VALUE AT RISK IN EMERGING MARKETS: EMPIRICAL EVIDENCE FROM TWELVE COUNTRIES

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ABSTRACT

This study focuses on the relative performance of three Value-at-Risk (VaR) estimation methodologies. The daily stock market index returns of twelve different emerging markets are used for the empirical analysis. In addition to the well-known methodologies, such as the historical simulation and GARCH-based ones, the extreme value theory (EVT) is also used to estimate the daily VaR. In this paper, we focus on EVT because it studies the non-linear estimation of the tails and we expect to find many extreme events when analysing the return distributions in these twelve emerging markets. We focus on the negative extreme events rather than on the positive ones. The daily VaR is forecasted at three different quantile levels: 90%, 97.5%, 99.9%; and competing methodologies are back-tested accordingly. The results indicate that the historical simulation and GARCH-based methodologies work better at lower quantile levels than they do at higher quantile levels, while VaR estimated using EVT is more accurate at higher quantiles. EVT provides better information about extreme events, especially when financial distress occurs in these economies.

Key words: Emerging markets; Value-at-Risk; Historical simulation; GARCH; Extreme Value Theory.

DEDICATION

I would like to dedicate this study to my beloved dad and mom, the angels in my life.

Mengying

To my family and fiancée, for their unconditional support and encouragement throughout this stage of my life. Helder

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1 INTRODUCTION

An emerging market economy is defined as an economy with a low-to-middle per capita income[1]. These countries constitute approximately 80% of the global population and represent nearly 20% of the world's economies. Emerging market economies (EMEs) are generally characterized as transitional, which means that they are in the process of moving from a closed market economy to an open market economy. An EME is also most likely receiving aid and guidance from large countries or world organizations, such as the World Bank and the International Monetary Fund.

One key characteristic of an EME is its increase in both local and foreign investment (portfolio and direct). A growth in the domestic investment in a country often indicates that the country has been able to build confidence within the local economy. Moreover, foreign investment is a signal that the world has begun to take notice of the emerging market. When international capital flows are directed toward an EME, the injection of foreign currency into the local economy adds volume to the country's stock market and long-term investment to the infrastructure. However, at the same time the economy starts to thrive, and the profit opportunities appear, the risk also starts to increase.

In the late 1990s, most of the twelve countries of the study experienced financial distress, and these emerging markets suffered billions of losses as a result of poor management of financial risks. Extreme events happened more frequently in emerging markets. VaR predicts the maximum expected loss over a period horizon with a certain quantile level. VaR was developed when financial disasters occurred in the 1990s and it played an increasingly important role in

[1] Term coined in 1981 by Antoine W. van Agtmael, International Finance Corporation, World Bank.

market risk management since then. Therefore, a precise estimate of VaR is a critical point in risk management. The sources of risk are very diverse and difficult to track using any methodology. In this study, we aim at analysing the efficiency of three different methodologies to calculate and predict VaR: the historical simulation methodology, GARCH-based methodology, and EVT.

The methodology comparison results generated from the back-testing show that the first two well-known methodologies give their best estimates of VaR at 90% quantile level. However, at a higher percent quantile level (97.5% and 99.9%), EVT works better on estimating VaR.

One of the first works that analyzed the problem of modelling VaR in emerging markets was done by Parrondo (1997). In this paper, he recognizes that emerging markets present a high instability which considerably decreases the efficiency of the usual statistical methods, and highlights the importance of developing a model which accounts for temporal correlation and discontinuities in a time series- Garch (1,1) process with Poisson-type jumps to characterize the jumps or discontinuities of the temporal behaviour of macroeconomic factors.[2]

A number of authors recognized the problems of VaR assumptions. Blanco and Dowd (2002) pointed out that traditional VaR methodologies tend to ignore extreme events and focus on risk measures that accommodate the whole empirical distribution of returns. An extensive literature has found problems when estimating VaR and capturing the risk of low-probability events assuming normality distributions of returns. It is in this scenario that EVT has brought more attention among researchers, and especially among practitioners. The basis of EVT is the extreme value theorem, a family of the central limit theorem, and this theorem tells us what the distribution of extreme values look like in the limit.

Gençay and Selcuk (2004) present some empirical results of VaR for nine emerging markets using EVT. They found that EVT-based VaR estimates are more accurate at higher

[2] Garch stands for Generalized Autorregresive Conditional Heteroscedasticity model.

quantiles, and that the daily return distributions have different moment properties at their right and left tails.

This study analyzes the risk associated with fat-tailed distributions when facing problems of not having a large data sample- the smaller the sample, the smaller the probability of finding extreme-events. Gençay and Selcuk (2004) based their study on different size-sample data for each one of the nine emerging countries trying to get the largest data as possible (as large as 27 years of daily data), and they proved that EVT-based VaR estimates work better than other models at higher quantiles. The empirical analysis presented in this study is based on a homogeneous size-sample (2411 observations = 9.6 years) of a larger number of countries (twelve).

The rest of the study is organized as follows. Section 2 describes each one of the methodologies used to calculate VaR, and the methods used to test the efficiency of each one when estimating VaR. Section 3 describes the data used in the empirical study. In Section 4, the results are presented for each one of the models, followed by a comparison of the three methodologies and an analysis of the results. Section 5 presents conclusions and recommendations.

2 VAR METHODOLOGY

VaR is one of the most important measures of risk used today for financial risk management. VaR is defined as the maximum expected loss over a given time horizon at a given quantile level (q). In mathematic terms, if P_t is the price of a financial asset on day t, a k-day VaR of a long position on day t is defined by:

$$P[P_{t-k} - P_t \leq Var(t,k,1-q)] = q. \tag{1}$$

There are two fundamental approaches for estimating VaR: parametric approach and non-parametric approach. We choose the historical simulation methodology among the non-parametric ones, because many large financial institutions compute VaR of their trading portfolios using the historical simulation methodology.[3] One of the reasons why this methodology has been so popular among practitioners is its accuracy, because it works well when a portfolio has substantial nonlinear components. However, the drawback of the historical simulation methodology is that it starts to react after a big movement. Among the parametric methodologies, the GARCH-based one is selected, because it provides a parsimonious model with few parameters, which usually fit economic time series very well. This methodology has become a prominent tool when analysing financial time-series, especially for the fact that this method address' the volatility clustering problem. Moreover, it is also related to chaos theory, because the nonlinearities behind chaos theory can be traced to the time variation in variances. Therefore, GARCH-based methodology explains some chaotic behaviour of financial markets.

[3] As mentioned in Pritsker, 2001

Due to the nature of the data under analysis (fat-tailed distributions), the Peaks Over the Threshold (POT) method is selected within EVT to estimate VaR of these emerging markets.

2.1 Historical Simulation

The historical simulation methodology assumes that the market risk factors in history will reflect changes of those in the future. It uses the actual quantile level as a VaR measure.

One advantage of the historical simulation methodology is that it does not make the assumption about the risk factors that will affect the portfolio's value, and in some sense, it is nonparametric. There is a key advantage since the empirical distribution of daily returns used in this study is fat-tailed, asymmetric and not normally distributed. In particular, the historical simulation methodology works well when the skew is not zero, and when the kurtosis is different from three. Hence, it does not require the estimation of volatilities and correlations, since historical volatilities and correlations have already been included in the data set.

Another advantage of the historical simulation methodology is that the historical simulation methodology is also very easy to implement. When it computes the empirical cumulative distribution function of the portfolio's value, the historical simulation methodology assigns equal weights, 1/N to each of the daily returns. This is equivalent to assuming that the daily return is identically independently distributed (i.i.d.) through time. This assumption is unrealistic; the volatility of the return is not constant over time. The volatility is time-varying, and may appear as a cluster over a specific period of time. Therefore, the assumption of i.i.d. tends to bias the VaR estimates.

Finally, another drawback of the historical simulation methodology is that it completely relies on the historical data to estimate the future value. Here, the Historical data is assumed to be representative of future events. If we just rely on the historical data, we may oversee some potential downturns of the market.

2.2 GARCH-based VaR

One of the drawbacks of the historical simulation methodology is that it responses too late to a big movement after it occurs. We select GARCH-based methodology, because it addresses this drawback. Volatility feedback is very significant in economic and financial terms, since it explains the negative relationship between the stock return and the risk involved. Considering that we analyse the stock return indexes, and that modelling time Variation in risk is paramount for the measurement of VaR, GARCH-based methodology should not only address the drawback of the historical simulation methodology, but also provide valuable information about the forthcoming risk. Many authors have pointed out that for most financial assets; volatility Varies in a predictable way. This Variation can be modelled by using time-series models such as GARCH.

We define the one-day logarithmic return of the stock index on day t as $x_t = \log(I_t) - \log(I_{t-1})$, where I_t is the value of the stock index today, and I_{t-1} is the value of the stock index yesterday.

$$x_t = \mu + \varepsilon_t, \quad \varepsilon_t | \Omega_{t-1} \sim D(0, \sigma_t^2) \tag{2}$$

$$\sigma_t^2 = c + a\varepsilon_{t-1}^2 + b\sigma_{t-1}^2 \tag{3}$$

ε_t follows a distribution with zero mean and σ_t^2 variance, conditional on any past information relevant (Ω_{t-1}). Thus, σ_t^2 is known as the conditional variance.

On the other hand, the unconditional variance of ε_t is constant.[4] The conditional volatility is an ARMA (1, 1) process which accounts for heteroscedasticity, i.e., and the fact that volatility is not constant implies that it varies in time yielding to periods with large volatilities and periods with small volatilities.

The GARCH-based methodology is non-linear; therefore, the parameters have to be estimated by maximizing the likelihood function, which involves a numerical optimisation. Also, the assumption that the residuals in this process are normally distributed and independent is not necessary. The parametric approach assumes that the market returns are normally distributed, but the evidence suggest that the returns are not normally distributed. Therefore, the maximum likelihood estimation of the GARCH-based methodology improve the traditional parametric approach.

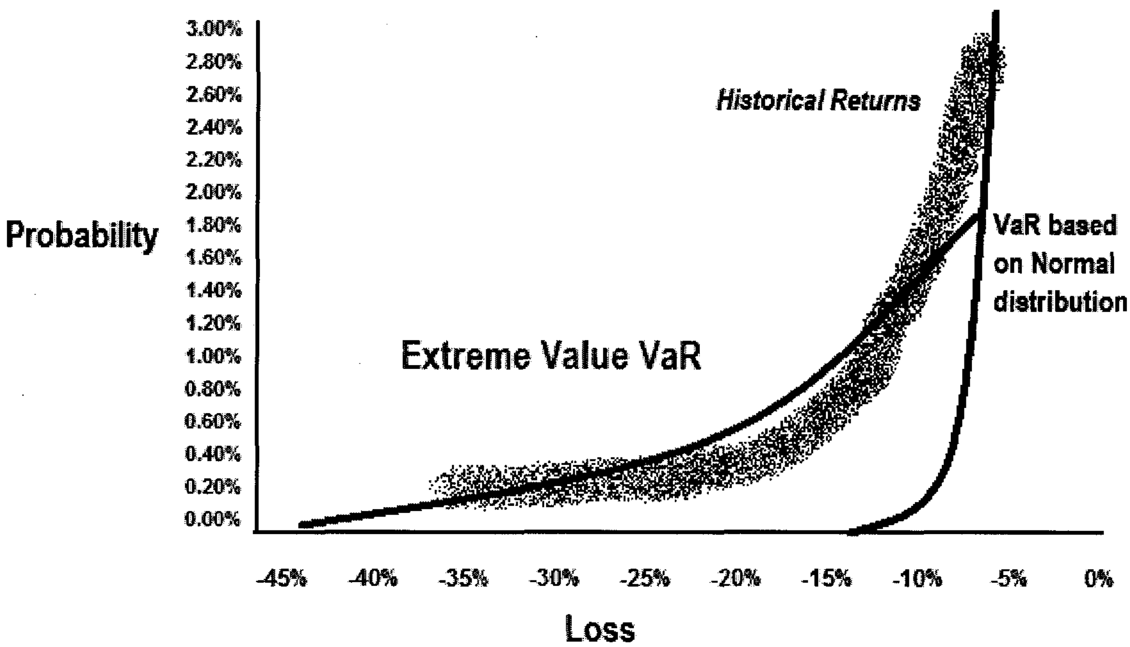
In addition, the distribution of real time series of financial returns has an obvious feature of fat tails, and the distribution is asymmetric. The drawback of assuming normality also includes the fact that the distribution focuses on the global density function of the return distribution; while it does not. As a result, the normal distribution neither ensure to fit the actual distribution, nor give a precise estimate of VaR. VaR estimation is based on the recognition that future changes of the market rates or portfolio values depend on historical data, and there is no need to assume the specific distribution of the return. Moreover, VaR estimation has a large requirement of historical data and may be biased when extreme events occur.

[4] $Var(\varepsilon_t) = \frac{c}{1-(a+b)}$

2.3 Extreme Value Theory (EVT)

EVT deals mainly with the extreme events, which deviates from the mean of probability distribution. The general theory is to assess the type of probability distributions generated by the process. This theory offers a framework of studying the behaviour of the tail distribution and describes more clearly about the extreme event. Besides this, EVT also studies the estimated quantile risk measure of financial time series. Therefore, EVT is also considered as a risk management tool.

Figure 2.1 Extreme Value - VaR Vs. Normal VaR



In this study, we introduce EVT by modelling the number of exceedances at a particular level of threshold. The analysis is based on twelve countries and assumes that we do not know the distribution of the returns; however, we can know what the tail distribution looks like by applying EVT. The general idea behind this is that we study the family of the extreme events of the return distribution. The family can be represented in a single parameterisation known as the Generalized Extreme Value (GEV) distribution. Subject to certain conditions, the distribution of the extreme event will be converged asymptotically to a Frechet distribution:

$$H_{\varepsilon,\mu,\sigma}(x) = \begin{cases} \exp(-[1+\xi(x-\mu)/\sigma]^{-1/\xi}, & if\ \xi \neq 0 \\ \exp(-e^{-(x-\mu)\sigma}), & if\ \xi = 0 \end{cases}. \tag{4}$$

Here, μ is the mean of the sample data, σ is the standard deviation of the sample data. The parameter that we are interested in is ξ, the tail index, which indicates how heavy the tail of the sample return distribution is. The higher the tail index, the heavier the tail.

2.3.1 The Distribution of the Exceedances (GPD)

The conditional excess distribution function $F_u(y)$ is defined as follow:

$$F_u(y) = P(x-u \leq y \mid x > u), \quad 0 \leq y \leq x_F - u). \tag{5}$$

Here, x is a random Variable; u is the value of given threshold. The value of the exceedances is defined as $y = x - u$. We verify that $F_u(y)$ can be written as:

$$F_u(y) = \frac{F(u+y)-F(u)}{1-F(u)} = \frac{F(x)-F(u)}{1-F(u)}. \tag{6}$$

When the value of the threshold u becomes sufficiently large, this excess function $F_u(y)$ can be approximated by the Generalized Pareto Distribution (GPD):

$$G_{\xi,\sigma}(y) = \begin{cases} 1-(1+\frac{\xi}{\sigma}y)^{-1/\xi}, & if\ \xi \neq 0 \\ 1-e^{-y/\sigma} & , if\ \xi = 0 \end{cases}. \tag{7}$$

Since $y = x - u$, GPD can also be expressed in terms of x as follows:

$$G_{\xi,\sigma}(x) = 1-(1+\xi(x-u)/\sigma)^{-1/\xi} \quad (8)$$

$$\text{with } x \in \begin{cases} [v,\infty], & if \xi \geq 0 \\ [v, v-\sigma/\xi], & if \xi < 0 \end{cases}$$

When ξ >0, the distribution is an ordinary GDP; when ξ =0, the distribution function of exceedances will follow an exponential distribution. If v =0, and σ =1, the distribution of exceedances is a standard GPD. Only the distribution with ξ >0 fits the model of fat-tailed distribution. Therefore, in this study, we will focus on the parameters with ξ >0.

We isolate $F(x)$ from Eq. (3) and get the formula as:

$$F(x) = (1-F(u))F_u(y) + F(u). \quad (9)$$

By substituting $F_u(y)$ in Eq. (9) for Eq. (7) we get:

$$F(x) = \frac{N_u}{n}(1-(1+\frac{\xi}{\sigma}(x-u))^{-1/\xi}) + (1-\frac{N_u}{n}). \quad (10)$$

For a given probability p:

$$VaR = u + \frac{\sigma}{\xi}((\frac{n}{N_u}p)^{-\xi} - 1). \quad (11)$$

By choosing a threshold value, we can estimate the parameters ξ and σ (ξ, σ) using the Maximum Likelihood Estimation (MLE).

For the sample $y_u = \{y_1, ..., y_n\}$ the log-likelihood function will be like:

$$L(\xi,\sigma \mid y) = \begin{cases} -n\log\sigma - (\frac{1}{\xi}+1)\sum_{i=1}^{n}\log(1+\frac{\xi}{\sigma}y_i), & if\ \xi \neq 0 \\ -n\log\sigma - \frac{1}{\sigma}\sum_{i=1}^{n}\log(1+\frac{\xi}{\sigma}y_i), & if\ \xi = 0 \end{cases} \quad (12)$$

2.3.2 Modelling the Fat Tails of the Stock Index

There are three methods that are useful to choose a threshold value: Sample Mean Excess plot, QQ-plot, and Hill-plot.

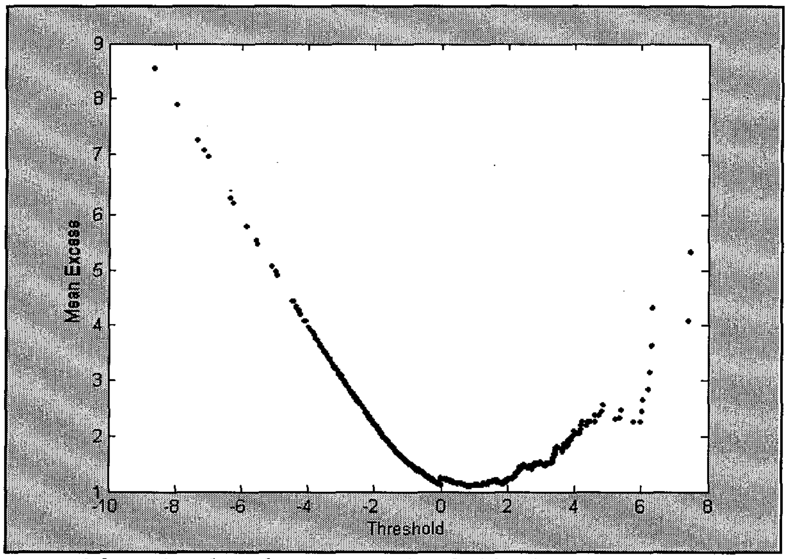
2.3.2.1 Sample mean excess plot:

The sample mean excess function is defined as: $(u, e_n(u))$, and

$$e_n(u) = \frac{\sum_{i=k}^{n}(x_i^n - u)}{n-k+1}, k = \min\{i \mid x_i^n > u\} \quad (13)$$

Here "$n-k+1$" is the number of exceptions exceeding a certain threshold (u) level, and "$x_i^n - u$" is the excess value of each observation. Hence, $e_n(u)$ is the mean of these excess values. For GPD, this function is linear. Figure 2.2 depicts the sample Mean Excess Function (MEF) for the stock index returns of India.

Figure 2.2 Mean excess plot



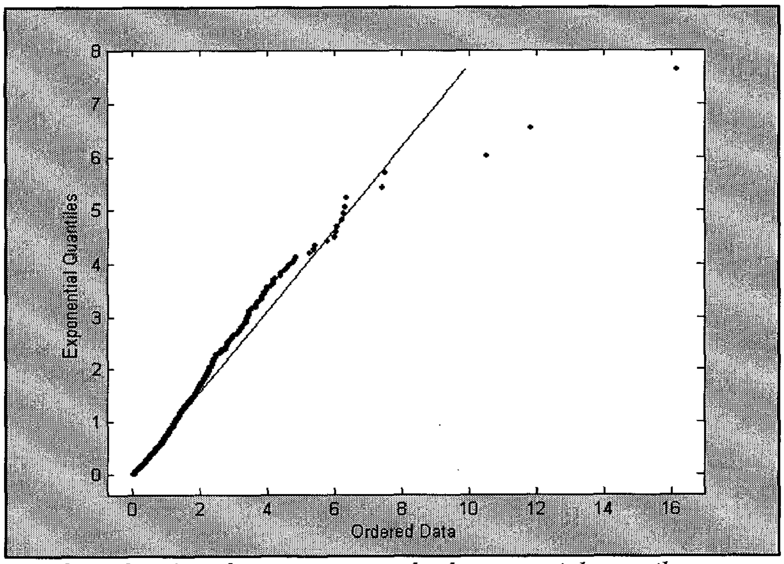
Note: The Mean Excess of the Indian loss data against threshold values.

The empirical MEF is a positively sloped straight line above the given threshold value u =1.8%, and the graph indicates that the returns of the stock index follows a GPD with a positive parameter ξ. Here, u will be determined within the range between 1.8% and 5%. Since the graph is roughly linear within this range. On the other hand, if MEF is a negatively sloped straight line, the distribution of the sample data will be short-tailed.

2.3.2.2 QQ-plot (quantile- quantile)

The actual quantiles are plotted against the exponential distribution quantiles to assess the tail of the distribution. If data comes from an exponential distribution, then the points of the graph would lie on a positively sloped straight line. If data comes from a fat-tailed distribution, the points will lie on a concaved line; if the data is from a thin-tailed distribution, the points will lie on a convex line. The scatter plot of India is shown in Figure 2.3 for illustration purposes.

Figure 2.3 QQ-plot



Note: QQ-plot of the Indian loss data against standard exponential quantiles.

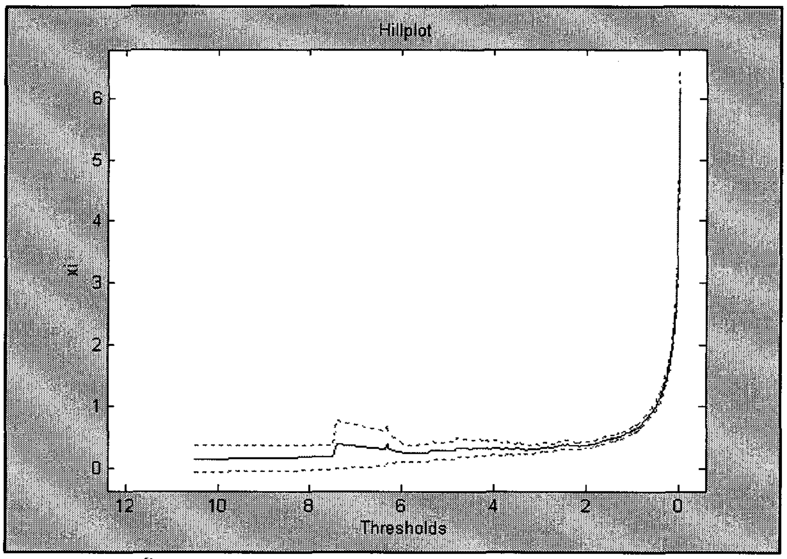
From Figure 2.3, we can conclude that the stock index returns come from a fat-tailed distribution, especially when u is set between 2% and 5%.

2.3.2.3 Hill-plot:

Hill (1975) proposed an estimator of ξ for cases in which ξ > 0. The tail index is plotted against the number of exceedances k. First, the data has to be ordered with respect to its value: $x_{1,n}, x_{2,n}, \ldots, x_{n,n}$, where $x_{1,n} > x_{2,n} > \ldots > x_{n,n}$. The function is defined as:

$$\xi = \frac{1}{k-1} \sum \ln x_{i,N} - \ln x_{k,N} \quad for\ k \geq 2. \tag{14}$$

Figure 2.4 Hill-plot with 95% confidence level.



Notes: Shape parameter ξ is plotted against different levels of threshold.

Therefore, the threshold value u will be selected between 2 and 6 based on Figure 2.4, for the shape of parameter ξ is fairly stable within this range.

By choosing a threshold value, we have a trade-off between bias and variance. If a low level of threshold is chosen, there will be more exceptions above this value, and the estimation will become more precise than VaR, which is estimated at a high level of threshold. However, the lower the threshold value, the more number of observations we will have. Those observations are located around the sample mean, the distribution centre. This is, in some sense, not consistent with the tail behaviour study of the distribution. This way, the estimation will be biased. The estimation of ξ is highly sensitive to the number of exceptions. Therefore, we try to use a combination of the three methods in order to determine the threshold level.

This method allows us to study the probabilities of the extreme event and the extreme quantile without talking into account the assumption of the return distribution. EVT also allows for asymmetry of the daily return distribution. EVT allows for the distribution of the returns from different countries to have different shapes on their left and right tails. Different values of ξ produce different information about the return features.

2.4 Back-testing VaR methodologies

In this study, we measure the relative performance of each methodology by comparing the actual failure rate with the expected failure rate. The actual failure rate is calculated as the number of exceedances divided by the total number of observations. The quantile level is the level at which we make a forecast of VaR. For example, if we choose to estimate VaR at a certain quantile level q, the lower return (1-q) will be the actual failure rate. Therefore, if the actual failure rate is higher than the expected failure rate, it means that the realized returns are higher than the predicted ones, and also implies that the forecasted VaR underestimates the return at a certain quantile level. In this study, a small actual failure rate is preferable at a given quantile level. However, small failure rate may not be desirable, for it is an indication of the underestimation of risks. From the perspective of the risk manager, small failure rate indicates excessive capital allocation; however, from the perspective of regulatory body, they only care about the excessive loss, which is indicated by the higher failure rate. In this study, our purpose is to meet the requirements of the regulatory body. Hence, the methodology works better if it gives a small actual failure rate.

In this study, our preference is a methodology that underestimates the risk for the reason of meeting the requirements of a regulatory body. If all methodologies underestimate VaR, the one that gives the least underestimation of VaR is preferred. For example, we estimate VaR at 90% quantile level, and the results estimated from the three methodologies above are 11%, 11.1%, and 11.2%, then the model with 11% gives the closest failure rate to the expected failure rate.[5]

The other back-testing method that we use is the Log-Likelihood Ratio test. As the Bernouli trials indicate, the number of exceptions x follows a binomial probability distribution:

$$f(x)=\binom{T}{x}p^{x}(1-p)^{T-x}. \tag{15}$$

Here, the expected value of x is $E(x)=pT$, and the Variance is $V(x)=p(1-p)T$. When T is large, we can use the central limit theorem and the binomial distribution will be approximated by the normal distribution.

$$z=\frac{x-pT}{\sqrt{p(1-p)T}}. \tag{16}$$

We use this binomial distribution to test whether the number of exceptions is acceptably small. For back-testing purposes, we face a trade-off between type 1 and type 2 errors. Type 1 occurs when we reject a model, which is actually acceptable; type 2 error occurs when we accept a model that has to be rejected. If a cut-off number of exceptions is low, there will be a high probability of type 1 error. On the other hand, if the cut-off number is high, there will be a high probability of type 2 error. Our goal is to get the lowest type 2 error for a given type 1 error.

Kupiec (1995) develops a confidence region of 95 percent for such a test. The regions are defined by the tail points of the Log-Likelihood Ratio:

$$LR_{uc}=-2\ln[(1-p)^{T-N}p^{N}]+2\ln\{[1-(N/T)]^{T-N}(N/T)^{N}\}. \tag{17}$$

[5] We list the actual failure rates from the historical simulation methodology, Garch-based methodology and EVT in Table 4.13.

Here, T is the number of exceptions, N is the number of total observations, and p is the actual failure rate. The Log-Likelihood Ratio is distributed Chi-square with one degree of freedom. The null hypothesis is that p is the true probability, and we will reject the VaR model if $LR_{uc} > 3.84$. Statistical decision theory shows that this test is the most powerful back-testing method among its class. In this study, we plug the actual failure rate of each country into the Log-Likelihood Ratio formula, and see whether we reject the VaR Methodology at the 95 percent confidence level . We use the same test for all three methodologies individually, and then count the number of "Not Reject" in order to accept the model at each of the three quantile levels: 90%, 97.5% and 99.9%.[6]

[6] The comparison results are shown in Table 4.12.

3 DATA DESCRIPTION

When investigating which countries to consider in the analysis, three elements were taken into consideration: the size of the sample, the frequency of the data (daily), and the fact that it is considered as an emerging economy. The countries considered in the analysis comprise six of the ten biggest emerging economies: Argentina, India, Indonesia, Mexico, South Africa, and South Korea. The other six countries are: Chile, Peru, Ghana, Czech Republic, Singapore, Kenya. These twelve countries represent a sample of different regions: South America, Central America, Africa, Asia, and Europe.

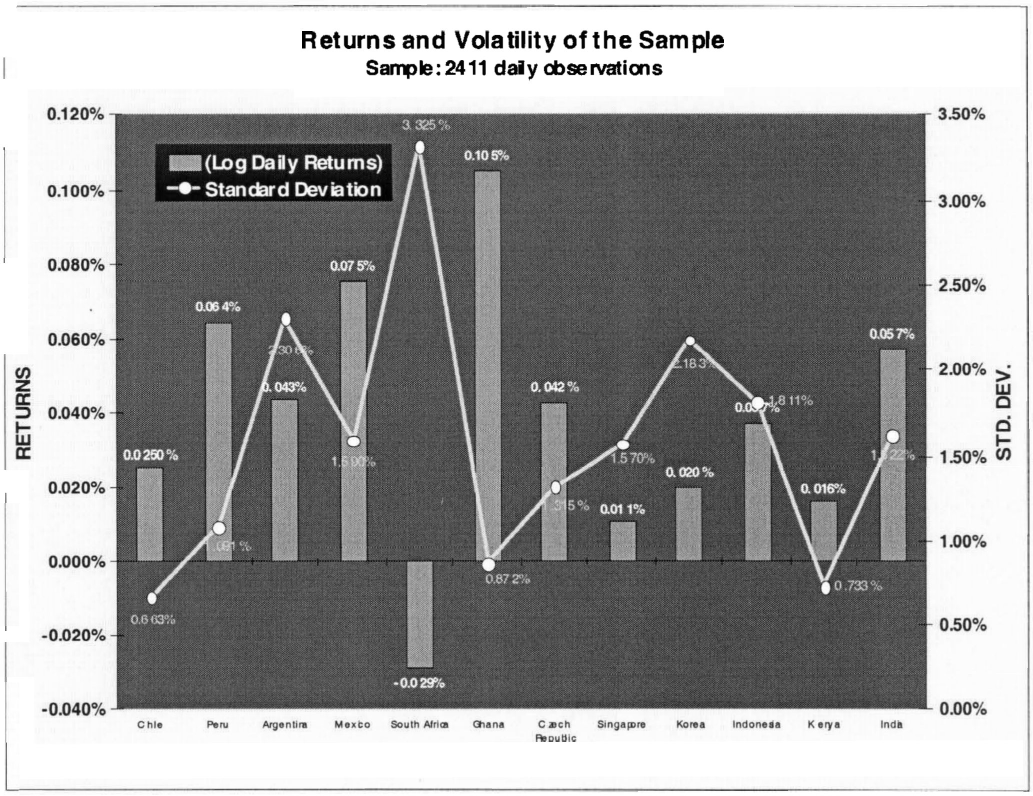
Table 3.1 Descriptive statistics of the daily returns.

Emerging Market	Stock Market Index	Mean	Max	Min	Std	Sk	Ku
Argentina	MERVALD	0.043%	16.117%	-14.765%	0.02306	-0.13086	8.39590
Chile	IGPA	0.024%	4.467%	-3.774%	0.00663	0.06282	7.09607
Czech Republic	HNWD	0.040%	15.079%	-14.876%	0.01314	0.05273	24.53014
Ghana	GGSEGSE	0.105%	12.185%	-9.227%	0.00873	2.72436	60.11888
India	SENSEX	0.058%	8.592%	-16.135%	0.01623	-0.71834	10.89401
Indonesia	JCI	0.037%	13.128%	-15.411%	0.01812	-0.13118	12.54745
Kenya	KNSMIDX	0.016%	4.648%	-4.950%	0.00733	0.30514	9.96166
Korea	KOSPI	0.018%	10.024%	-14.253%	0.02183	-0.14074	6.25915
Mexico	MEXBOL	0.076%	12.154%	-14.314%	0.01591	-0.06223	10.53535
Peru	IGBVL	0.064%	9.492%	-6.884%	0.01092	-0.13345	10.00432
Singapore	BTSRI	0.011%	18.428%	-12.982%	0.01571	0.58902	18.32404
South Africa	DDTI	-0.028%	38.963%	-26.775%	0.03317	0.38923	16.38969

Notes: Sample size=2411. Max=Maximum observed daily return; Min=Minimum observed daily return; Std=Standard deviation; Sk=Skewness; Ku=Kurtosis. Source: Bloomberg.

The data collected from these countries goes from October 15, 1996 to May 18, 2006.[7] From Figure 3.1, Ghana appears to have the most attractive combination of return and variance, followed by Mexico and India, while South Africa seems to have the worst return-variance ratio.

Figure 3.1 Data Statistics & Quantiles



Notes: The orange bars depict the average log daily returns of each country Stock Market Index.

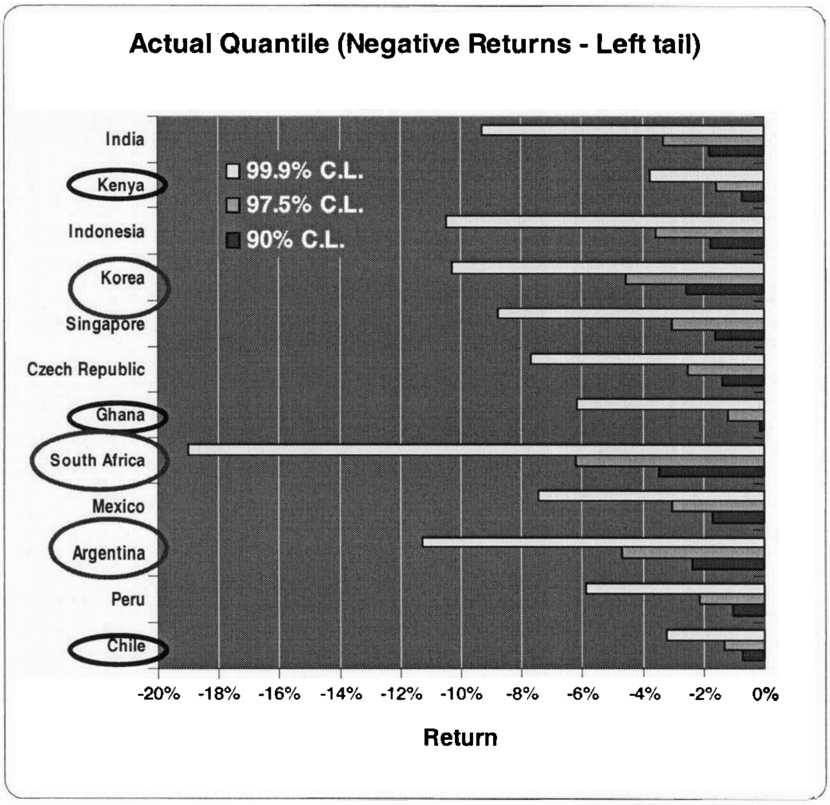
Chile, Kenya and Ghana are the countries with their 0.10 and 0.025 quantile returns above the –2% return-level. Kenya and Chile have their 0.001 quantile returns above the –4% return-level, while Ghana's exceed the –6% return-level barely. See Figure 3-2.

South Africa, Argentina, and Korea are the countries that have experienced the most extreme negative events. All three countries have their 0.10 and 0.025 quantile returns below the

[7] Graphs containing the Index evolution, the histograms, and statistics of the stock market index returns of each country is provided in the Appendix section.

–2% return-level and –4% return-level, respectively. Also, their 0.001 quantile returns are below the –10% return-level. See Figure 3-2.

Figure 3.2 Data Statistics & Quantiles (2)



Data Source: Aragones, Jose.R., Carlos Blanco, and Kevin Dowd. Learning curve extreme value VaR (p.3.)

4 ANALYSIS OF RESULTS

VaR rests on modelled volatilities of risk factors. It offers little guidance in exploring abnormal events outside the realm of normal statistical probability. This limitation may be overcome by using stress testing as a complimentary tool. A stress test examines the implications when the abnormal unexpected worst-case scenario does materialize.

4.1 Historical Simulation

Based on the index returns that we get from twelve emerging markets, we calculate VaR on a rolling window which consists of 500 data points. Under the historical simulation methodology, there is no need to make assumptions of the return distribution of the daily index return. First, we choose three quantile levels at which VaR is calculated: 90%, 97.5%, and 99.9%. Second, we calculate the VaR by setting up an event window which consists of 500 data points. For the daily VaR on the 501st day, in this case, October 15, 1996, data points from 1st and 500th are used. Third, in order to calculate the 502nd VaR we just move the rolling window one day ahead by descending the 1st data point and adding the 501st data point. Then, we use the same way as above to calculate VaR. Following this way, we calculate daily VaR from day number 501 (October 15, 1996) to day number 2411 (May 18th, 2006).

After calculating all of the daily VaRs for each country, we compare the actual failure rate with the expected failure rate. As mentioned above, it is preferred an smaller actual failure rate. The results from each country are shown in the Table 4.1.

Table 4.1 Actual failure rates of different emerging markets calculated at different quantile levels

Quantile Level	90%	97.5%	99.9%
Expected Failure Rate	10%	2.50%	0.10%
Country	Actual Failure Rate		
Chile	8.791%	1.936%	0.209%
Peru	5.704%	1.884%	0.157%
Argentina	9.262%	2.721%	0.209%
Mexico	8.268%	2.145%	0.105%
South Africa	9.576%	2.198%	0.157%
Ghana	7.954%	2.407%	0.366%
Czech Republic	9.314%	2.250%	0.209%
Singapore	7.221%	1.570%	0.209%
Korea	7.640%	1.936%	0.105%
Indonesia	8.163%	1.727%	0.209%
Kenya	9.733%	2.669%	0.262%
India	9.158%	2.355%	0.314%
Number of Good	12	10	0

Notes: Actual failure rate=T/N (T is the number of total observations; N is the number of exceedances). The actual failure rate of each country is calculated at three quantile levels: 90%, 97.5% and 99.9%. "Number of Good" means the number of countries in which the methodology gives a precise estimate of VaR.

From Table 4.1 the historical simulation methodology works better at a lower quantile level. At 90% quantile level, this methodology works for all of the countries. On the other hand, it does not work for any country at 99.9% quantile level.

Then we choose a certain quantile level at 99.9% and calculate daily VaR by changing the sample size from 500, to 750 and 1250 respectively. We compare the actual failure rate with the expected failure rate of each country, and then add them up the countries whose actual failure rate is less than the expected failure at a certain sample size. The results are shown in Table 4.2.

Table 4.2 Actual failure rates of different emerging markets calculated at different sample sizes.

Country	Actual Failure Rate		
Sample Size	500	750	1250
Chile	0.209%	0.000%	0.000%
Peru	0.157%	0.181%	0.000%
Argentina	0.209%	0.060%	0.000%
Mexico	0.105%	0.000%	0.000%
South Africa	0.157%	0.181%	0.172%
Ghana	0.366%	0.301%	0.258%
Czech Republic	0.209%	0.181%	0.000%
Singapore	0.209%	0.120%	0.000%
Korea	0.105%	0.120%	0.000%

Country	Actual Failure Rate		
Sample Size	500	750	1250
Indonesia	0.262%	0.000%	0.000%
Kenya	0.209%	0.000%	0.000%
India	0.314%	0.000%	0.086%
Number of Good	0	6	10

Notes: *Actual failure rate=T/N. T is the number of total observations; N is the number of exceedances. The actual failure rate of each country is calculated for a 99.9% quantile level at three sample size: 500, 750 and 1250. The Expected Failure Rate is 0.10%.*

From Table 4.2 we can conclude that as the sample size increases, the historical simulation methodology gives a more precise of estimating VaR.

A more powerful back-testing method is also used, the Log-Likelihood Ratio Test:

Table 4.3 Log-Likelihood Ratio Test Result for Historical Simulation Methodology

Historical Simulation methodology	90%		97.5%		99.9%	
	LR_{uc}	Decision 95% CL	LR_{uc}	Decision 95% CL	LR_{uc}	Decision 95% CL
Chile	(5.877)	NR	0.815	NR	1.533	NR
Peru	(7.146)	NR	(2.456)	NR	5.259	R
Argentina	(8.445)	NR	(2.293)	NR	1.533	NR
Mexico	(1.786)	NR	(1.057)	NR	(0.096)	NR
South Africa	(9.597)	NR	(1.397)	NR	0.378	NR
Ghana	1.324	NR	(2.283)	NR	7.660	R
Czech Republic	(8.668)	NR	(1.689)	NR	1.533	NR
Singapore	10.607	R	6.278	R	1.533	NR
Korea	4.946	R	0.815	NR	(0.096)	NR
Indonesia	(0.805)	NR	3.570	NR	1.533	NR
Kenya	(10.009)	NR	(2.395)	NR	3.194	NR
India	(7.963)	NR	(2.131)	NR	5.259	R
Total NR	10		11		9	

Notes: *Equation: $LR_{uc}=-2ln[(1-p)^{T-N}p^{N}]+2ln\{[1-(N/T)]^{T-N}(N/T)^{N}\}$. P is the expected failure rate; T is the number of total observations; N is the number of exceedances; LR_{uc} is the Log-Likelihood Ratio of the unconditional converge. **LR_{uc}** is calculated at three quantile levels: 90%, 97.5% and 99.9%. EVT is tested as **"Reject"**(R) or **"Not Reject"**(NR) within 95% confidence interval for each of the twelve countries. If **LR_{uc}** is greater than the critical value 3.84, it is rejected. On the other hand, EVT can not be rejected. "**Total NR**" indicates the total number of countries where EVT is not rejected.*

From Table 4.3, we can conclude that the historical simulation methodology woks well at all three quantile levels, since it is rejected no more than three times. Also, the relative performance of the historical simulation methodology is better at a lower quantile level (90%) than at a higher quantile level (99.9%).

4.2 GARCH-based VaR

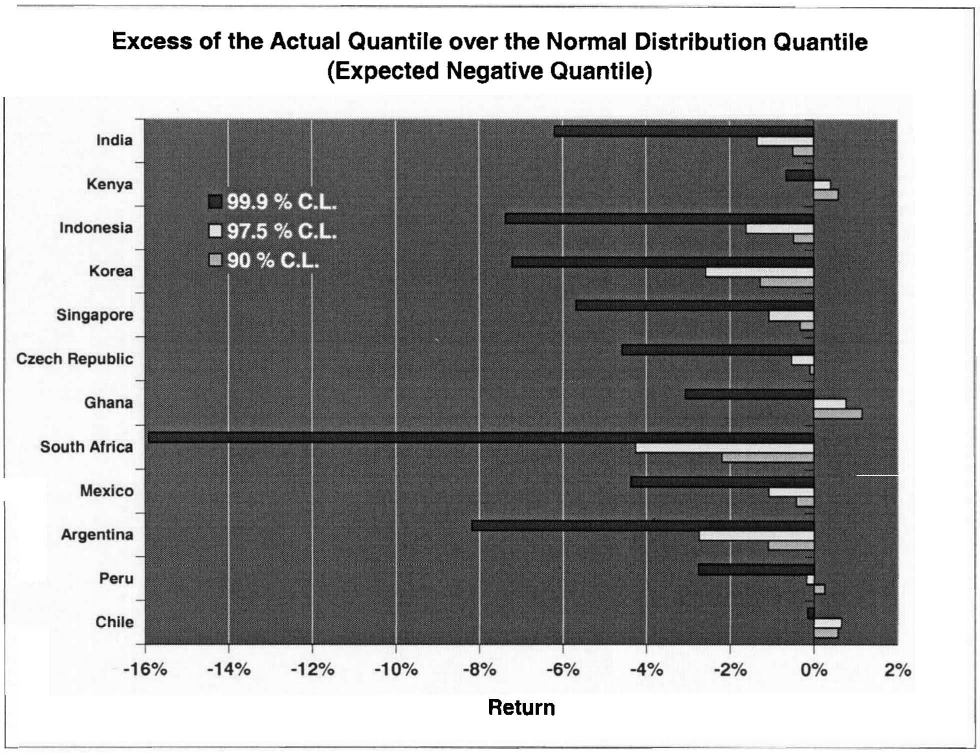
Traditional parametric VaR methods tend to ignore extreme events; they assume that the observations fit a normal distribution, but as Figure 4.1 shows below, the actual quantiles are much higher in absolute values than the ones predicted by the normal distributions for almost all countries at each quantile level. Only for four countries of the twelve being analysed in the study, and at the lowest quantile level (90%), the expected negative return predicted by the normal distribution (-1.28%) is not underestimating the actual negative return. The number of countries goes down to three when estimating VaR at a 97.5% quantile level. At the highest quantile level (99.9%), the normal distribution underestimates the negative expected return up to a 15.9% difference of negative return. Therefore, it is clear that we need to increase the efficiency of parametric methodology when estimating VaR.

Table 4.4 Values of the Excess over Normal Distribution Quantile

Stock market Index	Mean Returns	Actual Quantile (Left Tail)			Excess over Normal Distribution Quantile		
Country	(Log Daily Returns)	90.0%	97.5%	99.9%	90.0%	97.5%	99.9%
Chile	0.025%	-0.707%	-1.305%	-3.246%	0.574%	0.654%	-0.155%
Peru	0.064%	-1.031%	-2.138%	-5.853%	0.251%	-0.178%	-2.763%
Argentina	0.043%	-2.376%	-4.702%	-11.271%	-1.095%	-2.742%	-8.181%
Mexico	0.075%	-1.701%	-3.034%	-7.461%	-0.420%	-1.074%	-4.371%
South Africa	-0.029%	-3.476%	-6.214%	-18.991%	-2.194%	-4.254%	-15.901%
Ghana	0.105%	-0.128%	-1.191%	-6.154%	1.153%	0.769%	-3.064%
Czech Republic	0.042%	-1.373%	-2.499%	-7.669%	-0.092%	-0.539%	-4.578%
Singapore	0.011%	-1.611%	-3.030%	-8.770%	-0.329%	-1.070%	-5.680%
Korea	0.020%	-2.556%	-4.545%	-10.306%	-1.274%	-2.585%	-7.215%
Indonesia	0.037%	-1.765%	-3.573%	-10.450%	-0.483%	-1.613%	-7.360%
Kenya	0.016%	-0.694%	-1.553%	-3.749%	0.587%	0.407%	-0.659%
India	0.057%	-1.782%	-3.300%	-9.275%	-0.500%	-1.340%	-6.185%

Notes: Negative number of outliers over the expected number. The Quantile levels (Left Tail) of the Normal Distribution are: -1.28% (90%), -1.96% (97.5%), and -3.09% (99.9%).

Figure 4.1 Excess of the Actual Quantile over the Normal Distribution Quantile



Note: "C.L." = quantile level.

Traditional parametric VaR methodologies use Eq. (18):

$$Portfolio\ value * \{z(1-q) * \sigma\}. \tag{18}$$

Here, q is the level of confidence, and also the quantile level; z is the normal standard distribution. Hence, $z(1-q)=(1-q)$ is the percentile of the normal standard distribution, and σ is the standard deviation of the returns (volatility). Here, we assume a portfolio value of one monetary unit.

The GARCH model is used to estimate the conditional volatility[8] and improve the estimation of σ. The ultimate result should be a better estimation of the VaR, especially at the lowest quantile level.

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[8] $\sigma_t^2 = c + a\varepsilon_{t-1}^2 + b\sigma_{t-1}^2$

Table 4.5 shows the results by using GARCH-based methodology for each of the twelve stock index returns according to Eq. (3).

Table 4.5 Results of GARCH-based methodology

Country	Mean Equation	Variance Equation			L-likelihood	Persistence
	Coefficient	Coefficient	a	b		
Chile	0.000348 (0.002953)	0.0000015 (1.8E-07)	0.141247705 (0.00000)	0.828673 (0)	8,952.35	0.969920
Peru	0.00071500 (0.0001)	0.0000105 (0)	0.247974 (0)	0.679284 (0)	7,806.95	0.927258
Argentina	0.00112600 (0.0027)	0.0000147 (0)	0.117318 (0)	0.858327 (0)	5,976.57	0.975645
Mexico	0.00143200 (0)	0.0000053 (0)	0.12638 (0)	0.858528 (0)	6,886.89	0.984908
South Africa	0.00027300 (0.6702)	0.0004190 (0)	0.315382 (0)	0.350552 (0)	4,915.62	0.665934
Ghana	0.00043100 (0.0036)	0.0000040 (0)	0.109343 (0)	0.855477 (0)	8,433.23	0.964820
Czech Republic	0.00093100 (0)	0.0000065 (0)	0.108173 (0)	0.857194 (0)	7,307.83	0.965367
Singapore	0.00065400 (0.0027)	0.0000024 (0)	0.146135 (0)	0.861145 (0)	7,095.50	1.007280
Korea	0.00073500 (0.0338)	0.0000019 (0.00twelve)	0.05631 (0)	0.941929 (0)	6,089.00	0.998239
Indonesia	0.00100800 (0.0004)	0.0000053 (0)	0.098739 (0)	0.890825 (0)	6,633.59	0.989564
Kenya	-0.00032400 (0.0115)	0.0000028 (0)	0.125398 (0)	0.827966 (0)	8,686.26	0.953364
India	0.00139700 (0)	0.0000215 (0)	0.102371 (0)	0.819134 (0)	6,642.35	0.921505

Notes: *The numbers between parentheses are the t-stats of the parameter estimates. Persistence =* $a+b$.

The estimated coefficients of the variance equation of GARCH-based methodology using MLE are individually significant, and in almost all of the cases, the persistence parameters are close, but less than one – only South Africa and Singapore don't share this feature. The persistence parameter is the sum of $a+b$, and the higher it is, the higher the average daily variance will remain; on the other hand, the lower the persistence parameter is, the lower the conditional variance will decay after the shock moves it to a high level. At the same time, a persistence parameter, which is less than unity, indicates that the methodology is stationary.

Assuming that the mean of the return distributions equals zero, and after calculating all daily VaRs for each country by means of the conditional volatility, we calculate the actual failure rate in order to assess the efficiency of the GARCH-based methodology. The actual failure rate is calculated after estimating VaR using the conditional Variance as σ, and comparing this actual failure rate with the expected failure rate at each quantile level.

Table 4.6 Results of GARCH model (2).

Country	(Log Daily Mean Returns)	90%		97.5%		99.9%	
Chile	0.025%	8.50%	O	2.24%	O	0.29%	U
Peru	0.064%	6.43%	O	2.28%	O	0.58%	U
Argentina	0.043%	8.13%	O	3.07%	U	0.71%	U
Mexico	0.075%	7.42%	O	2.99%	U	0.54%	U
South Africa	-0.029%	6.51%	O	2.45%	O	0.50%	U
Ghana	0.105%	2.65%	O	1.53%	O	0.87%	U
Czech Republic	0.042%	7.42%	O	2.65%	U	0.50%	U
Singapore	0.011%	8.05%	O	2.41%	O	0.37%	U
Korea	0.020%	9.79%	O	2.78%	U	0.33%	U
Indonesia	0.037%	7.55%	O	2.61%	U	0.71%	U
Kenya	0.016%	6.84%	O	2.82%	U	0.75%	U
India	0.057%	7.22%	O	2.32%	O	0.33%	U
Number of "O"		12		6		0	

*Notes: **Actual Failure Rate** : Outliers / Observations (VaR forecasted using the normal probability distribution and the Conditioned Volatility estimated by the Garch model). O=Overestimates the Expected Failure Rate. U=Underestimates the Expected Failure Rate.*

An actual failure rate higher than the expected one indicates that the methodology consistently underestimates the return at the tail. This happens because the realized returns are higher than those that the model predicted, resulting in a failure rate higher than the expected one. We recognize the fact that even though an actual failure rate is lower than the expected one, it could be “good” for regulatory purposes, it does not mean that is doing the best estimation possible. The best estimation is achieved when the actual failure rate equals the expected failure rate. However, since we are interested in the left tail of distributions (losses), corresponding to a long-position portfolio, an actual failure rate lower than the expected is “good” enough.

The results in Table 4.6 show that the GARCH model works very good when estimating the VaR at the lowest confidence level (90%), works good in 50% of the countries at the 97.5% confidence level, and does not work at all when estimating daily VaR at the highest confidence level (99.9%). This shows that when trying to estimate extreme possible losses, GARCH-based methodology gives us mixed results at a 0.025 quantile level, and it fails to estimate losses at a 0.001 quantile level.

When using the Log-Likelihood Ratio Test to back-test this methodology, the results change drastically at the lower quantile levels.

Table 4.7 Log-Likelihood Ratio Test Result for GARGH-based Methodology.

GARCH-based Methodology	90%		97.5%		99.9%	
	LR_{uc}	Decision 95% CL	LR_{uc}	Decision 95% CL	LR_{uc}	Decision 95% CL
Chile	6.295	R	0.694	NR	5.753	R
Peru	38.618	R	0.488	NR	26.130	R
Argentina	9.949	R	2.992	NR	37.318	R
Mexico	19.339	R	2.204	NR	22.676	R
South Africa	36.723	R	0.028	NR	19.377	R
Ghana	198.506	R	10.668	R	53.874	R
Czech Republic	19.339	R	0.232	NR	19.377	R
Singapore	10.882	R	0.089	NR	10.549	R
Korea	0.121	NR	0.743	NR	8.025	R
Indonesia	17.437	R	0.125	NR	37.318	R
Kenya	29.680	R	0.976	NR	41.295	R
India	22.751	R	0.318	NR	8.025	R
Total NR	**1**		**11**		**0**	

Notes: *Equation:* $LR_{uc}=-2ln[(1-p)^{T-N}p^{N}]+2ln\{[1-(N/T)]^{T-N}(N/T)^{N}\}$. *P is the expected failure rate; T is the number of total observations; N is the number of exceedances;* LR_{uc} *is the Log-Likelihood Ratio of the unconditional converge.* $\mathbf{LR_{uc}}$ *is calculated at three quantile levels: 90%, 97.5% and 99.9% GARCH-based methodology is tested as "**Reject**" (R) or "**Not Reject**" (NR) within 95% confidence interval for each of the twelve countries. If* $\mathbf{LR_{uc}}$ *is greater than the critical value 3.84, it is rejected. On the other hand, GARCH-based methodology can not be rejected. "**Total NR**" indicates the total number of countries where GARCH-based methodology is not rejected.*

From Table 4.7, we can see that, at 90% quantile level, GARCH-based methodology works only for one country, Korea. However, it should be noted that the number of exceedances estimated by this methodology at this level of confidence is not larger than the expected, but too smaller; the method does not underestimate the risk, but overestimates it considerably. At 97.5%

quantile level, GARCH-based methodology estimates VaR well for almost all of the countries, except Ghana. At 99.9% quantile level, GARCH-based methodology is rejected among all twelve countries. The result is a little bit different from what we get by looking at the actual failure rate, however, generally speaking, GARCH-based methodology works better at lower quantile level than at higher quantile level.

4.3 Extreme Value Theory

Table 4.8 lists the estimated parameters of the tail index and the scale parameter, elements necessary to calculate the VaR (Table 4.10). The estimated index values range between 0.0386 (Argentina) and 0.3368 (South Africa). The negative stock return distribution in Czech Republic, Singapore, India, Indonesia have the first three moments but not have the fourth moment, since the estimated is around 0.25.

Table 4.8 Maximum Likelihood Estimates (MLE) of the parameters of the Generalized Pareto Distribution (GPD)

Country	$\hat{\xi}$	se($\hat{\xi}$)	$\hat{\sigma}$	se($\hat{\sigma}$)
Chile	0.06000	0.09370	0.00500	0.00065
Peru	0.08900	0.13020	0.01040	0.00180
Argentina	0.03860	0.08190	0.01930	0.00220
Mexico	0.20730	0.11900	0.00960	0.00150
South Africa	0.33680	0.15650	0.02090	0.00400
Ghana	0.13650	0.13490	0.01180	0.00210
Czech Republic	0.27340	0.11930	0.00750	0.00110
Singapore	0.21820	0.15620	0.01200	0.00240
Korea	0.10510	0.09690	0.13400	0.00170
Indonesia	0.24020	0.10670	0.01230	0.00170
Kenya	0.14850	0.07940	0.00520	0.00054
India	0.27910	0.19070	0.01280	0.00310

Table 4.9 Threshold returns, corresponding empirical quantiles and number of exceedances

Country	Threshold	Quantile	Exceedances
Chile	-0.0090	0.0531	128
Peru	-0.0200	0.0290	70
Argentina	-0.0300	0.0668	161
Mexico	-0.0250	0.0427	103
South Africa	-0.0600	0.0303	73
Ghana	-0.0100	0.0294	71

Country	Threshold	Quantile	Exceedances
Czech Republic	-0.0200	0.0473	114
Singapore	-0.0300	0.0253	61
Korea	-0.0350	0.0539	130
Indonesia	-0.0250	0.0560	135
Kenya	-0.0075	0.0867	209
India	-0.0350	0.0187	45

Table 4.10 Daily VAR Estimations

Country	90%	97.5%	99.9%
Chile	-0.0059	-0.0129	-0.0314
Peru	-0.0078	-0.0216	-0.0608
Argentina	-0.0223	-0.0493	-0.118
Mexico	-0.0175	-0.0304	-0.0795
South Africa	-0.0394	-0.0641	-0.1937
Ghana	0.0033	-0.012	-0.0607
Czech Republic	-0.0149	-0.0252	-0.0713
Singapore	-0.0158	-0.0301	-0.0863
Korea	-0.027	-0.0457	-0.1014
Indonesia	-0.0183	-0.0359	-0.1085
Kenya	-0.0068	-0.0146	-0.0404
India	-0.0178	-0.0314	-0.0929

Table 4.11 Log-Likelihood Ratio Test Result for EVT.

EVT-GPD	90%		97.5%		99.9%	
	LR_{uc}	Decision 95%CL	LR_{uc}	Decision 95%CL	LR_{uc}	Decision 95%CL
Chile	38.969	R	0.050	NR	0.873	NR
Peru	53.874	R	0.186	NR	0.074	NR
Argentina	3.978	R	0.318	NR	0.074	NR
Mexico	0.476	NR	0.001	NR	0.134	NR
South Africa	15.641	R	0.694	NR	0.134	NR
Ghana	7417.886	R	0.001	NR	0.134	NR
Czech Republic	4.051	R	0.089	NR	0.134	NR
Singapore	0.643	NR	0.009	NR	0.134	NR
Korea	4.340	R	0.028	NR	0.134	NR
Indonesia	0.685	NR	0.001	NR	0.134	NR
Kenya	0.446	NR	0.232	NR	0.074	NR
India	0.004	NR	1.239	NR	0.134	NR
Total NR	5		12		12	

Notes: *Equation:* $LR_{uc}=-2ln[(1-p)^{T-N}p^N]+2ln\{[1-(N/T)]^{T-N}(N/T)^N\}$. *P is the expected failure rate; T is the number of total observations; N is the number of exceedances;* LR_{uc} *is the Log-Likelihood Ratio of the unconditional converge.* $\boldsymbol{LR_{uc}}$ *is calculated at three quantile levels: 90%, 97.5% and 99.9%. EVT is tested as "**Reject**" (R) or "**Not Reject**" (NR) within 95% confidence interval for each of the twelve countries. If* $\boldsymbol{LR_{uc}}$ *is greater than the critical value 3.84, it is rejected; otherwise, EVT can not be rejected.*

Table 4.11 shows that EVT can not be rejected by Log-Likelihood Ratio Test in all countries at 99.9% quantile and 97.5% quantile levels. However, it is not rejected only in five countries at 90% quantile level. Therefore, we can conclude from this table that EVT is a methodology, which focuses on the tail behaviour of the return distribution.

4.4 Methodologies Comparison

When comparing the three methodologies together at the three quantile levels (90%, 97.5%, 99%), we follow the same approach that we used for each one of them to analyse the quality of the estimation: the actual-failure rate and the Log-Likelihood Ratio Test. Using the former back-testing method, we only consider those that produce an under-estimate of the risk according to the prediction of VaR. We rank the actual failure rates at three quantile levels for each country, and then choose the one that gives the closest rate to the expected failure rate. For instance, at 99.9% quantile level for Chile, the actual failure rates we get respectively from EVT, GARCH-based model and historical simulation are 0.17%, 0.29%, 0.21%. EVT is selected as the best model among the three methodologies, since it generates an actual failure rate that is closest to the expected failure rate. Following this rule, we compare the results from three models for each country at the three quantile levels. The model comparison results are shown in table 4.13. This table shows that, we can conclude that the historical simulation and GARCH-based methodology work for nine countries at 90% quantile level. They can give a more precise estimate of VaR at lower quantile level than at higher quantile level. As the quantile level increases, especially when it reaches 99.9%, VaR estimate from EVT dominates any other two methodologies; it works for ten of the twelve countries. This result indicates EVT focus on the tail behaviour of the stock return distribution and provides more information about the extreme events in emerging markets.

Table 4.12 Actual Failure Rate : Outliers / Observations (VaR forcasted using the normal probability distribution and the Conditioned Volatility estimated by the GARCH-based methodology)

Country	90.0%				97.5%				99.9%			
	EVT	GARCH	Historical	Comparison	EVT	GARCH	Historical	Comparison	EVT	GARCH	Historical	Comparison
Chile	14.02%	8.50%	8.79%	H	2.57%	2.24%	1.94%	G	0.17%	0.29%	0.21%	E*
Peru	8.79%	6.43%	5.70%	E	2.41%	2.28%	1.88%	E	0.12%	0.58%	0.16%	E*
Argentina	11.24%	8.13%	9.26%	H	2.32%	3.07%	2.72%	E	0.08%	0.71%	0.21%	E
Mexico	9.58%	7.42%	8.27%	E	2.49%	2.99%	2.15%	E	0.12%	0.54%	0.11%	H*
South Africa	7.67%	6.51%	9.58%	H	2.24%	2.45%	2.20%	G	0.12%	0.50%	0.16%	E*
Ghana	84.70%	2.65%	7.95%	H	2.49%	1.53%	2.41%	E	0.12%	0.87%	0.37%	E*
Czech Republic	8.79%	7.42%	9.31%	H	2.41%	2.65%	2.25%	E	0.12%	0.50%	0.21%	E*
Singapore	10.49%	8.05%	7.22%	G	2.53%	2.41%	1.57%	G	0.12%	0.37%	0.21%	E*
Korea	8.75%	9.79%	7.64%	G	2.45%	2.78%	1.94%	E	0.12%	0.33%	0.11%	H*
Indonesia	9.50%	7.55%	8.16%	E	2.49%	2.61%	1.73%	E	0.12%	0.71%	0.21%	E*
Kenya	10.41%	6.84%	9.73%	H	2.65%	2.82%	2.67%	E*	0.08%	0.75%	0.26%	E
India	10.04%	7.22%	9.16%	H	2.86%	2.32%	2.36%	H	0.12%	0.33%	0.31%	E*

(*) : The model underestimates the VaR at the corresponding Confidence Level, but it is the best approach among the three.

When using the Log-Likelihood Ratio Test back-testing method, at the 90% quantile level, EVT is not rejected in five countries, number which is less than the historical simulation and GARCH-based methodologies. EVT estimates VaR well for all twelve countries at 97.5% and 99.9% quantile levels, which is much better than the other two methodologies. From Table 4.13, the historical simulation methodology is not rejected in ten and eleven countries at 90% and 97.5% quantile level, respectively. GARCH-based methodology works in one country at 90% quantile level, and in ten countries at 97.5% quantile level.

Table 4.13 Log-Likelihood Ratio Test Result for comparison of three methodologies.

Confidence Level	90%			97.5%			99.9%		
Methodology	HS	GARCH	EVT	HS	GARCH	EVT	HS	GARCH	EVT
Chile	NR	R	R	NR	NR	NR	NR	R	NR
Peru	NR	R	R	NR	NR	NR	R	R	NR
Argentina	NR	R	R	NR	NR	NR	NR	R	NR
Mexico	NR	R	NR	NR	NR	NR	NR	R	NR
South Africa	NR	R	R	NR	NR	NR	NR	R	NR
Ghana	NR	R	R	NR	R	NR	R	R	NR
Czech Republic	NR	R	R	NR	NR	NR	NR	R	NR
Singapore	R	R	NR	R	NR	NR	NR	R	NR
Korea	R	NR	R	NR	NR	NR	NR	R	NR
Indonesia	NR	R	NR	NR	NR	NR	NR	R	NR
Kenya	NR	R	NR	NR	NR	NR	NR	R	NR
India	NR	R	NR	NR	NR	NR	R	R	NR
Total NR	**10**	**1**	**5**	**11**	**11**	**12**	**9**	**0**	**12**

Notes: $LR_{uc}=-2ln[(1-p)^{T-N}p^{N}]+2ln\{[1-(N/T)]^{T-N}(N/T)^{N}\}$. *P is the expected failure rate; T is the number of total observations; N is the number of exceedances. The methodologies were tested based on the Log-Likelihood Ratio of the unconditional converge (LR_{uc}) for each Confidence Level. The Methodology is either "**Rejected**" (**R**) or "**Not Rejected**" (**NR**) within 95% confidence interval for each of the twelve countries. "**Total NR**" indicates the total number of countries in which the respective Methodology was not rejected.*

5 CONCLUSIONS AND RECOMMENDATIONS

In this study, we investigate the relative performance of three methodologies of estimating VaR by using the daily stock market returns in twelve emerging markets. The empirical results show that the historical simulation and GARCH-based VaR methodologies work well in estimating VaR at 90% quantile level. However, as the quantile level increases, especially when the quantile level is 99.9%, the methodology comparison of two back-testing methods indicate that EVT dominates the other two in terms of VaR estimation. On the other hand, EVT, which captures more features of the return distribution, predicts a more precise VaR for more countries at 99.9% quantile level than at 90% quantile level. We conclude that EVT provides more information about the extreme events than other traditional methodologies.

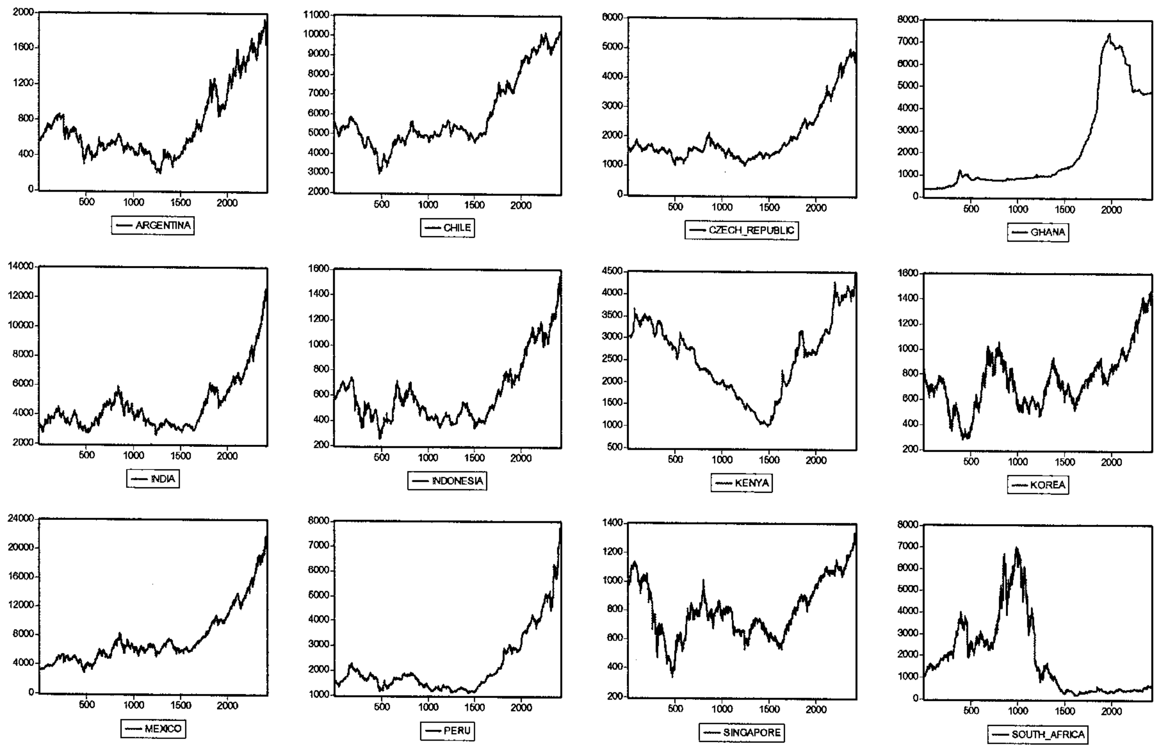
However, since the emerging market economies are different from developed countries in its economic dynamic structure, these economies are more sensitive to regime switches on short periods of time, especially when a financial crisis occurs, according to this study. This fact implies that any modelling exercise in emerging markets should take into account the changes of the environment.

Another thing that we should pay attention to is that we have a trade-off between variance and bias when choosing the threshold value. The higher the threshold value, the smaller the remaining number of exceedances to estimate VaR. On the other hand, the result of estimating VaR may be biased if more exceedances are left to estimate VaR. This also means that more data points around the mean of the distribution are chosen. We use a combination of three methods: Mean Excess Function, Hill-plot, and QQ-plot to estimate the threshold value. More attention should be put on the methodologies, and the choice of the threshold when estimating an optimal VaR. This is a topic that demands more investigation and study.

APPENDICES

Appendix A

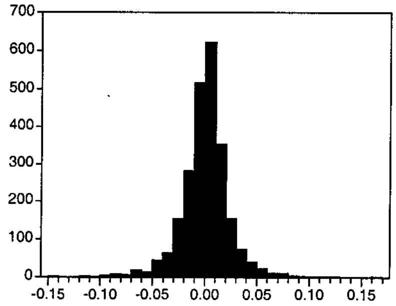
Graph 1 Stock Market Index of each country



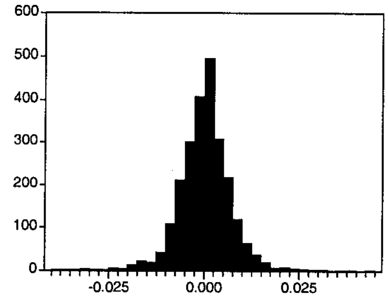
Notes: Data source collected goes from October 15, 1996 to May 18, 2006 (2412 observations)

Appendix B

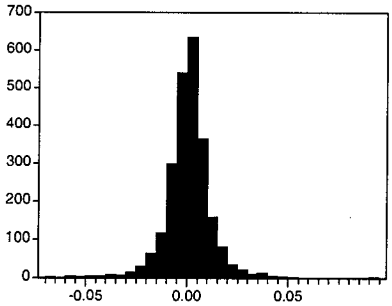
Graph 2 Histograms and statistics of Daily Index Returns



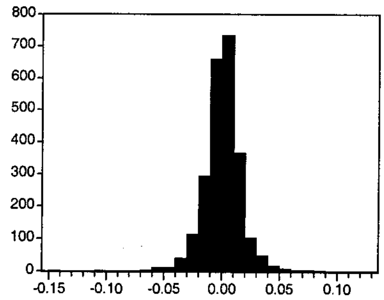
Series: R_ARGENTINA	
Sample 2 2412	
Observations 2411	
Mean	0.000432
Median	0.000123
Maximum	0.161165
Minimum	-0.147649
Std. Dev.	0.023064
Skewness	-0.130740
Kurtosis	8.384668
Jarque-Bera	2919.623
Probability	0.000000



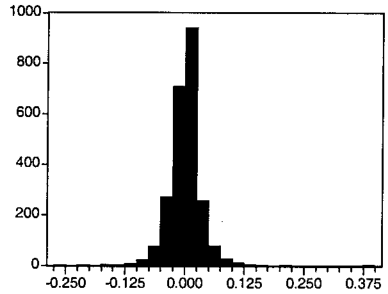
Series: R_CHILE	
Sample 2 2412	
Observations 2411	
Mean	0.000250
Median	0.000000
Maximum	0.044666
Minimum	-0.037735
Std. Dev.	0.006632
Skewness	0.063879
Kurtosis	7.080461
Jarque-Bera	1674.287
Probability	0.000000



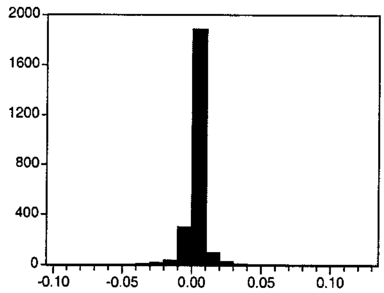
Series: R_PERU	
Sample 2 2412	
Observations 2411	
Mean	0.000643
Median	0.000270
Maximum	0.094920
Minimum	-0.068835
Std. Dev.	0.010914
Skewness	-0.133270
Kurtosis	10.02078
Jarque-Bera	4958.858
Probability	0.000000



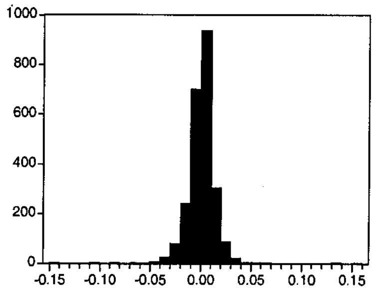
Series: R_MEXICO	
Sample 2 2412	
Observations 2411	
Mean	0.000753
Median	0.000937
Maximum	0.121536
Minimum	-0.143139
Std. Dev.	0.015900
Skewness	-0.062245
Kurtosis	10.54249
Jarque-Bera	5716.540
Probability	0.000000



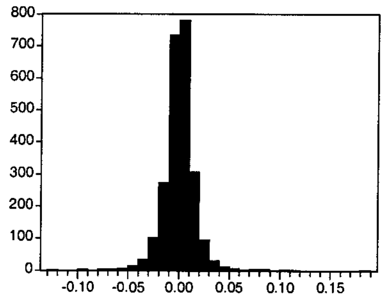
Series: R_SOUTH_AFRICA	
Sample 2 2412	
Observations 2411	
Mean	-0.000289
Median	0.000000
Maximum	0.389634
Minimum	-0.267748
Std. Dev.	0.033252
Skewness	0.382395
Kurtosis	16.25106
Jarque-Bera	17698.30
Probability	0.000000



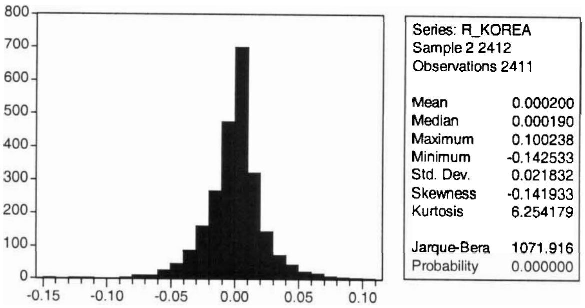
Series: R_GHANA	
Sample 2 2412	
Observations 2411	
Mean	0.001053
Median	0.000000
Maximum	0.121854
Minimum	-0.092266
Std. Dev.	0.008723
Skewness	2.726817
Kurtosis	60.21700
Jarque-Bera	331866.9
Probability	0.000000

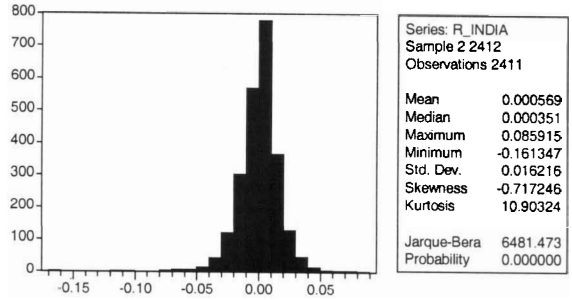


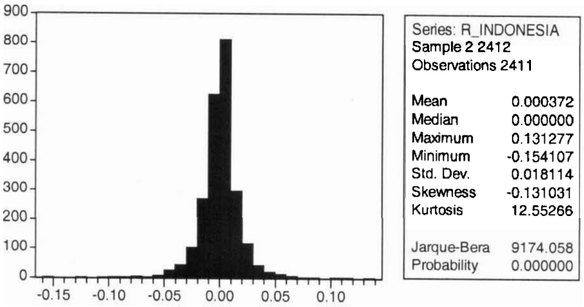
Series: R_CZECH_REPUBLIC	
Sample 2 2412	
Observations 2411	
Mean	0.000424
Median	0.000000
Maximum	0.150788
Minimum	-0.148764
Std. Dev.	0.013153
Skewness	0.053539
Kurtosis	24.43718
Jarque-Bera	46167.04
Probability	0.000000

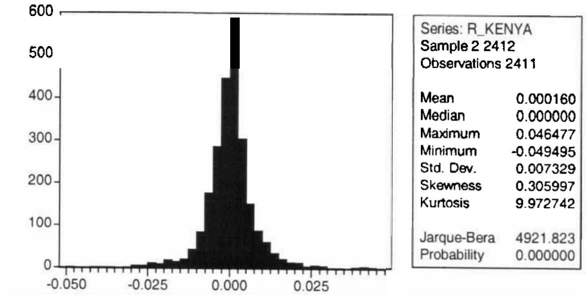


Series: R_SINGAPORE	
Sample 2 2412	
Observations 2411	
Mean	0.000106
Median	0.000000
Maximum	0.184280
Minimum	-0.129821
Std. Dev.	0.015696
Skewness	0.589650
Kurtosis	18.34601
Jarque-Bera	23797.65
Probability	0.000000



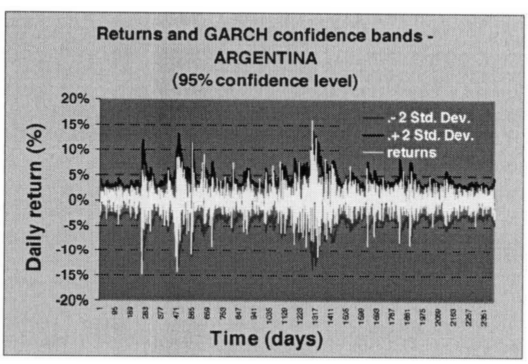


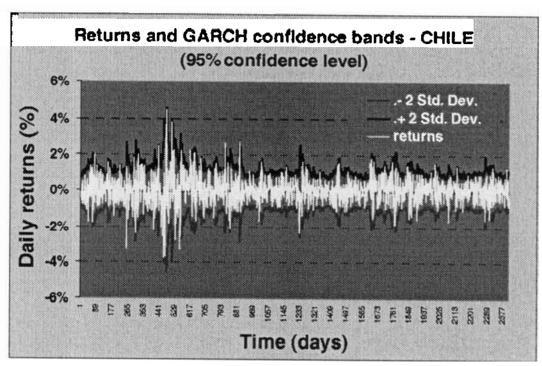


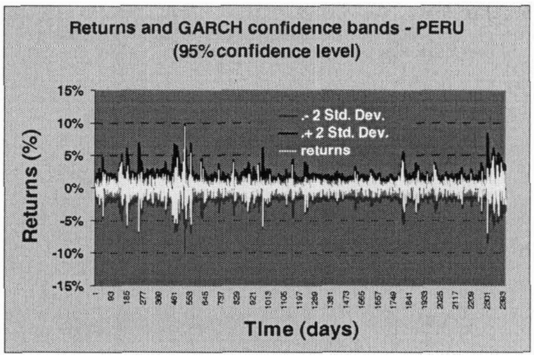


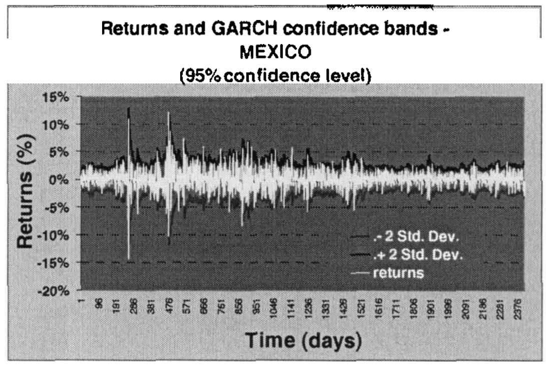
Appendix C

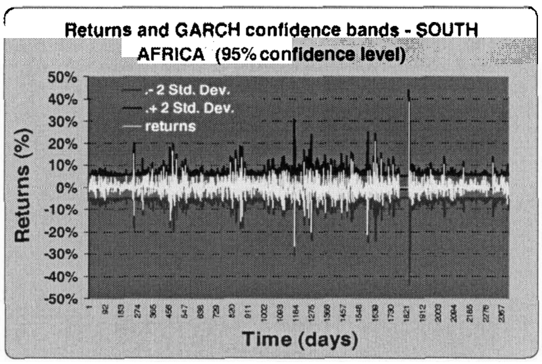
Graph 3 ARCH-based VaR estimate Confidence Bands

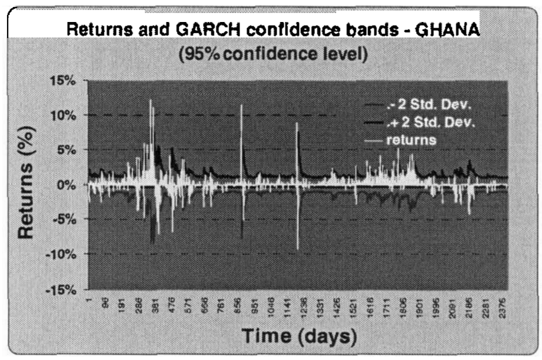


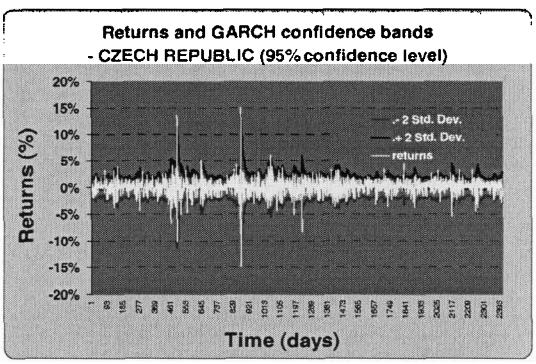


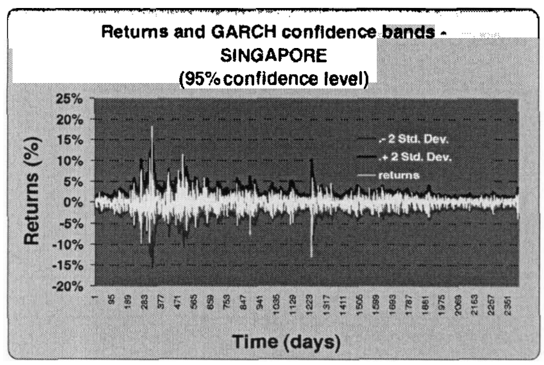

Returns and GARCH confidence bands - PERU
(95% confidence level)
15%
10%
5%
0%
-5%
-10%
-15%
Returns (%)
.- 2 Std. Dev.
.+ 2 Std. Dev.
returns
Time (days)

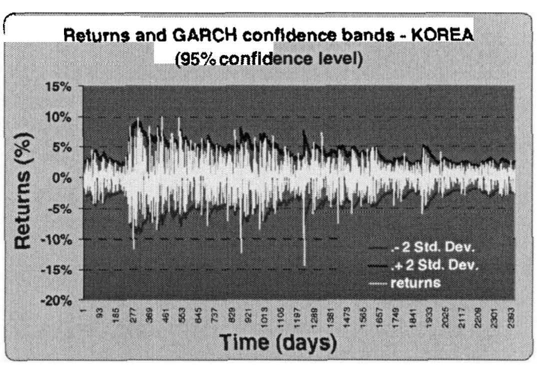

Returns and GARCH confidence bands - MEXICO
(95% confidence level)
15%
10%
5%
0%
-5%
-10%
-15%
-20%
Returns (%)
.- 2 Std. Dev.
.+ 2 Std. Dev.
returns
Time (days)

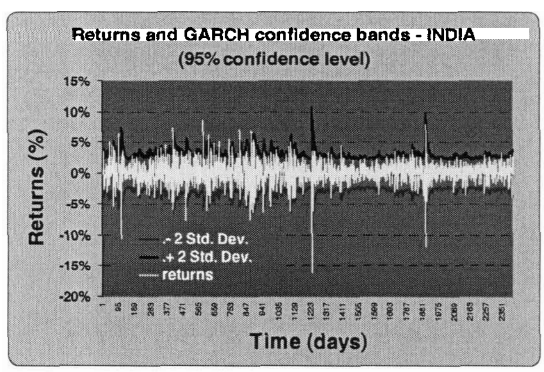

Returns and GARCH confidence bands - SOUTH AFRICA (95% confidence level)
50%
40%
30%
20%
10%
0%
-10%
-20%
-30%
-40%
-50%
Returns (%)
.- 2 Std. Dev.
.+ 2 Std. Dev.
returns
Time (days)

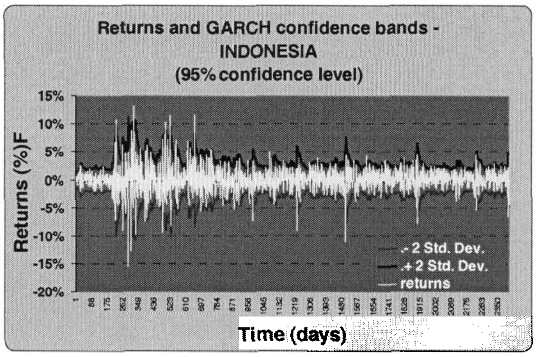

Returns and GARCH confidence bands - GHANA
(95% confidence level)
15%
10%
5%
0%
-5%
-10%
-15%
Returns (%)
.- 2 Std. Dev.
.+ 2 Std. Dev.
returns
Time (days)

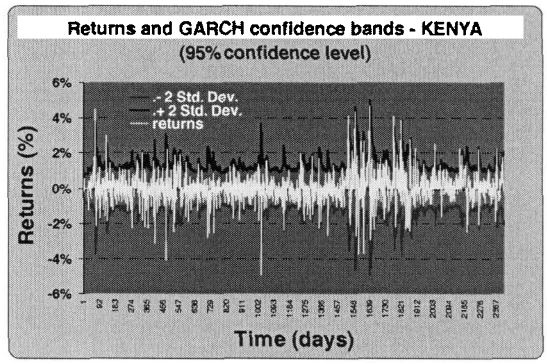

Returns and GARCH confidence bands - CZECH REPUBLIC (95% confidence level)
20%
15%
10%
5%
0%
-5%
-10%
-15%
-20%
Returns (%)
.- 2 Std. Dev.
.+ 2 Std. Dev.
returns
Time (days)


Returns and GARCH confidence bands - SINGAPORE
(95% confidence level)
25%
20%
15%
10%
5%
0%
-5%
-10%
-15%
-20%
Returns (%)
.- 2 Std. Dev.
.+ 2 Std. Dev.
returns
Time (days)


Returns and GARCH confidence bands - KOREA
(95% confidence level)
15%
10%
5%
0%
-5%
-10%
-15%
-20%
Returns (%)
.- 2 Std. Dev.
.+ 2 Std. Dev.
returns
Time (days)


Returns and GARCH confidence bands - INDIA
(95% confidence level)
15%
10%
5%
0%
-5%
-10%
-15%
-20%
Returns (%)
.- 2 Std. Dev.
.+ 2 Std. Dev.
returns
Time (days)





Notes: *In the graphs above, GARCH forecast of volatility indicates that volatility decreases progressively over time, not in an abrupt fashion..*

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