... that one justification for teaching mathematics is that it trains you to think... I don't think this particular reference was with respect to geometry necessarily. Is there training of the mind involved in doing algebra and trigonometry?
- P-21: I guess there is. It forces one to sit down and look at a system that is fairly rigid in its structure. You know, certain principles and invariants that if once understood can be applied to new situations. Maybe that is why the universities use mathematics as a ... as a hurdle... there must be something about the study of mathematics that indicates that a person is able to think in a logical, sequential fashion and
- I-22: You said that universities use mathematics as some kind of a hurdle...
- P-22: Certainly.
- I-23: ... some kind of screening device... even though the screening may be for subject areas which don't necessarily involve mathematics
- P-23: I think that they feel the thought processes that are necessary to be successful in mathematics at 11th grade level are sophisticated enough or are of the appropriate level of sophistication that they would indicate that person capable of a... doing the thinking and analysis.....
- I-24: Ok... I'm going to suggest an alternative viewpoint.... that your knowledge and understanding of history could also be used for such a screening process.
- P-24: I could be; but I think that a lot of people view history as a less logical and more 'memorization' or 'familiarity with a particular set of facts' - you know... most history doesn't seem to have a whole lot of logic to it... and it seems to me that... that that's why mathematics is chosen... because it is a fairly structured system...
- I-25: Your understanding of history, I would think, would have a lot to do with - you know, weighing, judging and seeing alternate viewpoints

and what you might call mature subjective viewpoints..... which is precisely what mathematics does not have..... which has to do with knowing the rules, having the skills.....

P-25: That's right...

I-26: ... in other words... from the point of view of intellectual maturity I wouldn't rate mathematics ... high - compared to history.

P-26: (pause) Well... I... probably what we are looking at is having different screens for different areas of study.... you know, I'm not sure the mathematics screen should be applied as widely as it is.

I-27: For example to nursing?

P-27: Yes. And I guess nursing is becoming more and more technical all the time - but certainly that mathematics screen has been there for much longer than we've been technologically....

I-28: Last night I was reading about this business about different conceptions and I came across... I mentioned two terms.... the logical system and the other one was the applied mathematics. These were described as bifurcated and integrated.

P-28: (laughter) I don't know what the hell that is...

I-29: You don't know what bifurcated is?

P-29:

I-30: Tell me about the curriculum that we mandate for secondary school students... what is your feeling about it... should it be.... based on the assumption that mathematics is a useful tool and taught in the context of applications or should we teach it as a logic system - as we mentioned about geometry... give us the main emphasis here.

P-30: I like the applications approach and I think that one thing that we have done in the curriculum is recognize things such as the everyday use of mathematics demands that we teach more skills in interpretation of data and display of data because if you really think

about it that is the type of mathematics that every person has to deal with on a day-to-day basis. If you pick up any paper or magazine or whatever and you are faced with a chart and you are being asked to make value judgment as to whether this conclusion is reasonable based on the evidence.. and you know... I think we have recognized that the purely algebraic approach that we used to see isn't useful for a lot of kids, and that we should give them a broader exposure to many areas of mathematics. I don't think we are emphasizing the structure of geometry to the extent we were in the past - certainly by that time we are starting to get the people who have decided they are going to specialize in the sciences and that type of thing. So it seems that the curriculum, at least to the end of grade 10 is more varied and to my way of thinking it's a broader mathematics education for everyone. I think in the past a lot of kids that rose to grade 11 had never done anything to do with data analysis - never done anything to any great extent in geometry other than seeing a few triangles that were applied to algebraic problems.

I-31: When you say 'In the past"... you mean 'Up to the present curriculum revision'?

P-31: Yes. Up to the present curriculum revision.

I-32: If you were King of Education and assuming that you have the power to make all things happen, how would you design the math curriculum?

P-32: I think number 1 I would want to have sufficient time to delve into the things that we are studying to more depth - particularly at the secondary school. We are trying to do too much and we have too little time. And consequently we are 'curriculum-bound' and many times when you would like to go to a greater depth in certain topics, you are forced to plow ahead because we have to get to point A in order that the kids are prepared to go on to the next level in the following year - that I see as the biggest problem. I don't think its such a problem in the elementary school. I think they have more leeway. They are not as curriculum bound. They have time to deal with manipulatives - that sort of thing... and I really think that the dramatic change in organizational structure between the elementary and secondary school is hurting mathematics education. Mathematics

in the elementary has time which is expandable and contractable as the teachers see fit. That doesn't happen in very many - well, I don't know of any secondary schools where mathematics isn't constrained to - you have this many hours and - I've never heard anyone tell me that any of the secondary school courses could be done to the degree that would please teachers in the time allowed.

I-33: So, there are two ways of doing that. One is to increase the amount of time available, and the other is to ease off on the content. What would be your preference?

P-33: I think more time. I think the way we are handling the content - there's a lot of things out there I think kids should be exposed to in mathematics and I like the way they have brought in data analysis. I like the way that geometry is in there. Certainly there has to be some algebra. You can't ignore algebra - you know - it's the backbone of mathematics. I think there has got to be more time. I don't believe that every subject in secondary school deserves equal amounts of time. I think English, I think mathematics deserves more time than welding and art and music and if a person turns out to be extremely good in welding or in art or music then we will get them more time - and we do get them more time. But in the core areas, certainly in grades 8, 9 and 10, if we are talking core curriculum, maybe this is where we have got to start exposing students to more - more in those areas.

I-34: It has been suggested that applications in mathematics are artificially constructed to serve as illustrations of the mathematics being taught - as opposed to the math being taught being realistic and sensible ways to solve the problems. Comment please.

P-34: Yeah... I think that is a problem. I think that a lot of the ways we have tried to apply algebra in particular has been very artificial. The word problems in particular. I mean - whoever cares if Mary is twice as old as John and last year she was - you know - three times as old.... it isn't real - and what we need is more information from the real world as to what kinds of mathematics is being done, and I guess again I go back to statistics and it is the easiest of the branches of mathematics that we study to go to a newspaper - daily and say look - here is the kinds of mathematics that....

- I-35: Does it not suggest - what you just said (name) - that the mathematical manipulations - factoring or whatever - may simply be... maybe they are not suitable curriculum content?
- P-35: Well, that could be and I guess with the advent of symbolic manipulators and that sort of thing there is certainly... and with calculators... I mean calculators have certainly lessened the need to drill algorithms in the elementary school. I think symbolic manipulators are going to lessen the need to drill factoring and division of polynomials and all that sort of thing. I don't think it goes away - because there are certain areas of science where that sort of mathematics is required... and we get a certain group of students - maybe an increasingly larger group of students - as we move into this information age as opposed to
- I-36: Somewhere around 17, 18, 19, 20 percent of high school graduates go to university.
- P-36: Yes...
- I-37: And of those, perhaps half, I would suspect, need the kind of mathematics they have been taking.
- P-37: Currently.
- I-38: Currently.
- P-39: I think we have to look ahead to...
- I-40: Does this not suggest then that there should be a greater division in terms of kinds of mathematics that kids take at high school - or does it?
- P-40: I think it does. I think we force a lot of kids to do algebra and they really had no intent of going on using that kind of mathematics...
- I-41: It would be unsuitable for perhaps 4/5ths of the kids?
- P-41: Yeah... I suppose 45% might be too high - but yeah.

- I-42: I recently had a conversation with a university professor who said words to this effect.... "Any teacher who sees him as a labourer who did as he was told in terms of teaching to exams - ought to be fired. What is your reaction to that one?"
- P-42: Yep... I don't think you teach to an exam. I think you cover the curriculum. But I think if teachers were to teach to an exam you've got a pretty bare-bones course. 'We will do this because it could be on the exam - and even though this is very closely related - they won't ask this, we'll leave it out - I wouldn't violently disagree with that! I think an exam is really, though, a measure of the curriculum that is being required for that particular subject and I guess if you are following a curriculum - then indirectly you are teaching to an exam - but....
- I-43: Ok...now... in what way do you see computers influencing content as opposed to methodology of future curriculum?
- P-43: (pause)
- I-44: I think you have answered that question in P- in previous comments....
- P-44: Well, I guess what I said earlier was that computers are going to allow symbolic manipulation of algebraic terms... they can solve equations. They can plot graphs that we may have laboured over in the past....
- I-45: We are talking about content now...
- P-45: By the fact that you have a computer - you're not going to teach as much factoring, you are not going to teach as involved... solving equations, systems of equations, you use the computer for graphing and you use the computer to get a result which is good enough - which is really the world situation anyway. Uhm... I think computers are going to be used.... or will influence what's taught to the same extent that calculators did with respect to logarithms - nobody teaches how to read log tables any more - or trig tables - or interpolation - so those are things that are being left out. We aren't

leaving out logarithms, we aren't leaving out trigonometry, but we aren't doing a lot of the really basic mundane things which, when you really get down to it where really manipulations that really didn't have a hell of a lot to do with the understanding of how logarithms could affect calculations or how trigonometry - you know - could affect or influence problem solving. I think computers can do the same thing - by cutting out a lot of the simple algebraic manipulations - at this point and I guess in geometry and design and that sort of thing they are certainly influencing whether you have to teach the depth of material that you've had to in the past....

I-46: Now... tell me about your own background experience with using computers.

P-46: My own experience... has been... essentially limited to use of a single computer in a classroom.

I-47: What did you do with it there?

P-47: Generally graphing techniques - uhm... probably grade 11 and 12 would be the area I've used in most. Seeing as that in the last fifteen years I guess, has been my area of concentration. But in teaching graphing techniques... in both grade 11 and 12... to have samples that can be quickly graphed and then to ask students to... "Alright you've seen how these samples work. Here is one for you to try. What do you predict is going to happen. Draw a graph of your own now. Does the computer verify what you have done?" Or conversely, I guess, give them a graph and ask them if they can derive the function that generates the graph.

I-48: Do you know any programming languages?

P-48: (pause) Basically no. I know a little bit of BASIC. I know how to spell 'Pascal'

I-49: (laughter)

P-49: But I am not a computer programmer. I've taken a course in programming, but I couldn't program a computer to save my life.

Participant 0.2
May 25, 1989

I-0: (Background explanation)

I-1: One of the readings that I came across said that you can perceive of mathematics as a formal academic discipline,

P-1: Yeah...

I-2: and applications tool.

P-2: Yeah...

I-3: ... or a form of artistic expression.

P-3: Yes...

I-4: Now, I wonder if you might sort of expound a little bit on each one of those three perspectives and then perhaps zero in on your own feelings about them. Does that make sense?

P-4: Ah ... yeah

I-5: We'll start with the academic discipline ... the formalized academic discipline. Can you comment on that a bit?

P-5: Well, I mean, obviously it is ... an abstract ah ... study. It is one of the purest I think, of the subject areas. And I guess that's what we mean, by a...

I-6: A pure abstract study?

P-6: Well, eh.. as a field of study it is what I consider sort of as a clean subject in that it can be done ... it is essentially intellectual ... ah study ... and intellectual discipline ... that certainly has its applications. ... but it is a...

- I-7: You were saying that the applications grow out of the formalized discipline rather than perhaps the other way around. You see, the other way to look at it is ... sorry...
- P-7: Well, ah, yeah, I suppose historically.. well it probably developed ways as a tool to help us analyze the physical world. But also, a lot of mathematics has been developed abstractly, you know, theoretically and later on found its applications. I believe, for example, matrices are one of those perfected - analyzed and perfected and developed as a mathematicians plaything and later the chemists came across it and said 'That's exactly the tool that we need to describe the lattice structure of crystals', or something like that. You know, of crystalline structure of matter.
- I-8: Ok. So what you are saying here is that this is an example of where an abstract mathematical concept found an application.
- P-8: Yeah ... and it probably goes both ways. I mean a lot of mathematics is developed that way, and the formal discipline, ah ... the mathematical theoretician couldn't care less about any applications that may be found for it, and pursues his little ... game. But then also, a lot of the mathematicians, a lot of the mathematics is developed simply of necessity to devise the model for the application ... if that makes any sense.
- I-9: It does. What I hear you saying is that sometimes its one way and sometimes the other way.
- P-9: Yeah...
- I-10: What's closest to your heart? I mean, when you think about mathematics do you think of the tool which solves engineering problems?
- P-10: No. I think of it as the abstract ... eh.. intellectual game...
- I-11: Ok.
- P-11: ... the fun of it... the...

- I-12: Is the intellectual exercise?
- P-12: Yeah ... yeah. I don't think too many students share that concept.
- I-13 Uh huh..
- P-13: ..but that's my ... you know its the love of the subject. Its the purity of it. It is the uhm ... it is, uhm, the intellectual ...uhm...
- I-14: When you think about mathematics, do you think about symbols, or do you think about numbers, numerical quantities? Or do you think about physical quantities?
- P-14: Whew! Uhm ... I don't know if I can separate them, but...
- I-15: When the word 'mathematics' popped into your mind, what comes foremost - is it an algebraic way of doing things or is an arithmetic way of doing things? Or is it something more concrete than that?
- P-15: Oh ... I guess its a lot of things combined ... not necessarily symbolic or numeric, but also just as a ... as an approach ... its perhaps the logic that ... your numbers and your symbols are a way of expressing it, but its the, its the attitude almost...
- I-16: Ok.
- P-16: ... that you, you approach the applications with the subject.
- I-17: Ok. Uhm... now. Let's take a further example of that for a moment. Remember at one time we were heavily into teaching the laws of mathematics ... you know ... the distributive law and all that stuff. Now ... can you describe to me in a few sentences how you would introduce this notion of the distributive law
- P-17: ...
- I-18: I mean, here you are ... your ... teaching assignment for today is to talk about the distributive law to a class that hasn't come across this before. How would you go about that?

- P-18: Hum... You would formalize it at the very end of it...
- I-19: Well, I don't know. I guess this is part of the question.
- P-19: Yeah...
- I-20: Would you do that?
- P-20: You know, I mean, I think you would approach something like that from the sort of point of view of having two different names for the same number ... the name being a symbol ... and you've got two different ways in which you can present this community of the same number ... either as $(5+2)3$ or as the 3 times 5 and the 3 times 2 ... yes that's correct ... and you get the same number ... so have two different ways of symbolizing the same number ... and I guess the phrasing that I prefer is two different names for the same number.
- I-21: Would you start off with $a(b+c)$?
- P-21: Oh God no. You got to start with concrete numbers.
- I-22: Start with concrete numbers?
- P-22: Oh Yeah. Sure.
- I-23: You see, I'm inclined to suggest that a lot of mathematics instruction takes place from the abstract to the concrete ... that ... the proof of that, in my view, is that the problems - quotation marks - always come at the end of the course.
- P-23: Yeah...
- I-24: In other words ... the applications come after the mathematics is studied. I mean that's the...
- P-24: Yeah.. yeah.. that could be true ... I hope that I'm not too guilty of that because I think that is an error in the way its taught. ... but I could perhaps see it being done by error ... because the teacher has got all of the concepts mastered ... and is trying to present those concepts in the most efficient manner, I suppose and will start

from the general rule to the specifics. ... but the student won't learn it that way ... that's not the best as far as I'm concerned. The best way of teaching it ... you'd have to start from the specifics and build up the pattern. You know, that's a whole other way of looking at math ... its just ... patterns ... so that you see that you can get the same number by this ... two different methods ... and after many, many examples, the student will be able to say ... 'Well, won't it always work?..' that if you have two numbers in brackets and you multiply by a third that you can multiply each one separately and then add.

- I-25: How do students perceive of mathematics? If you were to ask a student you know, 'What is mathematics. Tell me what its all about.' What would they say?
- P-25: Depending on the age level, but I think that a typical reaction would be as a hurdle.. something that we have to learn in order to get through school ... to graduate ... I'm sorry to see that attitude in a lot of students, but ... ah ... I think a lot of students have had bad experiences somewhere along the line and don't see the beauty in it. I think that as mathematics teachers that is something we have to try to instill in them ... enthuse them ... more and more I'm ... It's difficult, and I don't have the answers for it....
- I-26: An enthusiasm for the beauty in it...
- P-26: Yeah ... yeah ... you know, the love of the subject ... and both is you know, its applications and just the study for its own sake. You know ... the ... the...
- I-27: Now, you eluded to this, and I think its a bit of a universal truism ... that, you know, there's a lot of people who have rather a negative attitude towards what we call School Mathematics.
- P-27: Oh, yeah...
- I-28: Why is that?
- P-28: I wish I knew. Then maybe we could help fight it.
- I-29: You mentioned something about having a bad time of it.

- P-29: Well, yeah ... I think that perhaps...
- I-30: A lot of people that I talk to say that when I say words to this effect 'I teach math', they always say "I never did well in that'
- P-30: I know.
- I-31: Why is that?
- P-31: When I'm in a social context, and I'm asked what I do, and I say 'I'm a teacher' and you know what's coming next. ... and sometimes you almost hesitate because you are going to get that reaction sometimes. Ah ... why? ah ... I think that perhaps some of the concepts that they have been forced to learn were either done too early for them, or, you know, they just weren't intellectually ready to handle that level of abstraction.
- I-32: Uh hmm...
- P-32: Perhaps together with just not being presented correctly... it was just the wrong approach to them. For example, from the specific to the abstract ... ah specific to the ... rather than the other way around.
- I-33: Why does this seem to be more true with mathematics than with history, for example? I mean ... don't these people who teach history make the mistake of presenting things the wrong way?
- P-33: Yeah ... but mathematics is ... perhaps ... but mathematics is ... such a more precise ... ah ... study. Your answers are either right or wrong. When you're writing a paragraph ... some historical topic ... you can always manage to put in enough words to get some part marks. Whereas in mathematics the ... the ... at the more elementary level, your judged about whether or not the answer is right or wrong, and a student can have ... you know ... 90% of the.. the concept and still get the answer wrong ... so that there is a much higher frustration level with mathematics than I think with just about any other subject.
- I-34: This frustration is related to its need for preciseness, your suggesting.

- P-34: Yeah.. yeah. ... You know I also think mathematics is a more intellectual or something ... perhaps it accesses a different part of the brain or in a different way ... that students are ... they either got it at the time or they don't. ... I think that this is something that you would want to pursue ... ah ... as teachers' impressions of mathematics is how much ah ... well, I don't know how to word it, but, you know, 'How well do you agree or disagree with the statement that ah.. mathematics ... everybody can learn mathematics providing that they've got sufficient time', versus 'There is some innate ability in mathematics that some people have and some people just don't have.'
- I-35: Ok ... I hear your question ... The question runs something to this effect: 'That everyone can learn mathematics given sufficient time and opportunity' ... and the other approach says something to the effect that 'Mathematics requires some special ability which perhaps not everyone has in sufficient quantity' Is that right?
- P-35: Eh ... yeah, I suppose ... it's not as black and white as that, but...
- I-36: What's your answer to that by the way?
- P-36: Well, oh ... I don't have an answer to it. I have my ... my feeling, I guess is that there is a certain intellectual development or something that is perhaps there or not there in order to be really successful in mathematics ...
- I-37: Some do, and some don't...
- P-37: Well, I think perhaps its something that is there to varying degrees.. or perhaps it is there, but different individuals have a much more difficult time.. to access I guess or developing it.
- I-38: Generally speaking, mathematics is considered to be an important school subject ... ok?
- P-38: Uh huh...

- I-39: It's supposed to be important. Now, can you explain to me why this is so. Why is it that we teach mathematics? What is the purpose of the exercise?
- P-39: I guess I see it from two ways. One is that it is an important subject...
- I-40: Yes...
- P-40: ... because it is the model that we can use for so many other subjects. I mean in any of the sciences, particularly physical sciences, economics, a lot of psychology. Mathematics is your tool. It is the model we can use for a lot of the other subject disciplines...
- I-41: So ... in the other sciences we make use of mathematics models.
- P-41: Uh huh...
- I-42: And that constitutes at least a major reason for teaching it.
- P-42: Well, yeah ... yeah.. there is very few studies that you can pursue at the university, college ... trades level..
- I-43: Uh huh...
- P-43: ... that isn't going to use some form of mathematics.
- I-44: Well, what about all those people who don't do that? I would venture to guess that at least 80% of the population don't go to university.
- P-44: Yeah ... and therefore I ... I.. I question perhaps the need for them studying as much mathematics as they do. Ah ... you asked a question ... 'Why do we teach mathematics or why is mathematics ... considered one of the important subjects that everyone has to study ... One is that ... one reason that I just is gave is that it is the tool that we use for so many other disciplines ... but I also think mathematics is taught in the high school as the screening device in the ... because it does require ... a fairly high developed intellect ... you know at least there is a lot of abstraction to the subject ... it's a difficult subject and it requires a lot of work habits - good work habits - that a lot of the

universities in particular have been guilty of setting mathematics as one of their entrance requirements...

I-45: When you say 'Guilty', why do you use that word 'Guilty'?

P-45: Perhaps because I don't think its perhaps necessary. They are using mathematics as the screening device and say 'If you can handle Algebra 12 then you'll probably be successful in any of the other fields of academic study that ... well ... that your gonna take.. ah ... I say they are 'guilty' of that because ah ... I..I don't like that ... I don't approve ... of them using mathematics or any artificial screening device...

I-46: Why is it artificial?

P-46 Well ... I guess only in the sense that a lot of students will ... will go on, as you say ... and take courses of study where mathematics is not going to be utilized in any great extent ... and yet they have to pass that hurdle just to prove that they are quote unquote capable students.

I-47: You referred to using mathematics as a convenient screening device. One argument here would be something to the effect that learning mathematics teaches you to think logically...

P-47: Well ... I was just going to say, its ...

I-48: ... that there is benefit in learning it, not because of the actual mathematical content but because of the thinking...

P-48: Yup ... yup...

I-49: ... patterns that develop ... You endorse that?

P-49: Oh yes ... yeah...

I-50: You would say that there is a fair amount of ... or at least a substantial amount of transfer ... ah ... to other areas ... so for example a person who had mathematical training would be a better ... historian ... or something by virtue of the fact that he's had this mathematical training. Is that a valid...

- P-50: Well, yeah ... I agree that ... one of the major components of mathematics is its logic...
- I-51: And this logic is transferable?
- P-51: Well ... no ... that's where I'm not too sure ... I'm not too sure whether if you take someone through the mathematics courses and try and instill into them the. ... the logical way of approaching a problem ... or the logical formalized mathematical proofs ... the proofs of geometry, the proofs of triangles etc. ... I'm not too sure how transferable that is. ... I think that there is a perhaps ... a ... an assumption that it is transferable more than it is. ... and I guess that is why I say that universities are perhaps guilty of using mathematics as a screening device because they are going to assume that if a person is successful at mathematics then they must have the. ... highly developed sense of logic ... ah ... because of this study which will be transferable. ... and I'm not too sure that it will make them a better historian ... it may ... but I'd like to see some ... some research on that.
- I-52: Would you say that there were some areas that this transfer was more effective to ... we mentioned the word 'historian'. ... which is perhaps a bad example.
- P-52: Well, yeah...
- I-53: ... but perhaps some other field ... like biology perhaps - would one say that biology would benefit more than ... history from having previous mathematics training?
- P-53: I don't know.. I guess I'm not really qualified to say what a biologist has to ... think like..
- I-54: No...
- P-54: ...but certainly ... any ... any field where they had to classify, categorize, which a lot of biology is involving, I suppose ... ah ... anything that requires ah ... the same kind of logical approach ... sorting things out, trying to find what's relevant and not relevant...

- I-55: Now, supposing for the sake of argument you were King of Education....
- P-55: Oh jeez...
- I-56: ... for BC or something. Right? Now, what I'd like you to do ... you are totally unfettered ... you can put into place ... what you like ... know what I mean?
- P-56: Uhm hmm..;
- I-57: We're not talking about practicalities here. How would you design, or how would you change the mathematics curriculum? In other words, would you make it different in some way? Or would you leave it pretty well the way it is?
- P-57: ...
- I-58: Or.. you know ... what would you do?
- P-58:
- I-59: For a mathematics curriculum?
- P-59:
- P-60: The actual content of the mathematics curriculum, I guess you could get into a lot of minor details as to what should be included and what could be left out now ... the sequencing of them ... ah ... but if we are going to get all ... forget about practicalities and how it might be done....
- I-61: The purpose of the exercise is to make ... you know ... better students...
- P-61: Yeah ... ok ... but you know ... I ... I guess the secret would be to only get the students involved in a particular topic when they are ready for it ? ... when they are willing ... to do it? I guess we were mentioning before that the frustration that certain students ... that

certain people have with mathematics is because they are expected to cope with some concepts that are perhaps a little beyond them at the time ... and if we could find a way of avoiding that. ... that they will only go on into their ... to higher levels of difficulty ... of concepts when they are prepared ... though how you'd measure that I don't know.

I-62: So you're saying that ... the curriculum should perhaps be re-designed in such a way that the various concepts are used at what one would judge to be the most appropriate time in terms of the students' ability to deal with them.

P-62: Well ... Ideally, sure ... and I think that...

I-63 You don't think that is done now?

P-63: ... Oh ... I think it is trying to be done ... but its the average ... it's hitting the average student.

I-64: What about the actual concepts themselves ... ah ... what I'm suggesting is that maybe the content of the curriculum should be totally re-thought and large chunks of it should simply be removed and other things be put in its place. Do you feel that it is essentially 'Ok' now, or do you feel that it requires drastic revision in terms of what's actually in it?

P-64: ...

I-65: Let's take an example ... How important is factoring?

P-65: For the average student, not important at all. I mean ... god ... we are trying to teach the grade 9's how to factor trinomials...

I-66: Yes...

P-66: Uh ... why? You've got - what percentage of them? - perhaps 20% of them will go on and never have to do factoring of trinomials even in a mathematics course. But that type of algebraic manipulation is ... important I suppose ... but only at the time and for those students that are going to go into it. Unfortunately I think a lot of students want to

leave all doors open and figure that they are all going to go to university and will perhaps have to ... to do that kind of work so they are going to make sure they get the preparation for it ... but. ... you know the grade 8 or grade 9 teacher ... they're not interested in doing it. They see no application for it. ... and I'm very hard-pressed to convince them that it is something that is necessary ... except to say that this is one of the ... you know ... algebraic skills that they will have to be able to handle...

I-67: In order that what?

P-67: In order that when they get to the higher levels...

I-68: Yes..

P-68: ... of algebra...

I-69: ... yes...

P-69: ... that they will have the basic skills. The same reason they have to learn the multiplication tables. Basic arithmetic facts. That ... it's a bit of a drudgery at the time ... back in grade 1, 2, 3, 4. ... to learn your basic arithmetic facts, but it sure is going to be handy later on.

I-70: What about Euclidean geometry? ... For example, at one time we used to teach a lot of Euclidean geometry ... we devoted all of grade 10 to it at one point. Now I see only little vestiges left. I see.. you know.. triangle congruencies is in the curriculum there some place. I've forgotten exactly where ... in isolation from the rest of Euclidean geometry. Do you feel it's a good idea to have that in there ... or do you feel it could be taken out, or what?

P-70: Well, the emphasis perhaps being on logic ... now we talked about logic before, and I'm not ... I'm not sure how much logic we should try to develop ... but I guess geometry is one of the more convenient vehicles for trying to develop of a logical mathematical argument ... you are proving your congruent triangles, and proving your congruent parts from that etc. So in that context ... Euclidean geometry ... Also from the point of view of its application to design

... art ... nature uses so much geometry in it that I think we can appreciate it.

I-71: You have seen this film 'Donald in Mathemagic Land' Have you?

P-71: Uh ... no I haven't. And I know about it...

I-72: It's a Jim dandy - it's really fun.

P-72: Yeah ... yeah...

I-73: Alright ... uhmmm ... so we have talked about how you would see the curriculum change ... but you didn't come up with any particular changes you saw being needed except the fact that the timing might be improved upon....

P-73: Well ... yeah ... I think that would be ... the number one way of improving mathematics education ... to somehow be able to handle the timing better ... and some of the things I've heard suggested ... that girls ... and boys should be physically separated in mathematics classes because at the grade 5, 6, 7 level their intellectual development is significantly different ... that the girls are slightly more advanced and can handle some of these topics like distributive property and understand it as an abstract concept ... before girls ... ah ... before the boys can.

I-74: Do you think that's so?

P-74: Well ... well, I don't see why not. They certainly develop physically ...

I-75: A question out of the air. I was asking someone the other day about fractions, you see. And this person ... there are two ways of representing a fractional quantity ... one is you know like 1 slash 2 and the other one is 0 point 5. This person felt that one or the other of these came more natural.

P-75: I think the common fraction is more natural.

- I-76: You think that a student would have less difficulty dealing with a common fraction than a decimal.
- P-76: As a simple fraction ... yeah.
- I-77: We mentioned a half and point five. What happens if we get into something a little bit hairier ... like eleven sixteenths as opposed to point six seven eight or whatever the equivalent is. Does it still hold true there?
- P-77: I think perhaps as a concept as to what a fraction is ... eleven out of sixteen ... eleven parts out of a total of sixteen is perhaps easier for a students to grasp. As far as doing any computation is concerned, of course, you'd probably switch to the decimal form. Also for comparison purposes.
- I-78: If you were to go into a state of suspended animation as of this particular moment and you woke ten years from now and walked into a mathematics classroom in North Vancouver, what would you see that would be different?
- P-78: Hmm ... we're bouncing around here, aren't we?
- I-79: I'm trying to come up with specific questions...
- P-79: Yeah ... I know.. well tying in with ... because we are bouncing around sort of ... and I have a thought in the context of what I' saying ... I get lost ... but to answer both that question and what other curriculum changes would I see. There would certainly be a much greater emphasis on ... on estimation.
- I-80: Uhm hm...
- P-80: Because a lot of the computation ... even algebraic skills are perhaps going to be done by computer calculator so there is going to be much more ... need just to play just referee I guess, with the technology ... to say 'Does that sound reasonable' ...Before you do anything ... to have an idea before... to become better guessers. So that I can see that being done more in the classroom in the future as well as I guess a lot more technology in the classroom.

- I-81: Ok ... how about that ... what do you see the role of computers and technology in as being in mathematics education?
- P-81: Oh ... I guess the obvious answer is drill of basics. But I would hate to think that is going to be the primary use of computer technology in the classroom. Ah. ... they could be used to develop one's skills in estimation...
- I-82: So you'd have ... a computer program to, for example, that would say how much is so and so and you'd type in an estimate and it would confirm...
- P-82: I suppose ... or on the screen comes a regular shape ... estimate its area. Estimate its perimeter. All kinds of what we typically call word problems. probability problems are another good example of application for computers. ... the word problem ... the probability problem ... just pose the problem ... and ask for an approximate answer. 'What would you think would be close?' And ... then it would have its algorithm or something where the student could control it or it would just give them the correct answer.
- I-83: What about the symbolic manipulator? You mentioned this topic a little while ago. Do you see it having a role? Can you describe that role or not?
- P-83: In what context did I mention symbolic manipulator? ...
- I-84: Well, you mentioned calculators and I think you sort of threw in the symbolic manipulator at the same time. Maybe I was
- P-84 Oh!. ... Uhm ... well I guess in the way of algebraic manipulation. Distributive property. ah ... the whole solving of equations or something. That could easily be put on to a computer program so that ... ah ... the equation would be posed and the student would feed it in line by line.. ah ... feedback from the computer at each stage ... that type of drill I suppose ... rather being done with pencil and paper. There is not reason why you can't do it with pencil and paper, but for a while it will be the novelty of the technology which will have a certain...

- I-85: So you see the computer in this instance as something that would provide something that you might call a structured practise environment.
- P-85: Oh ... yeah ... yeah ... maybe you know more about it but ... I've heard that ... but apparently Alberta has apparently got just about all its high school mathematics curriculum computerized. Computer Assisted Instruction. I don't know how successful that would be ... I really don't know ...
- I-86: What's your own experience with computers?
- P-86: As applied to...?
- I-87: Well, just computers generally. You know, what have you done with them for example.
- P-87: Oh ... I've played around with computers for years and years and years. Both applications as well as programming.
- I-88: You program?
- P-88: Oh yeah ... Sure I...
- I-89: You program in BASIC?
- P-89: Ah ... Pascal...
- I-90: You program in Pascal? Ok. Good.
- P-90: Yeah ... I've only taken one university course and that was the very first computer that was brought into UVIC...
- I-91: Really...!
- P-92: They were teaching programming in FORTRAN ... you know the original version of Fort ran...
- I-93: Uh huh...

- P-93: ... but then when they started to come into the schools, I taught myself Apple BASIC and went from there to oh ... to Pascal.
- I-94: Uh huh...
- P-94: Uhmhm ... You know, so I'm certainly aware of the ... the difficulty in writing of programs that will be useful in the classroom.
- I-95: You think that computer programming as an intellectual exercise is a 'good' mathematical activity?
- P-95: Not necessarily a good mathematical activity ... it certainly is one of the better vehicles for teaching logic ... demanding logic, precision ... remember before, we were talking about one of the applications ... one of the uses of mathematics is of course being a tool, a vehicle for teaching logic ... well you don't have to do the mathematics proofs from geometry or algebra in order to do that. And how easy is it to ... to apply that logic that you learn in a mathematics course to any other form of logic ... certainly in computer programming might be a better vehicle for teaching the rigor and the discipline and the logic of carrying something through.
- I-96: Would you, if you became the king of mathematics education in BC - would you make computer programming, perhaps Pascal or something ... a part of the math curriculum?
- P-96: Perhaps a short unit ... for two reasons - one is that it would be vehicle for trying to develop in every student a better degree of logic and attention to detail and all the rest that goes with it ... Not a large unit because it has been my experience that a lot of students a .. lot of students cannot handle it. They become very frustrated with it ... and this is something we've got to try and keep to the bare minimum ... but one reason for perhaps doing it a little bit is as another vehicle for logic. ... but also because computers are such a ... currently, but in the future even more so, big part of everyone's activity... ... recreational that ah ... I think it would just be nice background for them to try and get some better idea as to how the computer works and how humans try to get the computer to do what they want.

- I-97: There is another way of looking at computers in mathematics; and that is the whole notion that you can use a computer language - or a short program or something as a means of expressing mathematics thoughts. In other words you would use a computer language as mathematics language. The most obvious example of that ... I mean, for example, if we were to say ... instead of saying $5!$, you would say `FOR X=1 TO 5: X=X*X` or whatever ... next. ... so that short program would be the equivalent expression as $5!$. Now that's ... here you have an example if a ... where this BASIC program might be a little more cumbersome, let's say..
- P-97: Yes, but...
- I-98: But what is also true is that it is possible in some languages to express things more succinctly.
- P-98: Uhm ... well, I'm not sure if it would express it more succinctly, but it would certainly, perhaps, and maybe this is what you're referring to, it would give a deeper understanding of what exactly ... what it is...
- I-99: So that `FOR X=1 TO 5:X=X*X` ... that is a better explanation or a better notation because its more obvious what it means?
- P-99: I wouldn't say its better ... but it is certainly an alternative expression. And if the student has difficulty understanding that abstract concept of factorial, then this would be another way of trying to get him to understand it.
- I-100: Would you like to use a computer for such a purpose in math?
- P-100:
- I-101: Or you don't have time? Or its just not necessarily the best way to do it.
- P-101: All of those I guess. Ah ... I mean, you have to have the computer in the classroom. ... you have to have the time ... with individual students that they can access that computer and have a teacher

provide some direction input... and direction and commentary as they are doing it.

I-102: What if they all had one built into their desk?

P-102: Ideally ... great. Yeah, I mean I could see ... I'm not being too specific here, but I could see that for many purposes, many opportunities of teaching mathematics it would be ideal to have a built in computer in every students desk.

I-103: Are you familiar with Logo or APL at all?

P-103: A little bit of Logo.

I-104: Yeah...

P-104: Yeah ... I mean, turtle graphics is one of the components of Pascal. No I shouldn't say components...

I-105: It can be ... some Pascal implementations have it.

P-105: Yeah ... the Macintosh ... you know, you just ask for turtle and you essentially got...

I-106: I've been floundering all over the place ... I haven't come up with any good questions. What have I left out? What should I ask you?

P-106: Students ... teachers impressions of math?

I-107: Conceptions.

P-107: Conceptions.

I-108: Well 'perceptions' maybe ...

P-108: ...

I-109: You see, the thesis is this ... at least the background thinking runs something like this: If one is going to implement some sort of change in curriculum then whatever mechanism by which you do that would

be more effective if in doing this planning for that you had some kind of an inventory of how teachers view the subject.

P-109: Yeah,

I-110: Now, the purpose of this exercise is for me to figure out how to find out what math teachers' perceptions, conceptions, of the subject is.

P-110: Yeah.

I-111: Because only after I figure out how to find this out can I go about finding out what it is. What I may find is that there is a whole range of different perceptions ... that your perception of mathematics is quite different from Jon Carrodus'.

P-111: Yeah. Yeah. But I'm sure we all have many perceptions...

I-112: Uhm Hum.

P-112: .. of what math is with perhaps different ... emphasis ... different emphasis ... on I guess maybe to summarize perhaps ... I view mathematics as an intellectual uhm ... study ... just for the fun of it ... when students ask, you know, the question comes up once or twice a year 'Why do we have to study this?' 'Because its fun. Okay...'

I-113: (Laugh) Do they believe that?

P-113: I don't know if they believe it ... but I hope they believe that I consider it ... that I believe it. But it ... One, it's fun. Secondly we teach mathematics because ... it is a very valuable tool ... the model of the universe if you want to...uhm we teach it for its logic ... as a vehicle for teaching logic ... a disciplined mind ... and ... we ... I guess the applications ... that's really the same thing as the model.

I-114: You know what I used to say when they asked me that question? I said 'Listen. You are not ... you don't know enough about it yet to understand the answer to the question, so I won't bother trying to explain it to you.'

P-114: Yeah. Yeah.

I-115: Do you buy that?

P-115: Oh Yeah ... sure ... and I try and tell the students that they know very, very little ... without trying to put them down. They know so very little of mathematics. I'm always trying to tease them with ... you know ... at the grade 8 level ... with imaginary numbers, caternarians ... what else, you know to let them be aware that there is so much more in the field that they've got no idea ... the extent of math. I always relate the story too ... sometimes I relate the story of two of my university professors ... a husband and wife team up from California ... that we had different courses from. This would be third and fourth year honours mathematics courses ... but they could not understand each other's work. They would be sitting there at the dinner table doing their preparation for the next lesson or something ... and they just didn't understand what each of them was doing. I found that at the time. ... a bit of a revelation ... an insight into the subject that is just so expansive ... that...

I-116: These people were taking different courses you say...?

P-116: No they were there for teaching ... different courses at the university.

I-117: Oh, I see ... and they didn't understand what the other was doing?

P-117: Yeah. Yeah. I mean here were the instructors, teaching you mathematics and they didn't understand each other's math. I forget what subject they were ... real analysis, more of an algebraist ... but they couldn't ... I mean they had a basic idea of what they were doing ... but they couldn't follow each other's work to any degree. Uhm ... so where does that leave us? Perceptions?

I-118: Well. We've gone for enough time here ... you don't have to feel that you have to say anything more. It's just that if you felt there was something ... you know.

P-118: Well ... I'll think of all the things I should have said after I leave.

Participant #0.3
June 17, 1989

- I-1: The first thing I'd like to comment on (participant's name), if you don't mind, is: "Mathematics is considered to be an important school subject." So the question is "Why is that?" Why is it that math is taught in school? Why is it an important subject?
- P-1: You really start off with the easy ones, don't you?
- I-2: (Laugh) I guess you could answer this one from several points of view - why is it historically in the curriculum? Why does society want it there now? Why do you think it should be here now? What use is it to the kids? You know ... why do we teach math?
- P-2: I guess the first thing that I ... when you first said that question I ... the first thing that came to my mind was the strong response that I get from parents at parent-teacher night ... and they say, "I wanted to come and see you and the English teacher because those are the two subjects ... so there is a ... the parents view Math and English, I think, as the two ... and I'm biased, I'd have to say Math is the most important subject that the kids take in that school.
- I-3: The parents ...
- P-3: The parents see it that way. That's been really common on just about every parent-teacher night. It's ahh ...
- I-4: From their perspective, that's a given?
- P-4: Yeah ... so historically ... ever since I can remember, they always been very concerned about how their child is doing in mathematics. You know ... more so than just about any other subject.
- I-5: Ok. Why do you think that they think so?
- P-5: I think they are going back to when they were in school. They remember so and so who is an old Math teacher. And it was definite. It was either right or wrong, and it was sort of a big divider. It was a mark of intelligence ... how you could handle patterns and thinking.

- I-6: You think it is a mark of intelligence?
- P-6: It is one of them. It's one of them.
- I-7: Uhm hum ...
- P-7: I think in Social Studies and English there is a subjective kind of evaluation; while mathematics is so objective that ... it's pretty hard to argue against. So ...
- I-8: What are the arguments for ...
- P-8: How do you mean?
- I-9: You said that it's pretty hard to argue against. What is it that is hard to argue against?
- P-9: Oh ... oh ... the evaluation. The parent thinks that he isn't getting a ... fair shake ... or a teacher ... they are bringing in their own personal biases ... on grading that kid ... and you can't do that in mathematics. The test scores speak for themselves. So there is a lot more objectivity.
- I-10: You would say that that is something that characterizes mathematics evaluation is that it is highly objective and not very subjective.
- P-10: Right ... and ... I think that's what parents, when they were kids sort of identified back with. They remember that old math teacher that they had and ... so-and-so, and ... I think they see the value of math more after they leave it kind of thing. A lot of parents will say, "Oh Gee, I was terrible in math ... and I don't want my kid to be like that."
- I-11: Both of the other people that I spoke to mentioned this particular point. That people tend to say, when you discuss with them what it is that you do, that "I didn't do well in math." Right? So, math is perceived by the population, if you like, as a difficult subject?
- P-11: Yeah. I think so. Yeah.

- I-12: And ... to what do you ascribe this? The fact that it is highly objective, or what? Why is it that math is perceived to be so difficult?
- P-12: ... ooh ... hmmm ...
- I-13: I mean, I would imagine that if you took the scores, the marks, if you like, handed out in English and the marks handed out in math, they'd be just about the same. There is the same number of A's and B's and C's and whatever. Surely the characteristics of a difficult subject would be one in which a larger number of people got worse scores. Wouldn't you think?
- P-13: I think, once you're ... uhm ... that's a hard one. In English, I guess, if you have to write that essay and you don't ... uhm ... you don't have a good handle on ... you can always put in a little extra time ... kind of thing?
- I-14: A little padding you mean?
- P-14: You can pad it somehow. You can always ... you can get into the next level. You can get by with it. An in mathematics it is a lot harder to just get by ... getting to that next level is a much more difficult step.
- I-15: Do I understand you to say that in mathematics you cannot progress to the next level until this one is mastered?
- P-15: That's, I guess, what I'm saying. Yes.
- I-16: It is a highly structured, sequential thing?
- P-16: Yeah. Yeah. More or less. Yeah.
- I-17: And that in other subjects that does not apply as much?
- P-17: Not as much. No. That is absolutely true.
- I-18: What real value is there for kids taking math? What do they really get out of it?

- P-18: I think there is a ... Well, I guess the highest level there is an intrinsic feeling that "I've done a good job. I really do understand what's going on here, and I can apply that to other mathematical problems." I was going to say "... other situations."
- I-19: Why did you not say that, (name).
- P-19: I just did.
- I-20: You said that you "... were going to say 'You can apply it to other situations'".
- P-20: Ah ... because I think here ... I always get the question, when you are doing the quadratic formula or something, the kids kind of ask you, ah ... "I'm never going to use this again in my life." And I have to agree with them. They are probably absolutely right. (Laugh)
- I-21: (Laugh) What answer do you give, by the way?
- P-21: So I say, well ... ahm ... I said. "What will you do when you leave school?" and they say "I don't really know." And as soon as they say that, I say, "Well, do you think you will go to university or college?" "Yeah, I might ... "
- I-22: Uhm hum ...
- P-22: "Might you take a math course or ..." "Well, I might". So I say, "Well, that's what we're not sure of either. We don't know what you are going to do. And if you are going to engineering or going into a field where there is a possibility that you will be working with a formula like this or maybe this one, and I bring up the Golden Gate Bridge and it has 33 simultaneous equations...."
- I-23: (laugh)
- P-23: ... or whatever it is. But ah ... I say "Yeah, you're right. For most of you, you probably won't use this. But some of you just may." They seem to accept that.

- I-24: Yes ... well ... yeah. This is what Zeke Peters [a retired teacher of common acquaintance] used to call 'Proof by intimidation'
- P-24: Yeah ... it's not a highly justified. You couldn't put it in a court of law, or anything (laugh).
- I-25: There is, of course, the argument that some things are worth knowing ... knowledge is worth knowing for its own sake.
- P-25: That's what I meant by the intrinsic kind of thing.
- I-26: Yeah ...
- P-26: They get a good feeling about getting it right. Seeing how it works. I think that when you get into that next level that I was talking about earlier ... and you say, "Hey, that's what I did back there. Just a little bit more complicated. I see the same kind of patterns here." And they ... the good math students, they really get off on that.
- I-27: Yeah. Ah ... tell me, when was the last time that, aside for the teaching situation, when was the last time that you needed to call upon some knowledge of mathematics you had? Could you give an example, or maybe two of that?
- P-27: Oh Boy! ... in my sort of every-day life?
- I-28: Yeah. You know. I mean, forget about the fact that you are a math teacher.
- P-28: Well, there is one. It's quite a while back, but. My dad phoned me up. He works down at the shipyards, and they had a cylindrical tank, and he said ... he gave me the ahm ... diameter of the tank. It was laying on it's side. It wasn't standing on its end. It was laying on its side ... and there was an opening at the top, and he knew the depth in there. I think it was ah ... some sort of oil or something ...
- I-29: Yeah ...

- P-29: And he said "We're arguing how many gallons is left in this tank. So he said, "I'll tell you the depth and I'll tell you the diameter. And he says, "Can you tell me ...
- I-30: How many gallons ...
- P-30: How many gallons is in there?" So I said, "Oh, I guess you could." Anyway, I played with these numbers and I gave him a number. I forget the exact dimension now. And he was bragging to all the guys down there that he'd been the closest ... because he ...
- I-31: (laugh)
- P-31: I don't know if he told them that I'd given him the answer. But I was a few gallons out. I forget. It was around a thousand gallons, or something. And I didn't really know the formula. I think I graphed it, or something. It was tricky doing it. Like ... I'd forgotten calculus ...
- I-32: You had a cylindrical tank of known depth and you wanted to know how many gallons, right?
- P-32: Yeah ... that's what he wanted to know. They were transferring it to something else and they wanted to know if it was going to fit in there. ... So that was one case.
- I-33: Do you consider yourself to be a mathematician, whatever that word means?
- P-33: A mathematician?
- I-34: Or the reverse ... What is a mathematician, and are you one?
- P-34: ... No ... I don't think I really am ... I'd say no.
- I-35: What is a mathematician, then?
- P-35: It's a guy like you, Harold ...
- I-36: (Laugh)

- P-36: (Laugh) No ... I'm serious. I'm not good at getting into that next level of thinking and extending the ... getting into the real abstract stuff ... I'm not very good at that. So you say, "Why am I a teacher?"
- I-37: Well ... One argument for teaching mathematics - I'm not advocating it here - I'm stating that some people feel that one reason for teaching mathematics is there is a transfer to other subject areas, ie: by learning mathematics or doing mathematics you develop a certain type of disciplined mind ... or perhaps you develop problem solving heuristics, or whatever, and that this ability is transferable to other disciplines. What do you feel about that?
- P-37: I feel there is a degree of truth in that, definitely. I feel the same way about computers. That ... programming in computing, that that has a transference to other levels, and if its true there, I don't see why it wouldn't be true in math as well.
- I-38: You said ".... To other levels". I'm talking about other subject
- P-38: I mean ... other subject. Yes. Other subjects.
- I-39: So, you would say that learning computer programming, or writing programs ...
- P-39: ... gives you a clearer understanding in other areas ... when you tackling problems in another area, think it helps your logical thinking ...
- I-40: Ah hmm ...
- P-40: Especially something like Pascal, where you have a top-down approach.
- I-41: These other areas that you are thinking about ... would they apply to the humanities, for example? Is there some kind of a transfer from ...
- P-41: ... I ...
- I-42: Computer programming to ...

- P-42: Yeah ... I think there probably is. But to a lesser degree. The more mathematical, or the more scientific, probably and the more analytical, then it would have a higher transference. And Social Studies and English maybe not quite as much. But there is still something there.
- I-43: Chemistry, for example ...it would be higher there.
- P-43: Yeah. that would be higher. Yeah.
- I-44: Uhm. Hum
- P-44: Yeah.
- I-45: Ah.... Some people feel that one of the main purposes served by mathematics in the school curriculum is to acts as a ... a filter, which colleges and universities use to select people by. Ah ... how do you ... do you feel that this is a valid selection process?
- P-45: ... How do you mean, like ... valid?
- I-46: Oh. Oh. For example, you can't get into nursing unless you have Algebra 12.
- P-46: Oh. Oh. Well, it's definitely that filtering process is definitely something that happens, and I think it is one of the main subjects they go by is mathematics.
- I-47: Yeah. Well, do you think there is another way to do things? Or would you rather see it done some other way? Why do you think they do that?
- P-47: I think it goes back to that earlier point. It's so objective. It pretty definite. It's pretty hard to argue against.
- I-48: Uhm hum. Its convenient.
- P-48: It's convenient ... it's more than a convenience thing.

- I-49: Now. Supposing that you were ... ok ... what do you think about the mathematics curriculum that you teach? Do you believe that it is appropriate? Would you like to see it done differently in some way? Or do you feel that the curriculum is basically ok?
- P-49: ... Well ... right now I'm teaching one block of nonacademic Math 10's.
- I-50: Ah hum.
- P50-: There's 25 kids in the class. Fifteen are designated as learning disabled. Of those 15 there is about five who aren't really learning disabled. They're really behaviour problems. Disordered individuals.
- I-51: Alright.
- P-51: And when you teach a class like that ... it really takes it out of you ... and when you question ... the value of that curriculum, and I see the kids in there with their doing ... uhm ... I really don't get a very good feeling about it.
- I-52: Should there be some other kind of math? For example, that they're doing ...
- P-52: Maybe ... yeah. Maybe they shouldn't take any. This particular group ... ah ... if I can get them sitting down at a desk, and if I can get them to bring a calculator ... ah ... if I can get them to stop throwing things in the waste paper basket ... these are great momentous occasions. And as far as ... when you try and get something across to them ... they are very under motivated. They ... the more concrete it is ... I've just finished a little trig with them ... and about half of them have picked up what a sine, a cosine and a tangent is ... and can figure out the second side ... you know, given an angle and a side, can figure out the second side, and so on ... or given another angle ...
- I-53: Is this stuff that you are talking about now in the curriculum?
- P-53: Yes it is in the curriculum. Yeap. And when you look at the level of what's expected of those guys ... uhm ... if I taught according to

what's in the curriculum ... and I have done a lot of it ... they would be ... of that 25 there would be five or six that would pass ... according to the curriculum. According to their level ... the expectations are just too high. Actually, I have an example that I did about two weeks ago. A fellow sells three cars for \$2000 each. So when I tell them that first ... that he sold them for \$2000 each ... and then I say, he bought them for \$6200. Did he have a profit or a loss? And half the class would not be able to figure that out and would argue with me on ... the a profit or a loss situation.

- I-54: The question that you just cited, (name), it seems to me is about a grade 4 or 5 level.
- P-54: Yeah, I would think it's about grade 4 or 5 ... and these are grade 10's. Its 10A, the math ... and I'm sure there's kids in there who are grade 11 or grade 12 age.
- I-55: You say the class is doing some basic trigonometry?
- P-55: This is the most basic trigonometry ...
- I-56: ... and at the same time you are asking them about profit and loss on the question you just mentioned.
- P-56: Yeah, that was another example. But ... they actually, I think, they made better progress on the trig because they could sort of visualize the concept maybe than they could over the buying and selling and percent discount and things like that.
- I-57: It's visual.
- P-57: Geometric and yeah ... visual.
- I-58: The trigonometry stuff there is perhaps not as difficult because its ...
- P-58: ... that's right. That's right. That's exactly it. But ... what was your original question? You were asking about the curriculum. Is it appropriate?
- I-59: Yeah.

- P-59: And this particular case. I don't know what you do with a group of kids like that. If you put those kids in an academic class. Ahm ... their self concept may go down because
- I-60: ... they experience failure?
- P-60: ... they see everybody else just acing this stuff and ... they're sort of at the bottom of the heap. They tend to be very withdrawn and you don't see to much of them because there's better kids in the class. You've got all these kids in one class the management problems, and the motivation problems in that class just becomes astronomical. From the teacher's standpoint of view. Now, maybe they don't get the failure as much, I mean I'm going to be giving mostly SG's in that class because, you know, there's 15 learning disabled designated in that confidential list that comes around ... they're pushed to the next grade and a lot of them don't deserve to be pushed to that next grade. But they're going to take nonacademic Math 11, when I would say there's a great percentage of them haven't master the Math 10.
- I-61: So you would say that this is a case where maybe they shouldn't be taking math at all?
- P-61: Yeah ... probably. But then you argue just as well that these guys need some basic math to get through a bit of life ...
- I-62: Like trigonometry.
- P-62: Well, like when they buy the car and ... when you take 6% of \$2000 or something, and you tell them, "Is it \$200 or is it \$20? Is it \$1200 ... well you get about half of them getting it right, on that particular question. If they do it in their head. You say, "I want 6% of \$2000 ...
- I-63: What about for the other classes. The non-general classes? Is the curriculum appropriate there?
- P-63: Yeah. The enriched stuff, I think is much better ... and it's more challenging, and the kids there are really dedicated and they work hard ... and ... I see more problem with the bottom end than I do at the top. We've just got a calculus course going this year.

I-64: Who teaches that?

P-64: (Name) does. And, you know, ... a lot of the kids ... they get an X block in the morning ... and a lot of the kids in that group seem to come up to the computer room as soon as its over, and they're sort of the cream of the school, the top 4 or 5 kids in the school, and they're hard workers and they have that math-computer kind of link and they, they love it. They just eat it up.

I-65: Supposing that you woke up tomorrow morning and found yourself minister of education ...

P-65: (Laugh) What kind of government is in power?

I-66: (Laugh) You are not constrained by realities. In other words, we are not talking about the difficulties of implementation here. You know, you are able to do whatever you saw fit. What would you do about the math curriculum?

P-66: That's hard to answer because I would probably do what would benefit me as a math teacher. I think I would be very selfish. (Laugh) Well, what would I do about the math curriculum? Right from the beginning to the end kind of thing?

I-67: Yes. You have some options. One of the things you can do is to change the content, in terms of topics studied. You can ... decrease the emphasis on symbolic manipulations and increase the emphasis on numerical manipulations or vice versa. You can say, for example, that grade 8 math doesn't exist any more. We're going to take a year off from math for a change. You could institute highly streamed courses. You would make many different, as opposed to one main math course. How do you see yourself changing it, given the absolute power to do this?

P-67: ... Uhm (laugh) Ah ... I think ...

I-68: You can opt out of the question if you like., You don't have to answer it.

- P-68: No ... I'm just trying to think. Off the top of my head. I guess I like the idea of an evaluation that goes right across the province ... at a particular grade level.
- I-69: An evaluation that goes right across the province?
- P-69: Yeah ... you know ... examining all the grade 10's ... all the grade 10's in the province do a similar kind of test.
- I-70: You'd have a provincial exam.
- P-70: Yeah ...
- I-71: At each grade level?
- P-71: Oh ... I don't know about each grade level. Not at each grade level, but some check, maybe and let results be known as to how your area is doing.
- I-72: What would be the purpose of this?
- P-72: I think there is a lot of areas that are fooling themselves as far as what they're accomplishing ...
- I-73: Are you suggesting that some teachers are not in fact accomplishing as much as they think they are?
- P-73: Yeah. I think that's probably true. And I think they probably know that. It wouldn't come as a big surprise. But just to let you know how you're doing with the rest of your colleagues. Where you fit in the big puzzle. I think you have a very isolated view of how you're doing in your little classroom. You don't really know what's going on a lot of the times. On a provincial basis.
- I-74: You feel that as a mathematics teacher, there is a certain level of isolation from colleagues in other schools and other district?
- P-74: Yeah. I think so. I think you get a little bit buried in your classroom.
- I-75: What about the kids. What would they get out of this exam?

- P-75: Lots of pressure and inconvenience. (Laugh) It wouldn't be gradeable or anything like that. Just a ...
- I-76: The purpose of the exercise, then, is to ... for mathematics teachers to know how they're doing ...
- P-76: how they're doing. I don't think we have a good handle on that. I think that (school district name) as a district is (inaudible). What would I change in the ... I'm still thinking about that.
- I-77: Well, for example. Recently, a unit on statistics and probability has been dropped in ...
- P-77: Dropped?
- I-78: Well, 'dropped in' - I mean, 'introduced'.
- P-78: Oh. Yeah.
- I-79: And at the same time over the few years we have seen a reduced emphasis on classic Euclidean geometry, which at one time was very high on the curriculum. Do you think that that was generally a good move, or what?
- P-79: I remember taking all that Euclidean geometry in grade 10. As a matter of fact, all of grade 10 was geometry. I loved that course. And I did well in that course.
- I-80: You loved that course.
- P-80: That was great! And I know kids beside me who just dreaded that thing. Who just couldn't do it.
- I-81: Was that a valuable thing do you think?
- P-81: I did much better in that than I did in my regular algebra... Was it valuable? It was for me. (Laugh)
- I-82: (Laugh)

- P-82: You know, this statistics. It's all personal preference. I don't like statistics. I took a Stats 304, I guess it was out at UBC and I didn't enjoy that course one little bit. Ah what ... I don't know. Things go in and out like the tide. I imagine we'll try stats for a while and get tired of it and then drop it. Somebody's going to start screaming we're not doing enough with the regular ...
- I-83: I guess my question is: "Statistics and Probability. It could be argued that ... here is one position that - you know Jim Swift's position on this - he says something to the effect that having a background knowledge of statistics and probability is important for a citizen by virtue of the fact that much of the political information that he receives is in a statistical form.
- P-83: Uhm Hum.
- I-84: So it could be argued that ... it could be argued ...
- P-84: Yeah.
- I-85: ... that it is a socially valuable thing.
- P-85: Yep. ... Yes. ... I remember also a few years ago, in one of my math classes, doing misleading graphs.
- I-86 Uh huh.
- P-86: And it so happened that there was a graph that came out in a little pamphlet from the district of (school district name). And it had outlined on it a pie chart, and it had on it percents of where the funding goes and how much goes to each area, and it had something, I forget what percent the ... was for education ... and they had all the facts and figures there and I looked at this and I thought: "This is the wrong size you know". I just looked at it - you know how you do your estimation kind of thing - and so I really worked it out, and their pie chart was out by about 11 or 12 degrees. And they were really out on a lot of them. So I took it to class and I used it, and the kids couldn't believe it, and then I showed them the pamphlet and ... I actually phoned up the district and talked to the guy ... and he got

very defensive about who drew the graph and so on. It was a great little exercise. So ... yeah. Interpretation of statistics. Yeah, I guess I'd have trouble arguing about that one.

- I-87: What about the value of classic geometry?
- P-87: Well ... I guess it's that type of thinking. Does it help you in other areas, or does it transfer well?
- I-88: And you believe that generally, it does.
- P-88: I think it generally does. But it's like the medicine. It's a bit painful. You know. There's areas of the brain here that have been untouched for some of these kids, and boy ... (chuckles) when you get ... you see we're opening doors here that have never been open before, so your going to get (laugh) (inaudible) ,, pay attention here.
- I-89: Some people feel that there is component of one's brain that makes a person a good math student. In other words, some people naturally do well at math, while others quote don't have it although they might to fine in other areas. Do you believe that mathematics is something - you know - that some people can do and others can't?
- P-89:
- I-90: Or is it something everyone can do if they want to?
- P-90: ... No ... I don't think its something that everybody can do if they want to.
- I-91: ...
- P-91: (Laugh) I wish that were true.
- I-92: What role do you see for computers in mathematics?
- P-92: ...
- I-93: You have access to computers on a regular basis.

P-93: Yep.

I-94: The question is, "In what ways do you use them?" Secondly, "In what other ways do you think you could use them - given the right resources and facilities?"

P-94: Ok. I've got some ideas on that. I had a grade 8 class last year, and I took them into the Mac lab, and I'd written a little program - it was just a drill and practice program. As a matter of fact, it was a program I'd written myself. It was not of high quality. It just gave you 10 questions - 10 addition - 10 - you chose the operation ... and depending on the number right and the speed that you did it, you got a little score. You got so many 'rad points' ... and to get a good score you had to add up your totals in adding, subtracting, multiply and divide ... and the kids got quite good at that. They really go for that ... and as change from the regular text book it was really good for them. They really enjoyed it. I took some measurements on how they improved from the first few minutes that they tried the program and until the last. I think there was one kid who went down, but the vast majority of the class really improved. I guess their keyboarding skills would have something to do with that too. The quality of software coming out is ... or CAI - is probably getting a lot better.

I-95: Have you seen any recently?

P-95: Well, my son's in grade 3 and he's learning his multiplications tables, and he's having a tough time ... so I heard about this Math Blaster, and we got that program (participant has a computer at home). And he finds that quite good. He will work on that computer doing Math Blaster ... practice and drill ... whereas he won't take out a piece of paper and write down his multiplication tables and practice them or flash cards. He doesn't want to listen to explanations of grouping or anything like that. But he sees the computer - and oh - no problem.

I-96: How long has he had access to it?

P-96: This has just started. We just got it about ten days ago.

I-97: Ok. So you see the computer as a useful device in mathematics by virtue of the fact that ...

- P-97: I can see the potential for it. But again here is my isolated view. In our school. We aren't doing it. It's not happening in our school. But the potential is sure there.
- I-98: You are now talking about a drill and practice type of activity.
- P-98: That's a drill and practice. Yeah.
- I-99: Uhm hum.
- P-99: Now, we just also bought - at the other end of it - we have a calculus program that has, I think it is 300 problems in it. It has a tutorial section, and it has a teaching kind of mode to it. I think there are 3 or 4 different modes in it. And (other teacher) and I looked at it and we were quite amazed at ...
- I-100: How well it was written ...
- P-100: ... how well it was done. You had parabolic curves and you had your tangent line, and by clicking the mouse you could pull the tangent line where you want it. And when you did it you see the slope change so nice. And those are things that a text book just has trouble in getting across maybe. The idea of motion in there. And the solution to the ...
- I-101: In this case, you are saying here, it's a program, its main benefit has something to do with the fact that the presentation was good as an audio-visual aid. Is that what you said?
- P-101: Yeah, you're getting the visual, you're getting the audio you're manipulating it. You can ask it to repeat. I guess if there is a teacher in the classroom, you're not going to put up your hand and say, "Give that to me again", whereas here - "Oh. I want see that again". You can go back in that computer. It's made for me as an individual - it's not the teacher giving it the same thing to the whole class at the same time at the same rate.
- I-102: So. You're saying this is a tutorial program. But it is of superior quality because it allows a person to interact with the software.

- P-102: Yeah - here is a lot of interaction. And not only that. I think the - I can see that kid - who's working on a terminal - and the kid next to him is working on a terminal - and he "Hey! Look at this! Look what I just got it to do". "Oh yeah. How did you do that?" That interaction doesn't happen in a regular classroom. If the teacher is dominating it.
- I-103: That's a good point. I like that point. But the other point too is that what you just said - 'Look what I got it to do'.
- P-103: Uhm Hm.
- I-104 So, what the key there is is that it is the student who does it rather than the computer who does it to the student.
- P-104: Oh Yeah. This programs a really interactive thing. We looked at it for maybe an hour, an hour and a half. The time just seemed - we could believe it. There was a couple of - the kids in the calculus class there, and "Try this. Do that:". You could hardly get a word in edgewise. I mean, everybody wanted to get their two bits in there. So it was a really exciting ...
- I-105: You tried it in class?
- P-105: No. No. No, this was just when we first got it in the mail, and we put it in and believe it or not, I don't think we have looked at that program since.
- I-106: How long ago was this?
- P-106: This was about - oh, a few months ago.
- I-107: Ok. Why haven't you looked at it since?
- P-107: (Name of teacher giving the calculus course) has been waiting for the overhead projection station. And we just got that... it's just arrived very recently. And the year's ending so ... We've got a workshop - Conti is coming in to do something with the overhead projection station

(discussion of Conti presentation)

P-108: Well, anyway. And I thought you know, from the stuff that I saw at the beginning, 10 years ago, when CAI was really quite rough, this is really a dandy program.

I-108: We are now talking about the nature of how mathematics could be taught. By using an audio-visual aid, if you want to call it that. Or a teaching device. But what about the content, or the material that is to be taught. Ie: the curriculum itself? The topics, or whatever. Or the way you solve problems. Do you see that as being influenced by the presence of computers?

P-108 Yes.

I-109: Ok. Could you talk about that a bit.

P-109: Ok. Do you mind if I get back to the same program? (Laugh)

I-110: Go ahead. (Laugh)

P-110: There's a step ... what you've got is, you got $f(x) =$ a polynomial equation, right? And when they do the derivative, you can see the number at the exponent exponent float down and they do a little subtraction thing and you know it moves to here - you see everything move, and it does it in sequence. And you can control the speed of that. You can replay it like I said. There is just more manipulation than you get in the regular classroom.

I-111: But again - this has to do with methodology, rather than content.

P-111: Yes. Oh - the content level here is of a high nature.

I-112: It is of a high nature, but it is of a traditional nature.

P-112: Yeah.

I-113: The question is, "Do you see the availability of computers changing the topics or content of topics or the way problems are solved. For example, is it possible that we would change over to a totally

numerical kind of treatment of mathematics as opposed to abstract treatment of mathematics? That's a question?

P-113 I guess the immediate response is that there is a tendency to go with the numerical because it's so much easier to program - so much easier to deal with.

I-114: Is it valid to do that. For example, in integration, I don't know if you've seen it, but I've written a little routine that draws a graph on the screen and you say, "Integrate from here to here", and it comes up with a numerical answer based on adding lots of rectangles. Is this valid mathematically? Or is it better to do it symbolically?

P-114: Oh, I think there's benefits both ways. I know that it's not the answer you're looking for, I know. (Laugh)

I-115: (Laugh)

P-115: I see them as being complementary, and what doesn't work for one kid, the second method's going to work.

I-116: In teaching, you should one proceed from the abstract - talk about generalities first - and then deal with quote word problems later, or is it better to start with concrete objectives and proceed to develop the abstract?

P-116: I suppose if you has a real tremendous enriched class that you could do the abstract first, but I think for the vast majority you have to do the concrete, and then try and extend that. That's what I would have to guess.

I-117: Tell me a little bit about your own background with computers.

P-117: My background? Oh - it goes back quite a way.

I-118: You teach Pascal, do you?

P-118: Well, I'm starting to.

- I-119: You are starting to teach Pascal? Do you speak Pascal? Can you program in it?
- P-119: Yeah. I can program in it a little bit. I don't go far enough to - I'm extending that each year. So, I'm going a little further each time.
- I-120: What about other languages? I mean, presumably you speak BASIC? Or do you?
- P-120: Yeah. What happened about 10 years ago, (another colleague) was teaching Computer Science 11 which is what it was called. And we had the Apple II+ (inaudible) machines, and I used just sort of go in there and look at what he was doing. I had no concept of what a computer was at all. And I used to go in after school and play with a few little things. Pretty soon he said, "You interested in teaching a course?" And I said, "Oh God no." I was shocked you know. Me do a computer course? He said "Try it. Try it." And so I did. Of course it was really ...
- I-121: That was programming in BASIC
- P-121: Mostly programming in BASIC. At that time we didn't do any word processing. Didn't know what a database was, or anything.
- I-122: You recently bought a Macintosh.
- P-122: Yeah. Yeah.
- I-123: What about spreadsheets, for example. Do you use them at all. You are familiar with spreadsheets?
- P-123: Yeah.
- I-124: Have you used them in mathematics classes at all?
- P-124: I haven't used spreadsheets in math classes at all.
- I-125: Do you see a place for that?
- P-125:

I-126: ...

P-126: Right now it's got very limited potential. We had a fellow come in from ... SFU. You probably know the guy. Was he a graduate?

I-127: Oh sure. Ofer.

P-127: Ofer. Sure. That's right. He did a little unit with the kids and was very successful. It was very good, and it was no problem. As far as to how you fit that in on a curricular basis, create the time for that ... you know ...

I-128: What I'm trying to get at is, "Should the curriculum change to ...

P-128: ... change to accommodate that kind of thing.

I-129: Exactly.

P-129: Uhm

I-130: ...

P-130 ... Probably not. With the changing technology there is benefits there I guess. ... (inaudible)

I-131: You know what a symbolic manipulator is?

P-131: Not too sure.

I-132: A symbolic manipulator is a computer program and you would type in $a = b + c$ and then you'd say, "Solve for c", and it comes up with a $- b$. Or you could type in an expression, and say "Here; factor it for me", and it comes up with all the factors. Right?

P-132: Yeah.

I-133: Now. Do you think that would be a useful thing to have in mathematics instruction? Or is it doing something the student should

be doing, and therefore ... it will mean that he doesn't have to learn it? Or does that matter?

P-133: This is very much like when the calculator was first introduced. I think we had quite a bit of resistance to kids using calculators. The idea is that once you get to use that calculator you can do those things faster, you can go into more depth, you can solve more difficult problems. And so on. I'd imagine that with the symbolic manipulator?, you'd be able to do more stuff. I think you'd still have to present what's going on there. The kids should have a good knowledge of math ... yeah, there's benefit in that.

I-134: Some people feel that the purpose of mathematics - its essence - is that it is a tool to solve real world problems - we are now talking about engineering applications, financial applications and so on. That's what it's all about. A tool, and you use it for something, and that's the main thrust. Other people feel that mathematics is an abstract thing that we study for its own sake. The applications and use is not really what it's all about. How do you stand on this?

P-134: Number 2 (laugh)

I-135: (Laugh) Is that right?

P-135: Yeah. Yeah. Well, ok. I'm a math major and when somebody asks you where do you use math in the real world, and what do you tell him? Basic estimation and so on. Sure, you catch that store clerk once in a while or you know what your tax is approximately but I think the more important thing is the intrinsic value - the thinking behind it.

I-136: Another person that I spoke to used the word beauty a lot in the interview. One of the things I hope to do here, by the way, is to see which key words people tend to use. Anyway, he used the word beauty a lot in describing mathematics. Do you associate mathematics with the word beauty?

P-136: Beauty? No I don't think so.

I-137: Here is another term that was used a lot. Disciplined mind.

- P-137: Yeah. I can identify with that much more than beauty. I like the idea of patterned thinking. Extension of thinking. Analogous events happening. You know, something with structure. Patterns.
- I-138: What is the biggest obstacle to teaching mathematics? The facilities in which you work? The curriculum you are required to present? Limitations in yourself? Limitations in the students? The nature of the subject? What are the hurdles?
- P-138: I don't think there is one of those that stands out. I think it is a combination. A little of each, kind of thing.
- I-139: What I'm trying to get at - some people tend to emphasize that the limitations are in the lack of motivation of the students ...
- P-139: Oh yeah. There is some of that. Definitely.
- I-140: And that is their biggest frustration. Other
- P-140: For one semester I've had this group of grade 10's and the type of thing - "Please sit down". I asked a kid the other day, ok, the bell had gone, "Please sit down", and I was getting my books out (some confusion) everybody is starting to open their books and this kid is still walking around visiting. Second time. "Please sit down." Continues to walk around the classroom. (some confusion). The kids got a plastic ball and he's bouncing it against the wall. And I said to him a third time. "Matt. Please. This is it. Sit down. Now." And ... ok it's page 354 question number 6. This kid is still bouncing the ball against the wall. I said, "Ok Matt. Now leave the room." What do you think he did when I said, "Now leave the room"?
- I-141: He bounced the ball against the wall.
- P-141: No. He went and sat down right away.
- I-142: Oh. I see.
- P-142: I said, " No. Sorry Matt. That was the previous instruction. Now it's 'Leave the room.'" "I'm sitting down." I said, "No. I'm not asking you to sit down. I'm asking you now to leave the room." I mean these

things. The motivation of that class is so bad. Of a hundred items, math is in the bottom 2 or 3 of that list of 100 items for those guys. So the motivation is really and extreme problem there. That's not so in an academic class. Now in the first semester I had an Algebra 11 class, and I had some really quite goofy kids and they give you positive (?) things like, "Boy, I really like you as a math teacher", and "I know I'm not a good math student but, boy, you make things really clear", and you know, you just need a couple of comments like that a semester and it keeps you going for the rest of the year. (laugh) So, you know, it varies. The motivation. If I coach the math contest group. We do that at lunch hours, and ... there's a really motivated group of kids. They're excellent. I mean - no problem. I remember a few years ago I taught night school. I did an 8-9-10. I had everybody in there from about 18 years of age to 65. And their motivation was so high. I mean, they would spend 2 or 3 hours a problem that they could get and they were missing some little thing. They were so highly motivated, and it was just - they were a ... the higher motivated the higher the degree that the teacher wants to be in there and ... and ... interact.

I-143: You have spoken about motivation on a number of occasions, and you've indicated that there is a very wide range. Is this range wider than in other subject areas?

P-143: I would say, "Definitely".

I-144: Definitely wider in mathematics?

P-144: In computer classes, my transfer has been mainly from math to computers, and one of the thing that I've really noticed in computers is a high degree of motivation, right across the board. Everybody seem to me motivated on a more equal basis. There is not the wide range there.

I-145: But the difference is that mathematics is not an elective. What about social studies for example? Do you feel that the range of motivation is as wide in Social Studies?

P-145: ... Oh ... I would guess not.

I-146: Why is it that it peculiar to math?

P-146: I think it is this (inaudible) that we were talking about before. You can BS your way into the next level in English or Socials. In math you just can't do it. You've got to master that previous level. And I think kids happen to be passed a lot easier in some of the other areas

I-147: You think that mathematics teachers are more particular and more stricter more objective than the other teachers?

P-147: Yeah ... I think ... kids are ...

I-148: What about that. Do you think that Social Studies teachers fiddle more and let kids through ...

P-148: Yeah. I do. I'd have to say I do. Yeah, I think they do. Where they don't really deserve to go on maybe. And we do it in mathematics too, but not to the same degree.

I-149: One of the things that I have to do is to - after listening and reading these interviews - (inaudible) is that I want to identify themes that seem to emerge.

P-149: Uh, huh.

I-150: And this particular one that you just mentioned here about the high degree of structure and specificity, if you like, is something that characterizes math. That is something that emerged. One of the other things that has come out is this notion of the question of transferability. So - how do people feel about that. Now you felt that there was a fairly high degree - a recognizable degree of transfer of intellectual discipline, if you like, to other subject areas, whereas the one of the other people that I spoke with did not feel this was the case. The question of what is the real purpose of school mathematics, in the case of the other two people, this business about the hurdle - the filter - was something which they emphasized quite a bit. That's what it's really all about.

P-150 Filtering people out.

I-151: Yeah.

P-151: Oh. I agree on that.

I-152: I think that you recognize that it is. I think that you said that. But I don't think that you attached - you didn't feel strongly that that was either good or bad. The sense that I got was that it is just the way it is, and it doesn't really bother me. Is that what you said?

P-152: ... Yeah ...

I-153: The other people. What I got from them is that this was not a good thing.

P-153: The filtering?

I-154: Yeah. I didn't hear you say, "It's not a good thing." But I heard you say something to the effect that maybe it has some validity.

P-154: Yeah. Oh, I think it does have validity. Yes.

I-155: So. I've drawn attention to three questions that we now have different opinions from different people on.

P-155: I think it's - what we're getting at here - sometimes I tell a kid that. "You're ah ... in for a job and there's is two of you left. And we have to make a decision between you and the other person. And maybe everything is about equal. And it shows - and they look at you marks, and all your marks are about the same, except in math. One of you is quite a bit higher than the other." And I said that way it could be used as a filtering kind of device. Maybe they'd take that extra person because they think that maybe they can organize their thinking a little better, or they - it's a measuring device - not that you'd ever use that math in your job. No you wouldn't use that math.

I-156: By the way, that is something that the other two people came up with too. The notion that the content that we teach is in fact in itself not terribly useful.

P-156: That's right. That's right. I'll go for that.

I-157: So we have agreement here. But what I hear from you is that one of the reasons for doing it is more on a personal level. You ought to this and learn the math because it is the way that you set yourself up ... for getting that job. Whereas the other people tended to have more of the 'world view'. In other words, with you it's more of a personal thing, whole the other one was more a world view. Anyway, I'm not saying which is right or wrong; but I'm just pointing out that not knowing where I'm going, or how to get there ...

P-157: Yeah.

I-158: ... what I've been able to do so far, I think, is to identify maybe 4 areas where I can point to differences in perception.

P-158: Uh huh.

I-159: And by doing a couple more interviews, I'm hoping to be able to identify a couple of more areas, and once having done that, I can say, I interviewed 20 people and we have so many people with that perspective, and so many people with ... and so we can develop some kind of grid.

(At this point we are interrupted and get off topic completely)

P-159: When (name of colleague who joined school as Mathematics Department Head) first came to (name of school), the first the thing we did was let's see now ... "Oh. You're still factoring polynomials, are you?"

I-160: He said?

P-160: Yeah. "Well, Jesus, we don't factor polynomials any more. God. Take those out of there." And I didn't know there was something wrong with factoring polynomials. I thought that that was what you did. You know. So that was sort of a shock. So. Ok. Throw those out. I didn't realize those were so bad. And in comes a lot of consumer ed stuff. We're going to learn about percent discounts and increases and so on. So in comes that , "And by the way, this shouldn't be at this

grade level. We'll bounce that up to here. Ahh. We'll have to re-write those tests, by the way ..."

I-161: (Laugh)

P-161: "Who's going to that? You do that. Ok." Ahh. Next semester. "Well, I think we'd better change this from this grade level to this level." And I mean it's been a state of constant change [The person being referred to left the school two years ago] So, maybe I feel that the answer isn't in flipping things around ... and we've done a lot of flipping.

I-162: You didn't like this?

P-162: Well ...

I-163: Why don't you like it? Because if you had done things in the way they ended up, in the first place ...

P-163: I mean we have gone cyclical in a lot of them. We've come back and changed them back in and flipped all over the place. It just lends to instability.

I-164: I think that's an important point. What you have said - what I heard you say - was that the business of stability ...

P-165: From the kids point of view. You've got a kid who's ... what's another one? I think in grade 11 we were doing simultaneous equations ...

I-166: Right.

P-166: ... and that was dropped down to grade 10. So what you have - you have a group of grade 10's who haven't done simultaneous equations. And then you drop it into grade 10, but those 10's have now gone to grade 11.

I-167: Ok.

P-167: So, they're in grade 11, and you now don't teach it in grade 11 any more because you dropped it to grade 10. I mean - these guys don't know what a simultaneous equation is. Maybe it doesn't matter.
(laugh)

I-168: What you said, I think, is that it is the implementation of the change. Because the implementation was badly planned perhaps.

P-168: Well - I don't know if it was badly planned. It just created havoc in .. it's implementation. I have a funny story. I've got to tell you this one.

I-169 Alright.

P-169: I'm teaching in a grade 11 class. The simultaneous equations was at the grade 11 level. I had this kid who is a really weak student, but he begged to be in there, and I said Ok. You know, give it a try. So he been doing (inaudible) unit. 40% 45% not doing too well. And we came to simultaneous equations, and he came to me after class and he said that, we'd been at them for 3 or 4 days, and into word problems and solving graphically and by comparison and all the different methods and so on ... and he said, "I'm really getting this Sir. I want you to know I'm really feeling good about this. I can see it he said. You've really done it. I've got this. It's really - good. I love this. But there is one little thing that's bothering me," he said, "What is that second equation doing under there? (Laugh)

I-170: (Laugh)

P-170: I had to tell you that. That's a true story.

I-171: Yeah. I believe you.

Participant #04.
June 20, 1989

- I-1: Mathematics is considered to be an important school subject ... or at least it is generally perceived to be an important school subject. The first thing I'd like you to comment on is whether or not you feel mathematics is of some higher or lower priority than other subjects, for example Social Studies, English, or whatever. Do you feel it of the same level of importance or does it have a special place?
- P-1: On the over-all scheme of things ... I think ... many of the subjects are important ... I hate to think, if, for example, Social Studies is supposed to help students to communicate, to get along with people ... to ... gather information and filter it ... interpret it properly ... learn to communicate, well, you'd make a very strong case that Social Studies is as important as any subject. English. If a person can't read or write ... what the hell can he learn? So I guess you almost have to put English at the top of the list.
- I-2: Ok
- P-2: Now, I would say mathematics is very important because a person has to be numerate. He has to understand about numbers. He has to understand about ... you know, how to do his dealings. I think it goes a little beyond that. So ... science. Is science important? Well, of course it's important. I think all the subjects are important. Mathematics, to me, you know, is definitely 'up there', but I hate to have to say, "Is it more important than Social Studies, is it more important than science?" I think probably somebody say, "Well, if you have to put it in order, would English be more important? Would language be more important ... I guess I'd have to say, "Yes". But after that, I would say that math is as important as anything else. So beyond the idea of the person being able to deal with his everyday affairs, I think it does ... you know other things ... like it teaches him ... discipline?
- I-3: Well, that is what I was going to ask you next (name). What is there about mathematics that makes it important?

- P-3: Well, as I say, probably the most immediate important thing is that a person becomes somewhat numerate. Not everybody goes to university and learns to apply algebra and ... statistics, calculus what have you, but they will, you know, at least learn how to ... how to deal with spending money and know what a discount is ... you know ... what do you call that kind of mathematics ... consumer kind of mathematics. You need that. Everybody needs that.
- I-4: Consumer mathematics as such is not something which is heavily emphasized in the high school curriculum.
- P-4: Well, maybe so. I don't think anybody would think - would say that it's not that important. I think, probably, teachers think that students ... probably pick it up without actually teaching it per se. I don't think you would find too many ... too many math teachers who would say that that kind of mathematics is not essential.
- I-5: No, but what I said was the curriculum does not ... specify things like budgeting and so on.
- P-5: I agree. There is definitely an attempt ... there has been an attempt at various times to introduce that kind of thing. And I think quite a few of us right now are definitely doing probably a better job even in our academic subjects ...
- I-6: Even though it's not in the curriculum?
- P-6: Even though - well I don't know. I think the curriculum, right now we even the provincial government has tried to introduce a course on consumer education.
- I-7: But that's a separate course. We're talking about making it part of mathematics. The kind of mathematics that people need. I'm talking about budgeting and looking after your finances. I'm talking about bank accounts and mortgages and all that sort of thing.
- P-7: Ok. I think that - personally speaking ... I bring the idea of ... a balance sheet, for example. When I teach negative numbers and I will actually have the date and the transactions that the person made, and have the person actually try to balance a balance sheet. Is he in the

red, or what? I bring that idea across. Percent. The idea of percent. Interest. We do the idea of discount. So a certain amount of that is done. But I guess, there seems to be, probably many math teacher perceive this as not being all that difficult, so that when the time comes if the student needs it, if he knows the algebra, the principles, he will be able to pick this up, sort of thing, I would think. See? I mean, to defend this thing ... some of these topics, I just wonder, you know, how ... whether the students actually relate to it that much until, in fact they go out there and start spending money and buying stuff ... get what I mean?

(interruption from person entering the room)

- I-8: What you have been talking about is what you might call the consumer math thing and so on. There is a whole host of other things ...
- P-8: Ok - you mentioned to me this thing of whether math is important. And I would say, you know, the idea is students would have to be numerate - they have to understand about numbers. They have to be able to deal with their affairs. I mean, most of my time is being spent in teaching them algebra because most of them - many of the students at high school are aspiring to become ... engineers ... chemists, physicists or accountants or what have you. So I guess, as a result, they get ... get ... what is the word ... but the curriculum as we have it, everything is kind of tied in. In grade 8 you'll do this, in grade 9 you'll do this and so on. Most of our teaching now, we have the new strand now. The data analysis. The geometry and the algebra.
- I-8: You were talking about algebra and geometry and data analysis and calculus and so on. Realistically ... realistically, only perhaps a fifth or so, of the population is going to go the university, whatever ... can use that stuff ... formally ...
- P-8: Right.
- I-10: So my questions is, what about all those things for a person who is not going to do that? Is it of value to them anyway?

P-10: Yeah. I think it is. Although, If you could ... if you could determine the student (inaudible). If you could say to a parent, "Look. There is no point in preparing ... for your child to take all these courses that are geared towards learning more math at the university. He's not cut out for that kind of thing. Then I think one could do other things. More beneficial to students.

I-11: But that's not the system we've got.

P-11: That's not the system we've got. I mean, if you were a parent and I came around and told you that, first of all it would be very presumptuous of me to say that, and nobody really knows a person's mind that well, to know that that's the way things are gonna be. So it seems to me the kind of society we have is that - the aspiration is to become a university graduate of some sort or another. To go as far as you possibly can in ... in education, never mind just mathematics. Now, mathematics may be one of the fields they choose and until they are beaten over the head that they can do it they want to try. And they want to do it. The parents push them, and we push them to an extent. If a student says to you, quite often, "Well, I think I want to try the nonacademic kind of math. (inaudible) Right away, we say "Hey, wait a minute. You're closing a lot of doors. Let's just make sure. Are you sure now?"

I-12: (name), what is it that characterizes mathematics in contrast to other subjects? In what way is it different from Chemistry, Social Studies, or whatever? What is its peculiarities?

P-12:

I-13: Now, I have some answers to that question from the other people, but I wonder what your perception is. What makes it different?

P-13: ... Well, I think that the subject matter makes it different. You know, I often wonder about myself ... we seem to get ... at least I seem to get ... where you kind of like things to follow in a logical kind of sequence. You hate inconsistencies. Now, do you become like that because you're working with the subject, or does the subject attract you because you are like that? I don't really have an answer to that.

- I-14: I personally think the latter. That's my perception.
- P-14: Which is because you are like that, the subject attracts you?
- I-15: Yeah.
- P-15: I mean, I've often given it a lot of thought, and I'll be damned if I can come up with an answer to that. You know, I mean, I ... I know that if I were to ... classify, say, my wife, I would definitely classify her as a non-. Ok, I call her a 'Micky Mouse'. The people who are not attracted by this and don't seem to ...
- I-16: Are you saying that exposure to mathematics education, exposure to the subject, would tend to make the person more logical?
- P-16: Yeah. I believe that.
- I-17: So, you believe that there is transfer of thinking 'styles' from mathematics to other subject areas?
- P-17: I swear ...
- I-18: ...
- P-18: I swear.
- I-19: And you would go as far as to say that is one benefit from teaching mathematics?
- P-19: Ok, I'm not sure that is a desirable quality.
- I-20: (laugh)
- P-20: But I definitely think it has that effect. To me, in the world where we are at, in our western world, if you want to say to a person, "You are illogical. You are not very consistent." I think the person would get very offended.
- I-21: That's probably true.

P-21: Most of us are relatively emotional a lot of the time. And if you want to tell a person - you know, "You're just too bloody emotional". I mean, I don't know what it is about the western world, maybe it's like that all over the world, but we would like to think that we can be logical about things, rational about things. And there is no doubt about it in my mind, that studying mathematics will affect you.

(the telephone rings and interrupts briefly)

P-21: As I was saying earlier on, I could almost swear that you can tell a person who has not had any training in science or mathematics ...

I-22: Now ...

P-22: ... by the way they reason, and by the assumptions they make. Well, this is one of the things that I ... quite often, you know, I accuse my students. I say, "You know, we are studying mathematics here, and yet, you know, the statement you just made is very un-mathematical". Now to be a very un-mathematical subject would be something like this. You hear a student making a statement like this; "Jesus, that test was really hard. Practically everybody failed." And when you pursue the thing further, and you say, "well, really? Who do you know that failed? Who have you talked to?" It turns out maybe they've talked to only one, maybe two, three four at most, and out of the four people they talked to they will make a statement about a class.

I-23: Now ... when you get into a conversation with people on a social basis, and they say, "What is it that you do?", and I say, "I am a math teacher" ...

P-23 I'm gonna start changing that.

I-24: (laugh)

P-24: I'm gonna start saying to people that I am a language teacher.

I-25: That's a very good point, by the way. I'd like to pursue that after.

P-25: I'll give you an example of that. This happened to me yesterday. I've been having a little trouble with my thyroid. I don't know how I got

an inflammation. We've done all kinds of things (inaudible) a scan, and so on. The eventual thing is you go and see a specialist. (inaudible) Although my family doctor told me by the time you get a chance to see him, the thing will probably be gone. He wasn't out by much. The guy kept feeling here and there and everywhere and there was just a tiny little nodule. But anyways, we started talking, you know, and he asked me what I do. And (inaudible) I said, "I'm a math teacher." "Oh, you are one of those smart types, are you?" "No really", I said, "I'm a math teacher, not a mathematician."

I-26: You think you are a mathematician?

P-26: No. I'm not. I don't think I'm that smart.

I-27: Can I diverge to that just a little bit? Why are you not a mathematician?

P-27: Ok. A mathematician to me is a guy that is so gifted that he will get a PhD in mathematics.

I-28: Is that so?

P-28: Yeah. You know, I don't know whether I would have been good enough to do that. I didn't go that far. So I would rather say that I'm a mathematics teacher I know I have a love for mathematics, appreciation for mathematics ...

I-29: Not all engineers have PhD's.

P-29: Who?

I-30: Not all engineers have PhD's.

P-30: They are not required to have it.

I-31: No, but you're not required to have it to be a mathematician.

P-32: Ok, I don't know. I don't consider myself to be a mathematician. Honestly. To me, a mathematician would be more like a guy at the university who has a PhD in math.

- I-33: Well, perhaps. I would define a mathematician as someone who uses mathematics, predominantly, as part of his profession.
- P-33: Ok. Well then, you know, we'd have to go back to our definition of terms. All I wanted to say to the guy, you know, I mean, hell, I'm only a mathematics teacher. I differentiate (inaudible) I'm not really making my living out of mathematics. I mean, I'm teaching mathematics, I'm helping students to learn math ...
- I-34: But you are primarily a teacher.
- P-34: Yeah. An the guys says, Well, what the hell did he say? What else did he say? Oh. Invariably this answer comes up. "I wasn't very good at math."
- I-35: Exactly what I was going to say.
- P-35: Ok. And I said, "Hell", I said, "It certainly didn't hold you back" You know, and he smiled. And it's very interesting, Harold, I have had ... this is almost like a replay with at least another four or five doctors, and, I mean, I don't change doctors that often, so I could almost say that practically every doctor that I have had has come up with the same deal. He wasn't any good at math.
- I-36: This is true of other people too. They very frequently - it doesn't matter who you talk to - a stranger - out of the blue, they say, "I wasn't very good at math."
- P-36: Yeah.
- I-37: So, mathematics is perceived by the population as being a difficult thing.
- P-37: Unfortunately. Unfortunately.
- I-38: Why do they perceive it this way? The point that I was making to another person here goes something to the effect that the same percentage of people get A's and B's and C's, or whatever, in math as

in other subject areas. But why is it that math is perceived to be so much more difficult?

P-38: ...

I-39: That's a real question. The question is "What is there about math that makes people think that it is hard?"

P-39: ...

I-40: Ok ...

P-40: Every time. Every time I encounter a person that thinks that mathematics is difficult ...

I-41: But what is there about mathematics ...

P-42: I don't really know that. The answer. But all I can tell you, Harold, is that it bothers me. It almost upsets me. No. I'm serious about that. And I have given it some thought. Because it upsets me. And many of the students that I have seem to encounter relatively good success in the subject, so why do they have trouble with math? Is it maybe that they keep going ... they keep studying mathematics up to a point where they can't do it or they find it too difficult. They say, "I'm finding that a little too hard, so I think I will leave that." So as a result, they forget all the good experiences they have had with math or what they have learned about math, and remember only where it ended? I'm not sure about that.

I-43: Some of the answers that I have had to that question from the other people run something like this: "One thing about mathematics that distinguishes it from other subjects is that everything is very much sequential or progressive."

P-43: I was gonna come to that.

I-44: The other thing that characterizes it, according to these people that I've talked to is that in the other subject areas - and Social Studies tends to be mentioned rather frequently as an example - is that you

can bullshit, whereas in math you can't ... bullshit. How do you feel about that?

P-45: ...

I-46: ...

P-46: Well ... I don't ... I wouldn't say that. I mean, I agree with the sequential thing. That is one of the things that makes mathematics different from the other subjects. And I use Social Studies as a contrast. Whereas in mathematics ... I'll give you an example which is a little out of date right now. Because there used to be a time, when if you couldn't factor polynomials, that would hold you back from, say, solving equations. Now with the calculator this is no longer true. I mean, as far as factoring polynomials is concerned, it's a very theoretical thing. It's no longer a problem solving equations. Right? I'm talking about quadratic equations. They just learn the formula. It's important to be able to multiply a polynomial together to get it in the $ax + b$ part ... right? But then you just plug it in. No problem. But it's definitely a sequential sort of a thing. No doubt about it. That's definite, because if you don't know about a , you can't learn about b . But in Social Studies, maybe ...

I-47: But you don't subscribe to the bullshit theory.

P-47: Not really. Because honestly, I mean, can you really ... I mean the students seem to have this deal where you can BS around, but I mean, if a question is stated accurately and precisely and the teacher asks you some particular thing. I mean, if somebody were to ask me about the climate in Russia right now, how can I BS my way around? If I've heard some things from the TV or I've read a book or something, either I know it or I don't know it. How can I BS my way around? If I'm supposed to relate the ...

I-48: There is a difference there. And that is that the information that you get about the climate in Russia, you may have obtained from a variety of sources - and socials is only one of them.

P-48: Exactly. Exactly.

- I-49: But generally speaking whether you can use the quadratic formula or something is something that you would only acquire in math class.
- P-49: But you're not BS'ing your way around. It may be that you picked up that information not out of the classroom. But to BS, to me, it means that you don't know anything and you can just give non-nonsensical information. I don't know that you can do that in Social Studies, or any field for that matter.
- I-50: One of the other things that is true of mathematics and that is that it is very much used as an entrance requirement to other fields. So cite a specific example: You have to have, I don't know, Algebra 11 or Algebra 12, whatever it is to go into nursing.
- P-50: Right.
- I-51: So it's used as an entry into other areas which may have nothing to do with mathematics.
- P-51: ...
- I-52: I mean, it could be argued that nursing is not a mathematics related activity.
- P-52: Ok, I would worry about that. Because, you know, why was that road blocked? Why was that done? I would hate like hell ... I mean ... Ok ... this happens quite often ... where you have a secretary who's supposed to type a math test for you. Now, when she went through her schooling, for what she was going to do, she didn't need any more math. Then it turns out she becomes a secretary in this school and has to type some math. And she doesn't know ... there are all kinds of symbols she can't do ... can't to the spacing properly ... why? She didn't do any math. Now in nursing, from what I understand, you have to be able to read thermometers, you have to be able to talk about scales ...
- I-53: That's pretty low level math. We're talking about algebra here.
- P-53: Ok. Algebra ... I don't know ... why was the thing made to begin with?

- I-54: I'm asking you ...
- P-54: I don't know why the thing was made ...
- I-55: You think it is a good way of doing it?
- P-56: I don't know that. Before I could make a comment on that I'd have to know what does a bloody nurse go through? Does she need it, or doesn't she need it? And if she doesn't then the mathematics should be more specialized to their training.
- I-57: The suggestion that I have had in the past as to why this is so, is that it simply is a convenient, objective way of filtering people.
- P-57: Well, then I think a big injustice is being done to mathematics.
- I-58: A big injustice is done to mathematics?
- P-58: Yes.
- I-59: And not to the student? To the person.
- P-59: And to the student. Both are being abused. Because mathematics is being used unjustly, and the student is being put through that bloody thing. I mean, I don't make that rule.
- I-60: Now, (name). Supposing that you woke up to tomorrow morning and you found that you had been appointed minister of education. Ok? We're not the slightest bit interested in the practicalities of this absurd situation, but you are it. You have the ability now to change the curriculum in three different ways. You can change the curriculum in terms of methodology, and you can change the curriculum in terms of content. Now the first question is, "How would you dictate or legislate, or whatever, methodology?"
- P-60: ...
- I-61: Or do you feel that basically, people are doing the right thing already. That's perfectly Ok.

- P-61 ... Well. Harold, that's a pretty tough thing to do. Because I'd want to know. Before I answer that I'd have to ... I'd really have to look into it. I mean, I know that basically speaking, I don't really think that there is that much wrong with our curriculum.
- I-62: There is two aspects to this. One is the content. The other is the methodology.
- P-62 Ok. The methodology? Would I change the methodology? What I would do, if it was in my power ... I would make sure ... that ... the people who are doing the teaching ... are ... first class. I would want to make sure that the people are carrying out whatever it is that they are doing - that they are dedicated, that they know their stuff and that they do, actually, are pedagogically sound.
- I-63: Do you think that this generally the case now, or do you feel that it is ... not good?
- P-63: I would like to see that improved.
- I-64: But you think that it's basically Ok?
- P-64: Well, I mean. You know.
- I-65: (inaudible)
- P-65: (inaudible) Well yeah. I think we are in pretty good shape. I mean compared to ... who are we comparing ourselves with?
- I-66: I don't know.
- P-66: I've just come back from a conference in Florida. And from what I understand, in comparison to the way things are in the United States, which is, you know, a relatively civilized country ...
- I-67: We're in pretty good shape.
- P-67: We're in relatively good shape. Now, you compare ourselves to some of the other places, yes - well, you want to say, "Well what about a

place like Japan?" well, from what I understand ... I'm not sure that if I had a son or a daughter that I would rather them have their system than ours. I don't think so.

I-68: What about the content? Do you think our topics are appropriate? Would you add to them, or subtract from them or change them?

P-68: Well, change yes ... mainly because with the coming of technology ... it's the sort of thing we've got to be conscious of almost instantaneously. I want to say daily, yearly. At least yearly. I mean at the end of every year ... but I think that this should happen even as you along. You may decide to change something from what you did last year.

I-69: Is the reason the curriculum is not taking into account the needs of technology?

P-69: Well, if it's not, then ... then it's not doing it's job. I mean, if I ... if I ... if I don't change ... something ... because of technology, I don't do because I don't have the insight to see it. But if I do have the insight that things have to be changed, I will do. And that's one of the reasons I attend this conference is because of that. Is to get an idea of where the hell are we going? ... What direction are we going?

I-70: I'm not trying to say that the things you have said aren't relevant, but the fact is that you haven't actually answered the question, which is, "Would you make any changes? If so, which changes? In content or topics."

P-70: ...

I-71: It's perfectly Ok to say, "No I wouldn't, because I think that what we've got is Ok That's all right.

P-71: Well, as I say, I think I did mention that what we are doing right now is reasonable. We're doing that data analysis, we're doing geometry, we're doing the algebra ... I mean, in order to change any more ... I would have to know a heck of a lot more. Like I said, I've just come back from a conference and the status of mathematics is, I think, in BC, at least in North Vancouver, in comparison, it seems to be in

pretty good shape. Now for me to change that would be pretty bloody presumptuous. So I would say, "Yes, I think it's reasonably good."

- I-72: One of the things that ... looking at these words that are highlighted here (referring to transcripts of previous interviews) ... one person tended to associate mathematics with the word beauty. Ok? Another one tended to associate it with the word intellectual level. Another word I heard in there somewhere or another was logic. Another word I heard was analytical. Do any of these words ring with you?
- P-72: Well, yeah - let's go back to the first one. The idea of beauty. Well that's ... Sure there is beauty in mathematics. But there is beauty in so many other things. Is there beauty in love? Is there beauty in poetry? Is there beauty in music? I think all these subjects have that. I mean, as far as being analytical, I definitely think it is. Analytical. And we talked about that earlier. I think that one of the things that it does is ... you see one of the things that I think is very important ... I mean, in your life, throughout your life ... you ... it ... is very important that you understand that if you are making an assumption, that you are making an assumption. You sometimes have to make assumptions. And if you are making an assumption then you had better know that you are doing it. So that if ... you know, how often I hear, you know ... a person who is aware of what he's implying about what he is saying, quite often will qualify his statement. Now what I find is that people who have not had this kind of training quite often will come out ... won't make a statement like that without qualifying, not realizing they are making quite a few assumptions. And I think that I ... to me ... and again we come back to the idea, "Are we like that because we're doing the bloody subject?" (inaudible) Because I know - with some people we are a bit of a pain.
- I-73: (laugh)
- P-73: You know, you sometimes you will get, "Well wait a minute, you just assumed something", and the people kind of "Hey, wait a minute. What you trying to do? Analyze what I'm talking about?" You know the people clam up.
- I-74: (name), what is the biggest obstacle to kids learning math?

- P-74: I would say ... one of the ... and not necessarily in that order ... one of it is the way it's presented. The other one is lack of discipline on the part of the students. That comes with the subject.
- I-75: It comes with the subject?
- P-75: The subject is like that. The factors that we talked about ...
- I-76: You're saying that in math you require a higher level of discipline?
- P-76: No. Yeah. Discipline, yes. The discipline. Remember we talked about math being cumulative?
- I-77: Yeah.
- P-77: And you have to retain a in order to learn b? Now. I mean, I may present a really nice lesson where they will learn about Pythagoras through a real nice ... lesson where they use manipulative materials ... in other words, they discover it by ... by ... almost by themselves. They're really quite involved. They are really excited about it. I've had lessons like that. Where you say, "Jesus, did they ever eat this stuff up. Did they ever learn." Then you find out, once you test them, you find out they maybe didn't learn as much as you think they did.
- I-78: That never happened with me!
- P-78: So. Eventually, even after they have picked this up, and they say, "Oh, isn't that nice. That's interesting." Well leave that topic to take another topic, and eventually you may match these two topics together. Well, what tends to happen is that while you are learning about b, they tend to forget about a.
- I-79: I just want to nail this down, just one more level. Do you feel that this is a ... lack, or a fault with the students, or is it something that comes with the territory?
- P-79: No, I think it comes with the territory. I think that forgetting is one of the fundamental thing of nature. And I don't know of anybody ... I don't know of anybody who doesn't forget. Let me get through to this ... and maybe, you know this ability to forget is very useful. I'm

serious about that. Supposing you have an unhappiness that has occurred. You'd like to forget, eh. You heard about this thing, Time heals all wounds. That is what happens. You tend to forget. So I don't know of anybody who doesn't forget. So, if you want to remember about that ... certain principles, certain facts about mathematics, there almost has to be a conscious effort to retain it. Or you're gonna forget it. And if you forget it, when you take the next level of mathematics, well you're a mess. And I .. I think that there are a lot of ... a lot of people who are actually very innovative ... very create ... but when it comes to the problem ... I don't know if you should call it a problem ... when it comes to the ... the point ... where they have to trust that thing to memory and remember it, they find that boring, and they won't do it. Now you can ... I have ... I think I have seen students who are actually very keen, very good math students ... but because they are relatively ... you want to call 'em lazy, you want to call 'em undisciplined, you want to call 'em ... I don't know what the hell you call 'em, but they don't seem to have that knack, that ability to say, "Oh, gee, yeah, I'm gonna remember that." And they rely ... they rely in remembering it in the same way as they remember who won last night's ... soccer game. And it seems like you might be able to get away with that in the lower grades where there is a hell of a lot of repetition, we don't learn a lot. But when it comes to a topic in higher algebra, by that I mean the grade 11, the grade 12, there is quite a bit. Practically every day there is something new. A lesson is a conscious effort to say, "I'm gonna remember that." (inaudible) And so many of our students ... they want to be there ... they want to be pick it up, but if it goes into their heads and remains there, fine. If not, (inaudible) Now, a student like that, his or her days are numbered. As far as success in mathematics is concerned. Now. Is that lack of discipline? I don't know. You tell me.

I-80: Tell me. Aside from the fact that you teach the subject, and obviously have to know or use mathematics as far as your daily vocation, when was the last time that you actually needed to call on your knowledge of mathematics for your own use?

P-80: Not that often.

I-81: Give me an example.

- P-81: An example of when I had to use it? Like I say. Very seldom. Probably it happened so long ago. About the only time that I would do that is if I'm building something. If I'm working out, you know something to do with carpentry, or ... well heck, if I do my office, when I did my shelves, I may want to do some computation, but not really high algebra. Most of it is arithmetic.
- I-82: Now. You know that my interest lies in computers. And specifically, one of the rationales that I have for doing what I'm doing, is that I believe that we are going to have to use computers in mathematics more than is the case now.
- P-82: Yes.
- I-83: But in order to allow that to happen one has to sort of plan the strategy by which it should take place. And in order for one to plan the strategy by which it should take place, you have to know the characteristics, the modes of thinking of the people you are dealing with, ie: the math teachers. Well that's what I'm doing, right? So, one of the questions I'm asking you now is, "In what way, if at all, do you see, that the fact that we now have easy access to computers, in what way do you think that, ought to, or might change methodology, content of the subject that is taught as mathematics in secondary school?"
- P-83: Well. Human beings, being what they are, unless they are nuts like you, who just seem to have that ... that innate curiosity about those bloody machines, they'd be like me. And I can see the wisdom of them.
- I-84: See the wisdom of them? Explain that a bit.
- P-84: I mean, I can see ... I have had instances ... seen people what they can do with the computer. Sure, maybe it's not gonna give you the exact answer to a problem (brief interruption) Maybe it's the same thing in science. Perhaps. In mathematics, the whole thing is laid out for us. You know, at the end you've got the provincial ... exam that the students have to write.

- I-85: (referring to highlighted words on transcript of previous interview)
Here look. Sufficient time, more time, more time, more time.
- P-85: I think it's a key issue. Because given enough time ... you know, many of the students will probably learn a hell of a lot more mathematics and they will enjoy it a lot more, if they could just learn it while they are there and take it again. In other words, if the repetition is necessary to learn a certain thing, as long as you do it for them, it's not as bad as having to go home and learn it on their own.
- I-86: Another point that was raised, (name) is the notion that a lot of stuff is introduced too early. When I asked this person, "What would you do, if you were minister of education? And he said "I would spread everything out more. Give people more time to assimilate things." And some topic or concept comes up, and he says, "I wouldn't do it in that grade. I'd do it in the next grade."
- P-86: Well, wouldn't that vary for different people?
- I-87: Well, of course. But I mean, the perception is that, speaking generally - with this one person - Ok - with that one person -
- P-87: I have to come back. If we're gonna take this on a global kind of thing, if we compare ourselves to other parts of the world like Europe, the Orient, we already are spreading ourselves quite wide. Some of the things that we do in grade 12, and some of these other countries, they've done it by the time they finish 9. So I don't know. It seems to me ... I wouldn't want to see where we are becoming more and more lax. I think students have the ability to master ... most of our students have the ability to master what we are doing, and maybe a little bit more. What I think is missing is a bloody work ethic.
- I-88: (laugh)
- P-88: They're watching too much bloody TV, and I don't think we're getting much cooperation from the parents any more.
- I-89: Do you think that is getting more so now?

- P-89: Absolutely. In fact, if we're gonna follow the states, it's gonna be worse than it is. And, this is coming back now to the social ... the social aspect of our ... of society ... you still have, I guess, the parents when the kid comes home and they say, "Eat your supper, and get up there and do your homework." I just wonder how common it is, or at least, you know, how much ... whether that has changed. I'm convinced that has changed a bit over the last ten, fifteen years.
- I-90: One final point. One of these people that I spoke with tended to dwell a lot on what you might term management problems. Classroom management problems. I think the allusion was that he spent an inordinate amount of time on discipline and not sufficient time, by default, on teaching mathematics.
- P-90: Well, Ok. Thank the Lord. I haven't had those problems yet. I haven't had those problems yet. And I hope I don't. But that would be an upsetting thing.
- I-91: Oh, it is upsetting. There is no question about it.
- P-91: But I find it kind of takes its toll. I work very hard. It almost seems that it is getting more and more and more demanding of you, the teacher, to make things innovate, interesting and so forth. And really, I, at least, work very hard. Being innovative, being enthusiastic, being excited and trying to do things in such a way that they would catch their attention. But I think we are definitely competing with the bloody television. (inaudible) There is a tendency for students to say, "Well, Jesus you know, you didn't motivate me, so I'm not that interested." Now, if you're the kind of teacher, I think, who thinks, "I've got something to say, and therefore I should have a captive audience that should listen to me, you're in for a shock. Now, in my case, if I don't get it because I'm very innovative, and because I make things interesting then (inaudible) rotten bastard. I demand that. I've been successful so far. I don't know for how long. But when the time comes that I'm not, I hope that I have enough sense to get the hell out of there.
- I-92: You will. (name), we have covered the topics that I wanted to mention. Is there anything that you feel has been left out? Keeping in mind, that what I'm primarily after is your attitude towards not only

the subject, the school subject, but your attitude towards math as a discipline or study. Is there something that you'd like to add?

- P-93: Well, the one thing that I think ... I wouldn't want this ... definitely part of taking mathematics ... a big part of doing mathematics is because I honestly believe that it does make you a better problem solver. And I would hope that in time - transfer isn't gonna happen just overnight - but in time I would hope, you know, he would, as I said, make you aware what is known, what you're assuming, and ... I don't know whether the students actually really appreciate all they pick up when they do math.
- I-94: Could you not achieve those same objectives - of logic, if you like - by teaching logic - as opposed to math?
- P-94: I'm sure you could, but isn't logic part of mathematics?
- I-95: You use logic in - yes. But there is such a thing as studying logic ... philosophy.
- P-96: Oh sure. Absolutely. All I'm saying is that that part is definitely coming out of the mathematics we have. Now if you want to do it a different way, no problem. But I think that definitely that thing comes out. Although, you know, the thing that bothers me quite a bit, and maybe it's natural, how illogical sometimes we do become, even when we are supposed to be the people who are supposed to be doing the bloody teaching.
- I-97: (laugh)
- P-97: Well, wouldn't we be worse? Maybe we would be worse without that kind of training.
- I-98: ...
- P-98: Another of the things that really disappointed me was when I went to SFU to get my masters' out there, and I found the politics that went on.
- I-99: Much worse than it is at a school.

P-99: My idea was, well you are a place of higher learning, and these people, they are intelligent and they are - you know. That sort of thing should be removed. But it's not. It seems like you say, even worse. So, I don't know. Is this a (inaudible) Perhaps. Or is it that sometimes, when our emotions take over, we lose that thing?

I-: ...

Here we drift off topic.

Participant #5
August 10, 1989

Participant's Questions:

Preliminary, exploratory:

- Q Are you the kind of person who likes to plan your life - a planned way or a spontaneous way?
- Q Are you interested in finding out how things work: how does a computer work? how does a car work - how do you fix things - do you say Let somebody else look after that. I know how to drive it I don't care what the carburetor is or the CPU on a computer.
- Q Do you think of mathematics as something beautiful in itself or as something practical?
- A I can't make that an either-or for myself.

Prompted questions:

- Q To what extent do you feel that teaching for understanding is a) desirable and b) practical?
- Q Do you believe in teaching for understanding?
- A Yes.
- Q Do you consider computer programming to be a valid mathematical activity?
- A Yes.

Proposed questions:

- Q1 Do you teach for understanding?
- A1 To a certain extent. As much as time permits. Other limitations are the number of students who do not want to understand. For some to be asked to understand is worse than to be told to do this and this. This is not intrinsic to children. It is something that society and the school systematically has taught them. They see that doing, not understanding is required on exams.

- Q2** Is there too much in the curriculum? Should some things be left out so that you can teach for deeper understanding. Give examples of what might be left out.
- A2** Yes, there is too much. Examples are factoring trinomials where the first coefficient is not 1, anything which is taught in 9 or 10 algebra just because you need it for algebra 11 rather than for any practical things that the students can see from mathematics it, such as algebraic fractions and the simplifying of them, possibly deductive logic in geometry (it is done at the moment in a haphazard way)
- Q3** What do you feel about teaching arithmetic skills versus using calculators?
- A3** I probably go for the calculators - along with an attempt to get kids to do simple things in their heads. Teach the simple things for understanding.
- Q4** What do you think about having a computer in the classroom? Would it be something for all students or just A students?
- Q5** What do you think about introducing mathematics history in the curriculum?
- A5** An excellent replacement for algebraic fractions and tough factoring.
- Q6** What do you think about students doing mathematics projects? Would you give your students an assignment to write about the biography of a mathematician or the development of a mathematical idea?
- A6** An excellent replacement for algebraic fractions and tough factoring.
- Q7** Would you give your students class time to do this?
- A7** To a limited extent. Limited because of logistics.
- Q8** Which is more important - the learning of techniques and details or an understanding and how to learn to learn.
- A8** Understanding and how to learn *should at all times be paramount, but for logistic and practical reasons is usually subsidiary* Simple things like class control. When you give students something that

requires thinking your lab management more quickly goes to pieces. If you give them something routine they will do it quietly. *Survival is necessary* This is the reason that (in the old days) they could have such large classes.

- Q9 Geometry has been presented as a way of teaching logic. Do you think that teaching computer programming would be a practical more modern alternative way to teaching logic?
- A9 Yes.
- Q9 Opinions of having deductive geometry back in the curriculum Have you taught it? Do you find it useful? What do you find it useful for?
- A9 Yes I have done it, and yes I found it useful for getting a different insight into how students think. Listening to their arguments teaches me more about my students. I have found that some who 'bombed out' in algebra were pretty good when it came to deductive reasoning.
- Q10 Integration of mathematics with science? should there be more or less?
- A10 More. Many topics are covered in both math and science - at different times and in different ways and the kids don't see the connection. More coordination is in order. We could generalize this to subjects other than science.
- Q11 Should the teaching of mathematics involve teaching processes and procedures or should it involve an overall view of what mathematics encompasses and involves? Power and diversity of applications? Out of 100 hours how much would you allocate for emphasis on each?
- A11 Perhaps 60% to processes and procedures. To get at this sort of thing would require a drastic change to the curriculum and we wouldn't do well on the international examinations any more. A lot of teachers wouldn't like it. It would take the rigour out of mathematics... we'd teach students to be appreciators of mathematics rather than calculators.
- Q12 To what extent is 'number theory' important?
- A12 It's not so much important for students to know about this sort of thing as it is for them to try and twist their heads around. Stretch

their minds. Challenge some of their assumptions about numbers. I find it is a good discussion item.

Q13 (From Sullivan Commission) Should secondary teachers be required to teach two subjects?

A13 Yes - but I don't want to! It means more work. I'm in favour of more work if some other work is taken away from me. But, yes.

Use of Computers in Math instruction

1. Audio visual aid

Should: Yes (✓) No ()

Reservations: It is always better if students do things for themselves.

I have: Yes (✓) No ()

If yes, give example Master Grapher.

If yes, how many hours in last year: 3 hours per class - 9 hours.

Associated problems: Logistical, lack of equipment.

2. Deliver Programmed Instruction

Should: Yes (✓) No ()

Reservations: Quality is important.

I have: Yes (✓) No ()

If yes, give example. Many, many.

If yes, how many hours in last year: Many, many.

Associated problems:

3. Assessment

Should: Yes (✓ {can be}) No ()

Reservations: It is easy to cheat. Must be used in conjunction with other ways.

I have: Yes (✓) No ()

If yes, give example

If yes, how many hours in last year: Many

Associated problems:

4. Administrative matters

Should: Yes (✓) No ()

Reservations:

I have: Yes (✓) No ()

If yes, give example

If yes, how many hours in last year:

Associated problems:

5. Provide exploratory models

Should: Yes (✓) No ()

Reservations: Not enough material available.

I have: Yes (✓) No ()

If yes, give example. Spreadsheet.

If yes, how many hours in last year: A little.

Associated problems: Software availability and lab time availability.

6. Other: Spreadsheets if doesn't come under previous category.

Should: Yes (✓) No ()

Reservations:

I have: Yes (✓) No ()

If yes, give example

If yes, how many hours in last year:

Associated problems:

7. Other: Programming

Should: Yes (✓) No ()

Reservations: You don't want to get carried away with it. Keep it simple.

I have: Yes () No ()

If yes, give example:

If yes, how many hours in last year: 10 hours maybe.

Associated problems: Hardware availability.

Teaching practice

Q To what extent is it desirable to make use of manipulative materials, models, films, videos etc. in teaching mathematics? What are the benefits? Problems?

A Yes. To a large extent. Availability and logistics.

Q Films and alternate presentation methods and manipulative materials I have used in class during the last year; frequency?

A Computer graphing. Films, no. Programming. Measure of distance. Next year students will build models.

Q What mathematics teaching aids such as films, manipulative materials, models etc. are available- in your school, from district and other sources?

A In our school we have quite a selection. At district level I know about a film or two.

Further Discussion:

Q Computers, because of their 'number crunching power' make it possible to solve mathematical problems using non-traditional techniques. Do you agree?

A Yes.

Q If yes, to what extent should we replace instruction in traditional techniques with instruction in these new approaches? What are the benefits? What are the problems? Personal experience with this (some, a lot etc.)?

A To some considerable extent. Problems are traditionalism and availability of materials. Benefits are better prepared students for any mathematics they might have to meet in life and make things more interesting and relevant to students. Oh, some or quite a lot perhaps.

Q Computers make it possible for students to explore dynamic mathematical models and formulate and test hypothesis.

A Yes.

Q If yes, to what extent should we replace instruction in traditional techniques with instruction in these new approaches? What are the benefits? What are the problems? Personal experience with this (some, a lot etc.)?

A ...

Q Scenario: A new topic has been added to the curriculum; one you know very little about. Packaged lessons have been prepared by the ministry or the BCTF or some other agency. All you have to do is to follow the instructions provided and assign suggested problems etc. Would you do this, or would you prefer to learn about the topic and formulate your own lesson plans?

A I would follow their suggestions the first time and then given the time available, I would probably improve on it.

Q I recently attended a symposium given by Richard Skemp at SFU. The thrust of his presentation was something to the effect that there are several different kinds of learning - skill learning such as carrying out of algorithms, closely related to which is habit learning and then there is learning for understanding which is necessary for the person to synthesize what he has learned to new situations - to adapt knowledge to novel situations. Learning for understanding is the building, modification and extending of schema or knowledge structures. It also involves the individual verifying and testing of hypotheses. He went on to say that if we taught this way we would find our teaching becoming much more effective. Do you agree with this?

A Yes.

Q Could you paraphrase what he said?
(He did. Quite a discussion followed. It would seem that participants was not familiar with my perception of what these terms meant.)

Q Do you do this?

A This was covered previously.

Q What are the problems with this kind of approach?

A This was covered previously.

Participant #06
Wednesday, August 16, 1989

Preliminary and exploratory questions:

(none)

Prompted questions:

Q What is the relative importance of teaching for understanding vs skills and proficiency in algorithms. Time allocation?

A 70-30 for understanding.

Q To what extent do you think that pupil teacher ratio is an important factor in math education?

A Very important.

Q What must we do to implement the Sullivan Commission Recommendations?

A We must, in part, make use of a much larger number of aids\paraprofessionals.

Q What do you think the goals of public secondary math instruction should be?

A

Proposed questions:

Q Do you have to have gone beyond a level of study in mathematics, and then look back from a new perspective, in order to really understand?

A Yes. It certainly seems to be the case with myself. This has implications for teaching and learning which we may not have addressed effectively.

Computer experience inventory

1. I use a computer for my personal and professional needs
 - Have done so for some time ()
 - Have recently started ()
 - Not yet to a significant degree ()

2. I can program in
 - Pascal no () cursory () some facility () quite versatile ()
 - BASIC: no () cursory () some facility () quite versatile ()
 - Logo: no () cursory () some facility () quite versatile ()
 - Other: no () cursory () some facility () quite versatile ()

3. I have am familiar with these application programs
 - Spreadsheets: no () cursory () some facility () quite versatile ()
 - Graph utility: no () cursory () some facility () quite versatile ()
 - Word Processing: no () cursory () some facility () quite versatile ()
 - Graphics: no () cursory () some facility () quite versatile ()
 - Others: no () cursory () some facility () quite versatile ()

Personal Mathematics Learning Inventory

Answers are paraphrased summaries.

- Q What (new) math have you learned since leaving university?
 A I have learned a lot more about the math I was supposed to have learned before! I have learned the things that I teach to a much greater degree of understanding than I ever had before.
- Q Which conferences courses, and workshops etc. have you attended in last 2 years?
 A Quite a few. (Respondent participates actively in professional matters.)
- Q Did you learn anything at these - pedagogy and math content?
 A Pedagogy: Some significant things. Reasonably satisfied. Math: I don't really know if I was 'supposed to know' some things I learned before. But an example would be graphing techniques.
- Q Can you mention any books that you have read recently, films you have seen recently having to do with mathematics? Do you stay reasonably current by reading?

- A With pedagogy, yes. I read *The Mathematics Teacher*. With respect to content, I don't know (for reasons discussed earlier).

Use of Computers in math instruction

1. Audio visual aid

Should: Yes () No ()

Reservations:

I have: Yes () No ()

If yes, give example: Graphing.

If yes, how many hours in last year: 4

Associated problems: None. I had assistance from colleague.

2. Deliver Programmed Instruction

Should: Yes () No ()

Reservations: On a relatively limited basis because interaction among people is vital. Cost effectiveness, if it exists, does not justify a large amount of this sort of thing. In remediation or making up for absences it might be OK.

I have: Yes () No ()

If yes, give example.

If yes, how many hours in last year:

Associated problems:

3. Assessment

Should: Yes () No () I really don't know.

Reservations: I am concerned with human interactions again.

I have: Yes () No ()

If yes, give example

If yes, how many hours in last year:

Associated problems:

4. Administrative matters

Should: Yes () No ()

Reservations: It's impersonal.

I have: Yes () No ()

If yes, give example: Computerized report cards.

If yes, how many hours in last year:

Associated problems:

5. Provide exploratory models

Should: Yes () No ()

Reservations: Not enough material available.

I have: Yes () No ()

If yes, give example: All our students type in a simple program in BASIC and use it to solve equations on a systematic trial basis. The kids type the program in, but the emphasis is not on programming, per se.

If yes, how many hours in last year: 10

Associated problems: None. Perhaps there might be equipment problems. I didn't have any problems partly due to assistance from cooperative colleague.

6. Programming. Not taught as such. (See Q5). Programming is a good math activity. Presumably you could use it to teach logic better than with geometry.

Should: Yes () No ()

Reservations:

I have: Yes () No ()

If yes, give example:

If yes, how many hours in last year:

Associated problems:

7. Other

Should: Yes () No ()

Reservations:

I have: Yes () No ()

If yes, give example:

If yes, how many hours in last year:

Associated problems:

Teaching practice

Q To what extent is it desirable to make use of manipulative materials, models, films, videos etc. in teaching mathematics? What are the benefits? Problems?

A Highly desirable. I am not aware of all the of the manipulative materials available. A big problem is time.

Q Films and alternate presentation methods and manipulative materials I have used in class during the last year; frequency?

A Some things - built models outside of class - but minimal use. I used a video in my calculus class that was taped from the Knowledge Network.

Q What mathematics teaching aids such as films, manipulative materials, models etc. are available- in your school, from district and other sources?

A We don't have any in the school. I don't know what is available on the district level.

Further Discussion:

Q Computers, because of their 'number crunching power' make it possible to solve mathematical problems using non traditional techniques. Do you agree?

A Yes.

Q If yes, to what extent should we replace instruction in traditional techniques with instruction in these new approaches? What are the benefits? What are the problems? Personal experience with this (some, a lot etc.)?

A You want to look at problem solving from many different points of view.

Q What about symbol manipulating applications?

I don't know the answer to that one. It's a curriculum decision that is still to be made. How much and what algebra do the kids need? It would seem that there is not much value in spending as much time as we do with many algebraic operations.

Q Computers make it possible for students to explore dynamic mathematical models and formulate and test hypothesis. Have you tried this?

A We have used the graphing packages and the programming activities I mentioned, but not for example, spreadsheet models.

Q If yes, to what extent should we replace instruction in traditional techniques with instruction in these new approaches? What are the benefits? What are the problems? Personal experience with this (some, a lot etc.)?

A To the extent that we able, consistent with time constraints, logistics, etc. but generally, more than we have so far.

- Q Scenario: A new topic has been added to the curriculum; one you know very little about. Packaged lessons have been prepared by the ministry or the BCTF or some other agency. All you have to do is to follow the instructions provided and assign suggested problems etc. Would you do this, or would you prefer to learn about the topic and formulate your own lesson?
- A Yes to both. But given the time constraints, I am more likely to do the former. Having used it, I would probably modify it after using it.
- Q I recently attended a symposium given by Richard Skemp at SFU. The thrust of his presentation was something to the effect that there are several different kinds of learning - skill learning such as carrying out of algorithms, closely related to which is habit learning and then there is learning for understanding which is necessary for the person to synthesize what he has learned to new situations - to adapt knowledge to novel situations. Learning for understanding is the building, modification and extending of schema or knowledge structures. It also involves the individual verifying and testing of hypotheses. He went on to say that if we taught this way we would find our teaching becoming much more effective. Paraphrase what I said.
- A (He explained what I said). He used the word 'work' instead of 'learning'. He was familiar with 'schema'.
- Q Do you agree with this?
- A My experience suggests that I disagree with Skemp. There is so much (habit and skill) learning in there (referring to the curriculum) that in order to cover it, we can't devote a lot of time to the other.

**APPENDIX B
PARAPHRASED SUMMARY STATEMENTS**

Participant 0.1

- S-1: I have difficulty in talking about math in the abstract sense.**
- S-2: Students do not see the connectedness between different topics in math.**
- S-3: Curriculum assumes use of manipulatives and introduction of technology.**
- S-4: Changes will be very slow in coming to the classroom.**
- S-5: Changes in methodology are necessary to change negative view of math.**
- S-6: Math is highly sequential and skill based.**
- S-7: Difficulties with math are related to its sequential character.**
- S-8: Children can often handle more difficult concepts earlier than we give them credit for.**
- S-9: Math should be oriented towards applications - the useful, not just mental gymnastics.**
- S-10: There is little or no transfer of deductive thinking.**
- S-11: Universities use math as a screening device.**
- S-12: The use of math a screening device is legitimate.**
- S-13: Maybe the use of math as a screening device has been over-applied.**
- S-14: Curriculum designers have recognized the value of a broader exposure to math - less emphasis on algebra and recognition of value of data analysis.**

- S-15: There is insufficient time to deal with topics to the desirable depth.
- S-16: There are too many things that are to be covered.
- S-17: There has to be more time made available.
- S-18: Math deserves relatively more time than some other subjects.
- S-19: Much math is artificial.
- S-20: Math should be more closely connected with what is needed in the real world.
- S-21: Calculators have lessened the need for drilling algorithms.
- S-22: Symbol manipulators will lessen the need for certain algorithms.
- S-23: There may be a need for more streaming.
- S-24: The present curriculum may be unsuitable for 80% of students.
- S-25: One should not 'teach to the exam'.
- S-26: Algorithms don't have a great deal to do with understanding.

Participant 0.2

- S-1 I view math as essentially an abstract study.
- S-2 Many math applications developed from theoretical considerations.
- S-3 Sometimes theory develops from applications.
- S-4 I identify with the intellectual-abstract view of math.
- S-5 I think of math as fun.
- S-6 Most students do not view math as fun.

- S-7 I love the subject partly because of its 'purity'.
- S-8 In teaching the abstract, one starts with concrete examples.
- S-9 Students perceive of mathematics as a hurdle.
- S-10 Many students have had negative math experiences.
- S-11 It is difficult to 'enthuse' students about math.
- S-12 A reason for negative attitudes is teaching a topic too early.
- S-13 Math is precise. Answers are right or wrong. This leads to a higher frustration level.
- S-14 Some students are more 'math talented' than others.
- S-15 Math is important because it has wide application in many disciplines.
- S-16 Math acts as a screening device.
- S-17 Math requires a highly developed intellect.
- S-18 Math involves a lot of abstraction.
- S-19 Math is a difficult subject.
- S-20 Math requires good work habits.
- S-21 I'm not sure math should be used as a screening device.
- S-22 The logic learned in math is useful in other fields.
- S-23 There is an assumption on the part of universities that problems solving skills-logic learned in math is transferable.
- S-24 Student readiness is very important in curriculum design.
- S-25 The present curriculum introduces some things too early for 'the average student'.

- S-26 There are many topics which simply aren't important; for example factoring.
- S-27 Many students do not see the relevance of much of the math we teach.
- S-28 Some students are hard to convince of the necessity of some math - as
- S-29 Geometry is a convenient vehicle for teaching logic.
- S-30 Geometry has applications in art and design.
- S-31 Girls can handle abstract concepts earlier than boys.
- S-32 Intellectual readiness is important.
- S-33 There is should be greater emphasis on estimation.
- S-34 Computers can be used to drill basic facts and improve estimating skills.
- S-35 The novelty factor of technology has an effect.
- S-37 Computer programming is a good vehicle for teaching logic.
- S-38 There is a connection between logic, rigour, discipline, attention to detail and carrying something through.
- S-39 There should be a short unit on computer programming in the curriculum.
- S-40 A lot of students cannot handle programming.
- S-41 Having computers in class requires a some considerable degree of teacher direction.

Participant 0.3

- S-1 Parents have always viewed math as an important subject.
- S-2 Math is the most important subject.
- S-3 Historically, being able to handle patterns has been a mark of intelligence.
- S-4 Evaluation in math is highly objective
- S-5 Parents are concerned with fairness of marks.
- S-6 In other subject areas it is possible to 'fudge' assignments - but not in math.
- S-7 Math is highly structured and linear.
- S-8 One of the valuable aspects of math is that it can make you feel that you have accomplished something when you 'get it'.
- S-9 You never know when you will need math being learned in school later.
- S-10 Not all students will need their math later.
- S-11 Students feel good when they gain insight.
- S-12 I am not a mathematician.
- S-13 There is transfer of heuristics from math.
- S-14 There is transfer of skills learned in computer programming to other areas.
- S-15 Writing computer programs helps logical thinking.
- S-16 There is a higher degree of transfer from logical-analytical subject areas than from the humanities.

- S-17 Math is used as a filter because it is so objectively evaluated which makes it a convenient measure.
- S-18 Teaching non academic students really takes it out of you.
- S-19 Maybe non-academic students shouldn't take any math.
- S-20 Expectations of non academic students is too high.
- S-21 Non academic students do better on visual material like trig and geometry.
- S-22 It is easy for poor students to experience failure and hence lowered self esteem.
- S-23 The curriculum for the academic students is generally appropriate and they work hard.
- S-24 I am sometimes motivated by what makes my life easier.
- S-25 Some of my colleagues do not do a good job. External examinations have their place for this reason.
- S-26 I feel isolated from other math teachers.
- S-27 I am concerned with introduction of new topics and new curricula. Sometimes they are just fads.
- S-28 I liked geometry when I was at school - and did well in it.'
- S-29 My school mates dreaded geometry.
- S-30 I did better in geometry than in algebra.
- S-31 I don't like statistics. I did not like the course in stat I took at university.
- S-32 There is general transfer from geometry to other areas, but it is painful.

- S-33 Some students don't like to think.
- S-34 Not everyone can do math well.
- S-35 Math is worth studying for its own sake, and because it makes you think.
- S-36 I don't associate beauty with math, but I do associate patterns and structure.
- S-37 Lack of student motivation is a big problem.
- S-38 There is a wider range of motivation in math than in other subjects.
- S-39 My math department head has made unnecessary and not well thought out changes.

Participant 0.4

- S-1 Math is not more important than other subjects. Communication - English is perhaps more important.
- S-2 Math helps you with everyday affairs.
- S-3 Math teaches you discipline.
- S-4 There is an implication in the curriculum that people pick up consumer math on their own.
- S-5 Too many math teachers would say that consumer math is not essential.
- S-6 Students have to be numerate.
- S-7 There are things we could do for nonacademic students that would be more beneficial than the present math.
- S-8 It would be presumptuous of me to 'close doors'.

- S-9 Society expects students to go as far as possible.
- S-10 I hate inconsistencies.
- S-11 Math attracts certain kinds of people.
- S-12 Exposure to math makes people more logical.
- S-13 Being logical is socially desirable.
- S-14 There is transfer of thinking styles from math to other areas.
- S-15 it is considered socially undesirable to be too emotional.
- S-16 You can tell if a person has had math training by the assumptions they make.
- S-17 I consider myself a teacher, rather than a mathematician.
- S-18 Mathematicians are smarter than I am.
- S-19 I have a love and appreciation for math.
- S-20 People perceive math as being difficult.
- S-21 It bothers me that people perceive of math as being difficult.
- S-22 Using math as a screen for courses that don't need math is an injustice to math.
- S-23 I don't think there is much wrong with the curriculum the way it is.
- S-24 It is important that everyone who teaches math is first class.
- S-25 Teachers should be dedicated.
- S-26 Teachers should be knowledgeable.
- S-27 I think there is room for improvement in teachers.

- S-28 The state of teaching and math education in BC is generally better than in the US
- S-29 I recognize that technology is important but I really don't know a great deal about it.
- S-30 There is beauty in math - but you find it also in all sorts of other things.
- S-31 An obstacle to the learning of math is the way it is presented.
- S-32 Math requires and teaches discipline.
- S-33 Good teaching encourages discovery.
- S-34 Students sometimes learn less than you think they do.
- S-35 It is natural for students to forget what they learn.
- S-36 Some students are lazy and undisciplined and don't put sufficient effort into remembering.
- S-37 At the end of the math course is the provincial exam.
- S-38 In many countries they have a much better math course than we have.
- S-39 We don't have a sufficiently strong work ethic. It has gotten worse.
- S-40 I have no discipline problems in my classes.
- S-41 I works very hard. I try to be innovative.
- S-42 Enthusiasm is very important in a teacher. We have to compete with TV
- S-43 I am a successful teacher, and when I cease to be, I'll quit.
- S-44 It bothers me that some teachers are illogical.
- S-45 There seems to be a lot of politics at university.

Participant 0.5

- S-1 I am interested in knowing how things work and how to fix things
- S-2 I think of mathematics both as something beautiful in its own rights as well as something practical.
- S-3 It is important to teach for understanding.
- S-4 Teaching for understanding is practical to a degree.
- S-5 I consider computer programming a valid mathematical activity.
- S-6 I teach for understanding as much as time permits.
- S-7 Some students do not want to understand.
- S-8 Students have learned that understanding is not really necessary, for example on exams.
- S-9 There is too much in the curriculum.
- S-10 The logic that we teach is done in a haphazard way.
- S-11 Much of algebra can be left out, for example factoring trinomials with first coefficient other than 1.
- S-12 Students should be taught to do simple things in their heads.
- S-13 Mathematics history should be in the curriculum.
- S-14 It would be good to have mathematics projects.
- S-15 Understand how to learn should at all times be paramount.
- S-16 Class control is a problem in teaching for understanding. Teachers must survive.

- S-17 We could have large classes in the old days because teaching for understanding was not expected.
- S-18 Computer programming would be a more practical modern alternative to teaching logic than Euclidean geometry.
- S-19 I found teaching deductive geometry useful for gaining insights into how students think.
- S-20 Some students who did not do well in algebra were pretty good when it came to deductive reasoning.
- S-21 There should be greater integration between mathematics and other subjects.
- S-22 We should teach students to be appreciators of mathematics rather than calculators.
- S-23 It is not very important for students to know number theory, but is useful for stretching their minds and challenging their assumptions.
- S-24 Processes and procedures vs understanding should probably be allocated 60-40 in terms of teaching time.
- S-25 I would take the rigour out of mathematics and teach students to be appreciators instead.
- S-26 I agree with the Sullivan Commission in that teachers should teach more than one subject.

Participant 0.6

- S-1 Pupil teacher ratio is crucial for effective teaching.
- S-2 Teaching for understanding requires much time spent with individual students.
- S-3 A great deal of guidance is required for discovery learning.

- S-4 A high degree of structure is required for discovery learning.
- S-5 Teaching logic is important.
- S-6 Euclidian geometry is not the only way to teach logic.
- S-7 Computer programming may be an effective way to teach logic.
- S-8 Euclidian geometry may not be important in teaching logic.
- S-9 Problem solving skills (heuristics) learned in mathematics are transferable to other areas.
- S-10 I am skeptical about recommendations of Sullivan commission.
- S-11 In the past, recommendation for reform have not been followed through with adequate planning and resources.
- S-12 Status is important to teachers.
- S-13 Mathematics is highly linear.
- S-14 Success is dependent on mastery of previous material.
- S-15 I view myself as a mathematician to some limited degree.
- S-16 I am first and foremost a teacher.
- S-17 Mathematical laws and knowledge preexist and are discovered, rather than being created.
- S-18 It is impractical for learners to create their own knowledge in least in the majority of cases.
- S-19 Discovery learning is not essential.
- S-20 Discovery learning isn't practical in our setting.
- S-21 Goal statements are important in deciding what we should do.

- S-22 Goals should be different for different students.
- S-23 There is a set of fundamental math knowledge and skills related to every day needs.
- S-24 The fundamental set should be a priority of the public school system.
- S-25 Much of algebra is probably excluded from the fundamental set.
- S-26 It is important to instill in students a greater appreciation for mathematics.
- S-27 It is important to instill in students a familiarity with the history of mathematics.
- S-28 It is important to instill in students a familiarity with the cultural implications of mathematics.
- S-29 Mathematics is a creative subject.
- S-30 Cooperative learning is one method of addressing the needs of individuals.
- S-31 Opportunity for success and immediate feedback leads to enjoyment.
- S-32 Opportunity for success and immediate feedback implies a high degree of structure.
- S-33 We have to have both 'useful' and 'abstract' mathematics in the curriculum.
- S-34 A lot of thought and research would be needed before I can make clear statements on what parts of the curriculum should be deleted and what should be added.
- S-35 I would need to do a lot of consulting with 'experts' before I can make clear statements on what parts of the curriculum should be deleted and what should be added.
- S-36 A lot of algebra could be eliminated.

- S-37 Factoring, long division of polynomials could be deleted.
- S-38 More consumer oriented things should be added.
- S-39 Probability and data analysis lead to an appreciation for math.
- S-40 Graphing techniques lead to an appreciation for math.
- S-41 Math history lead to an appreciation for math.
- S-42 I question the importance of many items in the curriculum.
- S-42. The details of curriculum specifics is not as important as attitudes and abilities fostered in students.
- S-43 Our role is not job training; except in so far as we prepare them for university.
- S-44 There should not be any specific mathematics requirement for university entrance.
- S-45 The public school system should decide what mathematics background our students should have.
- S-46 Post-secondary institutions should not be dictated what mathematics background our students should have.
- S-47 A person's performance in mathematics is not necessarily the only valid demonstration of his ability to learn and think.
- S-48 Programming is a mathematical activity.
- S-49 Our curriculum is far too full.
- S-50 We do not fit enough time to allow students to explore and use manipulative materials.
- S-51 We do not fit enough time to allow students to participate in cooperative learning approaches to problem solving.

- S-52 Much routine algebra which can be done by machines should be left out.
- S-53 The curriculum committee should investigate what is being done around the world.
- S-54 The curriculum committee should canvass opinions of people at all levels about views on essential and nonessential math.
- S-55 I would have liked to be on the curriculum committee.
- S-56 Some progress made by the recent curriculum revision, for example data analysis.
- S-57 I am not happy with the curriculum we have.
- S-58 The curriculum committee originally simply left everything in and added more to it.
- S-59 The curriculum committee did not do its job well.
- S-60 I feel a part of the curriculum designing process.
- S-61 The curriculum committee didn't listen to me.
- S-62 I was involved in writing letters etc. to the curriculum committee.
- S-63 It bothers me that I'm not more deeply involved in the curriculum design process.
- S-64 The curriculum is partly driven by the views of university professors.
- S-65 University professors really have no idea about secondary schools and their problems and goals.
- S-66 Those that go on to study math at the university could learn all the quote, 'essential' math they need for their university courses in about two months when they get there.

APPENDIX C DATA

At which school does participant teach?

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Data	A	A	A	A	A	A	A	B	B	B	B	B	B

1. Community view of mathematics

	1:positive			2:neutral			3:negative						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	3	3	1	3	3	3	3	1	3	1	1	1	1

Comments:

- 1.1 Many students do not have an aptitude for subject.
- 1.2 Irrelevance. The public does not understand needs.
- 1.3 Useful for further education. High proportion of oriental students in this school. These students achieve well in math because language is not a factor.
- 1.4 ...
- 1.5 Sequential and abstract nature of discipline and its presentation.
- 1.6 Perceived as boring and dry.
- 1.7 Perceived as irrelevant.
- 2.1 Useful, hard, required, important. Without it you are shut out.
- 2.2 Negative perception passed down from parents. It is seen as an arbitrary obstacle to further education.
- 2.3 Math is considered to be a very important subject.
- 2.4 Pragmatically useful. Manipulate numbers to live - earn more money.
- 2.5 The public is misguided - they think it teaches people to think.
- 2.6 Viewed as one of the most important. Don't know exactly why, but it is evident that they (the public, parents) do. I question relevance.

2a) Math marks as indicators of general learning ability

	1:good indicator			2:not particularly				3:poor					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	1	1	1	2	2	2	2	1	2	2	2

2b) Math marks as indicators of logical thinking

	1:good indicator			2:not particularly				3:poor					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	1	1	2	1	1	2	2	1	1	2	3	1

Comments:

- 1.1 ...
- 1.2 Not totally fair to some students. Not all bright students do well in math.
- 1.3 ...
- 1.4 I picked my baseball team on the basis of their elementary school mathematics performance.
- 1.5 The screening function dictates the curriculum. We make things difficult for this purpose. It is a poor scene when it used.
- 1.6 ...
- 1.7 No better than anything else.
- 2.1 It's a lousy way of screening people - atrocious.
- 2.2 It is being used for this probably unfairly.
- 2.3 ...
- 2.4 Could be, but not the way it's taught.
- 2.5 b) High school math; absolutely no.
- 2.6 ...

3 Transfer of logical thinking style from Euclidian geometry to other disciplines.

	1:yes			2:maybe some				3:little or none					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	2	2	3	3	0	3	3	3	3	2	3	2
Resp. 2*	2	2	2	3	3	0	3	3	3	3	2	3	2

* Response for 1.1 changed from 1 (yes) to 2 on basis of comment.

Comments:

- 1.1 Belief - no proof.
- 1.2 ...
- 1.3 We try to think the other way. Questionable.
- 1.4 ...
- 1.5 It would be nice if this were so - but there is no evidence that it is so.
- 1.6 Do not know.
- 1.7 ...
- 2.1 ...
- 2.2 I did my MA thesis was on this topic.

- 2.3 ...
 2.4 If you want to teach them to think logically, then (you should) teach them to think logically.
 2.5 ...
 2.6 ...

4 Computer programming as an alternative for teaching logical thinking

1: preferable 2: possibly 3: do not agree

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	1	0	0	1	2	3	2	1	1	2	3	3
Resp*	2	1	0	0	1	2	3	2	1	2*	2	3	2*

* Responses for 2.3 and 2.6 changed on the basis of comments. 1.5 was not changed despite that the comment might seem to justify it. This was because 1.5 was trying to be consistent with his previous strong stance on lack of transfer. Also, as later data will show, 1.5 comes out as favouring programming.

Comments:

- 1.1 ...
 1.2 ...
 1.3 The two are not parallel. The thinking involved is different. They may both achieve something - but these are different things.
 1.4 (the participant did not understand the question)
 1.5 If there is transfer. The kind of logic used in math is not necessarily the same as kind of logic that is used in other disciplines. I tend to believe that what you teach in math lives and dies in math.
 1.6 ...
 1.7 I have had students who are poor in math who are good in programming.
 2.1 ...
 2.2 There are many alternatives, including French. They share the same developmental approach. Programming could belong in the modern languages department - same kind of activities.
 2.3 (Participants questions value of programming - see question #5)
 2.4 ...
 2.5 Could not agree less. A (better) alternative - teach them to play poker. There are so many alternate ways to teach logical thinking.
 2.6 Seems to have more relevance. Perhaps equivalent.

5. Computer programming is a worthwhile mathematical activity

	1: agree			2: possibly				3: do not agree					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	3	1	0	1	1	3	1	2	3	2	3	1

Comments:

- 1.1 ...
- 1.2 One can get off track; preoccupied with programming. Not enthusiastic. More thought required.
- 1.3 ...
- 1.4 (Participant did not understand question.)
- 1.5
- 1.6 ...
- 1.7 ...
- 2.1 ...
- 2.2 ...
- 2.3 ...
- 2.4 Depends on program level: machine code, higher level language, or what.
- 2.5 It is not a math activity.
- 2.6 ...

6 It would be worthwhile for teachers to engage in dialogue

	1:agree			2:possibly				3:I doubt it					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	2	3	0	1	1	3	1	3	3	1	2	1

Comments:

- 1.1 ...
- 1.2 ...
- 1.3 We did this at the time of Sputnik and New Math. It didn't work.
- 1.4 We do this now. (Consider changing response)
- 1.5 ...
- 1.6 Math is like other subjects in that it is dynamic. Emphasis should be on problem solving.
- 1.7 I am too much of a realist. It can't happen.
- 2.1 ...
- 2.2 Not enthusiastic. Any changes would only be cosmetic. Curriculum should be remodeled - not restructured.

- 2.3 The topic of algebra is considered to be so absolutely necessary for post-secondary that (no possibility of change) exists in the foreseeable future, I hope.
- 2.4 A review of all subjects should be worthwhile - technology bears directly on math - there may be some areas that should be included.
- 2.5 Agree if you get the right group, but that may never be possible. Math teachers as a group never agree.
- 2.6 I don't know if we'd come up with something radically different. Curriculum hasn't changed appreciably.

7a) Stress appreciation

	1:agree			2:possibly			3:I doubt it						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	2	0	2	3	1	3	1	3	3	1	3	3

7b) Select students could catch up

	1:agree			2:possibly			3:I doubt it						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	2	0	2	3	1	3	1	3	3	1	3	3
Resp.*	1	2	3*	2	3	1	3	1	3	3	1	3	3

* 1.3 changed on the basis of comment.

Comments

- 1.1 ...
- 1.2 ...
- 1.3 a. It would mean that math education would not have a specific goal - hence this would not be well received.
b. Students would be deficient in skills.
- 1.4 ...
- 1.5 Too many people would have problems if topics are expanded - ie: those who aren't going on. We have a curriculum aimed at everybody but good for nobody. There are 4 groups of clients. They wouldn't be different subjects. Meaningfulness has to be tied together more.
- 1.6 I do it. I've talked about Gauss. I deal with enriched students.
- 1.7 Not particularly desirable.
- 2.1 ...
- 2.2 They would end up with a scattering of disjointed things.
- 2.3 a. (I would have a) different opinion if it was an alternate course.
b. Agree if (we are talking about) only algebra - but not the full spectrum of other math strands.

- 2.4 a. Relates to #6. (On clarification) wide curriculum - more topics, less depth - you're more likely to hit the kid's button.
b. It's done all the time by mature students at the university.
- 2.5 a. Waste of time. Students do not think the way adults do. Marvelous concept. At least 15 years too early in a person's life.
b. The thick hided university people want no part of it. They are very stubborn.
- 2.6 ...

8 Sufficiency of time

	1:insufficient			2:almost sufficient				3:sufficient					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	3	1	3	1	1	0	1	1	1	1	1	3	1

Comments

- 1.1 Plenty for some. Not enough for others.
- 1.2 We are rushing all the time.
- 1.3 ...
- 1.4 ...
- 1.5 ...
- 1.6 Yes for some. No for others.
- 1.7 Except at grade 12 where it is too heavy.
- 2.1 ...
- 2.2 ...
- 2.3 ...
- 2.4 ...
- 2.5 We have not been able to get the Ministry of Education to decide what the desired level is. There is always sufficient time for students who want to work. The majority of students do not want to work beyond the confines of the classroom.
- 2.6 Curriculum has too much in it. Some things could be left out - they are not important enough. Thinking skills could be retained.

9a) Teaching for understanding is generally impractical

	1:impractical			2:maybe			3:practical						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	3	1	1	3	1	1	0	2	3	3	3	1	2

9b) Curriculum priority of Teaching for understanding

	1:high			2:greater than now				3:not a priority					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	0	0	2	2	3	3	3	2	2	1	2.5	0	3

Comments:

- 1.1 Generally, teachers do teach for understanding. We do not test for understanding to the same degree that we teach for it.
- 1.2 At what cost in terms of time?
- 1.3 You try - but tests do not emphasis it. Same as Now - Not a high priority.
- 1.4 ...
- 1.5 Should be a priority, but there is no point in it. For what segment of the population? There are four groups of clients.
- 1.6 At higher levels of math. I'm not sure that those who perform really understand what they are doing. Even people who are trained in mathematics sometimes just 'turn the crank'.
- 1.7 Open to discussion. Practicality - yes, but depends on student. Can't be a priority. Priority for some kids is to survive.
- 2.1 Understanding is impractical if you have to cover the course - a priority decision.
- 2.2 ...
- 2.3 ...
- 2.4 (We should) re-examine what we are trying to do.
- 2.5 Impractical for the most part. (In a) small group it is practical. (In a) streamed (group it is) highly (practical). In unstreamed groups it is highly impractical - hence undesirable.
- 2.6 No matter what you do, some students won't understand - they have to do it by algorithm. It's black and white

10a) Discover Learning is desirable

	1: Yes			2: Possibly				3: No					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	2	0	0	2	2	0	2	1	2	2	0	1

10b) Discovery Learning is practical

	1: Yes			2: Possibly				3: No					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	3	3	2	2	1	3	2	2	1.5	1	0	2.5

Comments:

- 1.1 ...

- 1.2 There are many students for whom this is unsuitable. Determined by student ability.
- 1.3 Practical for a small group.
- 1.4 Desirable if you had the time.
- 1.5 (Participant considers himself a constructivist) Historically it has always been true that mathematics encounters a greater range of ability than any other subject.
- 1.6 ...
- 1.7 ...
- 2.1 There is not enough time to use discovery learning.
- 2.2 Discovery in learning is desirable, but it cannot be broadly applied.
- 2.3 ...
- 2.4 It works but takes longer - depends on stated goals.
- 2.5 (See comments for #9 - possible change entry)
- 2.6 It has its place - as much as possible - but we have to be efficient. It's time consuming.

11a) Heuristics are transferable within math

	1:yes			2:maybe				3:I doubt it					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	1	1	2	3	1	1	1	1	1	1	1	1

11b) Heuristics are transferable to other disciplines

	1:yes			2:maybe				3:I doubt it					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	1	2	3	1	1	1	1	2.5	1	2	1

Comments:

- 1.1 Transfer is largely dependent on transferee.
- 1.2 I am not familiar with research findings, but I suspect that there would be some transfer, particularly within math - if it is done right.
- 1.3 Yes, but less than within math. Systematic approach
- 1.4 I would hope so (I'm not sure participant understood question)
- 1.5 I doubt it very much. It is because there is no transfer that there are other disciplines - by definition.
- 1.6 ...
- 1.7 ...
- 2.1 ...
- 2.2 Ways of looking at problems are general and transferable.
- 2.3 ...

2.4 (Participant asked what 'heuristics were' - answer was 'Rules of thumb') Rules of thumb are generally quite transferable - they are short cut ways to get to understanding.

2.5 ...

2.6 ...

12a) & b) Calculator vs algorithms - arithmetic

	1: More			2: The same			3: Less						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.a	1	1	2	2	1	2	2	1	2	1	2	1	1
Resp.b	3	2	2	2	3	1	2	2	2	3	2	1	3

12c) & d) Computer vs algorithms - algebra

	1: More			2: The same			3: Less						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.c	1	1	0	0	1	2	2	1	1	1	2	3	3
Resp.d	3	2	0	0	3	2	2	2	2	3	2	2	2

12e) & f) Applications vs Euclidian - geometry

	1: More			2: The same			3: Less						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.e	1	1	2	1	1	2	1	1	1	2	1	1	2
Resp.f	2	2	2	0	3	2	3	2	3	2	2	0	2
Res.f*	2	2	2	2*	3	2	3	2	3	2	2	0	2

* Changed on basis of comments.

12g) Probability and Statistics

	1: More			2: The same			3: Less						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.g	1	1	2	2	0	2	2	2	1	2	2	0	1
Resp.g	1	1	2	2	1*	2	2	2	1	2	2	3*	1

* 1.5 and 2.5 changed on basis of comments.

12h) Number Theory

	1: More			2: The same			3: Less						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.h	2	2	2	2	3	2	2	2	3	2	2	0	2
Resp.h	2	2	2	2	3	2	2	2	3	2	2	3*	2

* 2.5 changed on basis of comment.

12i) Math history/social aspects

	1: More			2: The same			3: Less						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	2	1	1	1	1	2	1	1	2	1	1	1

12j) Personal finance

	1: More			2: The same			3: Less						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	1	2	1	1	1	2	1	2	1	1	1	2

Comments:

- 1.1 ...
- 1.2 I have a bias towards the topic of personal finance. Academic students get left out of this.
- 1.3 Those who use calculators a lot seem to do not as well. Use of calculators for simple questions waste time - the calculator becomes the master. The use of calculators should depend on level. Capable students can self-teach personal finance.
- 1.4 Not familiar with calculator/computer based algebra.
- 1.5 Re probability and statistics: practical, yes,; formal, no.
- 1.6 ...
- 1.7 There should be less work on logarithms.
- 2.1 You can't keep adding to the curriculum!
- 2.2 There is not a big place for personal finance in mathematics.
- 2.3 ...
- 2.4 a.&c. I can't see any pressing need for change. (referring to calculators/computers)
- 2.5 c. Algebra and computers are mutually exclusive from what I know about computer programs. The software I have seen is absolutely terrible.
f. Present arrangement is absolutely doomed to failure. Euclidian geometry should be concentrated in an elective course at grade 11 or 12 level.
g. Same as Euclidian geometry. Too early in their life.
h. Should come much later; after probability and stats. even.
j. To a level at which it becomes more meaningful. We put personal finance in slow learners...
- 2.6 i. At present there is nothing.

13a) Influence is unduly strong

	1:yes			2:not really			3:I doubt it						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	1	1.5	1	2	2	3	1	1.5	1	1	3	2

13b) Influence is desirable

	1:yes			2:not really			3:I doubt it						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	3	1	2	0	2	3	3	0	3	3	0	2

Comments:

1.1 b. No

1.2 ...

1.3 a. Strong doubt.

b. No. Those who want to succeed have to meet their requirements, and they do not bend. It is a fact.

1.4 They have a broader background than us.

1.5 There are 4 different 'markets'. University professors only deal with one group.

1.6 Our primary mandate is to prepare students to take a meaningful part in society.

1.7 b. Wouldn't be any, anyway. There is a committee in progress between senior science teachers and UBC to facilitate dialogue and identify weaknesses. This is desirable.

2.1 It is not our primary mandate to provide a screen for students for jumping through hurdles.

2.2 Maybe it is the wrong people (at university) who are exercising unduly strong influence.

2.3 The changes that are being initiated in the public school system are not being supported at the post-secondary level.

2.4 Math teachers should determine student needs.

2.5 The average college professor knows little of the high school curriculum and cares less. Quite removed - arrogant.

2.6 Unfortunately there is that goal up there... I don't really think that passing algebra 11 is a must for art history. The math hoop should be there for certain students. Other areas should count more.

14a) Clique

	1:yes			2:somewhat			3:no						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	1	1.5	1	2	2	3	1	1.5	1	1	3	2

14b) Conservative

	1:yes			2:somewhat			3:no						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	3	1	2	0	2	3	3	0	3	3	0	2

14c) Competent

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	3	1	3	2	0	2	3	0	0	2	3	1	1

Comments:

1.1 ...

1.2 ...

1.3 ...

1.4 We do not know if input is actually utilized.

1.5 b. There is the equality issue. They reflect what is politically acceptable public view. They do not buy 4 levels.

1.6 ...

1.7 ...

2.1 ...

2.2 ...

2.3 ...

2.4 Reaction time - lead time - for change in education is to slow.

2.5 They do not want status quo - always changing things... books etc. That type of activity attracts a certain kind of individual - class room escape artists. They love meetings - they don't shut people out intentionally. It's the nature of the activity that (eliminates much influence).

2.6 The type of people they have proven themselves in the field.

16 Relative emphasis - applications - theoretical - skill

_____ % vs _____ % vs _____ %

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp. a40	60	40	20	100	25	20	50	40	30	33	40	10	
Resp. b40	20	20	60	0	25	10	15	40	30	33	10	30	
Resp. c20	20	40	20	0	50	70	35	20	40	33	50	60	

* Participants assigned to categories as explained in Chapter 4.

Comments:

1.1 ...

- 1.2 Applications require basic skills.
 1.3 ...
 1.4 In some topics, few applications are available.
 1.5 ...
 1.6 ...
 1.7 ...
 2.1 You have to have a balance, but I don't know exactly what it is.
 2.2 ...
 2.3 ...
 2.4 We are in an applications oriented society - but maybe we shouldn't let that influence us.
 2.5 That's not too hard for me (?)
 2.6 Not happy about this - don't know.

17a) Linear structure

	1: definitely				2: only partly				3: not at all				
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	2	2	2	3	1.5	1	2	1	2	1	2

17b) Lack of relevance

	1: definitely				2: only partly				3: not at all				
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	2	2	2	3	1.5	1	2	1	2	1	2

17c) Teacher quality

	1: definitely				2: only partly				3: not at all				
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	2	2	2	3	1.5	1	2	1	2	1	2

17d) Student aptitude

	1: definitely				2: only partly				3: not at all				
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	1	2	0	1	2	3	1	2	2	1	1	1

17e) Student motivation

	1: definitely				2: only partly				3: not at all				
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	1	2	0	1.5	2	3	2	1	1	1	1	1

Comments:

- 1.1 (Greatest factor is) student variability.

- 1.2 ...
 1.3 ...
 1.4 Course content is not appropriate for many students and this affects their motivation. I never felt that math was difficult. Some students get turned off because of repetition.
 1.5 ...
 1.6 ...
 1.7 d. Very little e. Very little There is a cultural/sexual bias
 2.1 Holistic thinkers aren't as good at math as 'left-brain' people.
 2.2 It seems that students increasingly have difficulties in focussing - 'staying with it' - perseverance.
 2.3 ...
 2.4 a. Makes it easier.
 b. From the kids' perspective
 c. There is a tendency to 'use IE teachers' because 'its easy to teach math'.
 2.5 e. More than definite!
 The confusion caused by trying to broaden the field of math. Too many small topics.
 2.6 Attitudes of the general public. Parent: "I was poor at math myself" - a self-fulfilling prophesy.

18a) Curious

1:more than most 2:about the same as most 3:less than most

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	1	1	3	1	1	1	1	1	1	1	1	1

18b) Enthusiastic

1:more than most 2:about the same as most 3:less than most

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	0	1	1	1	1	1	1	1	2	3	1

18c) Frustrated

1:more than most 2:about the same as most 3:less than most

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	2	2	0	2	3	3	2	2	3	1	3

18d) Good teacher

1:more than most 2:about the same as most 3:less than most

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	1	1	2	0	0	1	1	1	2	1	1	1

18e) I'd rather do something else

1:definitely 2:would consider this 3:prefer to teach

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	3	2	2	2	3	3	3	3	2	3	3	1	1

18f) Other teachers are highly skilled

1:highly skilled 2:mostly good 3:many poor

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	2	2	2	2	2	0	2	3	2	3	2

Comments:

- 1.1 Frustrated at times.
- 1.2 Curious, but hesitant because of confidence. It may be time for a change.
- 1.3 Curious and enthusiastic about some things, but not so about others, for example sports.
- 1.4 d) We are always trying to improve.
- 1.5 I am selectively curious. I have seen hardly anyone I would describe as incompetent. (Their performance or effectiveness) is dictated by the system within which they are working.
- c) & d) Don't know.
- 1.6 I am better (than I otherwise would be) because of the energy I put into it. This compensates for lack of experience.
- 1.7 ...
- 2.1 I am critical of training of teachers of math, particularly at the elementary level, but also at the secondary. (To a large extent) those who have difficulty lack training. There is a feedback loop here. Untrained teachers influence poor attitudes.
- 2.2 ...
- 2.3 e) No way - I like what I do
- 2.4 c) I'm enjoying my new job a lot. (Participant recently joined a new district)
- d) Teaching is my third career.
- 2.5 ...

2.6 ...

19 Number cruncher applications

	1:definitely			2:maybe a little				3:no							
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6		
Resp.	1	1	3	1	1	2	1	1	1.5	1	1	3	1		

Comments:

- 1.1 ...
- 1.2 We'll lose them if we don't (referring to use of computers in general)
- 1.3 Takes a lot of time. We used to do 'base 2' in grade 8. It doesn't last.
- 1.4 ...
- 1.5 Absolutely
- 1.6 ...
- 1.7 ...
- 2.1 ...
- 2.2 'Definitely' is perhaps too strong; but 'Maybe a little' would indicate a lack of commitment.
- 2.3 ...
- 2.4 It's part of their world. It will be more real for them than it is for us.
- 2.5 ...
- 2.6 ...

20 Symbol manipulating applications

	1:definitely			2:maybe a little				3:no							
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6		
Resp.	1	2	1	0	1	3	1	1	1	1	1	2	2		

Comments:

- 1.1 We have done this here.
- 1.2 Not if it thwarts student's opportunity to understand concepts.
- 1.3 Understanding required as a prerequisite.
- 1.4 No opinion.
- 1.5 Definitely plus.
- 1.6 I think at high school level students would not have fundamental understanding of, for example, differentiation. Therefore, using computers may inhibit (development of?) understanding.
- 1.7 ...
- 2.1 ...
- 2.2 'Definitely' is perhaps too strong; but 'Maybe a little' would indicate a lack of commitment.
- 2.3 ...

- 2.4 Also data bases etc. that we haven't even touched on. It's a logical process.
- 2.5 It takes away all that I find desirable in math. The beauty, for example. It would make it more difficult to motivate.
- 2.6 They have their place - but the skill has to be learned manually first. Mechanics first.

21a) Personal computer use

0:no 1:recently started. 2:have done so for some time.

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	0	2	0	0	2	2	2	2	2	1	2	0	0

21b) Pascal

0:no 1:cursory 2:some facility 3:versatile

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	0	0	0	0	2	1	0	0	0	0	3	0	0

21c) BASIC

0:no 1:cursory 2:some facility 3:versatile

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	1	0	2	3	0	3	3	0	3	0	1

21d) Logo

0:no 1:cursory 2:some facility 3:versatile

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	1	0	0	2	0	0	0	0	0	3	0	0

21e) Other language

0:no 1:cursory 2:some facility 3:versatile

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	0	0	0	0	0	3	0	0	0	0	3	0	1

21f) Spreadsheets

0:no 1:cursory 2:some facility 3:versatile

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	3	0	1	1	2	2	3	0	2	3	0	1

21g) Graphing utility

	0:no					1:cursory				2:some facility				3:versatile			
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6				
Resp.	2	3	2	1	3	2	2	3	2	2	3	0	1				

21h) Word Processor

	0:no					1:cursory				2:some facility				3:versatile			
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6				
Resp.	0	3	0	0	2	3	3	0	2	2	3	0	1				

21i) Graphics

	0:no					1:cursory				2:some facility				3:versatile			
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6				
Resp.	0	2	0	0	1	3	2	0	3	2	3	0	1				

21j) Other applications

	0:no					1:cursory				2:some facility				3:versatile			
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6				
Resp.	0	0	0	0	2	0	0	3	0	0	3	0	0				

Comments:

- 1.1 I can follow instructions.
 1.2 ...
 1.3 c) Sometime in mid 70's
 1.4 ...
 1.5 ...
 1.6 e) Fortran.
 1.7 ...
 2.1 j) Data base.
 2.2 ...
 2.3 ...
 2.4 d) HyperCard
 e) Installed computers for the CBC in previous career
 2.5 ...
 2.6 ...

22 Which new math topics have you learned?

	1: hardly any					2:one or two				3: yes - several			
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	1	1	1	3	1	1	2	3	1	3	2	1

Comments:

- 1.1 Linear Programming.
 1.2 Computer use. Transformations (?)
 1.3 Some aspects of computers (in 70's)
 1.4 'Computer work'.
 1.5 Use of computer tools. Some aspects of quantum mechanics and probability & statistics.
 1.6 ...
 1.7 ...
 2.1 Various applications eg: computing.
 2.2 Linear Algebra, computers, electronics. Redone in an academic setting: Calculus, Probability and Statistics.
 2.3 ...
 2.4 Computer related stuff which has math components, plus 2 courses in logical thinking which is math related
 2.5 Many of the topics at the grade 12 level I have learned since leaving university by teaching them. Never took trigonometry - teach it through vector geometry(?). I am not a math trained person. I have a Master's degree in Latin.
 2.6 ...

23a) Conferences and workshops

	0:none			1:a couple			2:several						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	1	1	2	0	1	1	2	2	2	0	1

23b) New pedagogy

	0:not really			1:some			2:much						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	1	1	0	1	0	0	1	1	2	1	0	1

23c) New mathematics

	0:not really			1:some			2:much						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	0	0	0	0	0	0	0	1	2	1	0	1

23d) PSA attendance

	0:do not attend			1:occasionally				2:regularly					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	0	0	0	0	1	0	0	0	0	2	1	0	0

23 e) Read journals

	0:hardly ever			1:occasionally			2: regularly						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	0	1	2	2	1	2	2	2	0	0	0

23 f) Number of journals mentioned

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	4	2	1	2	1	1	4	1	1	2	0	0	0

Comments

- 1.1 V MT CUE Magazine, NCTM Research Journal
 1.2 V MT
 1.3 ...
 1.4 Arith. Teacher
 1.5 V ...
 1.6 SA
 1.7 MT Science Teacher, Gifted Ed. journals.
 2.1 MT
 2.2 V MT I try to avoid the politics of prof. organizations.
 2.3 V MT
 2.4 e) Math journals hardly ever.
 e) Computer magazines regularly.
 2.5 ...
 2.6 c) You can solve quadratic equations on a graph using a 'Carlile Circle'.
 Fascinating.

24 (i)a) Audio Visual Aid- should

	1:yes			2:maybe			3:no						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	1	1	1	1	1	2	2	1	1	1	3	2

24 (ii)a) Audio Visual Aid- I have

	1:yes, often			2:once or twice				3:no					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	3	3	3	3	3	3	3	3	2	3	1*	3	3

* 2.4: extensive

Comments:

- 1.1 a I'd love to. No equipment.
- 1.2 a No equipment.
- 1.3 a Time saving, but costly. No equipment.
- 1.5 a Is perhaps too teacher directed I do this in my dreams. There is no equipment.
- 1.7 a Not my style - confining.
- 2.1 a Students should have some interaction.
- 2.2 a Need equipment.
- 2.4 a 5-20 minutes per block all year.
- 2.5 a Haven't seen any decent software
- 2.6 a Availability. Mechanics. Cumbersome I have tried it in the past.

24 (i)b Programmed Instruction - should

	1:yes			2:maybe			3:no						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	2	1	1	3	3	1	1	1.5	3	2	3	3

24 (ii)b Programmed Instruction - I have done this

	1:yes, often			2:once or twice			3:no						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	2	3	3	3	3	3	3	1	3	2	3	3

Comments:

- 1.1 b Subject to availability of suitable software
- 1.2 b Can be tedious - motivation problems. Did arithmetic drills with modified students. Good software seems to be a problem.
- 1.6 b It has failed where tried
- 1.7 b Occasionally, for specific purposes.
- 2.1 b Its just one more instruction mode.
- 2.2 b I want to pick out specific things to be taught, and I have not found software that suits my purposes.
- 2.3 b Takes away motivation
- 2.4 b For specific students. Many programs are trash. I didn't like it. Too difficult to match students to programs.
- 2.5 b Haven't seen any decent software
- 2.6 b I don't find the software good enough. Not efficient enough under present circumstances - the need to book facilities etc.

24 (i)c) Assessment - should

	1:yes			2:maybe			3:no							
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6	
Resp.	1	1	1	2	3	3	2	2	1	3	2	3	2	

24 (ii)c) Assessment - I have done

	1:yes, often				2:once or twice				3:no				
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	3	3	3	3	3	3	3	3	3	3	2	3	3

Comments:

1.1 c. Security

1.2 c. On occasion

1.3 c. ...

1.4 c. Uncertain about this.

1.5 c. ...

1.6 c. Student facility with the terminal may affect the outcome.

1.7 c. ...

2.1 c. ...

2.2 c. ...

2.3 c. Inflexible.

2.4 c. Too difficult to monitor cheating.

2.5 c. Too impersonal

2.6 c. ...

24 (i)d) Exploratory models - should

	1:yes			2:maybe			3:no							
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6	
Resp.	1	2	1	1	1	1	1	1	1	1	1	3	2	

24 (ii)d) Exploratory models - I have done

	1:yes, often				2:once or twice				3:no				
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	2	3	2	1	3	3	3	1	3	1	3	3

Comments:

1.1 d. Dynamath, graphing

1.2 d. ...

1.3 d. I have sent my class to the lab.

1.4 d. ...

1.5 d. Explorations of functions.

1.6 d. ...

- 1.7 d. Great, but no facilities.
 2.1 d ...
 2.2 d. ...
 2.3 d. I expect it would be practical in terms of data analysis.
 2.4 d. Fantastic. I have tried spreadsheets. Found it was valuable and worked well.
 2.5 d. The computer should be in vocational programs and/or clubs.
 2.6 d. Accessibility

24 (i)e) Learn programming - should

	1:yes			2:maybe			3:no						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	2	1	2	1	1	2	3	2	3	2	3	3

24 (ii)e) Learn programming - I have done

	1:yes, often				2:once or twice				3:no				
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	3	2	1	3	3	3	3	3	2	3	3

Comments:

- 1.1 e. Logo and BASIC. Time constraints.
 1.2 e. Programming is not necessarily math connected
 1.3 e. I did some of this in the 70's. Only (suitable) for students of above average ability.
 1.4 e. Somewhere between 'maybe' and 'no'.
 1.5 e. ...
 1.6 e. ...
 1.7 e. As an option - kids don't really need that.
 2.1 e. Everyone should know a little about it. Simplistic (?)
 2.2 e. Not keen. Programming is an activity similar to learning a second language.
 2.3 e. There is already too much in the curriculum.
 2.4 e. ...
 2.5 e. ...
 2.6 e. Quest. #5 notwithstanding, programming should be an elective. It would be valuable for students but not necessarily in math.

The following is the number of hours actual use of computers in the classroom. for each use.

24 (iii)

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.a	0	0	0	0	0	0	0	0	16	0	?	0	0
Resp.b	50	5	0	0	0	0	0	0	3	0	?	0	0
Resp.c	0	0	0	0	0	0	0	0	0	0	?	0	0
Resp.d	50	10	0	7	200	0	0	0	0	0	?	0	0
Resp.e	17	2	0	3	30	0	0	0	0	0	?	0	0
Totals	117	17	0	10	230	0	0	0	18	0	?	0	0
Grp.*	1	2	3	2	1	3	3	3	2	3	1‡	3	3

*Groupings assigned on the basis of inspection of the data .

‡ 2.4 assigned to Group 1 on the basis of comments and interview.

25 a) Manipulatives are desirable

	1:very			2:beneficial			3:marginal						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	1	3	2	1	2	3	1	2	1	1	1.5	3

25 b) Manipulatives are practical

	1:very			2:somewhat			3:marginal						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	3	2	3	1.5	3	2	2	1	1	3	2

25 c) I use manipulatives

	1:frequently			2:occasionally			3:infrequently/never						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	2	2	3	2	2	2	3	2	2	2	2	3	3

25 d) Availability of manipulatives

	1:inadequate			2:sufficient			3:much						
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	1	2	1	1	2	3	1	1.5	1	1	1	1

Comments:

1.1 ...

1.2 ...

1.3 ...

1.4 ...

1.5 Inconvenience of access to materials.

- 1.6 a. Models - yes... others - no.
 1.7 b. Don't want any. My own methods of verbalizing seem to work.
 2.1
 2.2 d. Hard time finding 'good things'. I have looked at what is available.
 2.3 ...
 2.4 ...
 2.5 It's like the computer software. It is very inadequate for what I want to do. c) I don't ever do it.
 2.6 ...

26a) Years taught

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	17	22	28	27	15	2	12	27	25	25	10	35	14

26b) Predominant level

	1: junior high				2: senior high			3: all secondary levels					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	3	3	2	1	3	3	3	1	3	3	1	3	3

26c) Subject area

	1: mostly mathematics subjects							2: a significant amount of other subjects					
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	1	1	1	2	2	2	2	1	2	2	2	1

26d) Teaching preference

	1: mathematics				2: other subjects								
Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	1	1	1	2	2	2	2	1	1	1	2	1

26e) Preparation

Part.	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6
Resp.	1	1	1	1	1	1	1	1	1	1	1	1	1

Comments:

- 1.1 d. Enjoyment, interest.
 1.2 d. Interest.
 1.3 d. It is an easy thing.
 1.4 d. I like it and I do a better job in math than in other subjects.
 1.5 ...

- 1.6 d. Science and math complement each other.
- 1.7 d. Both science and math. Its boring to stick to one subject.
- 2.1 ...
- 2.2 d. I have the most to offer in math because I enjoy it.
- 2.3 c. science d. I think I have put a lot of energy into becoming qualified.
- 2.4 (see final comments, below)
- 2.5 c. Latin and math, 50-50. d. I would be full a time latin teacher if it was available. I wouldn't teach any math. e. More than adequately prepared.
- 2.6 d. because its structure makes it easy to teach. I find that marking in math is overly cumbersome, compared to Phys. Ed or Counselling. I would prefer to do these as well as math.

Final comments

- 1.1 ...
- 1.2 ...
- 1.3 ...
- 1.4 The development of logical thinking may be better served in some way other than Euclidian geometry. For example, puzzles and problems that would be of greater interest to the students. Better student-challenge match. I was afraid of computers up until a few years ago - but now I enjoy it. A colleague has been helpful. I can see that computers will play a big role if we have enough (of them). (Participant expressed dissatisfaction at the logistics of computer use.
- 1.5 I do not see any reason to separate math and science.
- 1.6 ...
- 1.7 ...
- 2.1 The math we teach is far too difficult too early.
- 2.2 ...
- 2.3 I think math is unique. It warrants some attention. 'The powers that be' should recognize this. Emphasize the time constraint - the amount to be 'covered'. A lot of research should go into revising the curriculum in light of technology. #6 notwithstanding, which refers to math teachers. We may discover in the next assessment a deterioration of student algebra skills as a result of the (newly) enlarged curriculum (This will lead to) a cry from the universities that students cannot do complex fractions, rational expressions and factoring by grouping.
- 2.4 Coming from industry, (I observe that) We (teachers) do not take ourselves seriously as professionals. These kinds of needs assessments (

referring this survey) should be done in all curricular areas on a regular basis. The speed of change is logarithmic. - and we haven't really looked at the curriculum for ten years. Math is too often taught in a way that precludes other possibilities. Logical thinking can narrow the range (?) There is a difference between 'right' answer and 'best' answer. The curriculum needs to be reevaluated more frequently to include topics that technology brings in. Math classrooms should have computer terminals at every desk.

2.5 ...

2.6 ...

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